Development of MHD Codes and Applications

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Topics of my works with TWJ over the last 25 years

- Acoustic instability (the first paper with TWJ)
- Development of TVD MHD code
- MHD cosmic-ray shocks
- MHD KH instability
- MHD (also HD) cloud propagation and collision
- MHD stellar and AGN jets
- Parker instability
- MHD turbulence (on-going)
- Shocks in clusters and the large-scale structure
- Cosmic ray acceleration at cosmological shocks
- Modeling of radio relics (on-going)
- ...

Development of TVD MHD code:
4 papers

NUMERICAL MAGNETOHYDRODYNAMICS IN ASTROPHYSICS: ALGORITHM AND TESTS FOR ONE-DIMENSIONAL FLOW

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ABSTRACT

We describe a numerical code to solve the equations for ideal magnetohydrodynamics (MHD). It is based on an explicit finite difference scheme on an Eulerian grid, called the total variation diminishing (TVD) scheme, which is a second-order–accurate extension of the Roe-type upwind scheme. We also describe a non-linear Riemann solver for ideal MHD, which includes rarefactions as well as shocks. The numerical code and the Riemann solver have been used to test each other.

the first code for astrophysical applications,
based on an upwind scheme

A DIVERGENCE-FREE UPWIND CODE FOR MULTIDIMENSIONAL MAGNETOHYDRODYNAMIC FLOWS

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$$\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial y}(B_x v_y - B_y v_x) = 0 , \qquad (1)$$

and

$$\frac{\partial B_y}{\partial t} + \frac{\partial}{\partial x}(B_y v_x - B_x v_y) = 0 . \qquad (2)$$

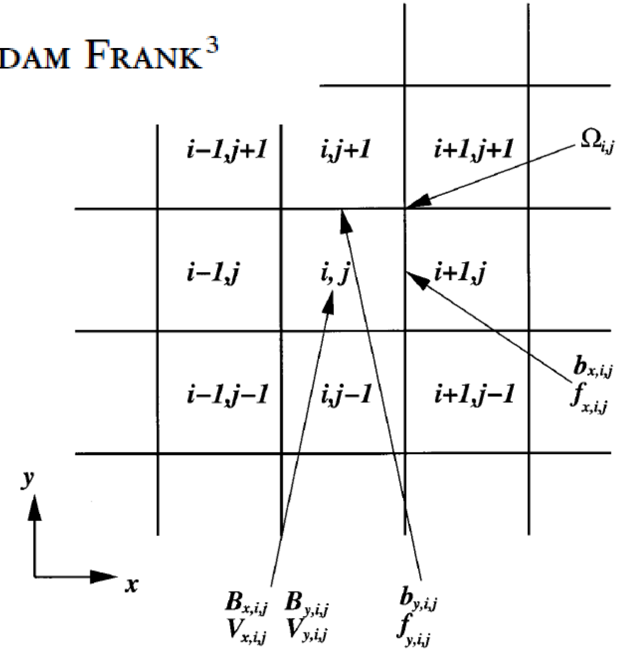
$$B_{x,i,j} = \tfrac{1}{2}(b_{x,i,j} + b_{x,i-1,j}) \qquad (3)$$

and

$$B_{y,i,j} = \tfrac{1}{2}(b_{y,i,j} + b_{y,i,j-1}) . \qquad (4)$$

$$\bar{f}_{x,i,j} = \frac{1}{2}(B^n_{y,i,j} v^n_{x,i,j} + B^n_{y,i+1,j} v^n_{x,i+1,j}) - \frac{\Delta x}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i+1/2,j} R^n_{k,i+1/2,j}(5) , \qquad (6)$$

$$\bar{f}_{y,i,j} = \frac{1}{2}(B^n_{x,i,j} v^n_{y,i,j} + B^n_{x,i,j+1} v^n_{y,i,j+1}) - \frac{\Delta y}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i,j+1/2,j} R^n_{k,i,j+1/2}(5) . \qquad (7)$$



$$\bar{\Omega}_{i,j} = \tfrac{1}{2}(\bar{f}_{y,i+1,j} + \bar{f}_{y,i,j}) - \tfrac{1}{2}(\bar{f}_{x,i,j+1} + \bar{f}_{x,i,j}) . \qquad (15)$$

$$b^{n+1}_{x,i,j} = b^n_{x,i,j} - \frac{\Delta t^n}{\Delta y}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i,j-1}) \qquad (16)$$

and

$$b^{n+1}_{y,i,j} = b^n_{y,i,j} + \frac{\Delta t^n}{\Delta x}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i-1,j}) . \qquad (17)$$

$$\oint_S \boldsymbol{b}^{n+1} \cdot d\boldsymbol{S} = (b^{n+1}_{x,i,j} - b^{n+1}_{x,i-1,j})\Delta y + (b^{n+1}_{y,i,j} - b^{n+1}_{y,i,j-1}) \Delta x = 0 , \qquad (18)$$

the first attempt to implement a flux CT scheme for to $\vec{\nabla} \cdot \vec{B} = 0$ an MHD code based on a upwind scheme

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THE MAGNETOHYDRODYNAMIC KELVIN-HELMHOLTZ INSTABILITY: A THREE-DIMENSIONAL STUDY OF NONLINEAR EVOLUTION

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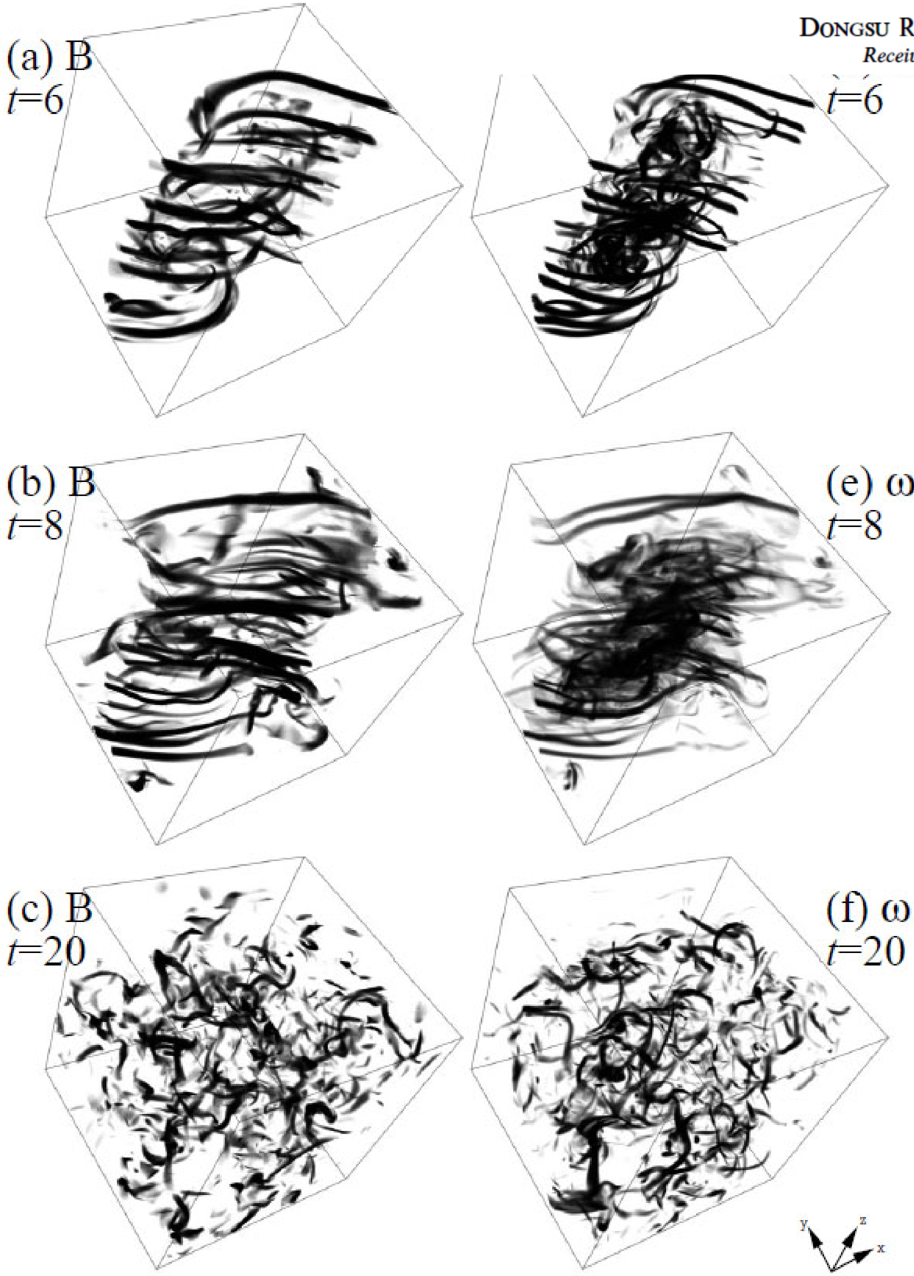

magnetic field strength and vorticity in 3D MHD KH instability

THREE-DIMENSIONAL MAGNETOHYDRODYNAMIC NUMERICAL SIMULATIONS OF CLOUD-WIND INTERACTIONS

G. Gregori,[1,2] Francesco Miniati,[2] Dongsu Ryu,[3] and T. W. Jones[2]

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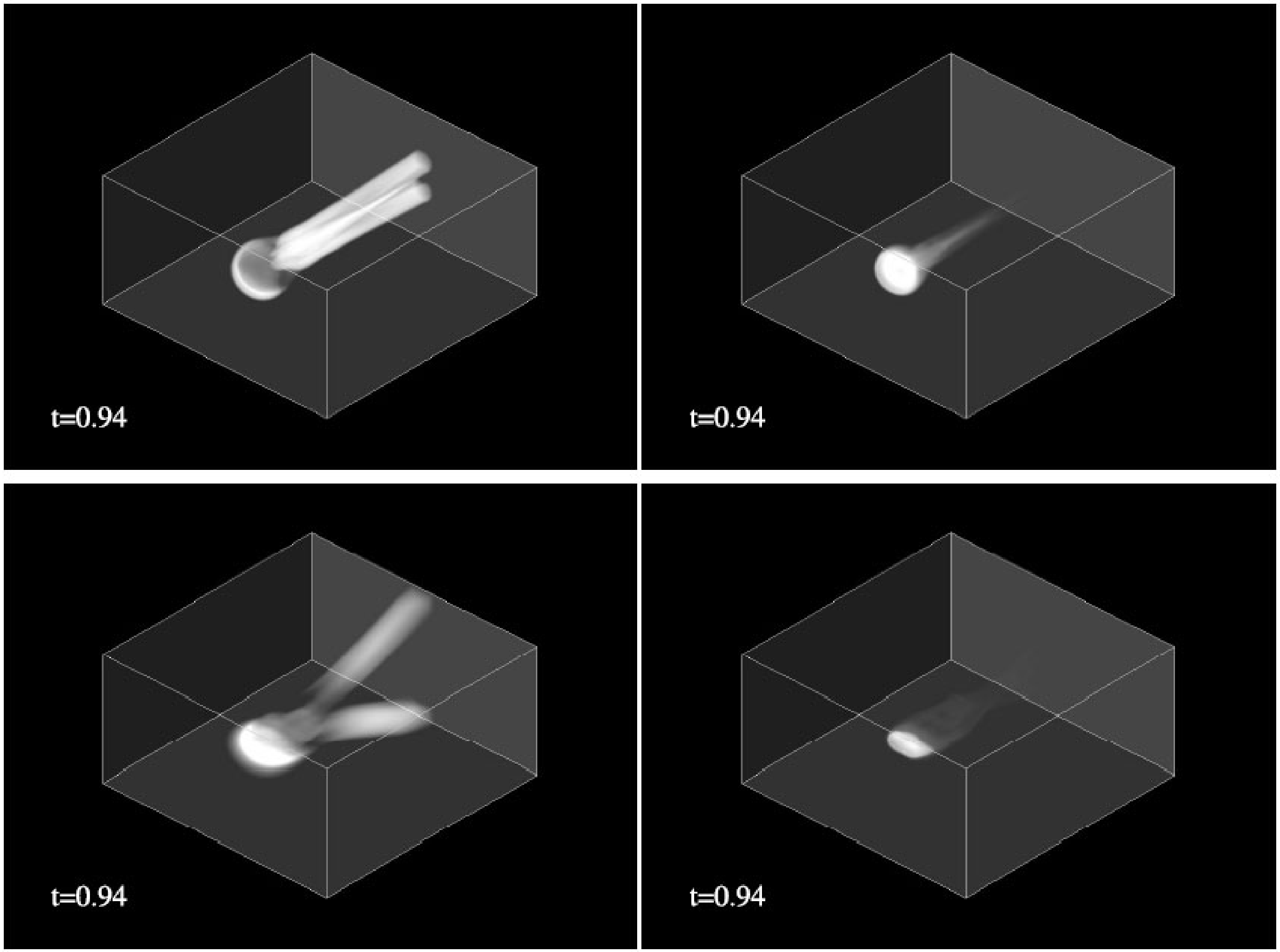

Fig. 9.—*Left column*: volume rendering of the magnetic pressure (log scale) for the $\beta = 100$ simulation (*top*) and $\beta = 4$ (*bottom*). *Right column*: volume rendering of the cloud density (log scale) for the $\beta = 100$ simulation (*top*) and $\beta = 4$ (*bottom*). Time is expressed in units of τ_{cr}.

SYNTHETIC OBSERVATIONS OF SIMULATED RADIO GALAXIES. I. RADIO AND X-RAY ANALYSIS

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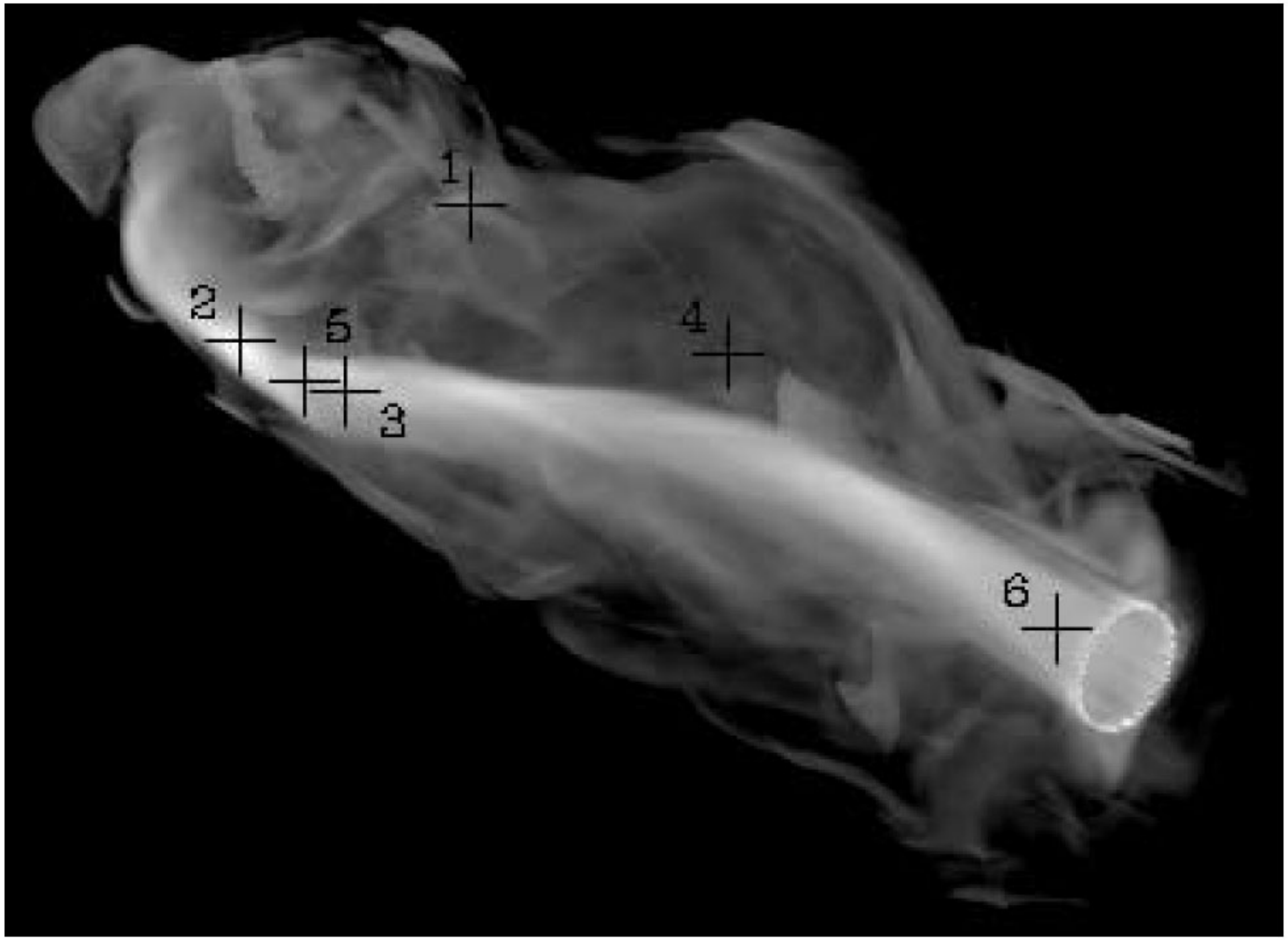

Fig. 7.—Gray-scale image of the control model 8 GHz synchrotron surface brightness.

THREE-DIMENSIONAL EVOLUTION OF THE PARKER INSTABILITY UNDER A UNIFORM GRAVITY

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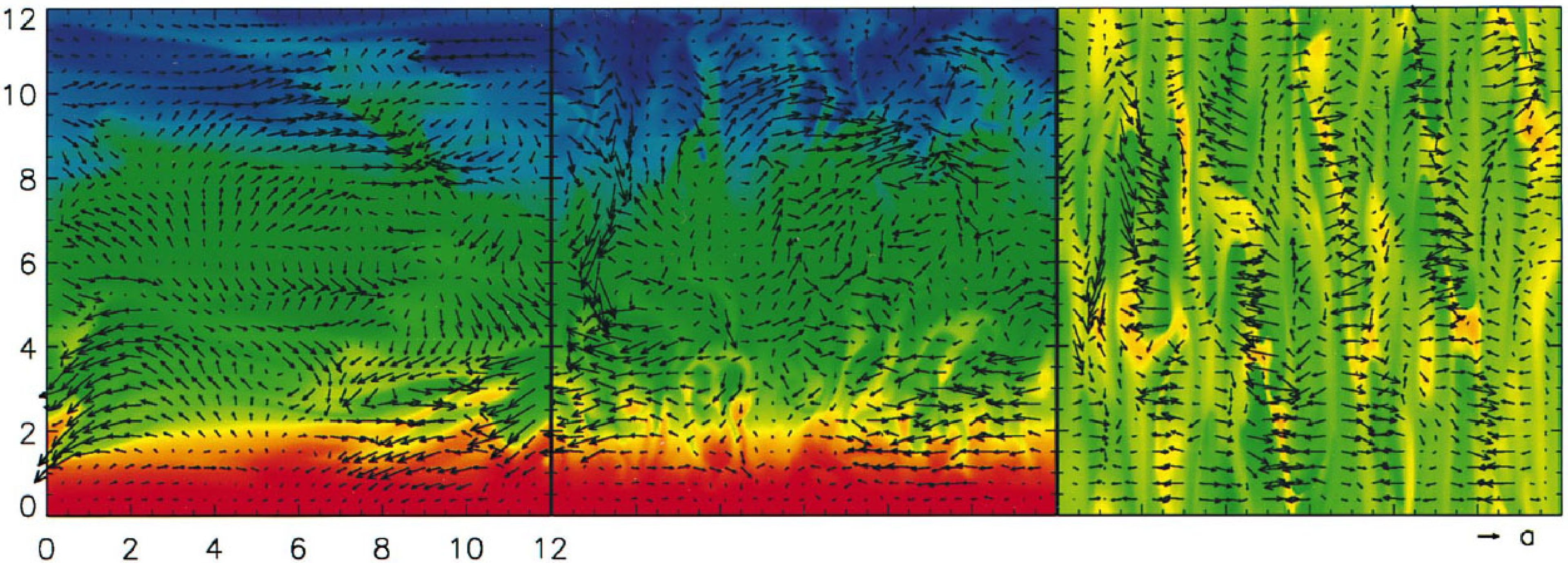


Fig. 3.—Images of density structure and velocity vectors on the three planes, $x = 6$ (*left*), $y = 6$ (*center*), and $z = 3$ (*right*) at $t = 40$ from the high-resolution simulation with 256^3 cells. The three planes are same as the sliced planes in Fig. 2. Colors are mapped from red to blue as density decreases. The unit of the velocity vectors is shown at the lower right corner of the image.

Vorticity, Shocks and Magnetic Fields in Subsonic, ICM-like Turbulence

David H. Porter[1], T. W. Jones[1,2], and Dongsu Ryu[3]

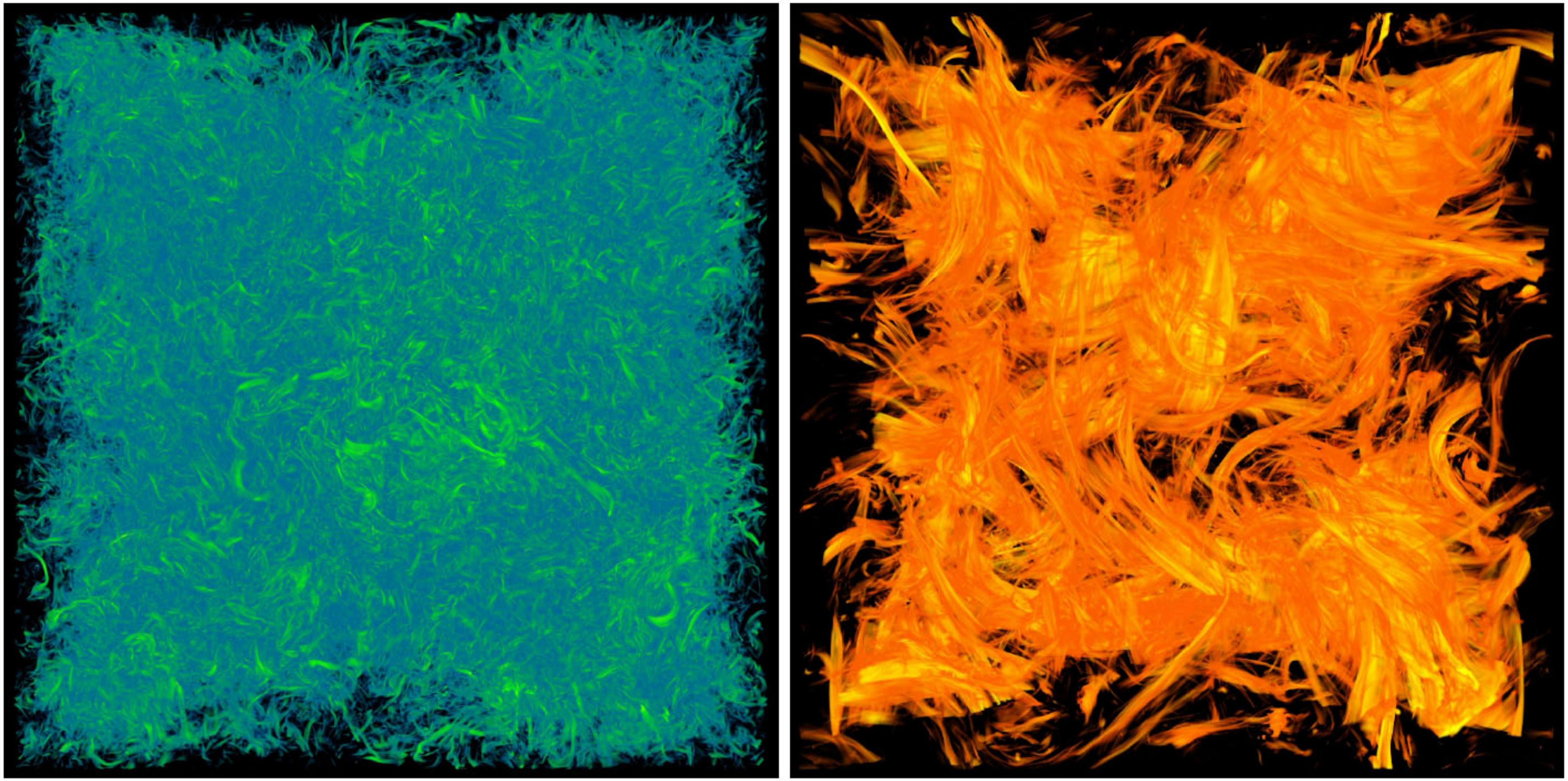


Fig. 4.— Magnetic energy density, E_B, volume renderings in the SK1 simulation at $t = 20$ (Left) and $t = 130$ (Right). "Cool" is weak; "hot" is strong. Opacities are chosen to isolate stronger fields.

Development of a New MHD Code based on WENO (Jang & Ryu)

$$\vec{q}_i^{\,n+1} = \vec{q}_i^{\,n} - \frac{\Delta t}{\Delta x}(\vec{F}^n_{i+\frac{1}{2}} - \vec{F}^n_{i-\frac{1}{2}})$$

$m = 1, 2, 3, 4, 5, 6, 7$ for fast-, Alfven-, slow-, entropy, slow+, Alfven+, fast +

1. $\alpha(m) = \max(|a_i(m)|, |a_{i+1}(m)|)$
2. $F_k^s(m) = \vec{L}_{i+\frac{1}{2}}(m) \cdot \vec{F}_k, \quad q_k^s(m) = \vec{L}_{i+\frac{1}{2}}(m) \cdot \vec{q}_k$
3. $\Delta\vec{F}_{k+\frac{1}{2}} = \vec{F}_{k+1} - \vec{F}_k, \quad \Delta\vec{q}_{k+\frac{1}{2}} = \vec{q}_{k+1} - q_k$
4. $\Delta F^{s\pm}_{k+\frac{1}{2}}(m) = \frac{1}{2}[\Delta F^s_{k+\frac{1}{2}}(m) \pm \alpha(m)\Delta q^s_{k+\frac{1}{2}}(m)]$
5. $F_i^{s\pm}(m) = \frac{1}{2}[F_i^s(m) \pm \alpha(m) q_i^s(m)]$

WENO5 using 5 stensils : $k = i-2, i-1, i, i+1, i+2$

for $\vec{F}_{i+\frac{1}{2}}$, using 6 cells : $i-2, i-1,\ i,\ i+1, i+2, i+3$, for $\vec{F}_{i-\frac{1}{2}}$: $i-3, i-2, i-1,\ i,\ i+1, i+2$

so, for $\vec{Q}_i$, using 7 cells : $i-3, i-2, i-1,\ i,\ i+1, i+2, i+3$

6. $\vec{F}^s_{i+\frac{1}{2}} = \frac{1}{12}[-\vec{F}^s_{i-1} + 7\vec{F}^s_i + 7\vec{F}^s_{i+1} - \vec{F}^s_{i+2}] - \varphi_N(\Delta\vec{F}^{s+}_{i-\frac{3}{2}}, \Delta\vec{F}^{s+}_{i-\frac{1}{2}}, \Delta\vec{F}^{s+}_{i+\frac{1}{2}}, \Delta\vec{F}^{s+}_{i+\frac{3}{2}})$
$+ \varphi_N(\Delta\vec{F}^{s-}_{i+\frac{5}{2}}, \Delta\vec{F}^{s-}_{i+\frac{3}{2}}, \Delta\vec{F}^{s-}_{i+\frac{1}{2}}, \Delta\vec{F}^{s-}_{i-\frac{1}{2}})$

$$\varphi_N(a,b,c,d) = \frac{1}{3}\omega_0(a-2b+c) + \frac{1}{6}(\omega_2 - \frac{1}{2})(b-2c-d)$$

$$\omega_0 = \frac{\alpha_0}{\alpha_0+\alpha_1+\alpha_2}, \quad \omega_2 = \frac{\alpha_2}{\alpha_0+\alpha_1+\alpha_2},$$

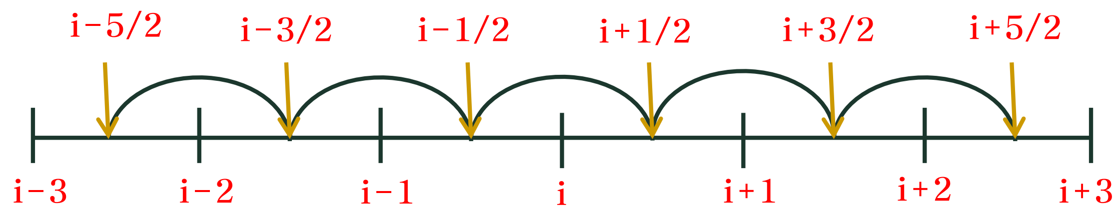
$$\alpha_0 = \frac{1}{\epsilon + IS_0}, \quad \alpha_1 = \frac{6}{\epsilon + IS_1}, \quad \alpha_2 = \frac{3}{\epsilon + IS_2}$$

$$IS_0 = 13(a+b)^2 + 3(a-3b)^2$$

$$IS_1 = 13(b-c)^2 + 3(b+c)^2$$

$$IS_2 = 13(c-d)^2 + 3(3c-d)^2$$

7. $F_{i+\frac{1}{2}}(m) = \vec{F}^s_{i+\frac{1}{2}} \cdot \vec{R}_{i+\frac{1}{2}}(m)$
8. $\vec{Q}_i = -(\vec{F}_{i+\frac{1}{2}} - \vec{F}_{i-\frac{1}{2}})$



5th-order (for space) WENO:
WENO-JW, WENO-M, WENO-Z

4th-order (for time) Runge-Kutta

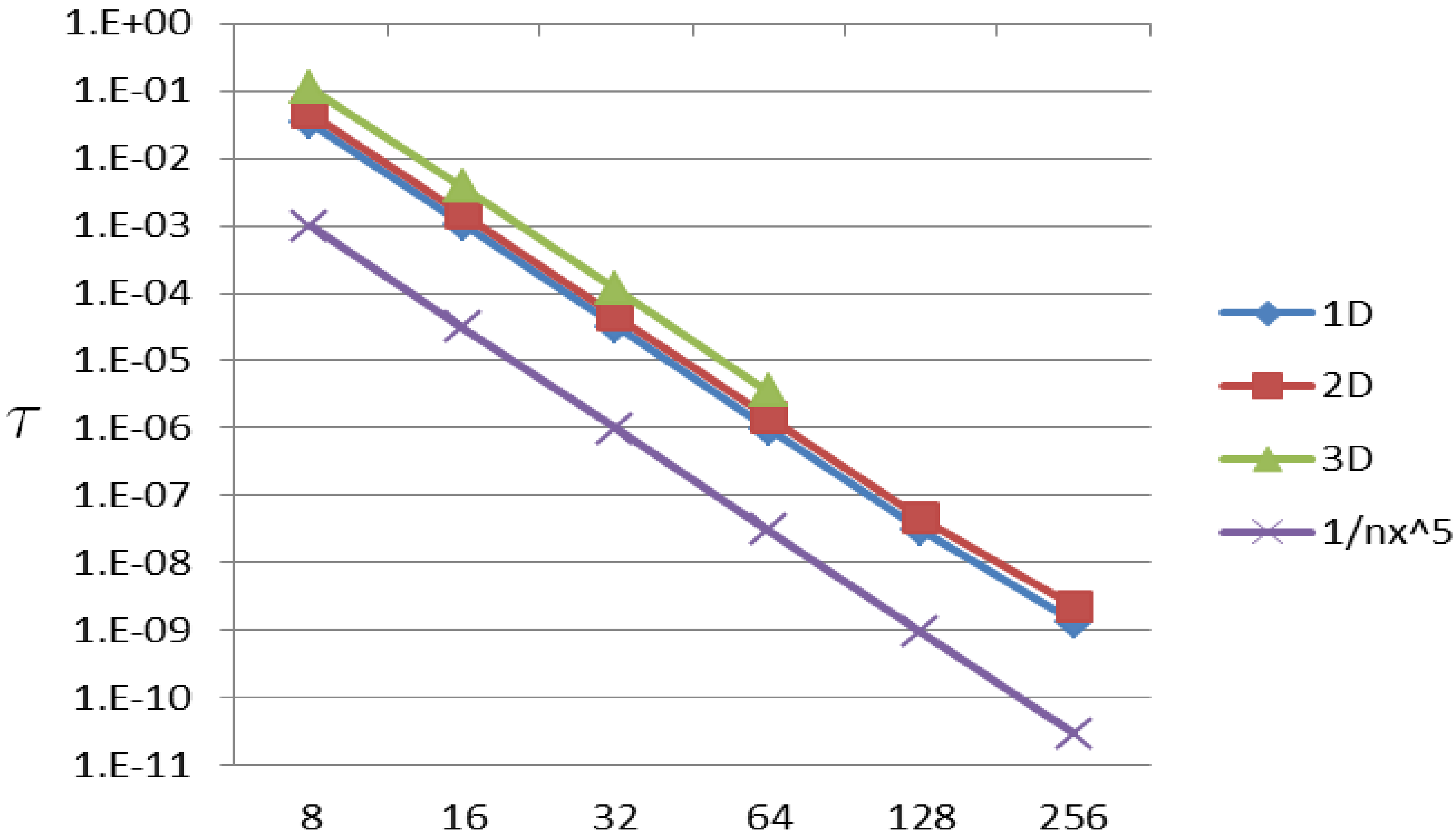
1. $\vec{q}_0 = \vec{q}_n$
2. $\vec{q}_1 = \vec{q}_0 + \frac{1}{2}\frac{\Delta t}{\Delta x}\vec{Q}_0$
3. $\vec{q}_2 = \vec{q}_0 + \frac{1}{2}\frac{\Delta t}{\Delta x}\vec{Q}_1$
4. $\vec{q}_3 = \vec{q}_0 + \frac{\Delta t}{\Delta x}\vec{Q}_2$
5. $\vec{q}_4 = \vec{q}_0 + \frac{1}{6}\frac{\Delta t}{\Delta x}(\vec{Q}_0 + 2\vec{Q}_1 + 2\vec{Q}_2 + \vec{Q}_3)$
6. $\vec{q}_{n+1} = \vec{q}_4$

a high-order implementation of the flux CT scheme for $\vec{\nabla} \cdot \vec{B} = 0$

Alfven wave decay test

decay of Alfven wave propagating diagonally in the computational domain

$$\tau = -\frac{1}{t} ln\left(\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)} \right)$$

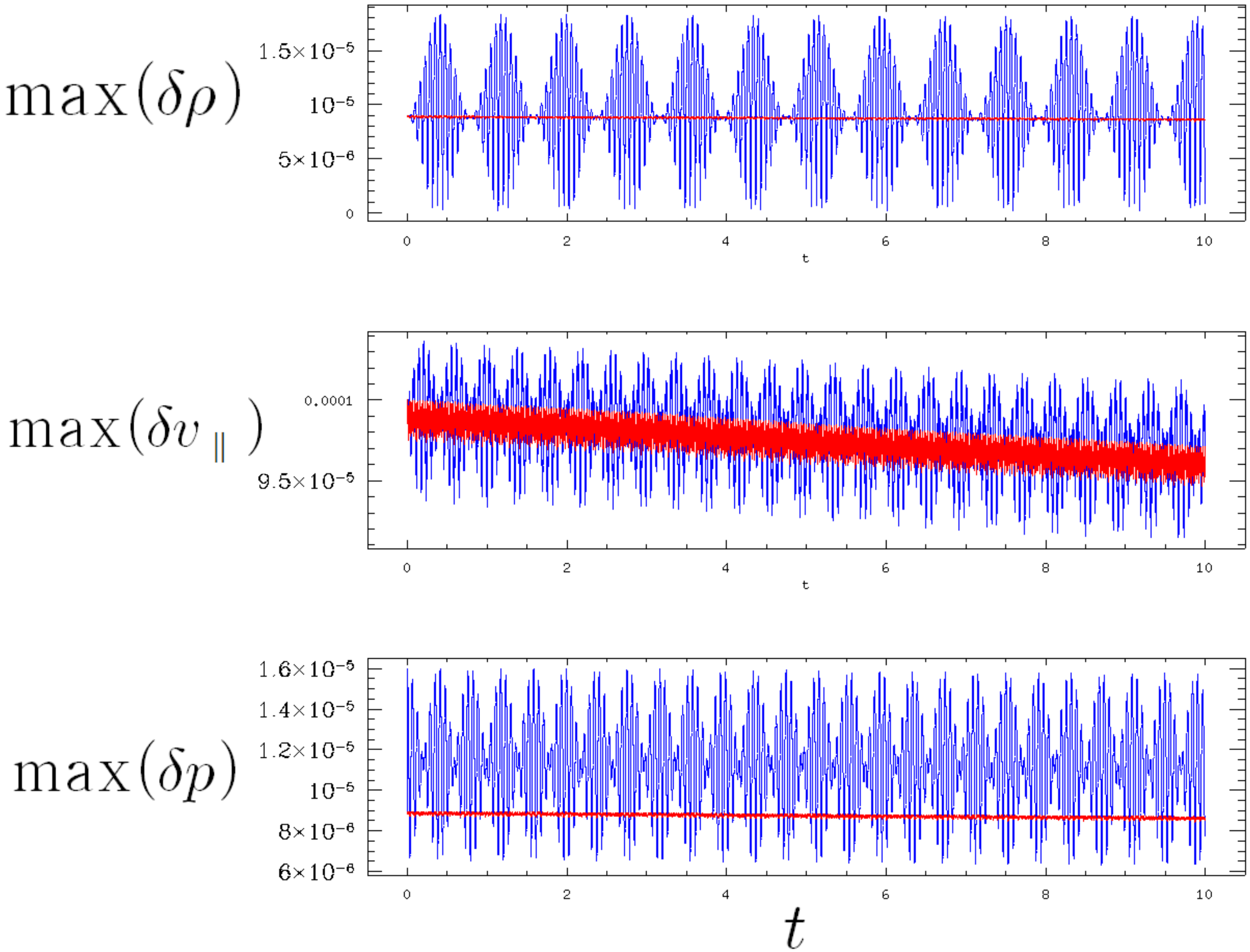


5th-order achieved !

no of zones along axes

2D fast wave decay test with β ~ 0.01

no of zones: 32^2



test with a second order implementation of flux CT

test with a fourth order implementation of flux CT

Thank you !