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Plasma Turbulence and Particle Acceleration

Peter H. Yoon
University of Maryland &
Kyung Hee University, Korea

- *Particle acceleration by turbulence is easy to describe.*

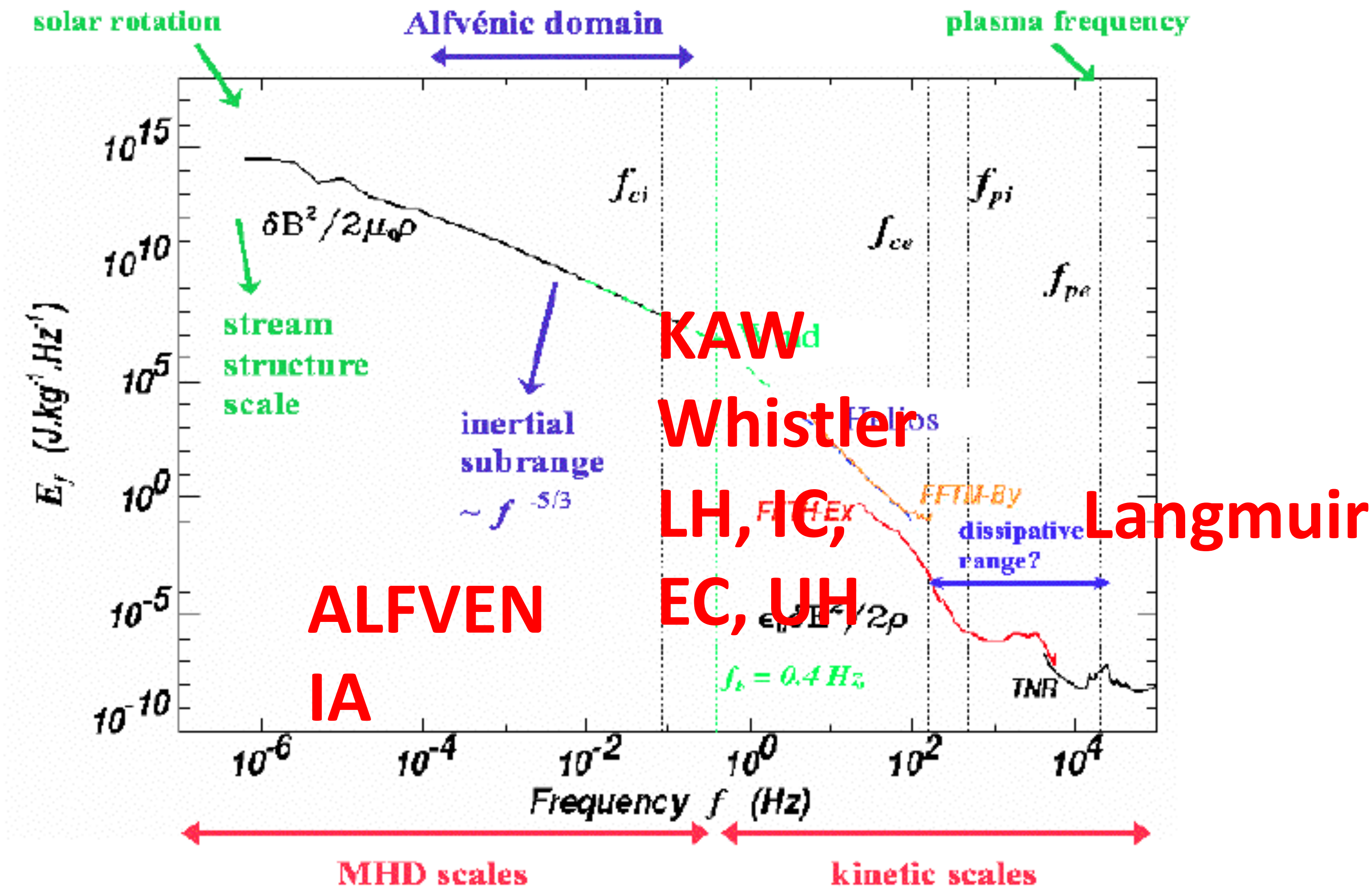
$$\frac{\partial f}{\partial t} = \frac{ie^2}{m_e^2}\frac{\partial}{\partial v_{\parallel}}\int dk \frac{|\delta E_k^2|}{\omega - kv_{\parallel}}\frac{\partial f}{\partial v_{\parallel}}$$

- *Describing the turbulence itself is hard.*

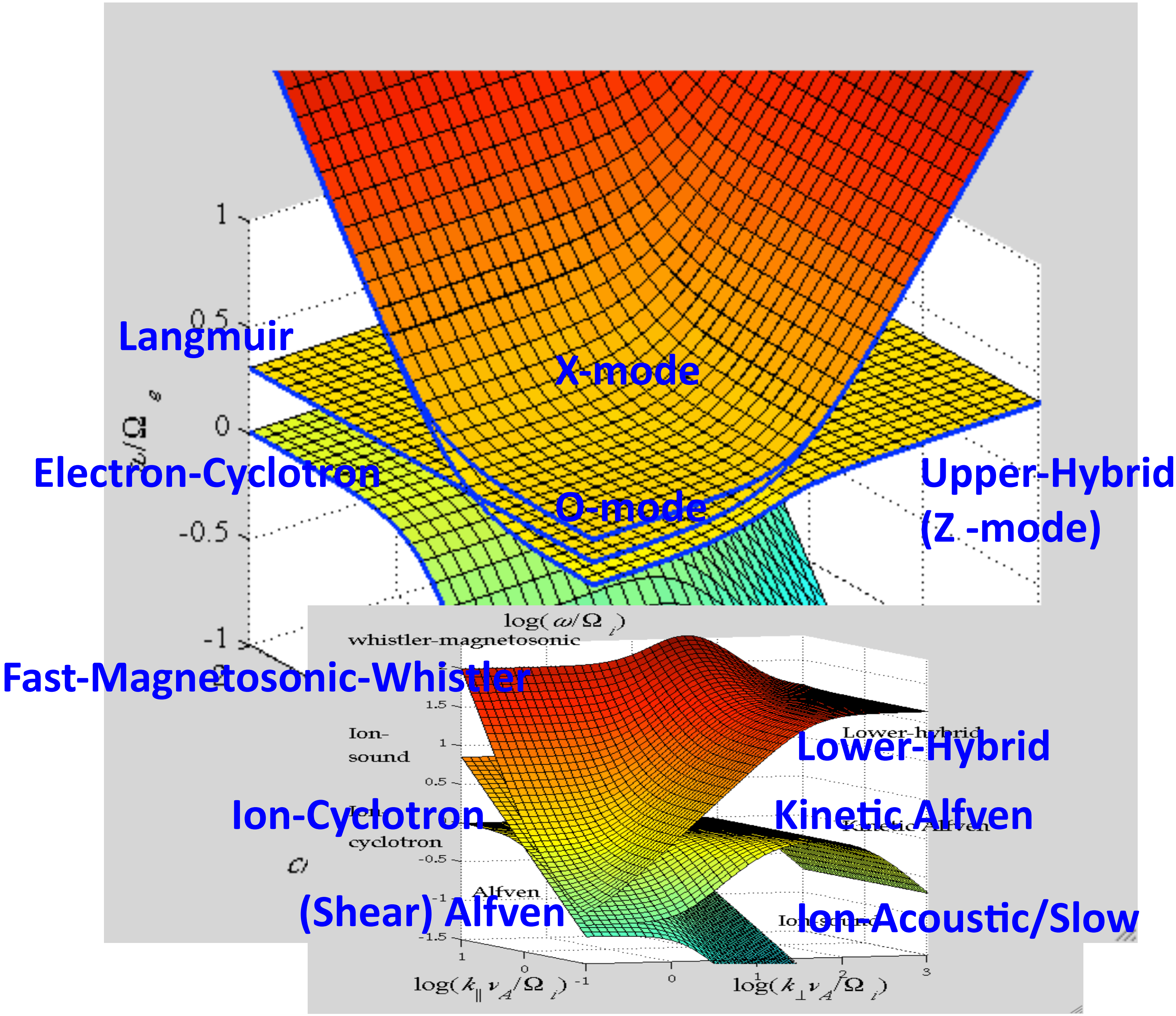
$$\frac{\partial |\delta E_k^2|}{\partial t} = ?$$

Global spectrum of solar wind electromagnetic fluctuations

- 2 months of data in the ambient solar wind near L1 -



[From Salem]

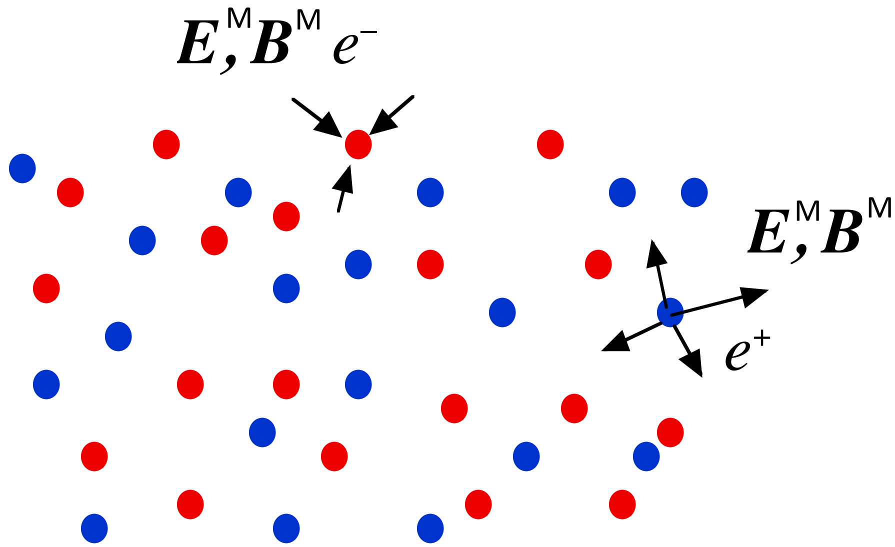


Langmuir
X-mode
Electron-Cyclotron
O-mode
Upper-Hybrid
(Z -mode)
Fast-Magnetosonic-Whistler
Lower-Hybrid
Ion-Cyclotron
Kinetic Alfven
(Shear) Alfven
Ion-Acoustic/Slow
log(ω/Ω i)
whistler-magnetosonic
Ion-
sound
cyclotron
Alfven
Lower-hybrid
Kinetic Alfven
log(k|| vA/Ω i)
log(k⊥ vA/Ω i)

Current State of Knowledge on Plasma Turbulence

	FLUID THEORIES	REDUCED KINETIC THEORIES	KINETIC THEORIES
UNMAGNETIZED	• Strong turbulence (Zakharov)		• Perturbative theory • Renormalized theory
MAGNETIZED	• MHD (Ideal, Resistive, Hall, etc) • Drift-wave turb. (Hasegawa-Mima)	• Drift Kinetic • Gyrokinetic	**?** [Perturbative kinetic theory for magnetized plasmas]

Nonlinear Plasma Interaction



$$\frac{d\mathbf{r}_i^a(t)}{dt} = \mathbf{v}_i^a(t),$$

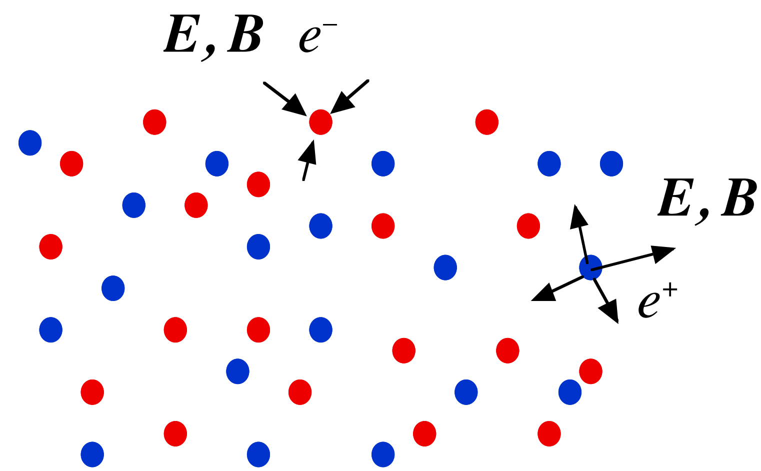
$$\frac{d\mathbf{v}_i^a(t)}{dt} = e_a \mathbf{E}[\mathbf{r}_i^a(t), t] + \frac{e_a}{c}\mathbf{v}_i^a(t) \times \mathbf{B}[\mathbf{r}_i^a(t), t],$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r}, t) = 0,$$

$$\nabla \bullet \mathbf{B}(\mathbf{r}, t) = 0,$$

$$\nabla \bullet \mathbf{E}(\mathbf{r}, t) = 4\pi \sum_a \sum_{i=1}^{N} \delta[\mathbf{r} - \mathbf{r}_i^a(t)],$$

$$\nabla \times \mathbf{B}(\mathbf{r}, t) - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r}, t) = \frac{4\pi}{c}\sum_a e_a \sum_{i=1}^{N} \mathbf{v}_i^a(t)\delta[\mathbf{r} - \mathbf{r}_i^a(t)].$$



Klimontovich function

$$N_a(\mathbf{r},\mathbf{v},t) = \sum_{i=1}^{N} \delta[\mathbf{r} - \mathbf{r}_i^a(t)]\delta[\mathbf{v} - \mathbf{v}_i^a(t)],$$

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \bullet \nabla + \frac{e_a}{m_a}\left(\mathbf{E}(\mathbf{r},t) + \frac{\mathbf{v}}{c} \times \mathbf{B}(\mathbf{r},t)\right) \bullet \frac{\partial}{\partial \mathbf{v}}\right] N_a(\mathbf{r},\mathbf{v},t) = 0,$$

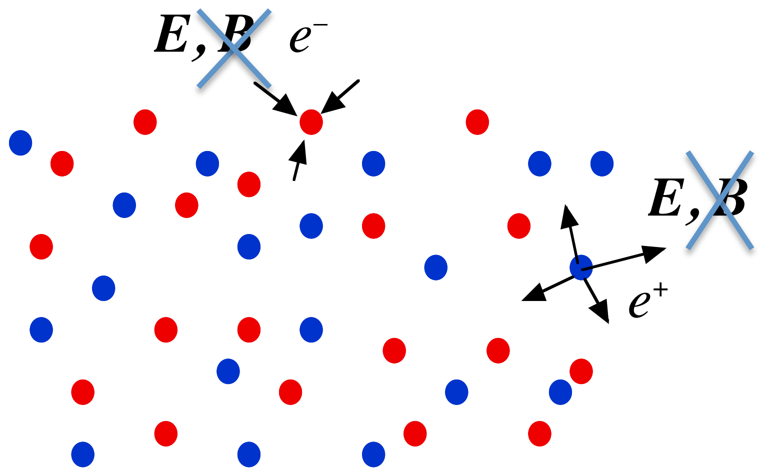
$$\nabla \times \mathbf{E}(\mathbf{r},t) + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t) = 0,$$

$$\nabla \bullet \mathbf{B}(\mathbf{r},t) = 0,$$

$$\nabla \bullet \mathbf{E}(\mathbf{r},t) = 4\pi \sum_a \int d\mathbf{v} N_a(\mathbf{r},\mathbf{v},t),$$

$$\nabla \times \mathbf{B}(\mathbf{r},t) - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t) = \frac{4\pi}{c}\sum_a e_a \int d\mathbf{v}\mathbf{v} N_a(\mathbf{r},\mathbf{v},t).$$

Electrostatic approximation



$$N_a(\mathbf{r},\mathbf{v},t) = \sum_{i=1}^{N} \delta[\mathbf{r} - \mathbf{r}_i^a(t)]\delta[\mathbf{v} - \mathbf{v}_i^a(t)],$$

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\bullet\nabla + \frac{e_a}{m_a}\mathbf{E}(\mathbf{r},t)\bullet\frac{\partial}{\partial \mathbf{v}}\right]N_a(\mathbf{r},\mathbf{v},t) = 0,$$

$$\nabla\bullet\mathbf{E}(\mathbf{r},t) = 4\pi\sum_a \int d\mathbf{v}N_a(\mathbf{r},\mathbf{v},t).$$

Separation into average and fluctuation

$$< N_a(\mathbf{r},\mathbf{v},t) >= f_a(\mathbf{r},\mathbf{v},t)$$

$$N_a(\mathbf{r},\mathbf{v},t) = f_a(\mathbf{r},\mathbf{v},t) + \delta N_a(\mathbf{r},\mathbf{v},t)$$

$$\mathbf{E}(\mathbf{r},t) = \delta\mathbf{E}(\mathbf{r},t)$$

Overview of Kinetic Plasma Turbulence Theory

$$\left[\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} + \frac{e_a E}{m_a}\frac{\partial}{\partial v}\right] f_a = 0, \quad \frac{\partial E}{\partial x} = 4\pi n \sum_a e_a \int dv\, f_a$$

$$f_a = F_a + \delta f_a, \quad E = \delta E$$

$$\left(\frac{\partial}{\partial t} + \frac{e_a}{m_a}\delta E\frac{\partial}{\partial v}\right) F_a + \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} + \frac{e_a}{m_a}\delta E\frac{\partial}{\partial v}\right)\delta f_a = 0,$$

$$\frac{\partial}{\partial x}\delta E = 4\pi n \sum_a e_a \int dv\,\delta f_a$$

Average over random phase: $$\frac{\partial F_a}{\partial t} = -\frac{e_a}{m_a}\frac{\partial}{\partial v} < \delta E \delta f_a >$$

Insert back to the original equation

$$\left(\frac{\partial}{\partial t}+\frac{e_a}{m_a}\delta E\frac{\partial}{\partial v}\right)F_a+\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}+\frac{e_a}{m_a}\delta E\frac{\partial}{\partial v}\right)\delta f_a=0$$

$$\frac{\partial F_a}{\partial t}=-\frac{e_a}{m_a}\frac{\partial}{\partial v}<\delta E\delta f_a>$$

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)\delta f_a=-\frac{e_a}{m_a}\delta E\frac{\partial F_a}{\partial v}-\frac{e_a}{m_a}\frac{\partial}{\partial v}\left(\delta f_a\delta E-<\delta f_a\delta E>\right)$$

Two time scales (slow and fast)

$$\delta f_a(x,v,t)=\int dk\int d\omega\delta f^a_{k,\omega}(v,t)e^{ikx-i\omega t}$$

slow: (v,t) — fast: $e^{ikx-i\omega t}$

$$\left(\omega-kv+i\frac{\partial}{\partial t}\right)\delta f^a_{k,\omega}=-\frac{ie_a}{m_a}\delta E_{k,\omega}\frac{\partial F_a}{\partial v}$$

$$-\frac{ie_a}{m_a}\frac{\partial}{\partial v}\int dk'\int d\omega'\left(\delta f^a_{k-k',\omega-\omega'}\delta E_{k',\omega'}-<^a_{k-k',\omega-\omega'}\delta E_{k',\omega'}>\right)$$

$$\left(\omega - kv + i\frac{\partial}{\partial t}\right)\delta f^a_{k,\omega} = -\frac{ie_a}{m_a}\delta E_{k,\omega}\frac{\partial F_a}{\partial v}$$

$$-\frac{ie_a}{m_a}\frac{\partial}{\partial v}\int dk'\int d\omega'\left(\delta f^a_{k-k',\omega-\omega'}\delta E_{k',\omega'} - <^a_{k-k',\omega-\omega'}\ \delta E_{k',\omega'} >\right)$$

- $\omega \to \omega + i\frac{\partial}{\partial t}$
- $K = (k,\omega), \quad g_K = -\frac{ie_a}{m_a}\frac{1}{\omega - kv + i0}\frac{\partial}{\partial v}$

$$f_K = g_K F E_K + \int dK' g_K (E_{K'} f_{K-K'} - < E_{K'} f_{K-K'} >)$$

- iterative solution: $f_K = f_K^{(1)} + f_K^{(2)} + \ldots$

- insert to Poisson eq: $E_K = -i\sum_a \frac{4\pi n e_a}{k}\int dv\, f_K$

$$\varepsilon(K)$$: linear dielectric response

$$0 = \left(1 + \sum_a \frac{4\pi n e_a i}{k} \int dv g_K F\right) E_K$$

$$+ \int dK' \sum_a \frac{4\pi n e_a i}{k} \int dv g_K g_{K-K'} F \left(E_{K'} E_{K-K'} - < E_{K'} E_{K-K'} >\right)$$

$$\chi^{(2)}(K'|K-K')$$: (second-order) nonlinear response

$$0 = \varepsilon(K) E_K + \int dK' \chi^{(2)}(K'|K-K') \left(E_{K'} E_{K-K'} - < E_{K'} E_{K-K'} >\right)$$

$$0 = \varepsilon(K) < E_K E_{-K} > + \int dK' \chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

$$0 = \varepsilon(K) < E_K E_{-K} > + \int dK' \chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

At this point we reintroduce the slow-time derivative

$$\varepsilon(K) < E^2 >_{k,\omega} \to \varepsilon\left(k, \omega + i\frac{\partial}{\partial t}\right) < E^2 >_{k,\omega} \to \left(\varepsilon(K) + \frac{i}{2}\frac{\partial \varepsilon(K)}{\partial \omega}\frac{\partial}{\partial t}\right) < E^2 >_{k,\omega}$$

$$0 = \frac{i}{2}\frac{\partial \varepsilon(K)}{\partial \omega}\frac{\partial}{\partial t} < E^2 >_K + \mathrm{Re}\,\varepsilon(K) < E^2 >_K + i\,\mathrm{Im}\,\varepsilon(K) < E^2 >_K$$
$$+ \int dK' \chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

- $\mathrm{Re}\,\varepsilon(K) < E^2 >_K = 0$ Dispersion relation
- $\frac{\partial}{\partial t} < E^2 >_K = -\frac{2\,\mathrm{Im}\,\varepsilon(K)}{\partial \mathrm{Re}\,\varepsilon(K)/\partial \omega} < E^2 >_K$ Wave kinetic equation

$$+ \mathrm{Im}\frac{2i}{\partial \mathrm{Re}\,\varepsilon(K)/\partial \omega} \int dK' \chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

Coupling $\frac{\partial F_a}{\partial t} = -\frac{e_a}{m_a}\frac{\partial}{\partial v} < \delta E \delta f_a >$ and $f_K = g_K F E_K$

we obtain the particle kinetic equation

$$\frac{\partial F_a}{\partial t} = -\frac{e_a}{m_a}\frac{\partial}{\partial v}\int dK g_K < E^2 >_K F$$

Formal equations of kinetic plasma turbulence theory

- $\mathrm{Re}\,\varepsilon(K) < E^2 >_K = 0$ — Dispersion relation
- $\frac{\partial F_a}{\partial t} = -\frac{e_a}{m_a}\frac{\partial}{\partial v}\int dK g_K < E^2 >_K F$ — Particle kinetic equation
- $\frac{\partial}{\partial t} < E^2 >_K = -\frac{2\,\mathrm{Im}\,\varepsilon(K)}{\partial \mathrm{Re}\,\varepsilon(K)/\partial\omega} < E^2 >_K$ — Wave kinetic equation

$$+\mathrm{Im}\frac{2i}{\partial \mathrm{Re}\,\varepsilon(K)/\partial\omega}\int dK' \chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

Three-body cumulant and closure of hierarchy. If E_k is plane wave (i.e., linear eigenmode)

$$0 = \varepsilon(K) E_K$$

Then by definition

$$< E_{-K} E_{K'} E_{K-K'} >= 0$$

But we are dealing with nonlinear theory, where

$$0 = \varepsilon(K) E_K + \int dK' \chi^{(2)}(K' | K-K') \left(E_{K'} E_{K-K'} - < E_{K'} E_{K-K'} > \right)$$

Let us write $E_K = E_K^{(0)} + E_K^{(1)}$ where $0 = \varepsilon(K) E_K^{(0)}$ then

$$0 = \varepsilon(K) E_K^{(1)} + \int dK' \chi^{(2)}(K' | K-K') \left(E_{K'}^{(0)} E_{K-K'}^{(0)} - < E_{K'}^{(0)} E_{K-K'}^{(0)} > \right),$$

$$E_K^{(1)} = -\frac{1}{\varepsilon(K)} \int dK' \chi^{(2)}(K' | K-K') \left(E_{K'}^{(0)} E_{K-K'}^{(0)} - < E_{K'}^{(0)} E_{K-K'}^{(0)} > \right)$$

Three-body cumulant

0

$$< E_{-K} E_{K'} E_{K-K'} > \approx < E^{(0)}_{-K} E^{(0)}_{K'} E^{(0)}_{K-K'} > + < E^{(1)}_{-K} E^{(0)}_{K'} E^{(0)}_{K-K'} >$$
$$+ < E^{(0)}_{-K} E^{(1)}_{K'} E^{(0)}_{K-K'} > + < E^{(0)}_{-K} E^{(0)}_{K'} E^{(1)}_{K-K'} >$$

But we are dealing with nonlinear theory, where

$$0 = \varepsilon(K) E_K + \int dK' \chi^{(2)}(K'|K-K') \left(E_{K'} E_{K-K'} - < E_{K'} E_{K-K'} > \right)$$

Let us write $E_K = E^{(0)}_K + E^{(1)}_K$ where $0 = \varepsilon(K) E^{(0)}_K$ then

$$0 = \varepsilon(K) E^{(1)}_K + \int dK' \chi^{(2)}(K'|K-K') \left(E^{(0)}_{K'} E^{(0)}_{K-K'} - < E^{(0)}_{K'} E^{(0)}_{K-K'} > \right),$$

$$E^{(1)}_K = -\frac{1}{\varepsilon(K)} \int dK' \chi^{(2)}(K'|K-K') \left(E^{(0)}_{K'} E^{(0)}_{K-K'} - < E^{(0)}_{K'} E^{(0)}_{K-K'} > \right)$$

Three-body cumulant

$$< E_{-K} E_{K'} E_{K-K'} > \approx -\frac{1}{\varepsilon(K')} \int dK'' \chi^{(2)}(K'' | K' - K'')$$

$$\times \left(< E^{(0)}_{K''} E^{(0)}_{K'-K''} E^{(0)}_{K-K'} E^{(0)}_{-K} > - < E^{(0)}_{K''} E^{(0)}_{K'-K''} >< E^{(0)}_{K-K'} E^{(0)}_{-K} > \right)$$

$$-\frac{1}{\varepsilon(K-K')} \int dK'' \chi^{(2)}(K'' | K - K' - K'')$$

$$\times \left(< E^{(0)}_{K''} E^{(0)}_{K-K'-K''} E^{(0)}_{K'} E^{(0)}_{-K} > - < E^{(0)}_{K''} E^{(0)}_{K-K'-K''} >< E^{(0)}_{K-K'} E^{(0)}_{-K} > \right)$$

$$-\frac{1}{\varepsilon(-K)} \int dK'' \chi^{(2)}(-K'' | -K + K'')$$

$$\times \left(< E^{(0)}_{K'} E^{(0)}_{K-K'} E^{(0)}_{-K'} E^{(0)}_{-K+K''} > - < E^{(0)}_{K'} E^{(0)}_{K-K'} >< E^{(0)}_{-K'} E^{(0)}_{-K+K''} > \right)$$

We then drop superscript (0)

Quasi-normal closure

$$< E_K E_{K'} E_{K''} E_{K'''} >= \delta(K + K' + K'' + K''')$$
$$\times\Big[\delta(K + K') < E^2 >_K < E^2 >_{K''} + \delta(K + K'') < E^2 >_K < E^2 >_{K'}$$
$$+ \delta(K' + K'') < E^2 >_K < E^2 >_{K'} + < E^4 >_{K;K+K';K+K'+K''}\Big]$$

ignore

$$< E_{-K} E_{K'} E_{K-K'} > \approx \frac{2\chi^{(2)}(K'|K-K')}{\varepsilon(K')} < E^2 >_{K-K'} < E^2 >_K$$
$$+ \frac{2\chi^{(2)}(K'|K-K')}{\varepsilon(K-K')} < E^2 >_{K'} < E^2 >_K$$
$$- \frac{2\chi^{(2)*}(K'|K-K')}{\varepsilon^*(K)} < E^2 >_{K'} < E^2 >_{K-K'}$$

$$< E_{-K} E_{K'} E_{K-K'} > \approx \frac{2\chi^{(2)}(K'|K-K')}{\varepsilon(K')} < E^2 >_{K-K'} < E^2 >_K$$

$$+ \frac{2\chi^{(2)}(K'|K-K')}{\varepsilon(K-K')} < E^2 >_{K'} < E^2 >_K$$

$$- \frac{2\chi^{(2)*}(K'|K-K')}{\varepsilon^*(K)} < E^2 >_{K'} < E^2 >_{K-K'}$$

$$\frac{\partial}{\partial t} < E^2 >_K = -\frac{2\,\mathrm{Im}\,\varepsilon(K)}{\partial\,\mathrm{Re}\,\varepsilon(K)/\partial\omega} < E^2 >_K$$

$$+\mathrm{Im}\frac{2i}{\partial\,\mathrm{Re}\,\varepsilon(K)/\partial\omega}\int dK'\chi^{(2)}(K'|K-K') < E_{-K} E_{K'} E_{K-K'} >$$

$$\mathrm{Re}\,\varepsilon(K) < E^2 >_K = 0$$

Dispersion relation

$$\frac{\partial F_a}{\partial t} = \frac{ie_a^2}{m_a^2}\frac{\partial}{\partial v}\int dK \frac{< E^2 >_K}{\omega - kv}\frac{\partial F_a}{\partial v},$$

Particle kinetic equation

$$\frac{\partial}{\partial t} < E^2 >_K = -\frac{2\,\mathrm{Im}\,\varepsilon(K)}{\partial \mathrm{Re}\,\varepsilon(K)/\partial\omega} < E^2 >_K$$

$$+\mathrm{Im}\frac{4}{\partial \mathrm{Re}\,\varepsilon(K)/\partial\omega}\int dK' \Bigg[\{\chi^{(2)}(K'|K-K')\}^2 \left(\frac{< E^2 >_{K-K'}}{\varepsilon(K')} + \frac{< E^2 >_{K'}}{\varepsilon(K-K')} \right) < E^2 >_K$$

$$-\frac{|\chi^{(2)}(K'|K-K')|^2}{\varepsilon^*(K)} < E^2 >_{K'} < E^2 >_{K-K'} \Bigg]$$

Wave kinetic equation

- Need to add spontaneous (discrete-particle) effects

- Need to add electromagnetic effects

Langmuir wave kinetic equation

$$\frac{\partial I_{\mathbf{k}}^{\sigma L}}{\partial t} = \frac{\pi \omega_{pe}^2}{k^2} \int d\mathbf{v} \delta(\sigma \omega_{\mathbf{k}}^L - \mathbf{k} \cdot \mathbf{v}) \left(\frac{ne^2}{\pi} f_e + \sigma \omega_{\mathbf{k}}^L I_{\mathbf{k}}^{\sigma L} \mathbf{k} \cdot \frac{\partial f_e}{\partial \mathbf{v}} \right)$$

Spontaneous & induced emission

$$+2 \sum_{\sigma', \sigma'' = \pm 1} \sigma \omega_{\mathbf{k}}^L \int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^L \delta(\sigma \omega_{\mathbf{k}}^L - \sigma' \omega_{\mathbf{k}'}^L - \sigma'' \omega_{\mathbf{k}-\mathbf{k}'}^S)$$

$$\times \left(\sigma \omega_{\mathbf{k}}^L I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} - \sigma' \omega_{\mathbf{k}'}^L I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} I_{\mathbf{k}}^{\sigma L} - \sigma'' \omega_{\mathbf{k}-\mathbf{k}'}^L I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} \right)$$

(*L* -> *L*+*S*) three wave decay

$$-\frac{\pi e^2}{m_e^2 \omega_{pe}^2} \sigma \omega_{\mathbf{k}}^L \sum_{\sigma' = \pm 1} \int d\mathbf{k}' \int d\mathbf{v} \frac{(\mathbf{k} \cdot \mathbf{k}')^2}{k^2 k'^2} \delta[\sigma \omega_{\mathbf{k}}^L - \sigma' \omega_{\mathbf{k}'}^L - (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}]$$

$$\times \left(\frac{ne^2}{\pi \omega_{pe}^2} (\sigma' \omega_{\mathbf{k}'}^L I_{\mathbf{k}}^{\sigma L} - \sigma \omega_{\mathbf{k}}^L I_{\mathbf{k}'}^{\sigma' L})(f_e + f_i) \right.$$

$$\left. + (\sigma' \omega_{\mathbf{k}'}^L - \sigma \omega_{\mathbf{k}}^L) I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} (\mathbf{k} - \mathbf{k}') \cdot \frac{\partial f_e}{\partial \mathbf{v}} - \frac{m_e}{m_i} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} (\mathbf{k} - \mathbf{k}') \cdot \frac{\partial f_i}{\partial \mathbf{v}} \right)$$

(*L* -> *L*+*e*) spontaneous & induced scattering

Ion-acoustic wave kinetic equation

$$\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}} = \frac{\pi\mu_{\mathbf{k}}\omega_{pe}^2}{k^2}\int d\mathbf{v}\delta(\sigma\omega_{\mathbf{k}}^S - \mathbf{k}\cdot\mathbf{v})\left[\frac{ne^2}{\pi}[f_e + f_i]\right.$$

$$\left.+\pi\sigma\omega_{\mathbf{k}}^L\left(\mathbf{k}\cdot\frac{\partial f_e}{\partial \mathbf{v}}_e + \frac{m_e}{m_i}\mathbf{k}\cdot\frac{\partial f_i}{\partial \mathbf{v}}\right)\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}}\right]$$

Spontaneous & induced emission

$$+\sum_{\sigma',\sigma''=\pm 1}\sigma\omega_{\mathbf{k}}^L\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^S\delta(\sigma\omega_{\mathbf{k}}^S - \sigma'\omega_{\mathbf{k}'}^L - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^L)$$

$$\times\left(\sigma\omega_{\mathbf{k}}^L I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} - \frac{\sigma'\omega_{\mathbf{k}'}^L I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^L I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}}\right)$$

(*S* -> *L*+*L*) three wave decay

Particle kinetic equation

$$\frac{\partial F_a}{\partial t} = \frac{\pi e^2}{m_e^2}\sum_{\sigma=\pm 1}\sum_{\alpha=L,S}\int d\mathbf{k}\frac{\mathbf{k}}{k}\bullet\frac{\partial}{\partial \mathbf{v}}\delta(\sigma\omega_{\mathbf{k}}^\alpha - \mathbf{k}\bullet\mathbf{v})\left(\frac{m_e\sigma\omega_{\mathbf{k}}^\alpha}{4\pi^2 k}F_a + I_{\mathbf{k}}^{\sigma\alpha}\frac{\mathbf{k}}{k}\bullet\frac{\partial F_a}{\partial \mathbf{v}}\right)$$

Transverse EM wave kinetic equation

$$\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma T}}{2} = \sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TLL}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})$$

$$\times\left(\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} - \frac{\sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} I_{\mathbf{k}}^{\sigma T}}{2} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma T}}{2}\right)$$

(*L*+*L* -> *T*) Harmonic emission

$$+\sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TLS}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})\left(\frac{\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S}}{\mu_{\mathbf{k}-\mathbf{k}'}} - \frac{\sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} I_{\mathbf{k}}^{\sigma T}}{2\mu_{\mathbf{k}-\mathbf{k}'}} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma T}}{2}\right)$$

(*L*+*S* -> *T*) Fundamental emission by decay

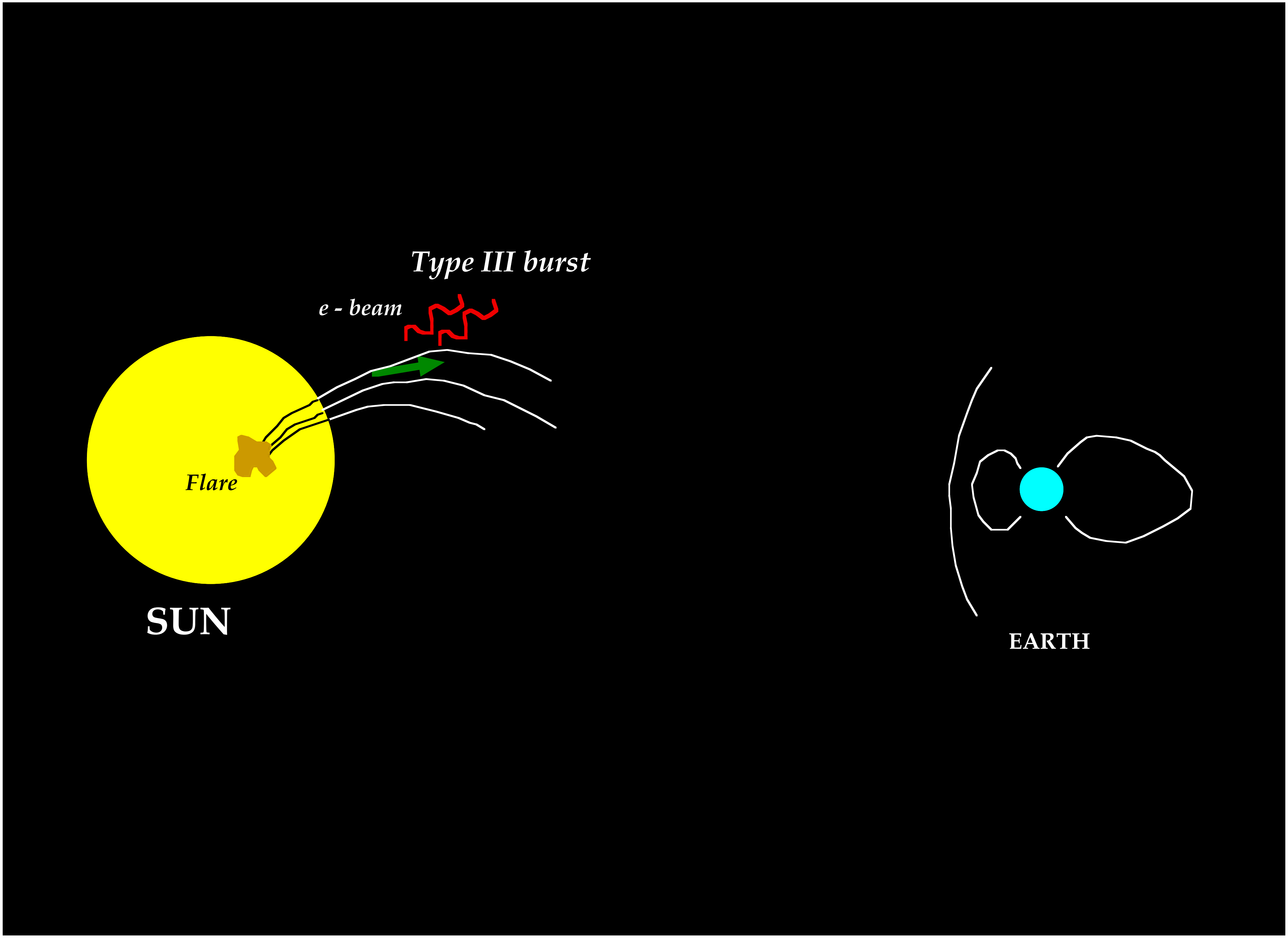
$$+\sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TTL}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{T} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})\left(\frac{\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' T} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} - \frac{\sigma'\omega_{\mathbf{k}'}^{T} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} I_{\mathbf{k}}^{\sigma T}}{2} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' T} I_{\mathbf{k}}^{\sigma T}}{4}\right)$$

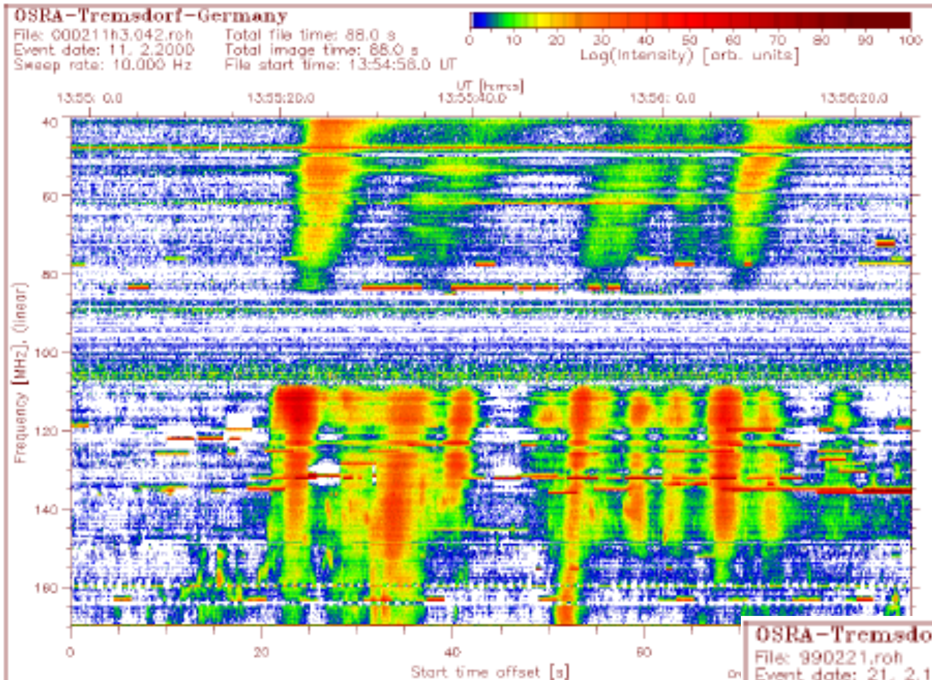
(*L*+*T* -> *T*) Higher-harmonic emission

$$+\sum_{\sigma'=\pm 1}\int d\mathbf{k}'\int d\mathbf{v}\, U_{\mathbf{k},\mathbf{k}'}^{T}\delta[\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$$

(*L*+*I* -> *T*) Fundamental emission by scattering

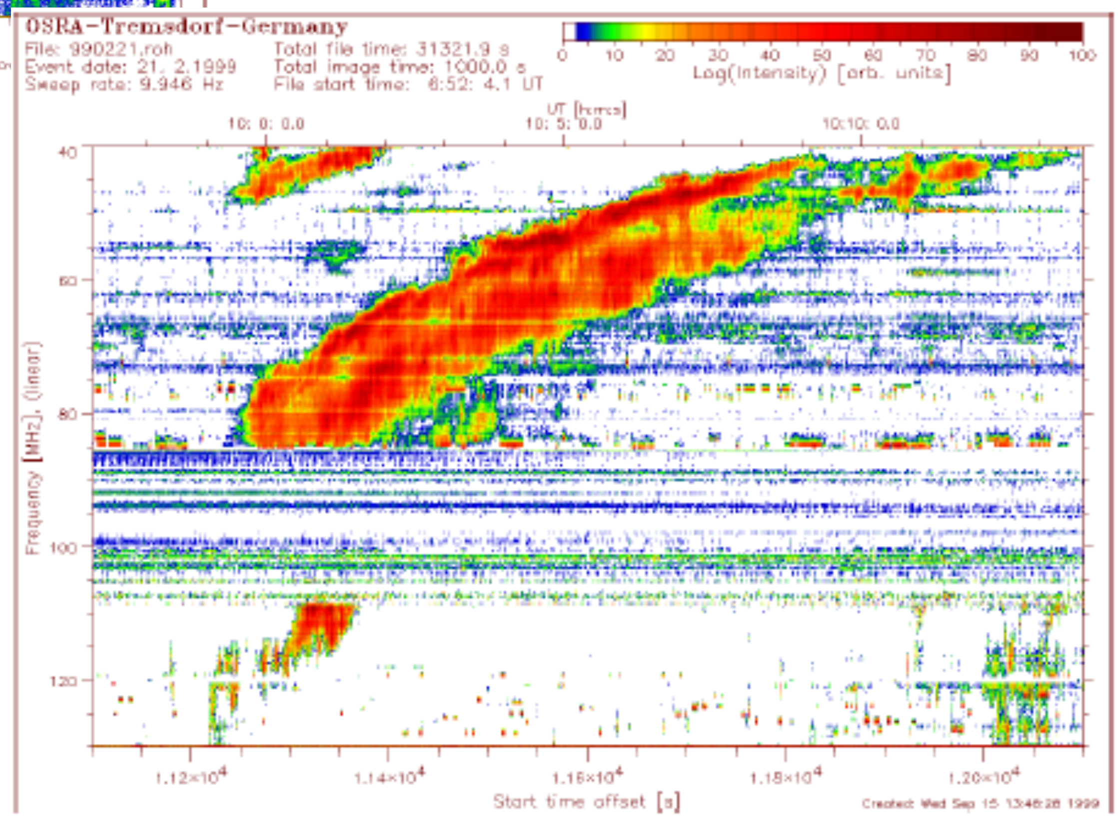
$$\times\left[\frac{ne^2}{\pi\omega_{pe}^2}\left(\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} - \frac{\sigma'\omega_{\mathbf{k}'}^{T} I_{\mathbf{k}}^{\sigma T}}{2}\right)(f_e + f_i) + \pi\frac{m_e}{m_i} I_{\mathbf{k}'}^{\sigma' L}\frac{I_{\mathbf{k}}^{\sigma L}}{2}(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial f_i}{\partial\mathbf{v}}\right]$$


Type III burst
e - beam
Flare
SUN
EARTH



Most type III bursts do not have the iconic F-H pair emission structure

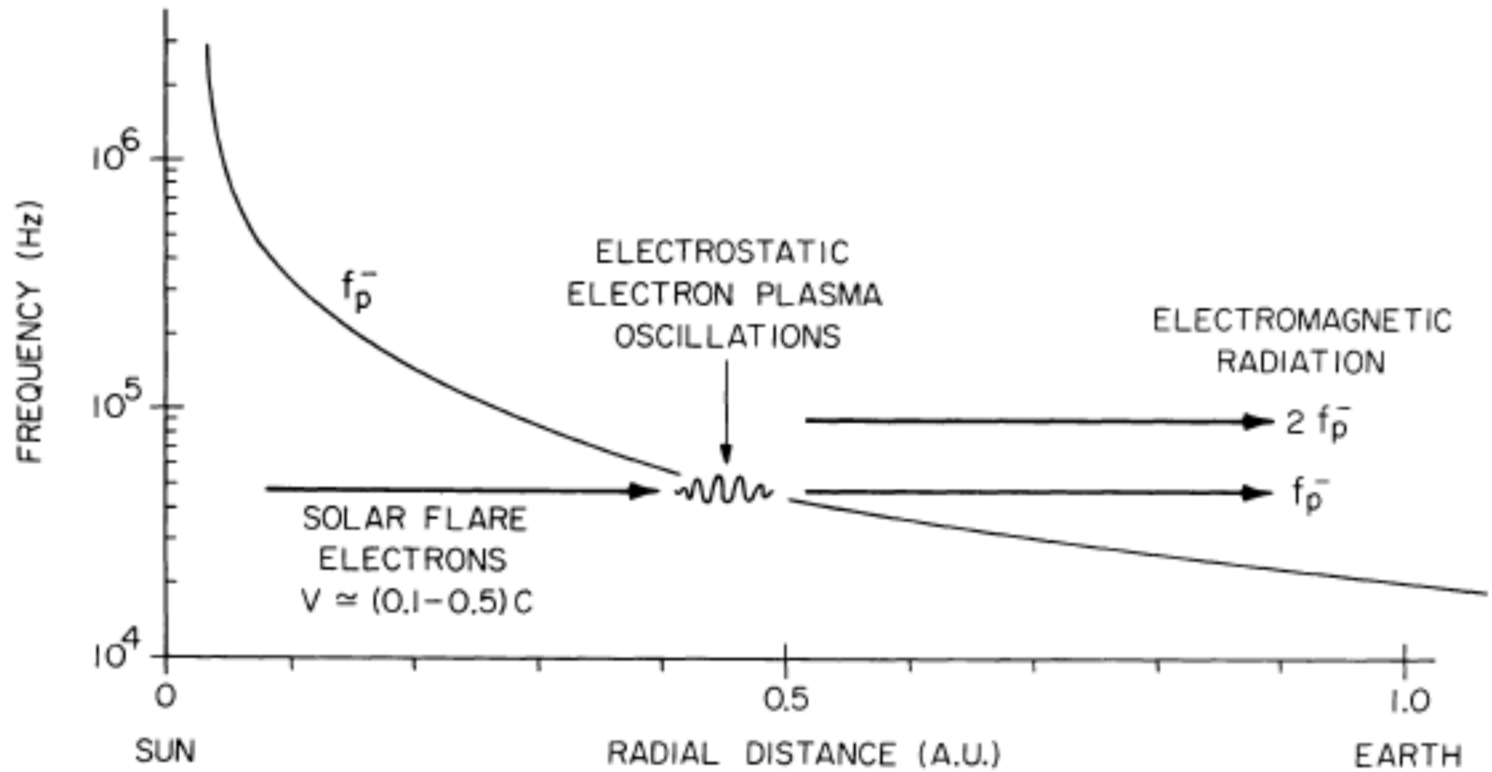
Type II emissions are somewhat more clearly identifiable with the beam-induced F-H pair emissions



Plasma emission scenario

(Ginzburg & Zeleznyakov, 1958; Melrose, 1970s; Robinson, Cairns, 1980 & 1990s)

- Electron **beam** produced during flare
- Generation of **Langmuir** waves
- **Backscattered** Langmuir waves by nonlinear processes
- **Harmonic** emission by merging of Langmuir waves
- **Fundamental** emission by Langmuir wave decay



FREQUENCY (Hz)
10^6
10^5
10^4
f_p^-
ELECTROSTATIC
ELECTRON PLASMA
OSCILLATIONS
ELECTROMAGNETIC
RADIATION
2 f_p^-
f_p^-
SOLAR FLARE
ELECTRONS
V ≃ (0.1–0.5) C
0
0.5
1.0
SUN
RADIAL DISTANCE (A.U.)
EARTH

Plasma emission scenario

- Despite six decades of research, rigorous demonstration of plasma emission based upon EM weak turbulence theory has never been done!

- **Recently plasma emission calculation based upon EM weak turbulence theory was done for the first time [Ziebell, Yoon, Gaelzer, Pavan, *ApJLett.*, 2014]**

Particle kinetic equation

$$\frac{\partial F_a}{\partial t} = \frac{\pi e^2}{m_e^2} \sum_{\sigma=\pm 1} \sum_{\alpha=L,S} \int d\mathbf{k} \frac{\mathbf{k}}{k} \bullet \frac{\partial}{\partial \mathbf{v}} \delta(\sigma\omega_{\mathbf{k}}^{\alpha} - \mathbf{k} \bullet \mathbf{v}) \left(\frac{m_e \sigma \omega_{\mathbf{k}}^{\alpha}}{4\pi^2 k} F_a + I_{\mathbf{k}}^{\sigma\alpha} \frac{\mathbf{k}}{k} \bullet \frac{\partial F_a}{\partial \mathbf{v}} \right)$$

Langmuir wave kinetic equation

$$\frac{\partial I_{\mathbf{k}}^{\sigma L}}{\partial t} = \frac{\pi \omega_{pe}^2}{k^2} \int d\mathbf{v} \delta(\sigma\omega_{\mathbf{k}}^{L} - \mathbf{k}\cdot \mathbf{v}) \left(\frac{ne^2}{\pi} f_e + \sigma\omega_{\mathbf{k}}^{L} I_{\mathbf{k}}^{\sigma L} \mathbf{k}\cdot \frac{\partial f_e}{\partial \mathbf{v}} \right)$$

$$+2 \sum_{\sigma',\sigma''=\pm 1} \sigma\omega_{\mathbf{k}}^{L} \int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{L} \delta(\sigma\omega_{\mathbf{k}}^{L} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})$$

$$\times\left(\sigma\omega_{\mathbf{k}}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} - \sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} I_{\mathbf{k}}^{\sigma L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} \right)$$

$$-\frac{\pi e^2}{m_e^2 \omega_{pe}^2} \sigma\omega_{\mathbf{k}}^{L} \sum_{\sigma'=\pm 1} \int d\mathbf{k}' \int d\mathbf{v} \frac{(\mathbf{k}\cdot \mathbf{k}')^2}{k^2 k'^2} \delta[\sigma\omega_{\mathbf{k}}^{L} - \sigma'\omega_{\mathbf{k}'}^{L} - (\mathbf{k} - \mathbf{k}')\cdot \mathbf{v}]$$

$$\times\left(\frac{ne^2}{\pi\omega_{pe}^2} (\sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}}^{\sigma L} - \sigma\omega_{\mathbf{k}}^{L} I_{\mathbf{k}'}^{\sigma' L})(f_e + f_i) \right.$$

$$\left. +(\sigma'\omega_{\mathbf{k}'}^{L} - \sigma\omega_{\mathbf{k}}^{L}) I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} (\mathbf{k} - \mathbf{k}')\cdot \frac{\partial f_e}{\partial \mathbf{v}} - \frac{m_e}{m_i} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma L} (\mathbf{k} - \mathbf{k}')\cdot \frac{\partial f_i}{\partial \mathbf{v}} \right)$$

Ion-acoustic wave kinetic equation

$$\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}} = \frac{\pi\mu_{\mathbf{k}}\omega_{pe}^{2}}{k^{2}}\int d\mathbf{v}\,\delta(\sigma\omega_{\mathbf{k}}^{S} - \mathbf{k}\cdot\mathbf{v})\left[\frac{ne^{2}}{\pi}[f_e + f_i]\right.$$

$$\left. + \pi\sigma\omega_{\mathbf{k}}^{L}\left(\mathbf{k}\cdot\frac{\partial f_e}{\partial \mathbf{v}}_e + \frac{m_e}{m_i}\mathbf{k}\cdot\frac{\partial f_i}{\partial \mathbf{v}}\right)\frac{I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}}\right]$$

$$+ \sum_{\sigma',\sigma''=\pm 1}\sigma\omega_{\mathbf{k}}^{L}\int d\mathbf{k}'V_{\mathbf{k},\mathbf{k}'}^{S}\delta(\sigma\omega_{\mathbf{k}}^{S} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})$$

$$\times\left(\sigma\omega_{\mathbf{k}}^{L}I_{\mathbf{k}'}^{\sigma' L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} - \frac{\sigma'\omega_{\mathbf{k}'}^{L}I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L}I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L}I_{\mathbf{k}'}^{\sigma' L}I_{\mathbf{k}}^{\sigma S}}{\mu_{\mathbf{k}}}\right)$$

Transverse EM wave kinetic equation

$$\frac{\partial}{\partial t}\frac{I_{\mathbf{k}}^{\sigma T}}{2} = \sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TLL}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})$$

$$\times\left(\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} - \frac{\sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} I_{\mathbf{k}}^{\sigma T}}{2} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma T}}{2}\right)$$

(*L*+*L* -> *T*) Harmonic emission

$$+\sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TLS}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{S})\left(\frac{\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S}}{\mu_{\mathbf{k}-\mathbf{k}'}} - \frac{\sigma'\omega_{\mathbf{k}'}^{L} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' S} I_{\mathbf{k}}^{\sigma T}}{2\mu_{\mathbf{k}-\mathbf{k}'}} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' L} I_{\mathbf{k}}^{\sigma T}}{2}\right)$$

(*L*+*S* -> *T*) Fundamental emission by decay

$$+\sum_{\sigma',\sigma''=\pm 1}\int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'}^{TTL}\delta(\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{T} - \sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L})\left(\frac{\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' T} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' T}}{2} - \frac{\sigma'\omega_{\mathbf{k}'}^{T} I_{\mathbf{k}-\mathbf{k}'}^{\sigma'' L} I_{\mathbf{k}}^{\sigma T}}{2} - \frac{\sigma''\omega_{\mathbf{k}-\mathbf{k}'}^{L} I_{\mathbf{k}'}^{\sigma' T} I_{\mathbf{k}}^{\sigma T}}{4}\right)$$

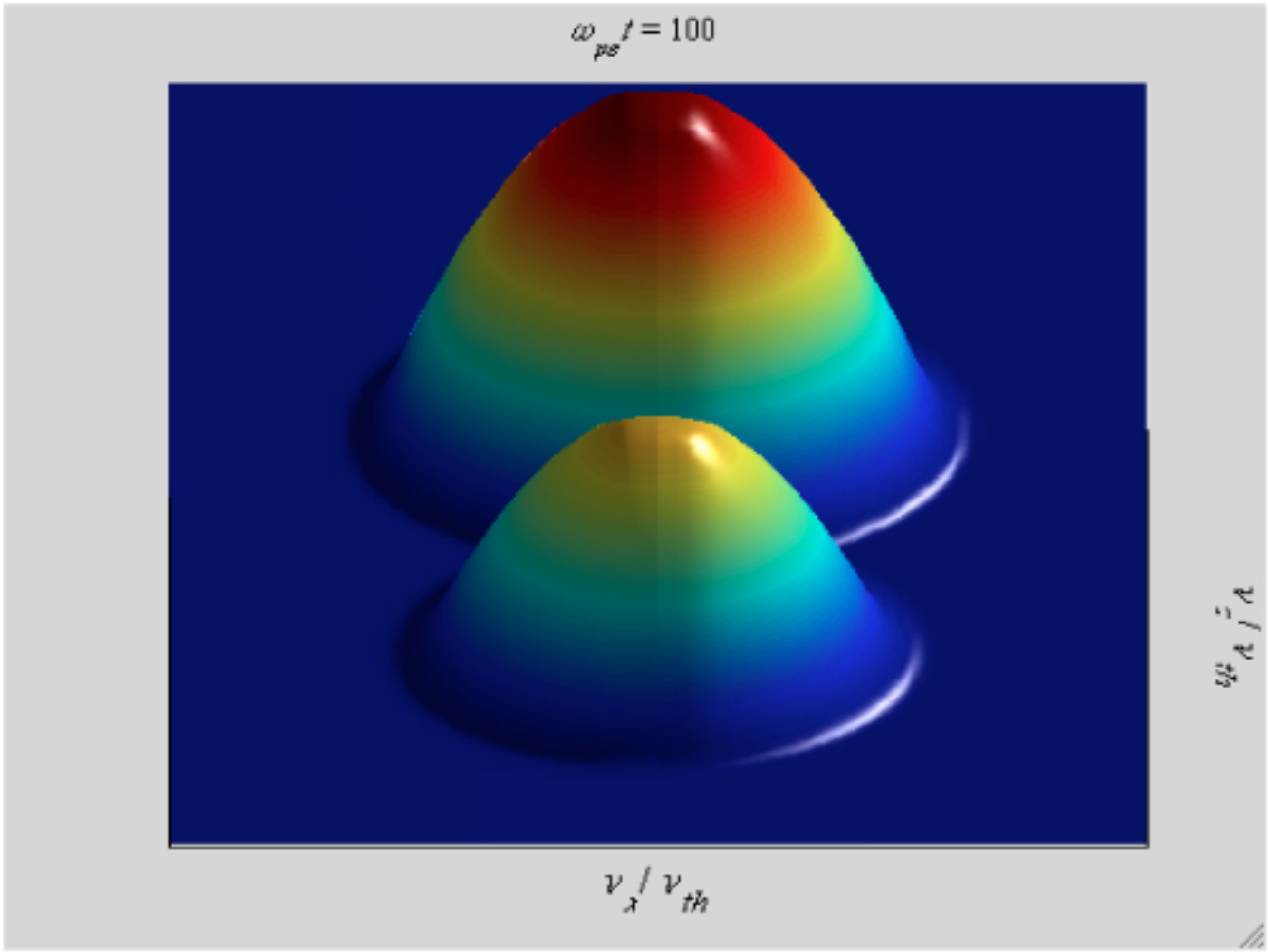
(*L*+*T* -> *T*) Higher-harmonic emission

$$+\sum_{\sigma'=\pm 1}\int d\mathbf{k}'\int d\mathbf{v} U_{\mathbf{k},\mathbf{k}'}^{T}\delta[\sigma\omega_{\mathbf{k}}^{T} - \sigma'\omega_{\mathbf{k}'}^{L} - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}]$$

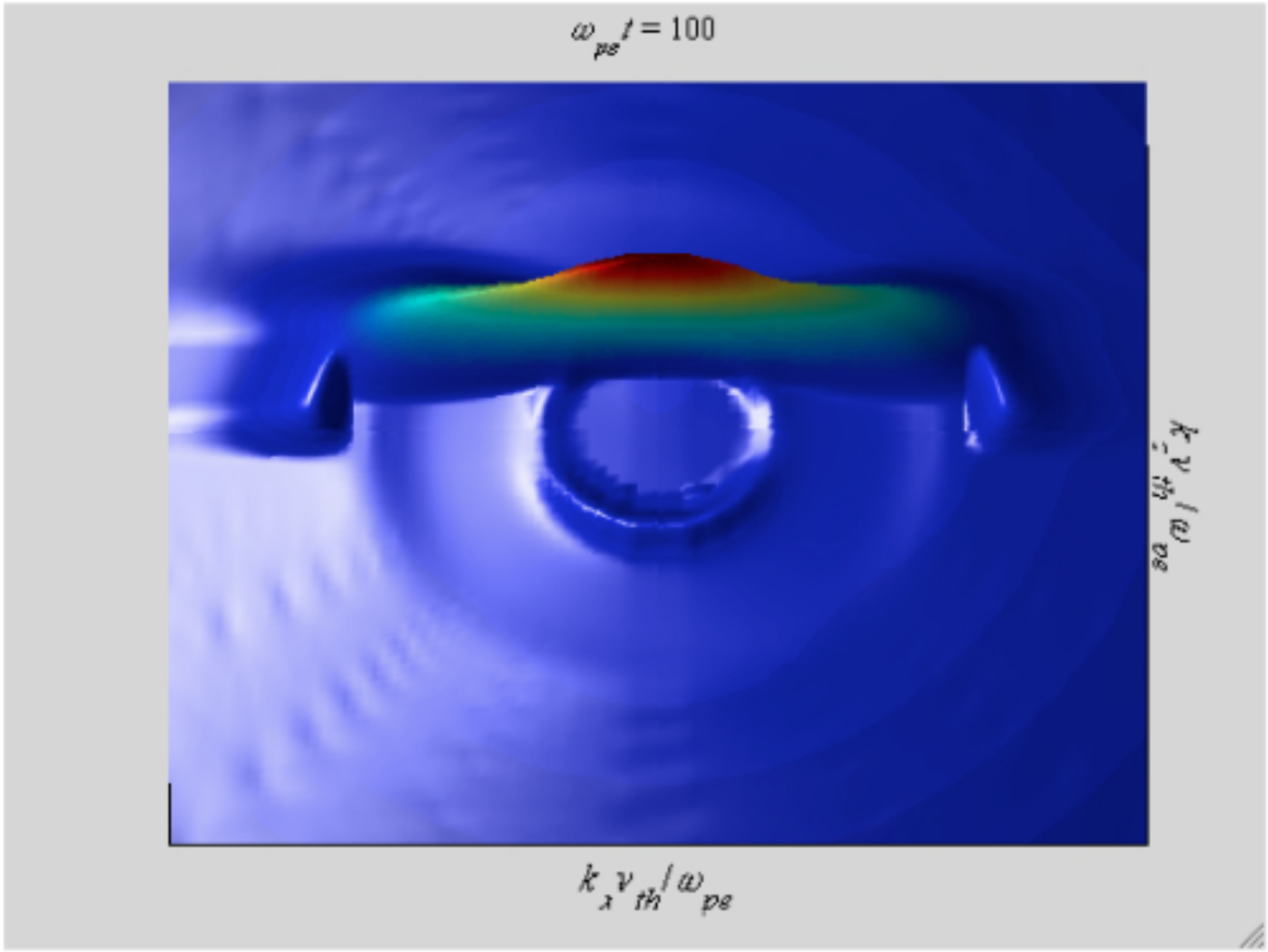
(*L*+*I* -> *T*) Fundamental emission by scattering

$$\times\left[\frac{ne^2}{\pi\omega_{pe}^2}\left(\sigma\omega_{\mathbf{k}}^{T} I_{\mathbf{k}'}^{\sigma' L} - \frac{\sigma'\omega_{\mathbf{k}'}^{T} I_{\mathbf{k}}^{\sigma T}}{2}\right)(f_e + f_i) + \pi\frac{m_e}{m_i} I_{\mathbf{k}'}^{\sigma' L}\frac{I_{\mathbf{k}}^{\sigma L}}{2}(\mathbf{k}-\mathbf{k}')\cdot\frac{\partial f_i}{\partial\mathbf{v}}\right]$$

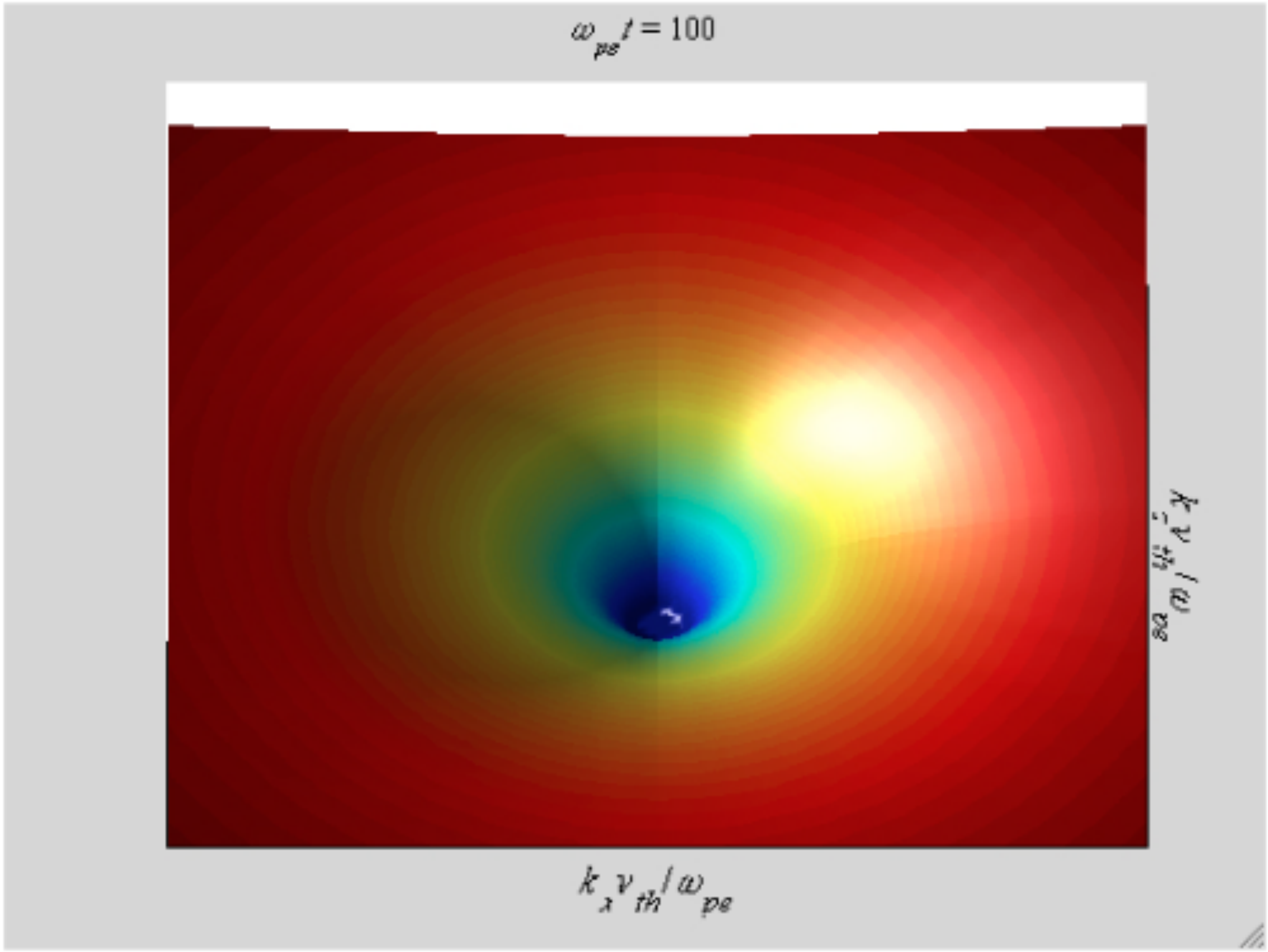
Electron Velocity Distribution Function



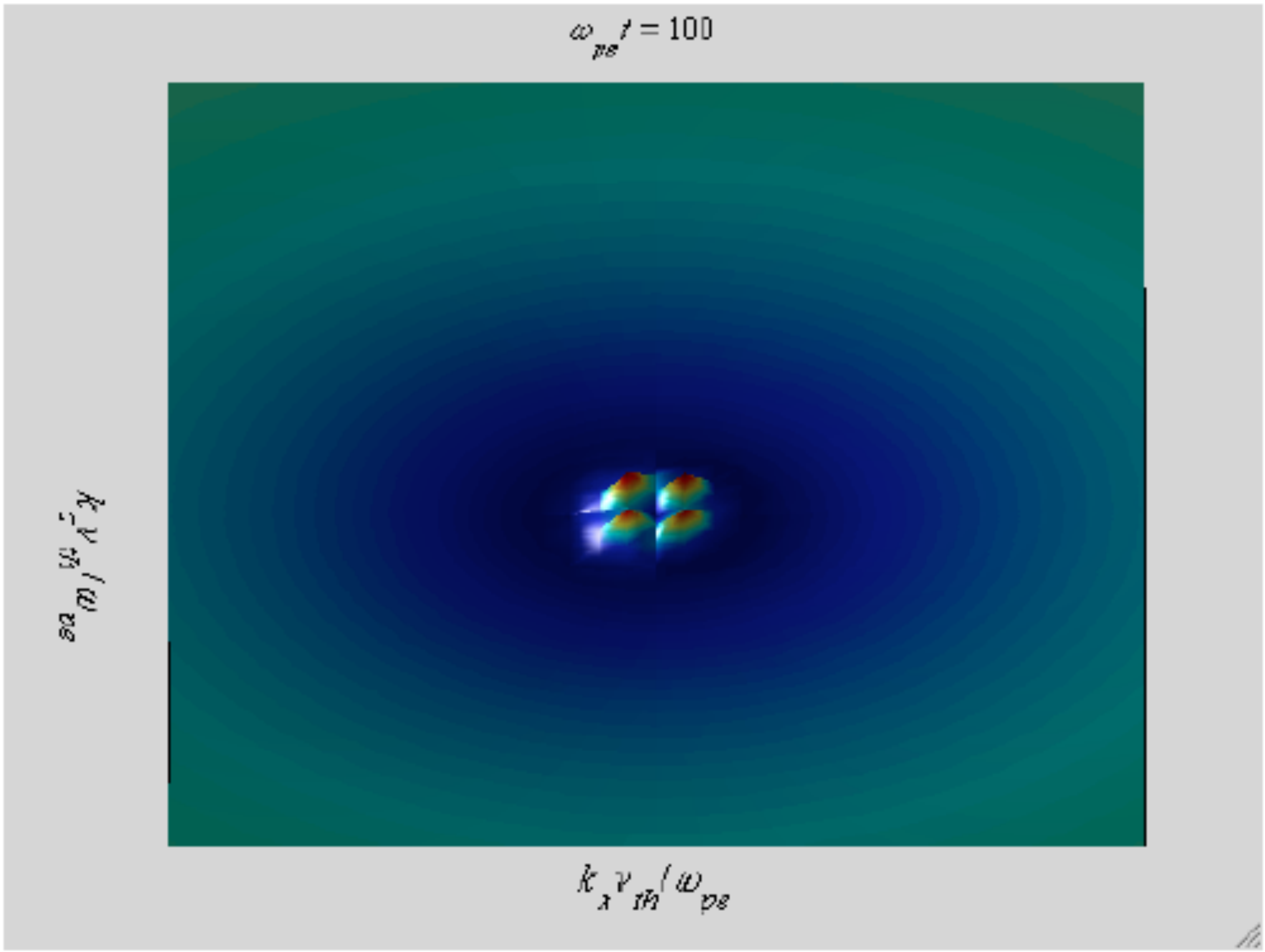
Langmuir Turbulence Spectrum

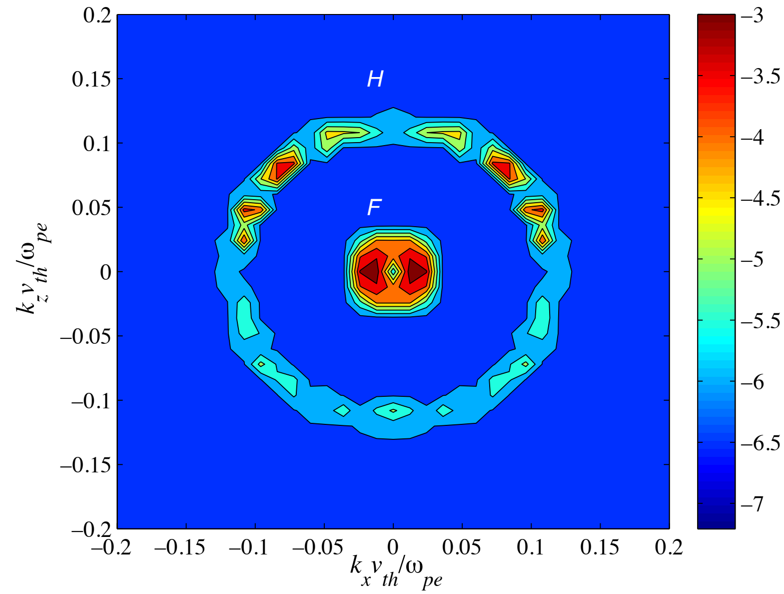


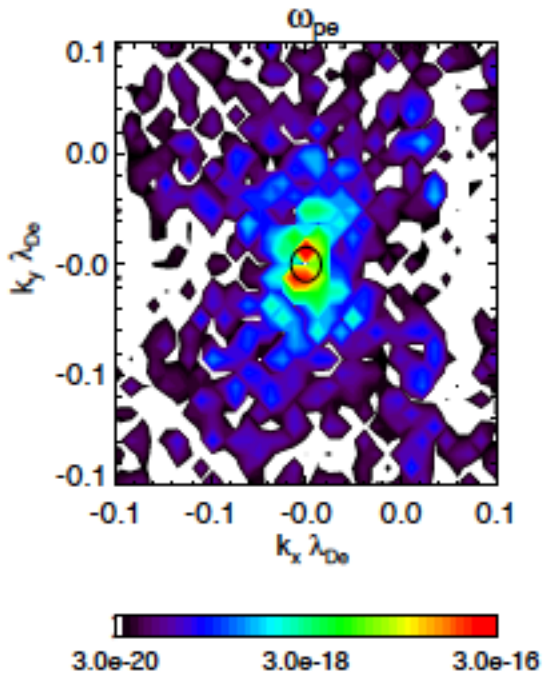
Ion-Acoustic Turbulence Spectrum



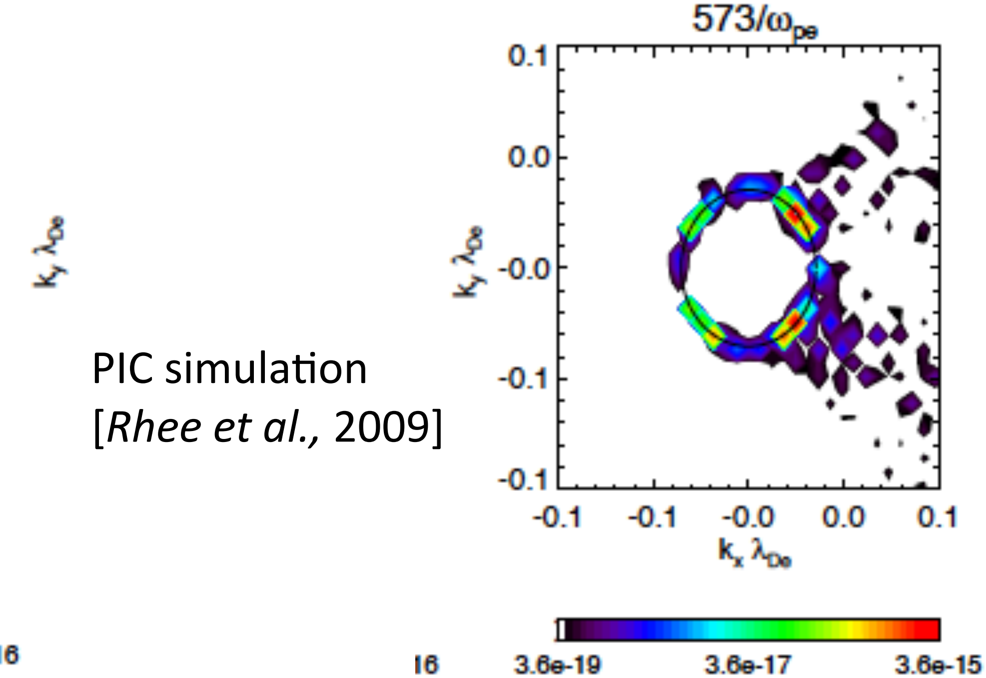
Electromagnetic Radiation Spectrum

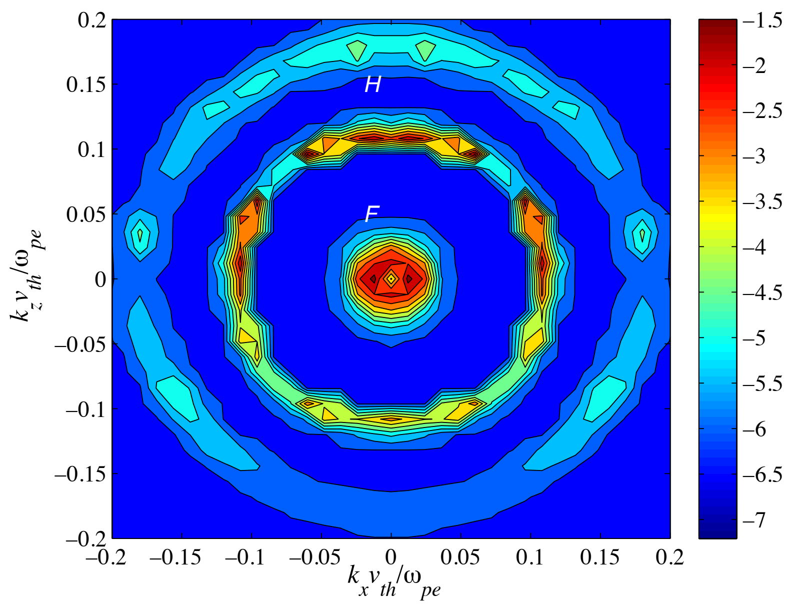


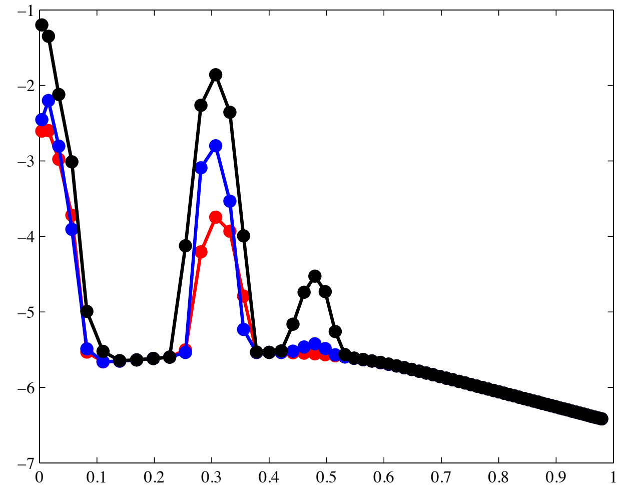




PIC simulation
[*Rhee et al.,* 2009]







PIC simulation [*Rhee et al.,* 2009]