MHD turbulence dynamo

UNIST

Kiwan Park, Dongsu Ryu, Dongho Park

▪ Necessity of studying magnetic fields in space

-Very common phenomenon. Astrophysical objects like Sunspot, Stellar rotation, Mass loss, etc. commonly show interaction of Magnetic fields & Conducting fluids.

▪ B fields play some important roles

1. B field contributes to the total pressure to stabilize gas clouds, and is essential to the star formation (removal of angular momentum)
2. But B field also reduces the star formation efficiency. (balanced with gravitation)
3. And B field controls the density and distribution of cosmic rays.

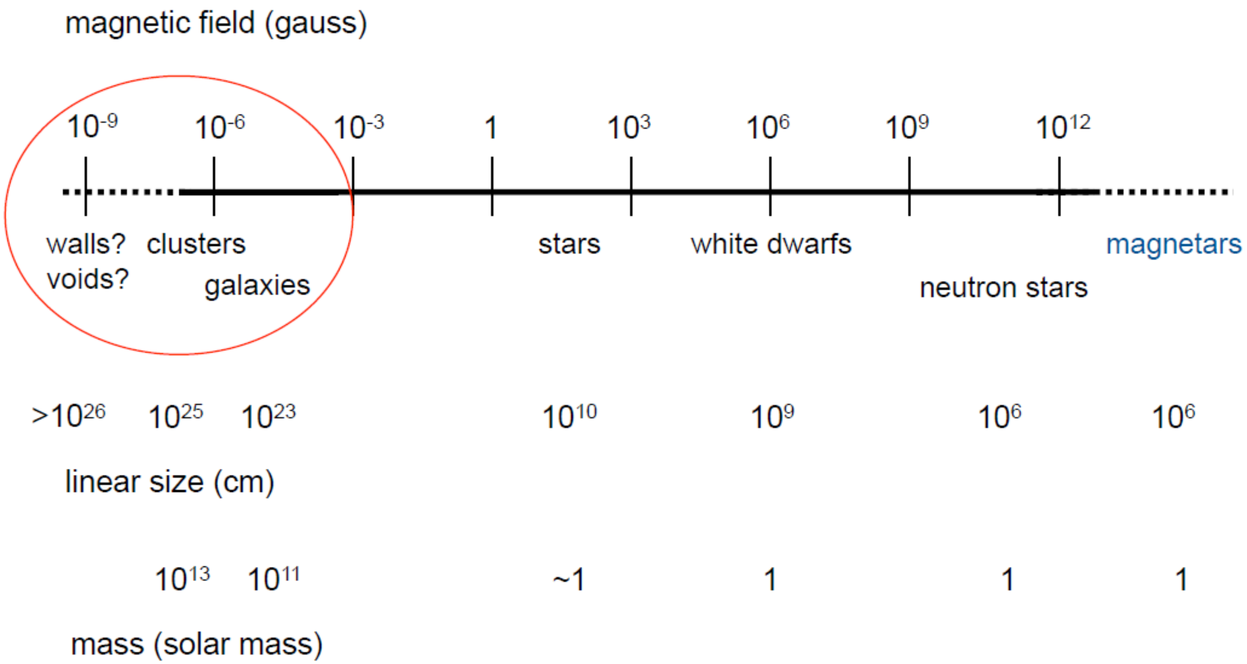
▪ Questions are…

1. What are the origin and evolution of Magnetic field?
2. And the annihilation of Magnetic field?

→ **MHD turbulence dynamo** & **magnetic reconnection**

- **MHD turbulence dynamo? mechanical energy → magnetic energy**
- **Magnetic reconnection? magnetic energy → kinetic & thermal energy (heating source for the ISM and halo gas)**

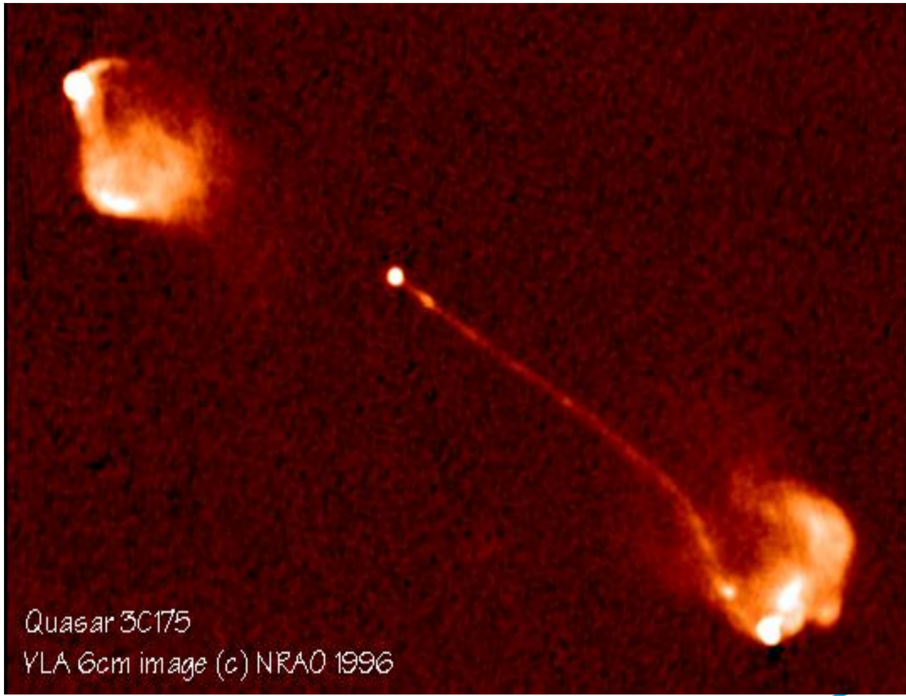
- **Magnetic fields has the wide range of magnitude:** $(10^{-9}\mathrm{G} - 10^{12}\mathrm{G})$

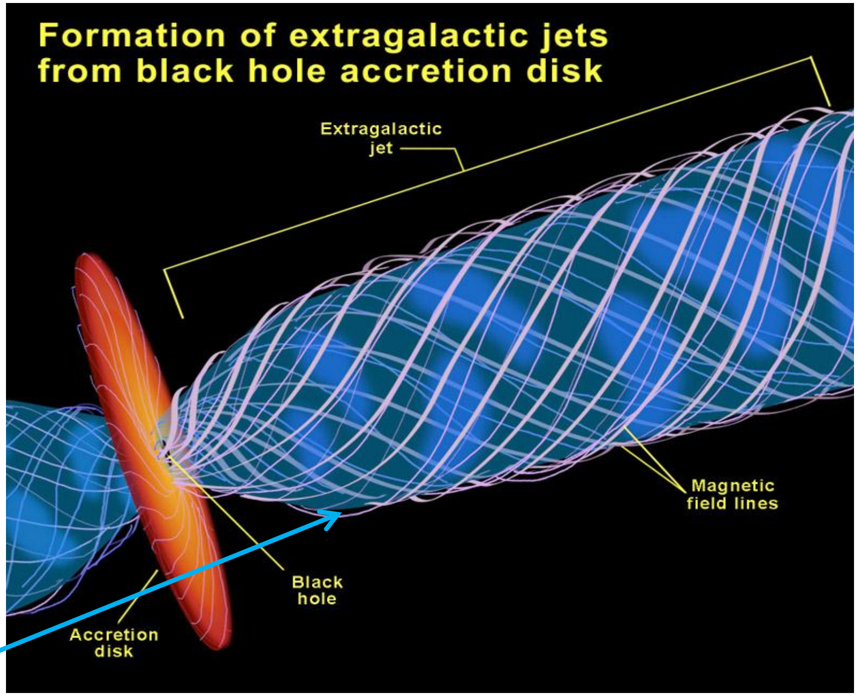


More examples
Spiral galaxy ~ 10 μG
Radio faint galaxy (M31, M33) & milky way galaxy ~ 5 μG
Gas rich spiral galaxy (M51) ~ 20-30 μG
Starburst galaxy (M82) ~ 50-100 μG

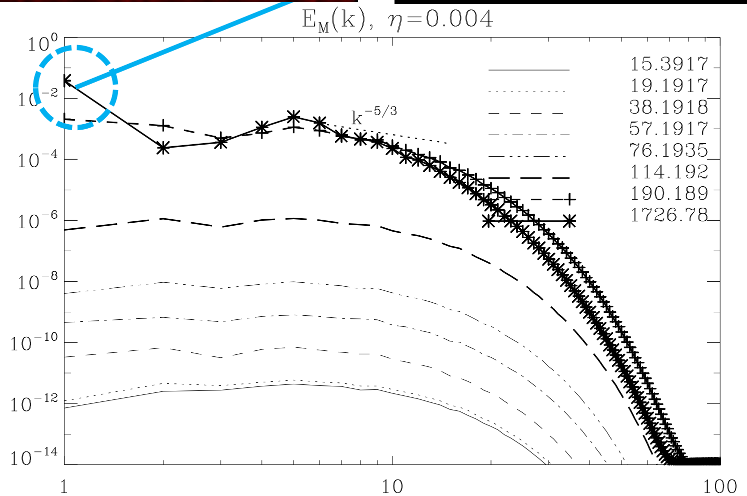
- **Magnetic field has huge range of scale**

  Ex) AGN Jet 3C175



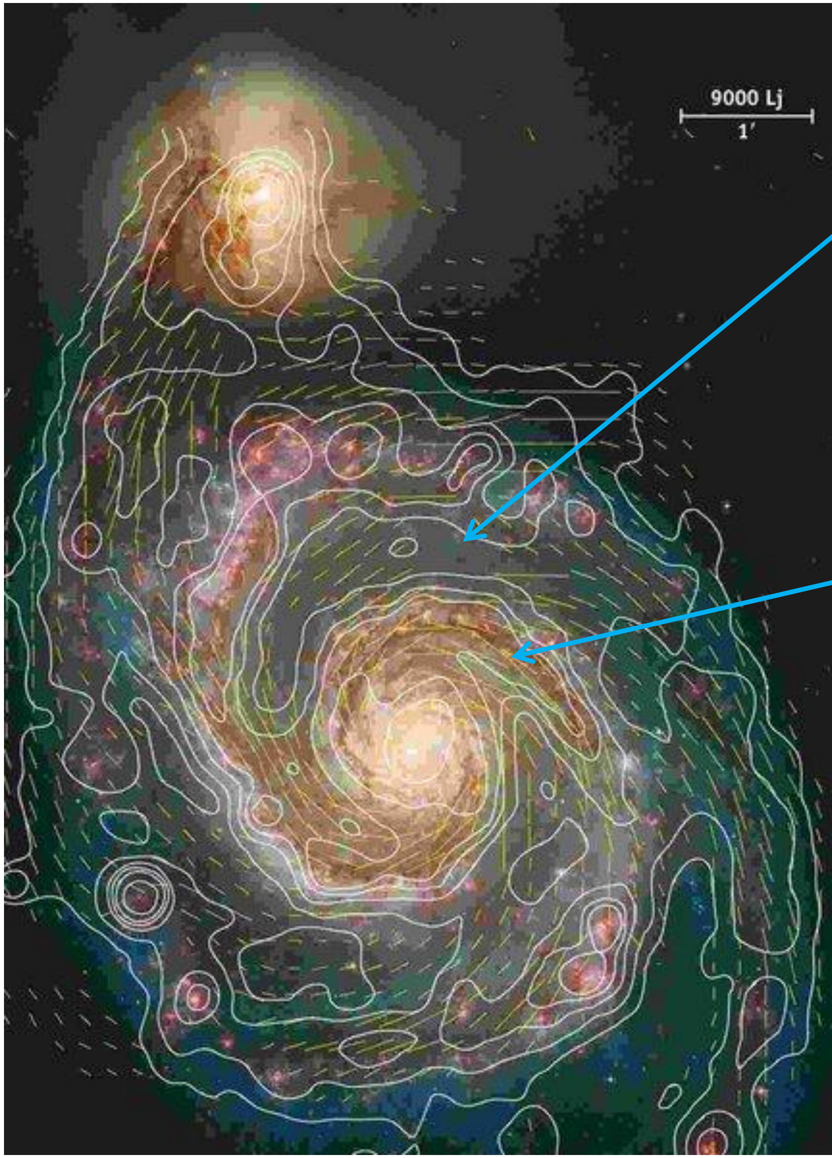


Large scale B field



Small scale B field

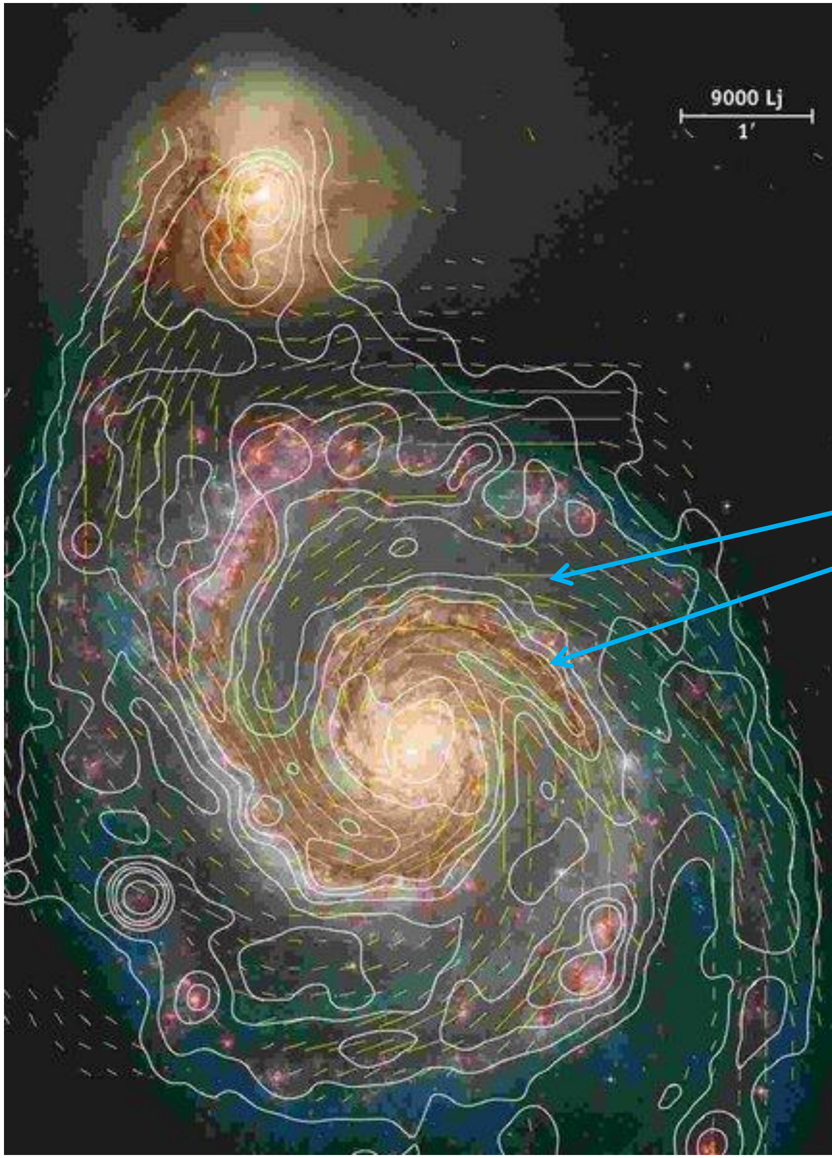
- Large scale B fields and small scale B fields are observed

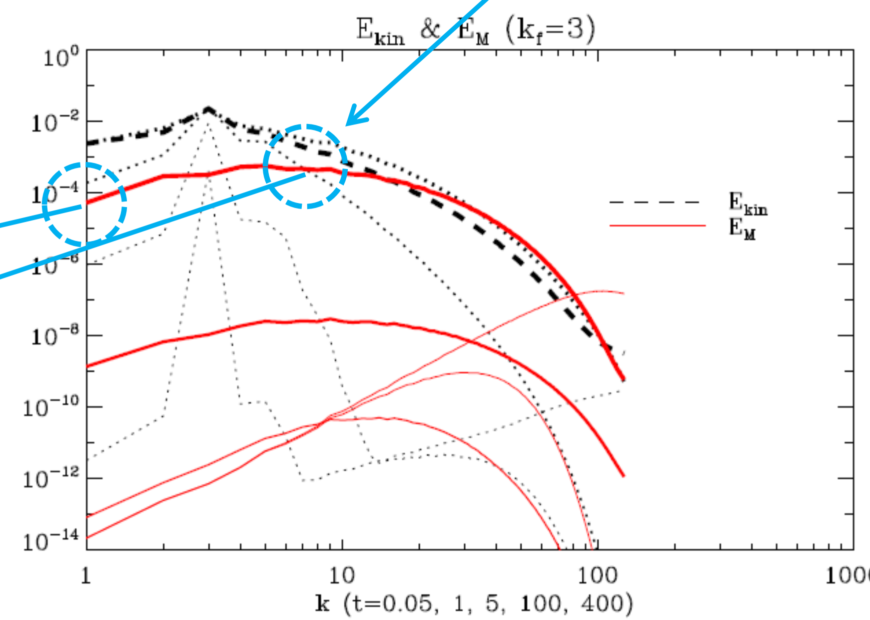


M51, Beck 2011

1. Ordered, regular large scale B-fields (10-15 μG) are observed between the optical spiral arms
2. Random turbulent small scale B fields (> 10-15 μG) are observed in the spiral arm.

Small scale peak





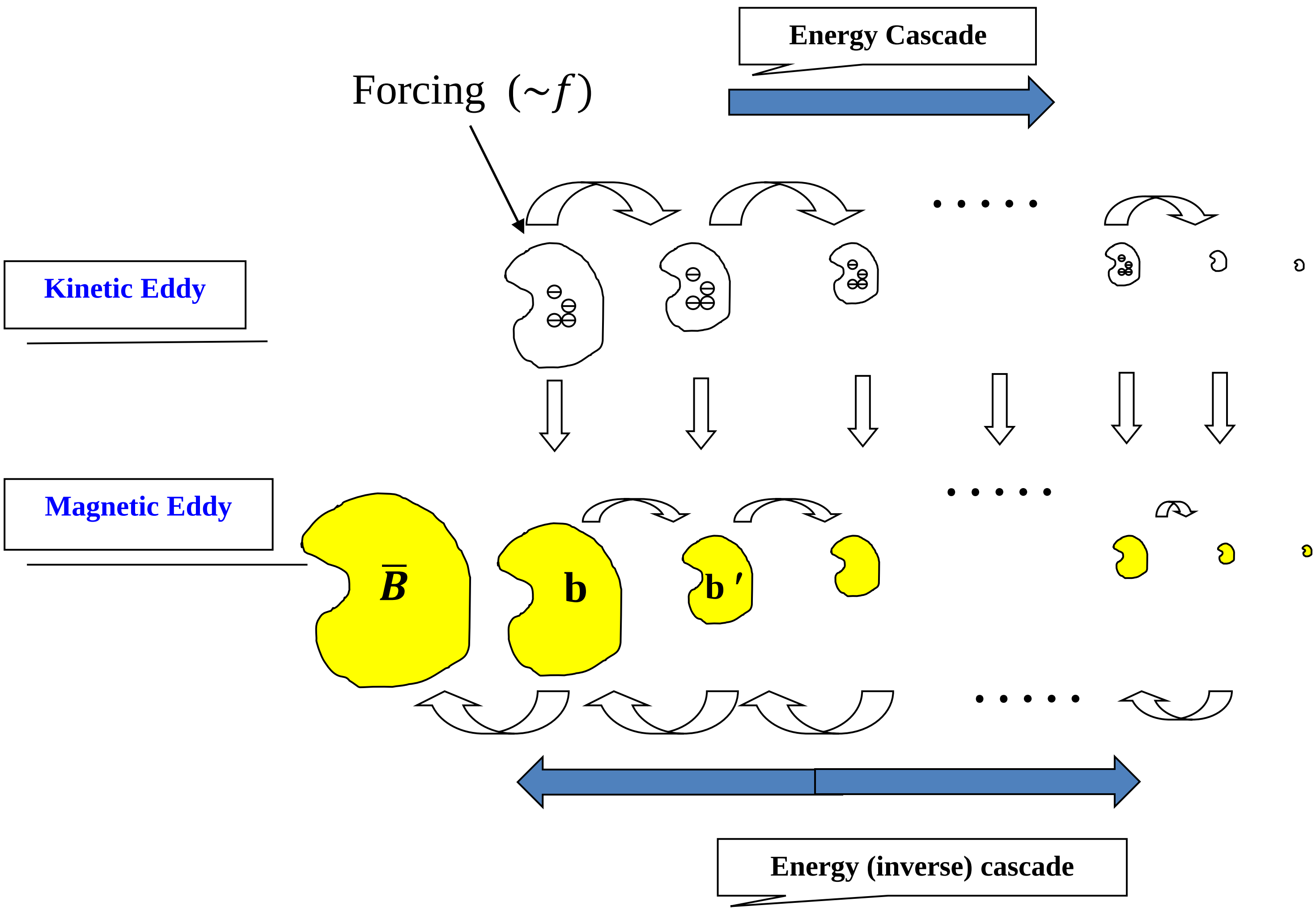
Beck 2011

- **Outline**

1. **Properties of large and small scale dynamo**

2. **Application of magnetic field dynamo to a galaxy (M51)**
   - **Small scale dynamo with high magnetic prandtl number & scaling factor**

▪ Formation of large or small scale B fields.



- **This plasma phenomenon can be explained by MHD equations like**

1. Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0$$

2. Momentum equation

$$\rho \frac{DV}{Dt} = -\nabla P + \frac{1}{c} J \times B + \nu \nabla^2 V \quad \left(\frac{D}{Dt} \equiv \frac{\partial \rho}{\partial t} + V \cdot \nabla \right)$$

3. Magnetic induction Equation

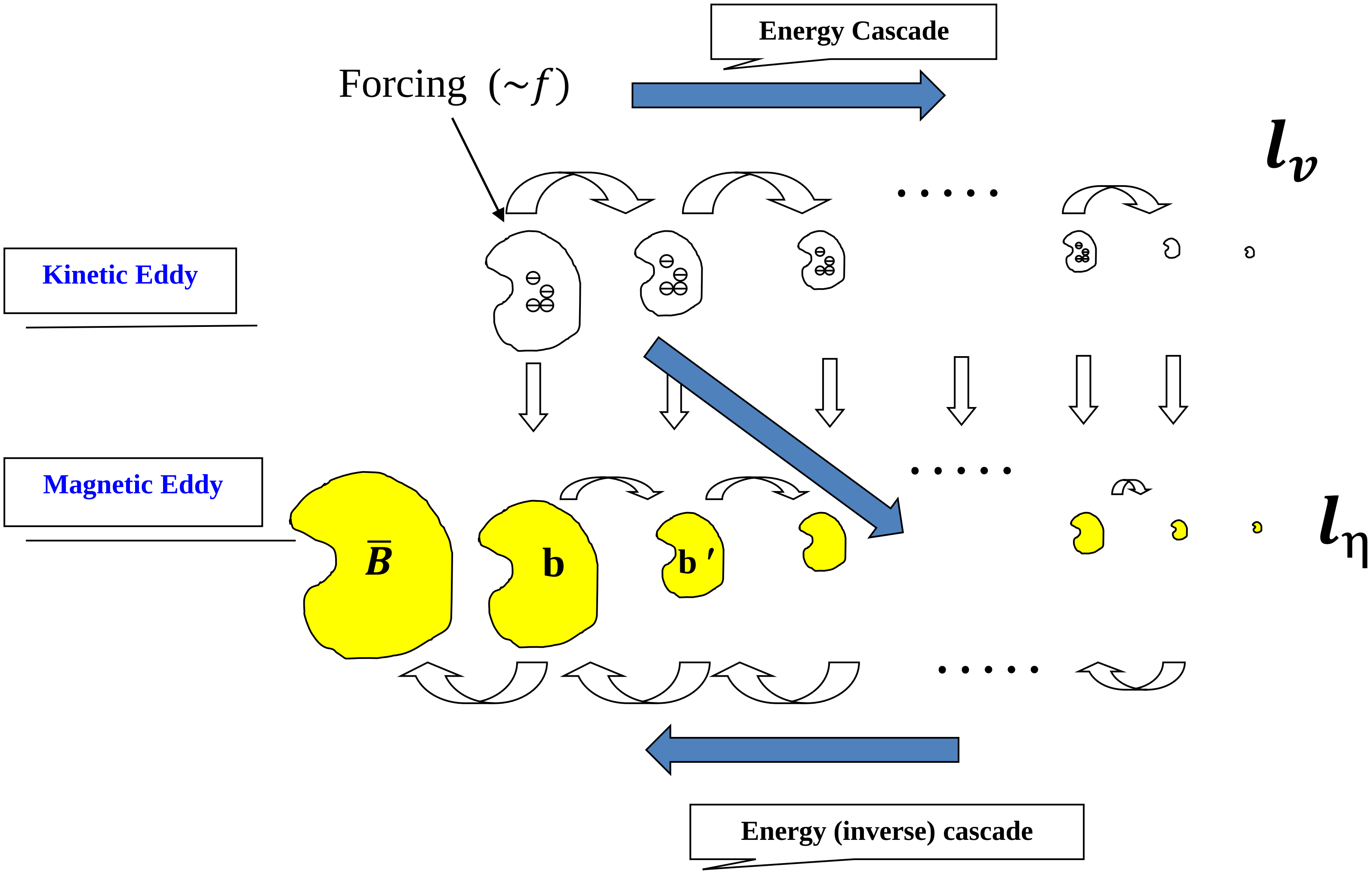
$$\frac{\partial B}{\partial t} = \nabla \times \underbrace{\langle V \times B \rangle}_{\mathrm{EMF}} + \eta \nabla^2 B$$

These Eqs need to be simplified…

A. Large scale B field btw the spiral arms

-If helical velocity(magnetic) field drives a system…

→ Inverse cascade of magnetic energy (ordered anisotropic)



- **To explain the large scale dynamo, we need to derive an analytic large scale $\bar{\boldsymbol{B}}$ field (magnetic induction) equation ☞ 2-scale mean field model**

$\bar{\boldsymbol{B}}$ (large) + $\mathbf{b}$ (fluctuation)

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$\bar{\boldsymbol{V}}$ (large) + $\mathbf{v}$ (fluctuation)

$$\rightarrow \frac{\partial \overline{B}}{\partial t} = \underbrace{\nabla \times \langle v \times b \rangle}_{\text{Source(small)}} + \underbrace{(\eta + \beta)\nabla^2 \overline{B}}_{\text{Diss.}}$$

- **Substitution of small scale v and b in $\langle \boldsymbol{v} \times \boldsymbol{b} \rangle$ using momentum and induction equation. And then use the vector identity**

$$\frac{\partial \overline{B}}{\partial t} = \nabla \times \langle v \times b \rangle + (\eta + \beta)\nabla^2 \overline{B}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} \sim \langle \overline{\mathrm{J}} \times \mathrm{b} + \mathrm{j} \times \overline{\mathrm{B}} \rangle \qquad \frac{\mathrm{db}}{\mathrm{dt}} \sim \nabla \times \langle \mathrm{v} \times \overline{\mathrm{B}} \rangle$$

$$\rightarrow \frac{\partial \overline{\mathrm{B}}}{\partial \mathrm{t}} = \underbrace{\nabla \times \alpha \overline{\mathrm{B}}}_{\text{Source}} + \underbrace{(\beta + \eta)\nabla^2 \overline{\mathrm{B}}}_{\text{Dissipation}}$$

$\boldsymbol{\alpha}$ has nontrivial value only when the fields v and b are helical ($\nabla\times\boldsymbol{v}\sim\boldsymbol{v}$ *or* $\nabla\times\boldsymbol{b}\sim\boldsymbol{b}$)

$$\begin{cases} \alpha \sim \int d\tau \big(\langle j \cdot b \rangle - \langle v \cdot \omega \rangle \big), \\ \beta \sim \int d\tau \langle v^2 \rangle \end{cases}$$

Kinetic helicity $\langle \boldsymbol{v} \cdot \boldsymbol{\omega} \rangle$

Magnetic helicity $\langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle$

Current helicity $\langle \boldsymbol{j} \cdot \boldsymbol{b} \rangle = \boldsymbol{k}^2 \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle$

- **Magnetic Helicity $\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$**

1. Conserved variable in the system

$$\frac{\boldsymbol{dH_M}}{\boldsymbol{dt}} = 0 \quad \rightarrow \langle \overline{\boldsymbol{A}} \cdot \overline{\boldsymbol{B}} \rangle + \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle = 0$$

2. Topological property of Magnetic Flux tube $\Phi_1 \cdot \Phi_2$ ($\Phi = \oint B \cdot dS$)

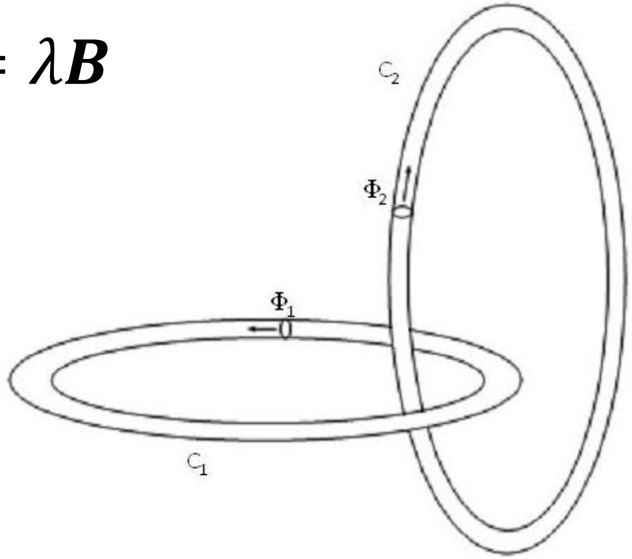
   - Wrapping around magnetic field itself, linking number

3. Minimum energy state

$$\int \left(\frac{v^2 + b^2}{2} - \frac{\lambda}{2} \boldsymbol{A} \cdot \boldsymbol{B} \right) dV = 0 \rightarrow \nabla \times \boldsymbol{B} = \lambda \boldsymbol{B}$$

4. Information on the magnetic structure

   - key to the solar dynamo



- **E_M(t), H_M(t) equation?**
  - **Scalar product of $\overline{A}$ and $\partial\overline{B}/\partial$t, $\overline{B}$ and $\partial\overline{B}/\partial$t**

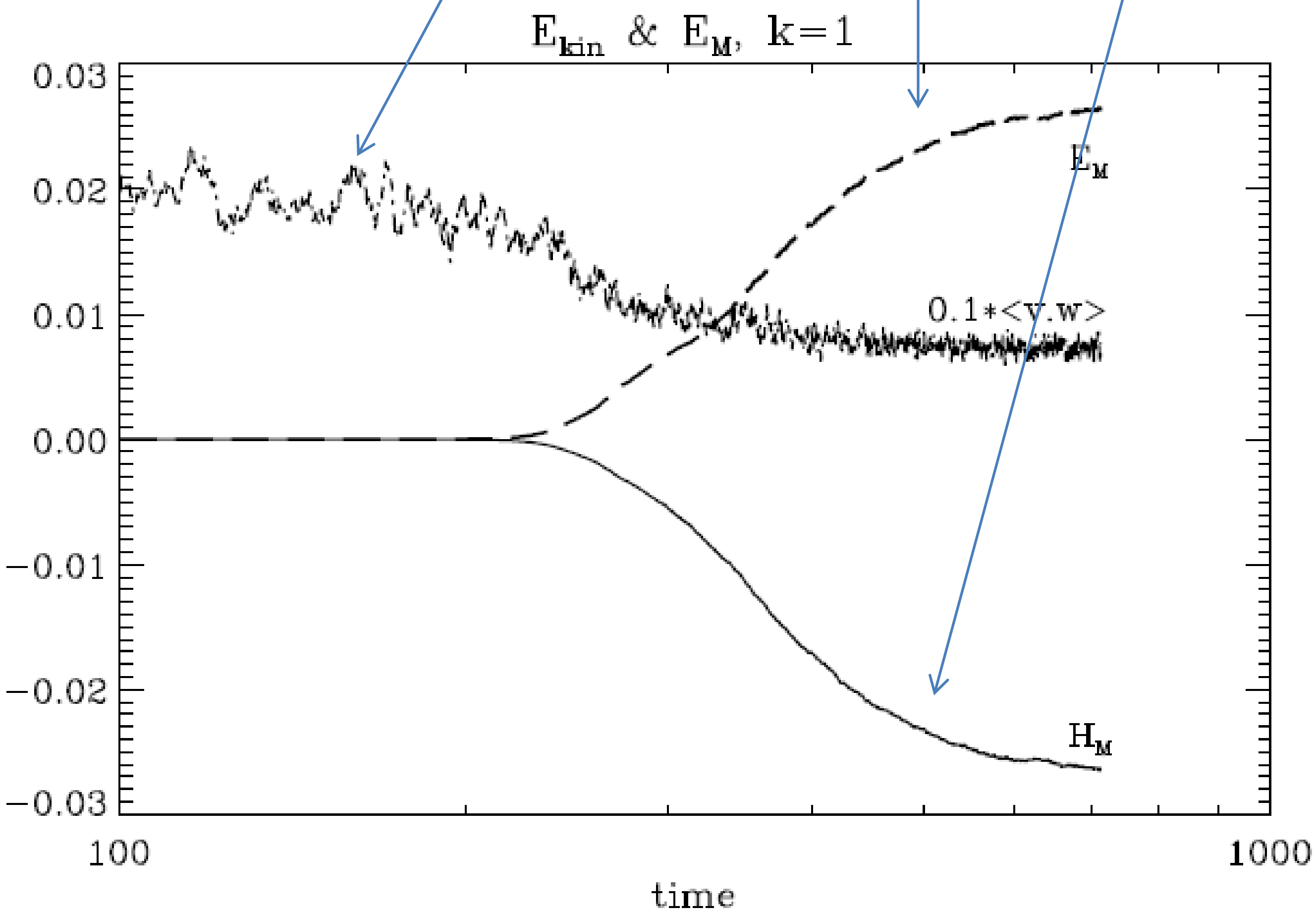
$$\overline{A}\cdot\left(\frac{\partial\overline{B}}{\partial t}=\nabla\times\alpha\overline{B}+(\beta+\eta)\nabla^2\overline{B}\right),\quad \overline{B}\cdot\left(\frac{\partial\overline{B}}{\partial t}=\nabla\times\alpha\overline{B}+(\beta+\eta)\nabla^2\overline{B}\right)$$

$$\Rightarrow\begin{cases} 2H_{M,L}=\underbrace{\left(H_{M0}+2E_{M0}\right)e^{2\int(\alpha-\beta-\eta)d\tau}}_{2}-\underbrace{\left(2E_{M0}-H_{M0}\right)e^{-2\int(\alpha-\beta-\eta)d\tau}}_{1} \\ 4E_{M,L}=\underbrace{\left(H_{M0}+2E_{M0}\right)e^{2\int(\alpha-\beta-\eta)d\tau}}_{2}+\underbrace{\left(2E_{M0}-H_{M0}\right)e^{-2\int(\alpha-\beta-\eta)d\tau}}_{1} \end{cases}$$

1. $\langle v\cdot\omega\rangle > 0:\ \int(\alpha-\beta-\eta)\,d\tau < 0 \Rightarrow H_{M,L}<0, E_{M,L}>0$
2. $\langle v\cdot\omega\rangle < 0:\ \int(\alpha-\beta-\eta)\,d\tau > 0 \Rightarrow H_{M,L}>0, E_{M,L}>0$

$$\alpha\sim\int d\tau\left(\langle j\cdot b\rangle-\langle v\cdot\omega\rangle\right)$$

If a system is driven by $\langle v \cdot \omega \rangle > 0$, $E_{M,L} > 0$, $H_{M,L} < 0$



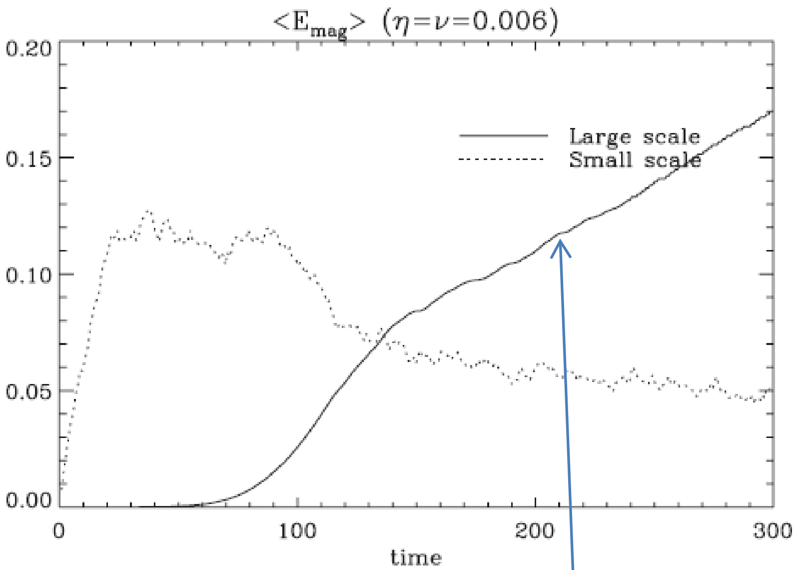
- **What if helical EM wave forces magnetic eddy?**

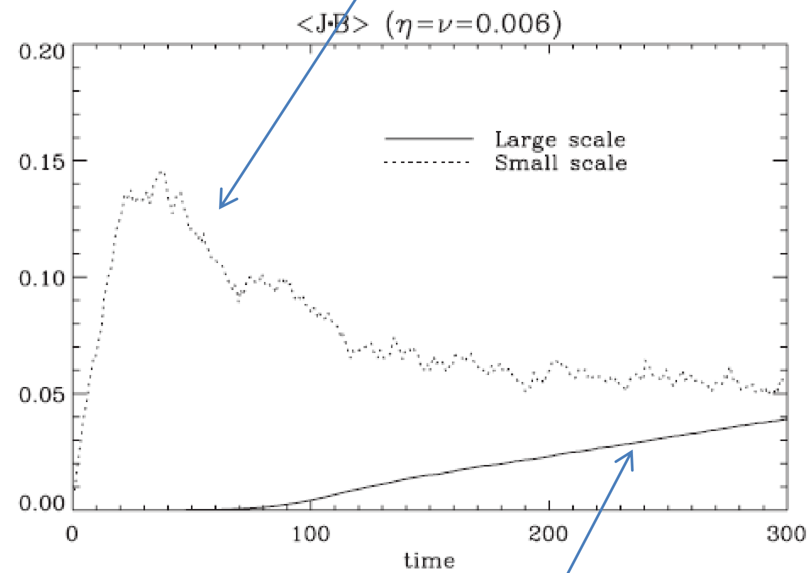
$$\Rightarrow \begin{cases} 2H_{M,L} = \underbrace{\left(H_{M0} + 2E_{M0}\right)e^{2\int(\alpha-\beta-\eta)d\tau}}_{2} - \underbrace{\left(2E_{M0} - H_{M0}\right)e^{-2\int(\alpha-\beta-\eta)d\tau}}_{1} \\ 4E_{M,L} = \underbrace{\left(H_{M0} + 2E_{M0}\right)e^{2\int(\alpha-\beta-\eta)d\tau}}_{2} + \underbrace{\left(2E_{M0} - H_{M0}\right)e^{-2\int(\alpha-\beta-\eta)d\tau}}_{1} \end{cases}$$

1. $\langle j \cdot b \rangle > 0 : \int(\alpha - \beta - \eta)d\tau > 0 \Rightarrow H_{M,L} > 0, E_{M,L} > 0$
2. $\langle j \cdot b \rangle < 0 : \int(\alpha - \beta - \eta)d\tau < 0 \Rightarrow H_{M,L} < 0, E_{M,L} > 0$

$$\alpha \sim \int d\tau \left(\langle j \cdot b \rangle - \langle v \cdot \omega \rangle\right)$$

$$\langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle > 0$$





$$\boldsymbol{E_{M,L}} > 0$$

$$\boldsymbol{H_{M,L}} > 0$$

■ Question…

Does small scale ⟨a·b⟩ suppress large scale dynamo?

- positive $\langle \mathrm{v} \cdot \nabla \times \mathrm{v} \rangle$ → negative $\langle \bar{\boldsymbol{A}} \cdot \bar{\boldsymbol{B}} \rangle$ in large scale

→ positive $\langle \mathrm{j} \cdot \mathrm{b} \rangle$ in small scale → decreasing $|\boldsymbol{\alpha}|$

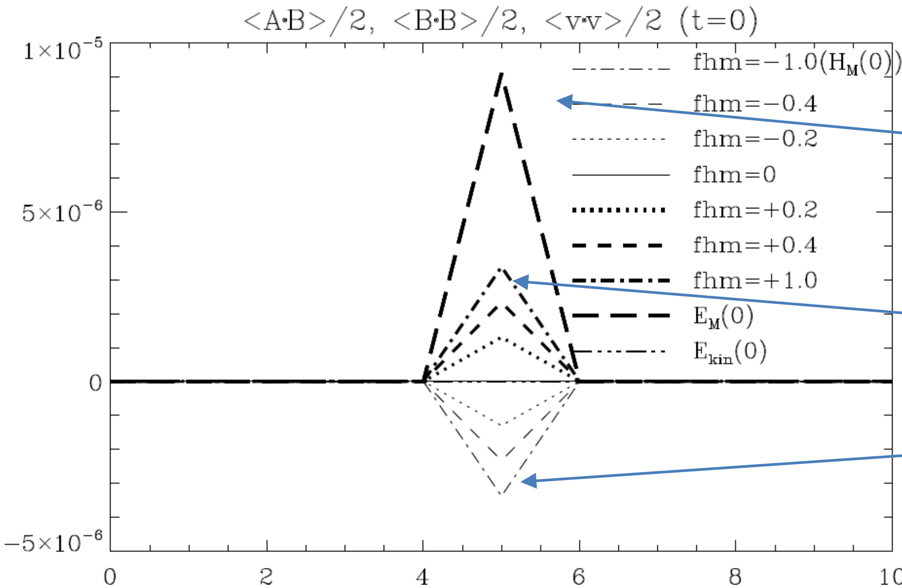
→ conservation of $\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$ and its redistribution

$$\frac{\partial \overline{\mathrm{B}}}{\partial \mathrm{t}} = \nabla \times \alpha \overline{\mathrm{B}} + (\beta + \eta) \nabla^2 \overline{\mathrm{B}}$$

$$|\boldsymbol{\alpha}| \sim \left| \boldsymbol{k}^2 \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle - \langle \mathbf{v} \cdot \boldsymbol{\nabla} \times \boldsymbol{v} \rangle \right| \boldsymbol{\tau}$$

☞ Shukurov et al. 2006 found the removal of $\langle a \cdot b \rangle$ in small scale enhances MHD dynamo.

But pre-existing $\langle a \cdot b \rangle$ also boosts to MHD dynamo.



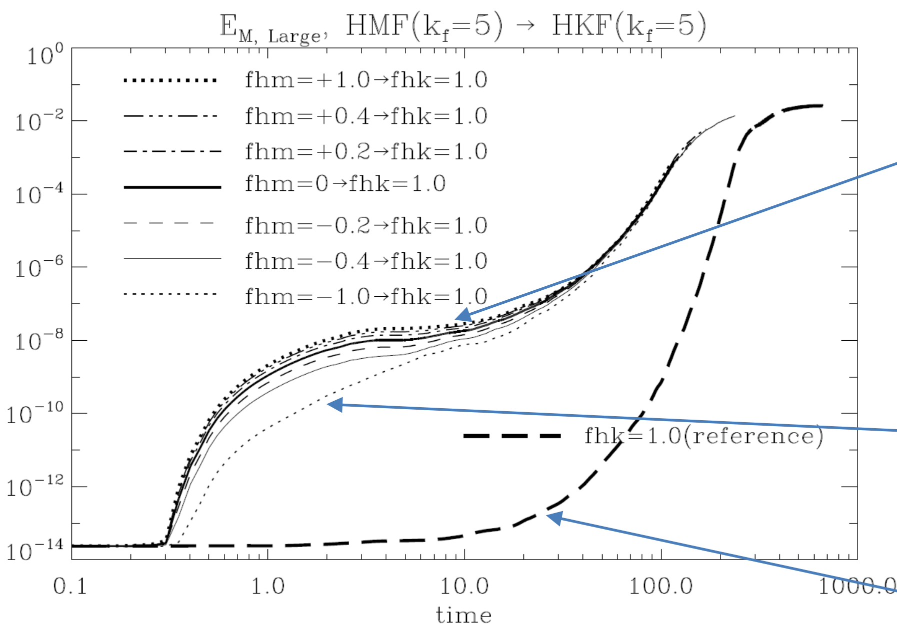
Large scale

small scale

1.8×10^{-5} (E_M, t=0)

$+3.4\times10^{-6}$ (H_M, t=0)

-3.4×10^{-6} (H_M, t=0)



$+3.4\times10^{-6}$ (H_M, t=0)

-3.4×10^{-6} (H_M, t=0)

0 (E_M, t=0)

Park. 2014

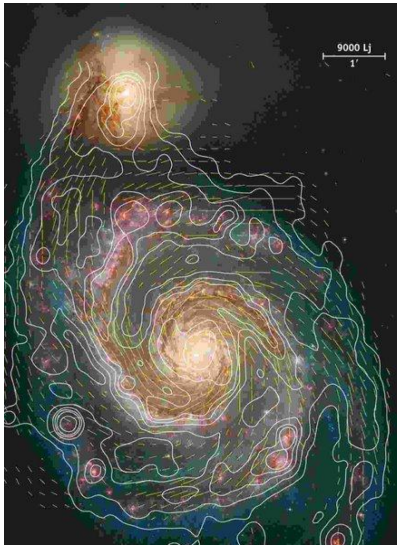
- Pre existing $\boldsymbol{b_0}$ field $\sim \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle$ plays the role of source in the system and modifies $\alpha\overline{\boldsymbol{B}}$ in induction equation.

$$
\begin{aligned}
|\alpha|\overline{\boldsymbol{B}} &\sim (\langle \boldsymbol{j} \cdot \boldsymbol{b} \rangle - \langle \mathbf{v} \cdot \nabla \times \boldsymbol{v} \rangle)\boldsymbol{\tau}\overline{\boldsymbol{B}} \\
&\rightarrow (\langle \boldsymbol{j} \cdot \boldsymbol{b} \rangle - \langle \mathbf{v} \cdot \nabla \times \boldsymbol{v} \rangle)\boldsymbol{\tau}\left(\overline{\boldsymbol{B}} + \boldsymbol{b_0} \boldsymbol{e}^{-\eta \boldsymbol{k}^2 \boldsymbol{t}}\right)
\end{aligned}
$$

- But more fundamentally related with the conservation of Magnetic energy and Magnetic helicity.

B. Magnetic fields in the optical spiral arms?

1. In the galaxy, helicity ratio is not high (5~15%).
Random, nonhelical small scale B fields exist
in the optical spiral arms.
→ Small scale dynamo, forward cascade of E_M

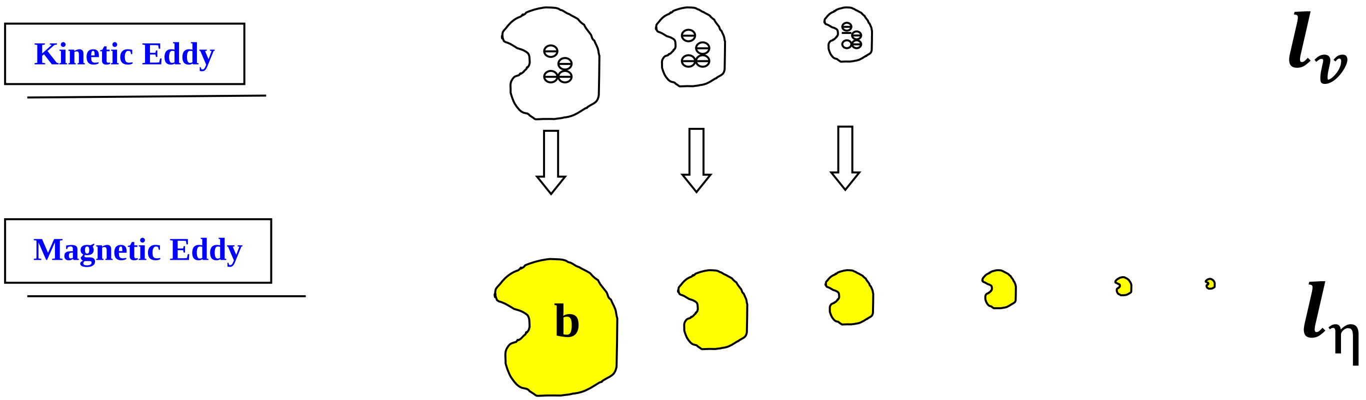


2. Also diffusivity scale is much smaller than viscous scale: $l_v \gg l_\eta$. Magnetic Prandtl number is very large.

$$Pr_M \equiv \frac{\nu}{\eta} = \left(\frac{l_v}{l_\eta}\right)^2 \gg 1$$

3. Magnetic diffusivity η in the induction equation can be neglected.

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B)$$



$$l_v \gg l_\eta$$

4. Non helical B field makes Lorentz force $J \times B$ dominant. → Strong correlation between V and B

$$\rho \frac{DV}{Dt} = -\nabla P + \frac{1}{c} J \times B + \nu \nabla^2 V$$

4a. Growing B fields constrain the motion of plasma through the magnetic pressure $-\nabla B^2/2$,

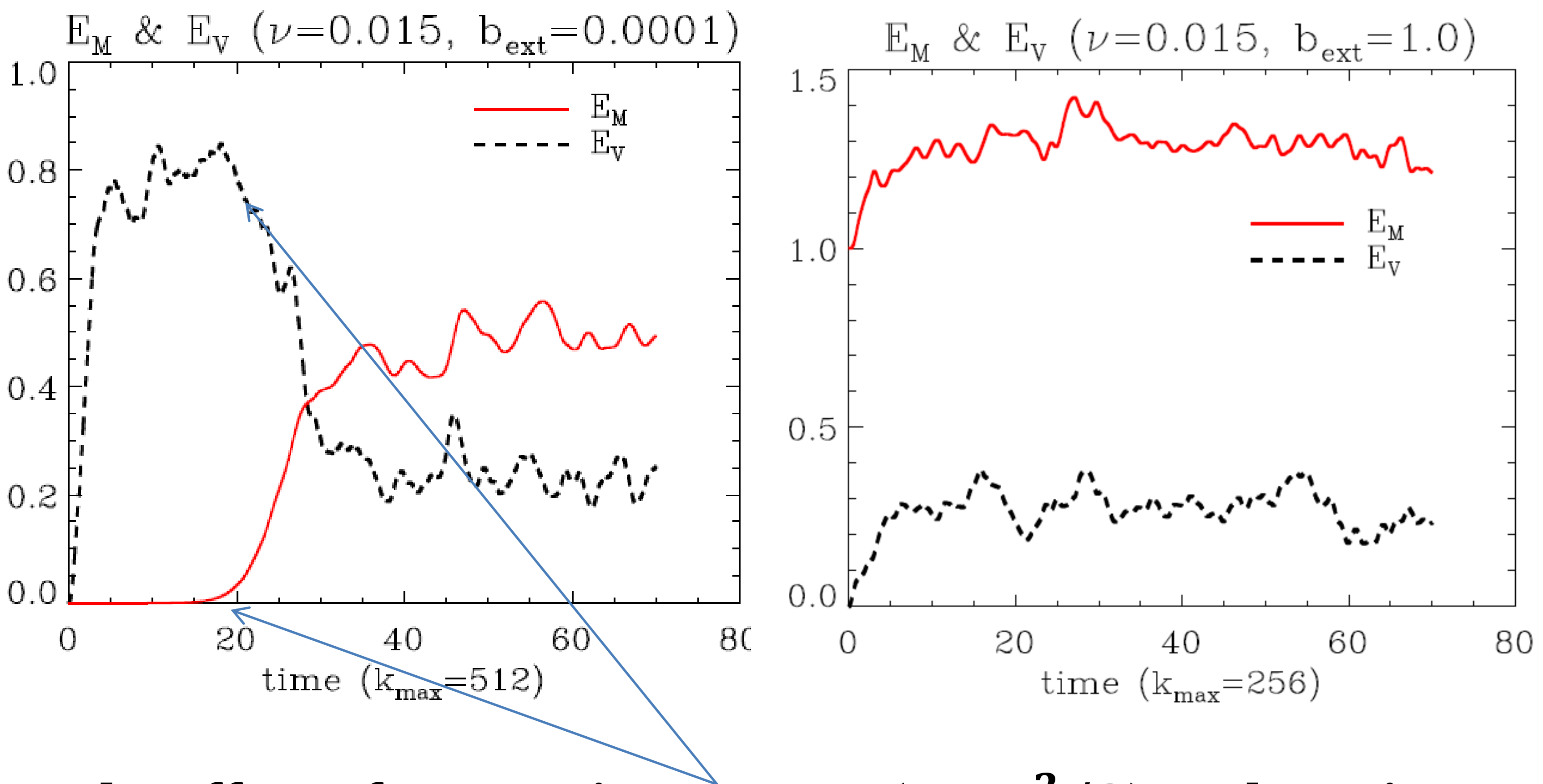
$$J \times B = -\frac{1}{2} \nabla B^2 + \mathrm{B} \cdot \nabla B$$

4b. Kinetic energy can be transferred to the magnetic eddy through $\mathrm{B} \cdot \nabla V$ increases.

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle = B \cdot \nabla \mathrm{V} - \mathrm{V} \cdot \nabla B$$

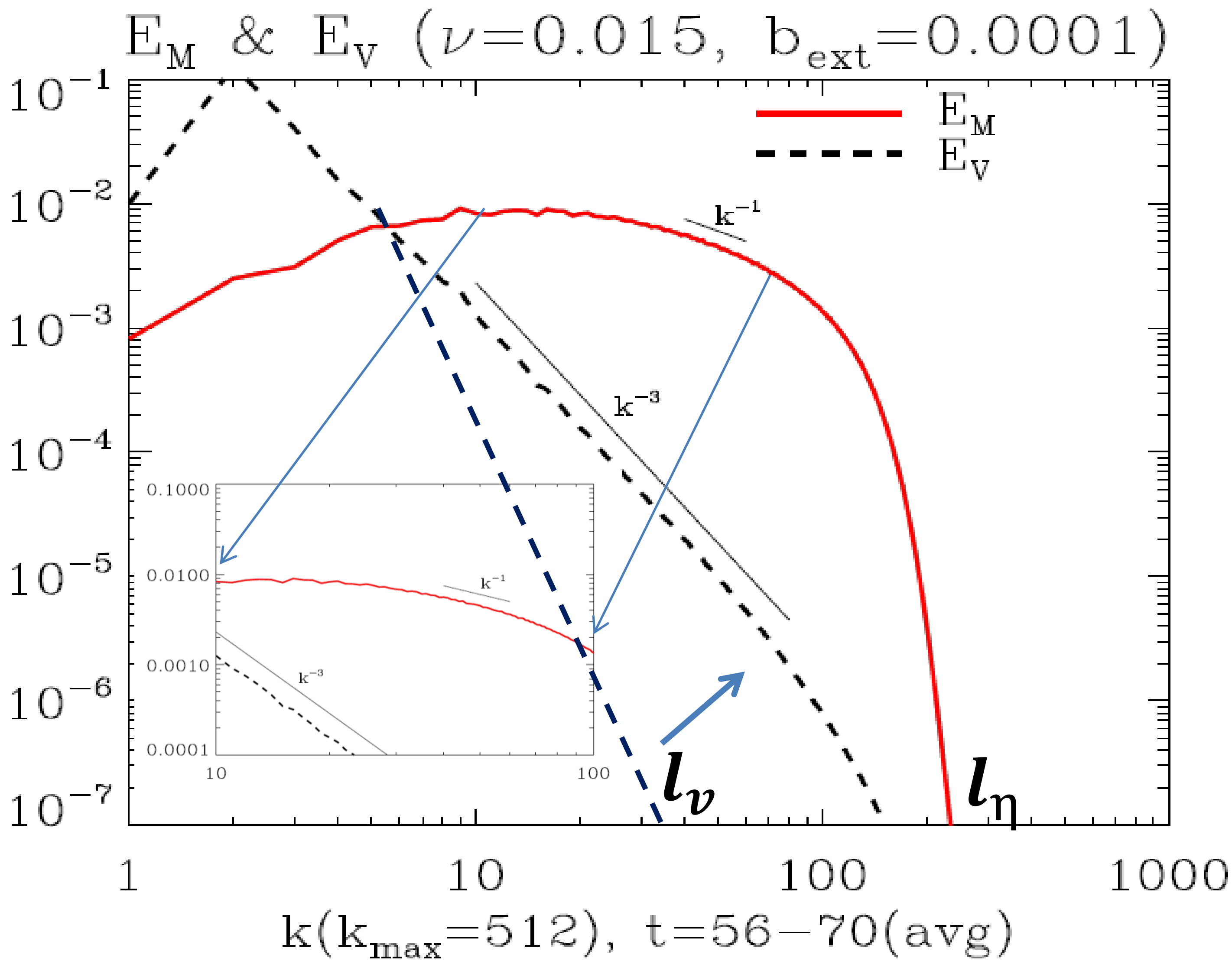
4c. In the subviscous regime, nonlocal energy transfer from the magnetic eddy to kinetic eddy increases through the magnetic tension $\mathrm{B} \cdot \nabla B$.

Results: $E_V(t)$ $vs.$ $E_M(t)$



The effect of magnetic pressure $(-\nabla B^2/2)$ and tension $(B \cdot \nabla B)$ is conspicuous with weak B_{ext}(0.0001).

5. Magnetic energy in subviscous regime is transferred to the kinetic eddies in the regime, which extends the viscous scale toward the resistivity scale.



6. In this scale, magnetic tension is balanced with dissipation effect.

$$B \cdot \nabla B \sim \nu \nabla^2 V$$

6a. In addition, magnetic energy transfer rate in stationary state is independent of scale.

$$\frac{E_M}{\tau_{cas}} = const$$

6b. From these two conditions, we obtain

$$\begin{cases} Weak\ B_{ext}\colon E_V = k^{-3}, E_M = k^{-1} \\ Strong\ B_{ext}\colon E_V = k^{-4}, E_M = k^{-1/2} \end{cases}$$

6c. The results are from the coupling between kinetic energy and magnetic energy. Kolmogorov's scaling factor is not valid anymore.

Summary

1. MHD turbulence dynamo sustains and amplifies B field in space.
2. Helical field cascades energy toward large scale, but nonhelical kinetic and magnetic field send magnetic energy toward smaller scale regime.
3. The effects of large scale dynamo and small scale dynamo in galaxy and ISM are seen in a mixed state.
4. When resistivity 'η' is an ignorably small value (high Pr_M), E_M is transferred to the kinetic eddies in the subviscous scale so that the viscous scale is extended to the resistivity scale.
5. Balance relation and stationary energy transfer rate explain the scaling factors of E_M and E_V in subviscous scale regime.