November 12th, 2014

8th KAW, Jeju Island, Korea

Whistler Waves and Electron Acceleration/Heating in the Planetary Magnetospheres and Interplanetary Medium

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1. Whistler Waves

- *N*: index of refraction,

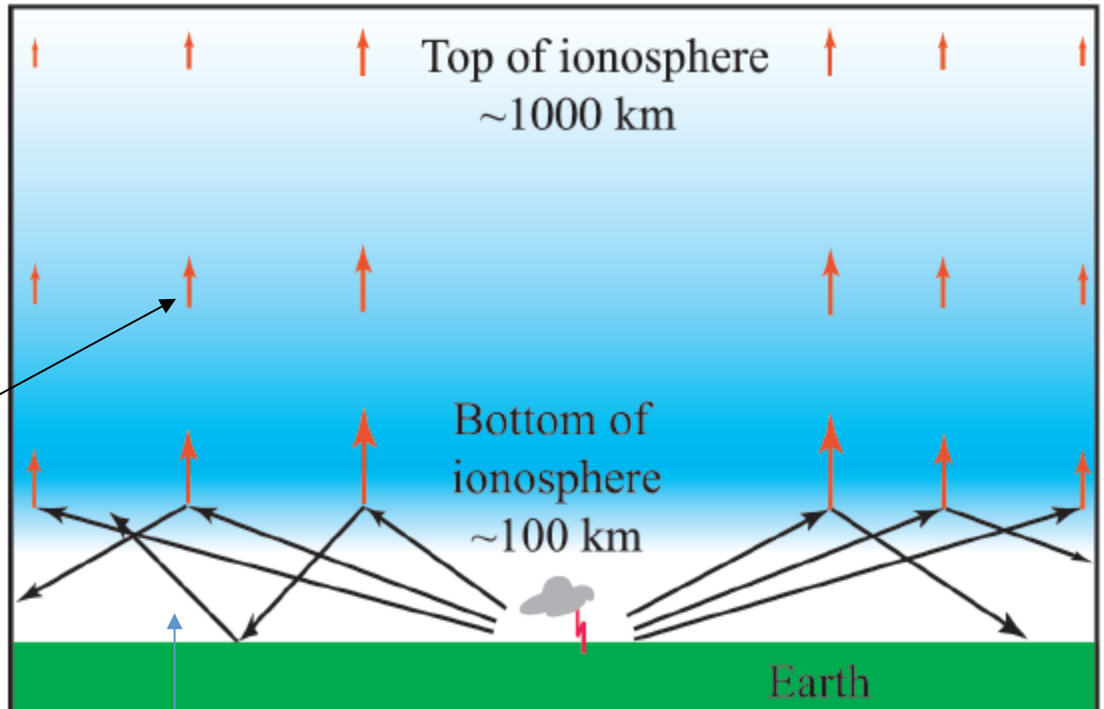
$$N = \frac{ck}{\omega}$$

- For vacuum, *N* = 1
- For whistler waves

$$N^2 = 1 - \frac{\omega_{pe}^2}{\omega(\omega - \Omega_{ce}\cos\theta)}$$

2. Atmospheric Whistler Wave Generation

Part of the radiation is launched into the magneto-sphere as a **whistler wave**.

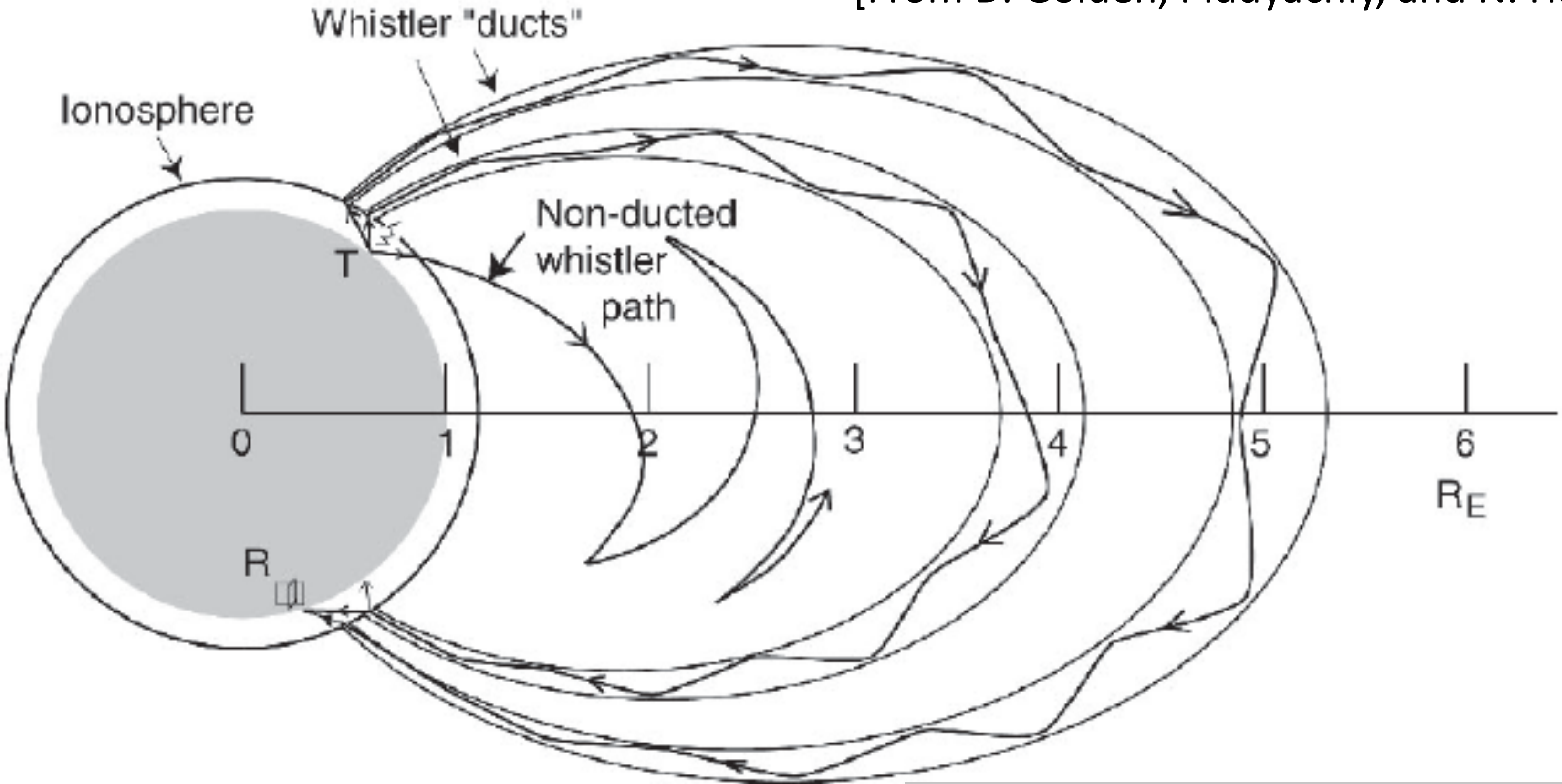


ELF/VLF range (300 Hz - 30kHz)

EM waves emitted by lightning discharge.

[From D. Golden, Piddyachiy, and N. Haque]

[From D. Golden, Piddyachiy, and N. Haque]



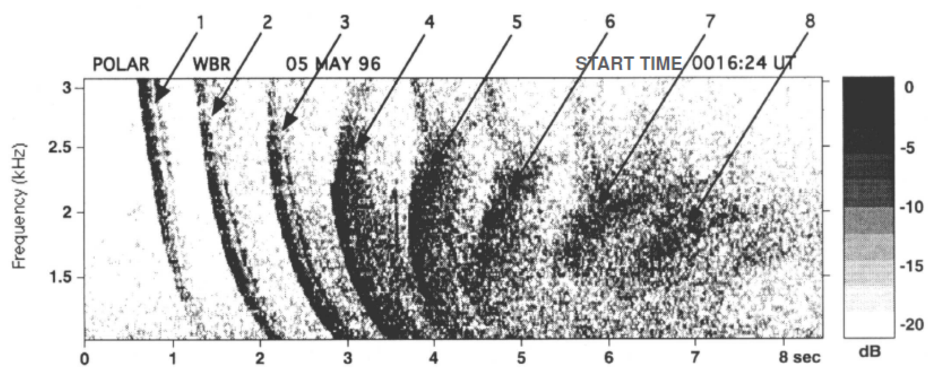
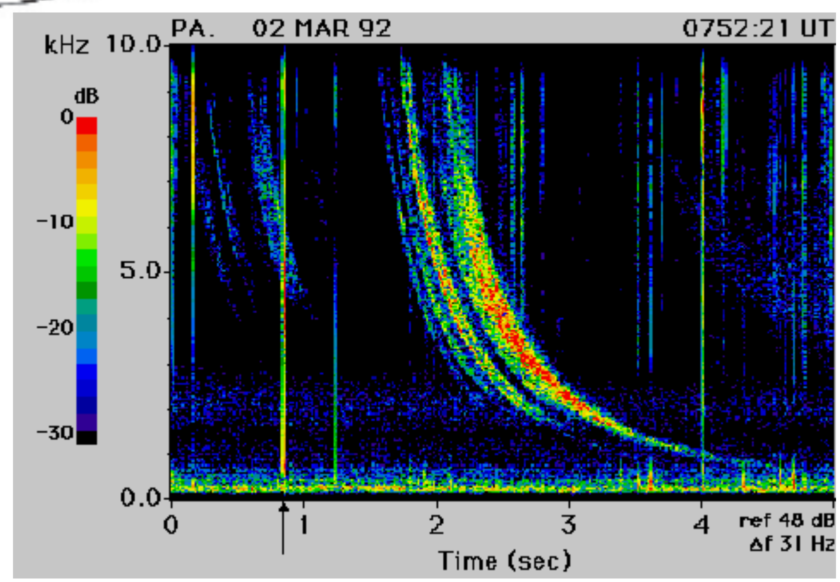
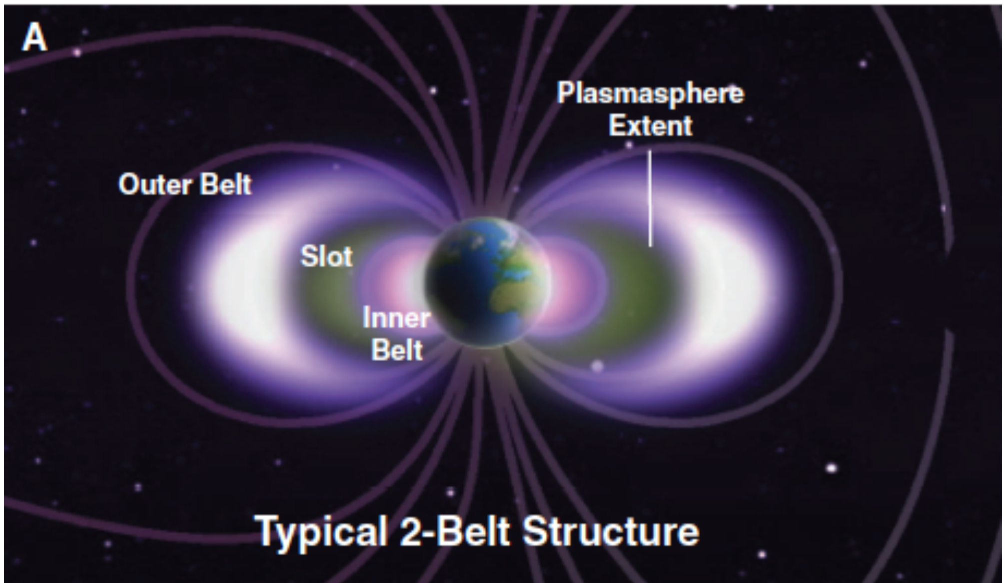


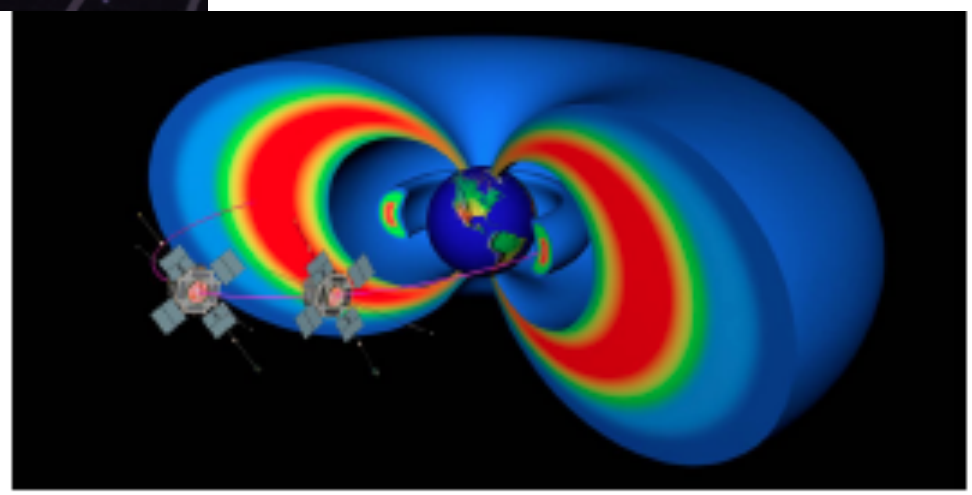
Figure 1.1: MR whistler observed aboard the POLAR satellite in the 8 sec interval starting at 0016:24 UT on May 5th 1996, showing 8 distinct components.



3. Whistler Waves in Radiation Belt

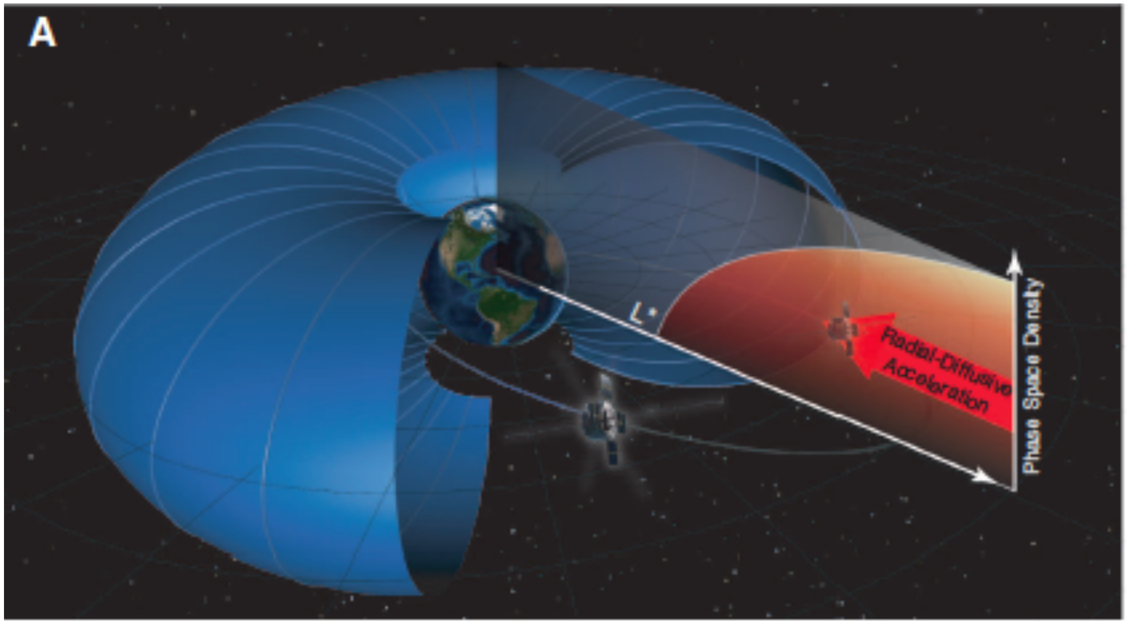


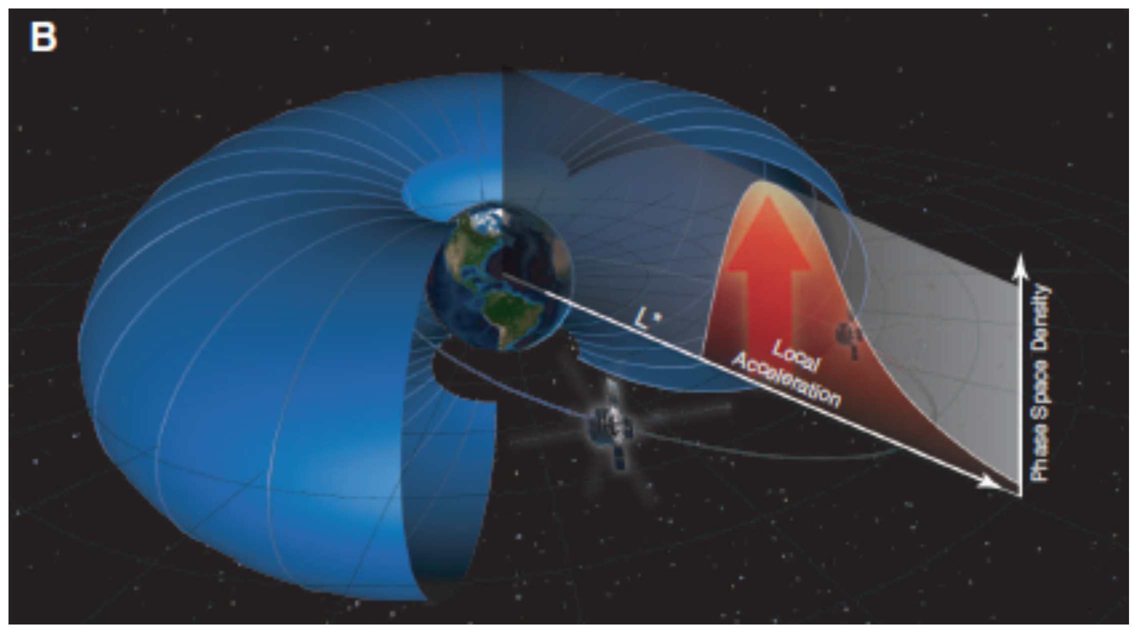
From Baker et al.]



[Van Allen Probes]

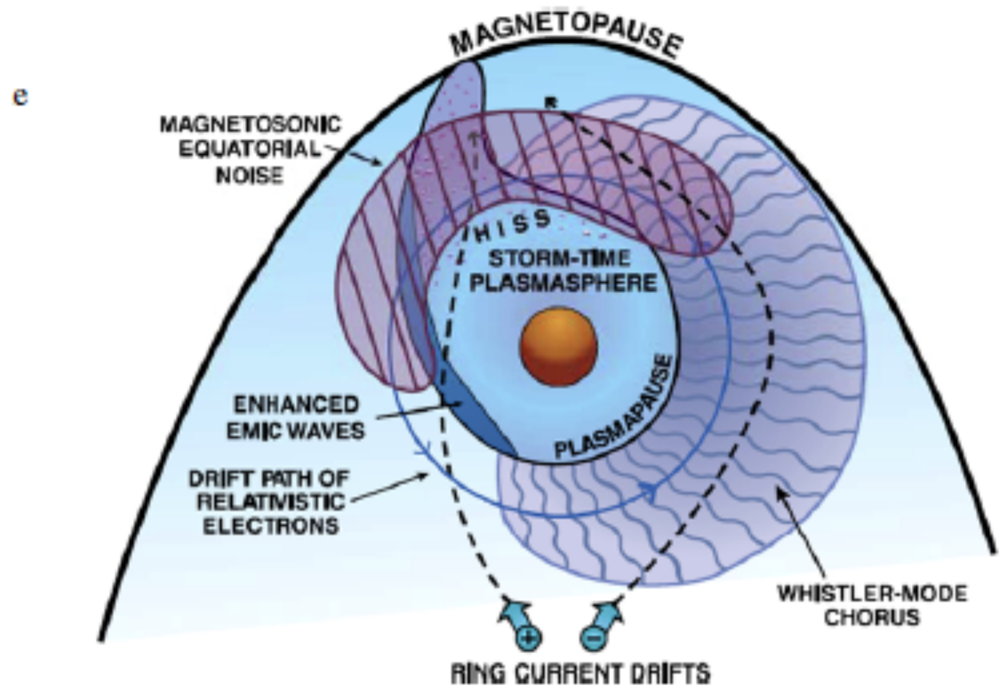
Fig 1. Radiation Belt Storm Probes Mission (RBSP)





[From Reeves et al.]

4. Whistler Wave Generation in Radiation Belt



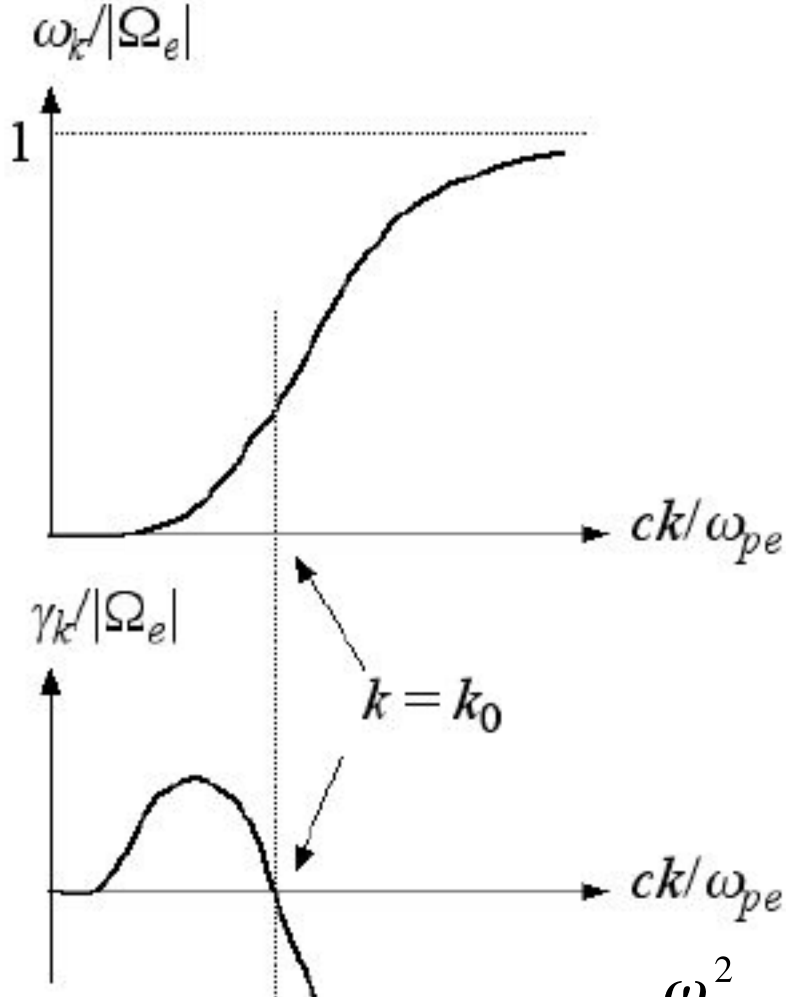
Perpendicular free energy source in the ions -> EM ion-cyclotron wave instability

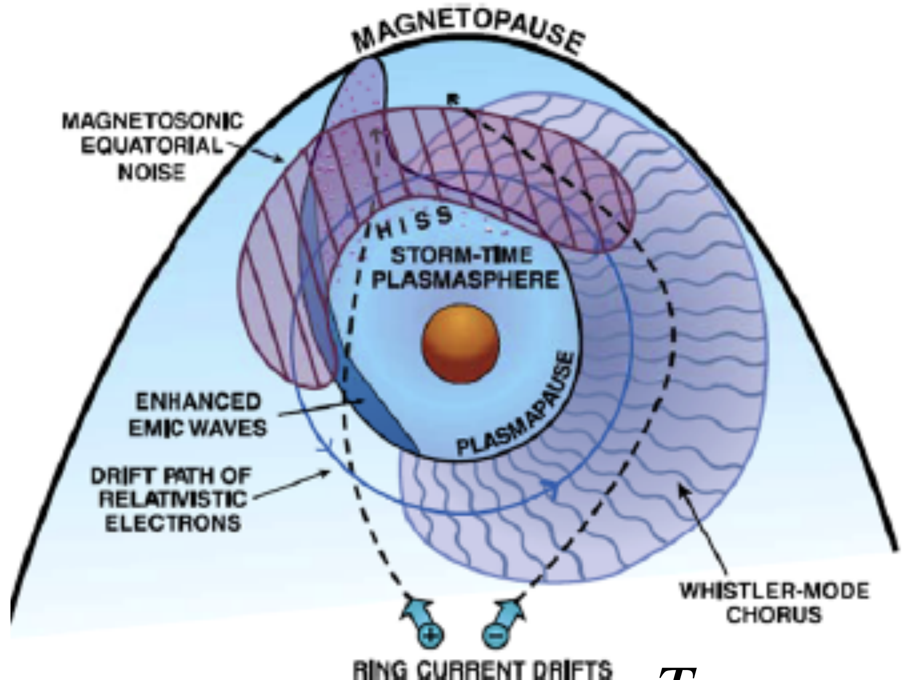
$$\frac{T_{\perp i}}{T_{\parallel i}} > 1$$

Perpendicular free energy source in the electrons -> whistler wave instability

$$\frac{T_{\perp e}}{T_{\parallel e}} > 1$$

- *Growth rate for whistler instability*





$$\frac{T_{\perp e}}{T_{\parallel e}} > 1$$

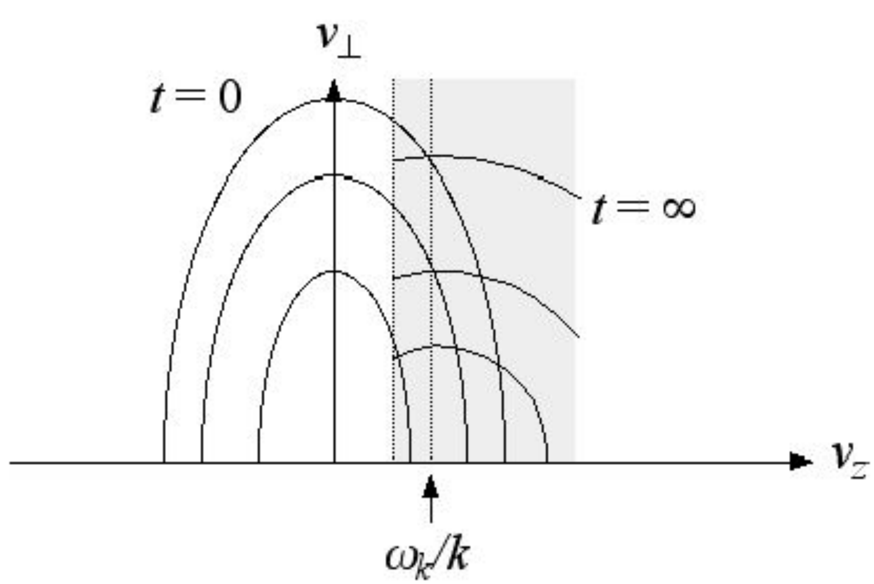
$$\gamma = \pi \frac{\omega_{pe}^2}{\omega^2} \int d\mathbf{v} \delta(\omega - kv_\parallel - \Omega_{ce}) \left(\Omega_{ce} f - \frac{k v_\perp^2}{2} \frac{\partial f}{\partial v_\parallel} \right)$$

5. Electron Acceleration by Whistler Wave

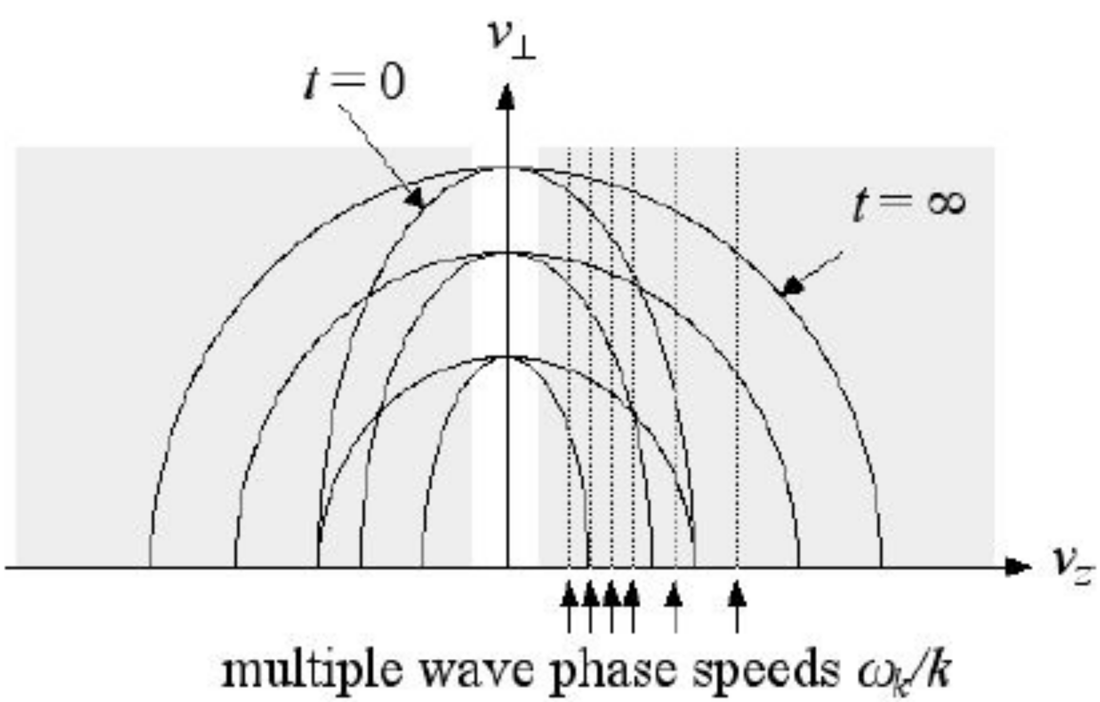
- *Wave-induced diffusion equation*

$$\frac{\partial f}{\partial t} = \frac{ie^2}{m_e^2}\int dk \frac{k^2}{|\omega|^2}\left[\left(\frac{\omega^*}{k} - v_\parallel\right)\frac{\partial}{v_\perp \partial v_\perp} + \frac{\partial}{\partial v_\parallel}\right]$$

$$\times \frac{v_\perp^2 |\delta E_k^2|}{\omega - kv_\parallel - \Omega_{ce}}\left[\left(\frac{\omega}{k} - v_\parallel\right)\frac{\partial f}{v_\perp \partial v_\perp} + \frac{\partial f}{\partial v_\parallel}\right]$$

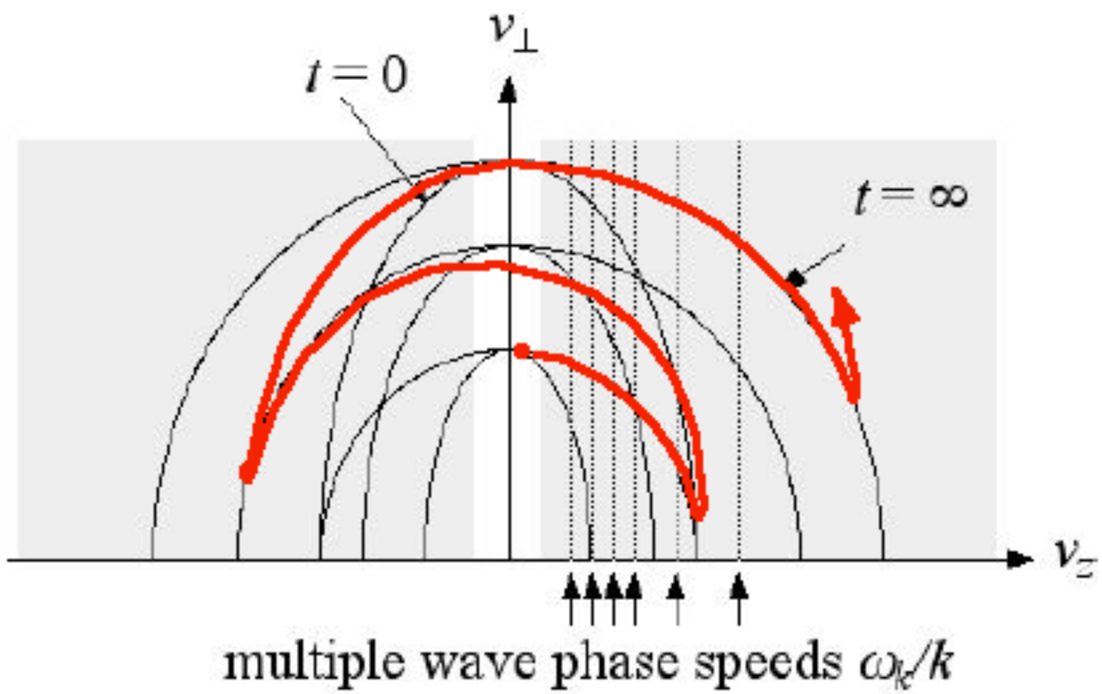


- *Electron acceleration by whistler waves through* ***resonant*** *wave-particle interaction*



Pitch-angle diffusion: Responsible for loss of radiation belt electrons

Energy diffusion: Responsible for accelerating electrons of mild energy level to relativistic levels.



$$\frac{\partial f}{\partial t} = \frac{ie^2}{m_e^2}\int dk\left[\left(1-\frac{kv_\parallel}{\omega^*}\right)\frac{\partial}{v_\perp \partial v_\perp}+\frac{k}{\omega^*}\frac{\partial}{\partial v_\parallel}\right]$$

$$\times\frac{v_\perp^2\,|\,\delta E_k^2\,|}{\omega - kv_\parallel - \Omega_{ce}}\left[\left(1-\frac{kv_\parallel}{\omega}\right)\frac{\partial f}{v_\perp \partial v_\perp}+\frac{k}{\omega}\frac{\partial f}{\partial v_\parallel}\right]$$

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\left(D_{vv}\frac{\partial f}{\partial v}-\frac{(1-\mu^2)^{1/2}}{v}D_{v\mu}\frac{\partial f}{\partial \mu}\right)\right]$$

$$-\frac{1}{v}\frac{\partial}{\partial \mu}\left[(1-\mu^2)^{1/2}\left(D_{v\mu}\frac{\partial f}{\partial v}-\frac{(1-\mu^2)^{1/2}}{v}D_{\mu\mu}\frac{\partial f}{\partial \mu}\right)\right], \qquad v=\sqrt{v_\perp^2+v_\parallel^2}, \mu=\frac{v_\parallel}{v}$$

- *Pitch-angle diffusion*

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)^{1/2} D_{\mu\mu}\frac{\partial f}{\partial \mu}\right],$$

$$D_{\mu\mu} = \frac{2\pi^2 e^2}{m_e^2}\int dk\delta E^2(k)\delta(kv\mu - \omega - \Omega_{ce})\left(\mu - \frac{kv}{\omega}\right)^2$$

If $D_{\mu\mu} = D$ = const

$$f(v,\mu,0) = \sum_{l=0}^{\infty} \int_{-1}^{1} d\mu' \left(l + \frac{1}{2}\right) P_l(\mu) P_l(\mu') e^{-\frac{l(l+1)Dt}{v^2}} f(v,\mu',0)$$

- *Energy (or v) space diffusion*

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 D_{vv}\frac{\partial f}{\partial v}\right],$$

$$D_{\mu\mu} = \frac{2\pi^2 e^2}{m_e^2}\int dk\delta E^2(k)\delta(kv\mu - \omega - \Omega_{ce})(1-\mu^2)$$

If $D_{vv} = v^\alpha$

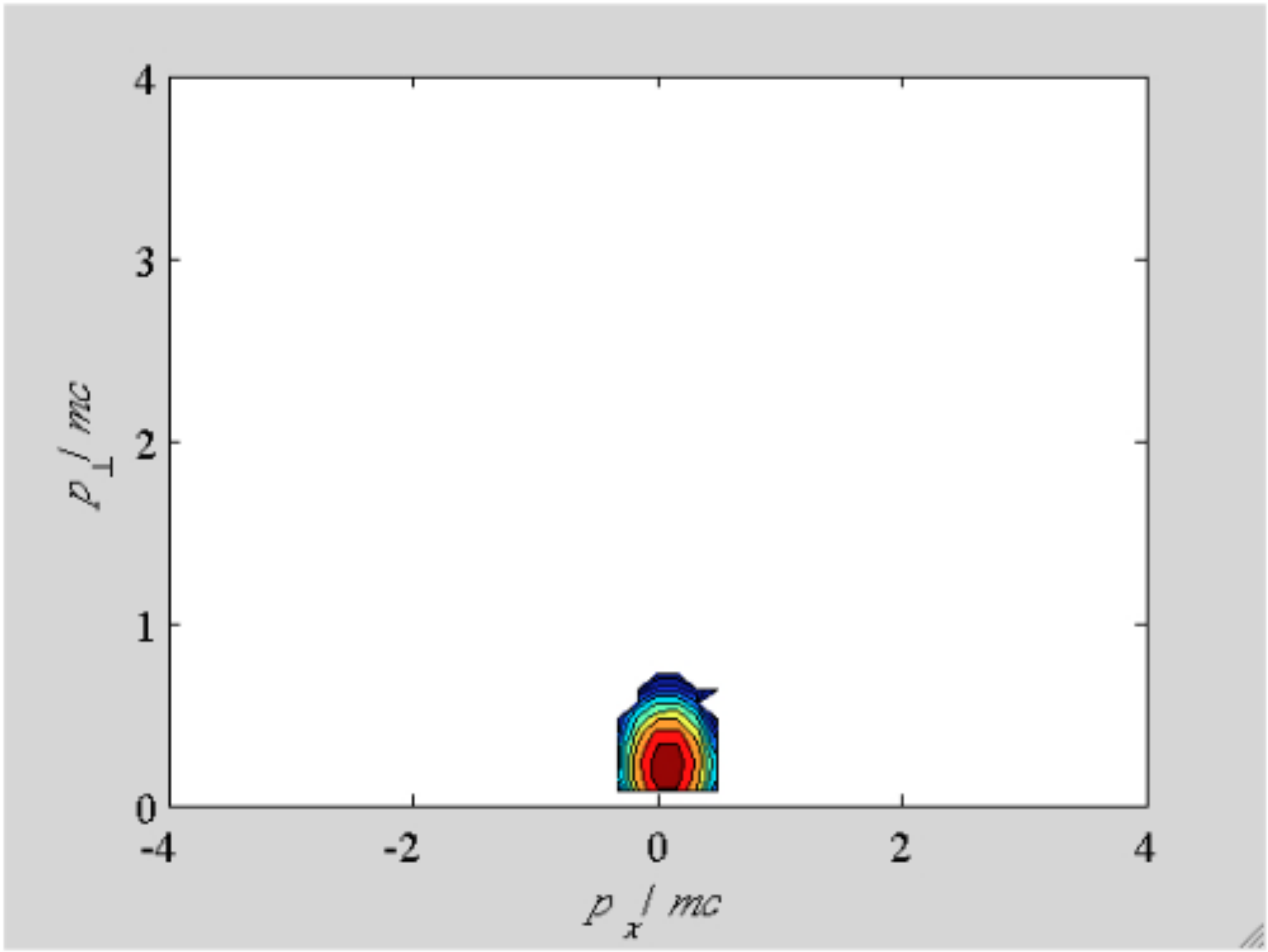
$$f(v,t) = \int_0^\infty dv' v'^2 G(v,v',t) f(v',0),$$

$$G(v,v',t) = \frac{(vv')^{-\frac{1+\alpha}{2}}}{(2-\alpha)t} I_{\frac{1+\alpha}{2-\alpha}}\left(\frac{2(vv')^{\frac{2-\alpha}{2}}}{(2-\alpha)^2 t}\right)\exp\left(-\frac{v^{2-\alpha}+v'^{2-\alpha}}{(2-\alpha)^2 t}\right)$$

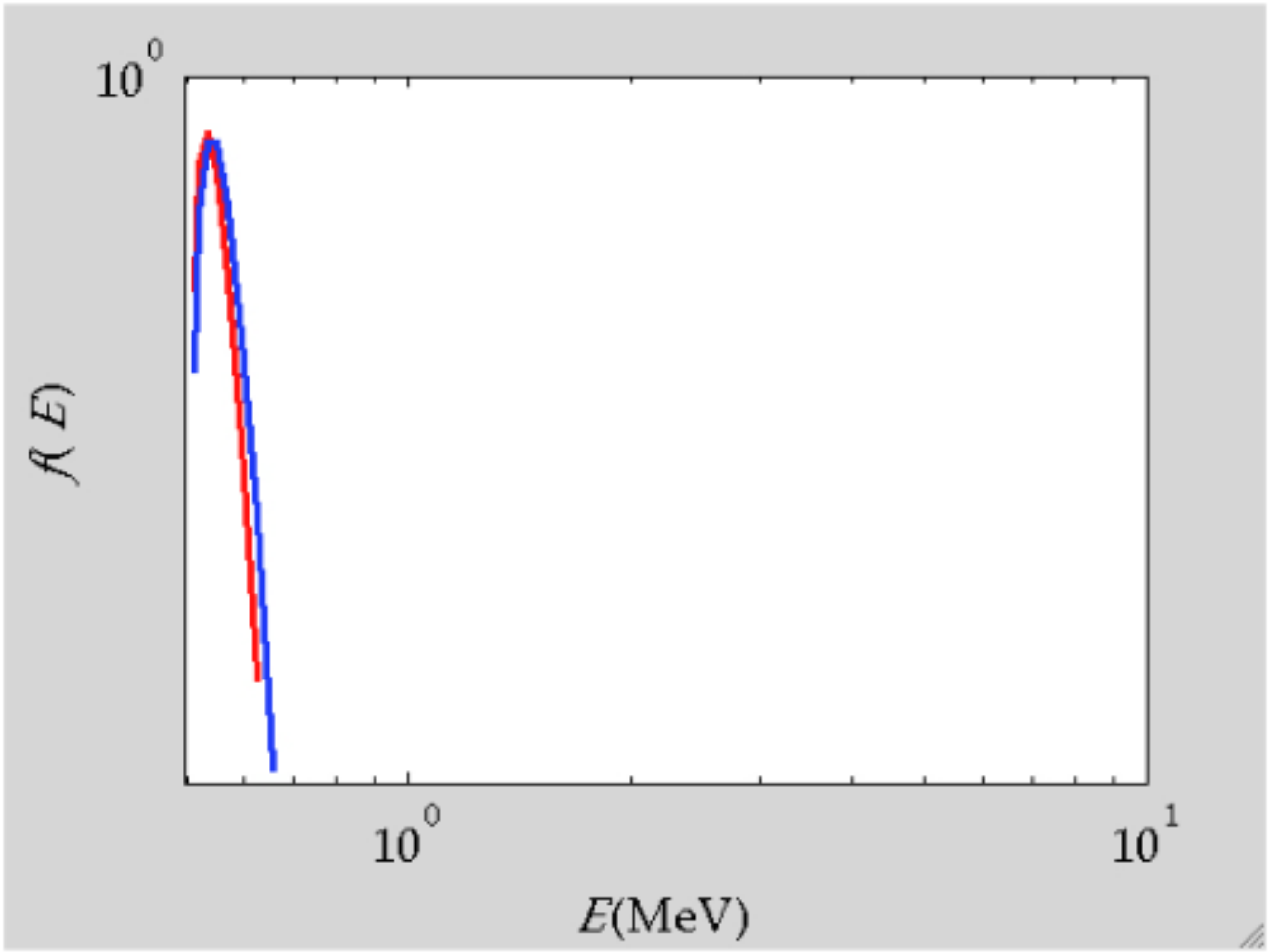
- *Example:*

$$f(v,0)=\frac{\delta(v)}{4\pi v^2} \qquad \alpha=-1 \quad \longrightarrow \quad f(v,t)=\frac{1}{4\pi^2}\frac{1}{3t}\exp\left(-\frac{v^3}{9t}\right)$$

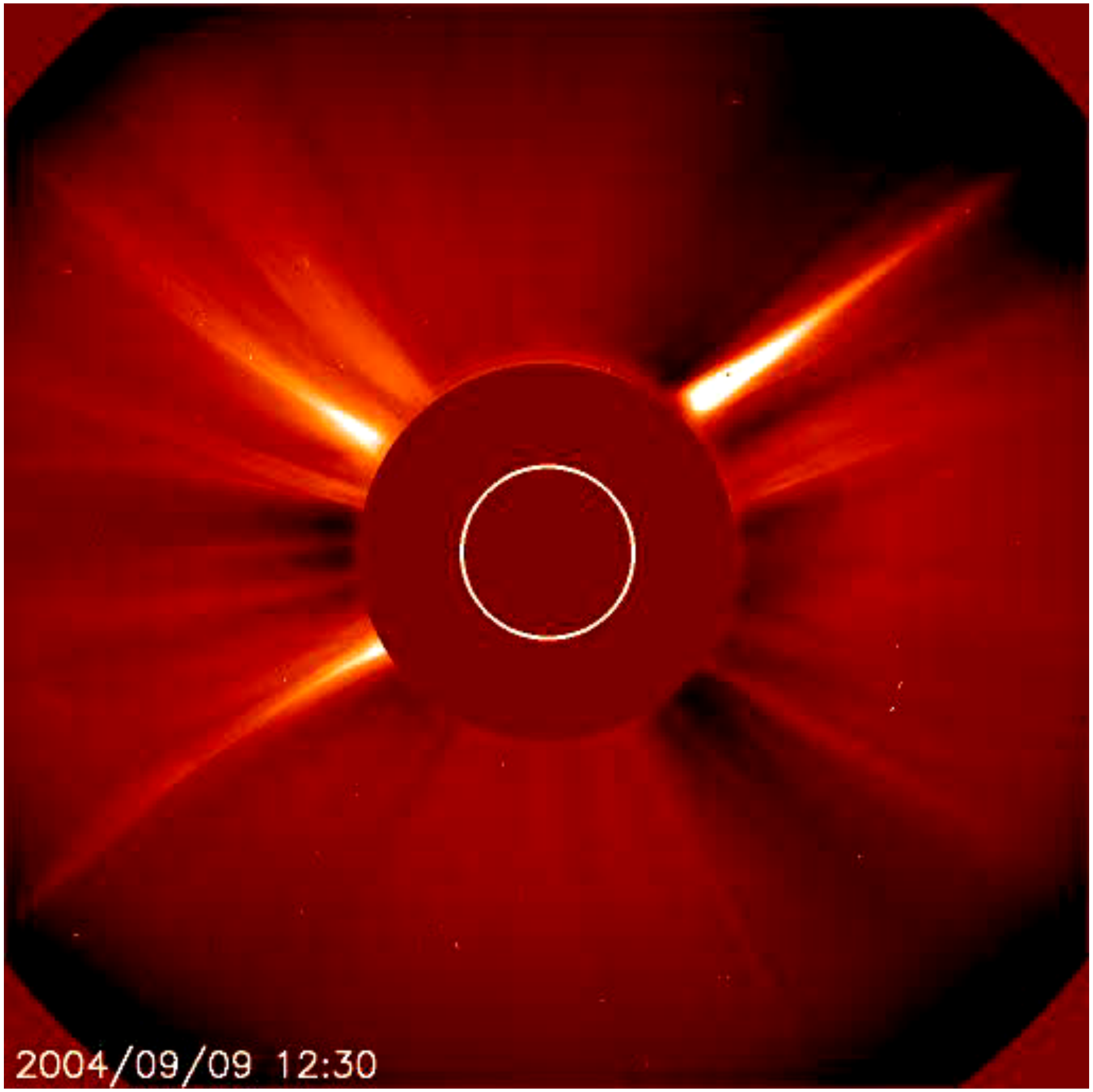
T = 0

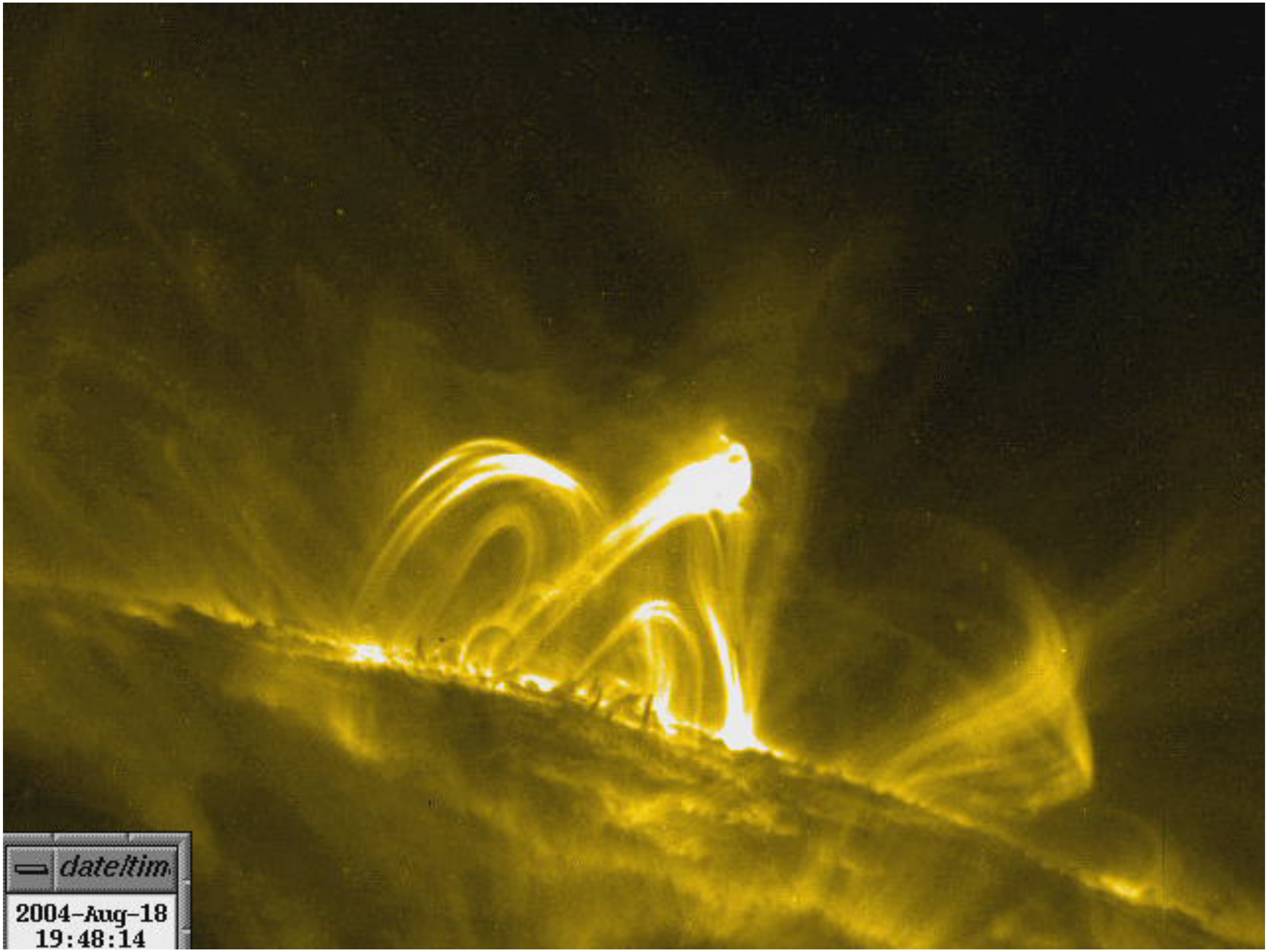


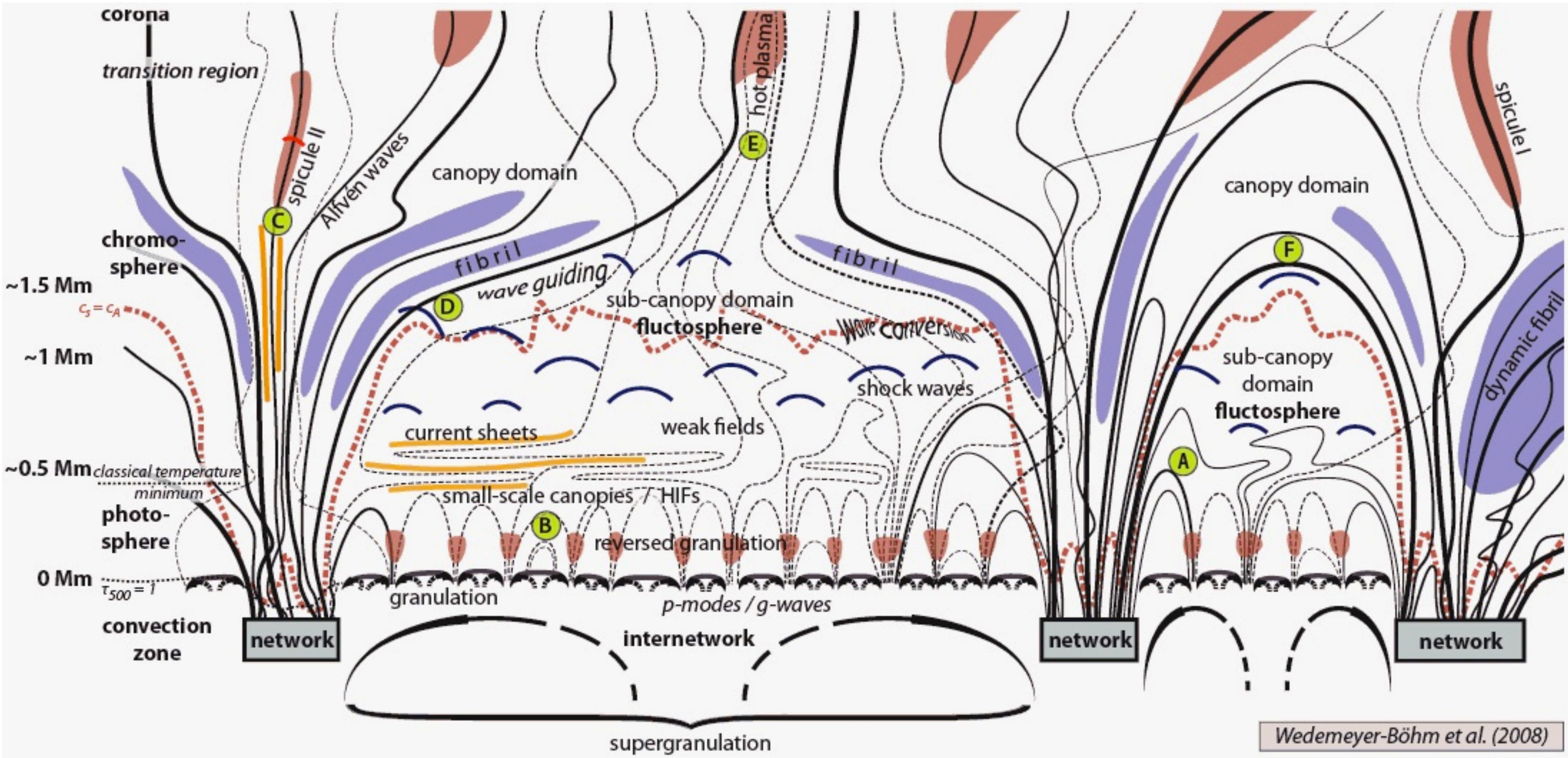
T = 1

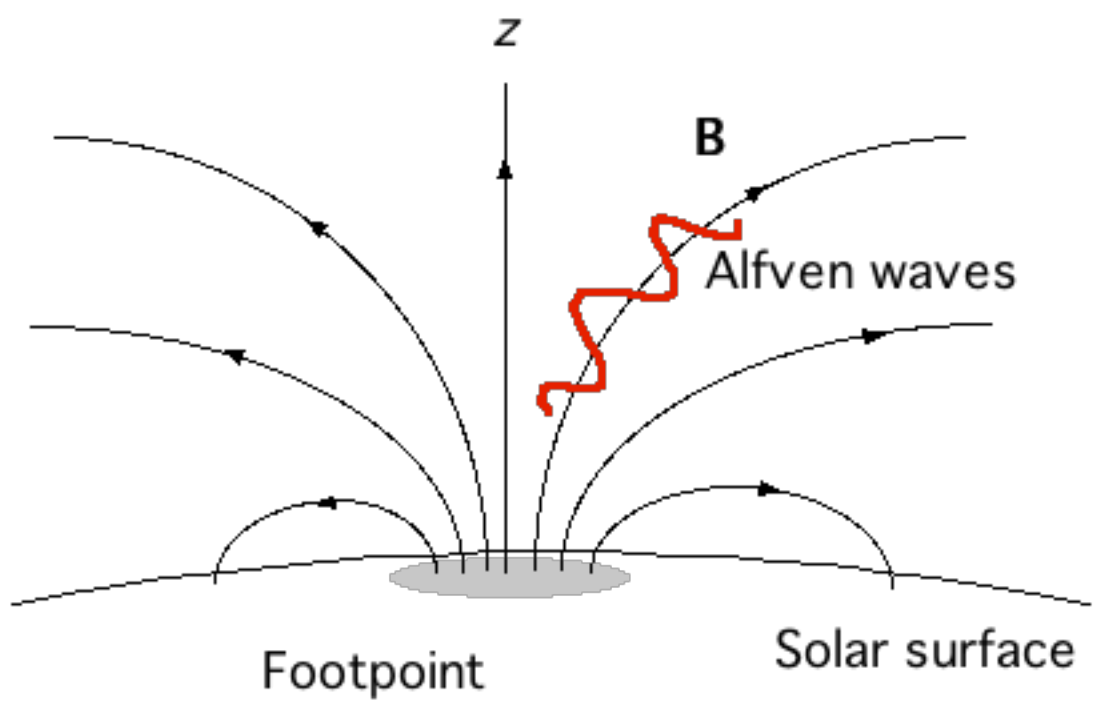


6. Whistler Waves in the Solar Wind

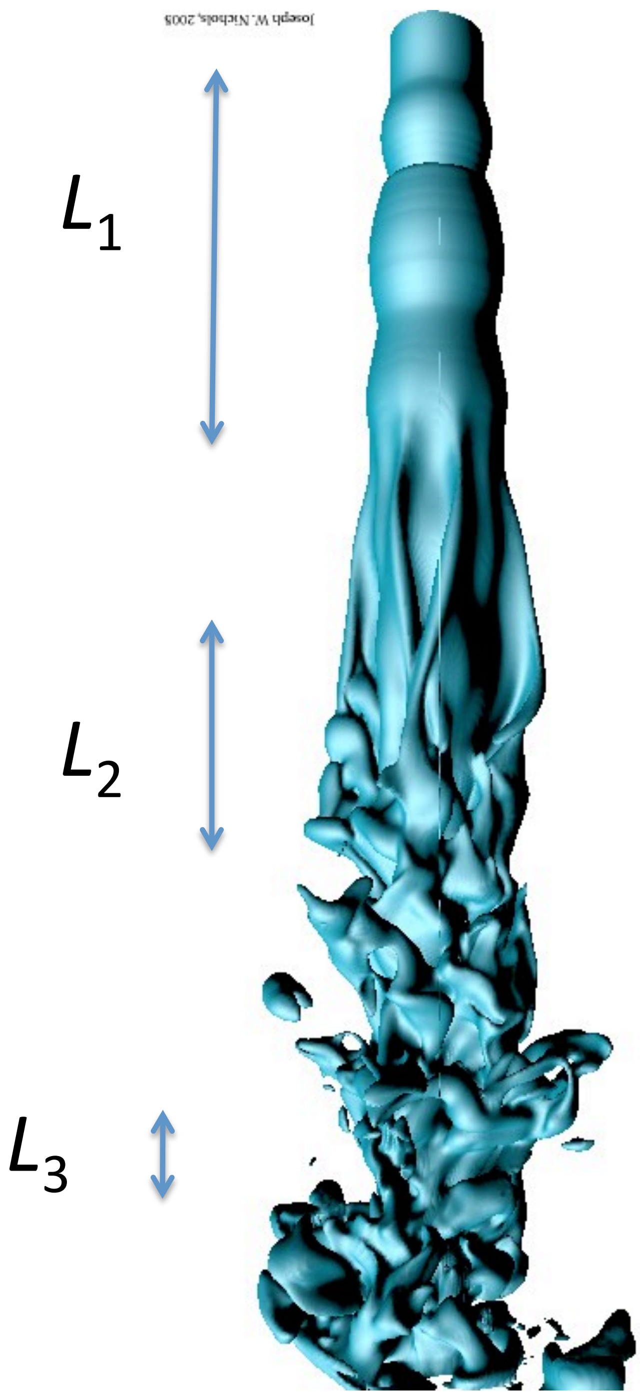


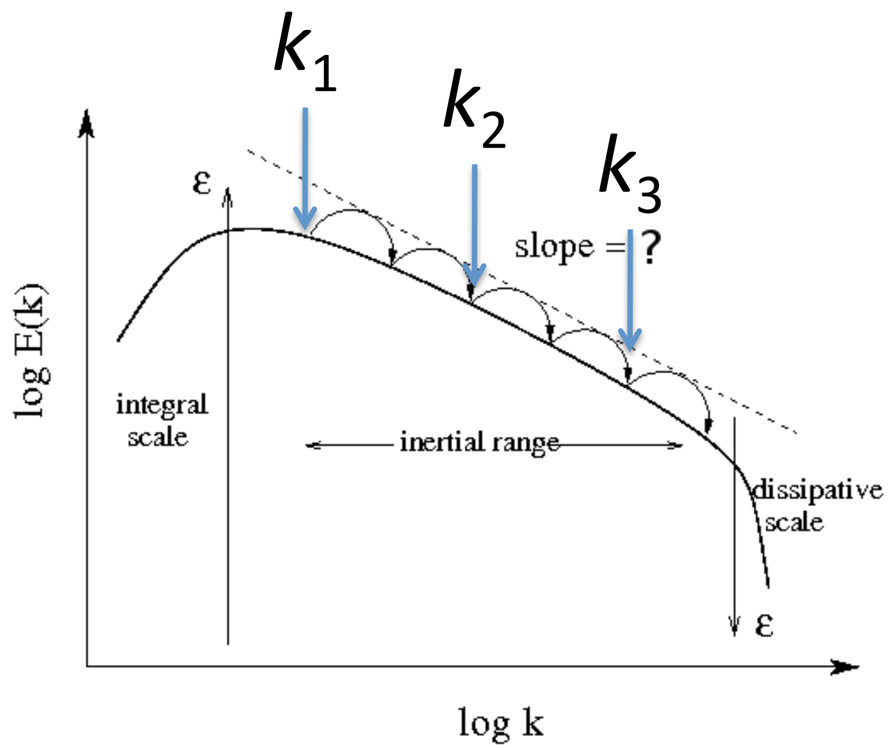

date/tim
2004-Aug-18
19:48:14


corona
transition region
spicule II
Alfvén waves
canopy domain
hot plasma
spicule I
chromo-
sphere
fibril
wave guiding
fibril
canopy domain
~1.5 Mm
$c_s = c_A$
sub-canopy domain
fluctosphere
wave conversion
~1 Mm
sub-canopy
domain
fluctosphere
dynamic fibril
shock waves
current sheets
weak fields
~0.5 Mm
classical temperature
minimum
small-scale canopies / HIFs
photo-
sphere
reversed granulation
0 Mm
$\tau_{500} = 1$
granulation
p-modes / g-waves
convection
zone
network
internetwork
network
network
supergranulation
Wedemeyer-Böhm et al. (2008)

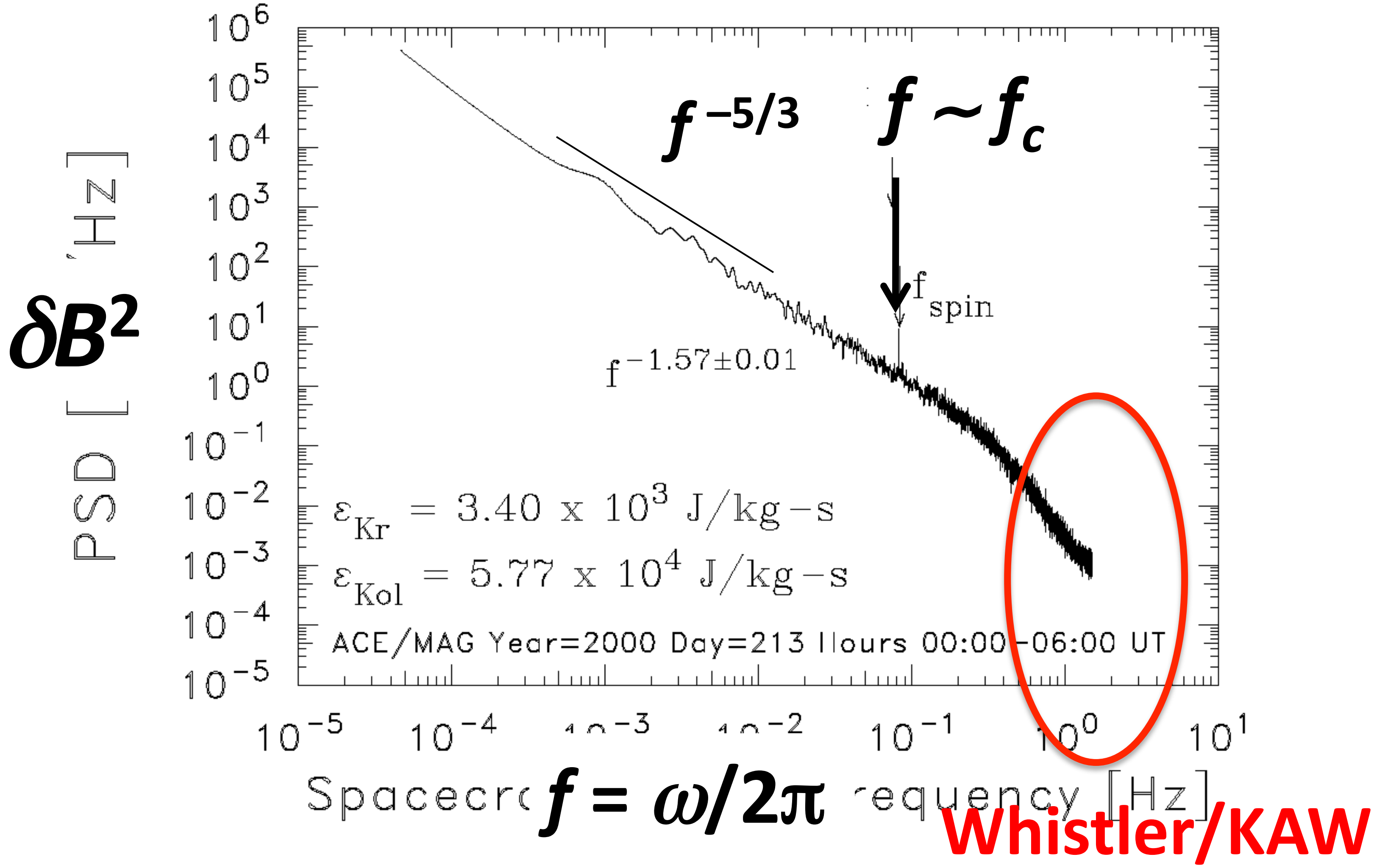

z
B
Alfven waves
Footpoint
Solar surface

Transition to turbulence: Cascade



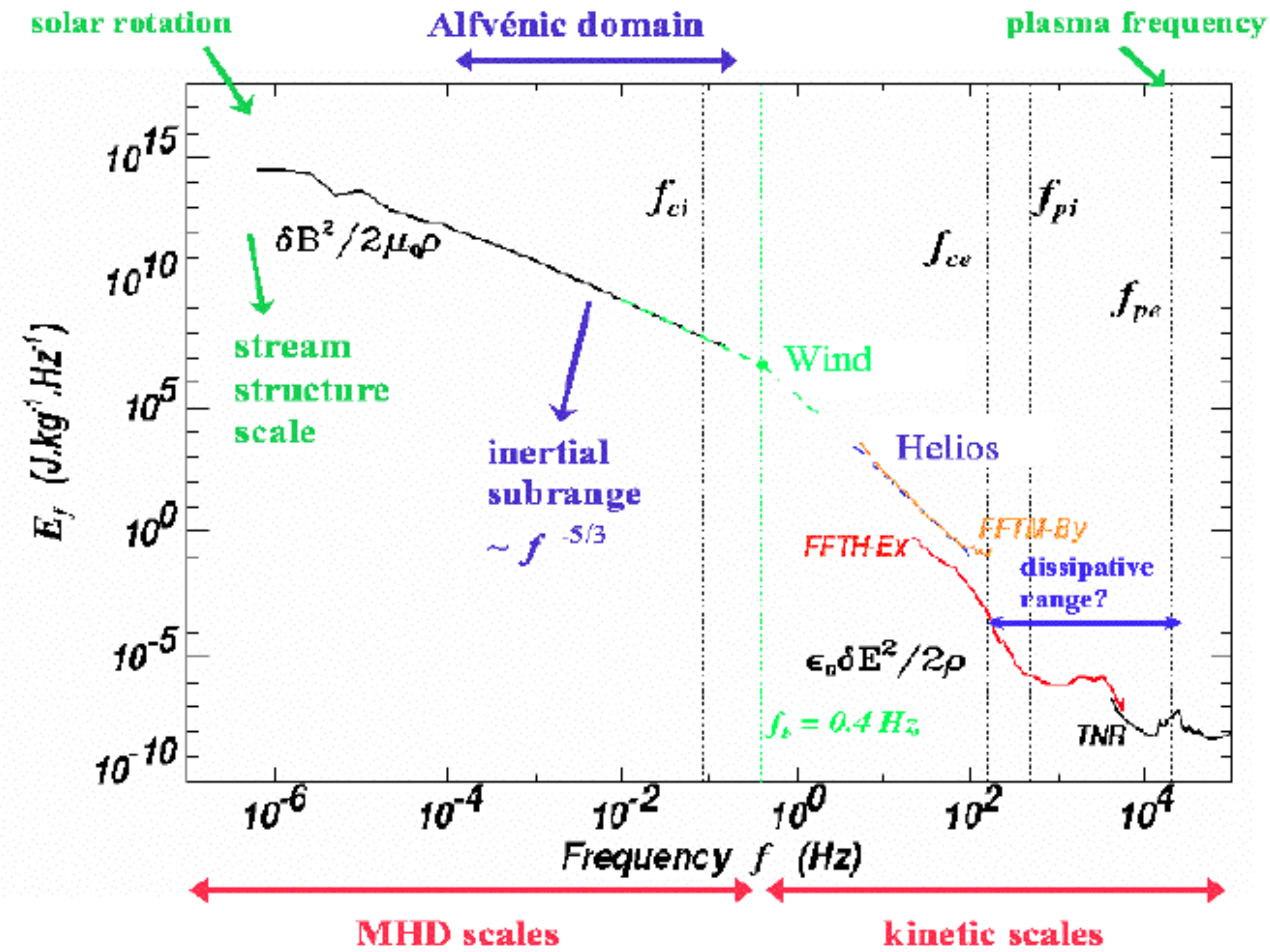


Typical Solar wind turbulence spectrum

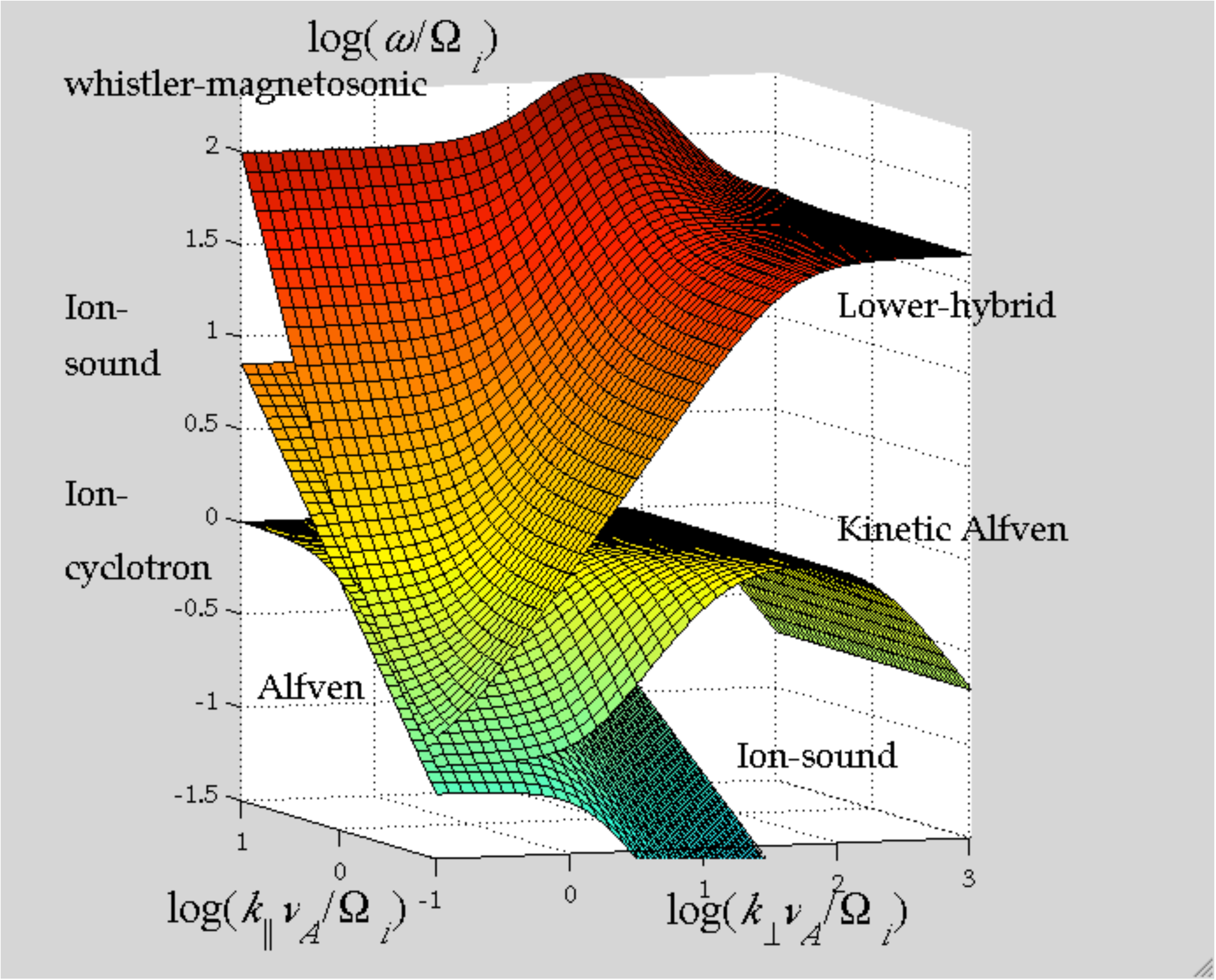


Global spectrum of solar wind electromagnetic fluctuations

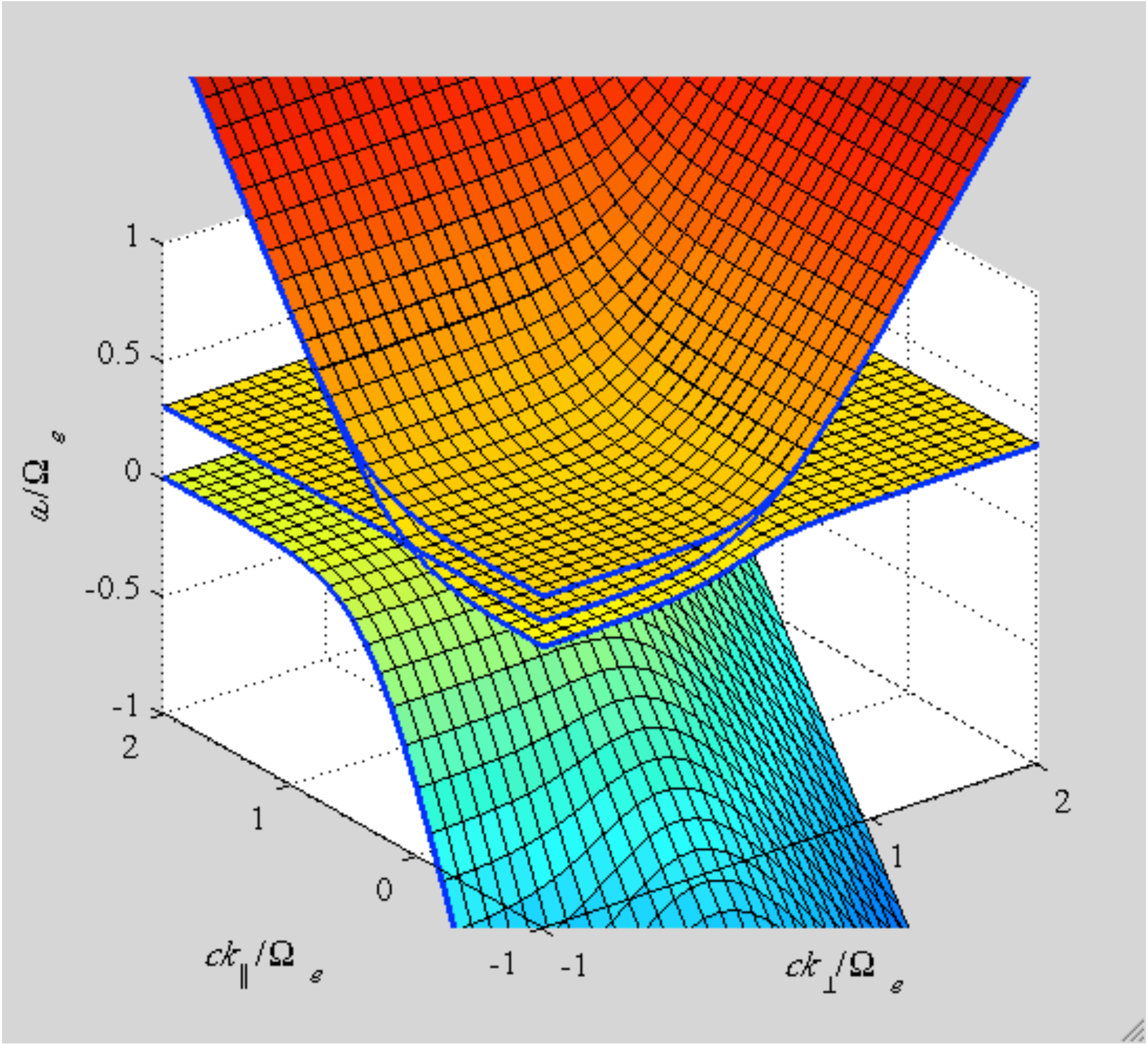
- 2 months of data in the ambient solar wind near L1 -



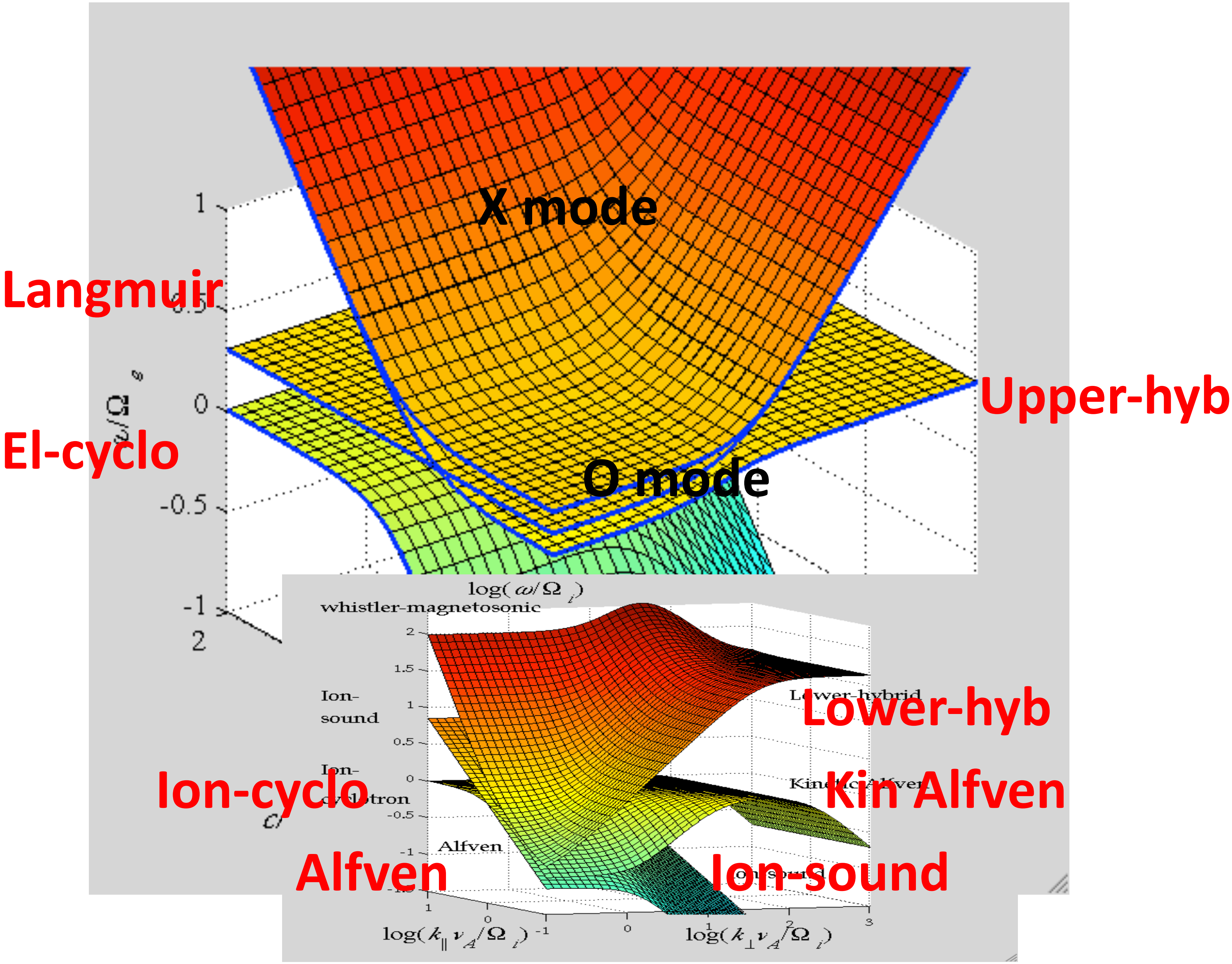
[From Salem]



log(ω/Ω i)
whistler-magnetosonic
2
1.5
Ion-
sound
1
0.5
Ion-
cyclotron
0
-0.5
-1
-1.5
Alfven
Lower-hybrid
Kinetic Alfven
Ion-sound
1
0
-1
0
1
2
3
log(k‖ vA/Ω i)
log(k⊥ vA/Ω i)



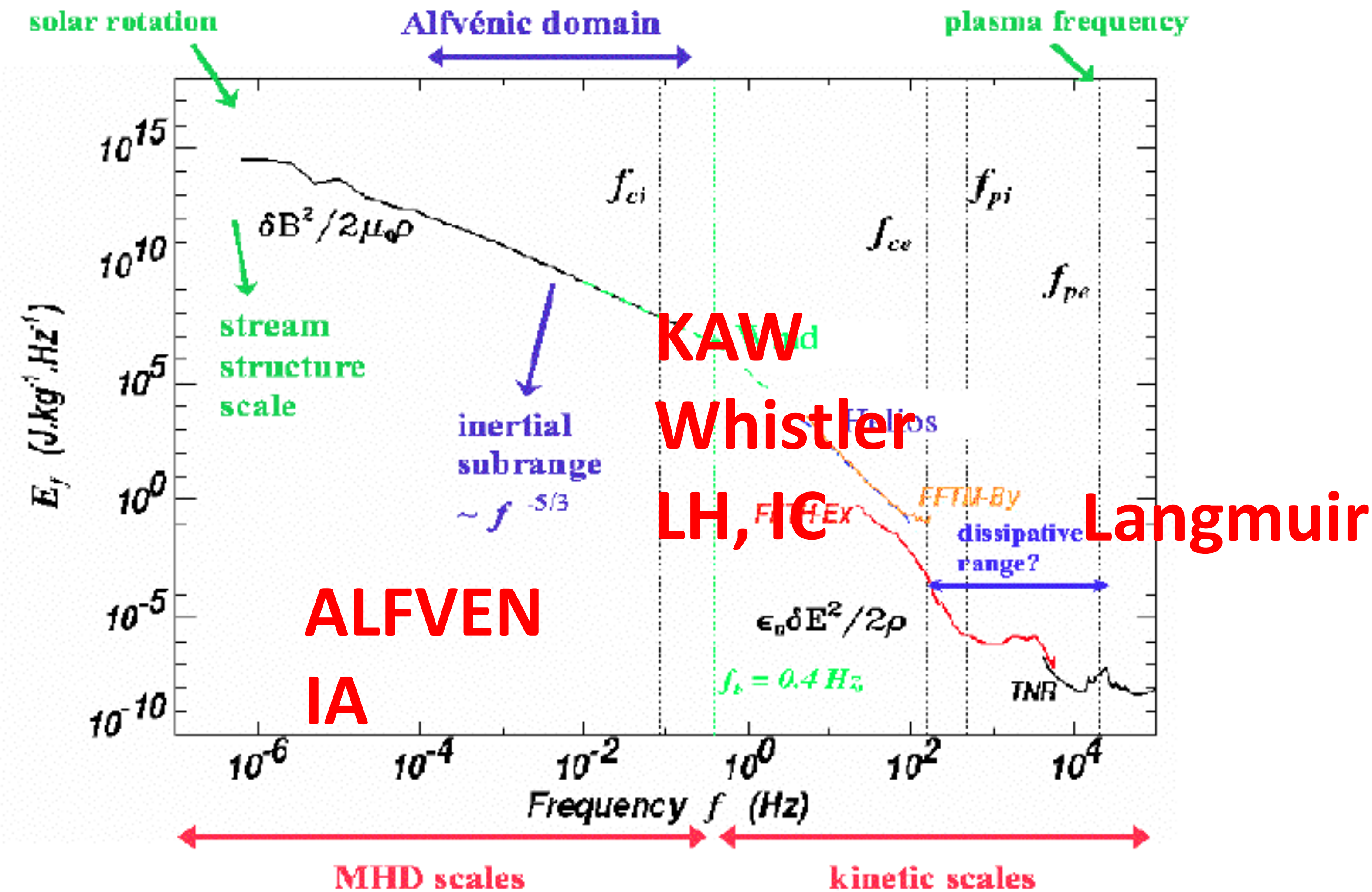
1
0.5
0
-0.5
-1
ω/Ω e
2
1
0
-1
ck∥/Ω e
-1
1
2
ck⊥/Ω e



X mode
Langmuir
Upper-hyb
El-cyclo
O mode
1
0
-0.5
-1
2
log(ω/Ω_i)
whistler-magnetosonic
Ion-
sound
Alfven
Lower-hyb
Kin Alfven
Ion-cyclo
Alfven
Ion-sound
log(k_∥ v_A/Ω_i)
log(k_⊥ v_A/Ω_i)

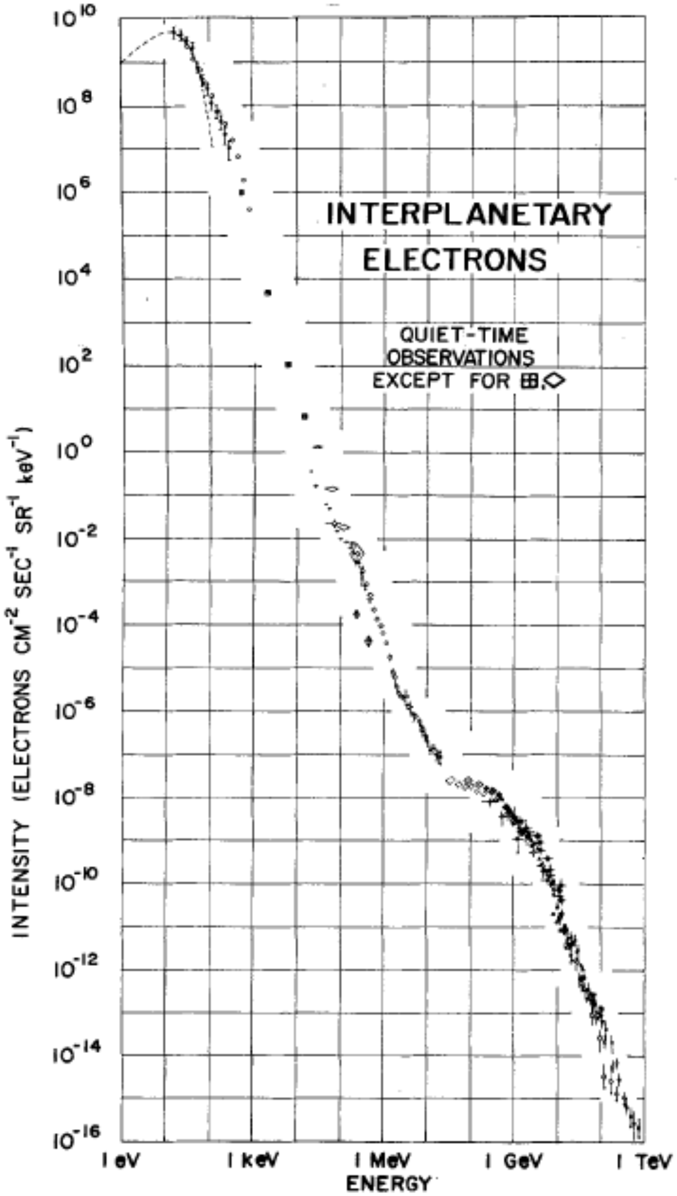
Global spectrum of solar wind electromagnetic fluctuations

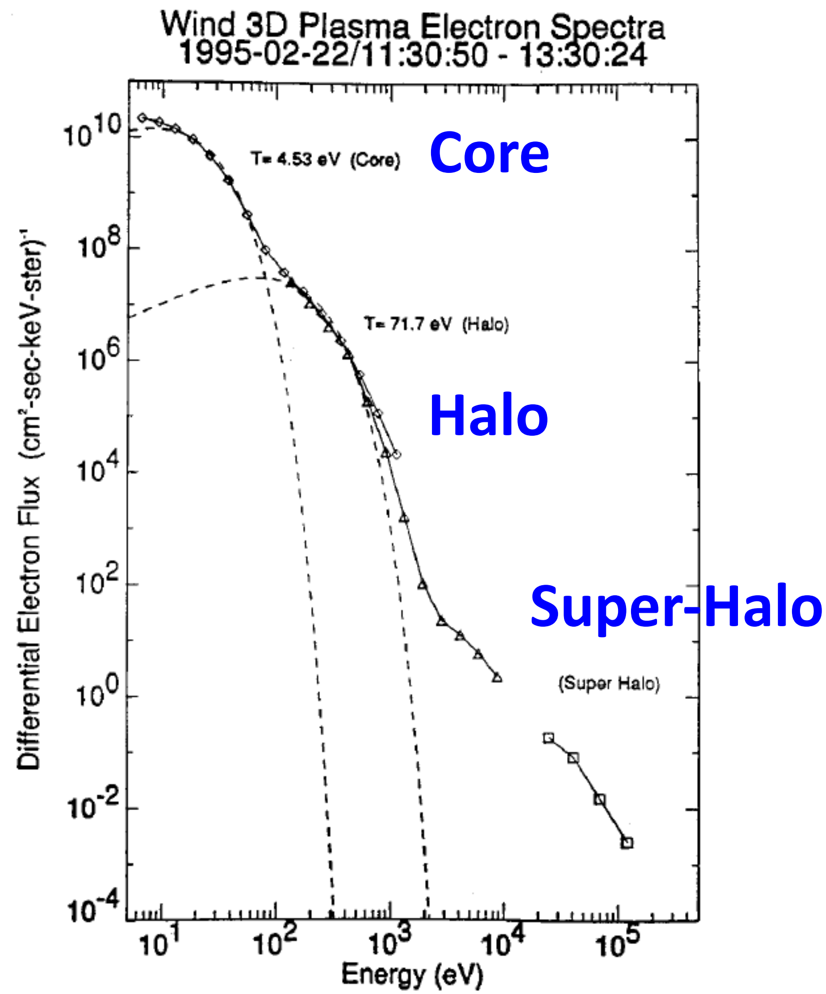
- 2 months of data in the ambient solar wind near L1 -



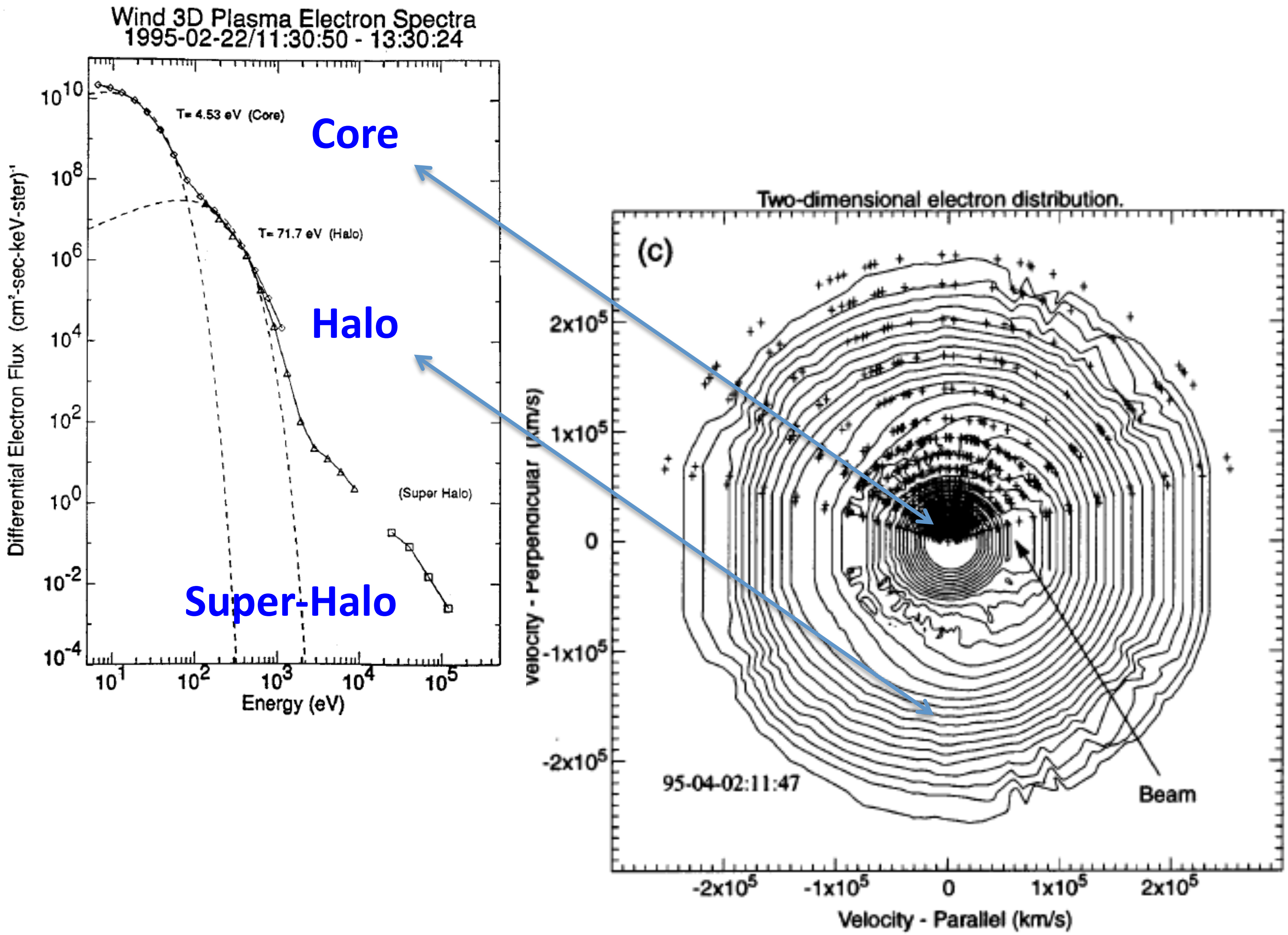
[From Salem]

7. Solar Wind Electrons





Lin, 1972



Wind 3D Plasma Electron Spectra
1995-02-22/11:30:50 - 13:30:24
T= 4.53 eV (Core)
T= 71.7 eV (Halo)
(Super Halo)
Differential Electron Flux (cm²-sec-keV-ster)⁻¹
Energy (eV)
Core
Halo
Super-Halo
Two-dimensional electron distribution.
(c)
95-04-02:11:47
Beam
Velocity - Perpendicular (km/s)
Velocity - Parallel (km/s)

- *Modeling solar wind electrons*

$$f = \frac{n_{core}}{n_{tot}} f_{core} + \frac{n_{halo}}{n_{tot}} f_{halo} + \frac{n_{s-halo}}{n_{tot}} f_{s-halo}$$

↑ Do not interact with any turbulence

↑ Wave-particle interaction with whistler turbulence

↑ Wave-particle interaction with Langmuir turbulence

8. Solar Wind Acceleration by Whistler Waves (Halo)

- *Fokker-Planck equation for halo electrons*

$$\frac{\partial f}{\partial t} = \frac{ie^2 k_m^2}{4\pi m_e} \int dk \left((\omega^* - k v_\parallel) \frac{\partial}{v_\perp \partial v_\perp} + k \frac{\partial}{\partial v_\parallel} \right)$$

$$\times \left(\frac{1}{\partial[\omega^2 \varepsilon_-(k,\omega)]/\partial\omega} \right)^* \frac{v_\perp^2 f}{\omega - k v_\parallel - \Omega_{ce}}$$

$$+ \frac{ie^2}{4 m_e^2} \int dk \left[\left(1 - \frac{k v_\parallel}{\omega^*} \right) \frac{\partial}{v_\perp \partial v_\perp} + \frac{k}{\omega^*} \frac{\partial}{\partial v_\parallel} \right]$$

$$\times \frac{v_\perp^2 \, | \delta E_k^2 |}{\omega - k v_\parallel - \Omega_{ce}} \left[\left(1 - \frac{k v_\parallel}{\omega} \right) \frac{\partial f}{v_\perp \partial v_\perp} + \frac{k}{\omega} \frac{\partial f}{\partial v_\parallel} \right],$$

- *Steady-State halo electron distribution*

$$f = C\exp\left(-\int dv\,\frac{1}{v}\,\frac{m_e c^2 k_m}{\pi}\,\frac{\Omega_{ce}^4}{\omega_{pe}^4}\,\frac{\int_{-1}^{1} d\mu\,\frac{1-\mu^2}{|\mu|\,\mu^2}}{\int_{-1}^{1} d\mu\,\frac{1-\mu^2}{|\mu|\,\mu^2}\,\delta E^2\left(-\frac{\Omega_{ce}}{v\mu}\right)}\right)$$

Whistler wave turbulence spectrum $\uparrow$ $\delta E^2(k)\big|_{k=-\Omega_{ce}/v\mu}$

via approximate wave-particle resonance condition

$$0 = \omega - kv\mu - \Omega_{ce} \approx -kv\mu - \Omega_{ce}$$

- *Steady-State Whistler Turbulence Spectrum*

$$\delta E^2(k) = \frac{k_m^2 m_e c^2}{2\pi} \frac{\Omega_{ce}^4}{\omega_{pe}^4} \frac{\alpha - 3/2}{\alpha + 1} \left(\frac{k^2 v_T^2}{\Omega_{ce}^2} + \frac{1}{\alpha - 3/2} \right),$$

$$\delta B^2(k) = \frac{k_m^2 m_e c^2}{2\pi} \frac{\alpha - 3/2}{\alpha + 1} \left(1 + \frac{1}{\alpha - 3/2} \frac{\Omega_{ce}^2}{k^2 v_T^2} \right)$$

- *Steady-State Halo Electron Distribution*

$$f = \frac{n}{\pi^{3/2} v_T^3} \frac{\Gamma(\alpha + 1)}{(\alpha - 3/2)^{3/2} \Gamma(\alpha - 1/2)} \left(1 + \frac{1}{\alpha - 3/2} \frac{v^2}{v_T^2} \right)^{-\alpha - 1}$$

9. Solar Wind Acceleration by Langmuir Waves (Superhalo)

- *Fokker-Planck equation for superhalo electrons*

$$\frac{\partial f}{\partial t} = \frac{ie^2 k_m^2}{2\pi m_e} \frac{\partial}{\partial v_\parallel} \int dk \left(\frac{1}{\omega \partial \varepsilon_\parallel (k,\omega) / \partial \omega} \right)^* \frac{v_\parallel^2 f}{\omega - k v_\parallel}$$

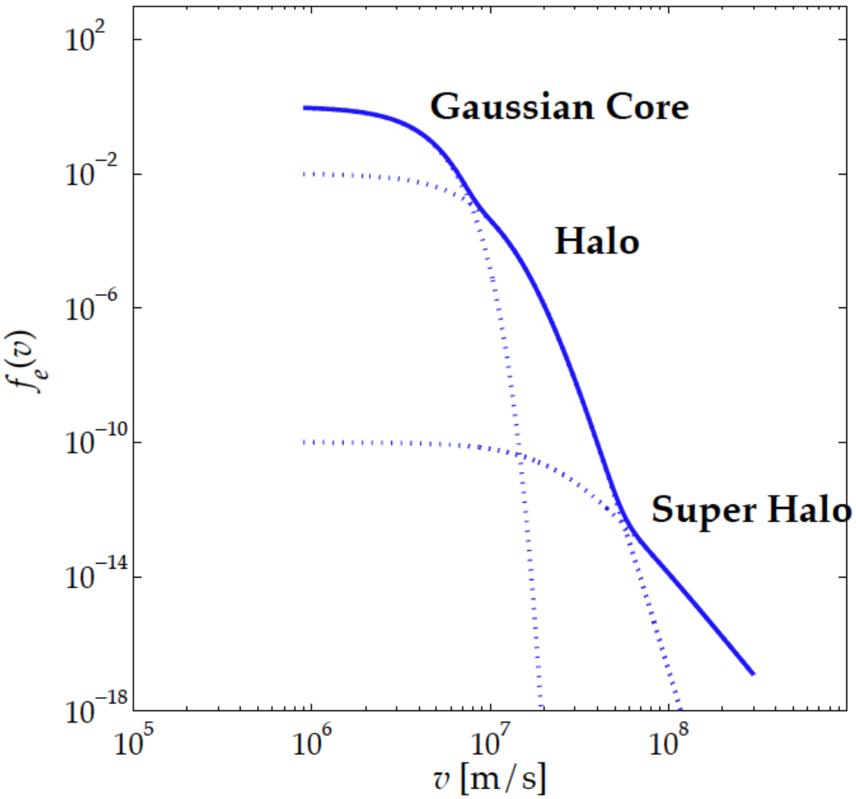
$$+ \frac{ie^2}{m_e^2} \frac{\partial}{\partial v_\parallel} \int dk \frac{|\delta E_k^2|}{\omega - k v_\parallel} \frac{\partial f}{\partial v_\parallel}$$

- *Steady-State Langmuir Turbulence Spectrum*

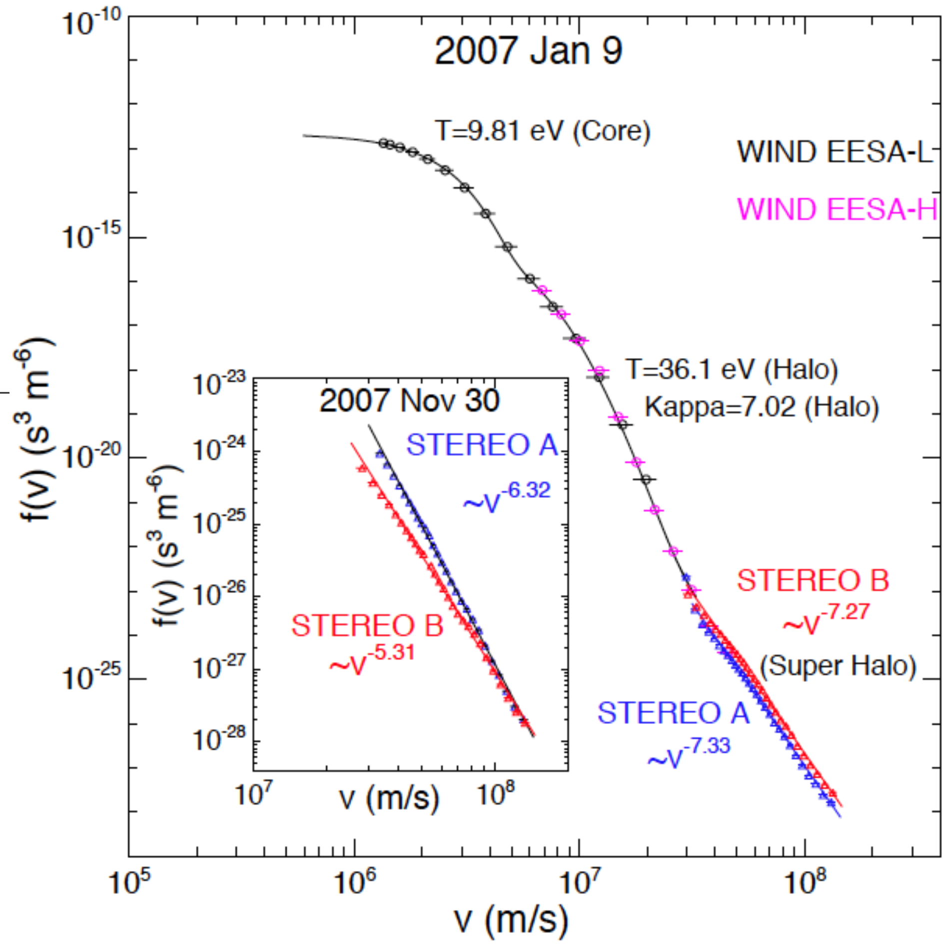
$$\delta E^2(k) = \frac{k_m^2 T_e}{4\pi} \frac{\kappa - 3/2}{\kappa + 1} \left(1 + \frac{1}{\kappa - 3/2} \frac{\omega_{pe}^2}{k^2 v_T^2} \right)$$

- *Steady-State Superhalo Electron Distribution*

$$f = \frac{n}{\pi^{3/2} v_T^3} \frac{\Gamma(\kappa + 1)}{(\kappa - 3/2)^{3/2} \Gamma(\kappa - 1/2)} \left(1 + \frac{1}{\kappa - 3/2} \frac{v^2}{v_T^2} \right)^{-\kappa - 1}$$



Gaussian Core
Halo
Super Halo
$f_e(v)$
v [m/s]



2007 Jan 9
T=9.81 eV (Core)
WIND EESA-L
WIND EESA-H
T=36.1 eV (Halo)
Kappa=7.02 (Halo)
STEREO B
~V^-7.27
(Super Halo)
STEREO A
~V^-7.33
2007 Nov 30
STEREO A
~V^-6.32
STEREO B
~V^-5.31
f(v) (s^3 m^-6)
v (m/s)

Summary

- Whistler waves are generated in the planetary (Earth) atmosphere and propagated through magnetosphere.
- Whistler waves are responsible for acceleration and loss of radiation belt relativistic electrons.
- Whistler turbulence is responsible for producing solar wind halo electrons.
- Langmuir turbulence is responsible for accelerating solar wind electrons to superhalo energies.

Challenge

- Turbulent acceleration is easy, as long as the turbulence intensity is GIVEN.
- Real challenge is how to describe turbulence.
- At present, only Alfvenic turbulence (MHD) and Langmuir turbulence (kinetic) are understood.
- There is no theory for whistler turbulence YET (for that matter, any other plasma turbulence, although I am working on it).