Scaling relations in MHD and EMHD Turbulence

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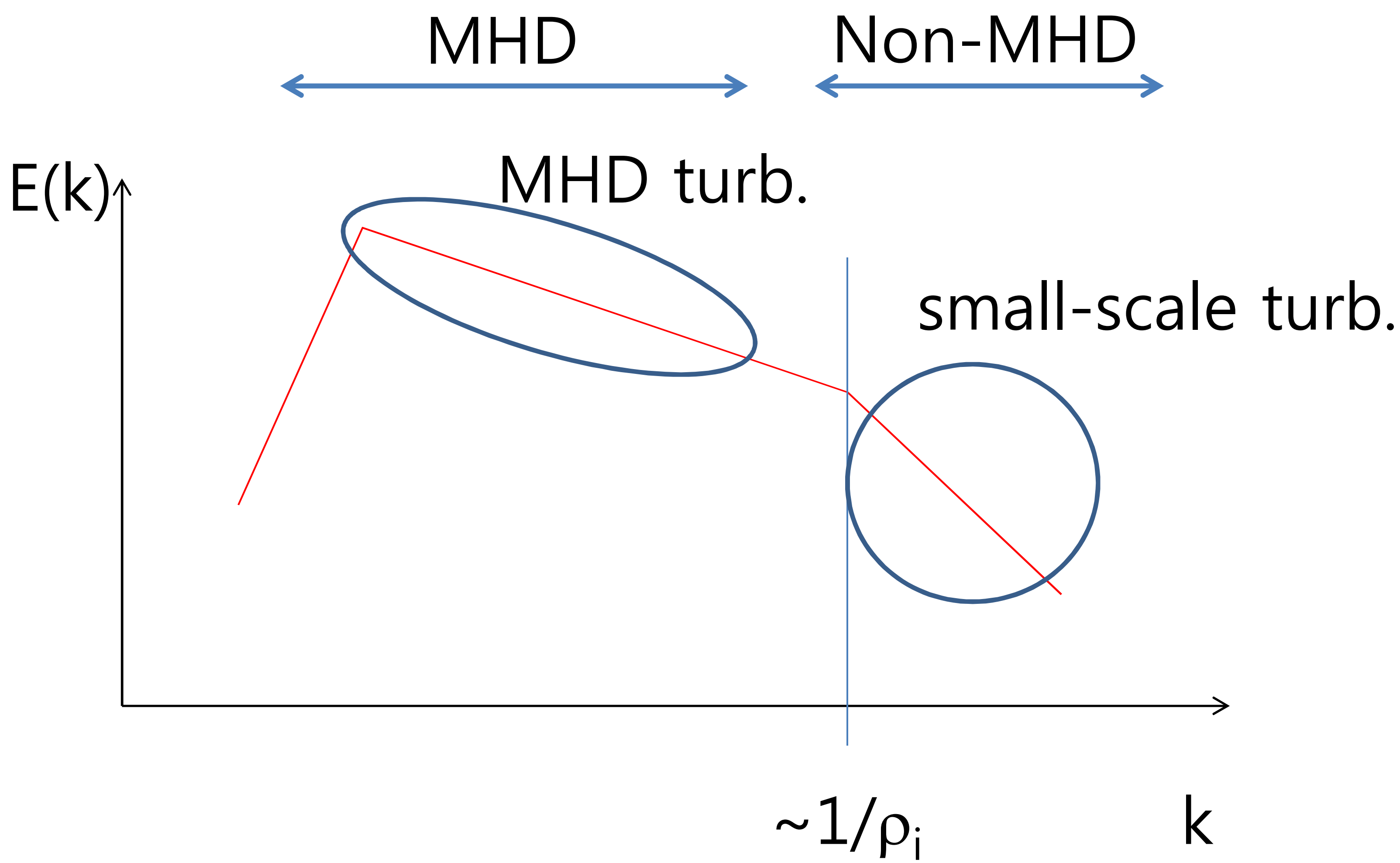
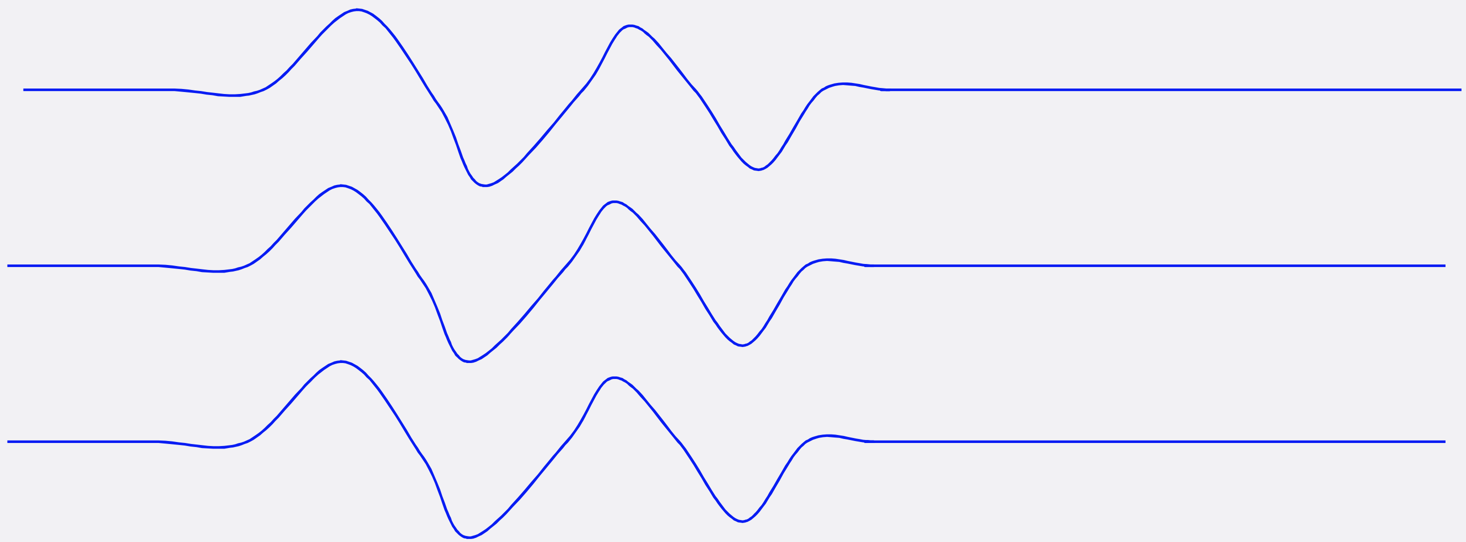
Outline



Topic 1. Strong MHD Turbulence

Alfven wave

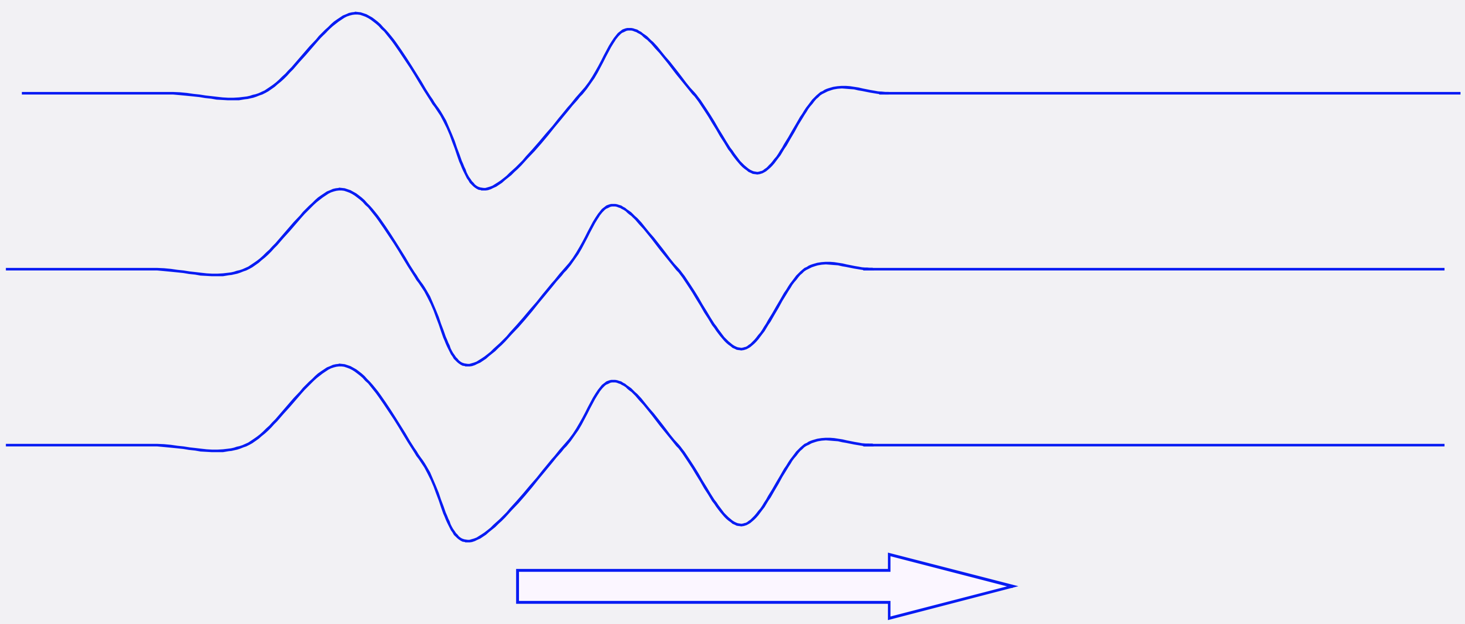


Suppose that we perturb magnetic field lines.
We will only consider Alfvenic perturbations.
(restoring force=tension)

We can make the wave packet move in one direction.
(We need to specify velocity)

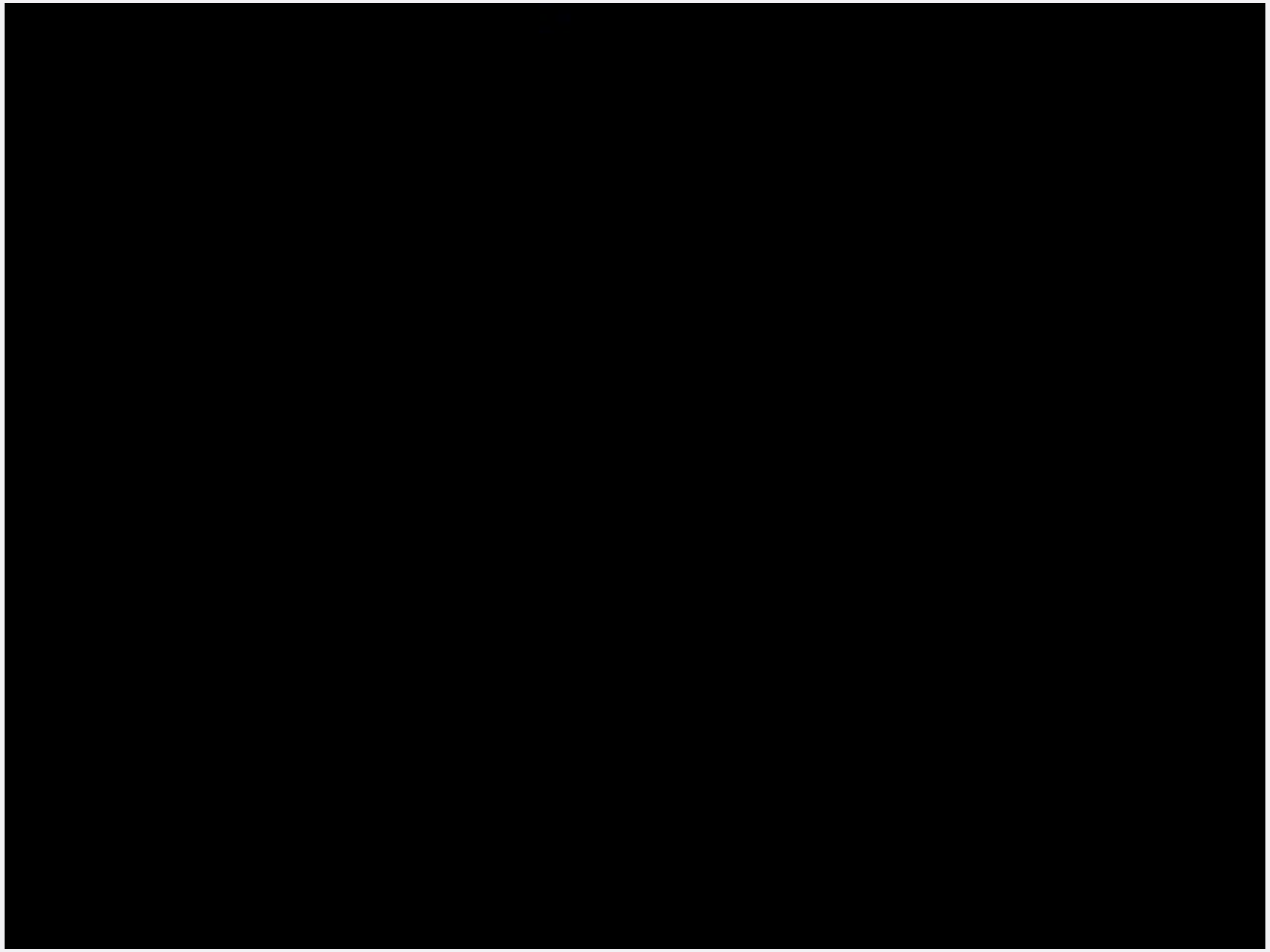
Dynamics of one wave packet

Suppose that this packet is moving to the right.
What will happen?



V_A: Alfven speed

One wave packet

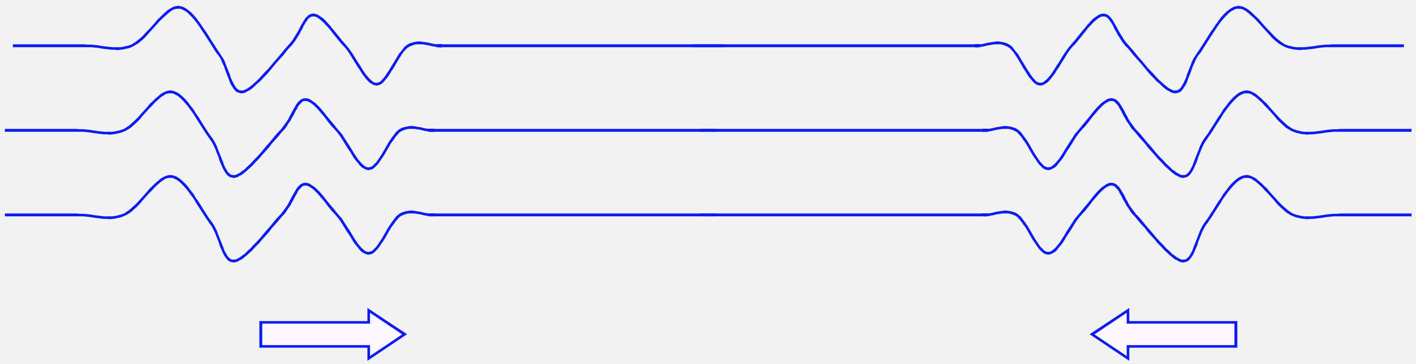


64^3

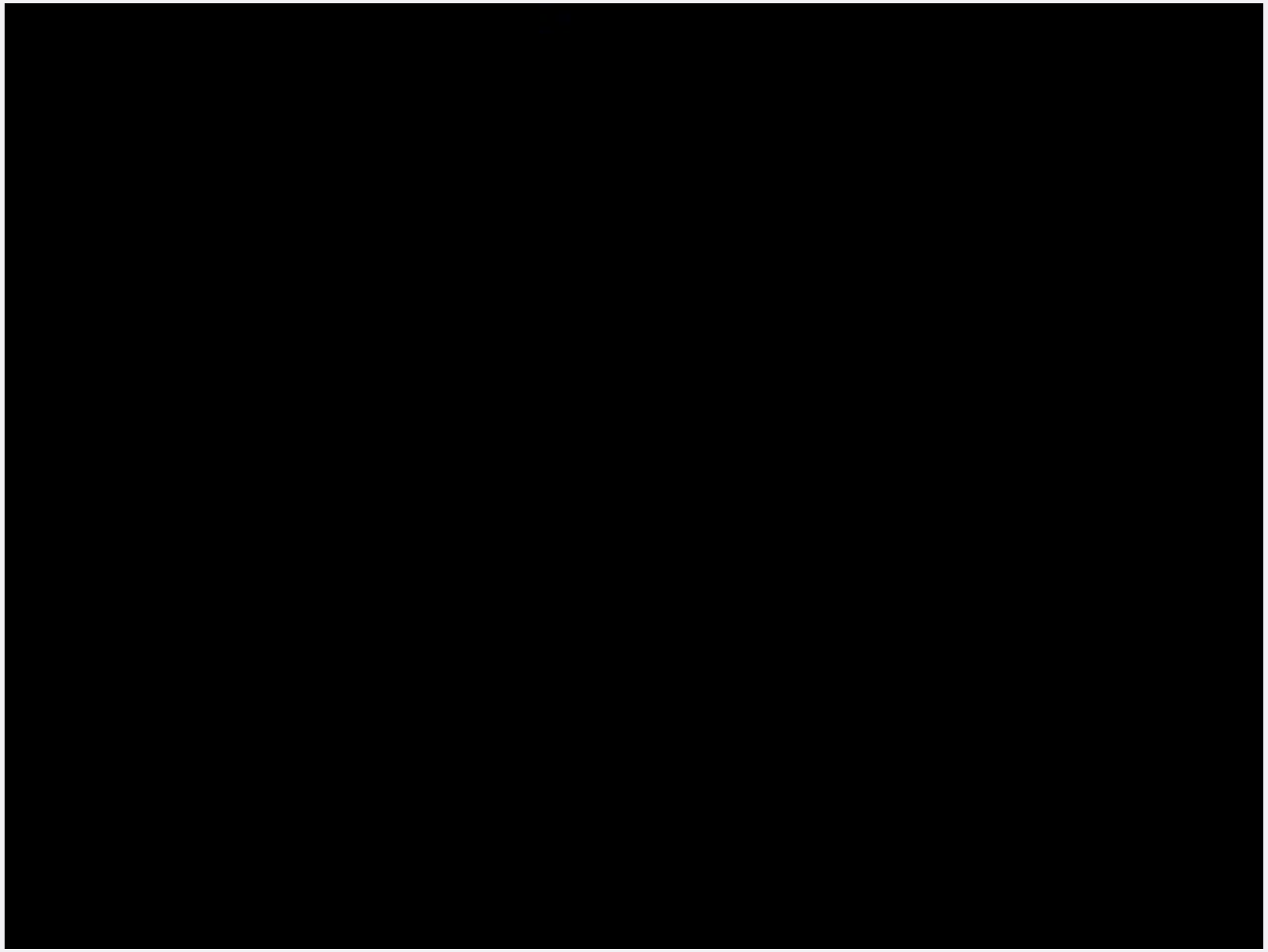
Nothing happens.

Dynamics of two opposite-traveling wave packets

Now we have two colliding wave packets.
What will happen?



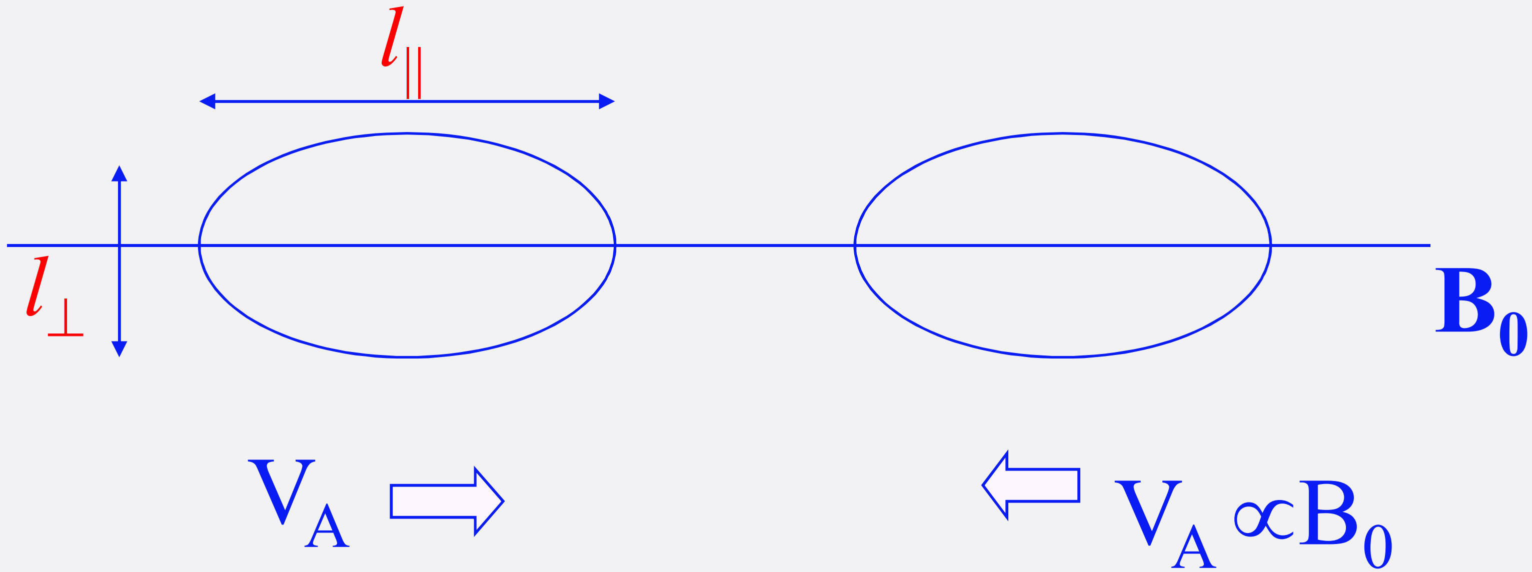
Two wave packets



This is something we call turbulence

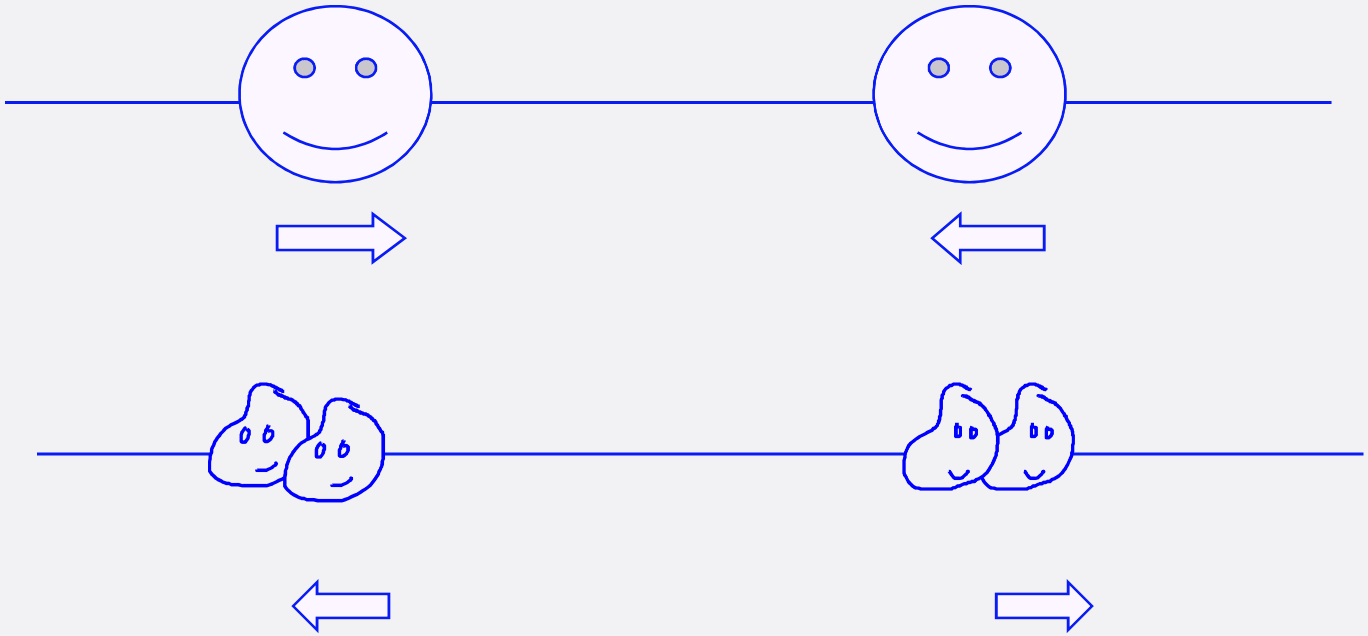
What happens?

What happens when two Alfvenic wave packets collide?

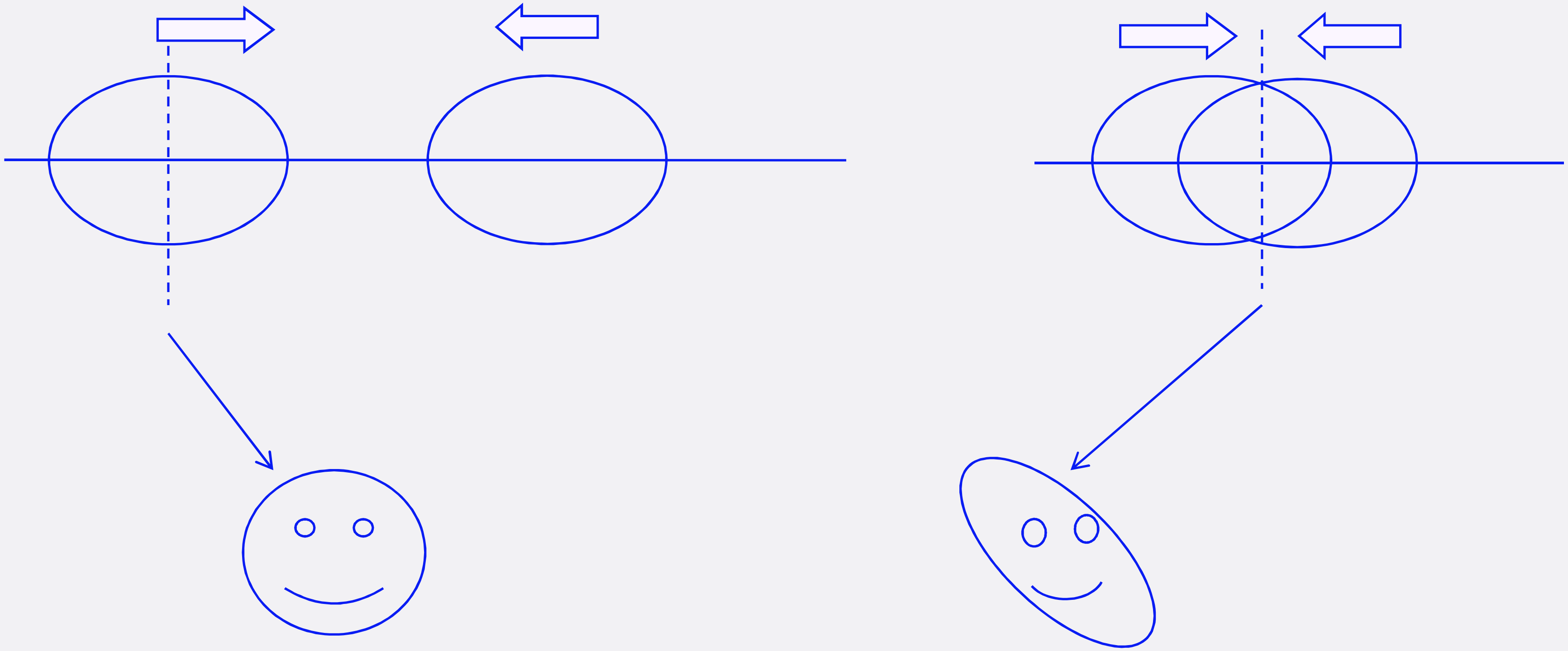


Goldreich & Sridhar (1995):

In strong turbulence, **1 collision is enough to complete cascade!**



1 collision is enough to complete cascade!

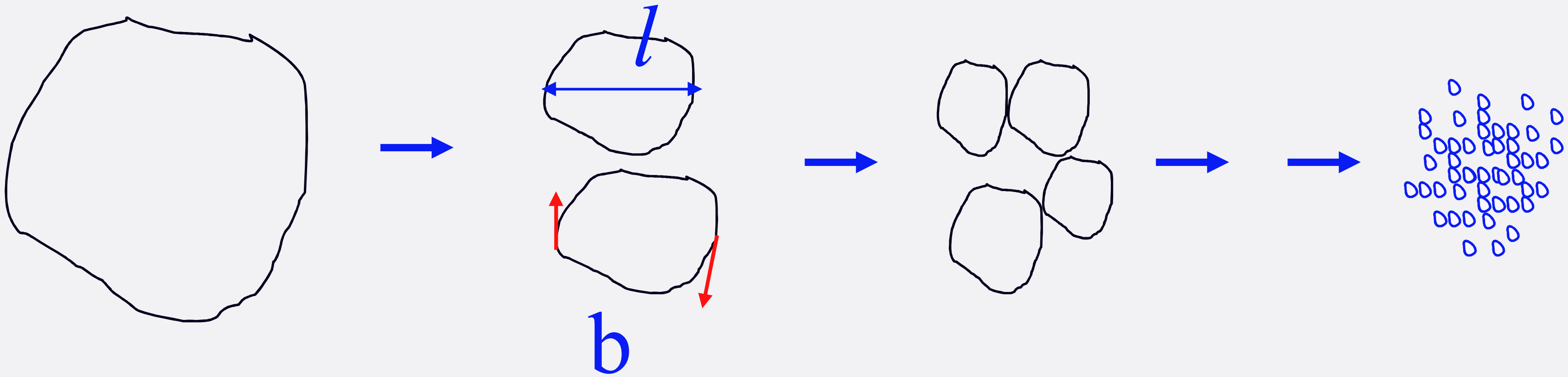


-Distortion time scale ~ $l_\perp / v_l$

-Duration of collision ~ $l_{||} / B_0$

$$t_w / t_{eddy} \sim (l_{||}/B_0)/(l_\perp / v) \sim (b\, l_{||}/ l_\perp B_0) \sim 1$$

Energy Cascade



$$b^2/t_{cas} = \text{constant}$$

Goldreich-Sridhar model (1995)

- Critical balance

$$\frac{l_\perp}{b_{\perp l}} = \frac{l_\parallel}{B_0}$$

- Constancy of energy cascade rate

$$\frac{b_{\perp l}^2}{t_{cas}} = \text{const}$$

$$\frac{b_{\perp l}^2}{(l_\perp / b_{\perp l})} = \text{const}$$

⇓

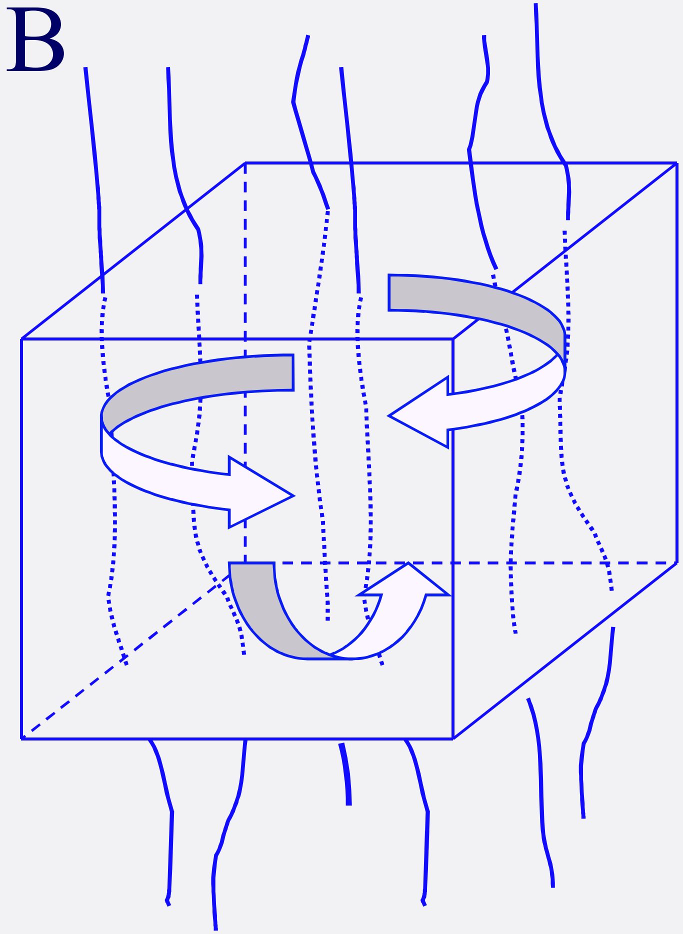
$$b_\perp \sim l_\perp^{1/3}$$

Or, $E(k) \sim k^{-5/3}$

$$l_\parallel \sim l_\perp^{2/3}$$

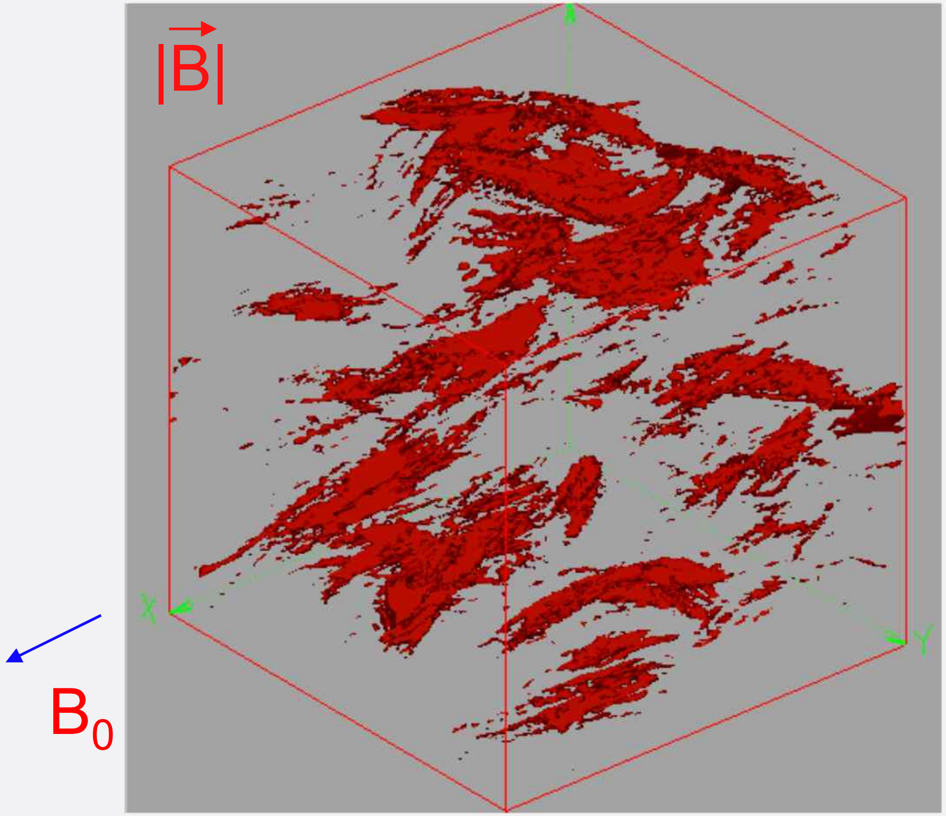
back

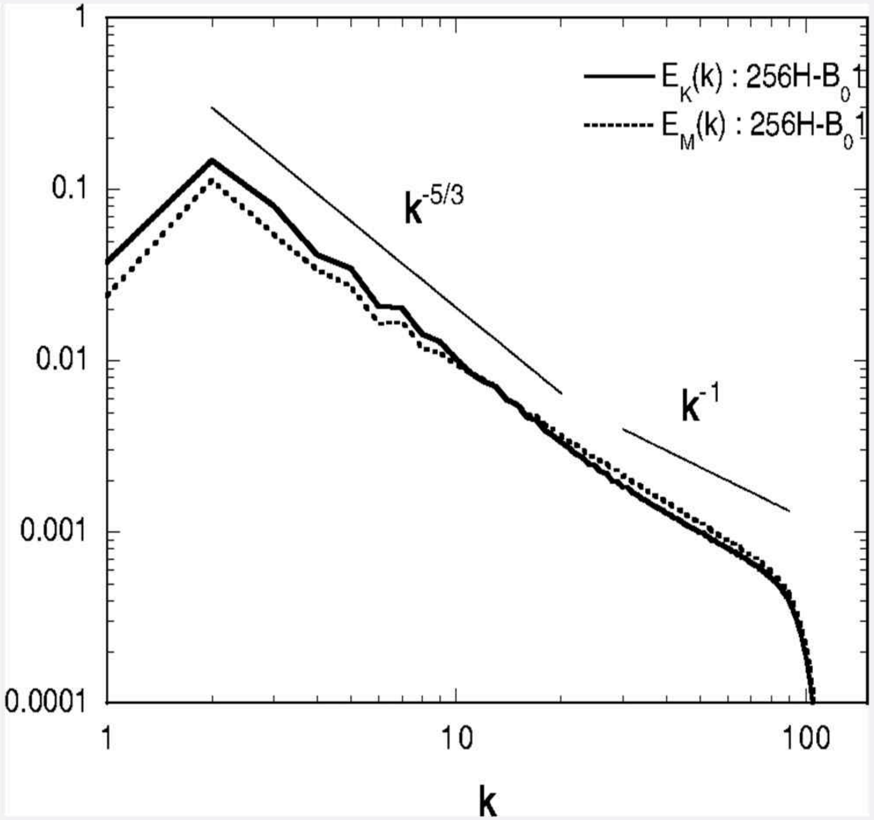
Numerical test: Cho & Vishniac (2000)



-pseudo-spectral method
-256^3

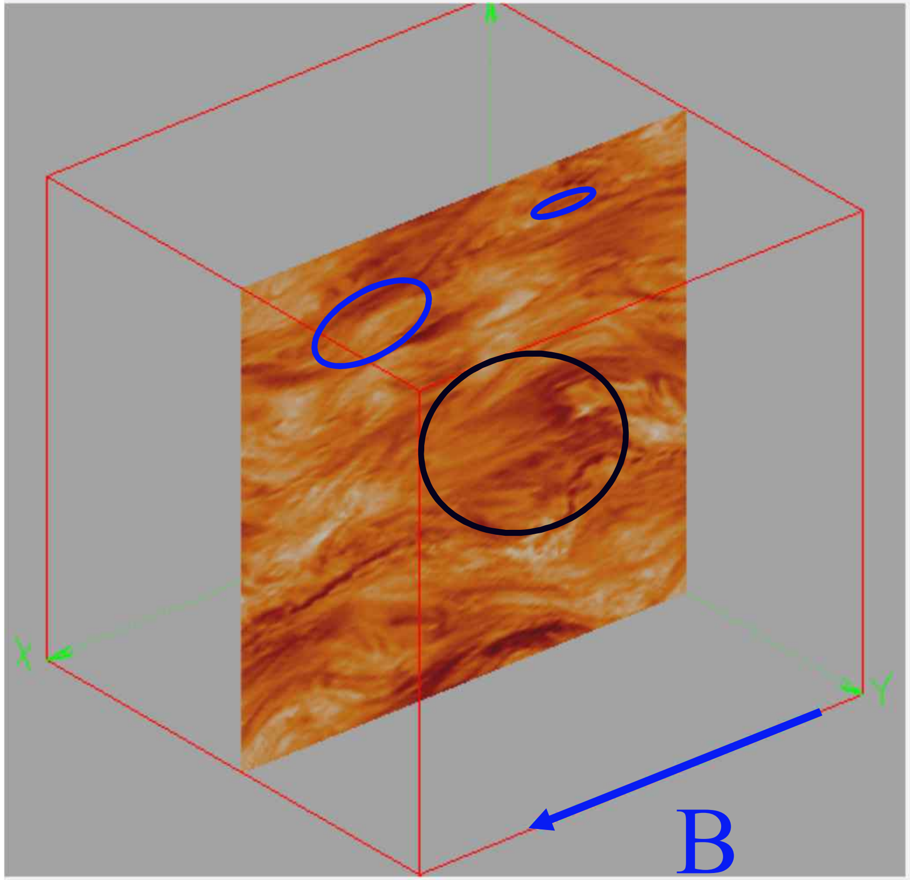
Spectra: Cho & Vishniac (2000)

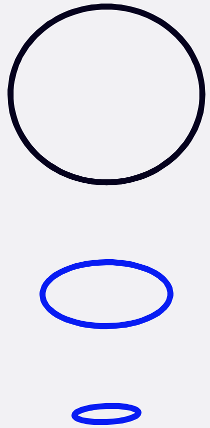




See also Muller & Biskamp (2000); Maron & Goldreich (2001)

Anisotropy

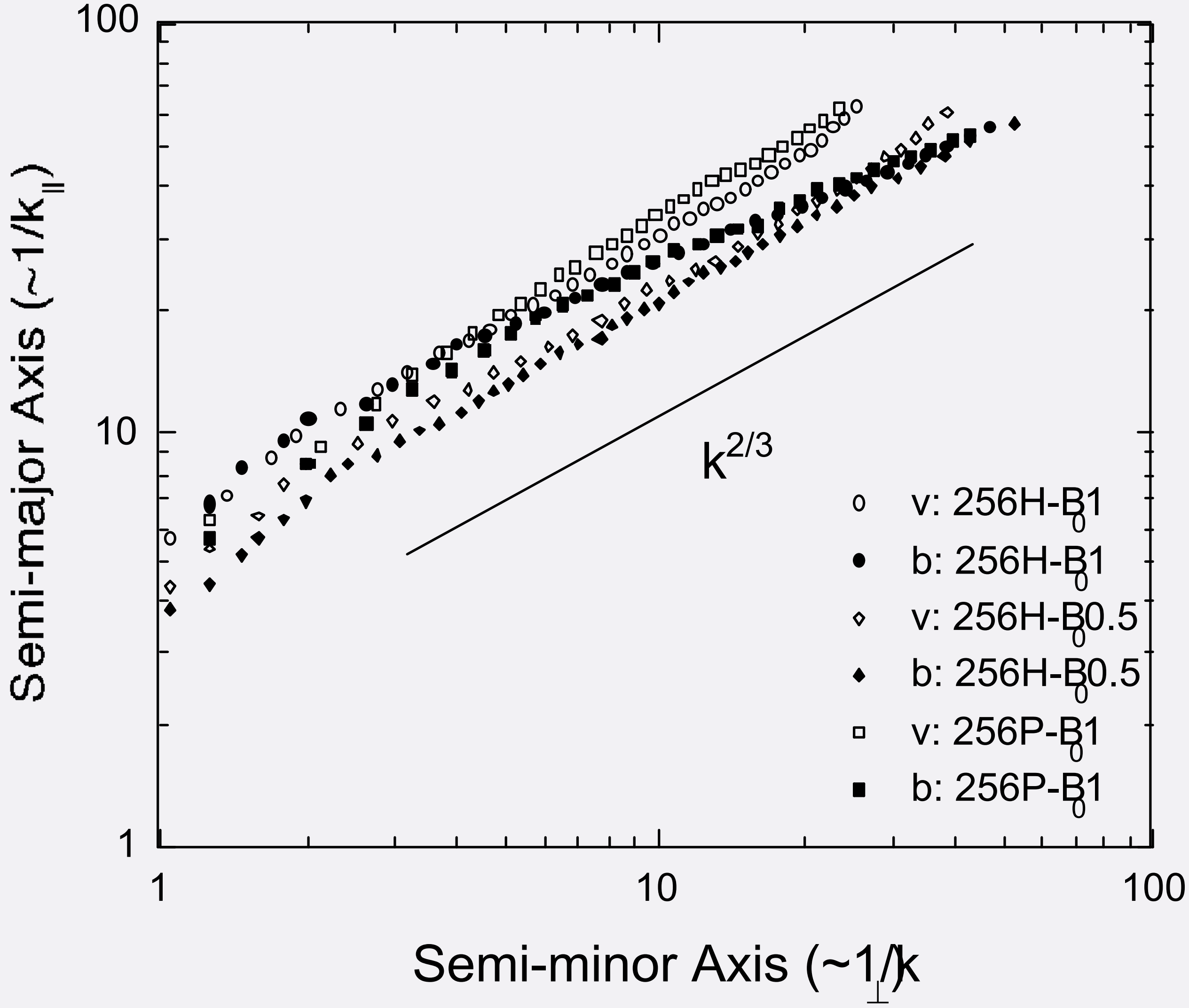




Smaller eddies are more elongated

=> Relation between parallel size and perp size?

Anisotropy: Cho & Vishniac (2000)



* Maron & Goldreich (2001) also obtained a similar result

Summary for Strong Alfvenic MHD turbulence

- Theory: Goldreich & Sridhar (1995, ApJ)

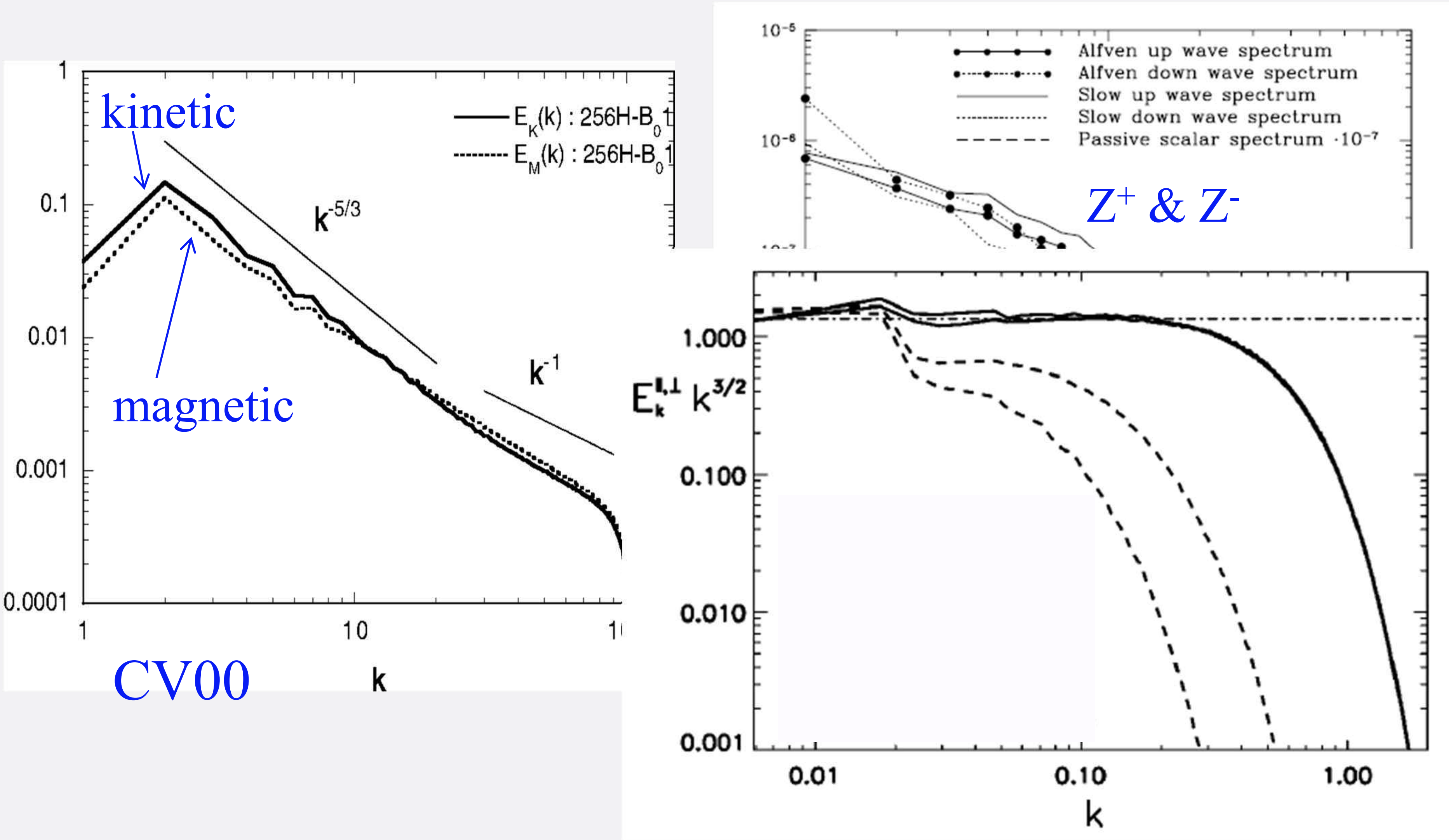
➔Kolmogorov spectrum +anisotropic structures

$E(k) \propto k^{-5/3}$

$k_{\parallel} \propto k_{\perp}^{2/3}$

- Numerical test: Cho & Vishniac (2000, ApJ)

Spectrum: Is the spectrum really a Kolmogorov?



Muller et al (2003)

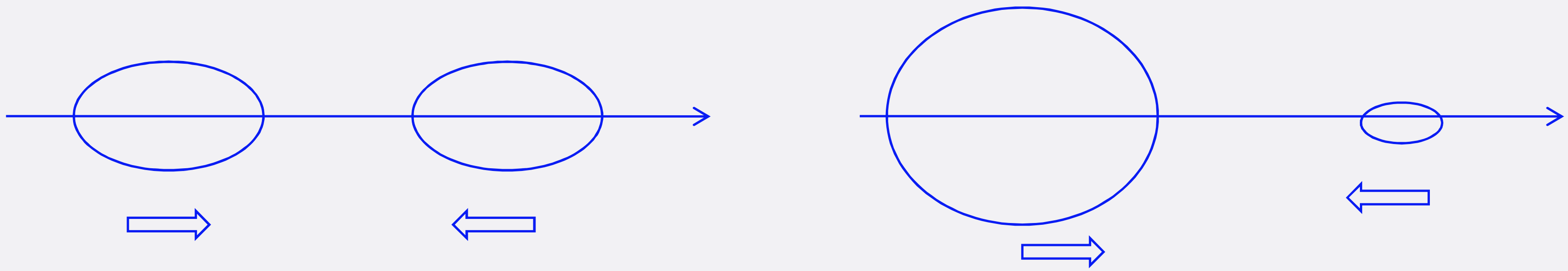
Cause? ➔ Alignment? Or something else?

-Boldyrev 05
-Beresnyak & Lazarian 06, 09
-Mason+ 06
-Gogoberidze 07
-Matthaeus+08
-Podesta & Bhattacharjee 10
-Podesta 11
-Beresnyak 11
-…

Locality

-Locality=interaction of similar size eddies

Non-locality



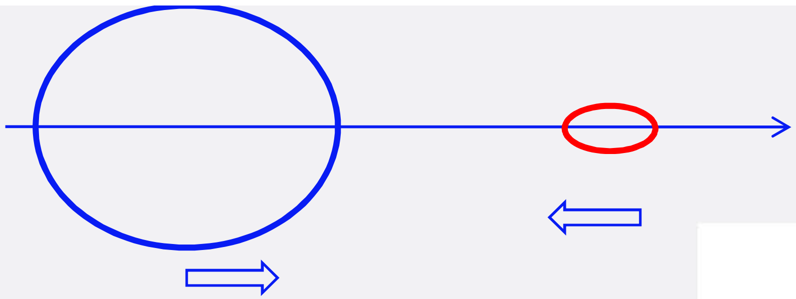
- Locality=interaction of similar size eddies

- In HD, locality is a fairly good approximation

(Verma+ 05; Alexakis+ 07; Mininni+08; Eyink & Aluie 08; Aluie & Eyink 08;…)

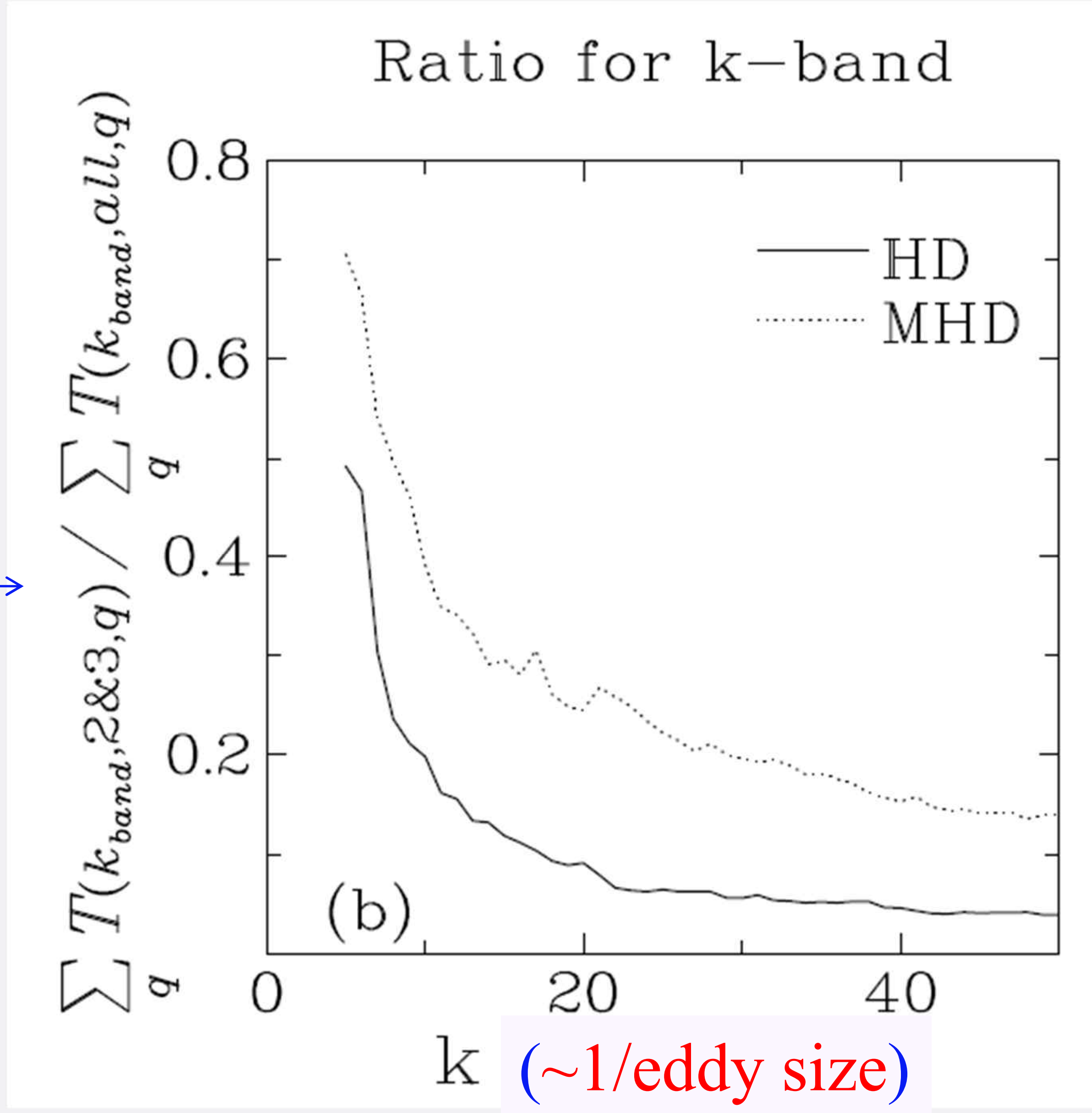
- In MHD, there have been some discussions

(Alexkis+ 05; Alexkis 07; Carati+ 06; Lessinnes+ 08; Yusef+ 09;
Aluie & Eyink 10; Beresnyak & Lazarian 10)



outer-scale eddy

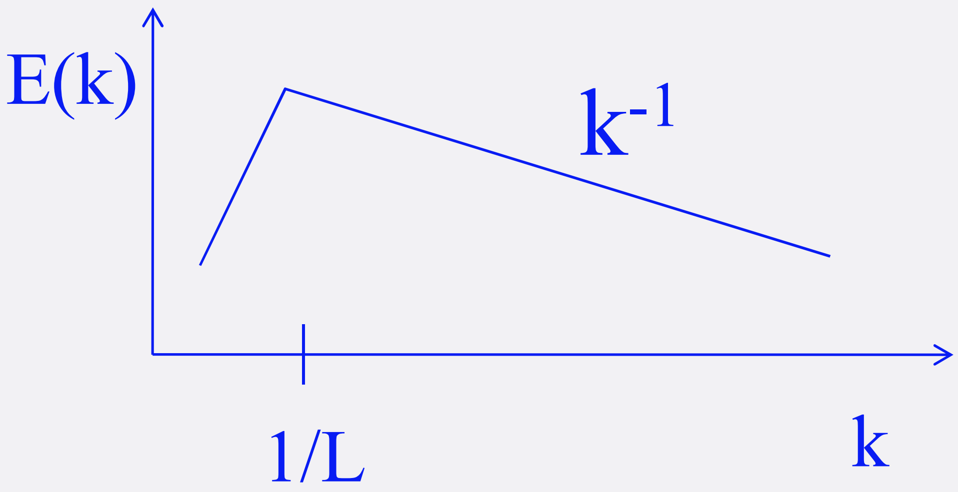
$$\frac{\text{Shear by the outer scale}}{\text{total shear}}$$



Cho (2010, ApJ)

Implication?

If the outer-scale shearing motions completely dominate other-scale motions, then the energy spectrum will be $E(k) \propto k^{-1}$ (Cho, Lazarian, & Vishniac 02,03)



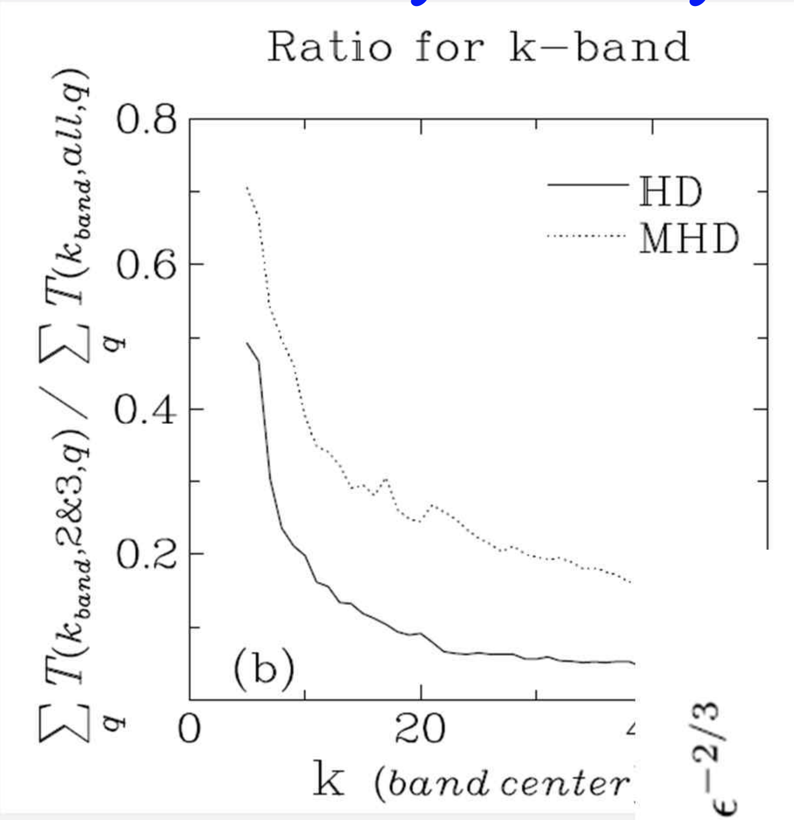
$$\frac{b_k^2}{t_{cas}} \sim \frac{b_k^2}{L/v_L} = constant$$

$$b_k^2 \sim kE(k) = constant$$

$$E(k) \propto k^{-1}$$

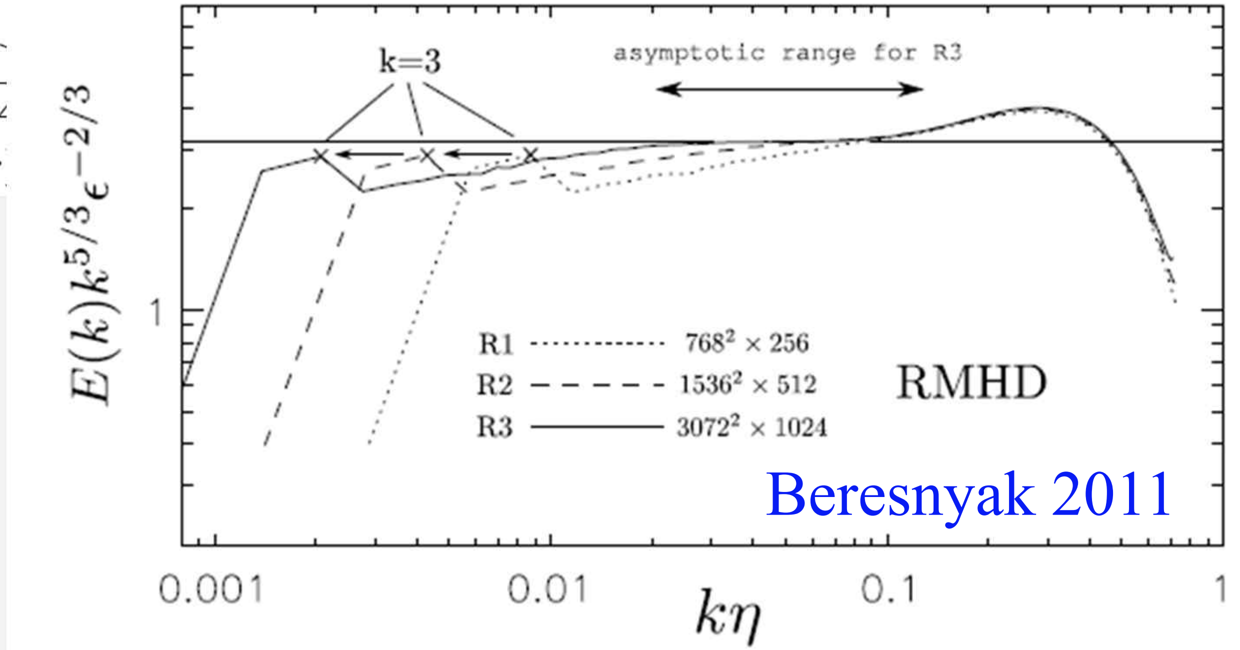
If the outer-scale shearing motions partially dominate other-scale motions, then the energy spectrum will become shallower than $k^{-5/3}$!

Non-locality on very small scales?



... the non-local effects of the outer scale will ultimately vanish on very small scales. (Cho 2010)

If this is true, we will have a $k^{-5/3}$ spectrum on very small scales



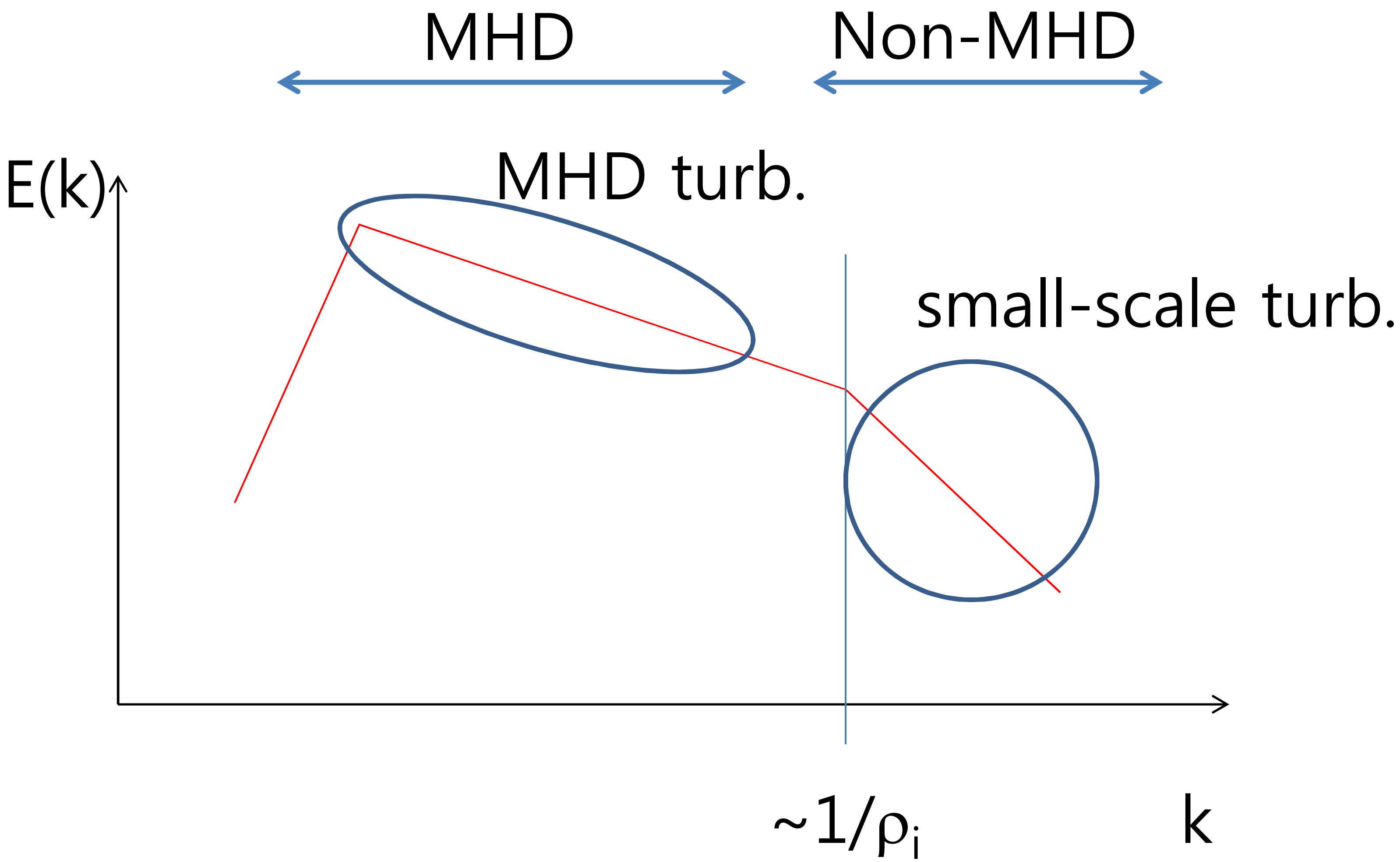
Beresnyak 2011

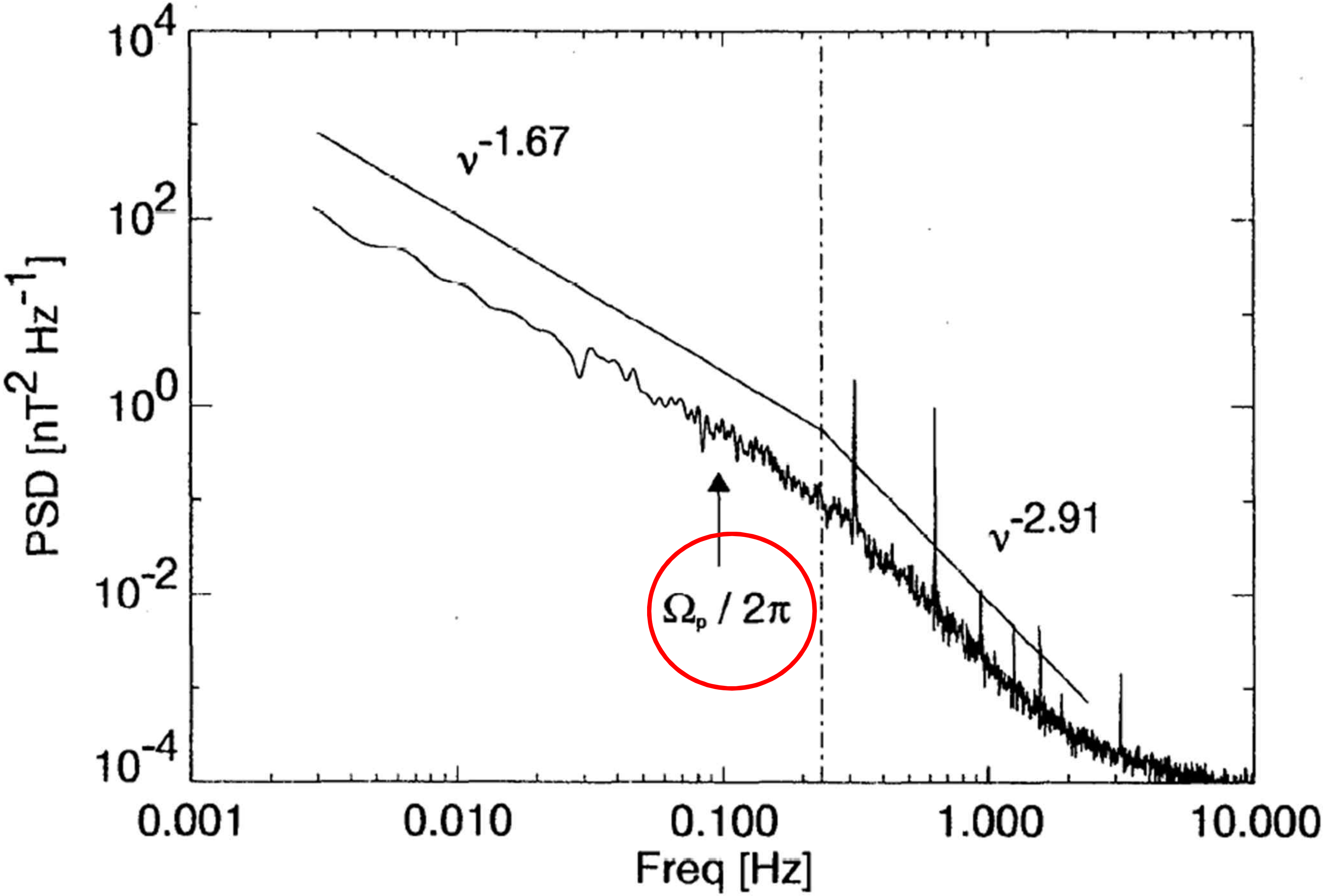
Conclusion?

- Non-locality can explain shallow energy spectrum
  * But, it may be a transient effect!

- I think non-locality and alignment etc. are related.

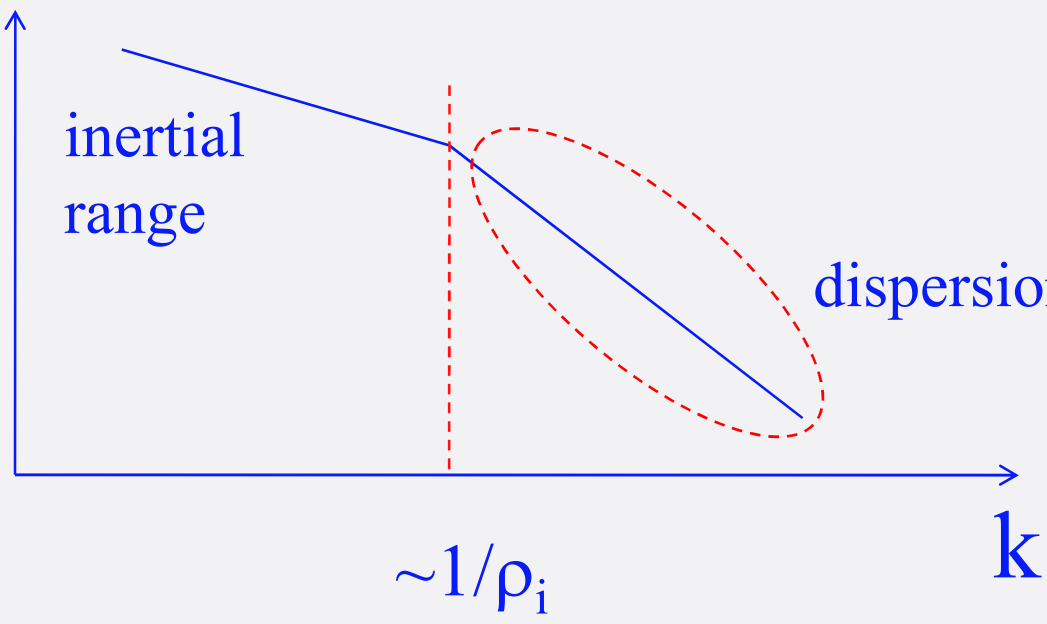
Topic 2. Small-scale turbulence





Leamon et al (1999)

- The power index below the break: between -2 and -4.
- This range is termed "dispersion range"
- Recent studies: Dmitruk & Matthaeus (2006); Schekochihin et al (2007), Howes et al (2008), Saito et al (2008), Gary, Saito & Li (2008), …



Electron MHD: Introduction

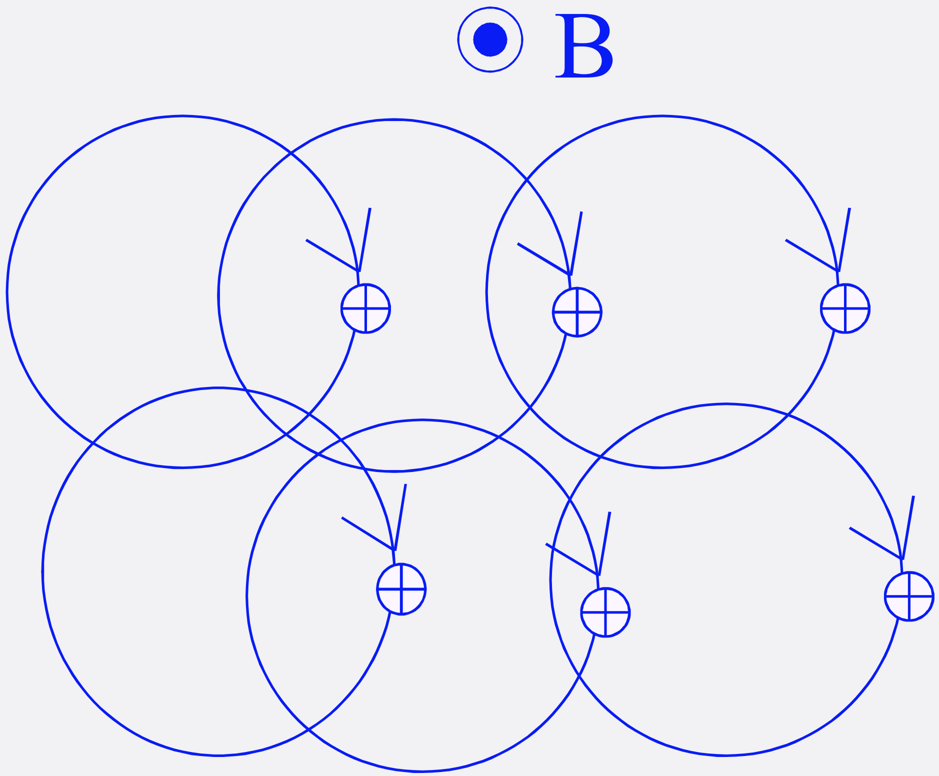
- How can we deal with small-scale physics?
- EMHD is a simple fluid-like description of small-scale physics

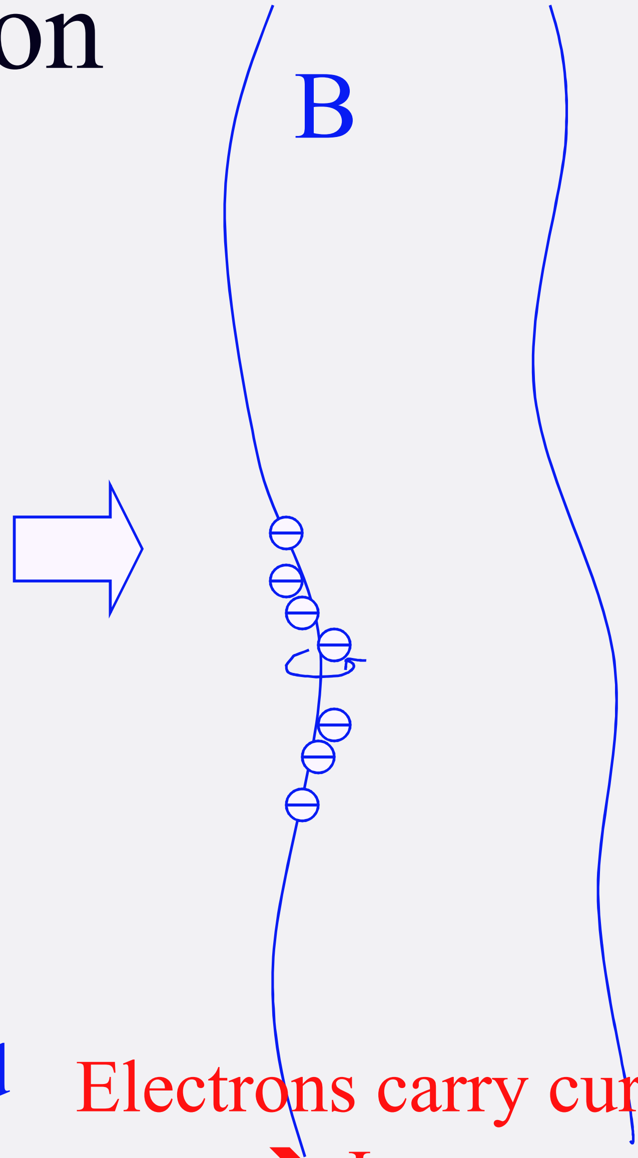
(Gyro-kinetic of PIC simulations would be better. But,…)

- The starting point is the magnetic induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

EMHD: Introduction





Protons ➔ smooth background

Electrons carry current

➔ J ∝ v

Electron MHD eq

$\mathbf{J} \propto \mathbf{v}$

$+$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},$$

0

$\mathbf{v} \propto \nabla \times \mathbf{B}$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

Ordinary incompressible MHD vs. EMHD turbulence

$$\frac{\partial \boldsymbol{v}}{\partial t} = -(\nabla \times \boldsymbol{v}) \times \boldsymbol{v} + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{v} + \boldsymbol{f} + \nabla P' ,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} ,$$

-Studied since 1960's

-Goldreich & Sridhar 1995

$E(k) \propto k^{-5/3}$

$k_{\|} \propto k_{\perp}^{2/3}$

-Numerical test:

Cho & Vishniac 2000

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

-Studied since 1990's

-Energy spectrum:

$E(k) \propto k^{-7/3}$

(Vainshtein 1973;
Biskamp-Drake 1990's)

-Anisotropy:

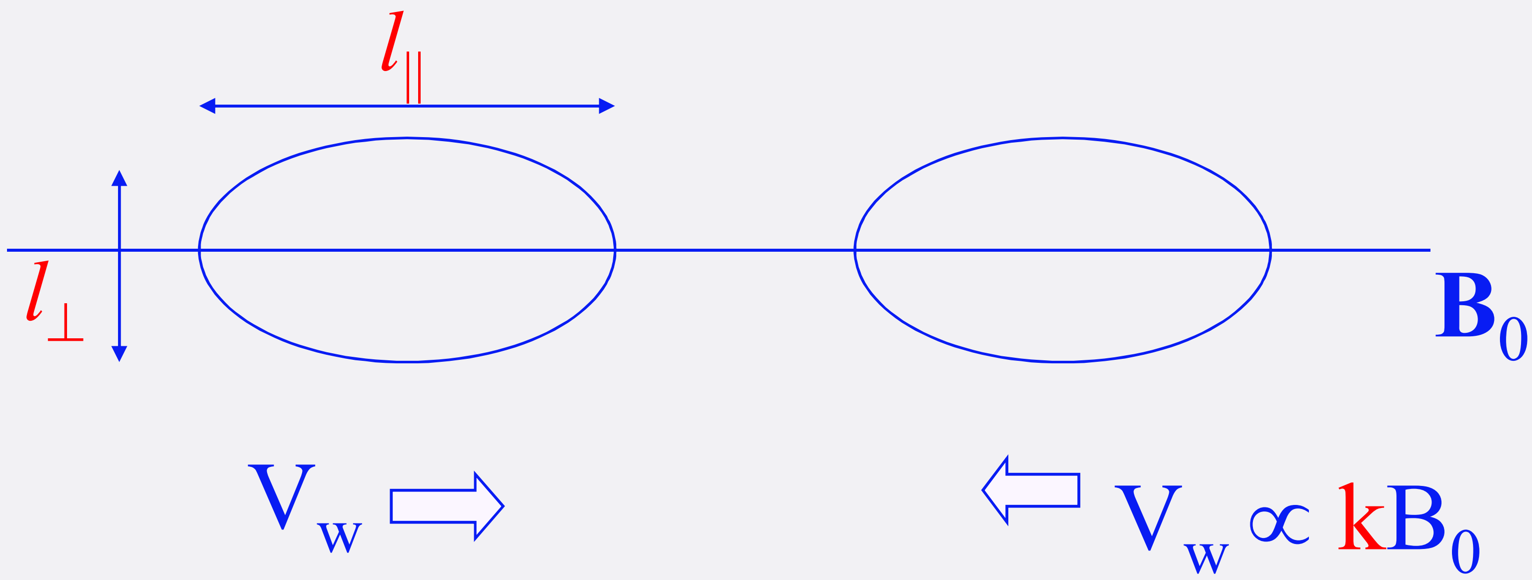
$k_{\|} \propto k_{\perp}^{1/3}$

(Cho & Lazarian 2004)

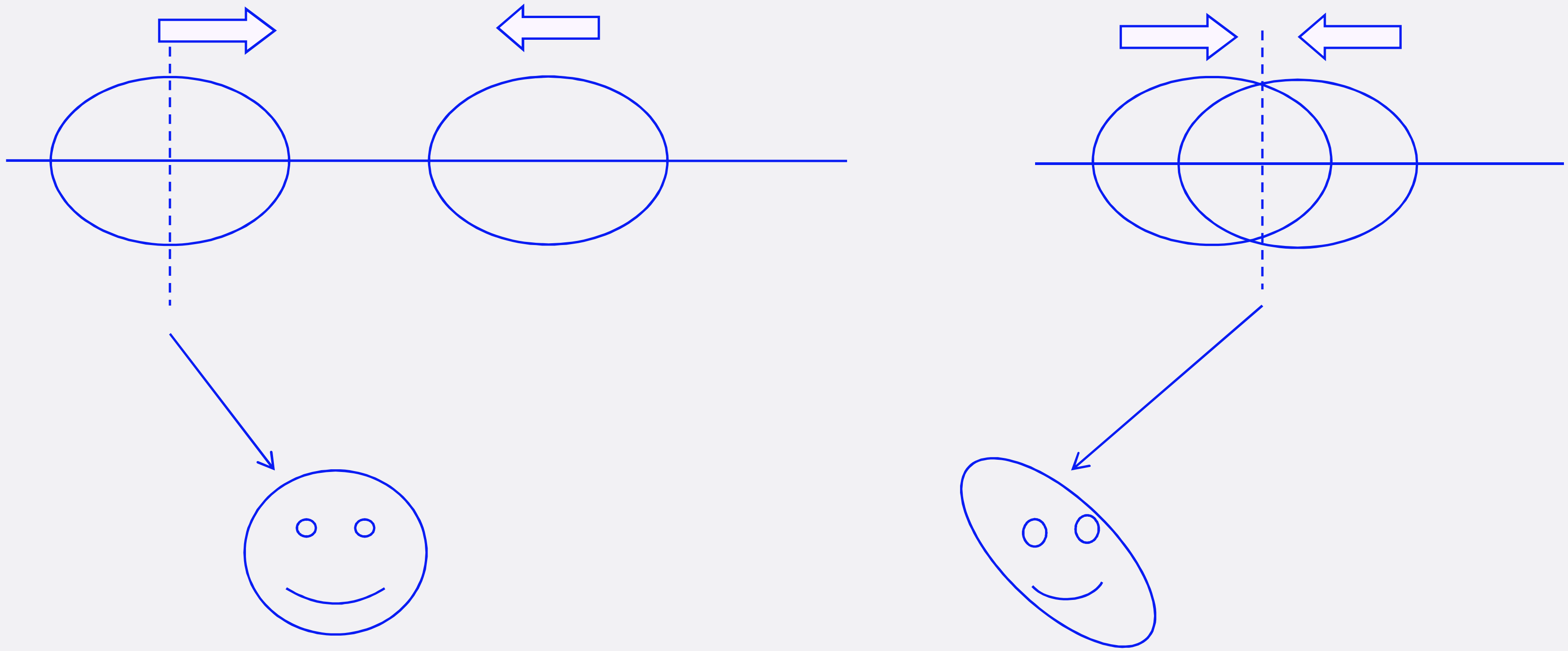
Scaling of EMHD turbulence

Consider two EMHD wave packets:

*Note: perturbations propagate along B



1 collision is enough to complete cascade!



-Distortion time scale ~ $l_\perp / v_l \sim l_\perp l_\perp / b_l$

-Duration of collision ~ $l_\perp l_\parallel / B_0$

$t_w / t_{eddy} \sim (l_\parallel / B_0) / (l_\perp / b) \sim (b\, l_\parallel / l_\perp B_0) \sim 1$

Cho & Lazarian (2004, ApJ)

- Critical balance

$$\frac{l_\perp^2}{b_{\perp l}} = \frac{l_\perp l_\parallel}{B_0}$$

- Constancy of energy cascade rate

$$\frac{b_{\perp l}^2}{t_{cas}} = \text{const}$$

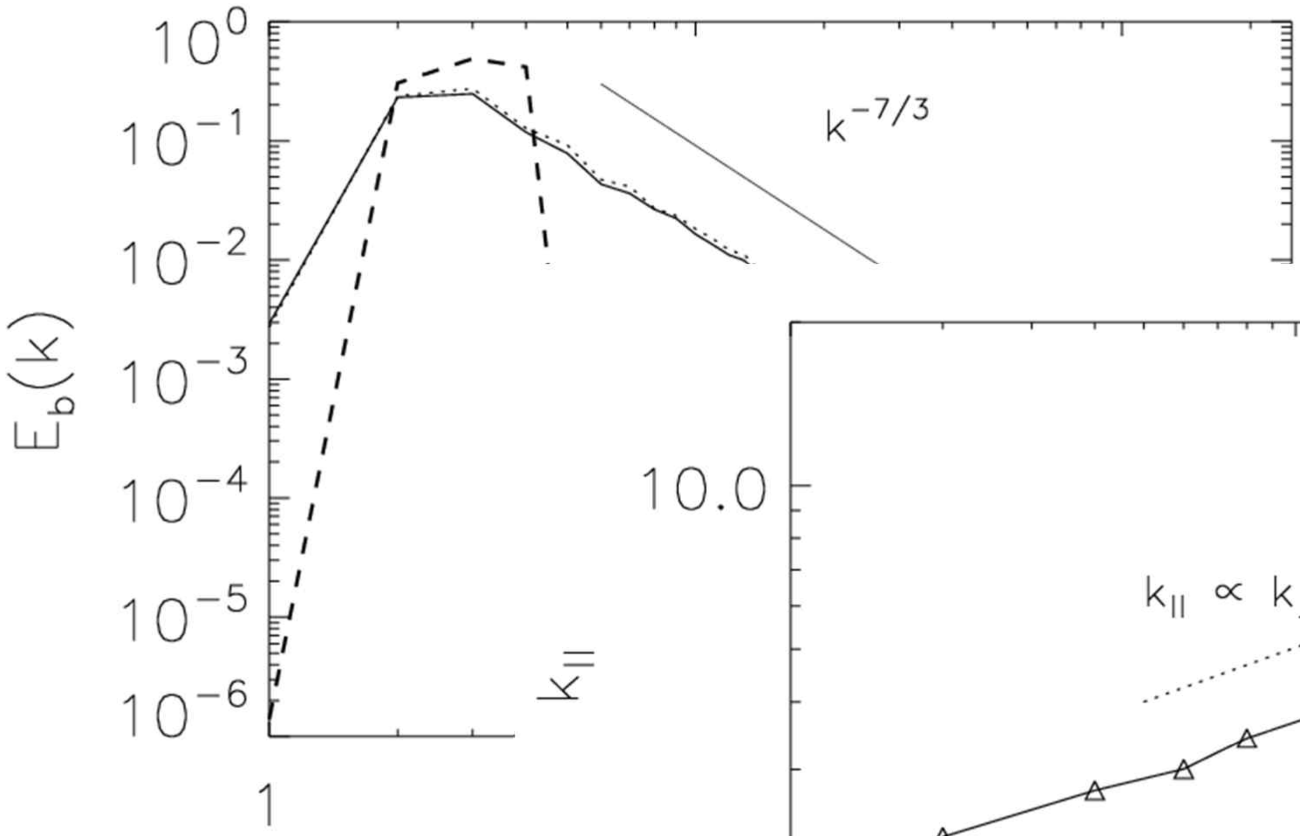
$$\frac{b_{\perp l}^2}{(l_\perp^2 / b_{\perp l})} = \text{const}$$

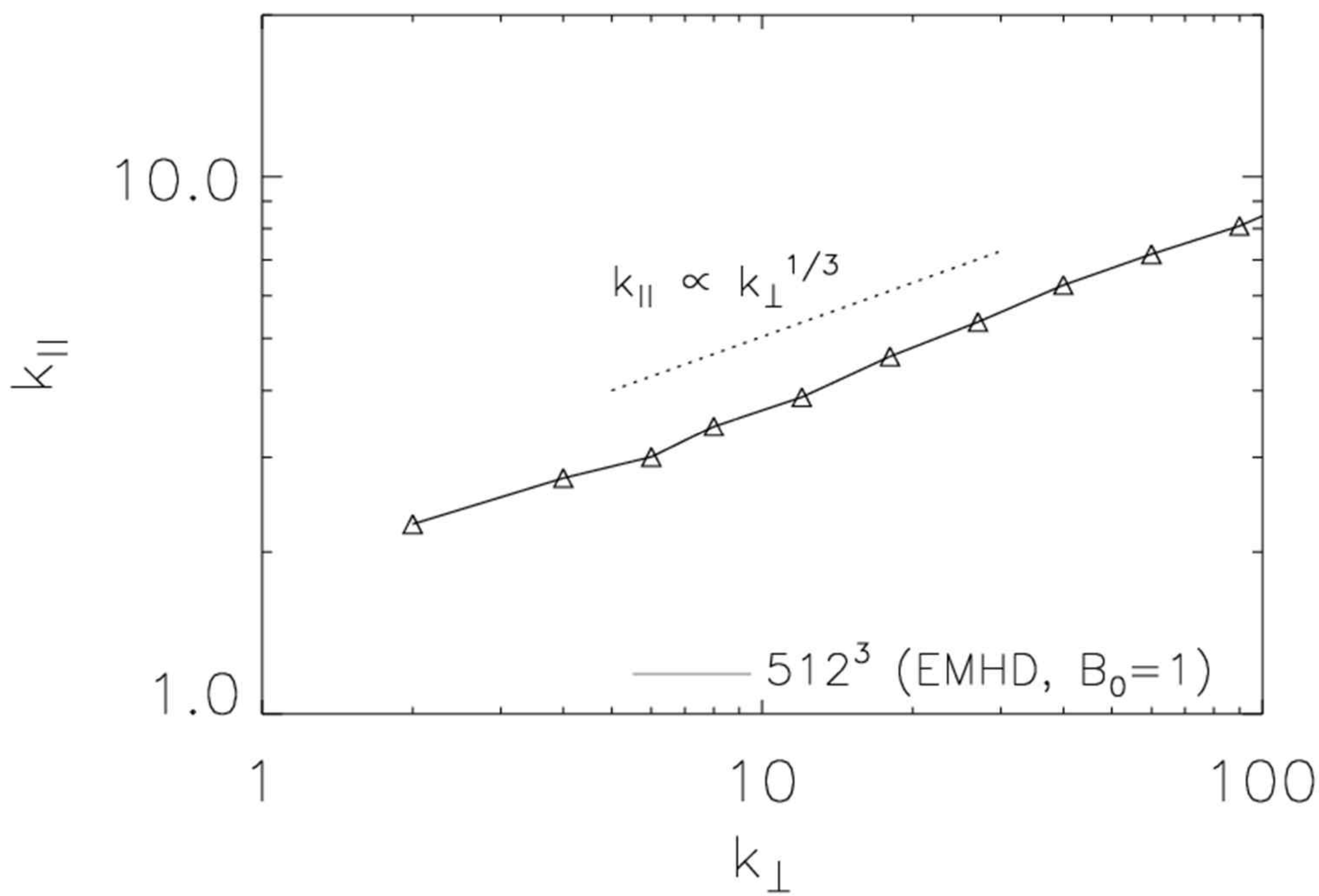
$$b_{\perp l} \sim l_\perp^{2/3}$$

Or, $E(k) \sim k^{-7/3}$

$$l_\parallel \sim l_\perp^{1/3}$$

Cho & Lazarian (2004, 2009):





512^3

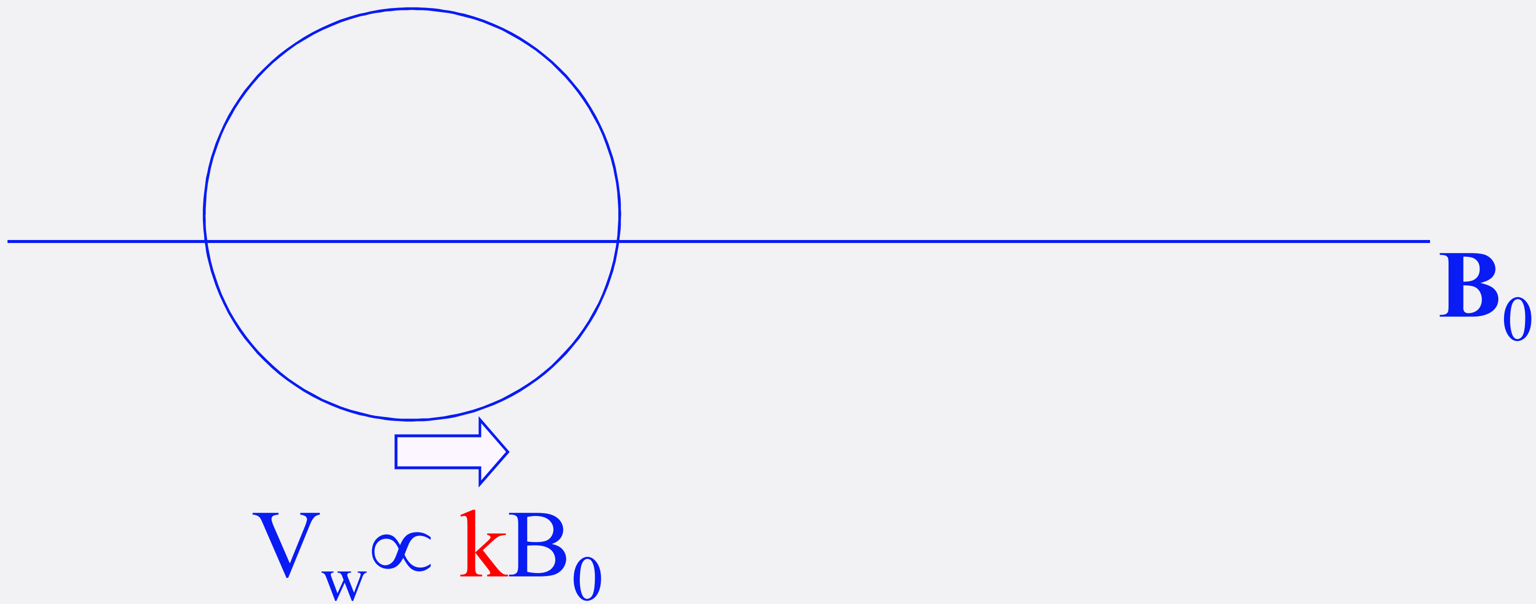
Summary for EMHD Turbulence

- Spectrum of B : $E(k) \sim k^{-7/3}$
- Anisotropy: $l_{||} \sim l_{\perp}^{1/3}$

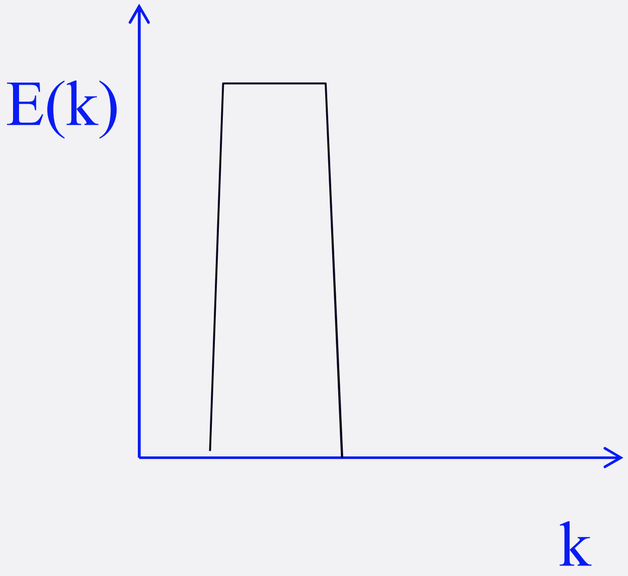
*We considered strong turbulence only.
For weak turbulence, see for example
Galtier & Bhattacharjee (2003)
+ Galtier's talk this afternoon

Topic 3. Scaling of EMHD wave packets
(≈completely imbalanced EMHD turb.)

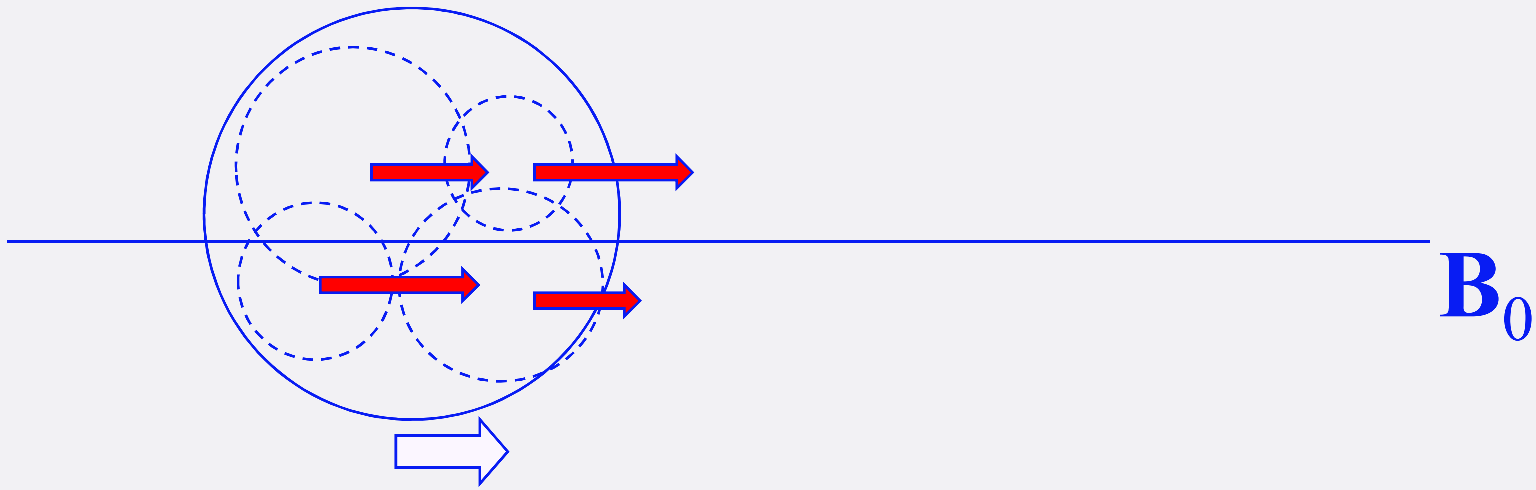
Consider one EMHD wave packet:



Energy spectrum at t=0:

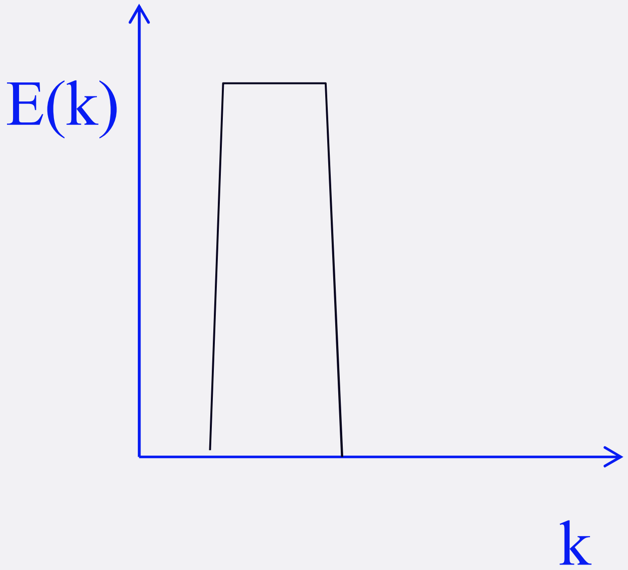


➔ Sum of sub-structures

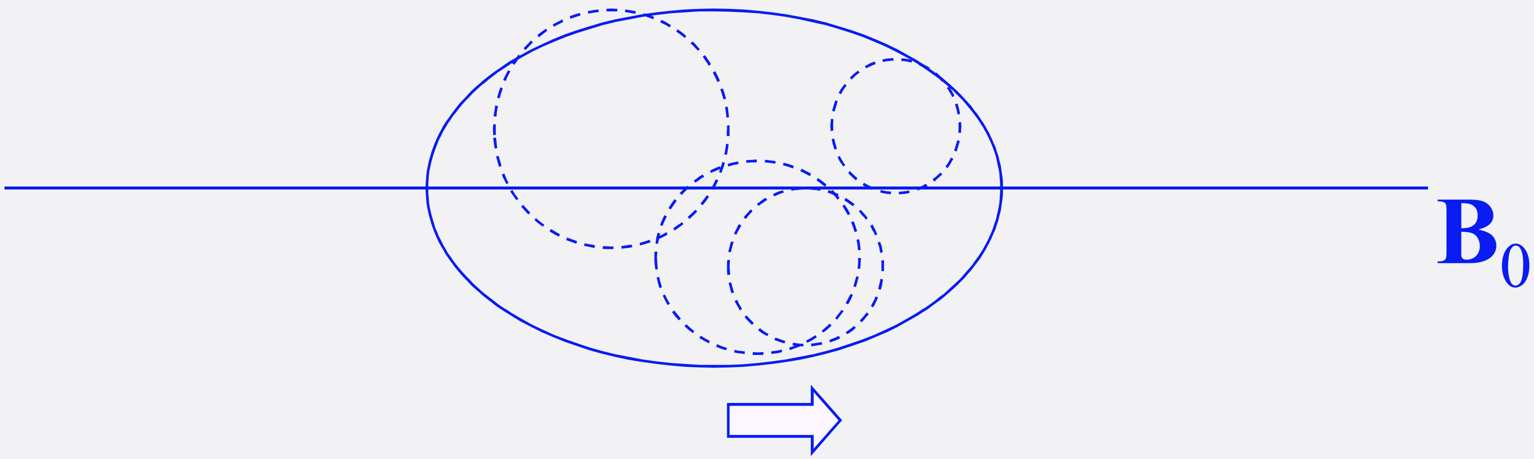


$V_w \propto kB_0$

Energy spectrum at t=0:

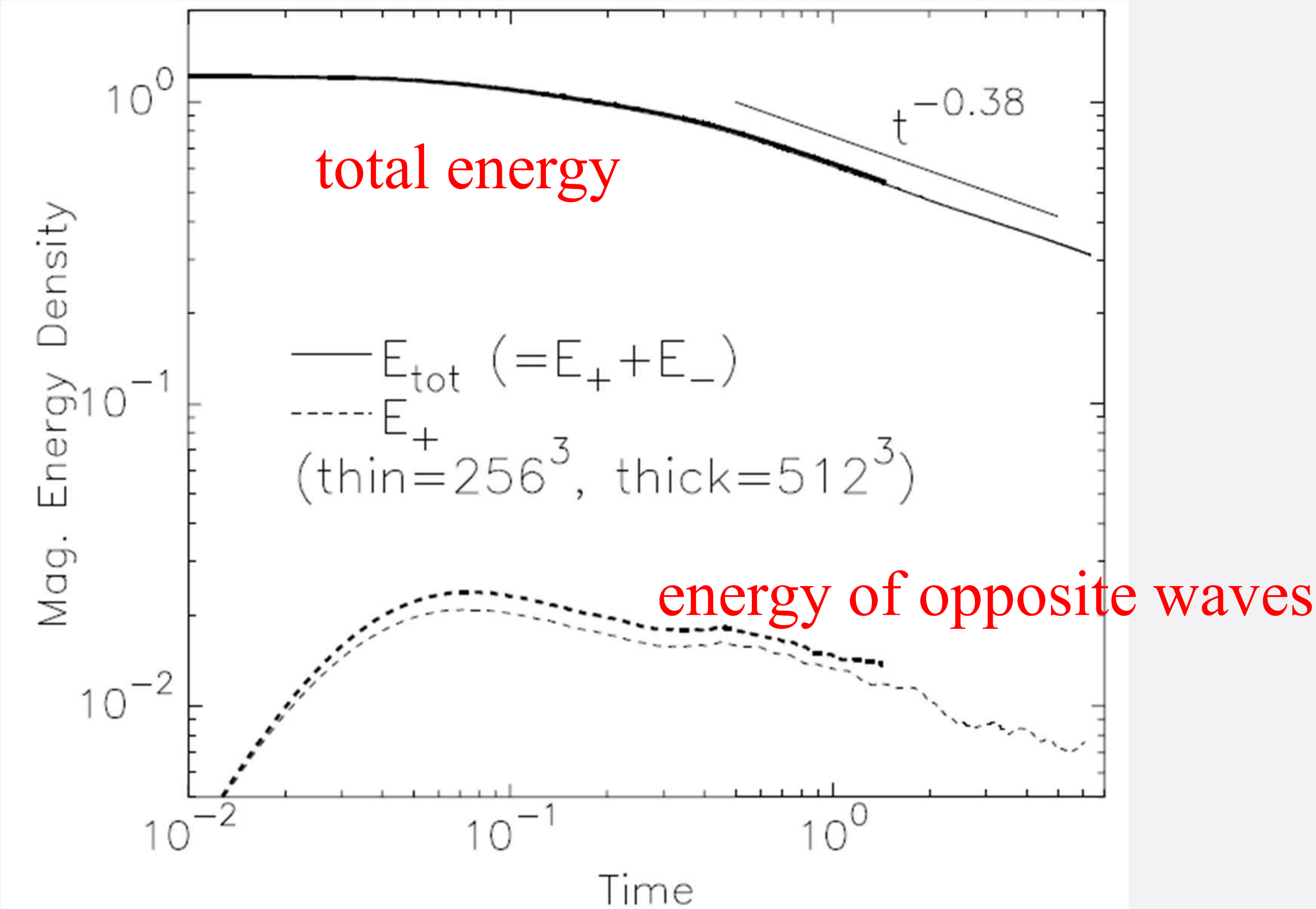


➔ Sum of sub-structures



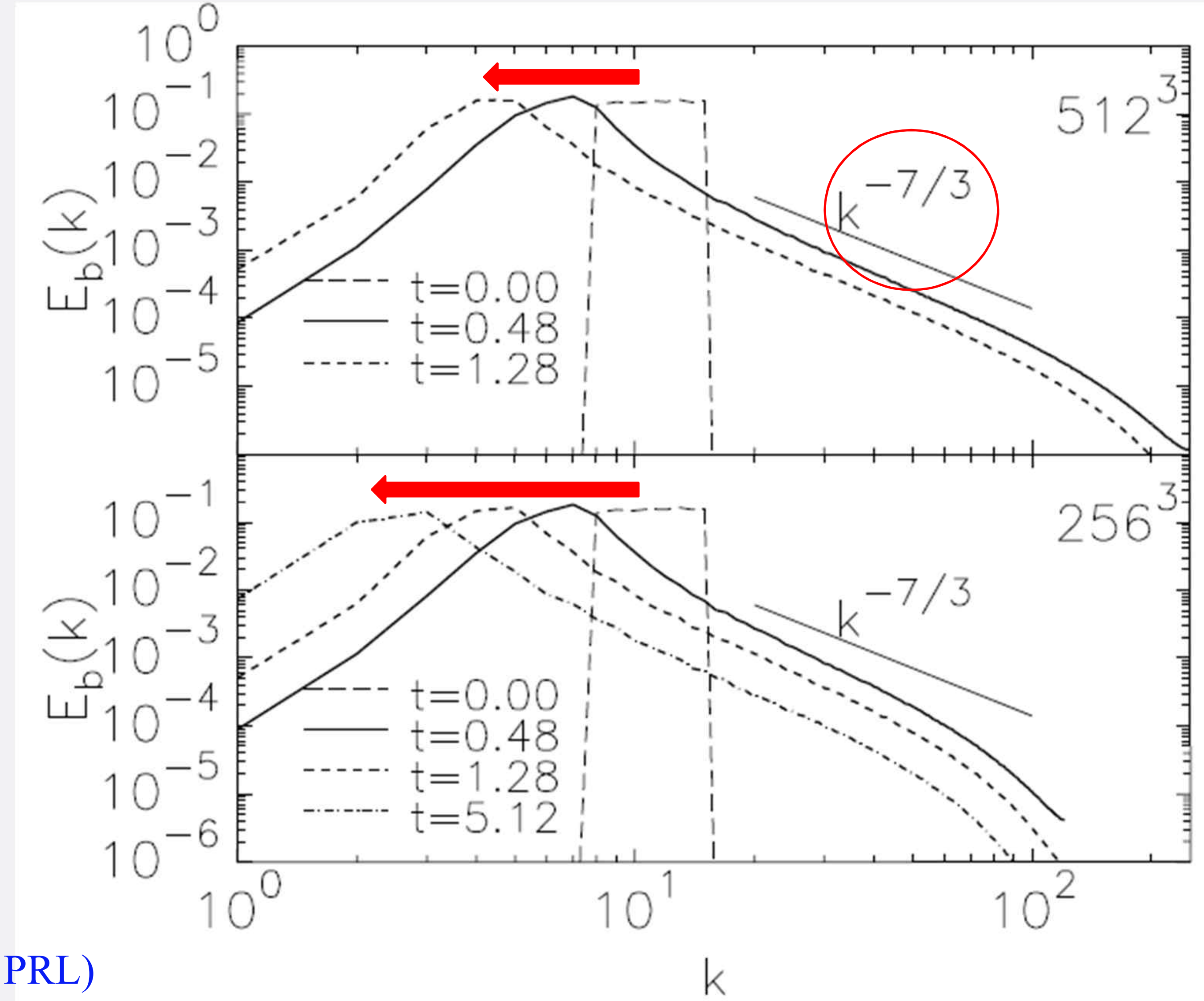
➔ Self-interactions can result in energy cascade

Magnetic energy decays!



This result is consistent with an earlier 2D result
by Ng, Bhattacharjee, et al (2003).

Energy spectrum shows an unexpected behavior!

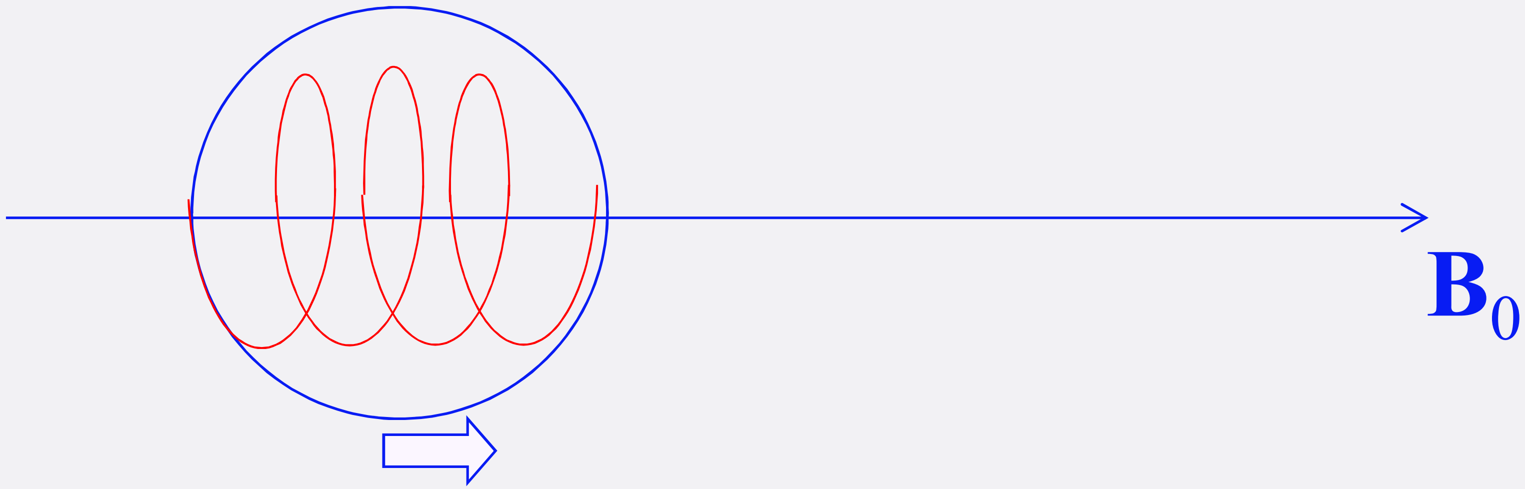


Cho (2011, PRL)

➔inverse cascade of energy!

Why is it happening?

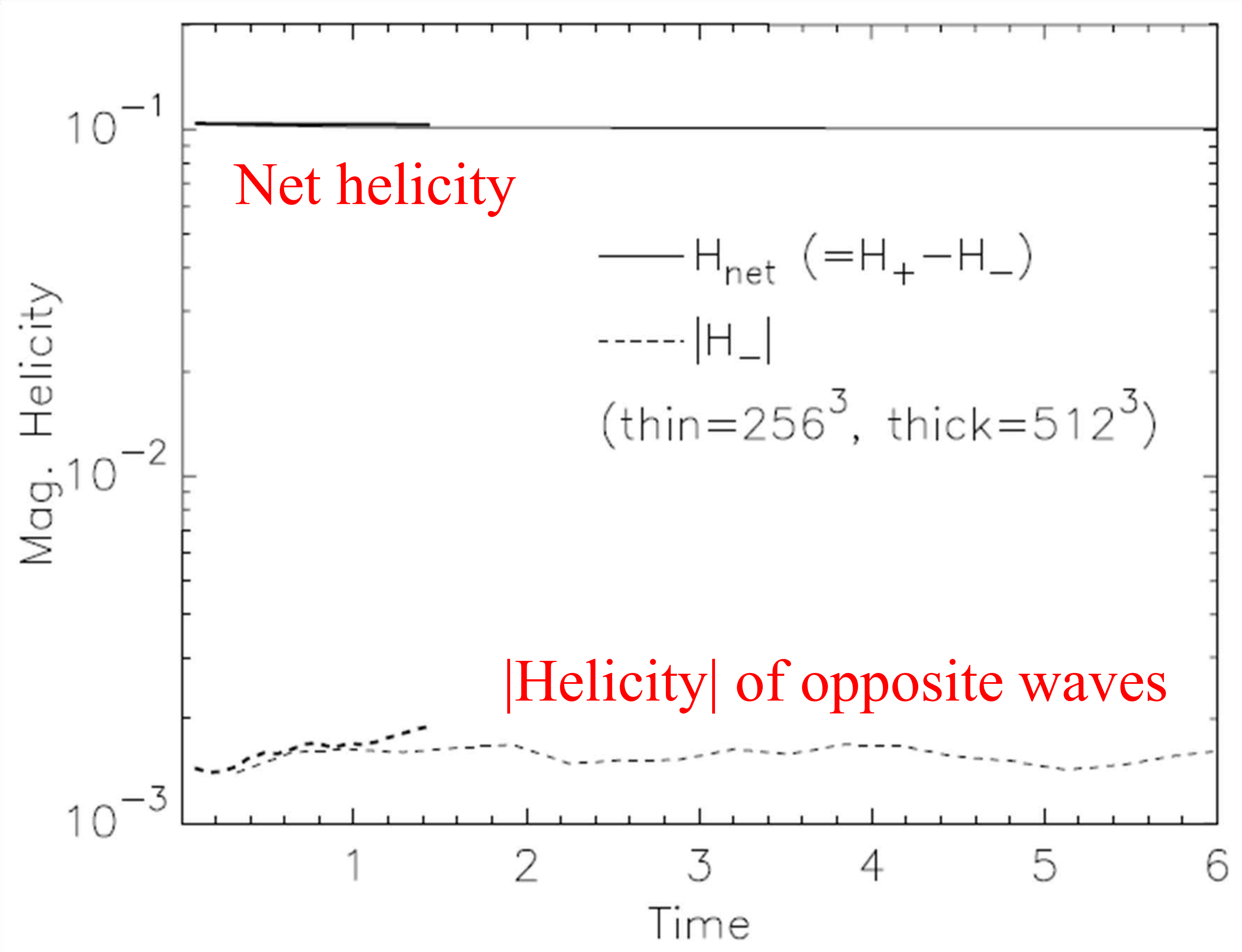
➔ Answer: magnetic helicity ($\propto \mathbf{A} \bullet \mathbf{B}$) conservation!



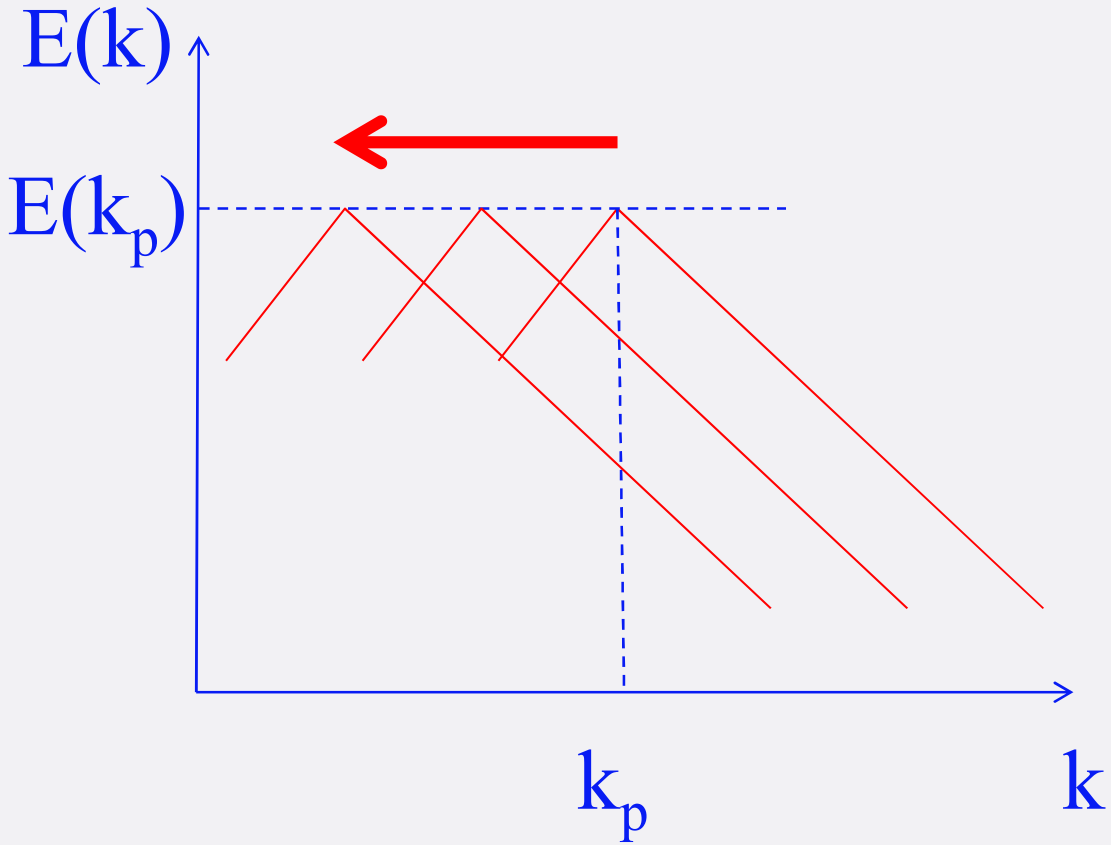
We can show that

1. magnetic field is helical
2. spectrum of magnetic helicity = E(k)/k

Conservation of magnetic helicity



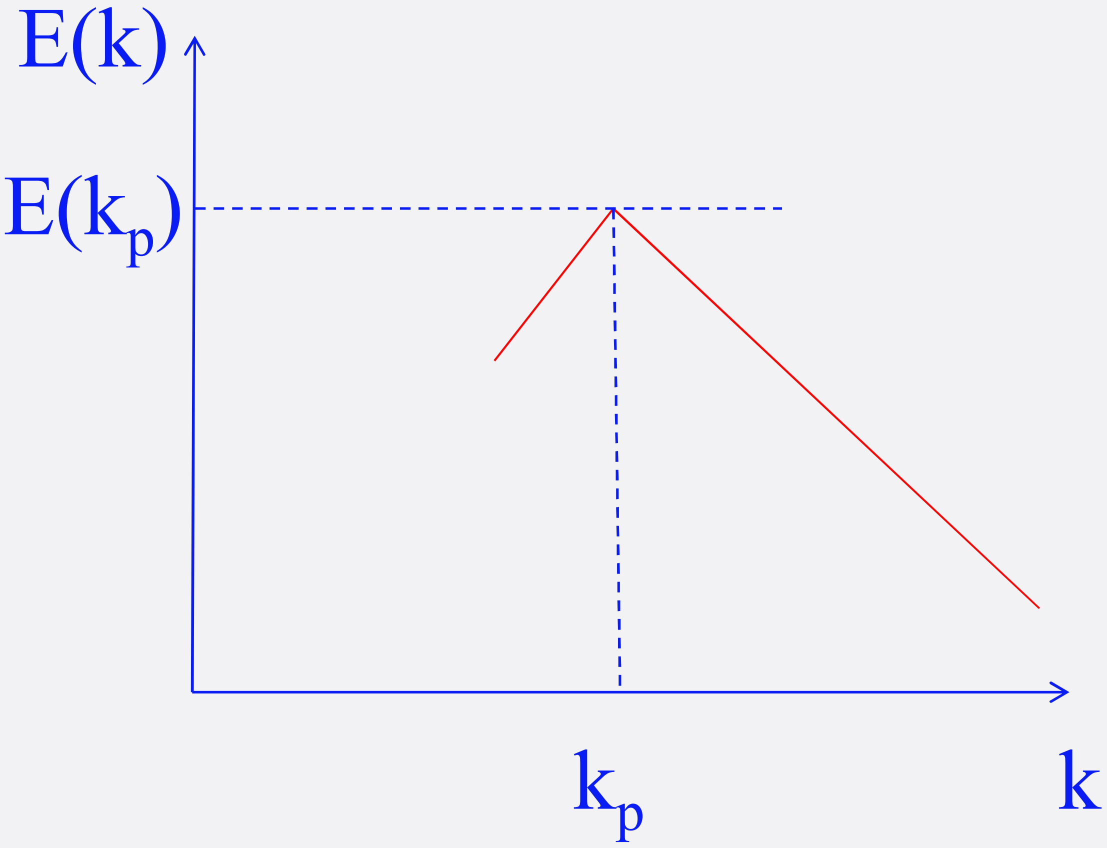
Implications of magnetic helicity conservation



Energy ≈ $k_p E(k_p)$

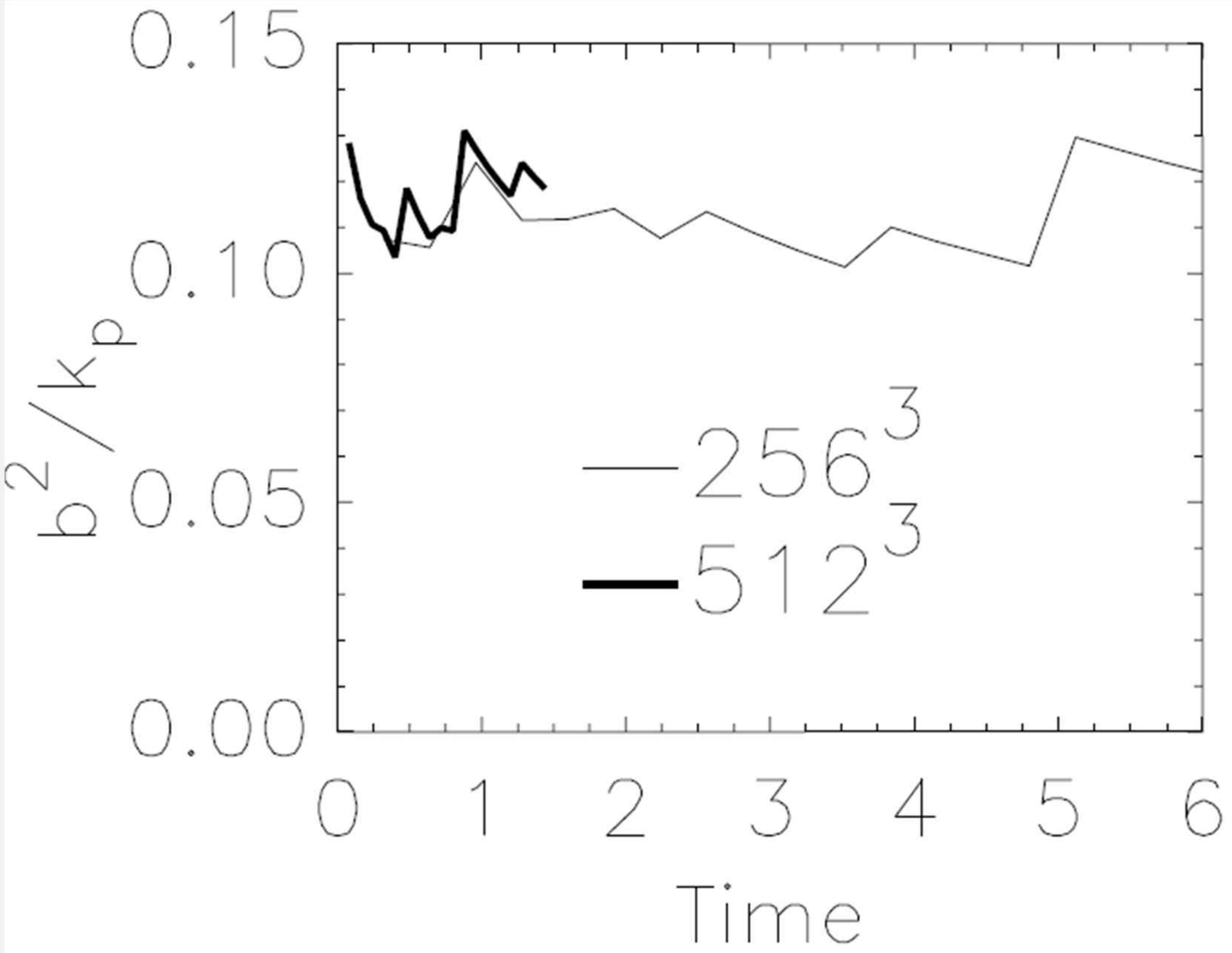
Helicity ≈ $k_p[E(k_p)/k_p] = E(k_p)$ ➔ $E(k_p)$ ≈ constant

Implications of magnetic helicity conservation



$\text{Energy} \approx k_p E(k_p)$

$E(k_p) = \text{constant}$ ➔ $k_p(t) \propto b^2(t)$



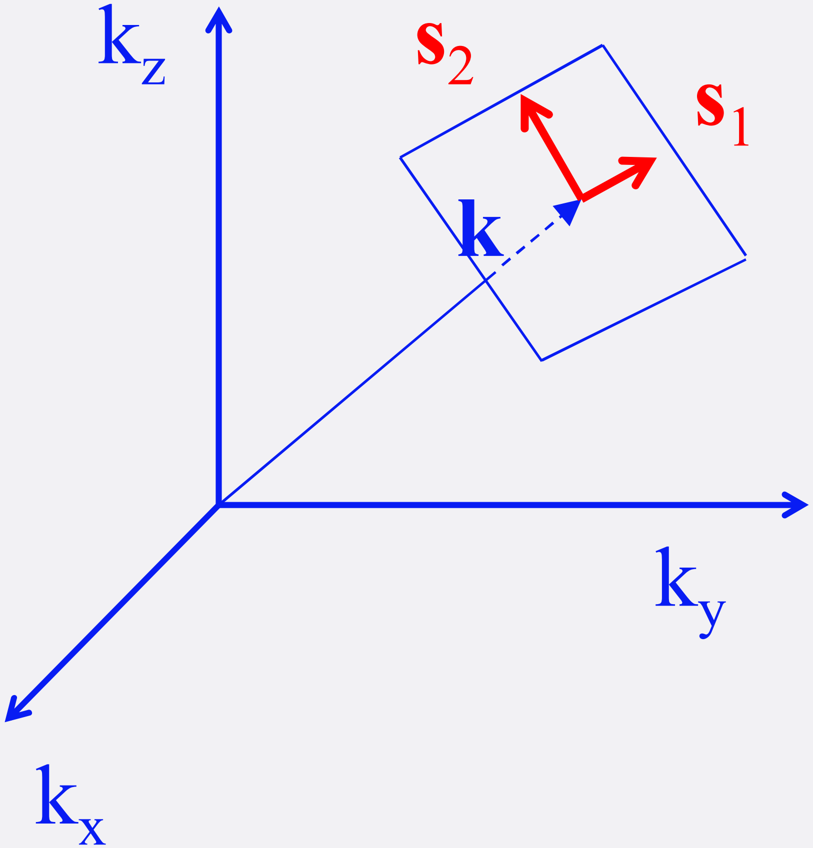
0.15
0.10
0.05
0.00
b^2/k_p
256^3
512^3
0
1
2
3
4
5
6
Time

Conclusion for small-scale turbulence

- Spectrum $\propto k^{-7/3}$ ← EMHD & GK
- Anisotropy = stronger than MHD ← EMHD
- Inverse cascade occurs due to helicity conservation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

$$\Rightarrow \frac{\partial \tilde{b}}{\partial t} = -i\boldsymbol{k} \times [(i\boldsymbol{k} \times \tilde{b}) \times \tilde{B}]$$

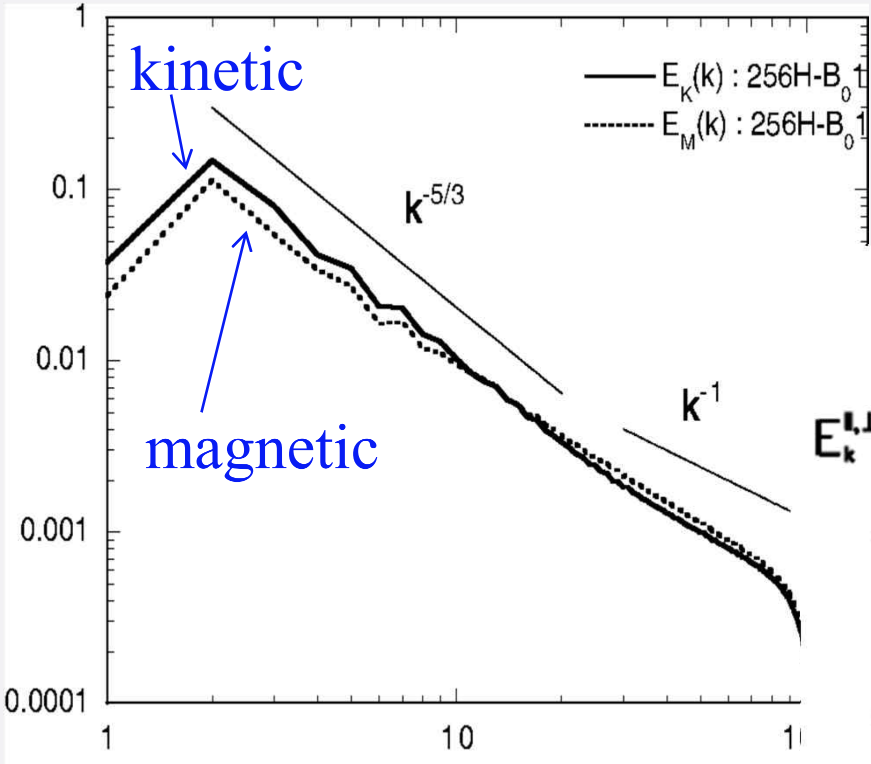


If $\tilde{\boldsymbol{b}} \propto \frac{(\boldsymbol{s}_1 + i\boldsymbol{s}_2)}{\sqrt{2}} \equiv \epsilon_+$,

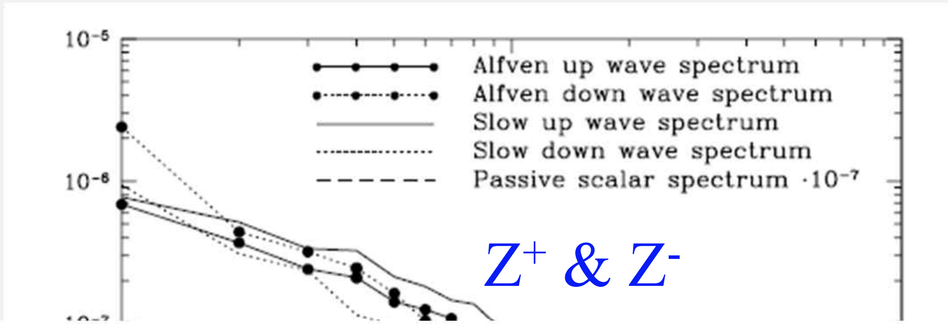
$$\frac{\partial(\tilde{b}\epsilon_+)}{\partial t} = -ikk_{\parallel}B_0(\tilde{b}\epsilon_+)$$

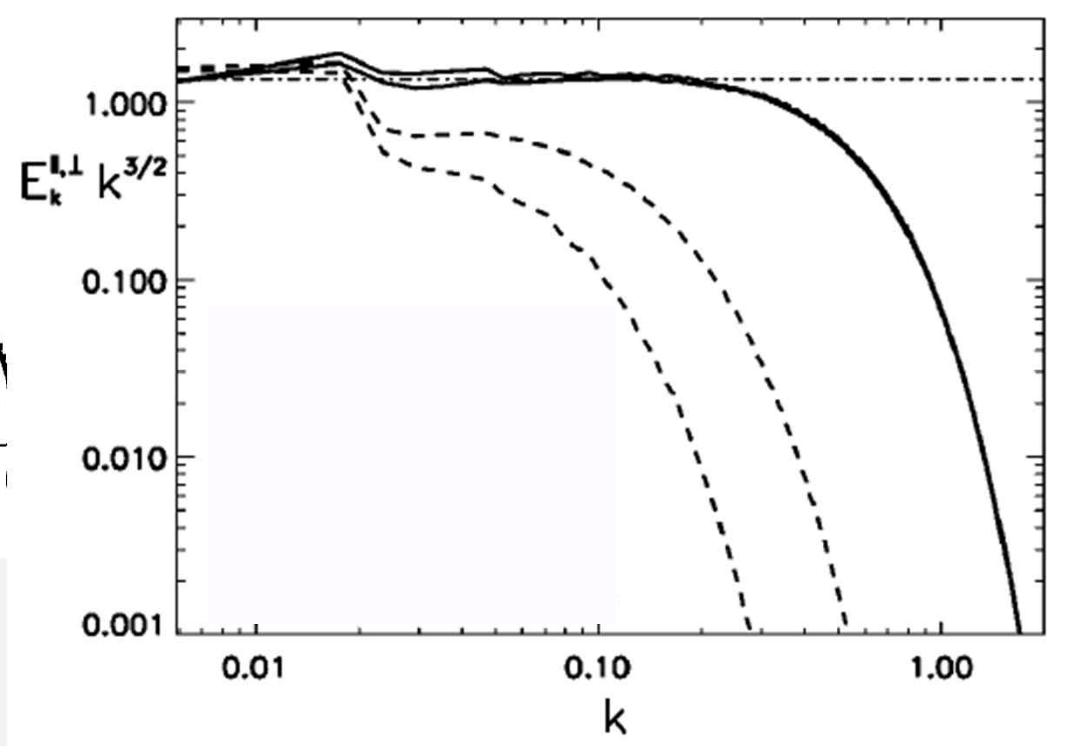
➔ Wave moving to +B direction

Spectrum: Is the spectrum really a Kolmogorov?

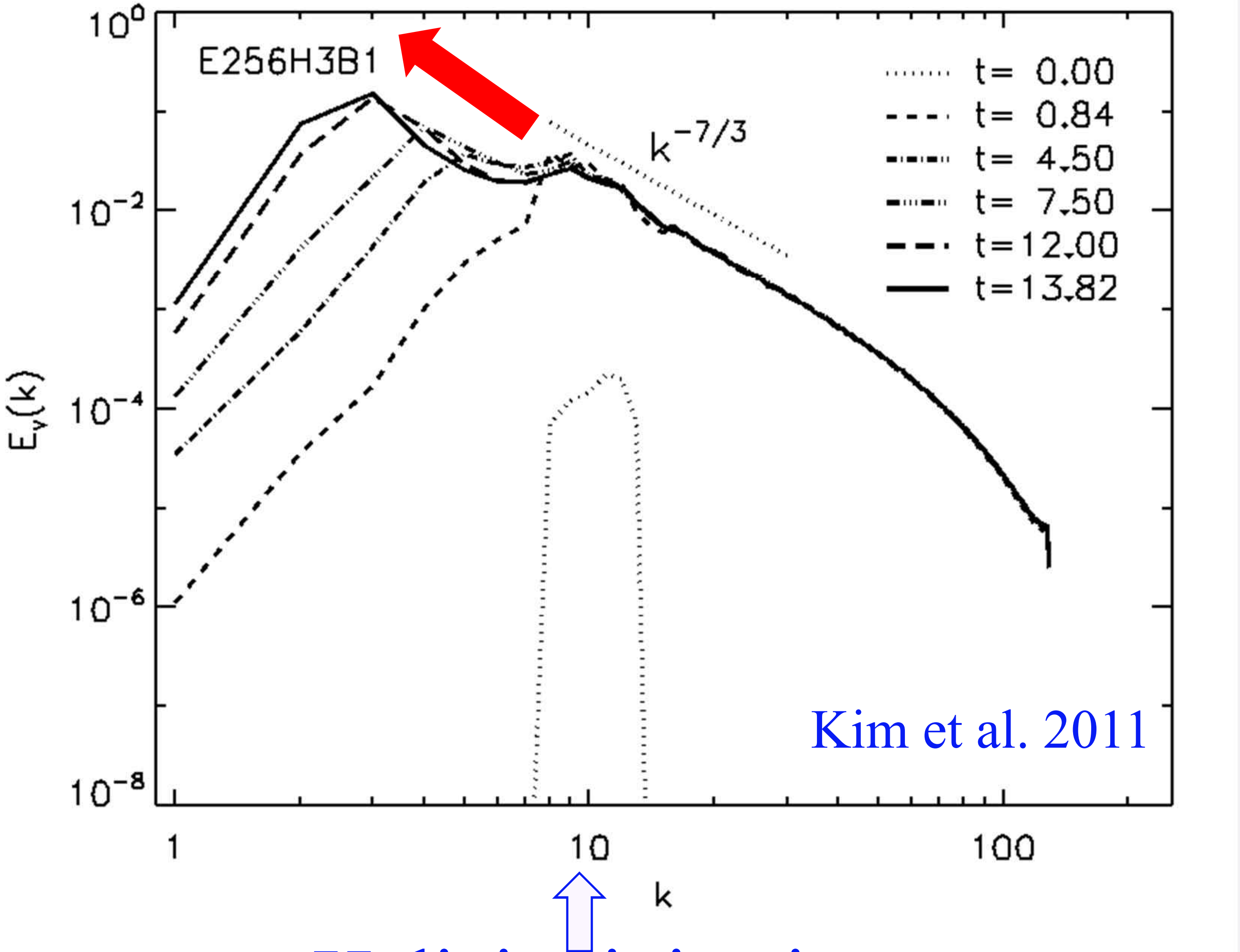


CV00





Muller et al (2003)



Helicity injection

Cf. Schekochihin+(2009)