Alfvénic Turbulence in Future Large Scale Fusion Plasmas

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Basics of Shear- Alfvén Waves

$$\delta E_{\parallel} = 0 \qquad \Rightarrow \frac{1}{c}\partial_t \delta A_{\parallel} = -\nabla_{\parallel}\delta\phi$$

ideal MHD

From $\vec{\nabla}\cdot\vec{\delta}J = 0 \qquad \Rightarrow \frac{m_i n_0 c}{B^2}\partial_t \underline{\underline{\nabla_{\perp}{}^2\delta\phi}} = \nabla_{\parallel} J_{\parallel} = \nabla_{\parallel}\nabla_{\perp}{}^2\delta A_{\parallel}$

$$\mathrm{v_A}^{-2}\omega^2 = k_{\parallel}^2$$

MHD : from $\vec{\nabla}\times$ of $\rho\frac{d}{dt}\vec{\mathrm{v}} = \vec{J}\times\vec{B}$

i.e. " Vorticity "

Gyrokinetics : " Polarization Density "

$$n_{\mathrm{pol}} = \nabla_{\perp}\cdot\left(\frac{m_i n_0 c}{B^2}\nabla_{\perp}\delta\phi\right)$$

Shear-Alfvén Continuum in Sheared Magnetic Field

- When driving is weak, shear-Alfvén wave DR. $\omega^2 = k_{\parallel}^2 \mathrm{v_A}^2$

  ➔In sheared magnetic field, with $k_{\parallel} = \dfrac{nq(r)-m}{qR} \cong \dfrac{k_\theta}{L_s}(r - r_{m,n})$

- For given n, m, linear D.R. is satisfied at least one radial position for any reasonably small values in $k_{\parallel}$

  ➔ as $k_{\parallel}(r)$ is varied as a function of r , ω assumes "continuum" of values , rather than an "eigenvalue" (discretized)

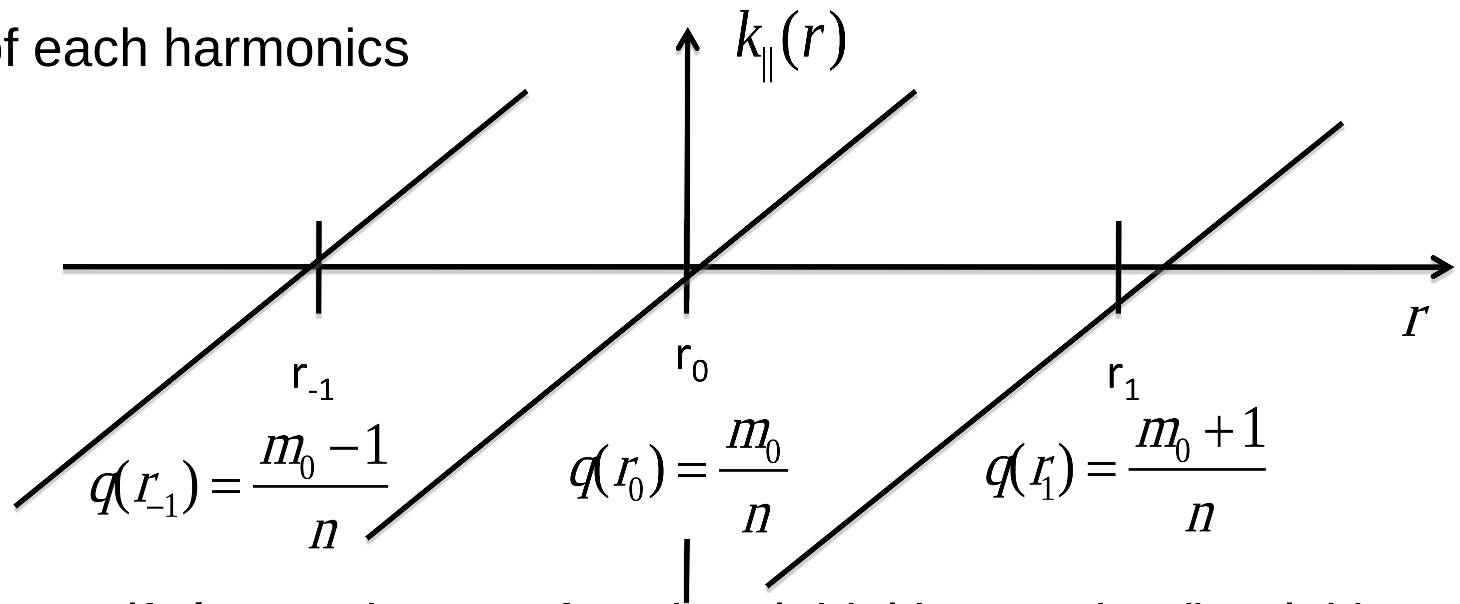
  ➔ Alfvén continuum ➔ initial wave packet will phase-mix and decay algebraically in time.

- Then what's the consequence of toroidal geometry?

  i.e., coupling between neighboring poloidal harmonics

Linear Coupling of Poloidal Harmonics

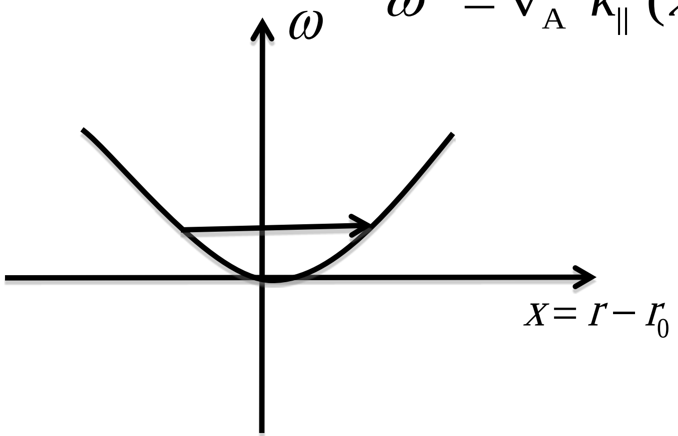
- $k_\parallel$ of each harmonics



- Shear-Alfvén Continuum of each poloidal harmonics (in Slab)

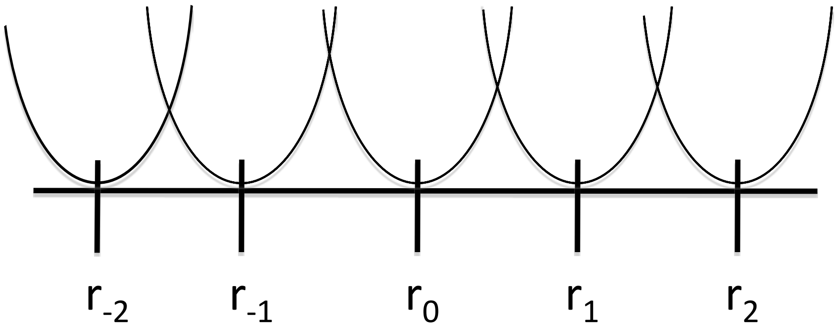
$$\omega^2 = \mathrm{v_A}^2 k_\parallel^2(x)$$

For $\forall \omega$, dispersion relation satisfied at one x.



Toroidicity-Induced Alfvén GAP

$$\omega^2 = k_{\|}^2 \mathrm{v_A}^2, \text{ with } k_{\|} \propto \frac{k_\theta}{L_s}(r - r_m)$$



➡ Each harmonics considered in sheared slab

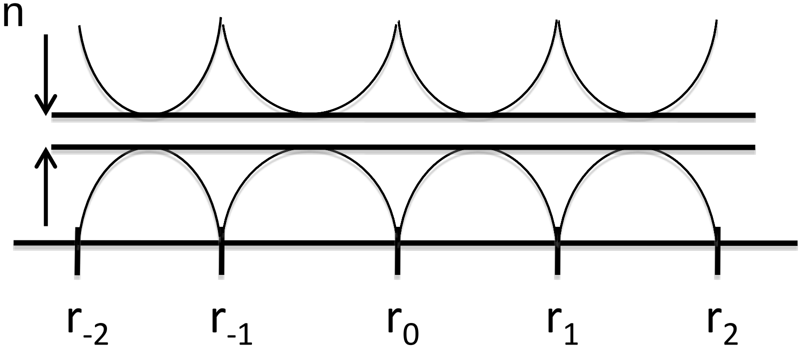
$$\mathrm{v}_A{}^2 \propto B^2 = \mathrm{const}$$

$$B = \frac{B_0}{1 + \frac{r}{R_0}\cos\theta}$$ induces toroidal GAP

→ (involves solving Matthieu Equation)

$$\omega^2 = \omega_A{}^2(r)$$

Toroidal Alfvén Gap



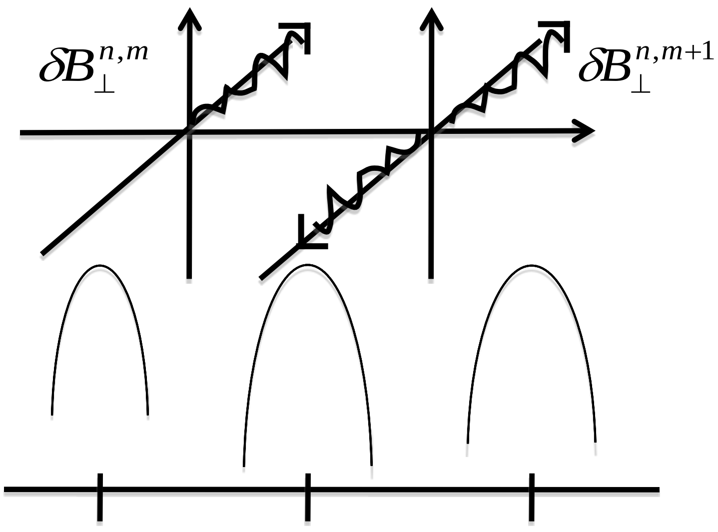
Continuum Dispersion Relation Not Satisfied for

$$\frac{\mathrm{v}_A}{2qR}(1-\varepsilon) < \omega < \frac{\mathrm{v}_A}{2qR}(1+\varepsilon)$$

Toroidal Alfvén Eigen modes

- At the midpoint between two adjacent rational surfaces
  $k_{\parallel} = \frac{1}{2qR}$ ~ GAP occurs near $\omega \simeq \frac{v_A}{2qR}$
- "Standing wave formation" from superposition of

  $\delta B_{\perp}{}^{n,m} e^{;\text{co}-\text{prop}}$ and $\delta B_{\perp}{}^{n,m+1} e^{;\text{counter}-\text{prop}}$



- This "TAE" modes can be excited via resonance with energetic ions.

- … AE Zoo accommodates TAE, BAE, GAE, CAE, HAE, EAE, LSAE, RSAE, …, and

  Nonconventional AE !

Brief Summary of TAE research in Magnetic Fusion

- Energetic particle driven Alfvénic instabilities such as Toroidal Alfvén Eigenmodes (TAE's) :

  [*Cheng, Chen, Chance. Ann. Phys., '85*]

  observed in many magnetic fusion plasma experiments,

  expected to be excited in future experiments (eg., ITER)

  and limit their performances.

- Predicting their behavior for future devices:

  important plasma physics research

  for success of magnetic fusion energy.

Brief Summary of TAE research in Magnetic Fusion

- The most unstable modes

  : toroidal mode number n which satisfies $k_\theta q \rho_E \sim 1$ (where, $k_\theta = \frac{nq}{r}$, ρ_E is Larmor radius of a typical energetic ion, r is local minor radius, $q = \frac{B_\phi}{B_\theta}\frac{r}{R}$ is a safety factor, B_ϕ and B_θ are toroidal and poloidal magnetic field respectively).

- For present day tokamaks, $n \sim \frac{r}{q^2 \rho_E} \gtrsim 1$, due to their modest size machine and TAE's with a low mode number (n=1,2,…) have mostly been observed.
- Most nonlinear theories on TAE's have considered the low mode number TAE's which are easier to study.

Brief Summary of TAE research in Magnetic Fusion

- For the low mode number TAE's, the dominant nonlinear saturation mechanism is believed to be a single wave trapping of energetic ions by TAE [*Berk et al., PRL, '96*].
- Eigenmode structure of a TAE is mostly described by MHD equations for bulk plasmas (excluding energetic ions) which ignore kinetic effects.
- Nonlinear evolution due to this single wave trapping has been illustrated using a simpler bump-on-tail instability example [*Berk et al., PRL, '99*], and steady improvements of model have been achieved including phase space dynamics *[Lesur and Diamond, This meeting, '01*].
- To date, this paradigm has been widely accepted in the magnetic fusion energy plasma physics community.

TAE's in Future Devices?

- For future magnetic fusion experiments such as ITER the most unstable TAE's , $n \sim \frac{r}{q^2 \rho_E}$ will be on the order of 10.
- Prevailing nonlinear theories for low-n TAE's can no longer be applied.
- Nonlinear mode coupling between different TAE's, rather than single wave trapping of energetic ions should dominate in nonlinear regime.
- In addition, kinetic effects such as nonlinear Landau damping of nonlinearly generated beat waves via thermal ions (which were neglected in the aforementioned single wave trapping theories) should be considered.

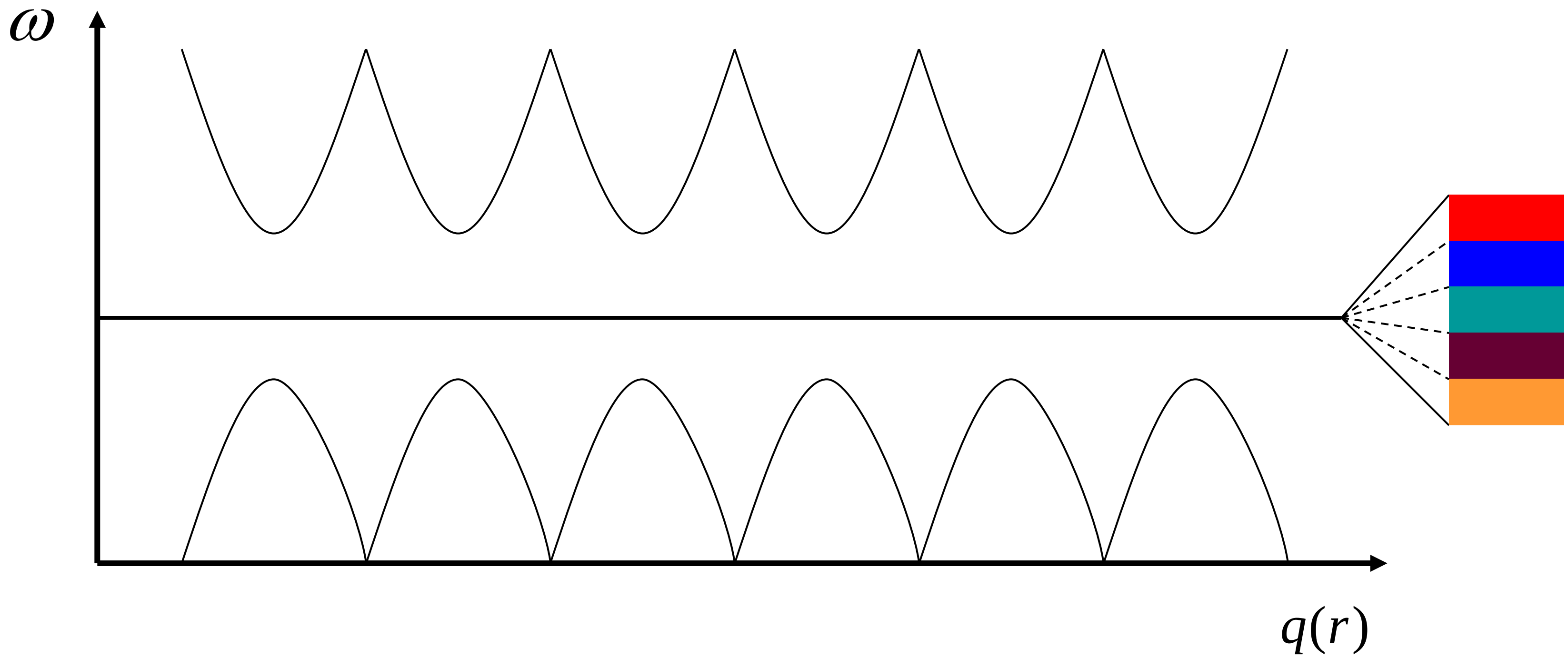
TAE's in Future Devices?

- Such a work has been initiated using the weak turbulence nonlinear mode coupling formalism [*Sagdeev and Galeev, Nonlinear Plasma Theory, '69*] in [*Hahm and Chen, Phys. Rev. Lett, '95*].

- But, it has only received a modest level of attention.

Translational Invariance

- If $\frac{v_A}{qR_0}$ is uniform in r, $\sim N(q(a)-q(0))$ modes have the same eigenfrequency (degenerate).



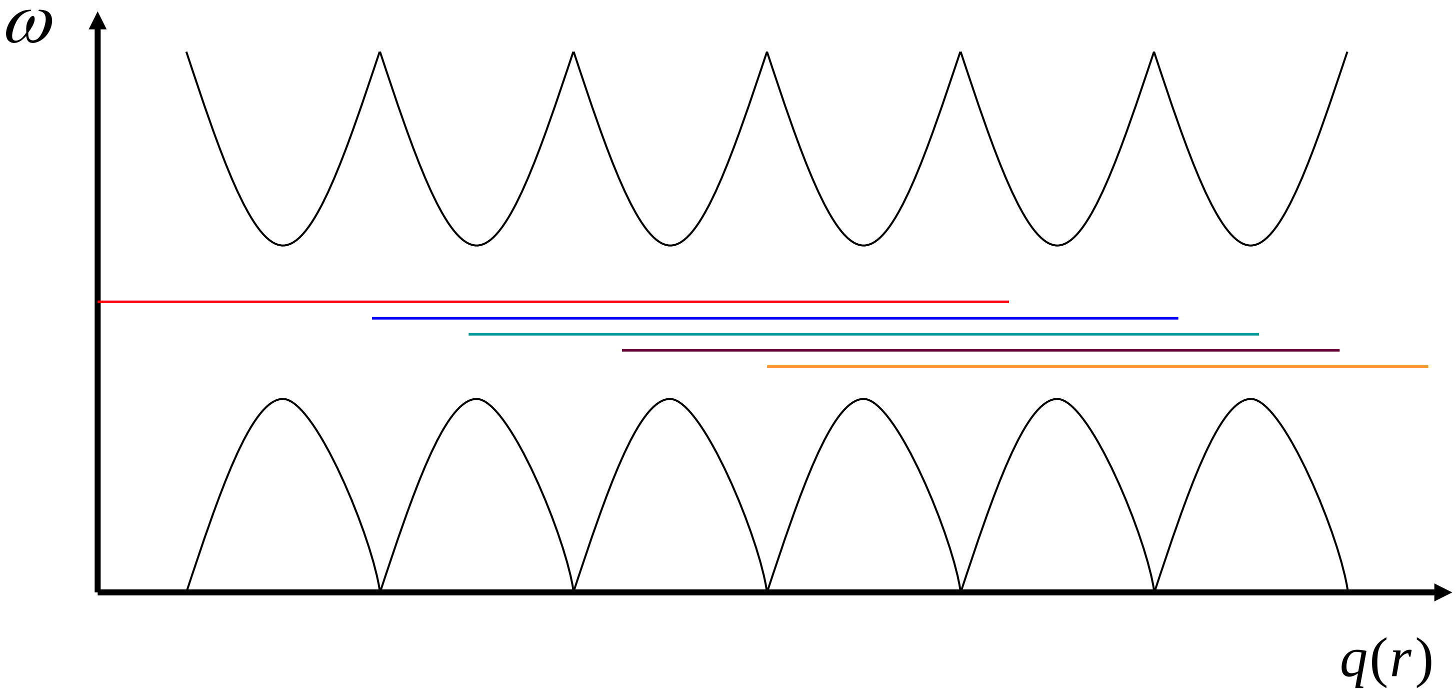
For single-N,

Quasi-Translational Invariance

- With equilibrium variation, degeneracy is broken.

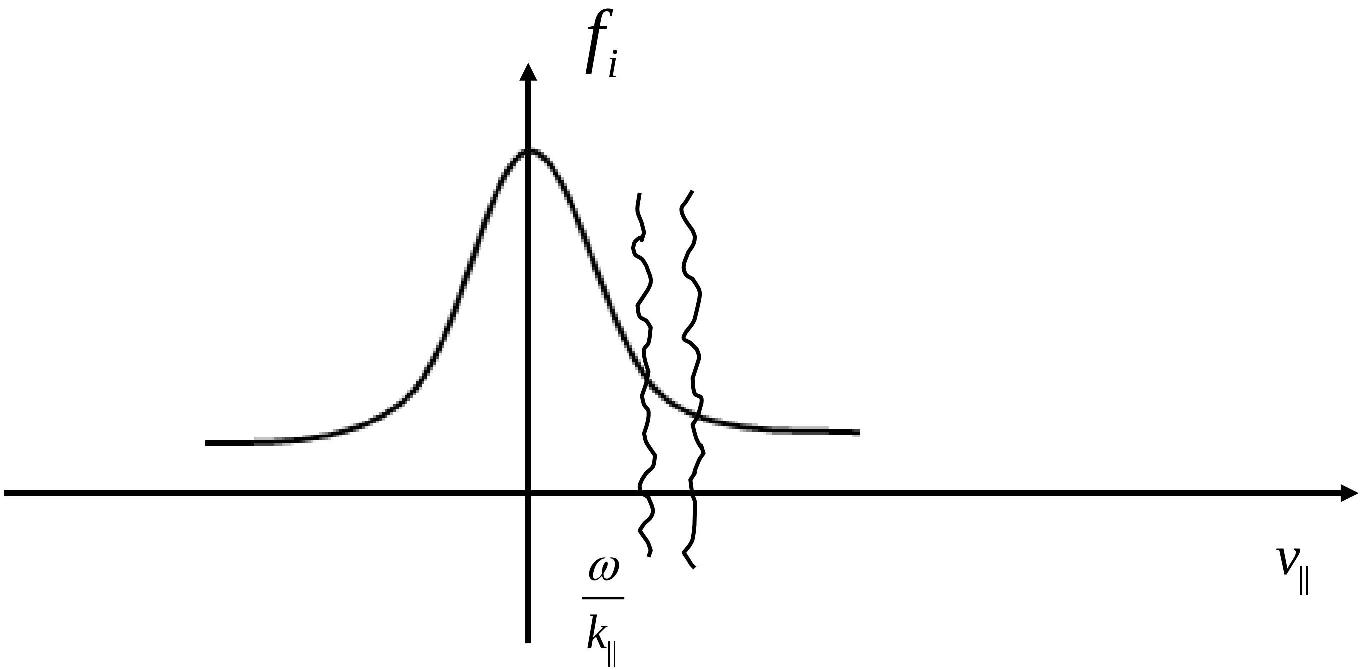
  Each TAE's has slightly different eigenfrequency.

  "High-N TAE" still contains many poloidal harmonics.



$C_s^2/v_A^2 \beta < 10\%$ in tokamaks

➔ $v_{Th,i} << \frac{\omega}{k_{\parallel}} = v_A$ of shear Alfvenwave



Nonlinear Saturation Mechanism for High-N TAE

" Ion Compton Scattering "

TAE ω

TAE ω'

$\delta J_\perp \times \delta B_\perp$ Nonlinearity →

" BEAT WAVE "

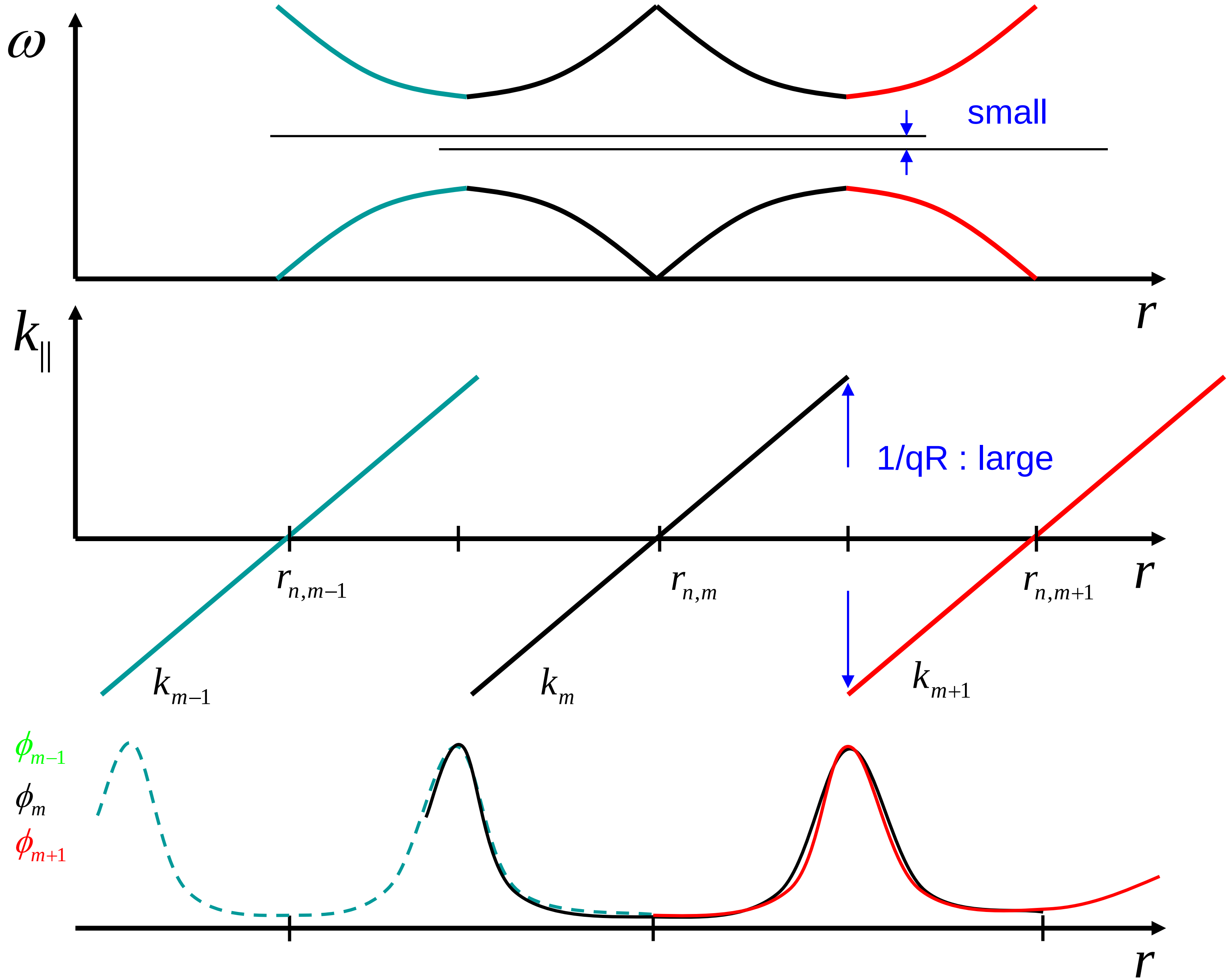
$\omega'' = \omega - \omega'$

$k''_\parallel = k_\parallel - k'_\parallel$

- If $\dfrac{\omega''}{k''_\parallel} \sim v_{Ti} << v_A$ (Bulk), via Compton Scattering, fluctuation energy is transferred to lower frequency mode, eventually absorbed by linearly stable mode near lower continuum.

→ " Look for Nonlinear Coupling Channel " via beat wave with low phase velocity which gives $\dfrac{\omega''}{k''_\parallel} \sim v_{Ti}$!

Generation of Low Phase Velocity Beat Wave



Nonlinear Interaction of TAE's → Sound Wave-like Density Fluctuation

	Test mode (TAE)	Background mode (TAE)	Beat wave (Sound wave)
Wave Vector	$\vec{k}$	$\vec{k}'$	$\vec{k}'' = \vec{k} - \vec{k}'$
Frequency	ω	ω'	$\omega'' = \omega - \omega' \lesssim \varepsilon\omega$ small
$k_\parallel$ @ gap	$\frac{1}{2qR}$	$-\frac{1}{2qR}$	"$\frac{1}{qR}$" large
• Single – n :			
Toroidal Mode Number	n	n	0
Poloidal Mode Number	m	$m+1$	-1
• For multi – n :			
Toroidal Mode Number		n'	$n - n' << n$
Poloidal Mode Number		$m'+1$	$m - m' - 1 << m$

* This talk : $10^1 \leq N << 10^2$

Weak Turbulence Nonlinear Analysis *[Hahm&Chen, PRL '95]*

- 3rd Order Perturbation Theory : $\frac{\gamma_L}{\omega_A} << 1$

- 1st order : Test TAE ($\vec{k}$) ideal MHD

$$\frac{\partial}{\partial t}\psi_{\vec{k}}^{(1)} = -\hat{\mathbf{n}}\cdot\nabla\phi_{\vec{k}}^{(1)}$$

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi_{\vec{k}}^{(1)} = -\mathrm{v}_\mathrm{A}^2\hat{\mathbf{n}}\cdot\nabla\nabla_\perp^2\psi_{\vec{k}}^{(1)}$$

- 2nd order : Nonlinear Interaction of two TAE's, $\begin{pmatrix}\vec{k}\\ \vec{k}'\end{pmatrix}$.

→ Sound-wave-like density fluctuation, $\vec{k}''$.

$$\left(\frac{\partial}{\partial t} + \mathrm{v}_\parallel\hat{\mathbf{n}}\cdot\nabla\right)\delta f_{\vec{k}''}^{(2)} = \left\{\left(\frac{\delta\vec{B}}{B_0}\cdot\frac{\partial}{\partial t}\vec{\mathrm{v}}_E\right)_{\vec{k}''} + \frac{|e|}{M_i}\hat{\mathbf{n}}\cdot\nabla\phi_{\vec{k}''}\right\}\frac{\partial f_0}{\partial \mathrm{v}_\parallel}$$

$\delta J_\perp \times \delta B_\perp$ NL

" $\omega'' = k_\parallel''\mathrm{v}_\parallel$ Resonance important."

- 3rd order : Nonlinear Evolution of Test TAE ($\vec{k}$),
  in the presence of density fluctuation $\delta n_{\vec{k}''}^{(2)}$
  and other TAE's $\phi_{\vec{k}'}^{(1)}$.

$$\mathrm{v}_A^2\hat{\mathbf{n}}\cdot\nabla\nabla_\perp^2\psi_{\vec{k}} + \frac{\partial}{\partial t}\nabla_\perp^2\phi_{\vec{k}} + \sum_{\vec{k}'}\nabla\cdot\left(\frac{\delta n_{\vec{k}''}^{(2)}}{n_0}\frac{\partial}{\partial t}\nabla_\perp\phi_{\vec{k}'}^{(1)}\right) = 0.$$

Nonlinear Evolution of TAE in the presence of density fluctuation

$$v_{A0}^2\left(\hat{n}\cdot\nabla\nabla_\perp^2\psi_{\vec{k}}+\sum_{\vec{k}'}\nabla\psi_{\vec{k}'}\times\hat{n}\cdot\nabla\nabla_\perp^2\psi_{\vec{k}''}\right)+\frac{\partial}{\partial t}\nabla_\perp^2\phi_{\vec{k}}+\sum_{\vec{k}'}\nabla\phi_{\vec{k}'}\times\hat{n}\cdot\nabla\nabla_\perp^2\phi_{\vec{k}''}+\sum_{\vec{k}'}\nabla\cdot\left(\frac{\delta n_{\vec{k}''}^{(2)}}{n_0}\right)\frac{\partial}{\partial t}\nabla_\perp\phi_{\vec{k}'}=0$$

- Recall; Vorticity Equation is $\vec{\nabla}_\perp\cdot\delta\vec{J}_\perp^{\rm pol}+\nabla_\parallel\cdot\delta J_\parallel=0$

  where " $\delta\vec{J}_\perp^{\rm pol}=\frac{1}{B_0}\hat{n}\times\rho\frac{d\vec{v}_E}{dt}$ " depends on "number density."

- Other nonlinearities are subdominant for $k'_\perp k''_\perp\rho_s^2<<\frac{\omega'}{\Omega_{ci}}$

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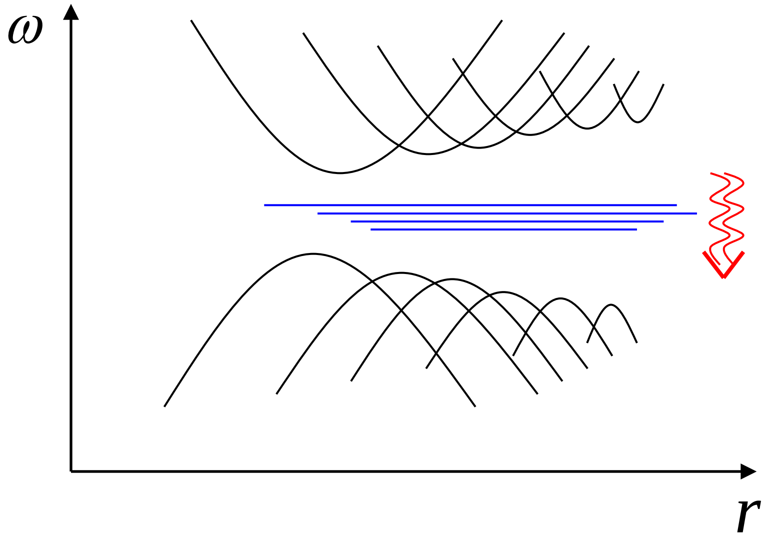
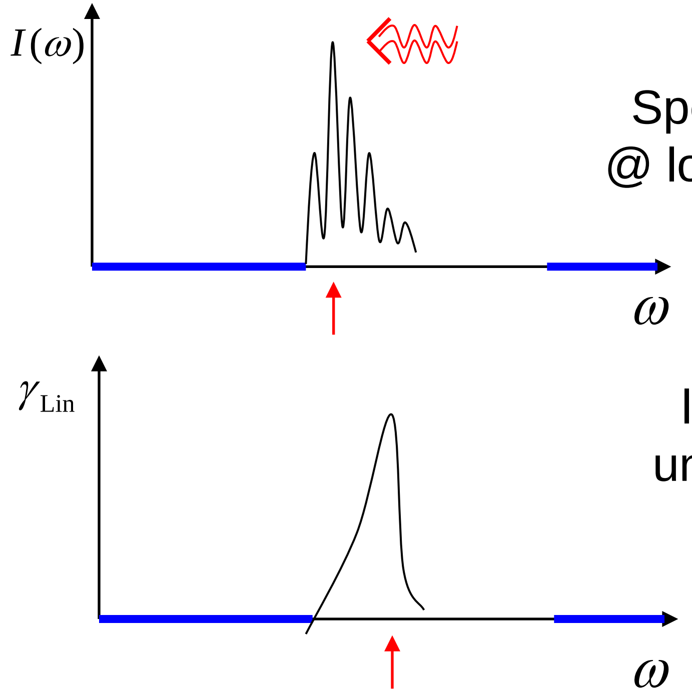
* Multiplying $\phi_{\vec{k}}^*$, take imaginary part of the radial average, we get

$$\frac{\partial}{\partial t}I_{\vec{k}}=\gamma_L(\vec{k})I_{\vec{k}}-\sum_{\vec{k}'}M_{\vec{k},\vec{k}'}I_{\vec{k}'}I_{\vec{k}}$$

$$I_{\vec{k}}\equiv\left\langle\left|\nabla_\perp\phi_{\vec{k}}\right|^2\right\rangle,\quad M_{\vec{k},\vec{k}'}\equiv\frac{\omega'}{2}\frac{\chi_e^2\,\mathrm{Im}\,\chi_i}{\left|\chi_e+\chi_i\right|^2}\frac{M_i}{B_0}$$

" Consequence of Ion Compton Scattering "





Spectrum peaks
@ lower frequency

than
linearly most
unstable mode.

Fluctuation Energy is transferred to
Lower Frequency due to Ion Compton Scattering.

At Nonlinear Saturation

$$``\left(\frac{\delta B_r}{B_\varphi}\right)^2 \simeq \frac{1}{4\pi}\left(1+\frac{T_e}{T_i}\right)^2\left(\frac{\bar{\gamma}_L}{\omega_A}\right)\left(\frac{r}{R_0}\right)^4"$$

- Magnitude : $\frac{\bar{\gamma}_L}{\omega_A} < \frac{\bar{\gamma}_A^{\text{Max}}}{\omega_A} \lesssim 10^{-2}$

  $\frac{r}{R_0} \sim 10^{-1}$ ➡ $\frac{\delta B_r}{B_\varphi} \lesssim 10^{-3}$

- Scaling : $\frac{\delta B_r}{B_\phi} \propto \left(\frac{\bar{\gamma}_L}{\omega_A}\right)^{\frac{1}{2}}$: weak turbulence theory

  (⬌ $\delta B_r \propto \gamma_L^2$; à la single wave trapping) Berk, Breizman, Fu, W. Park(not H. Park), Wu, White, Rosenbluth …

Our Mechanism :

- More relevant for High-N Multi-mode Overlapping Case.
  Also for stronger drive.

Single Wave Trapping (Berk-Breizmann et al.; Old version)

- Resonant particles are trapped in the potential well produced by TAE.
- The potential well will last an auto-correlation time, ω_{ac}^{-1}.
- A trapped particle will transverse a closed orbit in, ω_b^{-1}, mixing hot-particle distribution function.

$\rightarrow$ Require : $\underline{\omega_b > \omega_{ac}}$

Nonlinear Saturation : $\omega_b \sim \gamma_{\mathrm{Lin}}$

- It also requires a well-defined potential well in space,

$$\underline{\Delta X_{\text{particle, excursion}} < \Delta X_{N,M}}$$

,where $\Delta X_{N,M}$: distance between neighboring rational surfaces.

Weak Turbulence Theory

Requires

$$\omega_b < \omega_{ac}$$

and

$$\Delta X_{\text{ptl. exc.}} > \Delta X_{N,M}$$

Chirikov Criterion

Validity Regimes

* For Quasi-Linear Theory (Weak Turbulence Expansion)

- $\Delta X_{\text{ptl. exc.}} > \Delta X_{N,M}$

$$\Rightarrow \quad \frac{\delta B_r}{B_\phi} > \frac{1}{8q^4\hat{s}}\left(\frac{r_0^2}{R_0\rho_\alpha}\right)\cdot\frac{1}{N^4}$$

- $\omega_b < \omega_{ac}$

$$\Rightarrow \quad \frac{\delta B_r}{B_\phi} < \frac{1}{8q^4\hat{s}}\left(\frac{r_0^2}{R_0\rho_\alpha}\right)\cdot\frac{4\varepsilon^2\hat{s}^2}{N^2}$$

* For Single Wave Trapping

$\Delta X_{\text{ptl. exc.}} < \Delta X_{N,M}$, (Then, $\omega_b > \omega_{ac} \rightarrow 0$)

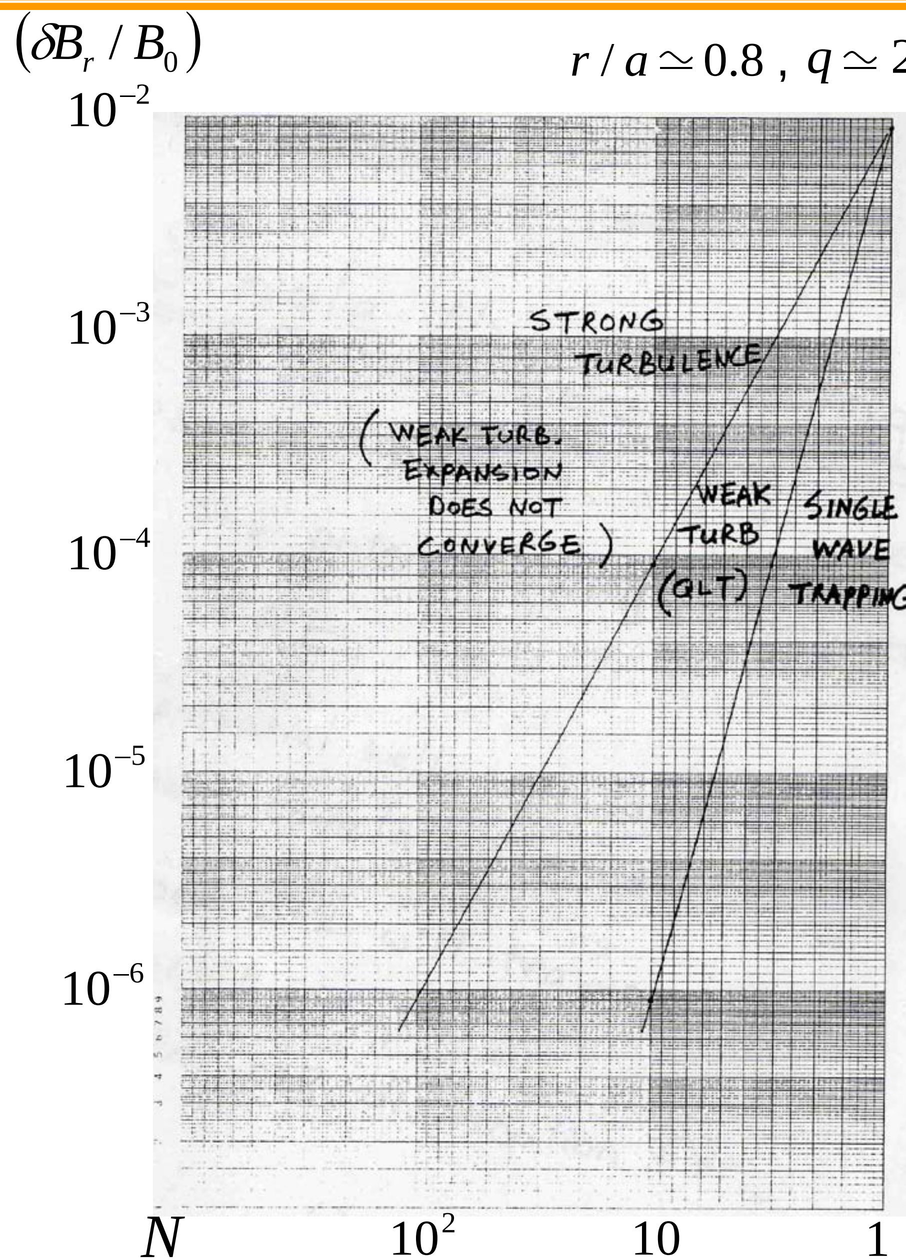
PLOT : in $\left(\frac{\delta B_r}{B_\phi}, N\right)$ space

Approximate equilibrium parameters for ITER pretor.

[Candy, Rosenbluth, NF '95]

ITER PRETOR

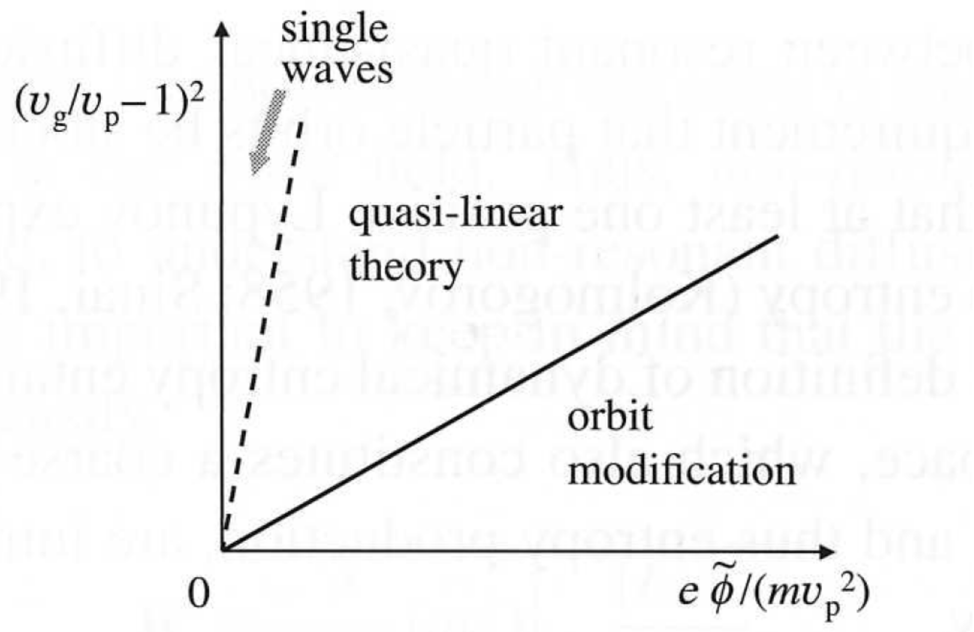
$r/a \simeq 0.8$, $q \simeq 2$ dense mode packing



from T.S. Hahm and L. Chen, APS-DPP, ’95

Range of applicability for the quasi-linear theory

See “Modern Plasma Physics Vol.1:
Physical Kinetics of Turbulent Plasmas”
by Diamond and Itohs
for more details in physics (without MFE-TAE jargons)



Range of applicability for the quasi-linear theory. Amplitude is normalized by the particle energy at phase velocity on the horizontal axis. The vertical axis shows the magnitude of dispersion, i.e. the difference between the group velocity and phase velocity.

Incompressible MHD Turbulence

Tutorial at Theorié-Fest, Aix-en-Provence, 2007 by P.H. Diamond.

Connects :

- Weak Turbulence Kinetic Theory : Sagdeev & Galeev '67
- Scaling Derivation : PHD & Craddock: Comments in Plasma Phys '90
  Lazarian & Vishniac: Ap. J. '99

and

- Lengthy Crank : Goldreich & Sridhar: Ap. J. '95

Common Theme :

Nonlinear Interaction of Low Frequency Beat Wave
with
Particles or Eddys

	Weak Turbulence Theory of Incompressible MHD Turb.	**WTT of Toroidal Alfvén Eigenmodes in Tokamaks**
High Freq. Shear-Alfvén Waves interact nonlinearly with	Eddys	Thermal Ions
allowed by	Low Freq. Beat Wave produced by Counter-Prop$^{\underline{n}}$	Low Frequency Beat Wave produced by "standing" TAEs (formed linearly by Counter-Prop$^{\underline{n}}$ of neighboring harmonics)
resulting in	Alfvén Effect (down by $\frac{\Delta\omega_k}{k_\parallel \mathrm{v}_A}$) and Scale-dependent Anisotropy (Intermediate Turbulence : G&S Ap. J. '97 $\simeq$ Anisotropic I-K '65)	Down-shift of Frequency $k_\parallel$ determined by equilibrium $\vec{B}$ geometry $k_\perp$ by linear drive (energetic particles)
Theory breaks down	when $k_\parallel \mathrm{v}_A = \mathrm{v}_\perp / \ell_\perp$ i.e, small scales first	with non-overlapping island (violation of Chirikov Criterion) in phase-space. i.e. large scales first
turning into	Critically Balanced Cascade (G&S '95 '97)	Single-Wave Trapping $\rightarrow$ Hole-Clump Pair in phase-space. Berk-Breizman Paradigm

Gyrokinetics and Polarization Density

Ref. Hahm-Lee-Brizard, Phys Fluids '88

- Vorticity Equation can be derived from

$$\frac{dn_e}{dt} = \frac{1}{|e|}\nabla_{\|} j_{\|e}$$

$$-\quad \frac{d}{dt}N_{\text{gyrocenter}} = -\frac{1}{|e|}\nabla_{\|} j_{\|i}$$

⬅ From moments of Gyrokinetic Equation

$$\frac{d}{dt}\text{"}N_{\text{pol}}\text{"} = \frac{1}{|e|}\nabla_{\|} J_{\|}$$

- Physical Meaning of Polarization Density?
- Gyrokinetics

Conclusions

- Alfvén waves have been studied from different angles.

  – MHD Turbulence Community : Fixation with k-spectra

  …

  – MFE Theory Community : Fixation with zoology

  TAE, GAE, RSAE, LSAE, …

  -- For future fusion devices, nonlinear kinetic mode coupling theory should be developed.

  – Nonlinear Gyrokinetic Theory-based Extensions may bridge some gaps.