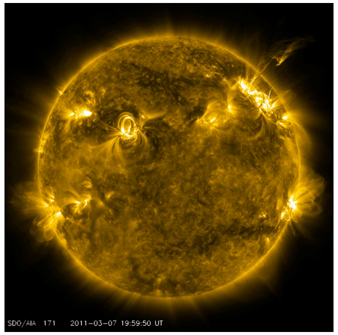
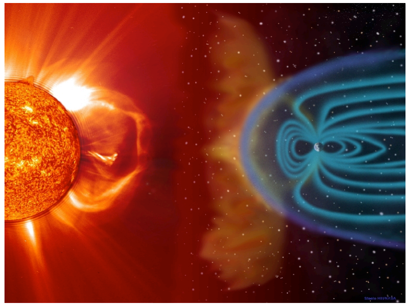
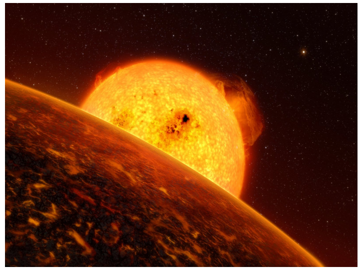
Fundamental Processes of Astrophysical Turbulence

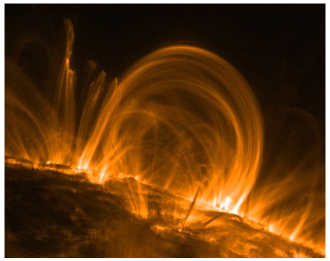
6th Korean Astrophysics Workshop

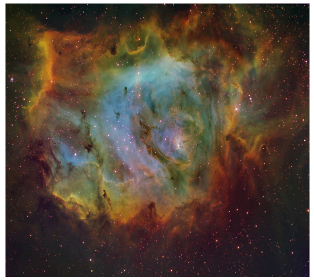
Pohang, Nov. 2011



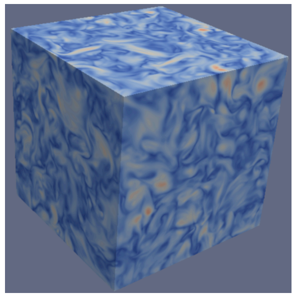








« Hall / MHD Turbulence:
Applications (mainly) to Solar Physics »



Sébastien **GALTIER**

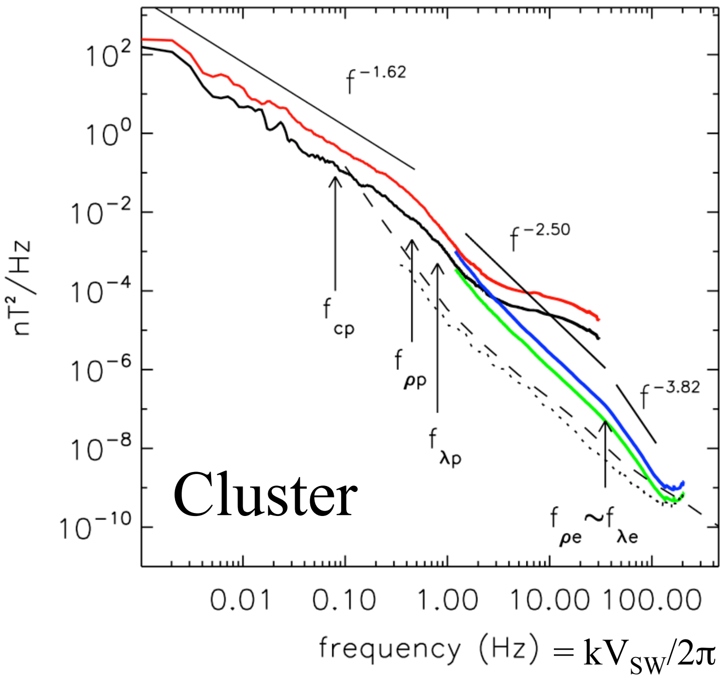
IAS INSTITUT D'ASTROPHYSIQUE SPATIALE

UNIVERSITÉ PARIS SUD
Comprendre le monde, construire l'avenir®

IUF

Some issues in MHD turbulence

Solar wind
magnetic spectrum



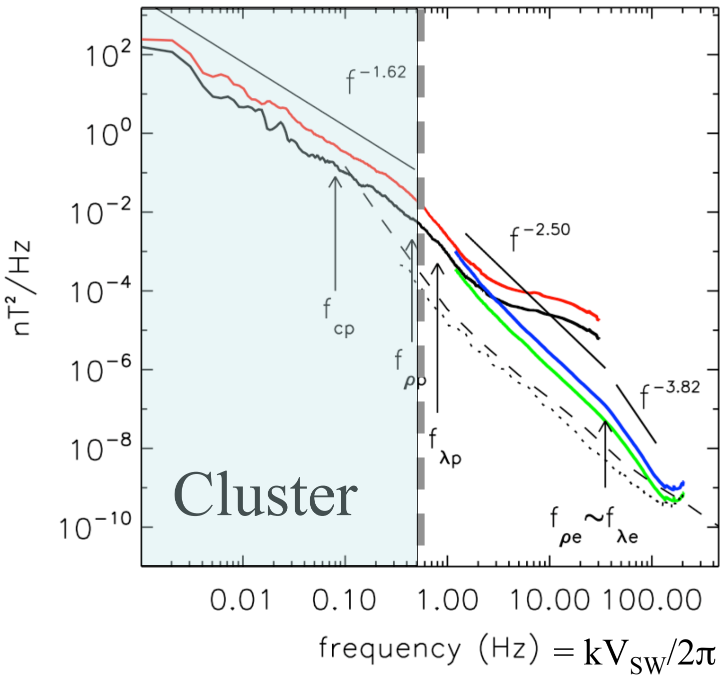
[Sahraoui et al., PRL, 2009]

Some issues in MHD turbulence

• Anisotropy & critical balance in MHD

→ towards a rigorous treatment ?

Solar wind
magnetic spectrum

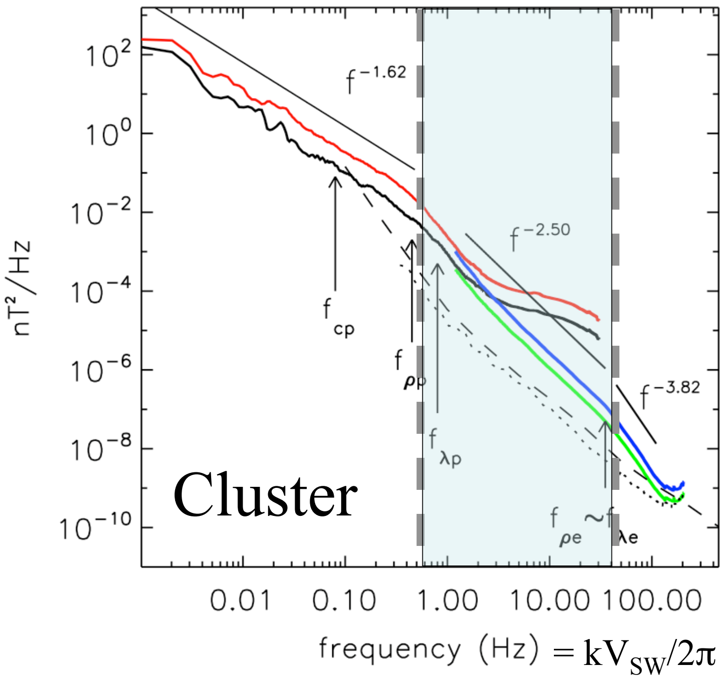


[Sahraoui et al., PRL, 2009]

Some issues in MHD turbulence

- Anisotropy & critical balance in MHD
  - → towards a rigorous treatment ?
- Transition to dispersive MHD
  - → from MHD to $k^{-2.5}$ in the SW

Solar wind magnetic spectrum



[Sahraoui et al., PRL, 2009]

Some issues in MHD turbulence

- Anisotropy & critical balance in MHD

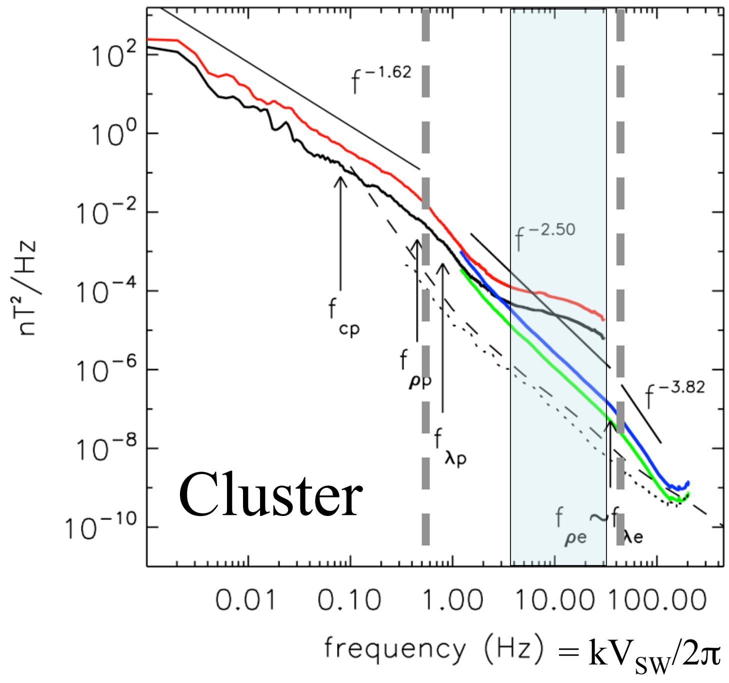
  → towards a rigorous treatment ?

- Transition to dispersive MHD

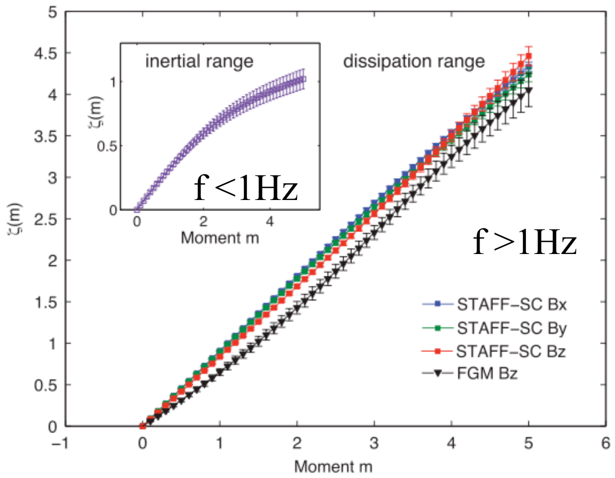
  → from MHD to $k^{-2.5}$ in the SW

- Intermittency in dispersive MHD

Solar wind
magnetic spectrum



[Sahraoui et al., PRL, 2009]



[Kiyani et al., PRL, 2009]

Some issues in weak MHD turbulence

- Whistler wave turbulence

    → predictions

Some issues in weak MHD turbulence

- Whistler wave turbulence

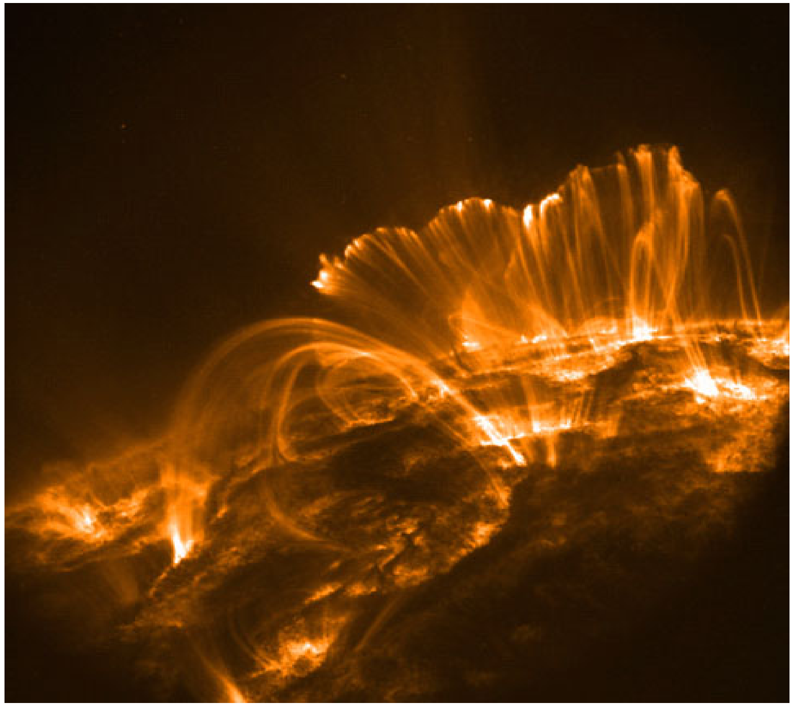
  → predictions

- Role of the slow mode ($k_{//}=0$) in MHD

  → 3D MHD DNS

- Direct observations

  → in solar magnetic loops ?



Alfvén wave packets

The main difference between neutral fluids and MHD is the presence of Alfvén waves [Alfvén, Nature, 1942]

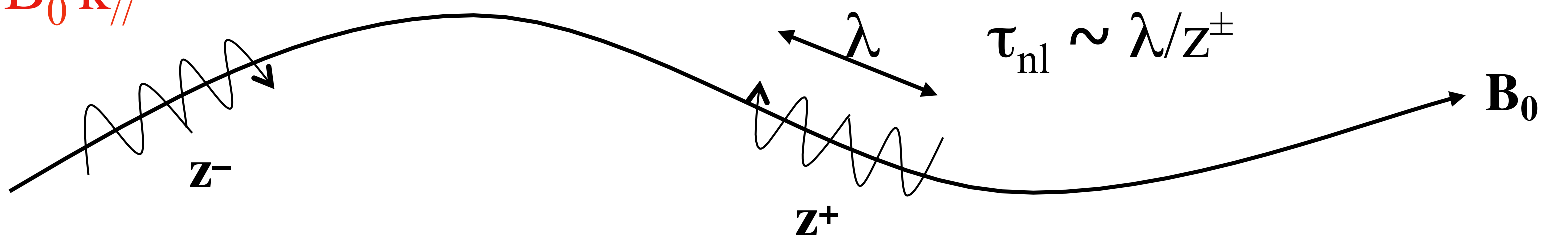
Wavepackets interact nonlinearly on a crossing time: $\tau_A \sim \lambda / B_0$

$$\begin{cases} \partial_t \mathbf{z}^{\pm} \mp \mathbf{B_0} \cdot \nabla \mathbf{z}^{\pm} = -\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} - \nabla P_* \\ \nabla \cdot \mathbf{z}^{\pm} = 0 \end{cases}$$

$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$ are the Elsässer fields

[Elsässer, PR, 1950]

$\omega_A = B_0 \, k_{//}$

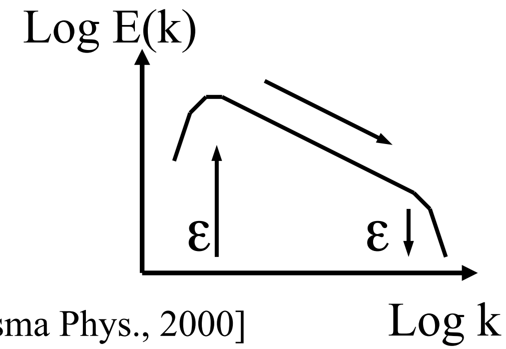


Many collisions between wavepackets

Isotropic MHD

$$\varepsilon \sim z_\lambda^4/(\lambda B_0) \quad \rightarrow \quad E(k) \sim (\varepsilon B_0)^{1/2}\, k^{-3/2}$$

[Iroshnikov, Sov. Astron., 1964; Kraichnan, Phys. Fluids,1965]



1) Problem: isotropy is assumed !
2) Solution: weak turbulence theory [Galtier et al., J. Plasma Phys., 2000]
3) Heuristic result not compatible with the 4/3's exact law :

[Politano & Pouquet, PRE, 1998]

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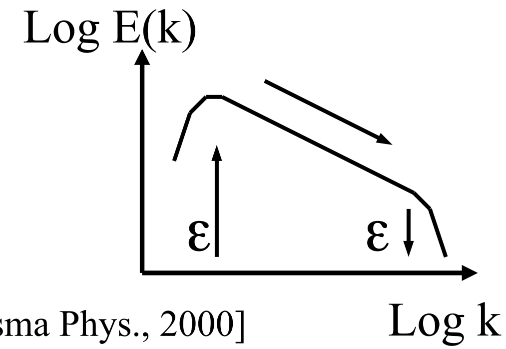

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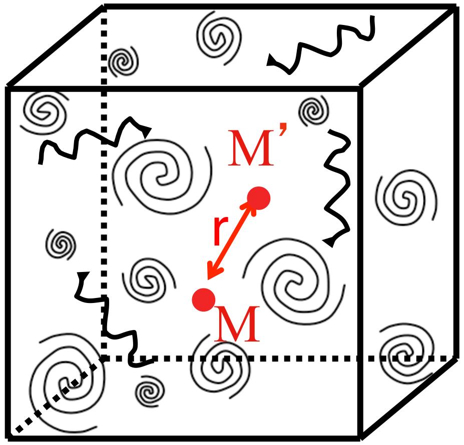
RIGOROUS :



- Homogeneous medium (Re ➔ +∞) :

$$-\frac{1}{4}\nabla_{\mathbf{r}} \cdot \mathbf{F}^{\pm}(\mathbf{r}) = \varepsilon^{\pm} \qquad \mathbf{F}^{\pm}(\mathbf{r}) = \langle \delta \mathbf{z}^{\mp} (\delta \mathbf{z}^{\pm})^2 \rangle$$

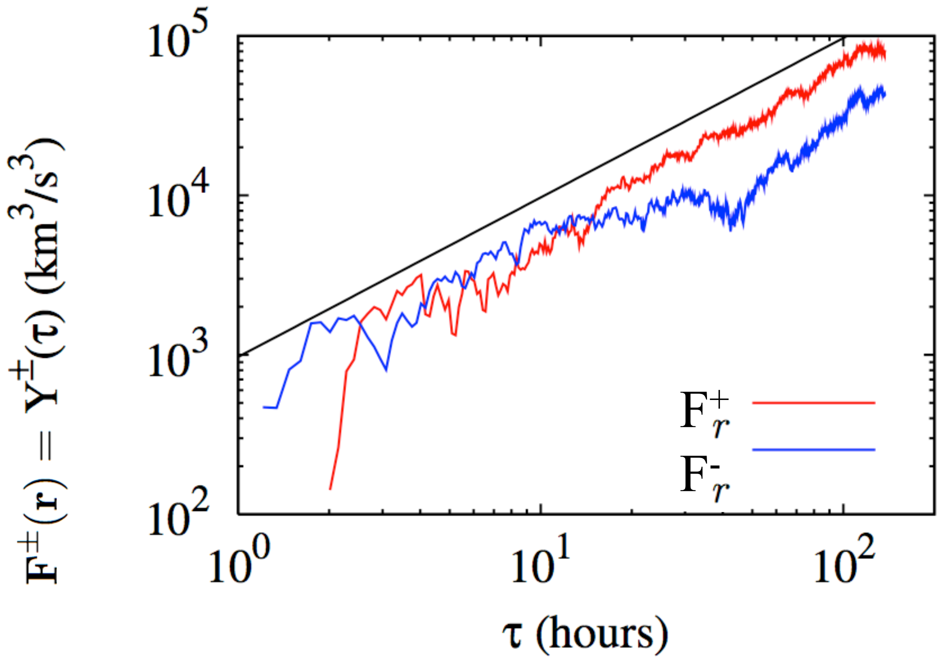
- Isotropy :

$$-\frac{4}{3}\varepsilon^{\pm} r = \langle \delta z_r^{\mp} (\delta z^{\pm})^2 \rangle$$

➔ compatible with $E(k) \sim k^{-5/3}$

4/3's exact law and the solar wind

[Sorriso-Valvo et al., PRL, 2007]



$$-\frac{4}{3}\varepsilon^{\pm} r = \langle \delta z_r^{\mp} (\delta z^{\pm})^2 \rangle$$

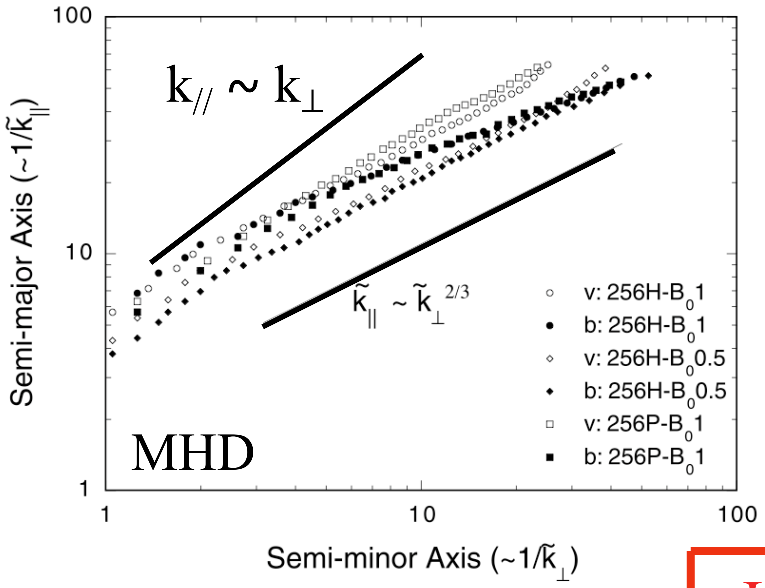
$$\mathbf{F}^{\pm}(\mathbf{r}) = \langle \delta \mathbf{z}^{\mp} (\delta \mathbf{z}^{\pm})^2 \rangle$$

$\varepsilon^{\pm}$ measure the local heating

Anisotropy is important and could change the conclusion

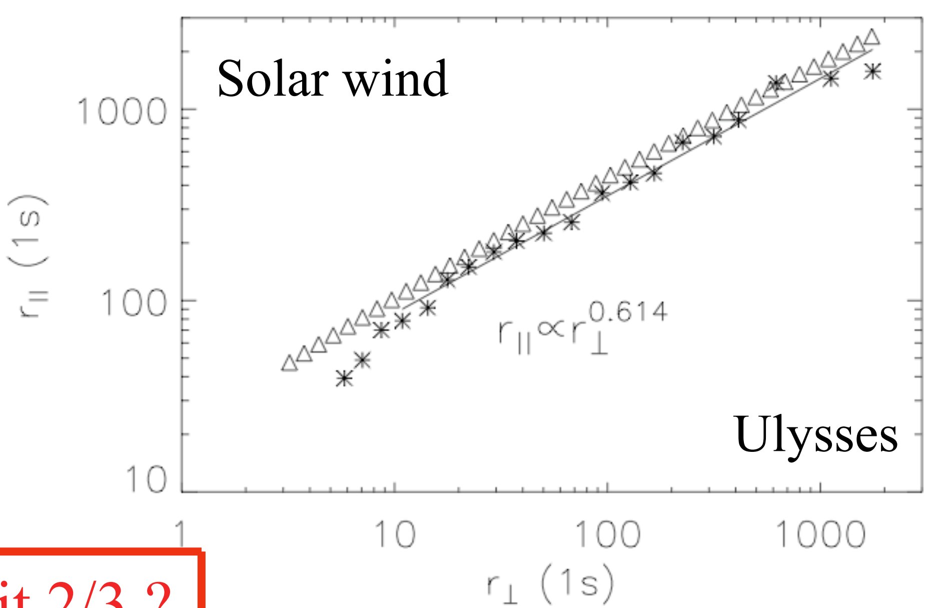
Anisotropic MHD

- Critical balance : $\tau_{nl} \sim \tau_A$ at all scales [Goldreich & Sridhar, ApJ, 1995]
  - ➔ it is a heuristic model
  - ➔ $E(k_\perp) \sim k_\perp^{-5/3}$ and $E(k_{//}) \sim k_{//}^{-2}$
  - ➔ increase of anisotropy at small scales : $k_{//} \sim k_\perp^{2/3}$



[Cho & Vishniac, ApJ, 2000]

Is it 2/3 ?



[Luo & Wu, ApJ, 2010]

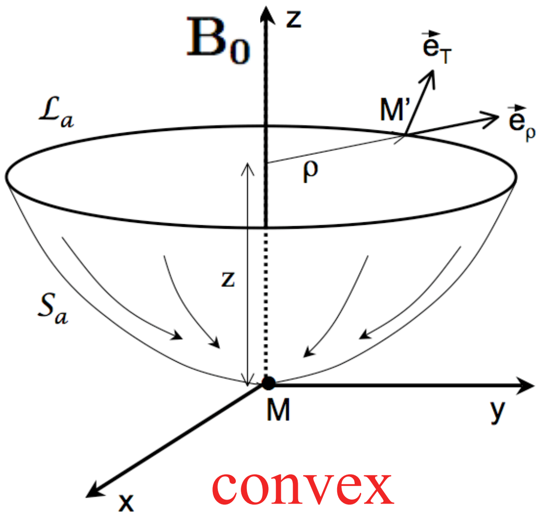
Anisotropic MHD

$\exists$ a scaling relation between $\perp$ and // correlation length scales

Is there a Kolmogorov law in space correlation ?

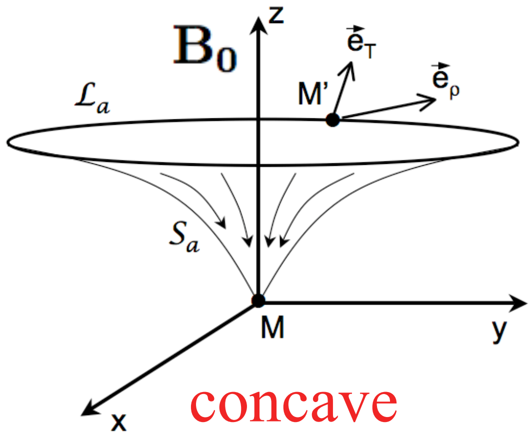
Anisotropic MHD

Let's consider a class of (local) axisymmetric MHD turbulence



convex

a>o



concave

a<o

[Galtier, ApJ, 2009]

$$\mathbf{F}^{\pm}(\mathbf{r}) = F_T^{\pm}\mathbf{e_T}$$

$$\mathbf{F}^{\pm}(\mathbf{r}) = \langle \delta\mathbf{z}^{\mp}(\delta\mathbf{z}^{\pm})^2 \rangle$$

$$z = f_a(\rho) = A\rho^{1+a}$$

a : degree of anisotropy

a=0 : 3D isotropic case
a=-1: 2D isotropic case

$$-4\varepsilon^{\pm} = \nabla \cdot \mathbf{F}^{\pm}(\mathbf{r}) = \frac{1}{\rho}\frac{\partial(\rho F_\rho^{\pm})}{\rho} + \frac{\partial F_z^{\pm}}{\partial z}$$

Anisotropic MHD

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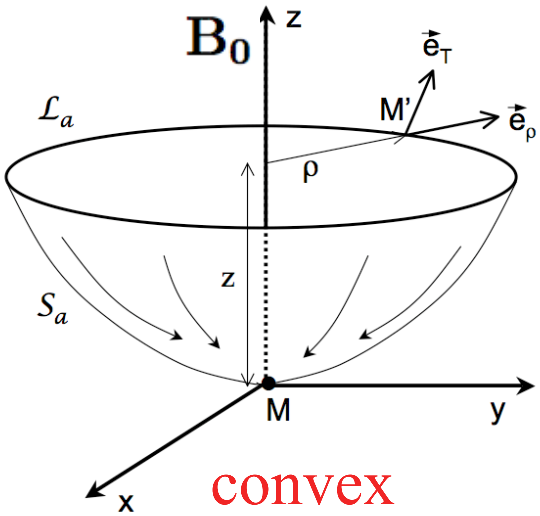
[Galtier, ApJ, 2009]



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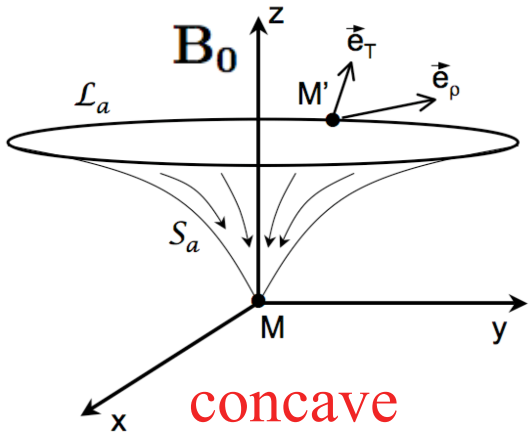

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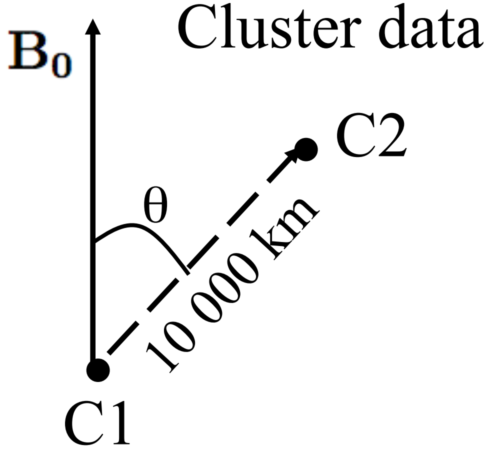
[Galtier, 2011]

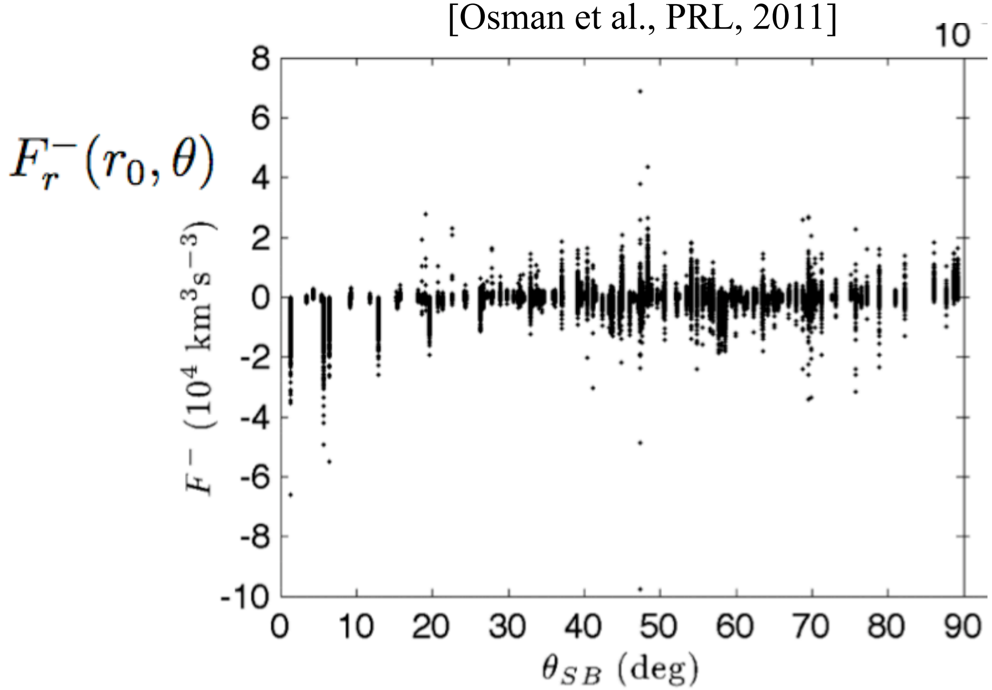
$$\mathbf{F}^{\pm}(r,z) = -\frac{4}{3+a}\varepsilon^{\pm}\left(\rho\,\mathbf{e}_\rho + (1+a)z\,\mathbf{e_z}\right)$$

Exact vectorial law for a class of axisymmetric turbulence

Solar wind turbulence



[Osman et al., PRL, 2011]



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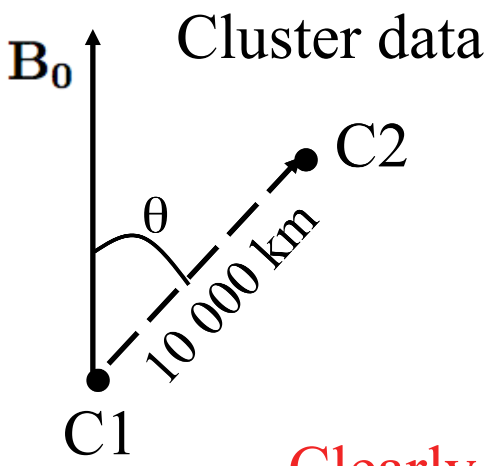

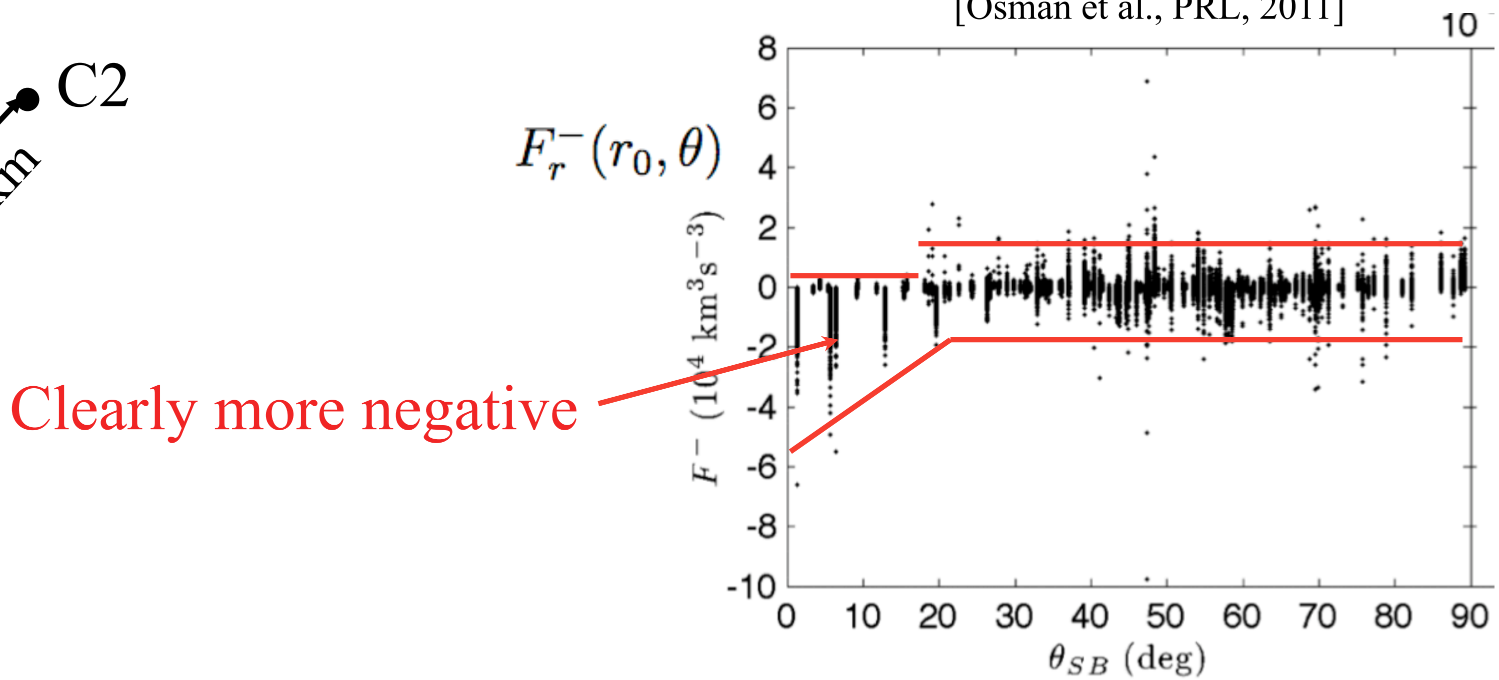

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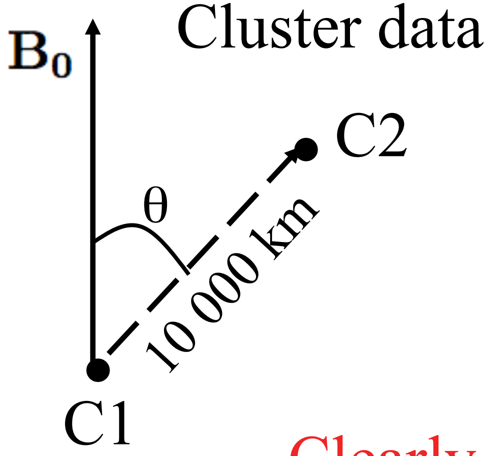

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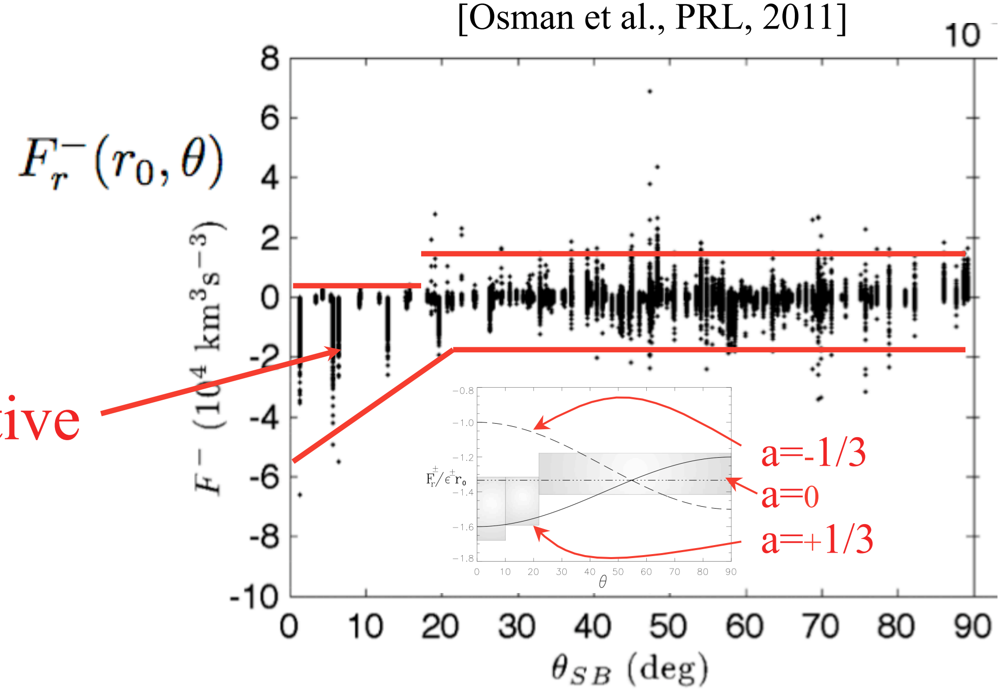
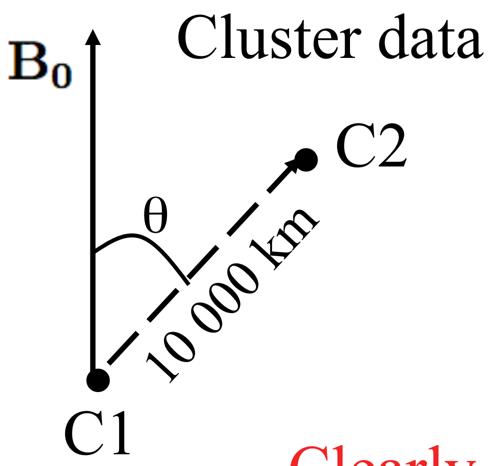


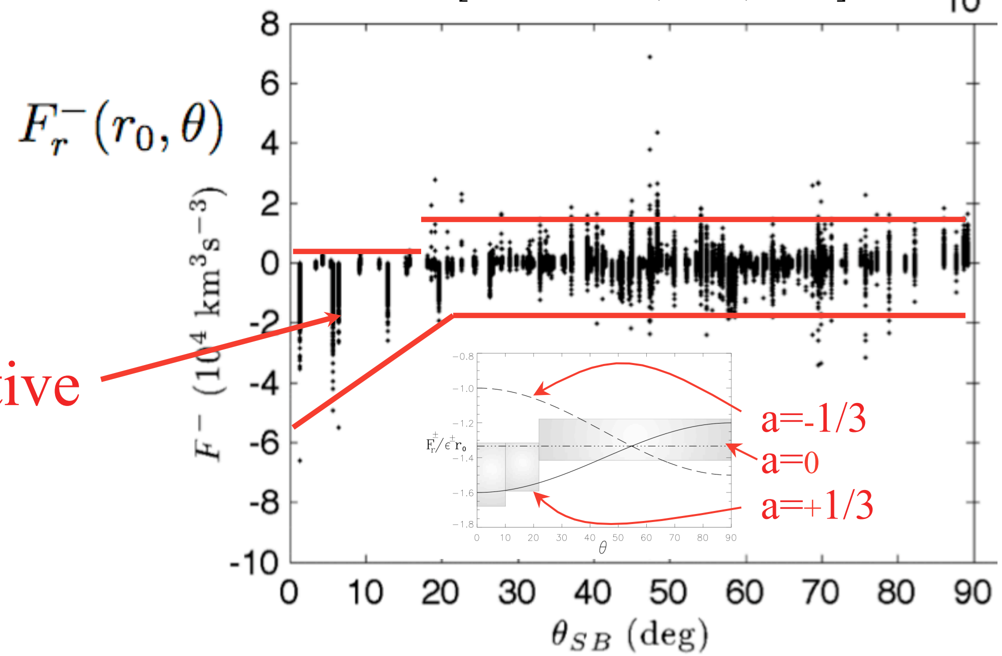
Theory says: $F_r^{\pm}(r,\theta) = -\frac{4}{3+a}\varepsilon^{\pm} r(1 + a\cos^2\theta)$

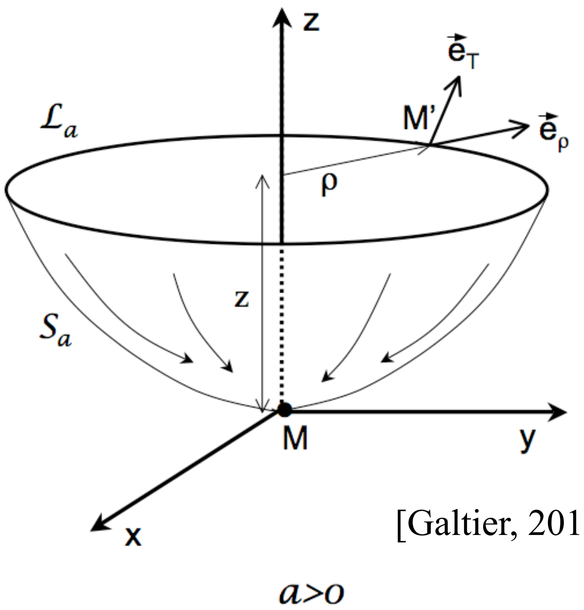
Solar wind turbulence



Clearly more negative

[Osman et al., PRL, 2011]





[Galtier, 2011]

Theory says: $F_r^{\pm}(r,\theta) = -\dfrac{4}{3+a}\varepsilon^{\pm} r(1 + a\cos^2\theta)$

Compatible with a convex turbulence

Hall MHD turbulence

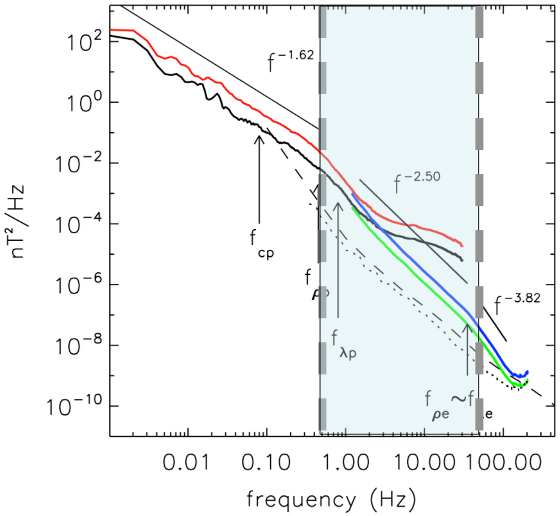
$$\begin{cases} (\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P_* + \mathbf{b}\cdot\nabla\mathbf{b} + \nu\Delta\mathbf{v} \\ (\partial_t + \mathbf{v}\cdot\nabla)\mathbf{b} = \mathbf{b}\cdot\nabla\mathbf{v} - \boxed{d_I \nabla \times [(\nabla\times\mathbf{b})\times\mathbf{b}]} + \eta\Delta\mathbf{b} \end{cases}$$

Kolmogorov's law for **Hall** MHD : [Galtier, PRE, 2008]

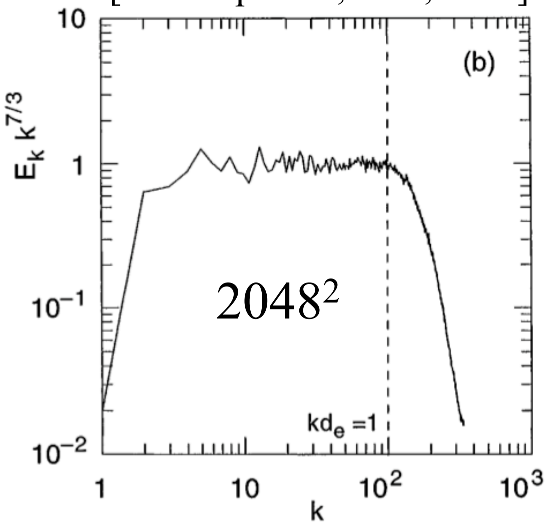
$$-\frac{4}{3}\varepsilon^T r = \langle[(\delta\mathbf{v})^2 + (\delta\mathbf{b})^2]\delta v_r\rangle - 2\langle[\delta\mathbf{v}\cdot\delta\mathbf{b}]\delta b_r\rangle \Big\} \quad E \sim k^{-5/3}$$

$$+ \ ??$$

[Sahraoui et al., PRL, 2009]



[Biskamp et al., PRL, 1996]



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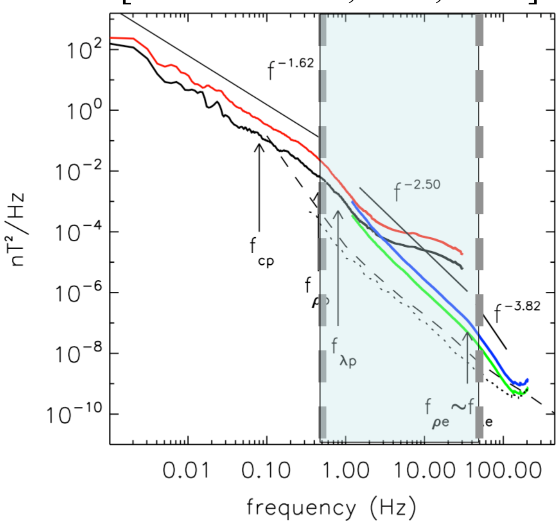
$$+4d_I\langle[(\mathbf{J}\times\mathbf{b})\times\delta\mathbf{b}]_r\rangle \Big\} \; E^b \sim k^{-7/3}$$

[Sahraoui et al., PRL, 2009]



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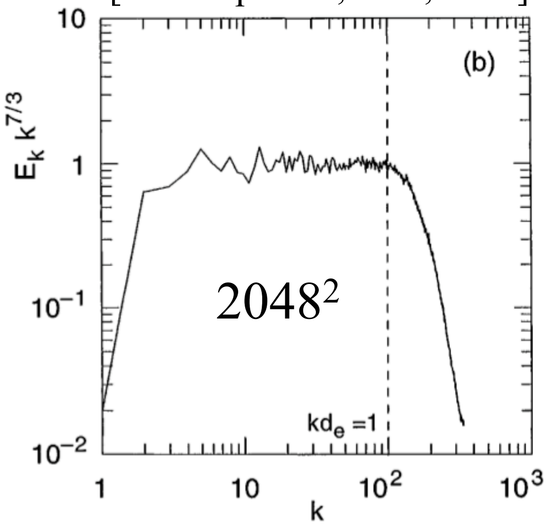

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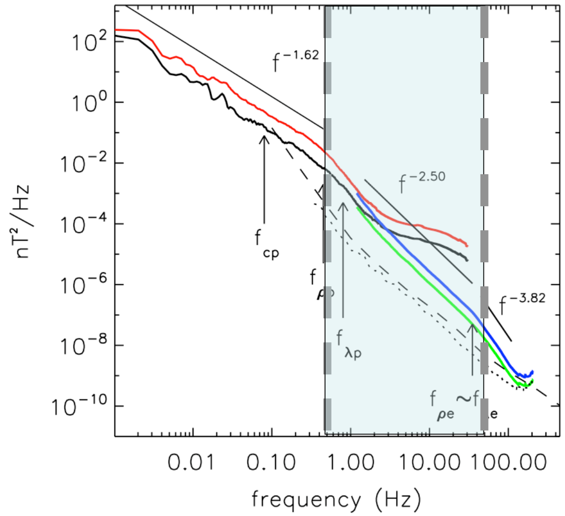
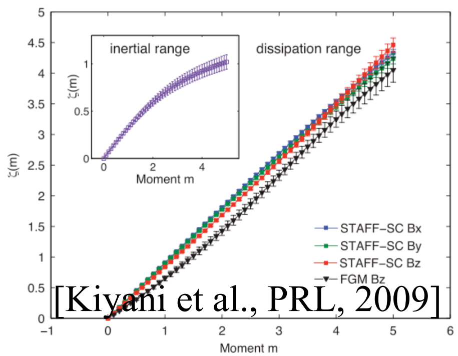


But 7/3 is not 2.5 and :



[Kiyani et al., PRL, 2009]

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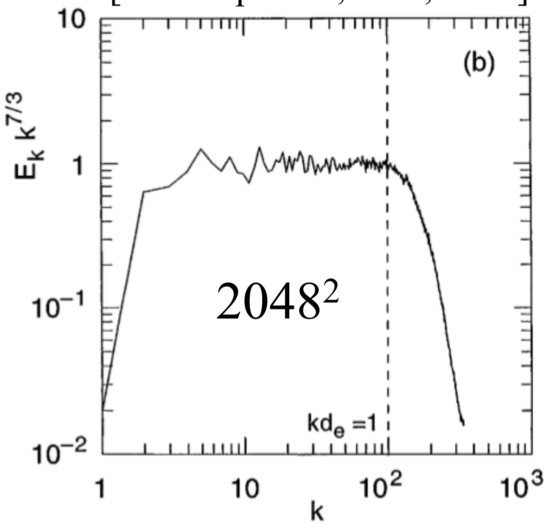

Whistler wave turbulence

[Galtier, J. Plasma Phys., 2006]

- Expansion in ε: **B** (**x**,t) = $B_o \, \mathbf{e}_{//} + \varepsilon \mathbf{b}(\mathbf{x},t)$ with $0 < \varepsilon << 1$
- Complex helicity decomposition (whistler waves): $\omega_w = d_i B_0 \, k_{//} k$
- Leads to the wave amplitude equation: (three-wave interactions)

$$\partial_t a^s_\Lambda = \frac{\epsilon}{4d_i} \int \sum_{\substack{\Lambda_p,\Lambda_q \\ s_p, s_q}} \xi^{s\,2}_\Lambda \frac{\xi^{s_q}_{\Lambda_q} - \xi^{s_p}_{\Lambda_p}}{\xi^s_\Lambda - \xi^{-s}_\Lambda} \underbrace{M^{\Lambda\Lambda_p\Lambda_q}_{\substack{s\, s_p\, s_q \\ -k\, p\, q}}} a^{s_p}_{\Lambda_p} a^{s_q}_{\Lambda_q} e^{-i\Omega_{pq,k}t} \delta_{pq,k} \, d\mathbf{p} \, d\mathbf{q},$$

matrix of 9 indices (but symmetrical)

k

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[Galtier, J. Plasma Phys., 2006]

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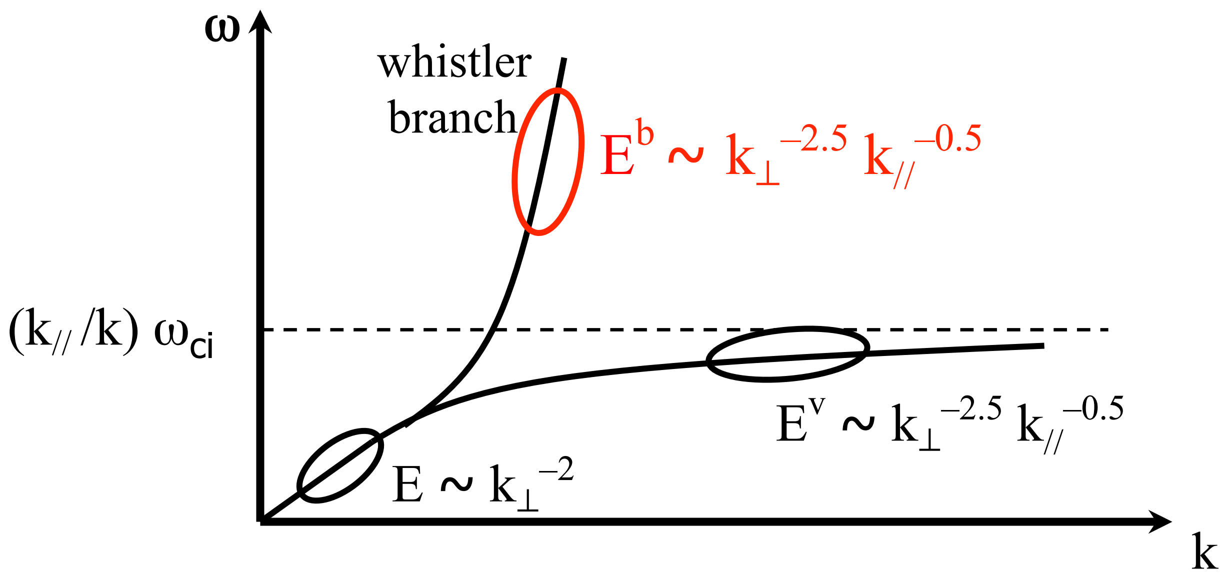
⋮

matrix of 9 indices (but symmetrical)

- Exact solutions:

$(k_\perp >> k_{//})$



Solar wind turbulence

absence of intermittency

+

steep (-2.5) magnetic spectrum

are currently only compatible with

whistler wave turbulence

Alfvén wave turbulence

[Galtier et al., J. Plasma Phys., 2000]

- Asymptotic theory $(B_0 \rightarrow +\infty;\ \tau_A \ll \tau_{nl})$:

$(k_\perp \gg k_{//})$

$$\frac{\partial E^{\pm}(k_\perp, k_\parallel)}{\partial t} = \frac{\pi \varepsilon^2}{B_0} \iint_\Delta \cos^2\phi \sin\theta \, \frac{k_\perp}{q_\perp} E^{\mp}(q_\perp, 0) \left[k_\perp E^{\pm}(p_\perp, k_\parallel) - p_\perp E^{\pm}(k_\perp, k_\parallel) \right] dp_\perp dq_\perp$$

→ no transfer along $\mathbf{B_0}$, hence : $E^{\pm}(k_\perp, k_\parallel) = E^{\pm}(k_\perp) f_{\pm}(k_\parallel)$

→ exact stationary solutions:

$E^{\pm}(k_\perp) \sim k_\perp^{n\pm}$

$n_+ + n_- = -4$

Alfvén wave turbulence

[Galtier et al., J. Plasma Phys., 2000]

- Asymptotic theory ($B_0 \to +\infty$; $\tau_A << \tau_{nl}$) :

($k_\perp >> k_{//}$)

$$\frac{\partial E^{\pm}(k_\perp, k_\parallel)}{\partial t} = \frac{\pi \varepsilon^2}{B_0} \iint_\Delta \cos^2\phi \sin\theta \, \frac{k_\perp}{q_\perp} E^{\mp}(q_\perp, 0) \left[k_\perp E^{\pm}(p_\perp, k_\parallel) - p_\perp E^{\pm}(k_\perp, k_\parallel)\right] dp_\perp dq_\perp$$

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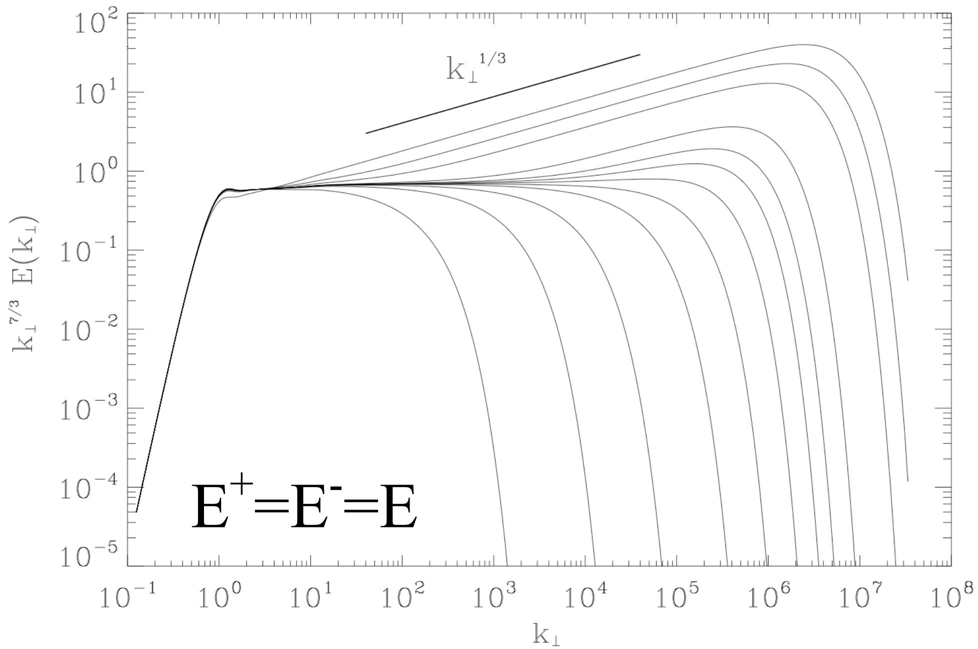
$E^{\pm}(k_\perp) \sim k_\perp^{n\pm}$

$n_+ + n_- = -4$

→ anomalous scaling in $k_\perp^{-7/3}$

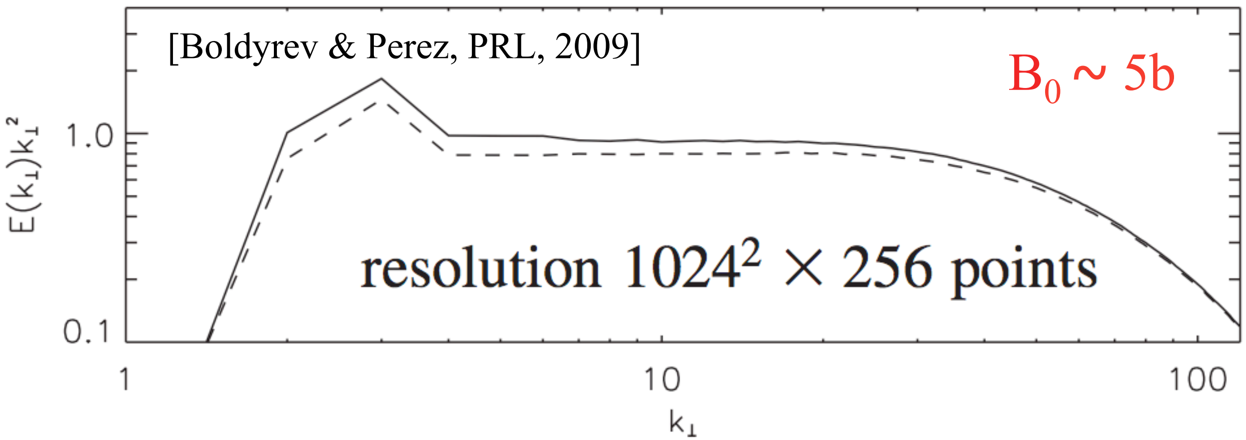
Generic behavior !

[Connaughton et al., Phys. D, 2003]

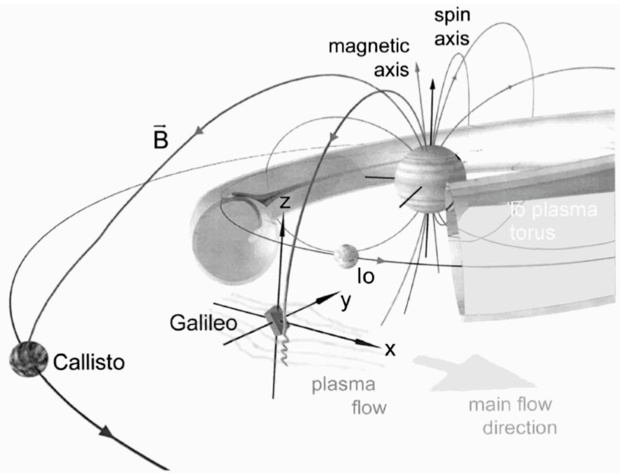


Alfvén wave turbulence

- First signature with direct numerical simulations: [Bigot et al., PRE, 2008]



- Indirect signature in the Jupiter's magnetosphere

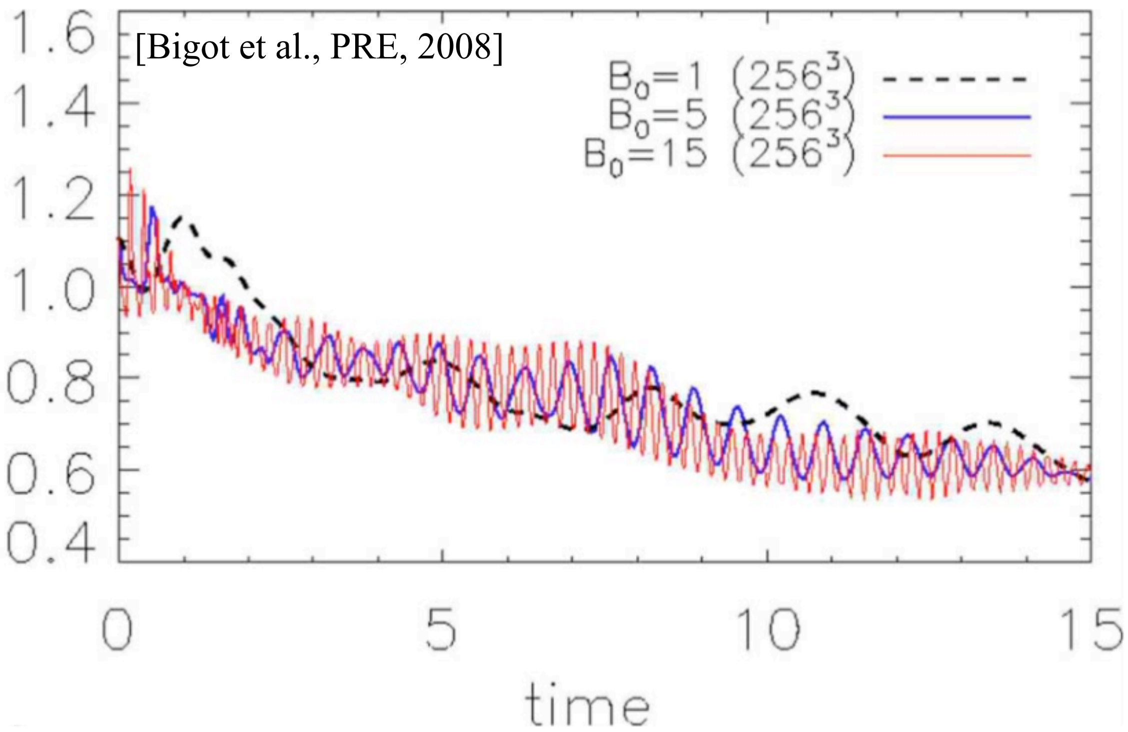


[Saur et al., A&A, 2002]

Alfvén wave turbulence

Equipartition is expected but never reached !

Alfvén ratio: E^u / E^b



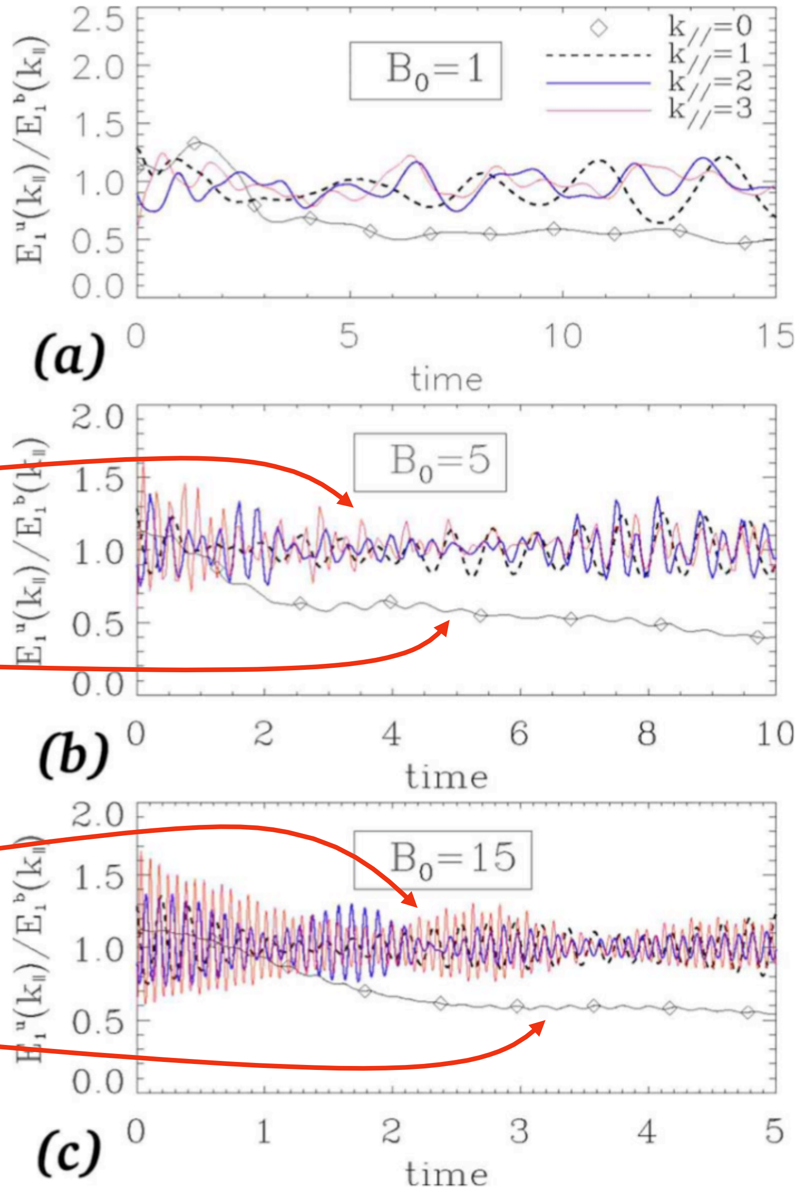
Why ?

Alfvén wave turbulence may be hidden

$E^u(k_{//}) \,/\, E^b\,(k_{//})$

[Bigot et al., PRE, 2008]

512^2x64



3D modes ($k_{//}>0$)

Slow mode ($k_{//}=0$)

$\omega_A = B_0\, k_{//}$

Wave turbulence

Strong turbulence

Alfvén wave turbulence

- Role of the slow mode ($q_{//}=0$ is not weak turbulence):

$$\frac{\partial E^{\pm}(k_\perp, k_\parallel)}{\partial t} = \frac{\pi \varepsilon^2}{B_0} \iint_\Delta \cos^2\phi \sin\theta \, \frac{k_\perp}{q_\perp} \underbrace{E^{\mp}(q_\perp, 0)} \left[k_\perp E^{\pm}(p_\perp, k_\parallel) - p_\perp E^{\pm}(k_\perp, k_\parallel) \right] dp_\perp dq_\perp$$

dynamics may be given
by strong turbulence

[Galtier et al., J. Plasma Phys., 2000]

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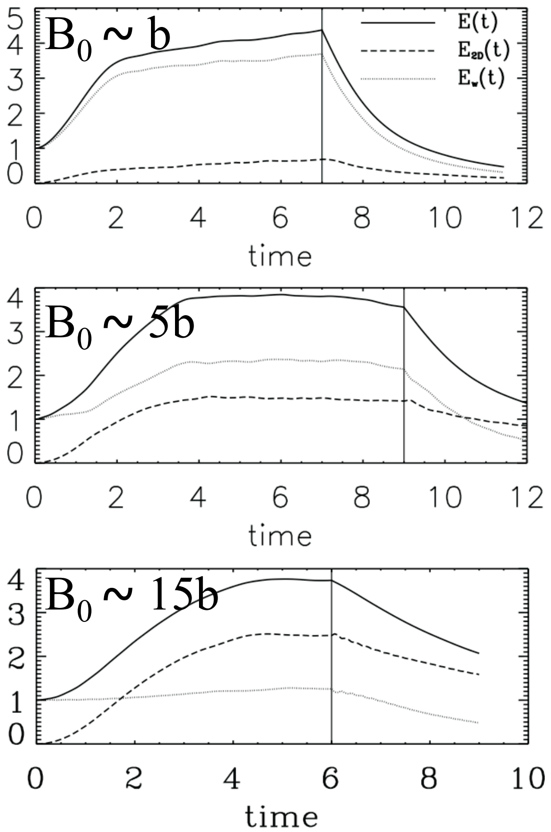
512^3 [Bigot et al., PRE, 2011]



dynamics may be given by **strong** turbulence [Galtier et al., J. Plasma Phys., 2000]

Alfvén wave turbulence

- Role of the slow mode ($q_{//}=0$ is not weak turbulence):

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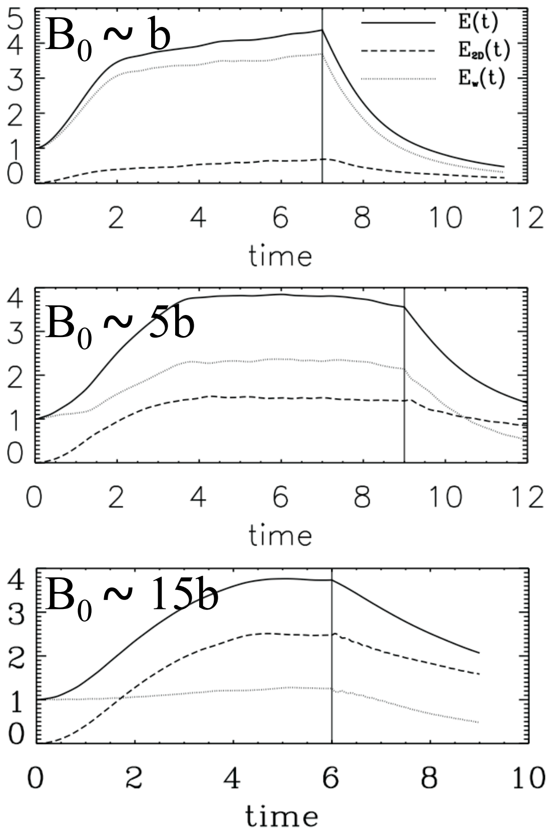


dynamics may be given by strong turbulence [Galtier et al., J. Plasma Phys., 2000]

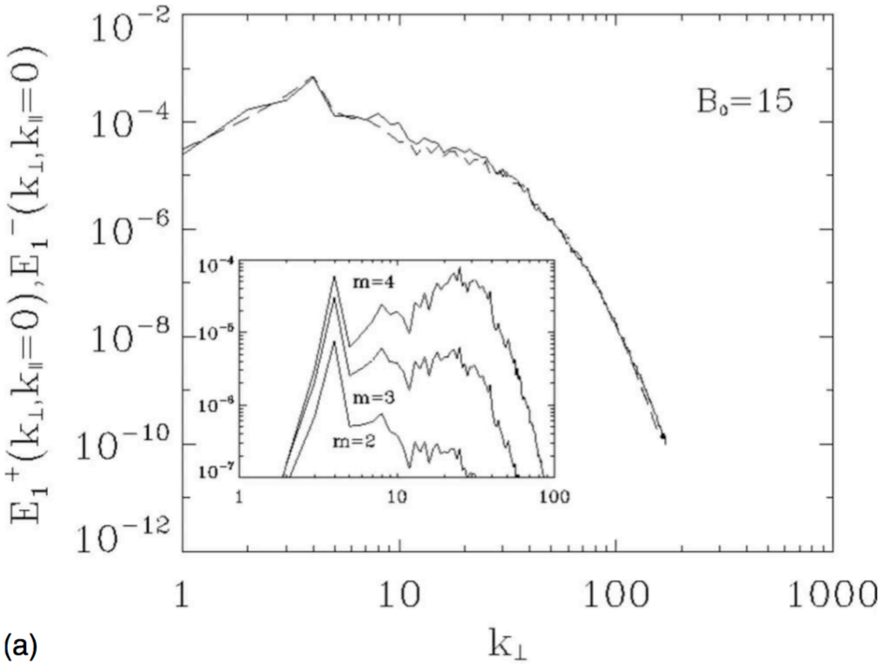
$n_{2D} + n_W = -4$ always valid

$E^{\pm}(k_\perp, k_{//}=0) < E^{\pm}(k_\perp, k_{//}>0)$: $n_{2D} = n_W = -2$

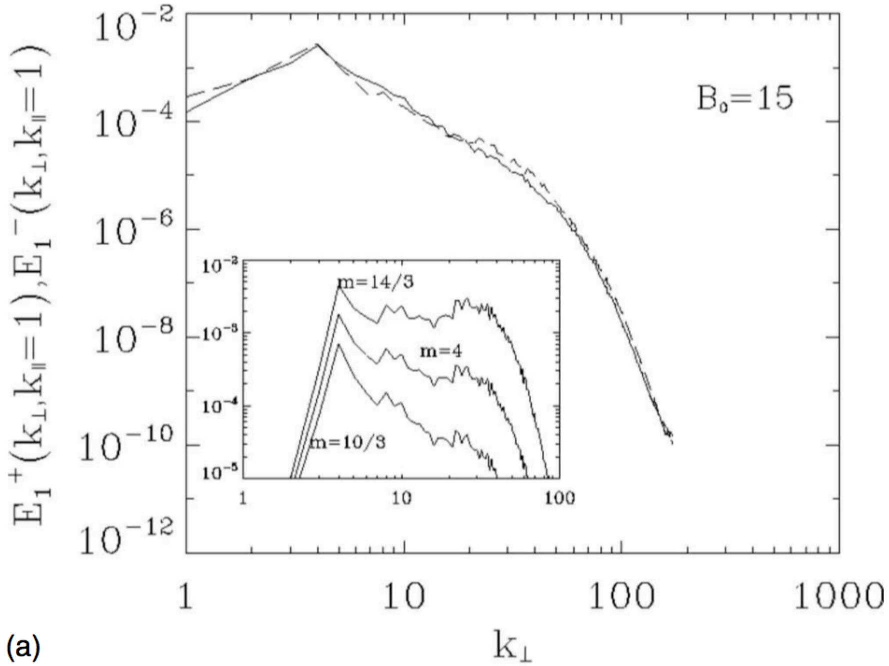
$E^{\pm}(k_\perp, k_{//}=0) > E^{\pm}(k_\perp, k_{//}>0)$: $\begin{cases} n_{2D} = -5/3 \\ n_W = -7/3 \end{cases}$

~ Kolmogorov scaling (with intermittency)

Alfvén wave turbulence



(a)



(a)

[Bigot et al., PRE, 2008]

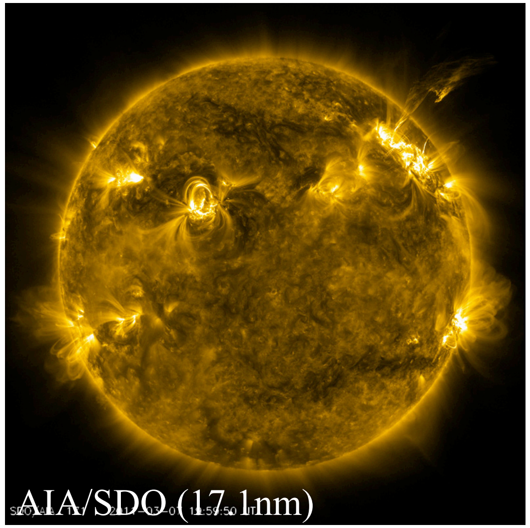
$n_{2D} \approx -5/3$

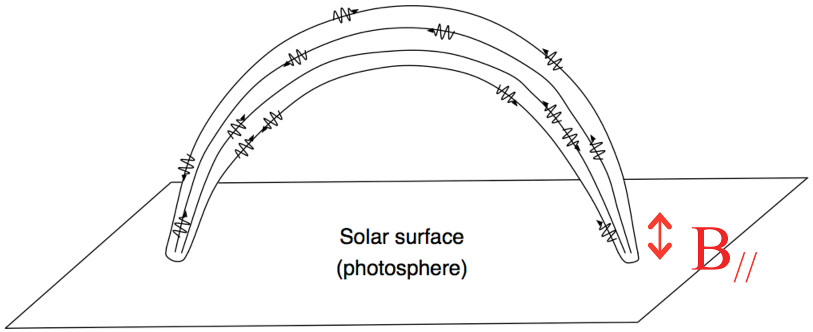
$n_W \approx -7/3$

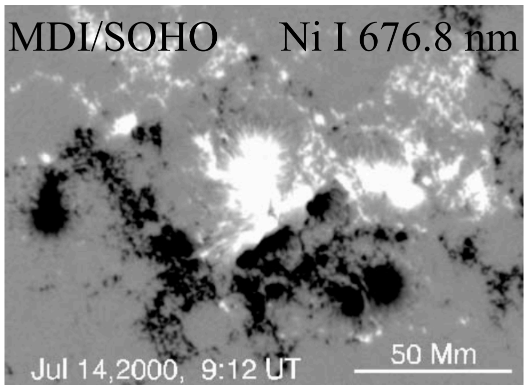
Solar Alfvén wave turbulence

- Alfvén waves have been detected on the Sun
  [Cargill et al., Nature, 2011]

- Direct signature of weak MHD turbulence







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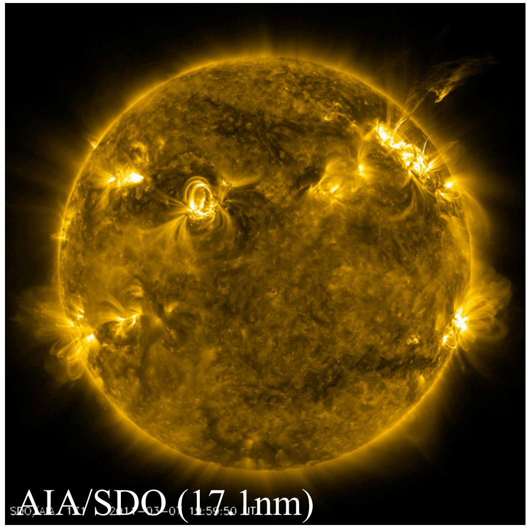

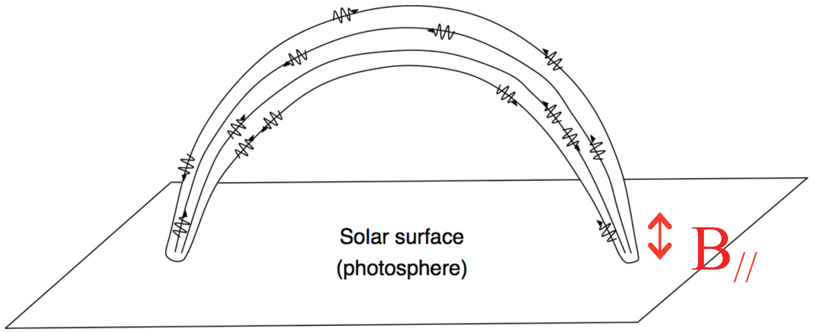

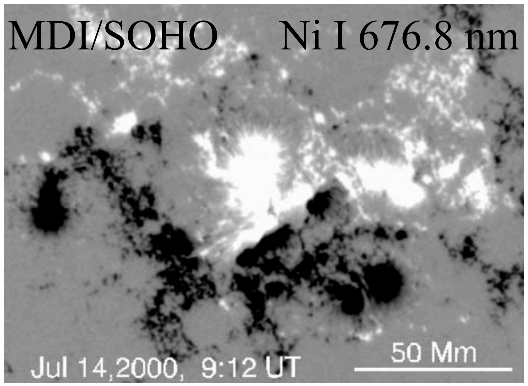

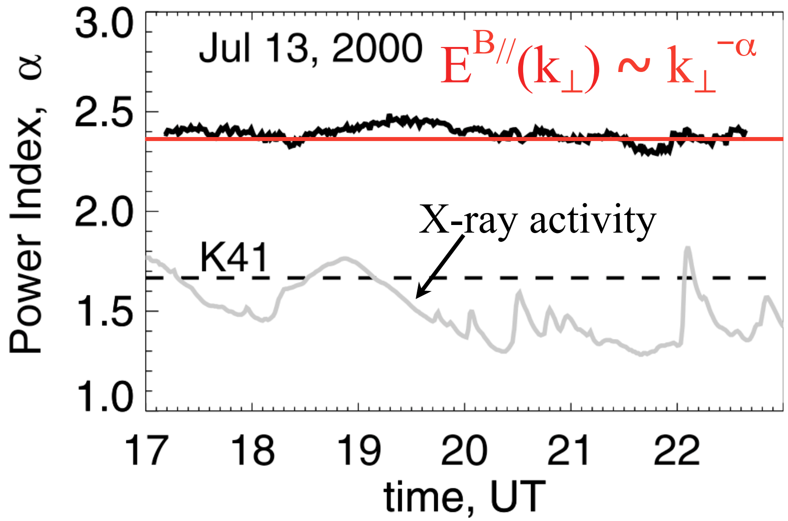

[Abramenko, ApJ, 2005]

Compressible turbulence

PRL **107,** 134501 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Exact Relation for Correlation Functions in Compressible Isothermal Turbulence

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(Received 24 March 2011; published 23 September 2011)

Compressible isothermal turbulence is analyzed under the assumption of homogeneity and in the asymptotic limit of a high Reynolds number. An exact relation is derived for some two-point correlation functions which reveals a fundamental difference with the incompressible case. The main difference resides in the presence of a new type of term which acts on the inertial range similarly as a source or a sink for the mean energy transfer rate. When isotropy is assumed, compressible turbulence may be described by the relation $-\frac{2}{3}\varepsilon_{\text{eff}}r = \mathcal{F}_r(r)$, where $\mathcal{F}_r$ is the radial component of the two-point correlation functions and ε_{eff} is an effective mean total energy injection rate. By dimensional arguments, we predict that a spectrum in $k^{-5/3}$ may still be preserved at small scales if the density-weighted fluid velocity $\rho^{1/3}\mathbf{u}$ is used.

<u>Motivations</u>: simulations, models and absence of Kolmogorov theory

[Kadomtsev et al., Sov. Phys, 1973; Kritsuk et al., ApJ, 2007; Federrath et al., A&A, 2010]

Compressible isothermal turbulence

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \qquad P = C_s^2 \rho$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \mu \Delta \mathbf{u} + \frac{\mu}{3} \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f},$$

$$\begin{cases} E = \rho u^2/2 + \rho e \\ e = C_s^2 \ln(\rho/\rho_0) \end{cases}$$

- Homogeneity
- 3D direct cascade
- ε becomes constant when Re → +∞
- Stationary turbulence

→ $$\frac{\mathcal{R}(\mathbf{r}) + \mathcal{R}(-\mathbf{r})}{2} = \langle E \rangle - \frac{1}{4}\langle \delta(\rho \mathbf{u}) \cdot \delta \mathbf{u} \rangle - \frac{1}{2}\langle \delta\rho \delta e \rangle,$$

$\mathcal{R}(\mathbf{r}) \equiv \langle \rho \mathbf{u} \cdot \mathbf{u}'/2 + \rho e' \rangle \equiv \langle R \rangle$: 2 points correlation function

Exact relation for compressible turbulence

Source term (S) modifies ε → ε_{eff}

from pressure

$$-2\varepsilon = \langle(\nabla' \cdot \mathbf{u}')(R - E)\rangle + \langle(\nabla \cdot \mathbf{u})(\tilde{R} - E')\rangle$$
$$+ \nabla_{\mathbf{r}} \cdot \left\langle\left[\frac{\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u}}{2} + \delta\rho\delta e - C_s^2\bar{\delta}\rho\right]\delta\mathbf{u} + \bar{\delta}e\delta(\rho\mathbf{u})\right\rangle,$$

Exact relation for compressible turbulence

Source term (S) modifies $\varepsilon \rightarrow \varepsilon_{eff}$

from pressure

$$-2\varepsilon = \langle(\nabla' \cdot \mathbf{u}')(R - E)\rangle + \langle(\nabla \cdot \mathbf{u})(\tilde{R} - E')\rangle + \nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{\delta(\rho \mathbf{u}) \cdot \delta \mathbf{u}}{2} + \delta\rho\delta e - C_s^2 \bar{\delta}\rho \right] \delta \mathbf{u} + \bar{\delta} e \delta(\rho \mathbf{u}) \right\rangle,$$

ISOTROPIC TURBULENCE

$R - E$ is mainly negative

: direct cascade

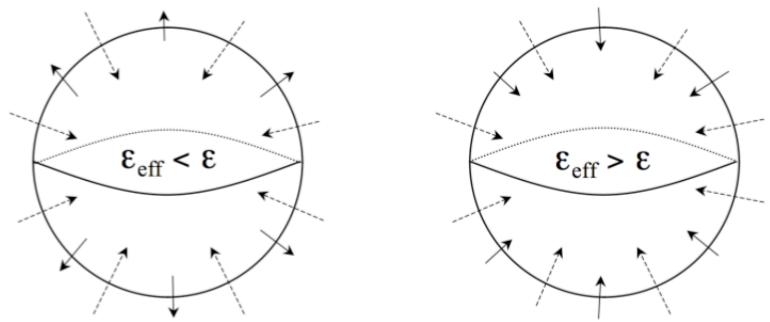

FIG. 1: Dilatation (left) and compression (right) phases in space correlation for isotropic turbulence. In a direct cascade scenario the flux vectors $\mathcal{F}$ (dashed arrows) are oriented towards the center of the sphere. Dilatation and compression (solid arrows) are additional effects which act respectively in the opposite or in the same direction as the flux vectors.

Exact relation for compressible turbulence

Source term (S) modifies $\varepsilon \rightarrow \varepsilon_{\text{eff}}$

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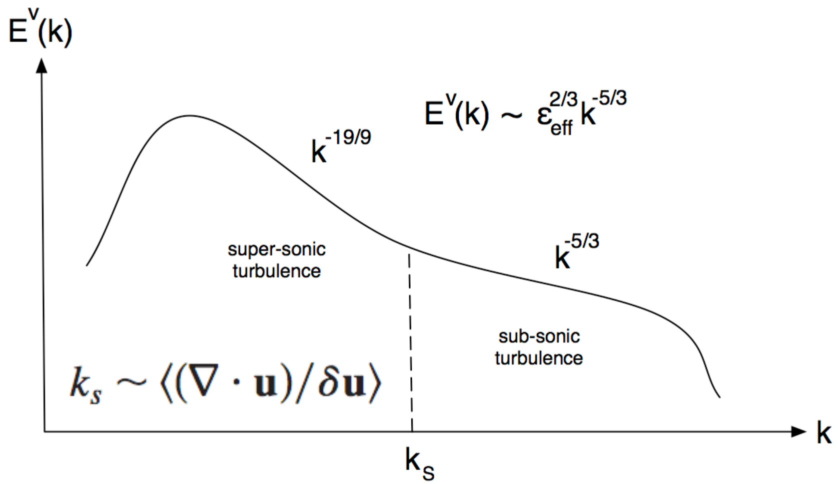
$$-2\varepsilon = \langle(\nabla' \cdot \mathbf{u}')(R - E)\rangle + \langle(\nabla \cdot \mathbf{u})(\tilde{R} - E')\rangle + \nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u}}{2} + \delta\rho\delta e - C_s^2 \bar{\delta}\rho \right] \delta\mathbf{u} + \bar{\delta} e \delta(\rho\mathbf{u}) \right\rangle,$$

ISOTROPIC TURBULENCE

$$\mathcal{S}(r) \simeq -\langle\bar{\delta}(\nabla \cdot \mathbf{u})[\tfrac{1}{4}\delta(\rho\mathbf{u}) \cdot \delta\mathbf{u} + \tfrac{1}{2}\delta\rho\delta e]\rangle$$

$\sim r^{2/3}$

$v \equiv u\, \rho^{1/3}$



$R - E$ is mainly negative

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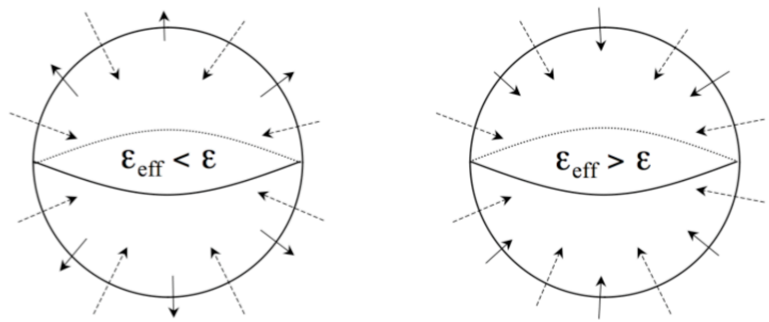

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Summary

- Vectorial law for a class of axisymmetric MHD turbulence
- Dispersive solar wind turbulence: Hall MHD fits well
- Is -7/3 more universal than -2 in Alfvén wave turbulence ?
- Direct signature of solar Alfvén wave turbulence
- Exact relation for compressible turbulence (70 years after K41)

  → $v = u\rho^{1/3}$ is justified; compatible with some DNS

- Future: Solar Orbiter (ESA, 2017)