PIC simulation study of the parametric instabilities of Alfven-ion-cyclotron waves and their turbulence in a low beta plasma

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Outline

1. Introduction

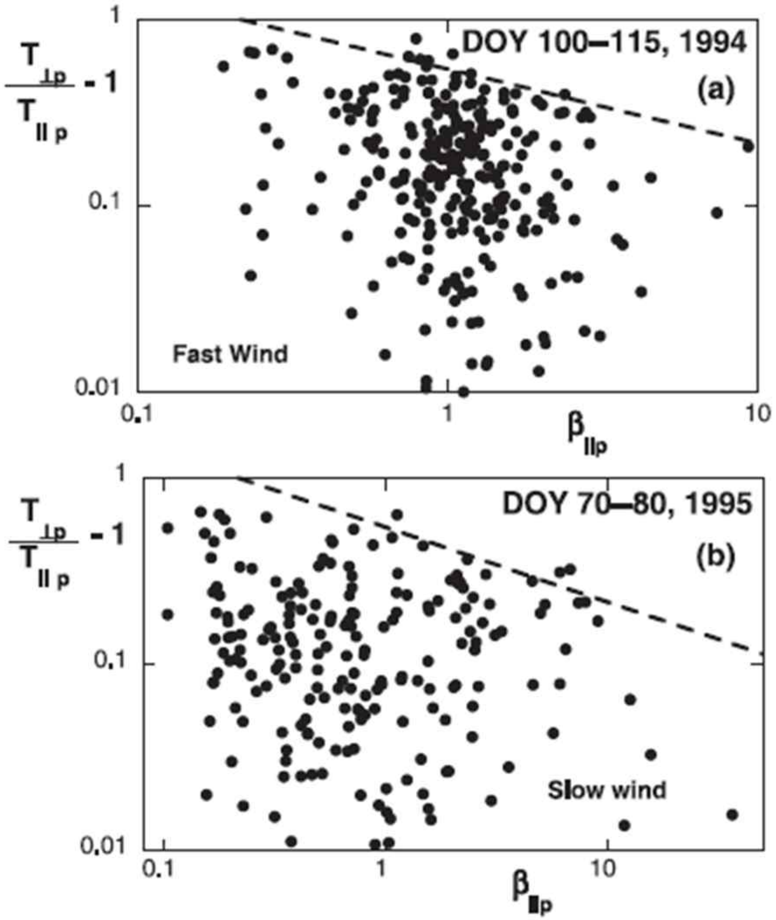
   - Space observation

2. Simulation Results

   - with Initially imposing Alfvenic perturbation
   - with anisotropic temperature

3. Summary

EMIC Instability in Solar Wind



Temperature anisotropy in the solar wind (Ulysses data anayses). [Gary et al., JGR(2009)]

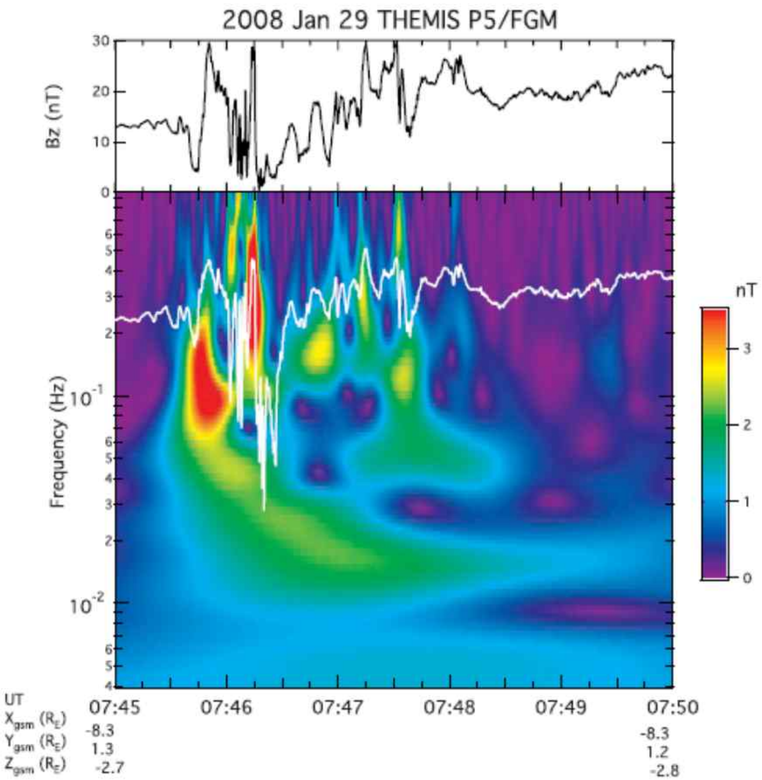
- $T_{\perp i}/T_{\parallel i}>1$ [Marsch et. al, 1982; Neugebauer et al.,2001;Gary et al. 2002]

⇒ Electromagnetic ion-cyclotron (EMIC) instability can be excited

  - Anisotropic ion kinetic energy with ambient magnetic field

- EMIC mode is observed in space.

EMIC Turbulence in Magnetotail

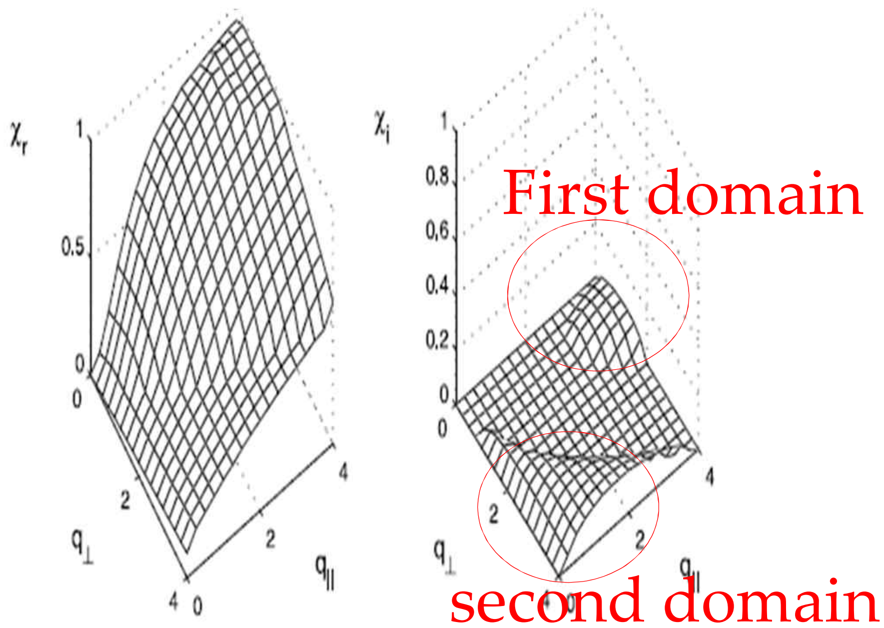


Wavelet analysis of observation data by THEMIS satellite [Lui et al., JGR(2008)].

- Turbulence observed in the magnetotail.
- The inverse cascade begins at high frequencies and settles at low frequencies.
- EMIC instability occurs in two distinct frequency domains.
- These instabilities can be explained by EMIC modes induced by ion drift.

EMIC Instability Induced by Ion Drift

- The wave spectrum is characterized by a gap between the two unstable modes.
- EMIC instability occurs in two distinct frequency domains.
- The first domain corresponds to waves propagating nearly along the magnetic field.
- The second regime corresponds to waves propagating mainly perpendicular to the magnetic field.



The linear dispersion curve of the EMIC instability induced by ion drift [Mok et al., JGR(2010)].

Excitation of EMIC waves in simulation

Two methods to excite Alfven-ion-cyclotron waves:

- Initially imposing Alfvenic perturbation : using Walen condition

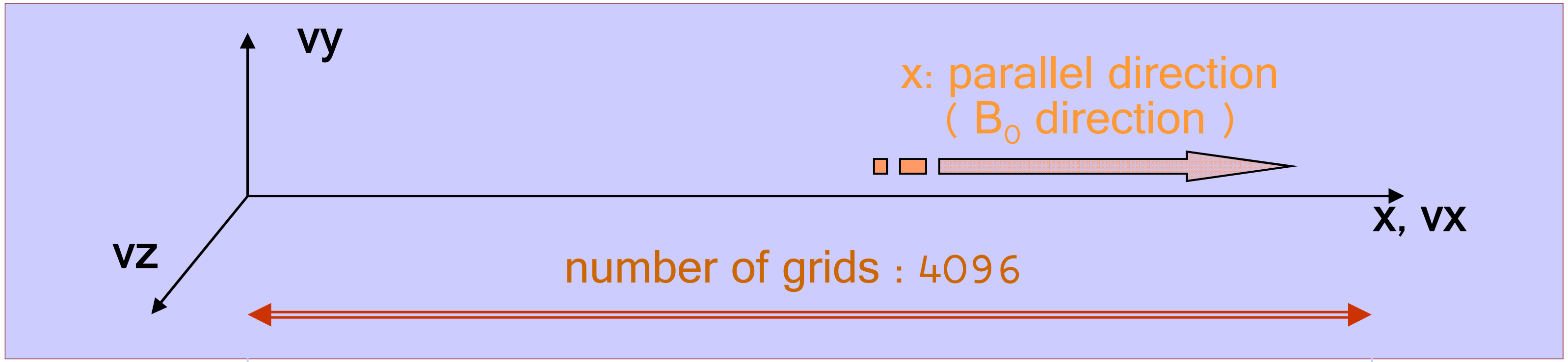
$$b_p = \sum_{k_0=k_1}^{k_2} b_{k_0}^h \exp[i(\omega_0 t - k_0 x + \phi_{k_0})],$$

$$v_p = \sum_{k_0=k_1}^{k_2} v_{k_0}^h \exp[i(\omega_0 t - k_0 x + \phi_{k_0})], \qquad where, \quad k_0^2 = \frac{\omega_0^2}{1 \pm \omega_0}$$

- Anisotropic temperature

1D PIC Simulation with Alfvenic Perturbation

Initially imposed Alfvenic perturbation with isotropic temperature plasma



- $m_i/m_e = 16$
- Strength of the background magnetic field : $\Omega_e/\omega_{pe} = 0.5$
- Initial temperature(isotropic Maxwellian distribution)

  : $\beta_e = 0.08,\ \beta_i = 0.1$
- Number of particles : 4000/cell for each species

Initially Imposed Alfvenic Perturbation

$$\delta \boldsymbol{B}(x, t=0)=\sum_{n=1}^{15}\{\delta B_{L_n}[\cos(k_{L_n}x)\hat{\boldsymbol{y}}+\sin(k_{L_n}x)\hat{\boldsymbol{z}}]+\delta B_{R_n}[\cos(k_{R_n}x)\hat{\boldsymbol{y}}-\sin(k_{R_n}x)\hat{\boldsymbol{z}}]\}$$

$$\left(\frac{\delta B_{L(R)_n}}{B_0}\right)^2 = w_0 \exp\left(-\frac{\kappa_{L(R)_n}^2}{\Delta^2}\right), \quad w_0 = 1 \text{ and } \Delta = 0.25.$$

$$\kappa_{L(R)_n} = \frac{k_{L(R)_n} v_A}{\Omega_i} \quad \text{and} \quad \omega_{L_n} = \frac{k_{L_n} v_A}{\sqrt{1 + k_{L_n}^2 v_A^2/\Omega_i^2}},$$

$$\omega_{R_n} = k_{R_n} v_A \sqrt{1 + \frac{k_{R_n}^2 v_A^2}{\Omega_i^2}}.$$

Table 1. Parameters for fluctuation spectrum.

n	$\kappa_{L(R)_n}$	ω_{L_n}/Ω_i	ω_{R_n}/Ω_i
1	0.798	0.624	1.02
2	0.828	0.638	1.08
⋮	⋮	⋮	⋮
15	1.23	0.775	1.94

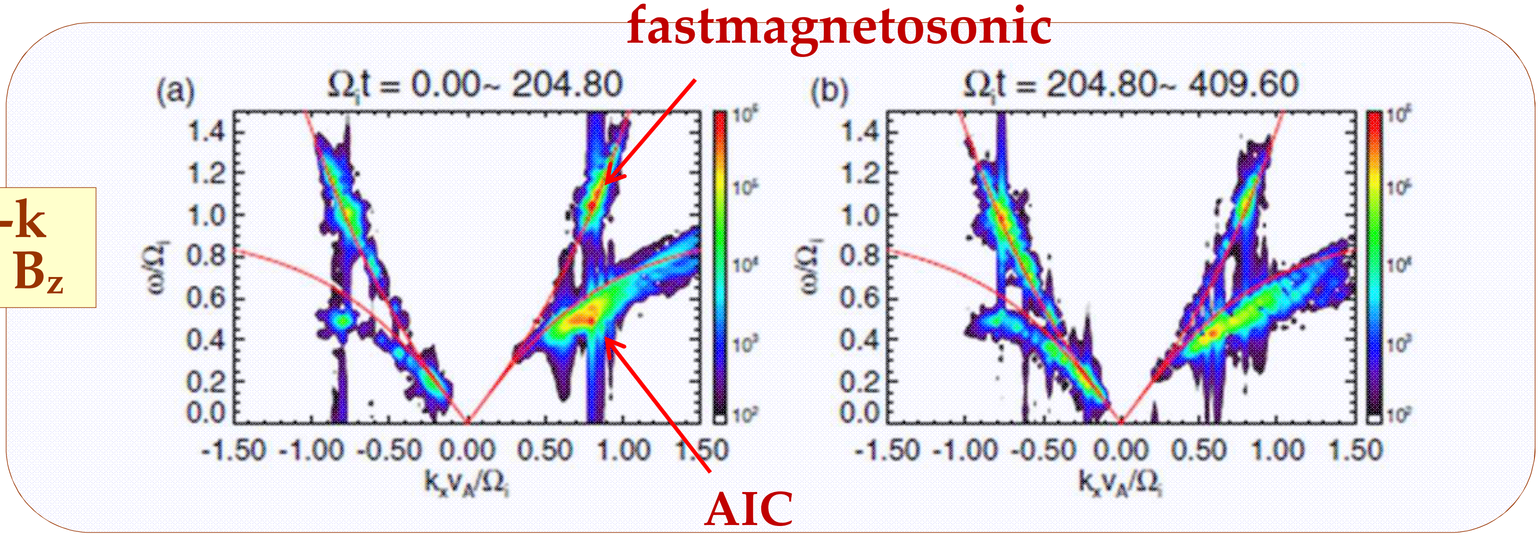
* *k :positive*

$$\delta \boldsymbol{v}_i = \sum_{n=1}^{15}\left\{\frac{\delta B_{L_n}}{B_0}[\cos(k_{L_n}x)\hat{\boldsymbol{y}}+\sin(k_{L_n}x)\hat{\boldsymbol{z}}] \times \left(\frac{(\omega_{L_n}-\Omega_e)({\omega_{L_n}}^2 - c^2{k_{L_n}}^2)}{(\omega_{pe}^2+\omega_{pi}^2)k_{L_n}} - \frac{\omega_{L_n}}{k_{L_n}}\right)\right.$$
$$\left.+\frac{\delta B_{R_n}}{B_0}[\cos(k_{R_n}x)\hat{\boldsymbol{y}}-\sin(k_{R_n}x)\hat{\boldsymbol{z}}] \times \left(\frac{(\omega_{R_n}+\Omega_e)({\omega_{R_n}}^2 - c^2{k_{R_n}}^2)}{(\omega_{pe}^2+\omega_{pi}^2)k_{R_n}} - \frac{\omega_{R_n}}{k_{R_n}}\right)\right\},$$

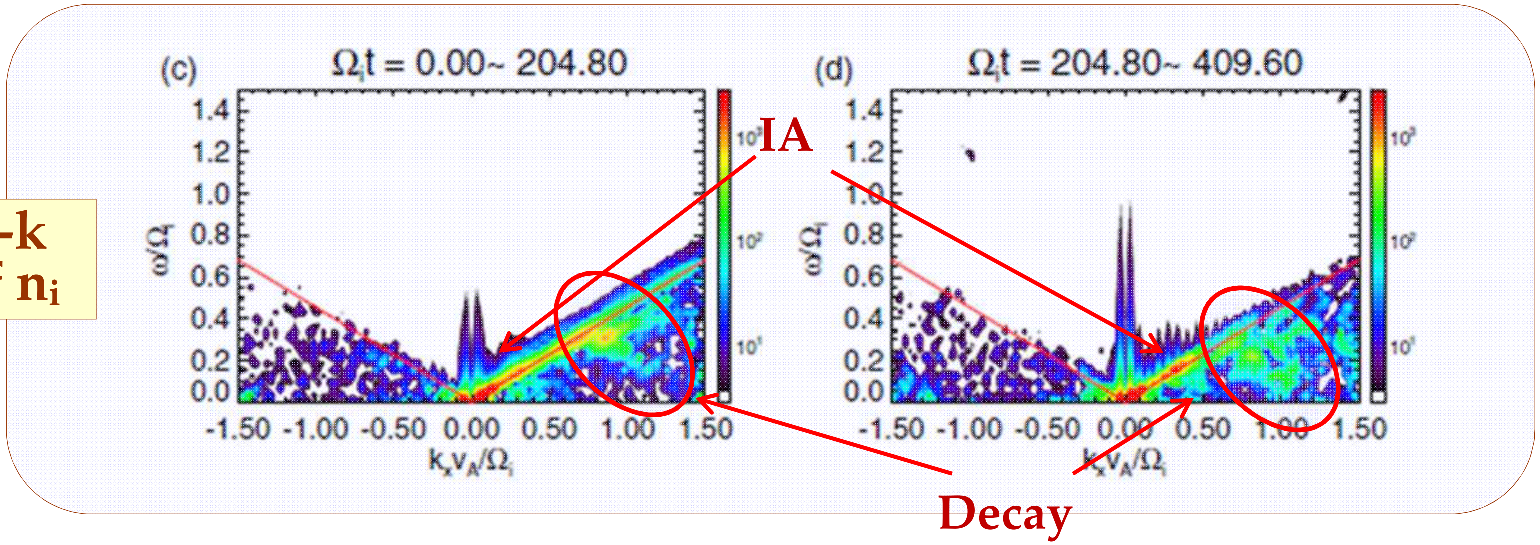
$$\delta \boldsymbol{v}_e = \sum_{n=1}^{15}\left\{\frac{\delta B_{L_n}}{B_0}[\cos(k_{L_n}x)\hat{\boldsymbol{y}}+\sin(k_{L_n}x)\hat{\boldsymbol{z}}] \times \left(\frac{(\omega_{L_n}-\Omega_i)({\omega_{L_n}}^2 - c^2{k_{L_n}}^2)}{(\omega_{pe}^2+\omega_{pi}^2)k_{L_n}} - \frac{\omega_{L_n}}{k_{L_n}}\right)\right.$$
$$\left.+\frac{\delta B_{R_n}}{B_0}[\cos(k_{R_n}x)\hat{\boldsymbol{y}}-\sin(k_{R_n}x)\hat{\boldsymbol{z}}] \times \left(\frac{(\omega_{R_n}+\Omega_i)({\omega_{R_n}}^2 - c^2{k_{R_n}}^2)}{(\omega_{pe}^2+\omega_{pi}^2)k_{R_n}} - \frac{\omega_{R_n}}{k_{R_n}}\right)\right\}.$$

Alfven-cyclotron & fast-magnetosonic waves

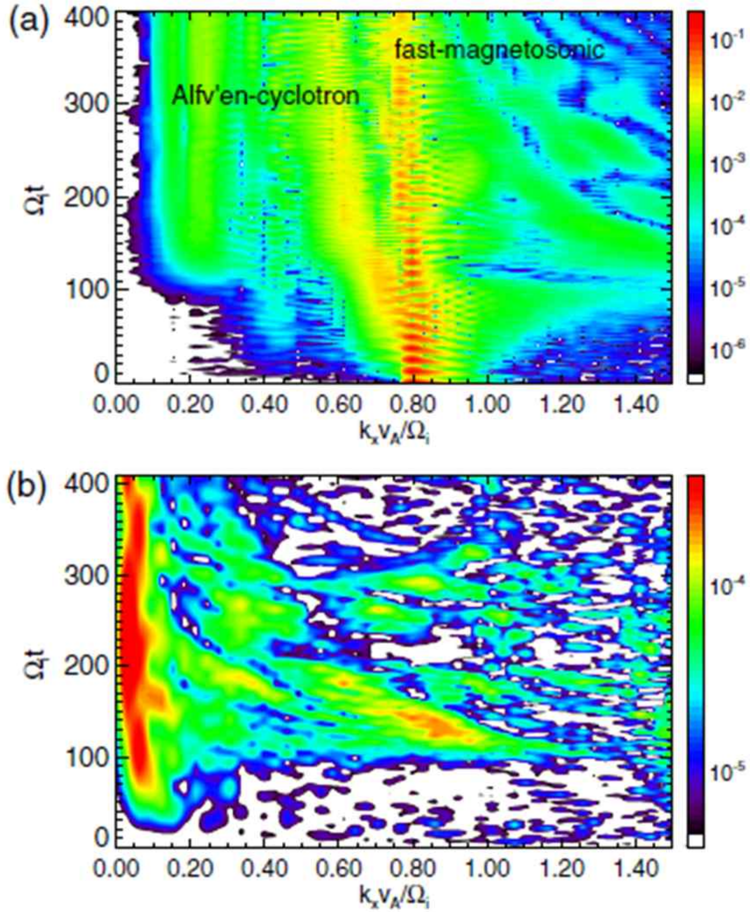
ω-k of B_z



ω-k of n_i

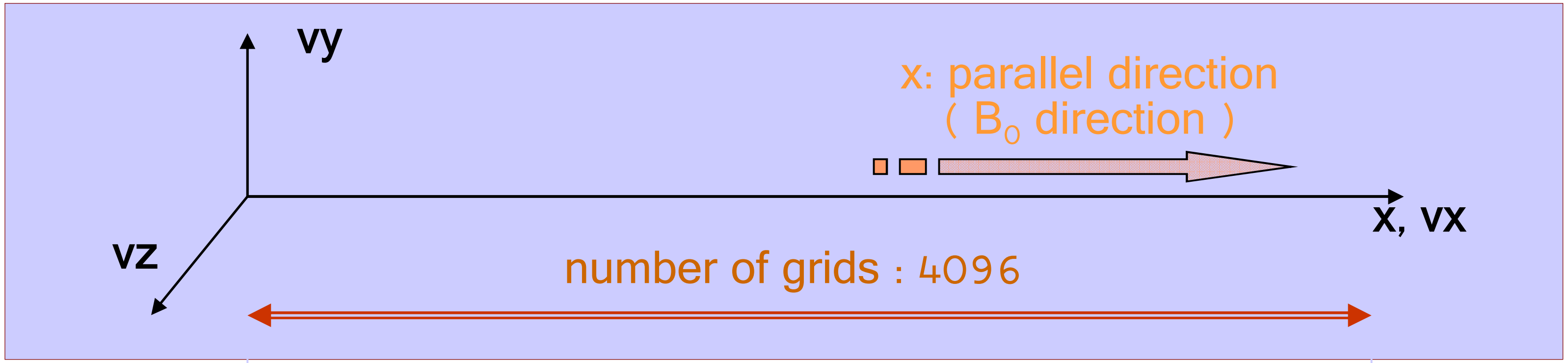


Time Evolution of Excited Modes



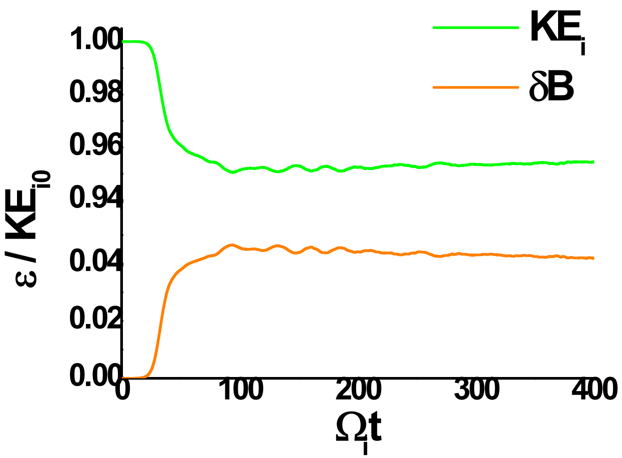
1D PIC Simulation with Anisotropic T

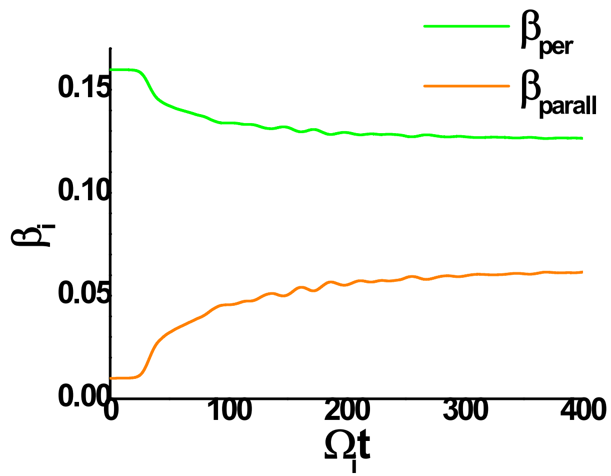
Magnetized plasma with anisotropic ion temperature.

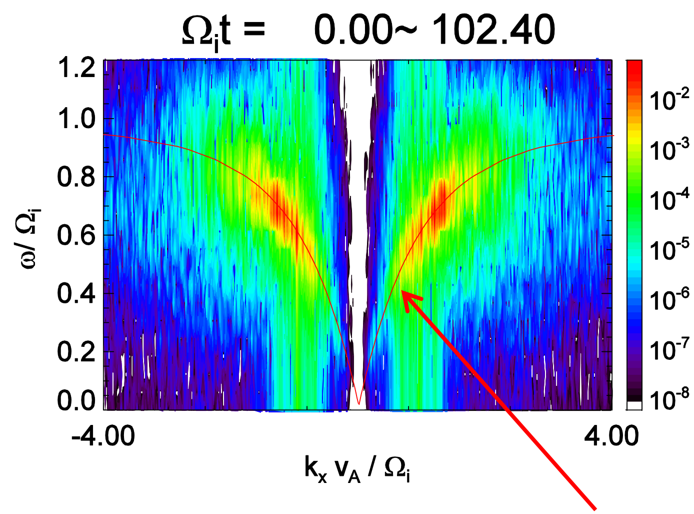


- $m_i/m_e = 25$
- Strength of the background magnetic field : $\Omega_e/\omega_{pe} = 0.5$
- Initial temperature(with Maxwellian distribution)
  - electron (isotropic) : $\beta_e = 0.04$
  - ion (anisotropic) : $\beta_{\perp i} = 0.16$, $\beta_{\parallel i} = 0.01$
- Number of particles : 4000/cell for each species

Energy & Temperature, Excited AIC Mode



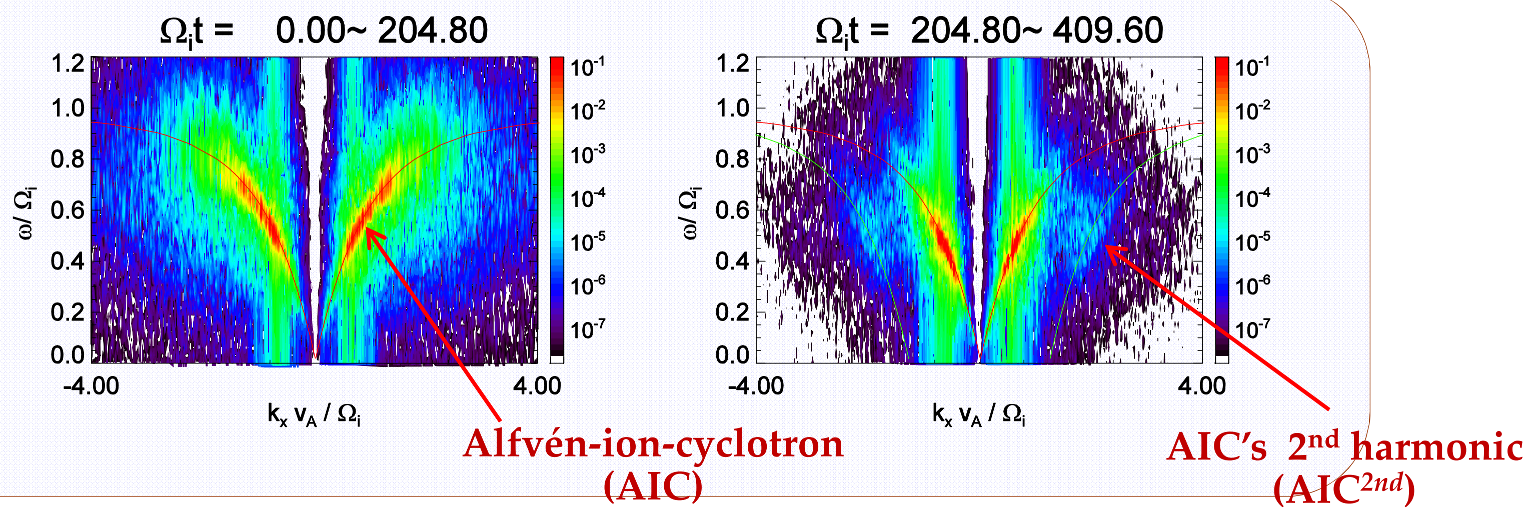




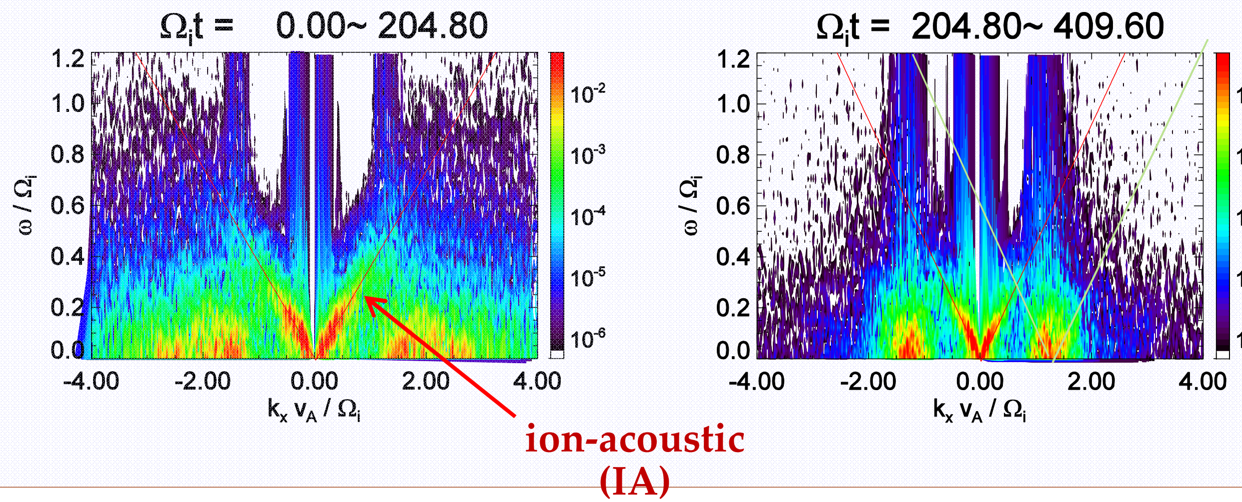
Alfvén-ion-cyclotron (AIC) dispersion curve

Excited Modes at Different Times

ω-k of B_z



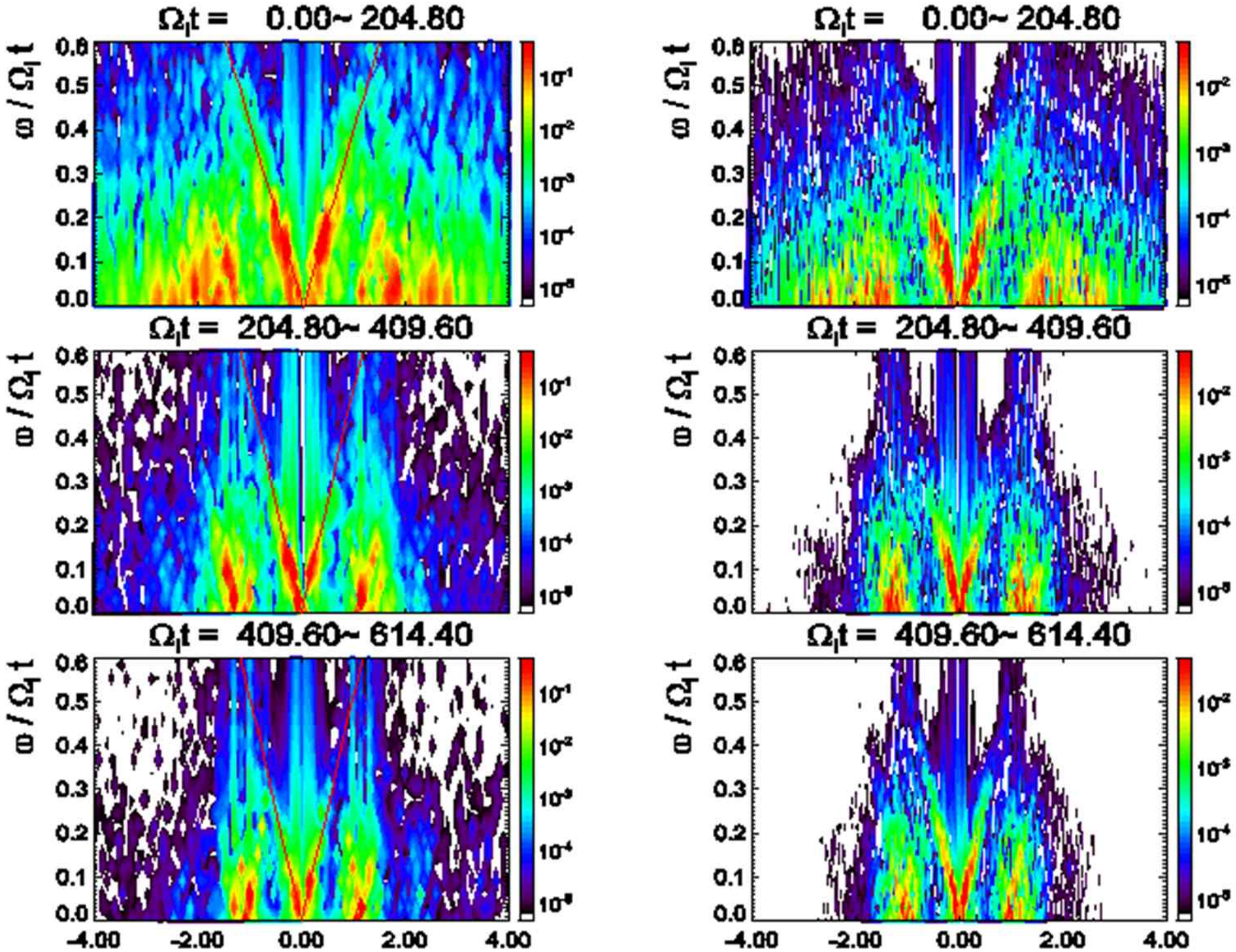
ω-k of n_i



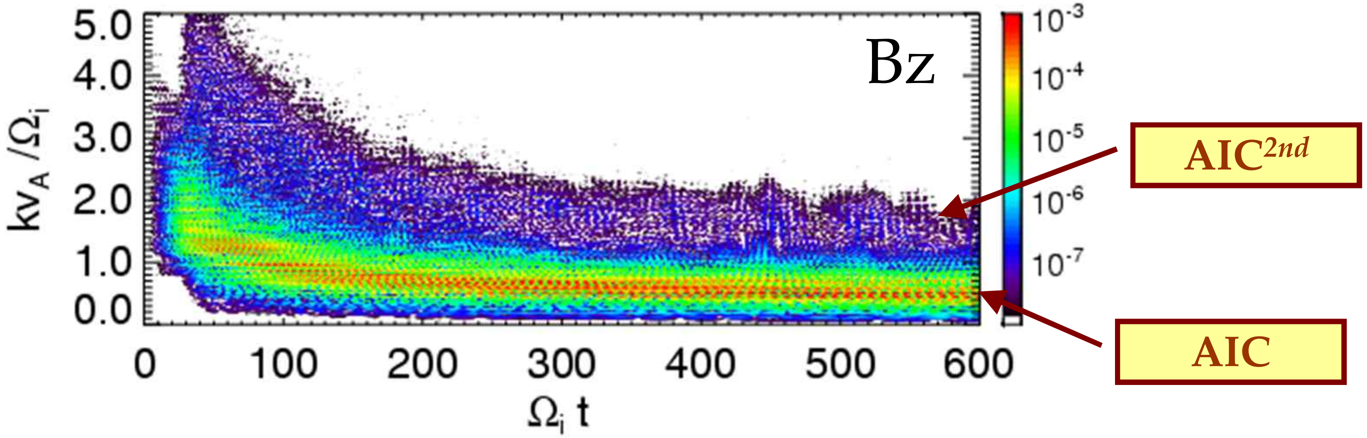
IA^{2nd} Mode for Different Number of Grids

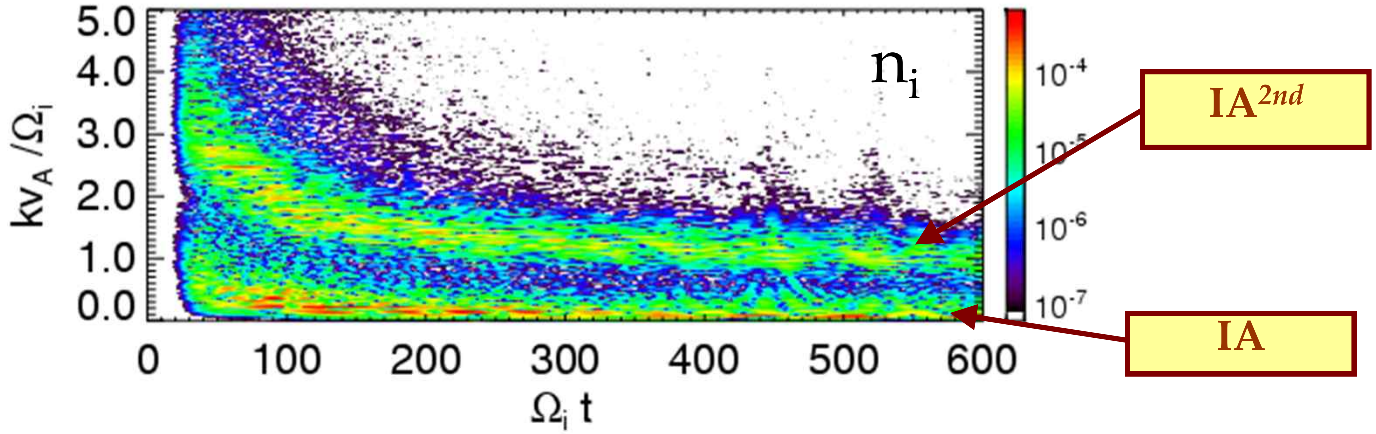
nx=4096

nx=16384

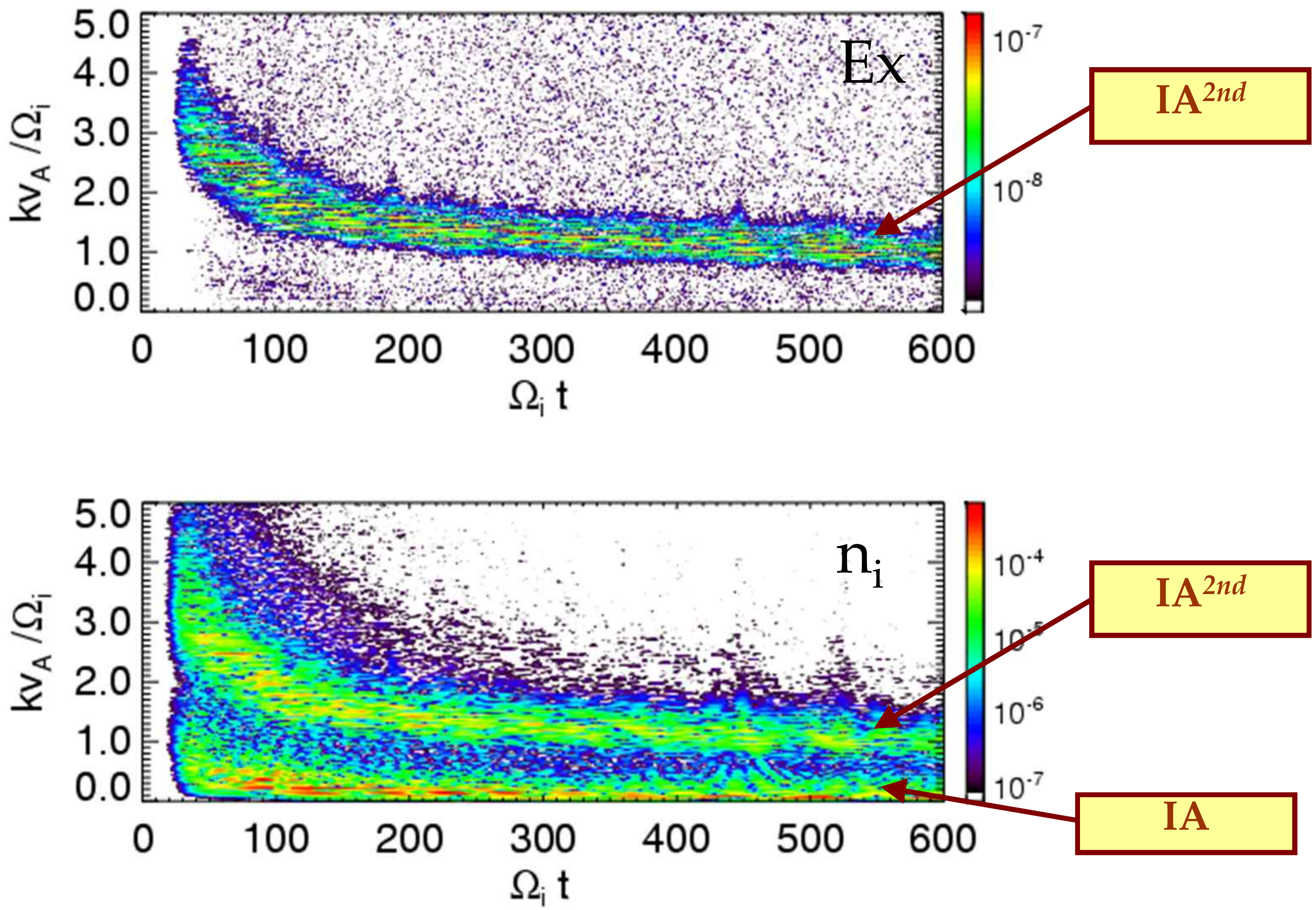


Inverse Cascade





Electrostatic Field and Ion Density



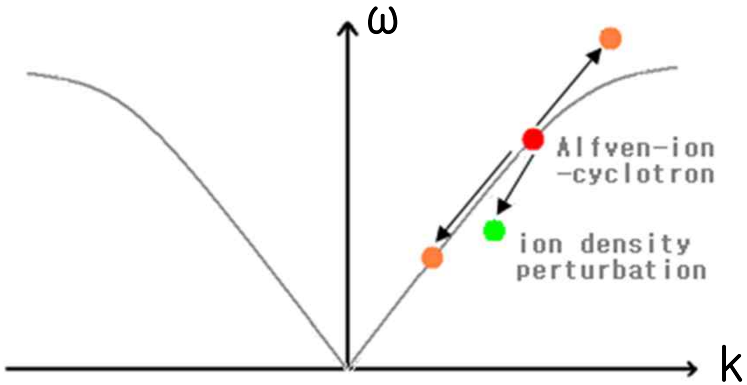
Parametric Instability

Alfvén waves (k_0,ω_0)

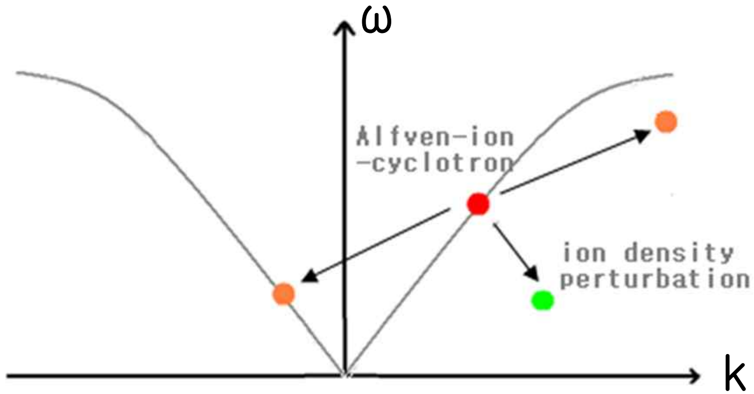
$\rightarrow$ density mode (k,ω)+ Alfvén waves$(k_0+k,\omega_0+\omega)$+ Alfvén waves$(k_0-k,\omega_0-\omega)$

Plasma beta(β) \ Polarization	Left-hand polarized	Right-hand polarized
small β	**modulational & decay**	decay
large β	beat	modulational

modulational inst.



decay inst.



Governing Eqs.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \vec{V}] = 0$$

$$\frac{d\vec{V}}{dt} = \frac{1}{\rho}[\nabla \times \vec{B}] \times \vec{B} - \frac{1}{2\rho}\nabla P$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} = -\vec{V} \times \vec{B} + \frac{1}{\rho}[\nabla \times \vec{B}] \times \vec{B} - \frac{1}{2\rho}\nabla P_e$$

Zeroth order solution

$$\overrightarrow{B_\perp} \equiv \begin{vmatrix} B_y \\ B_z \end{vmatrix} = \frac{1}{\sqrt{2}}\left[A_0 \exp i(k_0 x - \omega_0 t)\vec{e}_j + c.c. \right]$$

$$where, \quad \vec{e}_j \equiv \frac{1}{\sqrt{2}}\left[\vec{e}_y + \hat{i}\cdot \vec{e}_z \right] \qquad j = \begin{matrix} R \\ L \end{matrix}$$

$$\overrightarrow{V_\perp} = -\frac{k_0}{\omega_0}\overrightarrow{B_\perp}$$

$$k_0^2 = \frac{\omega_0^2}{1 \pm \omega_0} \qquad \begin{matrix} R \\ L \end{matrix} \ polarization\ vector$$

1st order solution

$$\overrightarrow{B_\perp'} \equiv \begin{vmatrix} B_y' \\ B_z' \end{vmatrix} = \frac{1}{\sqrt{2}}\left[A_+ \exp i(k_+ x - \omega_+ t)\hat{e}_j + c.c. + A_- \exp i(k_- x - \omega_- t)\hat{e}_j + c.c. \right]$$

$$k_\pm = k_0 \pm k, \quad \omega_\pm = \omega_0 \pm \omega$$

$$-\left[\omega^2 - k^2 C_s^2\right] n_1 = -k^2 \left[A_+ A_0^* + A_-^* A_0 \right]$$

$$-\left[\omega_+^2 - k_+^2 (1 \pm \omega_+)\right] A_+ = \frac{1}{2} k_0 k_+ \left[1 \pm \omega_+ - \frac{\omega}{k}\frac{k_0}{\omega_0} - \frac{\omega}{k}\frac{k_0}{\omega_0}\frac{\omega_+}{\omega_0} \right] n_1 A_0$$

$$-\left[\omega_-^2 - k_-^2 (1 \pm \omega_-)\right] A_-^* = \frac{1}{2} k_0 k_- \left[1 \pm \omega_- - \frac{\omega}{k}\frac{k_0}{\omega_0} - \frac{\omega}{k}\frac{k_0}{\omega_0}\frac{\omega_-}{\omega_0} \right] n_1 A_0^*$$

[Terasawa et al., JGR(1986)]

Dispersion Eq. for Parametric Instability

$$L_-(L_+D+R_+B_+) + L_+L_-D = 0$$

$$X = \frac{\omega}{\Omega_i} \quad and \quad Y = \frac{kv_A}{\Omega_i}$$

$$L_\pm = Y_\pm - \frac{X_\pm}{\phi_\pm}$$

$$R_\pm = Y_\pm \left(X_0 - \frac{YX_0^2}{Y_0} + \frac{X_\pm}{\phi_\pm} \right) \frac{1}{2\phi_0}$$

$$B_\pm = \pm A \frac{X\phi_m \left(X\phi_\pm X_0^2 - Y_0\phi_0 X_\pm^2 \right)}{Y_0 Y_\pm}$$

$$D = \phi_0\phi_+\phi_-\beta_e Y^2 - X^2 \left[\left(\frac{\delta B_\perp}{B_0} \right)^2 + \phi_0\phi_+\phi_- \left(1 - \beta_i \frac{Y^2}{X^2} \right) \right]$$

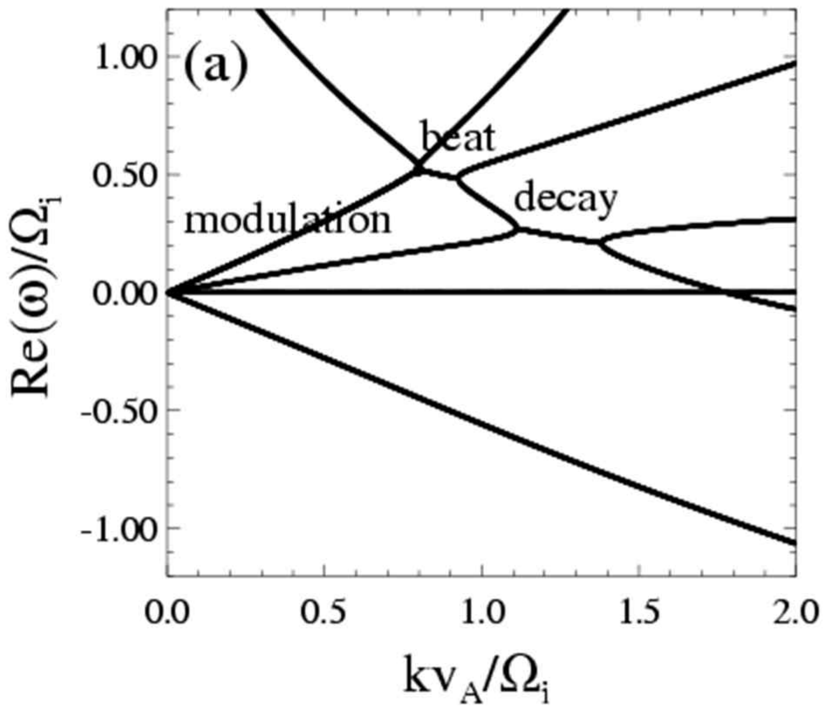
$$X_\pm = X_0 \pm X,\ Y_\pm = Y_0 \pm Y,\ \phi_0 = 1 - X_0,\ \phi_\pm = 1 - X_\pm$$

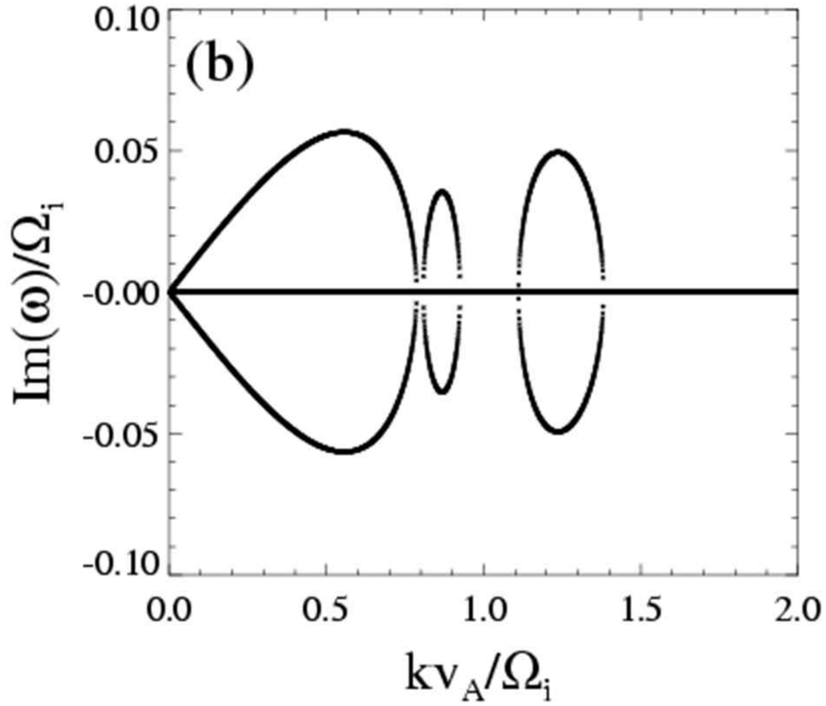
[Araneda et al., JGR(2007)]

Numerical Solution

- Using the parameters from the simulation results

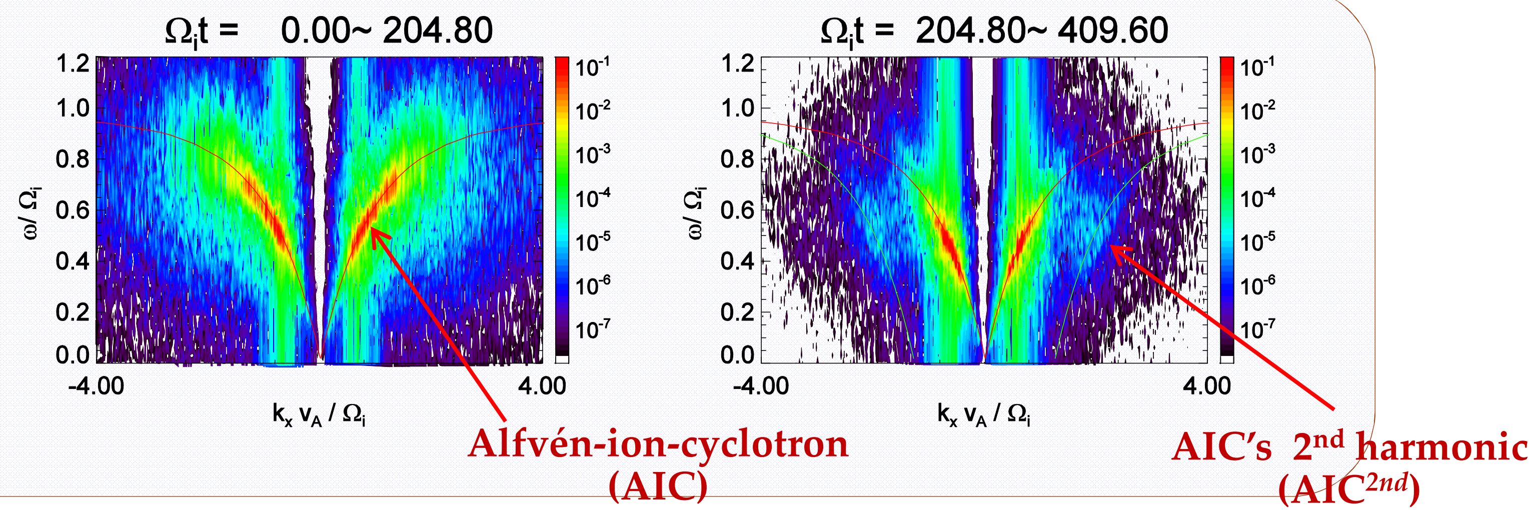
$$A = 0.2,\ \ X_0 = 0.54,\ \ Y_0 = 0.8,\ \ \beta_e = 0.08,\ \ \beta_i = 0.34$$



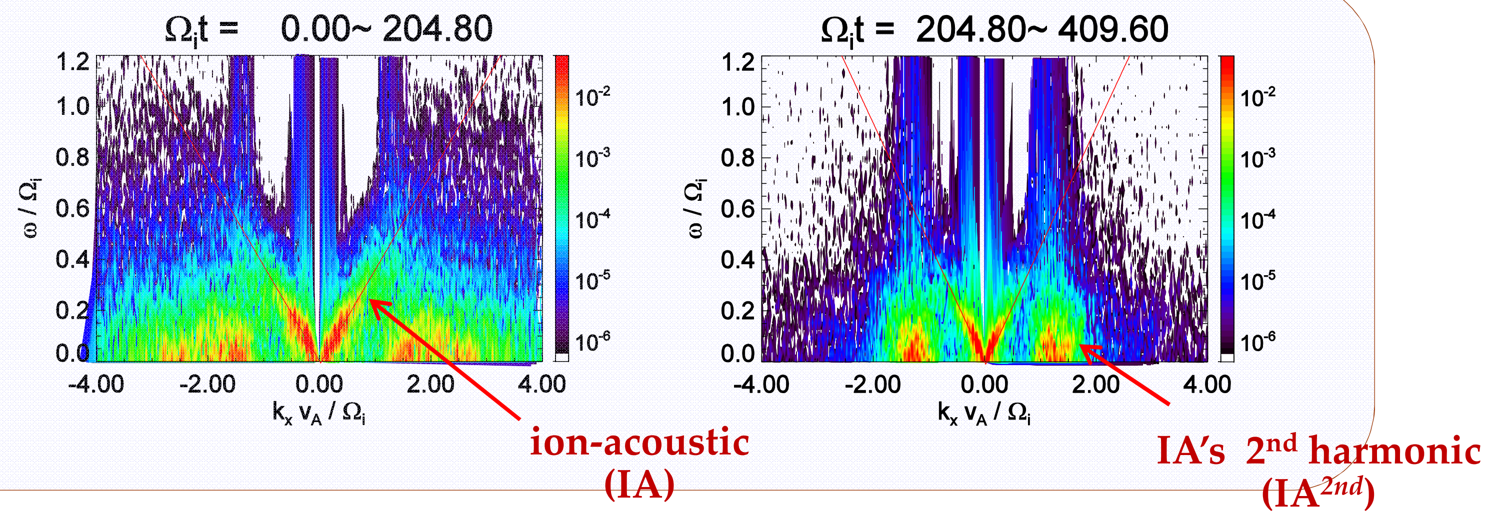


Excited Modes at Different Times

ω-k of B_z



ω-k of n_i



Bicoherence Analysis

Three-wave interaction

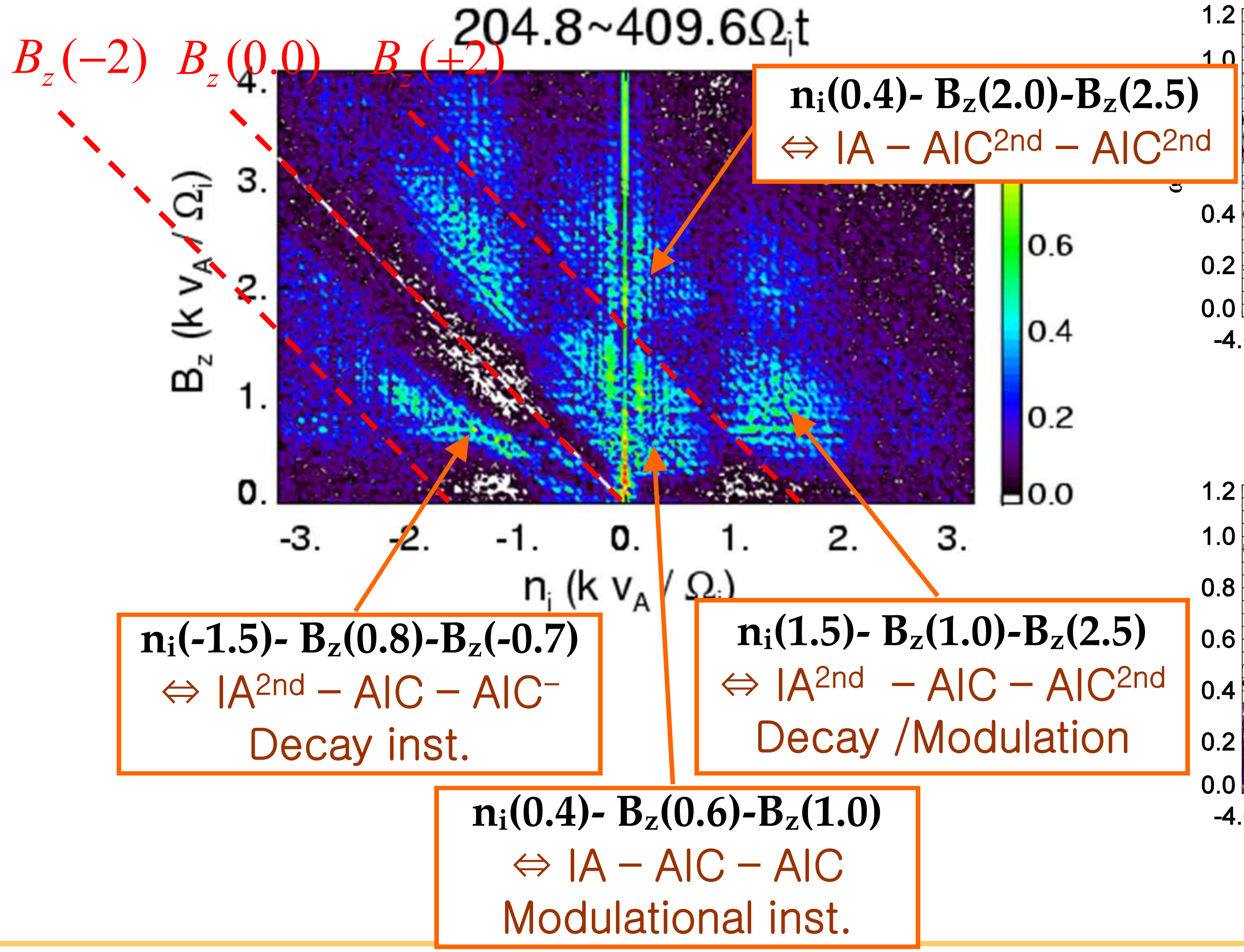
$$k_1 + k_2 = k_3$$

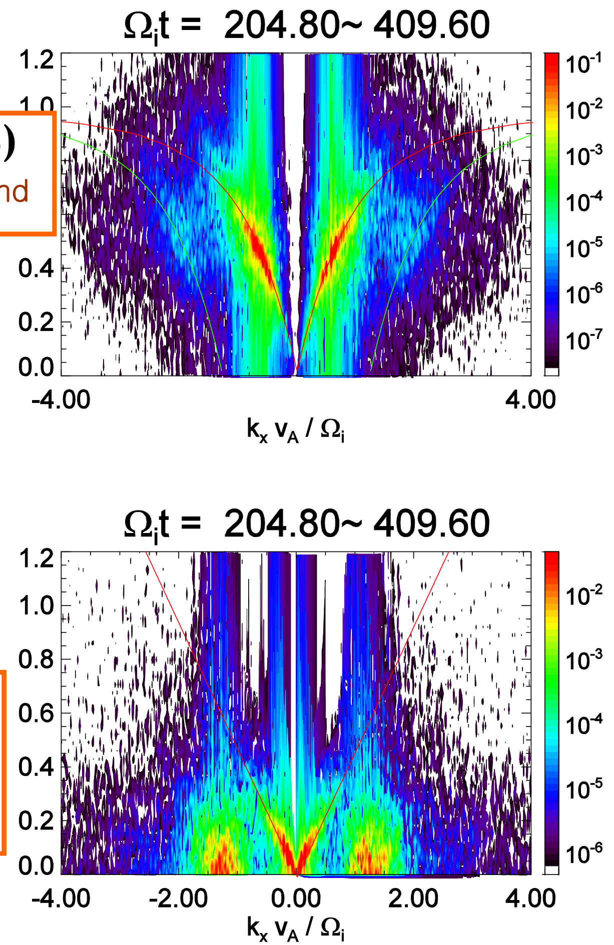
$$\omega_1 + \omega_2 = \omega_3$$

$$b(k_1, k_2) = \frac{|< f^{(1)}{}_{k_1} f^{(2)}{}_{k_2} f^{(3)}{}_{k_1+k_2} >|}{\sqrt{<| f^{(1)}{}_{k_1} f^{(2)}{}_{k_2} |^2><| f^{(3)}{}_{k_1+k_2} |^2>}}$$

$$f^{(1)}{}_{k_1} + f^{(2)}{}_{k_2} \rightarrow f^{(3)}{}_{k_1+k_2} \quad or \quad f^{(3)}{}_{k_1+k_2} \rightarrow f^{(1)}{}_{k_1} + f^{(2)}{}_{k_2}$$

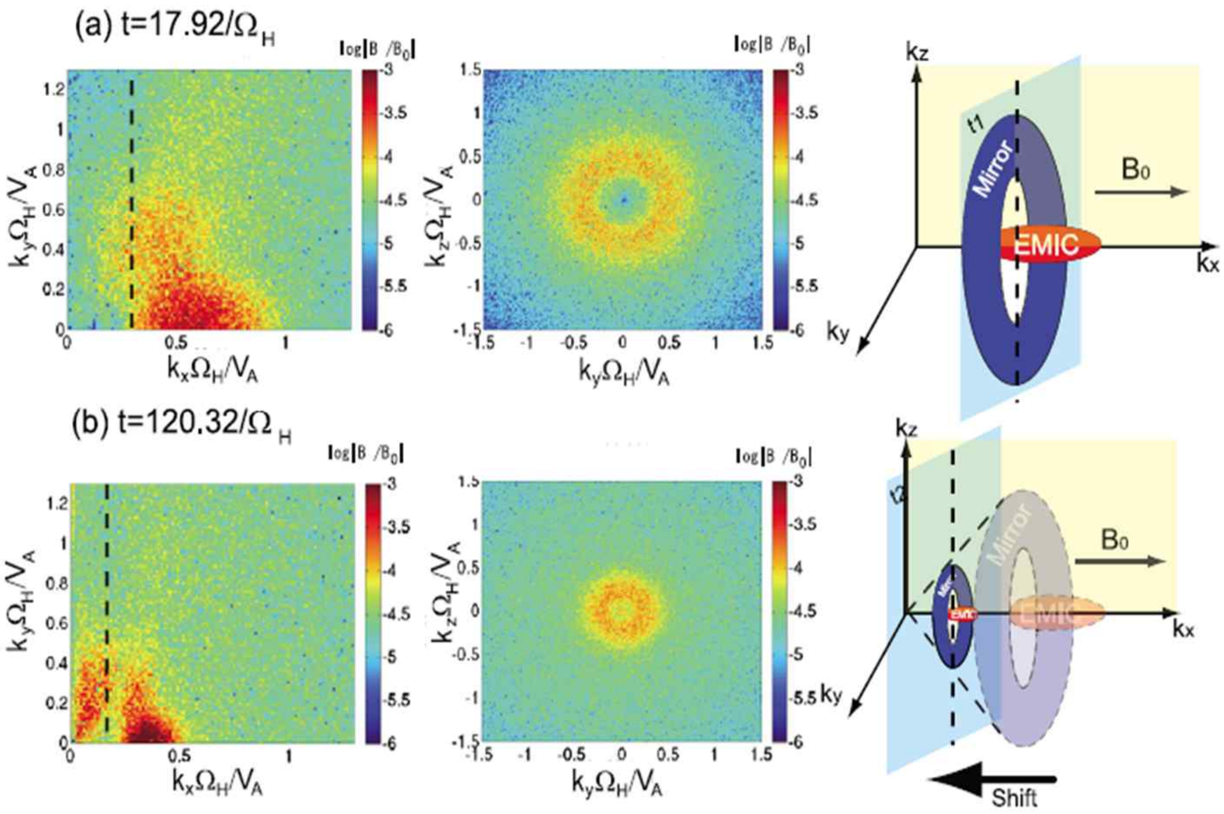
Bicoherence Spectrum





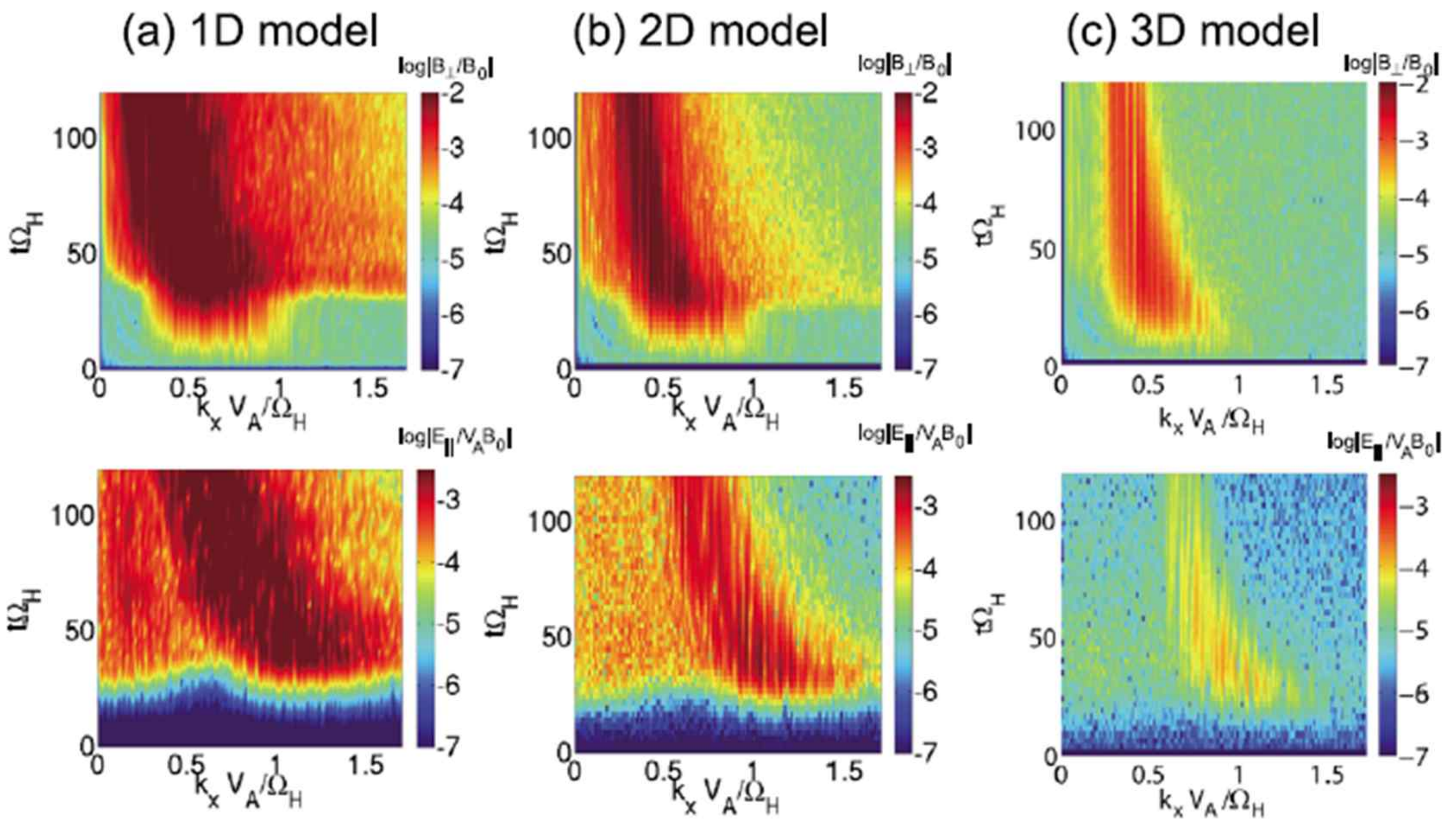
Anisotropic Temperature

- 3D Hybrid Simulation studies with $T_{\perp i} / T_{\parallel i} = 4$
  ; particle ions and fluid electrons
- EMIC & mirror modes are excited. *[Shoji et al., JGR(2009)]*



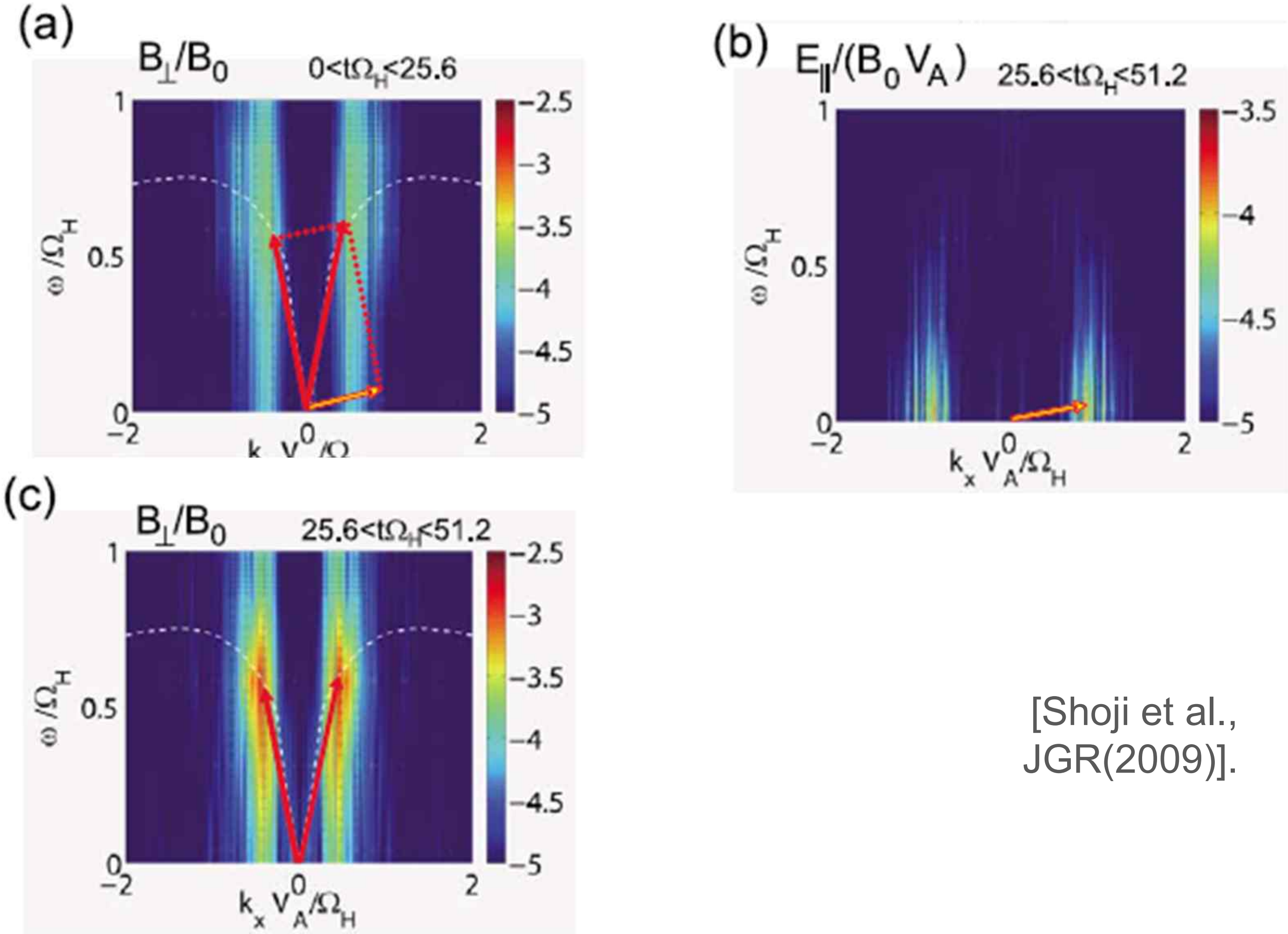
1~3D Hybrid Simulations

Simulation shows similar results regardless of dimension.



[Shoji et al., JGR(2009)].

Indication of 2nd harmonic IA mode



[Shoji et al., JGR(2009)].

Summary

- Alfvén-ion-cyclotron(AIC) waves and their turbulence are studied by using PIC simulations.

- The excited Alfvén ion-cyclotron(AIC) waves are unstable for the modulational and decay instability. By the modulational instability AIC waves are decomposed into an ion-acoustic(IA) wave and a lower k AIC wave. The decay instability brings AIC wave to be decomposed into the IA's 2^{nd} harmonic mode(IA^{2nd}) and another AIC wave propagating in opposite direction.

- The 2^{nd} harmonic mode of AIC waves are induced by nonlinear three-wave interaction between AIC and IA^{2nd} *modes*. The excited AIC harmonic mode decays into a lower-k AIC^{2nd} and IA waves.

- The inverse cascade takes place via the three-wave interaction of AIC waves.

  Modulational instability may be a key underlying mechanism for the inverse cascade process of the AIC turbulence.