Stochastic Acceleration by Turbulence

Application to Solar Flares

Vahe Petrosian

Stanford University

With

Qingrong Chen

Reuven Ramaty
High Energy Solar Spectroscopic Imager (RHESSI)
Acceleration Mechanisms

General Remarks

1. Parallel (to $B$) Electric Field
   *Unstable; Leads to Turbulence*

2. Stochastic Acceleration *(Fermi 1949)*
   $2^{\text{nd}}$ order Fermi: *Scattering by Turbulence*

   Accl. Rate/Scattering
   $$\left(\frac{v_A}{v}\right)^2$$

3. Acceleration in converging flows; *Shocks*
   First order Fermi:
   $$\frac{\delta p}{p} \sim \frac{u_{sh}}{v}$$

   Most likely scattering agent is Turbulence
I. A Brief Review of Acceleration in Solar Flares

Some Critical Observations

II. Stochastic Acceleration Model

Basic Equation and Important Coefficients

III. Application to RHESSI Total Spectra and Images

Determination of Turbulence Characteristics
I. A Brief Review of Solar Flares

Some Critical Observations
Energy Release
(Reconnection)

- Acceleration
- Heating
- Turbulence
- Radiation (non-thermal)

Radiation (thermal)

Impulsive

Gradual

Turbulence?
Basic Model

Acceleration Site and Process

Flare loop reconnection. site

CME shock

2nd order Fermi?

1st order Fermi?

Seed Particles?
I. Observed Signatures to be Explained by the Acceleration Model

1. Radiative Signatures of Electrons: $X$-rays

2. Nuclear Lines and Pion Decay Continuum by Energetic Protons and Ions: $\Gamma$amma-$\gamma$ays

3. Solar Energetic Particles or SEPs Observed at 1 A.U.

CONCENTRATE ON MINIMAL REQUIREMENTS
I. Observations Supporting This Model

Bremsstrahlung by Electrons

Hard vs Soft X-ray Structure

Distinct Looptop and Footpoint Sources

YOHKOH RHESSI

Bremsstrahlung by Electrons

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Pohang, Korea 2011
I. Observed Signatures of Acceleration

2. Nuclear Lines and Pion Decay by Protons and Ions

*Most recent data obtained by Fermi*

2010, June 12

2011, March 7-8
I. Observed Signatures of Acceleration

Electron vs Proton Acceleration Rates

\[(\varepsilon_p/\varepsilon_e)_{\text{Flares}} < 0.3\]
\[(\varepsilon_p/\varepsilon_e)_{\text{SEPs}} \sim 3\]
\[(\varepsilon_p/\varepsilon_e)_{\text{CRs}} \sim 100\]

Flares More Efficient Electron Accelerators
I. Observed Signatures of Acceleration

3. Solar Energetic Particles or **SEPs** Observed at 1 A.U.

**Spectra and Isotopic Enhancements** (3He/4He)

![Graph showing fluence vs. kinetic energy for SEPs](image)

Mason et al.

![Graph showing 3He/He vs. 4He fluence](image)

Ho et al. 2006
II. Stochastic Acceleration Model

Basic Equation and Coefficients
II. Particle Acceleration and Transport

The Kinetic Equation

Fokker-Planck Equation for Gyrophase Average Dist. \[ f(t, s, E, \mu) \]

\[ \frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial s} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 \left[ D_{pp} \frac{\partial f}{\partial p} + D_{p\mu} \frac{\partial f}{\partial \mu} \right] + \frac{\partial}{\partial \mu} \left[ D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_{\mu p} \frac{\partial f}{\partial p} \right] - \frac{1}{p^2} \frac{\partial}{\partial p} (p^2 \dot{\hat{p}}_L f) + \dot{S} \]

1. Isotropic if \[ D_{\mu\mu} \gg v/L \text{ and } D_{pp}/p^2 \]

Define \[ F(p, s, t) = \frac{1}{2} \int_{-1}^{1} d\mu f(p, \mu, s, t) \text{ and } Q(p, s, t) = \frac{1}{2} \int_{-1}^{1} d\mu S(p, \mu, s, t) \]

\[ \frac{\partial F}{\partial t} - \frac{\partial}{\partial s} \kappa_s \frac{\partial F}{\partial s} = \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^4 \kappa_p \frac{\partial F}{\partial p} - p^2 \dot{\hat{p}}_L F \right) + \dot{Q}(p, s, t) + \frac{\partial}{\partial s} (\ldots...) \]

Where \[ \kappa_s = \frac{v^2}{8} \int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}} \text{ and } \kappa_p = \frac{1}{2p^2} \int_{-1}^{1} d\mu (D_{pp} - D_{\mu p}/D_{\mu\mu}) \]

With acceleration and scattering times \[ \tau_{ac} = 1/\kappa_p \text{ and } \tau_{sc} = 8\kappa_s/v^2 \]

2. If \[ D_{pp}/p^2 \gg D_{\mu\mu} \]

then \[ \tau_{ac} = p^2/\langle D_{pp} \rangle \ll \tau_{sc} \]

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II. Particle Acceleration and Transport

3. If Homogeneous \textit{(or spatially averaged)}

and defining \( N(E) dE = 4\pi p^2 F(p) dp \) we get

\[
\frac{\partial N(E)}{\partial t} = \frac{\partial^2}{\partial E^2} [D_{EE} N(E)] - \frac{\partial}{\partial E} \left[ (A(E) - \dot{E}_L(E)) N(E) \right] - \frac{N(E)}{T_{esc}(E)} + \dot{Q}(E)
\]

Diffusion \quad Accel. \quad Loss \quad Escape

\[
D_{EE} = c^2 \beta^2 D_{pp} \quad A(E) = \frac{1}{p^2} \frac{d(c^2 \beta^2 D_{pp})}{dp}
\]

\[
T_{esc}(E) \simeq \tau_{cross} \left(1 + \tau_{cross}/\tau_{sc}\right)
\]

\[
\tau_L \equiv E/\dot{E}_L
\]

\[
\dot{E}_L = 4\pi r_0^2 \ln \Lambda mc^3 n/\beta + (4/9) r_0^2 c \beta^2 \gamma^2 B^2
\]
II. Wave Particle Interactions

e.g. parallel propagating waves

\[ D_{ij} \propto \sum_r \sum_{n=-\infty}^{n=\infty} \int W(k) d^3k \delta \left( \omega(k_r) - k_{r,||} \nu \mu - n\Omega/\gamma \right) (J_n, J_{n-1}, ...) \]

Resonance Condition

\[ \omega = \mu \nu k + \frac{\Omega_i}{\gamma}. \]

Dispersion Relation

\[ (ck)^2 = \omega^2 \left[ 1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega - q_i/|q_i|\Omega_i)} \right]. \]
II. Wave Particle Interactions

*Spectrum of Turbulence (assumed)*

The diffusion coefficients $D_{EE}, D_{\mu\mu}$ or $\tau_{ac}, \tau_{sc}, T_{esc}$ are related to:

1. **Turbulence parameters:**
   - Injection and damping wave numbers
   - Inertial range spectral index:

$$k_{\text{min}}, k_{\text{max}} \text{ and } q$$

2. **Plasma parameters:**
   - Plasma and gyro-frequencies
   - Turbulence energy density

$$\alpha = \left( \frac{\omega_{pe}}{\Omega_e} \right) \propto \left( \frac{\sqrt{n}}{B} \right) \text{ and } u_{\text{turb}} \sim 8\pi \delta B^2 \sim \rho v_{\text{turb}}^2$$

$$\tau_p^{-1} = \left( \frac{\pi}{2} \right) \Omega_e \left( \frac{u_{\text{turb}}}{B^2/8\pi} \right) (q - 1) \left( \frac{c k_{\text{min}}}{\Omega_e} \right)^{q-1}$$
Two Important Aspects
IMPORTANT ASPECT 1

Define

\[ R_1 = \left( \frac{D_{pp}}{p^2} \right) / D_{\mu\mu} \quad \text{and} \quad R_2 = \left( \frac{D_{\mu p}}{p} \right) / D_{\mu\mu} \]

For High Energy Protons and Relativistic Electrons

Alfven and Fast Mode

\[ R_1 = \left( \frac{v_A}{v} \right)^2 \ll 1 \]

\[ \kappa_p = \frac{1}{2p^2} \int_{-1}^{1} d\mu \left( D_{pp} - D_{\mu p}^2 / D_{\mu\mu} \right) \]

\[ \tau_{ac} = 1 / \kappa_p \ll \tau_{sc} \]

But for highly magnetized plasmas or at low energies

\[ R_1 \gg 1 \]

\[ \tau_{ac} = p^2 / \langle D_{pp} \rangle \gg \tau_{sc} \]

Acceleration by Turbulence More Efficient than Shock

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Accel/Scatt Ratio $R1$

The diagram shows a graph with axes labeled $KE$ (MeV) on the y-axis and $\alpha$ on the x-axis. The graph includes contours labeled with values such as 100, 10, 1, 0.5, 0.1, and 0.01. A red line with the label $\mu = 0.3$ intersects these contours.
In general

\[(D_{pp}/p^2) : (D_{\mu p}/p) : D_{\mu \mu} = [x_j^2] : [x_j(1 - \mu x_j)] : [(1 - \mu x_j)^2] \text{ with } x_j = (\beta_{ph,j}/\beta)^2\]

Thus when a **Single MODE** dominates, then the **Acceleration Rate for the Isotropic case**

\[
\kappa_p = \frac{1}{2p^2} \int_{-1}^{1} d\mu (D_{pp} - D_{\mu p}^2/D_{\mu \mu}) \to 0
\]

This however does not affect the nonisotropic \( R_1 > 1 \)
III. Stochastic Acceleration in Flares

1. Electron vs Proton Acceleration and Spectra
III. Stochastic Acceleration in Flares

2. SEPs and He3/He4 Acceleration

He3, He4 Fluence Ratios

**Observations**

**Model Results**

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III. Stochastic Acceleration of Electrons in Solar Flares

Testing the Acceleration Model
Back to The Basic Model: 
Relating Electrons and Photons

\[ F_{LT}(E) = \nu(E)N(E) \]

Connected by Escaping Process

\[ F_0(E) = F_{LT}(E)\left(\frac{\tau_{\text{cross}}}{T_{\text{esc}}}\right) \]

\[ F_{FP}(E) = \nu N_{FP} = \frac{\nu(E)}{E_L(n)} \int_{E}^{\infty} \frac{N(E')}{T_{\text{esc}}(E')} \, dE' \]

HXR: \[
\begin{align*}
I_{LT}(\epsilon) & = \frac{n V}{4\pi R^2} \int_{\epsilon}^{\infty} \left\{ \frac{F_{LT}(E)}{F_{FP}(E)} \right\} \sigma(\epsilon, E) \, dE \\
I_{FP}(\epsilon) & = \frac{n V}{4\pi R^2} \int_{\epsilon}^{\infty} \left\{ \frac{F_{LT}(E)}{F_{FP}(E)} \right\} \sigma(\epsilon, E) \, dE
\end{align*}
\]

Regularized Inversion of Photon Images to Electron Images

\[ I(x, y; \epsilon) = \frac{a^2}{4\pi R^2} \int_{E=\epsilon}^{\infty} N(x, y) F(x, y; E) Q(\epsilon, E) \, dE \]
\[ J(x, y; q) \, dq = \int_x \int_y \int_{\epsilon=q}^{\infty} D(q, \epsilon) I(x, y; \epsilon) \, d\epsilon \, dx \, dy \]

RHESSI produces count visibility, Fourier component of the source

\[ V(u, v; q) = F^2(J(x, y; q)) \equiv \int_x \int_y J(x, y; q) e^{2\pi i (ux + vy)} \, dx \, dy \]

Defining electron flux visibility spectrum and count cross section

\[ W(u, v, E) = a^2 \int_x \int_y N(x, y) F(x, y; E) e^{2\pi i (ux + vy)} \, dx \, dy \]
\[ K(q, E) \, dq = \int_{\epsilon=q}^{\infty} D(q, \epsilon) Q(\epsilon, E) \, d\epsilon \]

We get

\[ V(u, v; q) = \frac{1}{4\pi R^2} \int_q^{\infty} W(u, v; E) K(q, E) \, dE \]

Regularized inversion produced \textit{smoothed} electron flux visibility spectrum

\[ \| V_{[u,v]} - K \cdot W_{[u,v]} \|^2 + \lambda_{[u,v]} \| W_{[u,v]} \|^2 = \text{minimum} \]

\[ N(x, y) F(x, y; E) = \frac{1}{a^2} \int_u \int_v W(u, v; E) e^{-2\pi i (ux + vy)} \, du \, dv \]

\textit{Piana et al. 2007}
Results From Regularized Inversion of Images

(1) LT image gives the *accelerated* electron spectrum

\[ N(E) = F_{LT}(E)/v \]

(2) FP images give the *effective* spectrum

\[ F_{FP} = \frac{v}{\dot{E}_L} \int_{E}^{\infty} \frac{N(E')}{T_{esc}(E')} \, dE' \]

\[ T_{esc} = N(E) \times \frac{(dF_{FP}\dot{E}_L/v)}{dE} \]

From these spectra we can derive the *escape time*

\[ T_{esc}(E) = \frac{\tau_L(E)(F_{LT}/F_{FP})}{\delta_{FP}(E) + 2/(\gamma^2 + \gamma^2)} \], with \( \tau_L = E/\dot{E}_L \)

and determine the mean and *turbulence scattering times*

\[ \tau_{scat} \approx \tau_{cross}^2/(T_{esc} - \tau_{cross}) \quad \text{and} \quad \tau_{scat}^{\text{turb}} \approx \tau_{scat}(1 + \tau_{scat}/\tau_{scat}^{\text{Coul}}) \]

Applications and Results

We apply the inversion to images of two flares

1. 2003 November 3 Flare (X3.9 class)
2. 2005 September 8 Flare (M2.1 class)

and evaluate the escape and scattering times
and compare with stochastic acceleration model parameters.
2003 Nov 3 Flare (X3.9 class)

LT source detected up to 100-150 keV

(Chen & Petrosian, in preparation)

HXR images by MEM_NJIT

Electron flux images by MEM_NJIT
2003 Nov 3 Flare: Model Parameters

Electron Spectra

Time Scales

LT+FPs: $\delta = 2.25 \pm 0.08$
$E_b = 92.6 \pm 4.4$
$\delta = 2.88 \pm 0.02$

FPs: $\delta = 2.06 \pm 0.07$
$E_b = 91.3 \pm 3.4$
$\delta = 2.83 \pm 0.02$

LT: $\delta = 2.99 \pm 0.03$

$L = 10^9 \text{ cm}$
$n_{LT} = 0.5 \times 10^{11} \text{ cm}^{-3}$
\[
\frac{\partial N(E)}{\partial t} = \frac{\partial^2}{\partial E^2} \left[ D_{EE} N(E) \right] - \frac{\partial}{\partial E} \left[ (A(E) - \dot{E}_L(E)) N(E) \right] - \frac{N(E)}{T_{esc}(E)} + \dot{Q}(E)
\]
2005 Sep. 8 Flare: HXR/Electron Images

HXR:
2 loops

Electron
2005 Sep. 8 Flare: Model Parameters

Electron Spectra

Time Scales

LT+FPs: $\delta=3.86\pm0.03$
FPs: $\delta=3.51\pm0.03$
LT: $\delta=4.78\pm0.06$

$T_{\text{esc}} \propto E^{\kappa}$, $\kappa=0.22\pm0.14$

$\tau_{\text{cross}} = \frac{L}{V}$

$T_{\text{secat}} \propto E^{-\lambda}$, $\lambda=0.88\pm0.23$

$L=10^9$ cm
$n_{\text{LT}}=3\times10^{10}$ cm$^{-3}$

Two loops summed
2006 Sep. 8 Flare: A Complete Model

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Comparison With Stochastic Acceleration Model

$q = 4.0 - 1.7$, $\alpha = 0.98$

Timescale

Energy (keV)

Flare 2005/09/08
Some Required Values for the Parameters of the Stochastic Acceleration Model

\[ q = 3, \quad \text{and} \quad \alpha \sim 1 \left( n \sim 10^{10} \ \text{cm}^{-3}; b \sim 300 \ \text{G} \right) \]

A scattering time of \( \tau_{\text{scat}} \sim 2 \times 10^{-2} \ \text{s} \)

Requires \( \tau_p^{-1} \sim 50 \ \text{s}^{-1} \) and

\[
\frac{u_{\text{turb}}}{B^2/8\pi} \left( \frac{c k_{\text{min}}}{\Omega_e} \right)^2 \sim 3 \times 10^{-9}
\]

which means

\[ u_{\text{turb}} \ll \frac{B^2}{8\pi} \text{ for } c k_{\text{min}}/\Omega_e > 10^{-4} \]

Thus, we need to have a small fraction of magnetic energy in form of sub-Alfvenic turbulence
SUMMARY and CONCLUSIONS

1. Several observations *require trapping* of the accelerated particles in the acceleration Site. The most likely candidate for this is *turbulence*.

2. This turbulence can in addition *accelerate* particles and this acceleration is the most efficient acceleration mechanism at *low energies and highly magnetized plasmas*.

3. Relative acceleration of *electrons/protons; He3/He4* and relative spectra of *Looptop/Footpoint* sources can be reproduced by this process.

4. The newly developed *regularized inversion technique* can be used to determine some of the acceleration parameters directly from the *RHESSI* X-ray data.
He3, He4 Fluence Distributions


Variation of He3/He4 Spectra

Spectra of “gradual” SEPs

Second Stage Acceleration

Chen, VP and , 2011
SEPs: P, He3, He4 and Heavy Ions
I. Observed Signatures of Acceleration

1. Solar Energetic Particles or SEPs Observed at 1 A.U. 

*Enhancements of 3He and Heavy Ions*

Reames et al
I. Observed Signatures of Acceleration

Electron vs Proton Acceleration Rates: SEPs

\[
\left( \frac{\varepsilon_p}{\varepsilon_e} \right)_{SEPs} \sim 3
\]

Cliver and Ling 2007

Dist. of SEP Ratios

\[
\text{Energy Ratio: } \log \left[ \frac{E_e(>0.5\text{MeV})}{E_p(>10\text{ MeV})} \right]
\]
$e$ vs $p$: Dependence on Magnetization

$$\alpha = \left(\frac{m_e}{m_p}\right)^{1/2} / \beta_A \propto n^{1/2} / B$$

$$\tau_p^{-1} = \left(\frac{\pi}{2}\right) \Omega_e \left(\frac{u_{\text{turb}}}{B^2/8\pi}\right) (q - 1) \left(\frac{ck_{\text{min}}}{\Omega_e}\right)^{q-1}$$

[Graphs showing the distribution of electron and proton densities as a function of energy (E/keV) on a log scale, with different values of $\tau_p^{-1}$ and $\alpha$.]
Regularized Inversion of Photon Spectra to Electron Spectra

Start with the detected photon count spectrum

\[ J(\varepsilon') = \int I(\varepsilon) D(\varepsilon', \varepsilon) d\varepsilon = n \int_\varepsilon^\infty K(E, \varepsilon') F(E) dE; \text{ with } K(E, \varepsilon') = \int_{\varepsilon'}^\infty D(\varepsilon', \varepsilon) \sigma(E, \varepsilon) d\varepsilon \]

Invert this to get the electron flux by minimizing the matrix

\[ \| J - K . nF \|^2 + \lambda \| nF \|^2 \]

Inversion of the total spectra \((Piana et al. 2003; Kontar et al. 2005)\)

\[ I_{\text{Tot}} = I_{\text{LT}} + I_{\text{FP}} = \frac{nV}{4\pi R^2} \int_\varepsilon^\infty v N_{\text{eff}}(E) \sigma(\varepsilon, E) dE \]

Gives the effective electron flux or density spectrum \(N_{\text{eff}}(E)\)
Regularized Inversion of Photons to Electrons

1. Inversion of total spectra (Piana et al. 2003; Kontar et al. 2005)

\[ I_{\text{Tot}} = I_{\text{LT}} + I_{\text{FP}} = \frac{nV}{4\pi R^2} \int_{\epsilon}^{\infty} vN_{\text{eff}}(E)\sigma(\epsilon, E)dE \]

Gives the total effective radiating electron spectrum

\[ N_{\text{eff}}(E) = N(E) + \frac{1}{\dot{E}_L} \int_{E}^{\infty} \frac{N(E')}{T_{\text{esc}}(E')}dE', \quad \text{or} \quad \frac{d(N/v)}{dE} \frac{N}{v} - \frac{N/v}{\dot{E}_L T_{\text{esc}}} = \frac{d(N_{\text{eff}}/v)}{dE} \]

Solve for the accelerated electron spectrum

\[ N(E) = N_{\text{eff}}(E) - v(E) \int_{\eta(E)}^{\infty} \frac{N_{\text{eff}}(E)}{v(E)} e^{\eta - \eta'} d\eta' \]

with \( d\eta = \frac{dE}{\dot{E}_L T_{\text{esc}}} \)

Requires a knowledge of escape time!
II. Transport Coefficients of Electrons and Ions

1. Large Scale $B$ field:
   a. Adiabatic Invariance
      \[
      \frac{\partial \ln B}{\partial s} \times \frac{\partial}{\partial \mu} [(1 - \mu^2) f]
      \]
   b. Synchrotron Losses
      \[
      E_L = (4/9) r_0^2 c \beta^2 \gamma^2 B^2
      \]
      Important only for electrons only

2. Background Plasma Density $n$:
   a. Inelastic Coulomb
      \[
      E_L = 4\pi r_0^2 n m c^2 \ln \Lambda / \beta
      \]
      (e-e, e-p, p-p, p-e)
   b. Elastic Brem. And Nuclear Interactions (Lines and Pions)

3. Background Photons (Inverse Compton)
   Important only for electrons only and at low $B$

4. Plasma Waves or Turbulence: Scattering and Acceleration Rates
   \[
   \propto \Omega \left( \frac{\varepsilon_{\text{turb}}}{B^2} \right)
   \]
   Also Energy diffusion, Pitch angle changes and diffusion

   Most Important Parameter
   \[
   \alpha = \omega_p / \Omega \propto 1 / \beta_A \propto \sqrt{n / B}
   \]
I. Observed Signatures of Acceleration

**Electron vs Proton Acceleration Rates: Flares**

\[
\left( \frac{\varepsilon_p}{\varepsilon_e} \right)_{\text{Flares}} < 0.3
\]

*Dist. of Flare Ratios*