Stochastic Acceleration by Turbulence
Application to Solar Flares

Vahe Petrosian

Stanford University

With

Qingrong Chen

Reuven Ramaty
High Energy Solar
Spectroscopic
Imager (*RHESSI*)

Acceleration Mechanisms
General Remarks

1. Parallel (to *B*) Electric Field

   Unstable; Leads to Turbulence

2. Stochastic Acceleration *(Fermi 1949)*

   2[Nd] order Fermi: Scattering by Turbulence

   Accl. Rate/Scattering $(v_A/v)^2$

3. Acceleration in converging flows; *Shocks*

   First order Fermi: $\delta p/p \sim u_{sh}/v$

   Most likely scattering agent is Turbulence

Outline

I. A Brief Review of Acceleration in Solar Flares

Some Critical Observations

II. Stochastic Acceleration Model

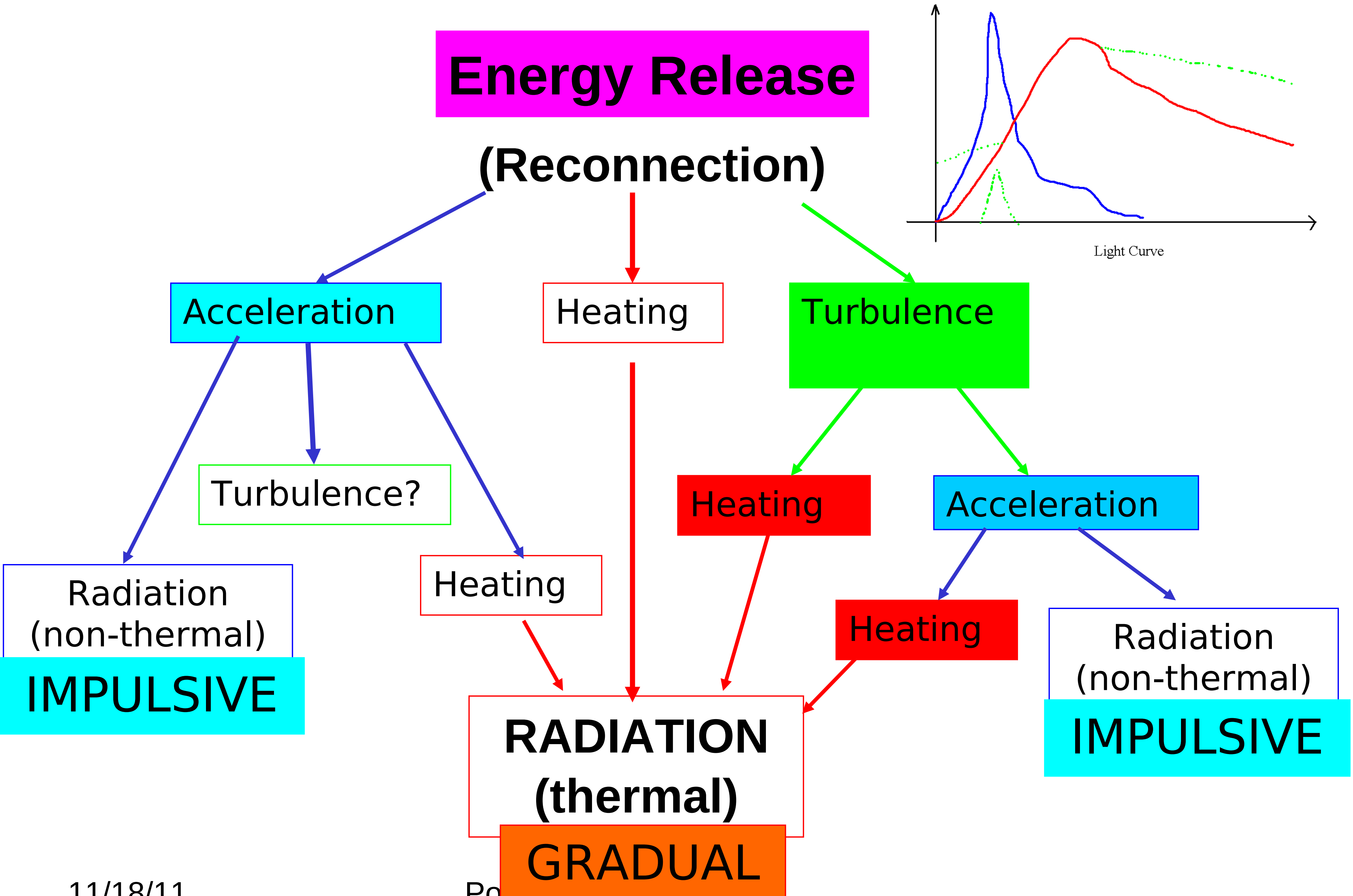
Basic Equation and Important Coefficients

III. Application to *RHESSI* Total Spectra and Images

Determination of Turbulence Characteristics

I. A Brief Review of Solar Flares

Some Critical Observations

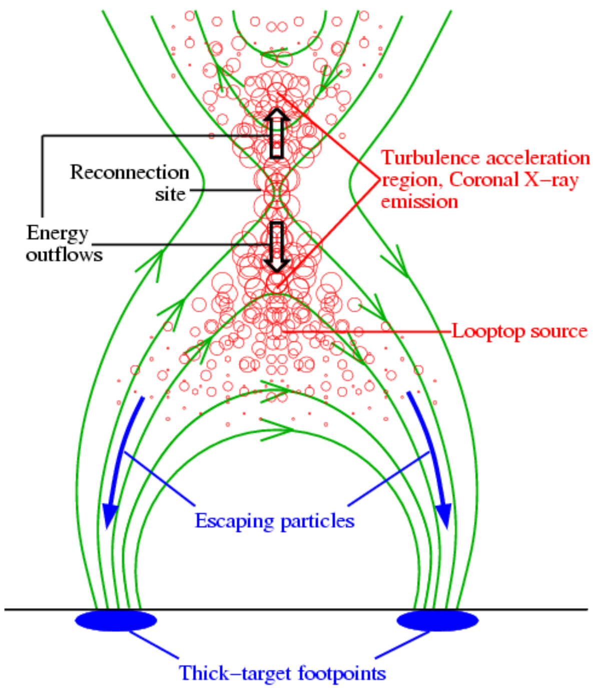

Energy Release
(Reconnection)
Light Curve
Acceleration
Heating
Turbulence
Turbulence?
Heating
Acceleration
Radiation
(non-thermal)
IMPULSIVE
Heating
Heating
Radiation
(non-thermal)
IMPULSIVE
RADIATION
(thermal)
GRADUAL

Basic Model

Acceleration Site and Process

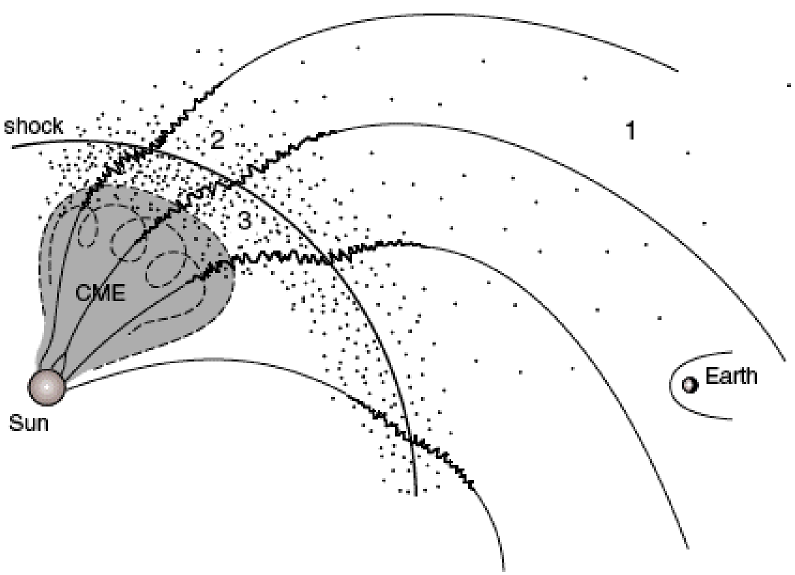
Flare loop reconnection. site

2^{nd} order Fermi?



CME shock

1^{st} order Fermi?



Seed Particles?

I. Observed Signatures to be Explained by the Acceleration Model

1. Radiative Signatures of Electrons: ***X-rays***

2. Nuclear Lines and Pion Decay Continuum by Energetic Protons and Ions: ***Gamma-rays***

3. Solar Energetic Particles or ***SEPs*** Observed at 1 A.U.

CONCENTRATE ON MINIMAL REQUIREMENTS

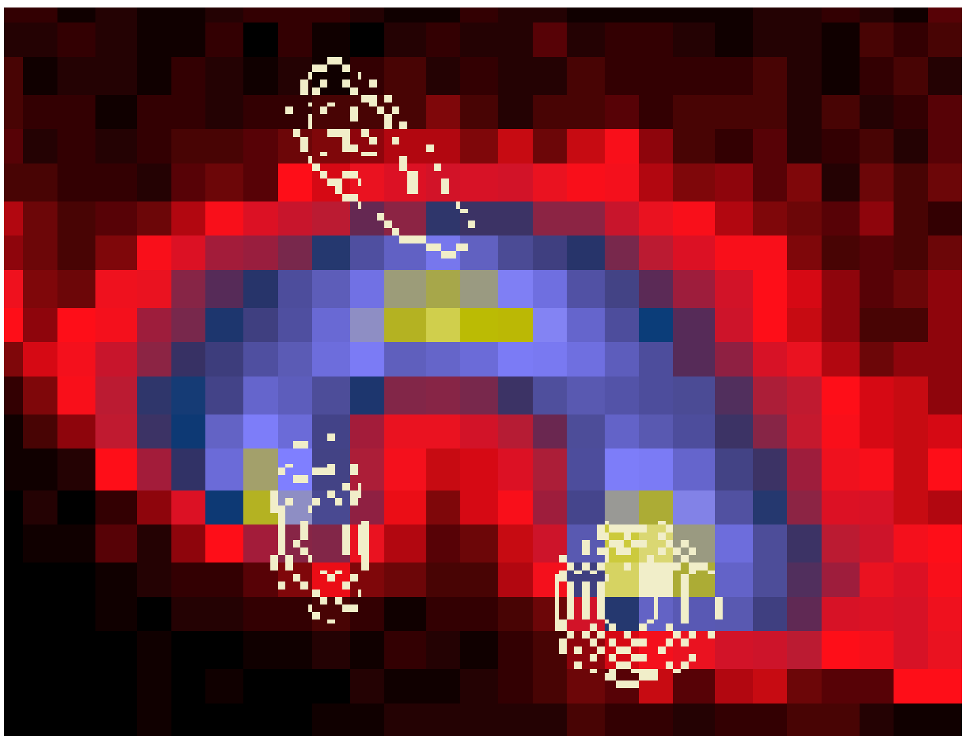
I. Observations Supporting This Model

Bremsstrahlung by Electrons

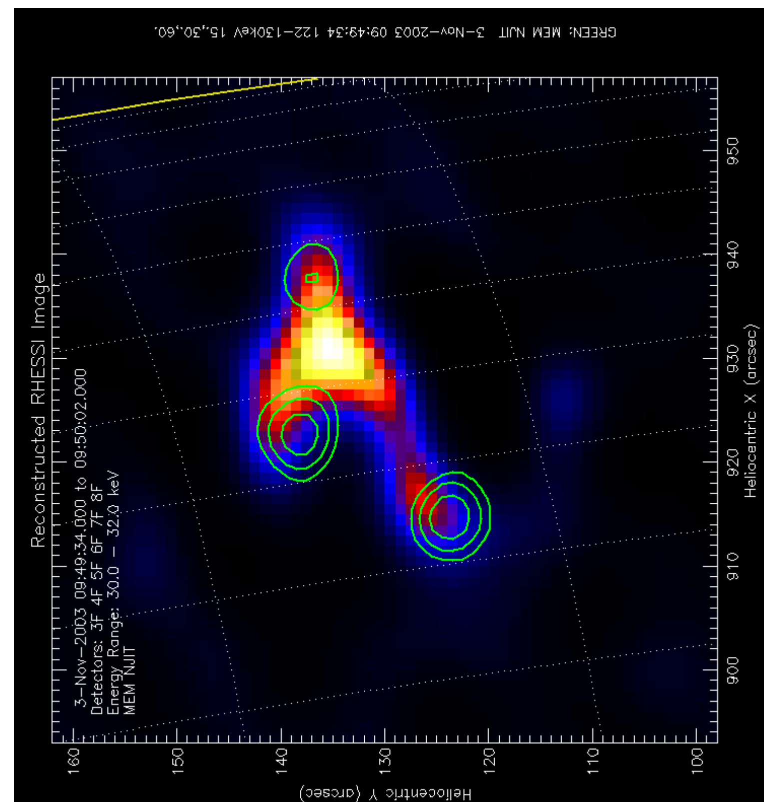
Hard vs Soft X-ray Structure

Distinct Looptop and Footpoint Sources

YOHKOH



RHESSI

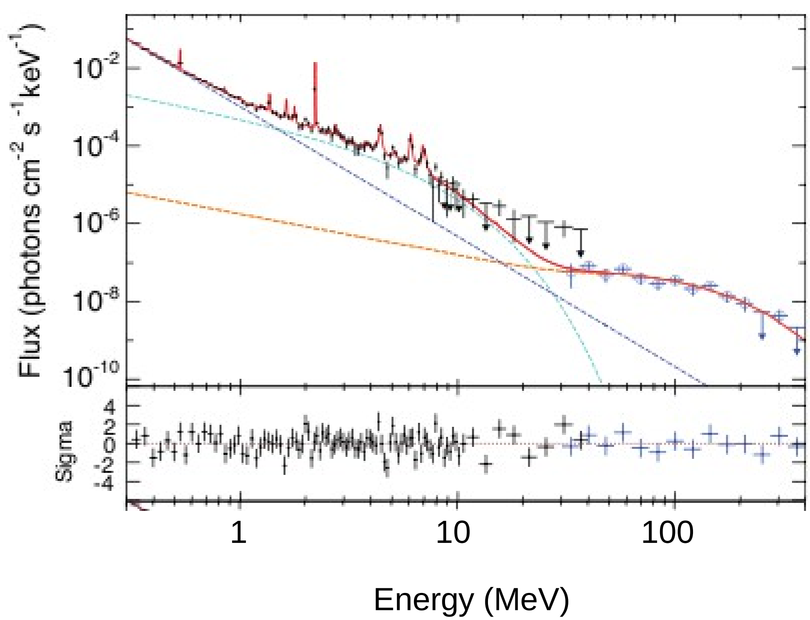


I. Observed Signatures of Acceleration

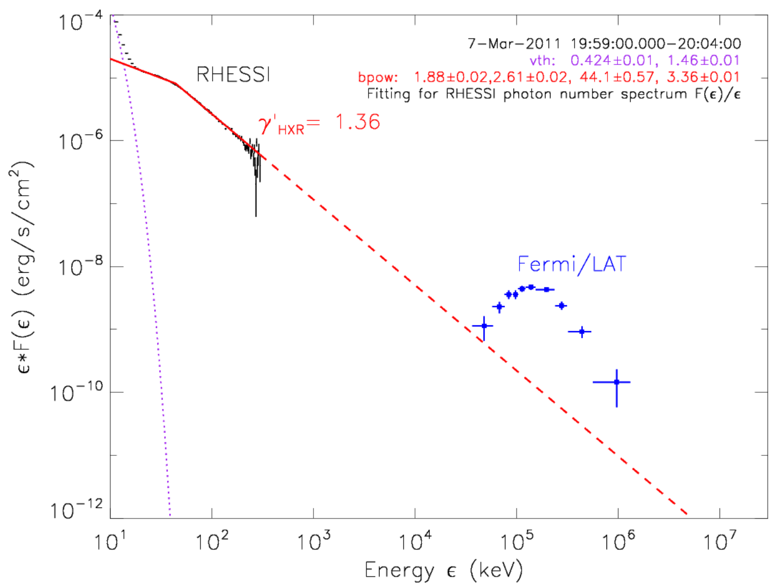
2. Nuclear Lines and Pion Decay by Protons and Ions

Most recent data obtained by Fermi

2010, June 12



2011, March 7-8



I. Observed Signatures of Acceleration

Electron vs Proton Acceleration Rates

$$(\varepsilon_p/\varepsilon_e)_{\text{Flares}} < 0.3$$

$$(\varepsilon_p/\varepsilon_e)_{\text{SEPs}} \sim 3$$

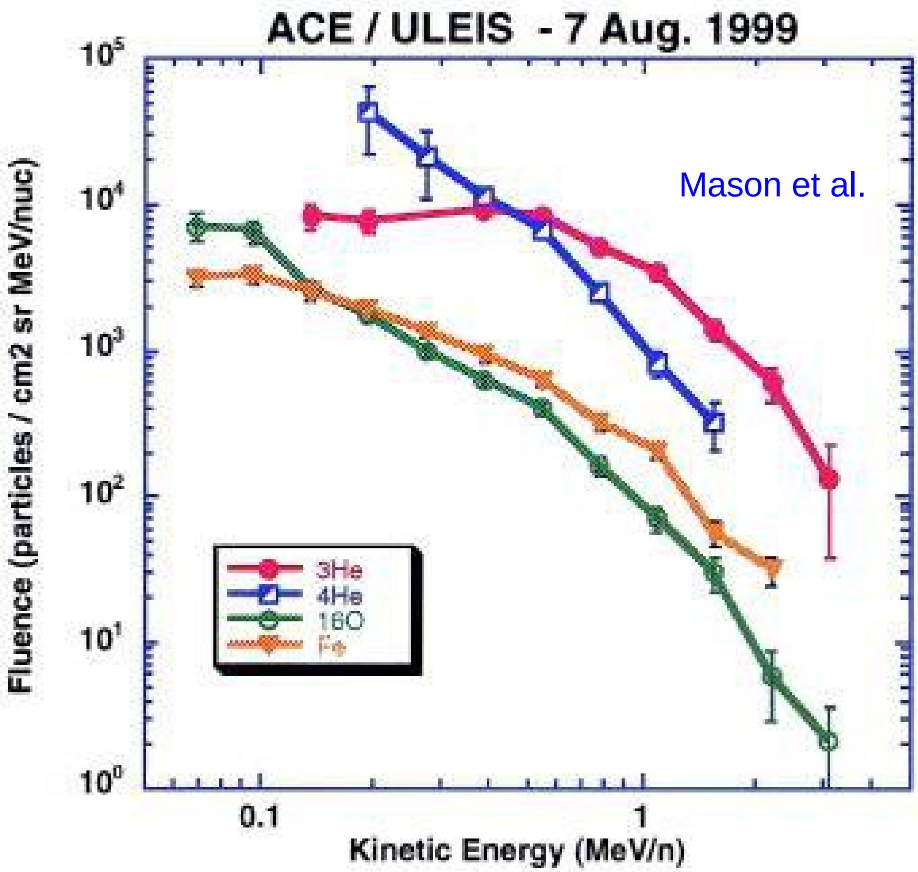
$$(\varepsilon_p/\varepsilon_e)_{\text{CRs}} \sim 100$$

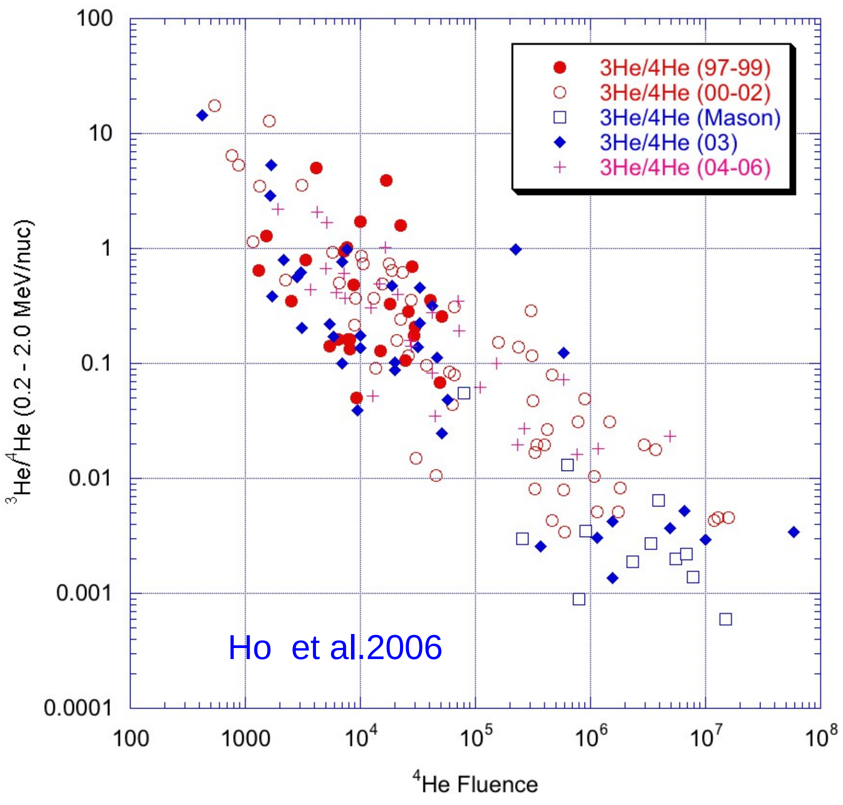
Flares More Efficient Electron Accelerators

I. Observed Signatures of Acceleration

3. Solar Energetic Particles or ***SEPs*** Observed at 1 A.U.

Spectra and Isotopic Enhancements (3He/4He)





II. Stochastic Acceleration Model

Basic Equation and Coefficients

II. Particle Acceleration and Transport

The Kinetic Equation

Fokker-Planck Equation for Gyrophase Average Dist. $f(t,s,E,\mu)$

$$\frac{\partial f}{\partial t}+v\mu\frac{\partial f}{\partial s}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p}+D_{p\mu}\frac{\partial f}{\partial \mu}\right]+\frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}+D_{\mu p}\frac{\partial f}{\partial p}\right]-\frac{1}{p^2}\frac{\partial}{\partial p}(p^2\dot{p}_L f)+\dot{S}$$

1. Isotropic if $D_{\mu\mu}\gg v/L$ and D_{pp}/p^2 Define $F(p,s,t)\equiv\frac{1}{2}\int_{-1}^{1}d\mu f(p,\mu,s,t)$ and $\dot{Q}(p,s,t)\equiv\frac{1}{2}\int_{-1}^{1}d\mu\dot{S}(p,\mu,s,t)$

$$\frac{\partial F}{\partial t}-\frac{\partial}{\partial s}\kappa_s\frac{\partial F}{\partial s}=\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\kappa_p\frac{\partial F}{\partial p}-p^2\dot{p}_L F\right)+\dot{Q}(p,s,t)+\frac{\partial}{\partial s}(.......).$$

Where $\kappa_s=\frac{v^2}{8}\int_{-1}^{1}d\mu\frac{(1-\mu^2)^2}{D_{\mu\mu}}$ and $\kappa_p=\frac{1}{2p^2}\int_{-1}^{1}d\mu(D_{pp}-D_{\mu p}^2/D_{\mu\mu})$

With acceleration and scattering times $\tau_{ac}=1/\kappa_p$ and $\tau_{sc}=8\kappa_s/v^2$

2. If $D_{pp}/p^2\gg D_{\mu\mu}$ then $\tau_{ac}=p^2/<D_{pp}>\ll\tau_{sc}$

II. Particle Acceleration and Transport

3. If Homogeneous *(or spatially averaged)*

and defining $N(E)dE = 4\pi p^2 F(p)dp$ we get

$$\frac{\partial N(E)}{\partial t} = \frac{\partial^2}{\partial E^2}\left[D_{\rm EE} N(E)\right] - \frac{\partial}{\partial E}\left[\left(A(E) - \dot{E}_{\rm L}(E)\right)N(E)\right] - \frac{N(E)}{T_{\rm esc}(E)} + \dot{Q}(E)$$

Diffusion Accel. Loss Escape

$$D_{EE} = c^2\beta^2 D_{pp} \qquad A(E) = \frac{1}{p^2}\frac{d(c^2\beta^2 D_{pp})}{dp}$$

$$T_{\rm esc}(E) \simeq \tau_{\rm cross}(1 + \tau_{\rm cross}/\tau_{\rm sc})$$

$$\tau_L \equiv E/\dot{E}_L$$

$$\dot{E}_L = 4\pi r_0^2 \ln\Lambda mc^3 n/\beta + (4/9) r_0^2 c\beta^2\gamma^2 B^2$$

II. Wave Particle Interactions

e.g. parallel propagating waves

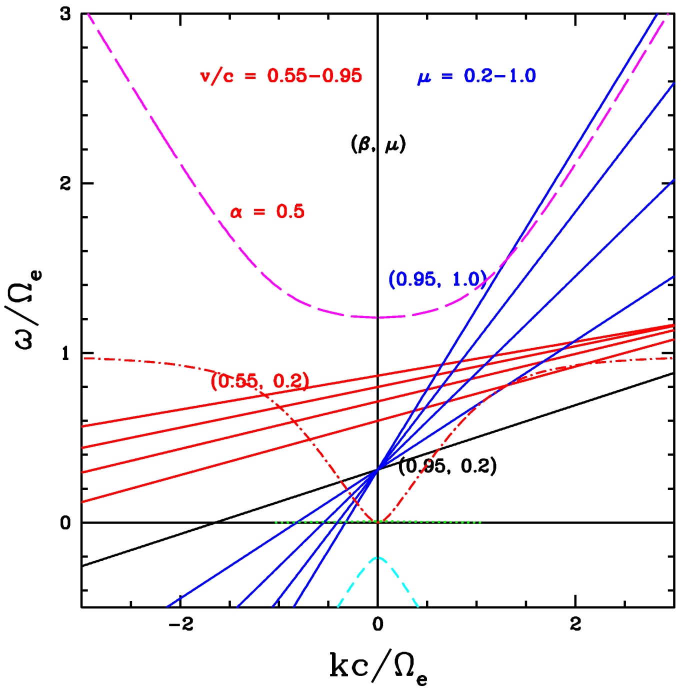
$$D_{ij} \propto \Sigma_r \Sigma_{n=-\infty}^{n=\infty} \int W(\mathbf{k}) d^3 k \delta \left(\omega(k_r) - k_{r,||} v\mu - n\Omega/\gamma \right) (J_n, J_{n-1}, \ldots)$$

Resonance Condition

$$\omega = \mu v k + \frac{\Omega_i}{\gamma}.$$

Dispersion Relation

$$(ck)^2 = \omega^2 \left[1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega - q_i/|q_i|\Omega_i)} \right].$$



II. Wave Particle Interactions

Spectrum of Turbulence (assumed)

The diffusion coefficients $D_{EE}, D_{\mu\mu}$ or $\tau_{ac}, \tau_{sc}, T_{esc}$
are related to:

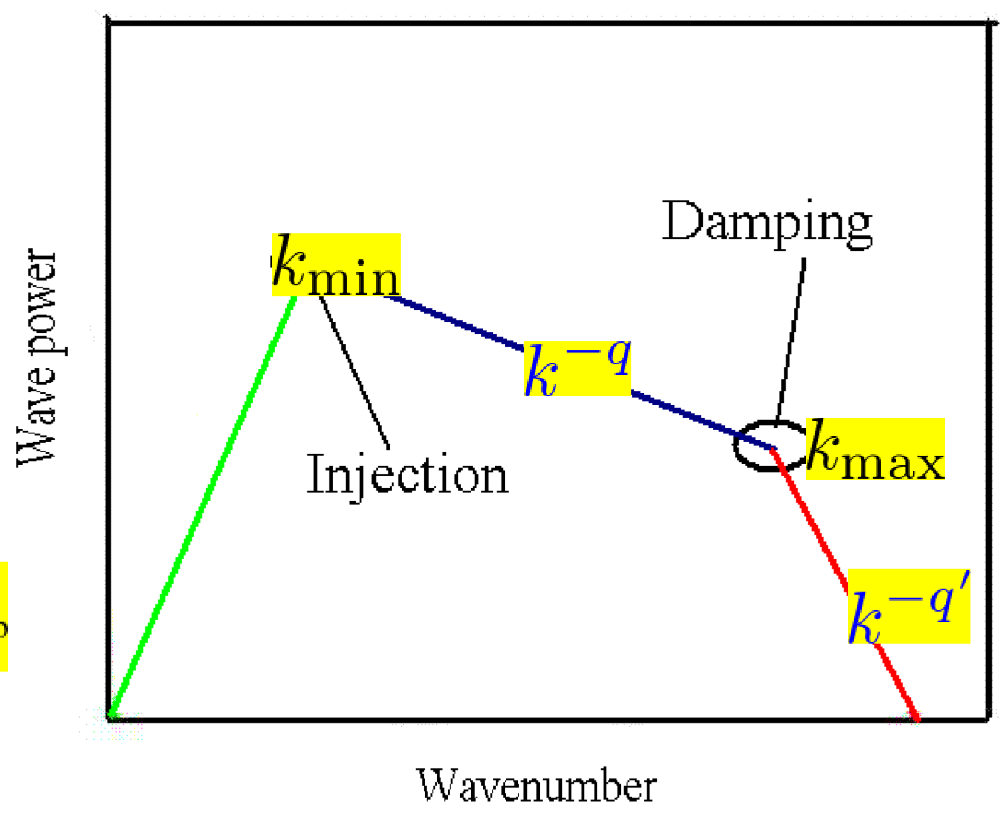
1. Turbulence parameters:
Injection and damping wave numbers
Inertial range spectral index:

$$k_{\min}, k_{\max} \text{ and } q$$

2. Plasma parameters:
Plasma and gyro-frequencies
Turbulence energy density

$$\alpha = \left(\frac{\omega_{pe}}{\Omega_e}\right) \propto \left(\frac{\sqrt{n}}{B}\right) \text{ and } u_{\rm turb} \sim 8\pi\delta B^2 \sim \rho v_{\rm turb}^2$$

$$\tau_p^{-1} = \left(\frac{\pi}{2}\right)\Omega_e\left(\frac{u_{\rm turb}}{B^2/8\pi}\right)(q-1)\left(\frac{ck_{\min}}{\Omega_e}\right)^{q-1}$$



Two Important Aspects

IMPORTANT ASPECT 1

Define $R_1 = (D_{pp}/p^2)/D_{\mu\mu}$ and $R_2 = (D_{\mu p}/p)/D_{\mu\mu}$

For High Energy Protons and Relativistic Electrons

Alfven and Fast Mode $R_1 = (v_A/v)^2 \ll 1$

$$\kappa_p = \frac{1}{2p^2}\int_{-1}^{1} d\mu (D_{pp} - D_{\mu p}^2/D_{\mu\mu})$$

$$\tau_{ac} = 1/\kappa_p \ll \tau_{sc}$$

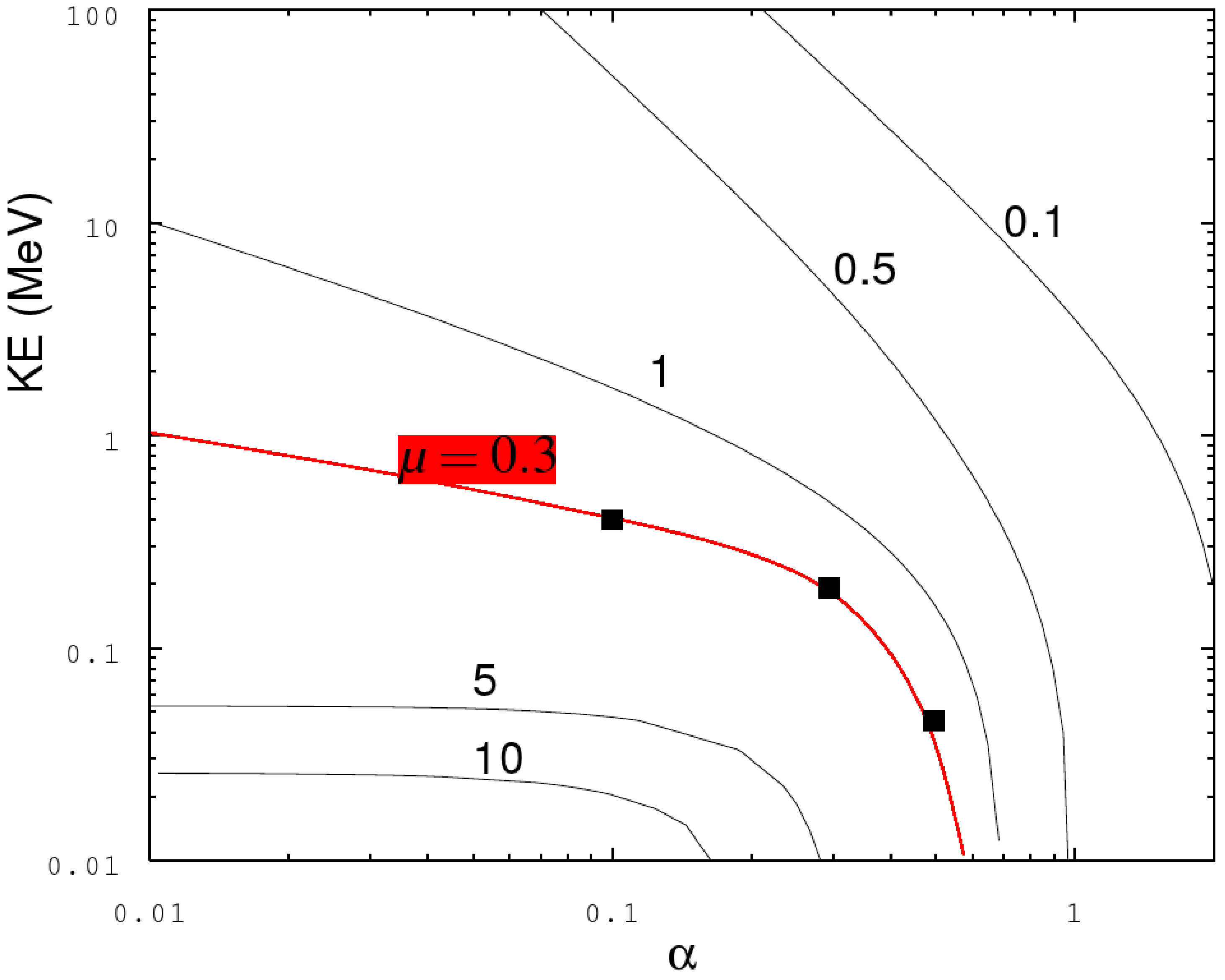
But for highly magnetized plasmas or at low energies

$R_1 \gg 1$

$$\tau_{ac} = p^2/ < D_{pp} \gg \tau_{sc}$$

Acceleration by Turbulence More Efficient than Shock

Accel/Scatt Ratio *R1*



IMPORTANT ASPECT 2

In general

$$(D_{pp}/p^2) : (D_{\mu p}/p) : D_{\mu\mu} = [x_j^2] : [x_j(1-\mu x_j)] : [(1-\mu x_j)^2] \text{ with } x_j = (\beta_{ph,j}/\beta)^2$$

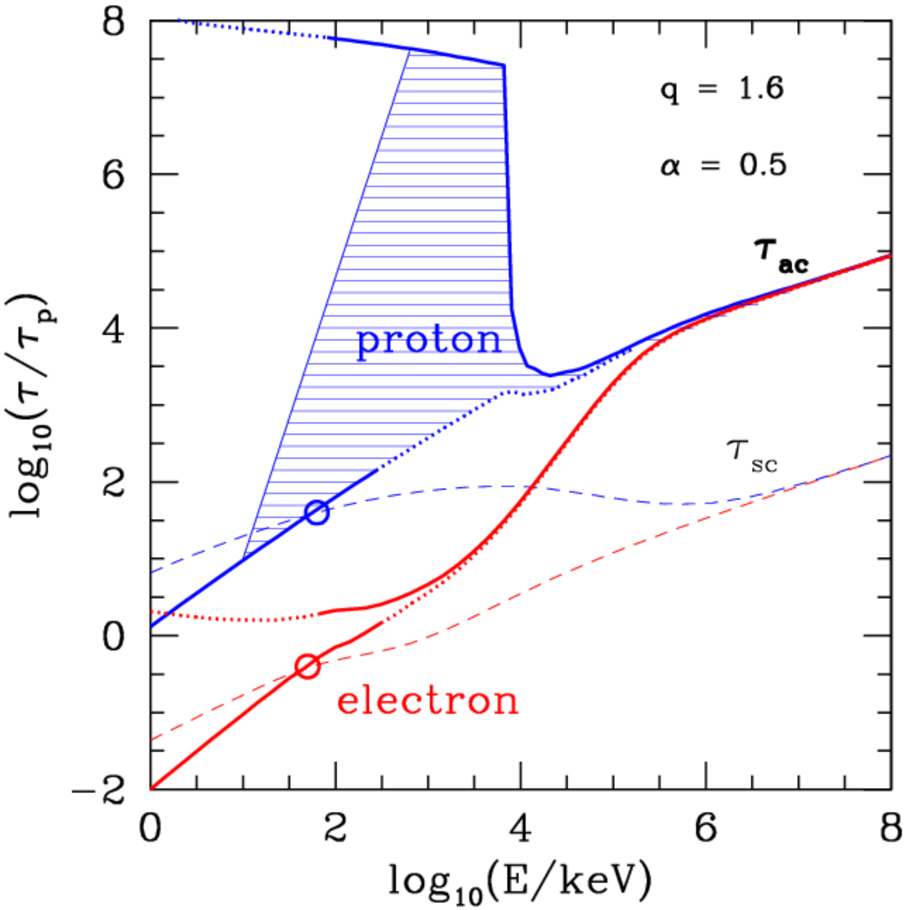
Thus when a **Single MODE** dominates, then the *Acceleration Rate for the Isotropic case*

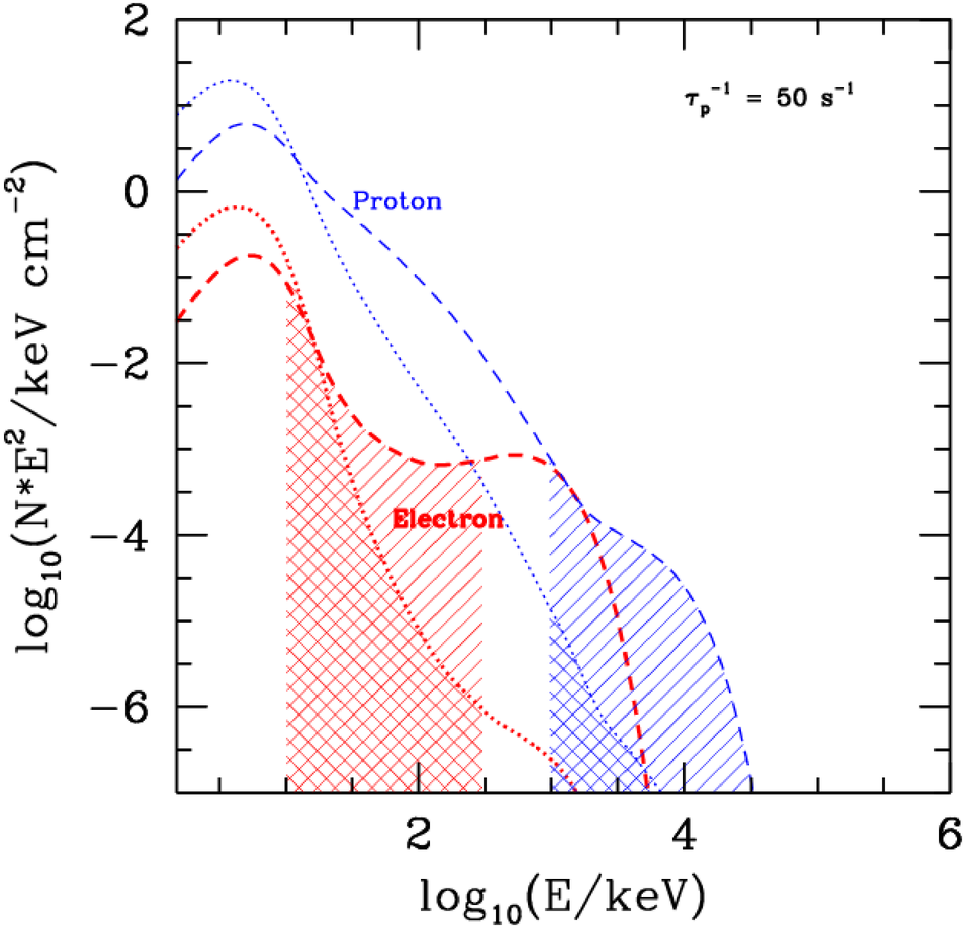
$$\kappa_p = \frac{1}{2p^2}\int_{-1}^{1} d\mu (D_{pp} - D_{\mu p}^2/D_{\mu\mu}) \to 0$$

This however does not affect the nonisotropic $R_1 > 1$

III. Stochastic Acceleration in Flares

1. Electron vs Proton Acceleration and Spectra

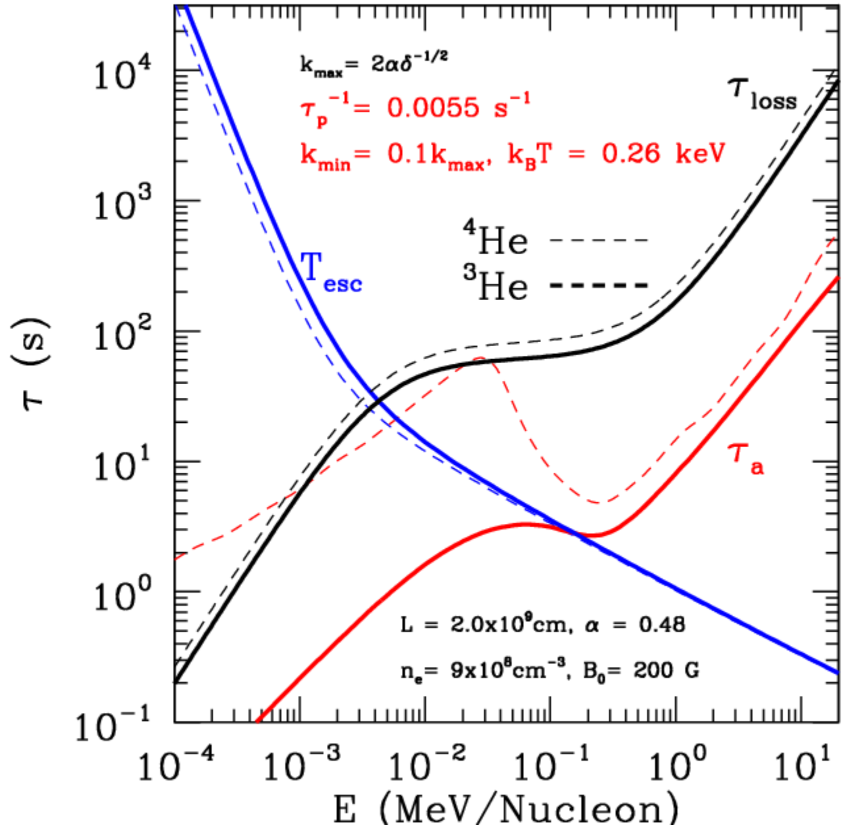




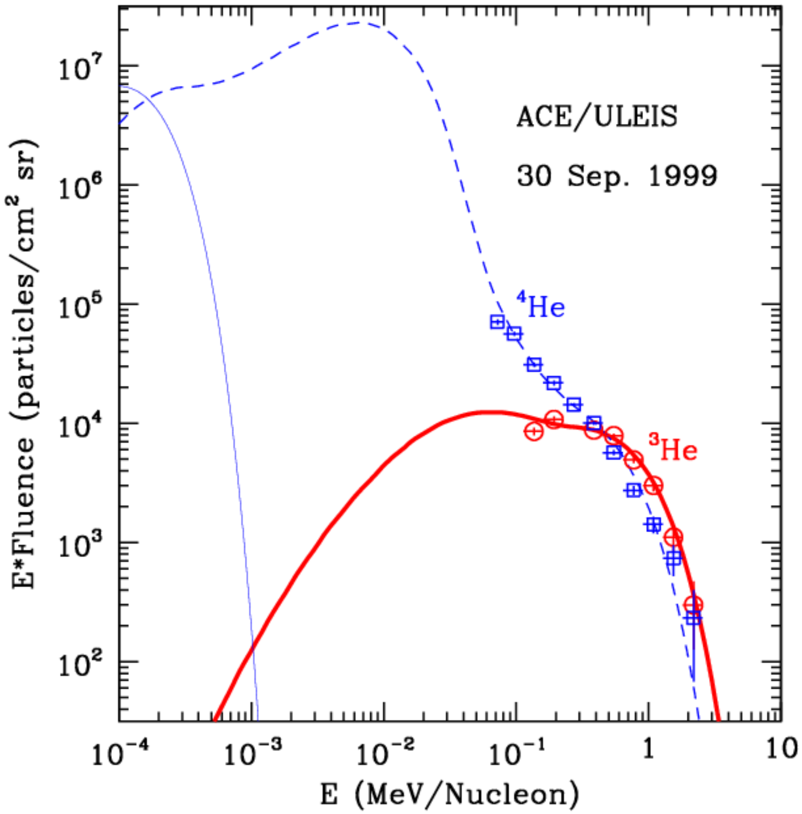
III. Stochastic Acceleration in Flares

2.SEPs and He3/He4 Acceleration

Timescales

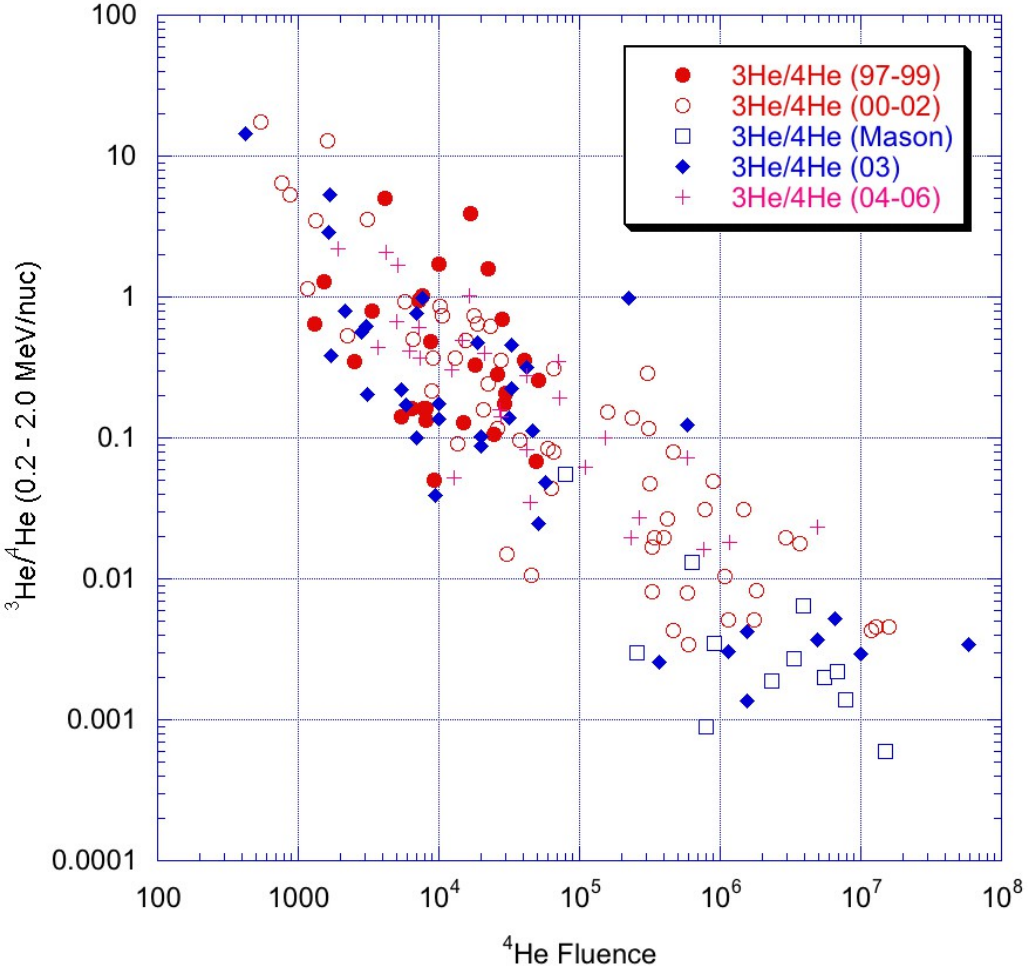


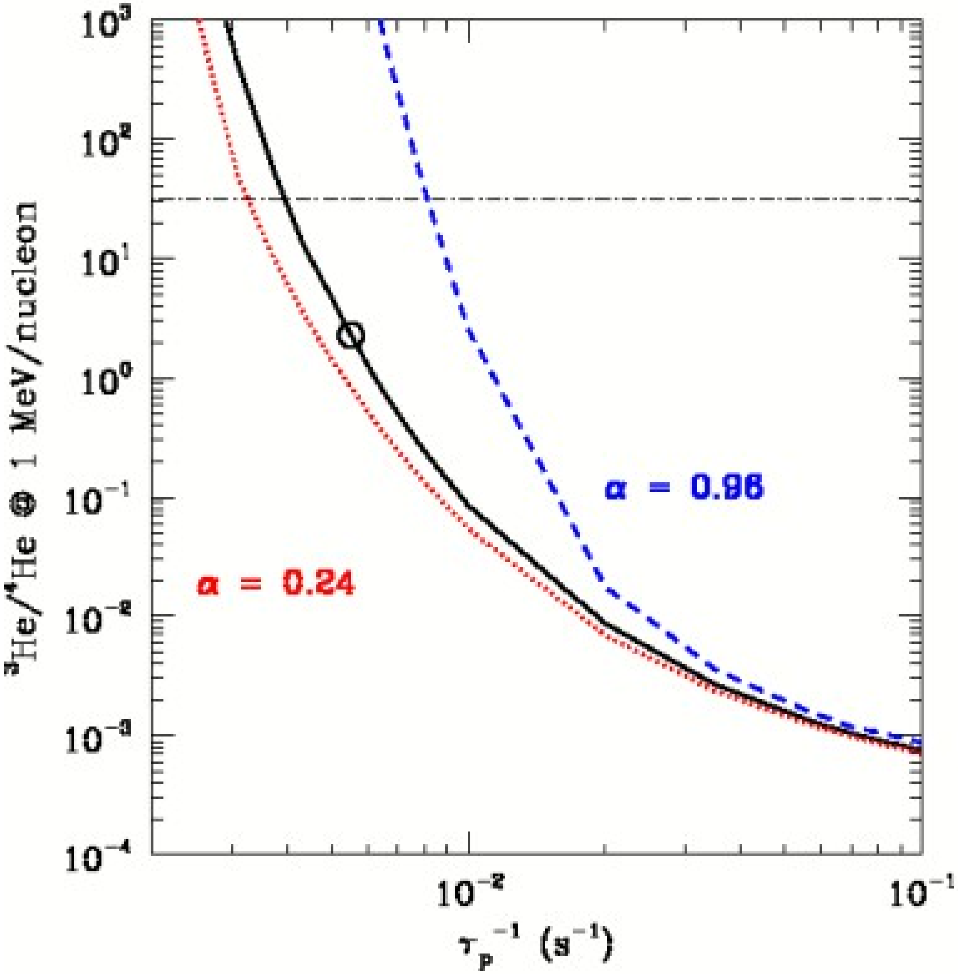
Spectral Fits



Liu, VP and Mason, 2004, ApJ

He3, He4 Fluence Ratios



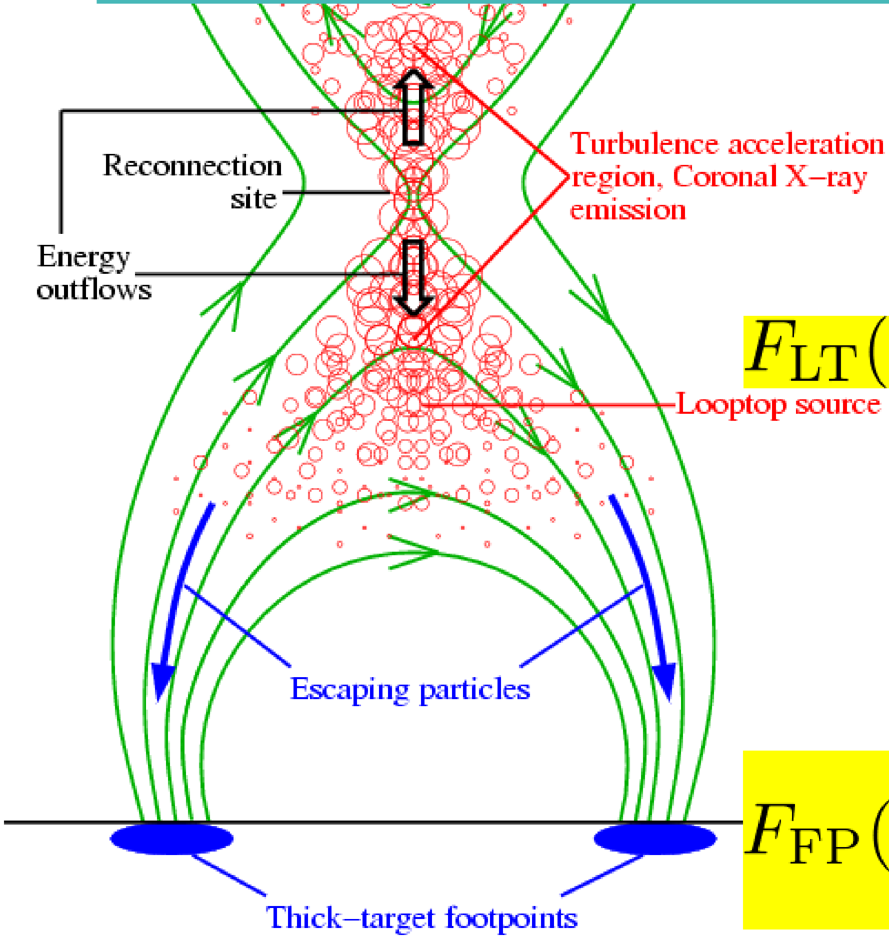


Observations *Model Results*

III. Stochastic Acceleration of Electrons in Solar Flares

Testing the Acceleration Model

Back to The Basic Model:
Relating Electrons and Photons



See Petrosian & Chen (2010) ApJ Letters, 2010, 712, 131

$$F_{\rm LT}(E) = v(E)N(E)$$

Connected by Escaping Process

$$F_0(E) = F_{\rm LT}(E)(\tau_{\rm cross}/T_{\rm esc})$$

$$F_{\rm FP}(E) = vN_{\rm FP} = \frac{v(E)}{\dot{E}_{\rm L}(n)} \int_E^\infty \frac{N(E')}{T_{\rm esc}(E')} {\rm d}E'$$

HXR: $$\left\{ \begin{array}{c} I_{\rm LT}(\epsilon) \\ I_{\rm FP}(\epsilon) \end{array} \right\} = \frac{nV}{4\pi R^2} \int_\epsilon^\infty \left\{ \begin{array}{c} F_{\rm LT}(E) \\ F_{\rm FP}(E) \end{array} \right\} \sigma(\epsilon, E) dE$$

Regularized Inversion of Photon Images to Electron Images

$$I(x,y;\epsilon) = \frac{a^2}{4\pi R^2} \int_{E=\epsilon}^{\infty} N(x,y)\overline{F}(x,y;E)Q(\epsilon,E)\,dE \qquad J(x,y;q)\,dq = \int_x \int_y \int_{\epsilon=q}^{\infty} D(q,\epsilon)I(x,y;\epsilon)\,d\epsilon\,dx\,dy$$

RHESSI produces count visibility, Fourier component of the source

$$V(u,v;q) = \mathcal{F}^2(J(x,y;q)) \equiv \int_x \int_y J(x,y;q)e^{2\pi i(ux+vy)}\,dx\,dy$$

Defining electron flux visibility spectrum and count cross section

$$W(u,v;E) = a^2 \int_x \int_y N(x,y)\overline{F}(x,y;E)e^{2\pi i(ux+vy)}\,dx\,dy \qquad K(q,E)\,dq = \int_{\epsilon=q}^{\infty} D(q,\epsilon)Q(\epsilon,E)\,d\epsilon$$

We get $V(u,v;q) = \frac{1}{4\pi R^2} \int_q^{\infty} W(u,v;E)K(q,E)\,dE$

Regularized inversion produced *smoothed* electron flux visibility spectrum

$$\left\| \boldsymbol{V}_{[u,v]} - \boldsymbol{K} \cdot \boldsymbol{W}_{[u,v]} \right\|^2 + \lambda_{[u,v]} \left\| \boldsymbol{W}_{[u,v]} \right\|^2 = \text{minimum}$$

Fourier Transform Gives $N(x,y)\overline{F}(x,y;E) = \frac{1}{a^2} \int_u \int_v W(u,v;E)e^{-2\pi i(ux+vy)}\,du\,dv$

Piana et al. 2007

Results From Regularized Inversion of Images

(1) LT image gives the accelerated electron spectrum

$$N(E) = F_{\rm LT}(E)/v$$

(2) FP images give the *effective* spectrum

$$F_{\rm FP} = \frac{v}{\dot{E}_{\rm L}} \int_E^\infty \frac{N(E')}{T_{\rm esc}(E')} {\rm d}E'$$

$$T_{\rm esc} = N(E) \times \frac{(dF_{\rm FP}\dot{E}_L/v)}{dE}$$

From these spectra we can derive the *escape time*

$$T_{\rm esc}(E) = \frac{\tau_{\rm L}(E)(F_{\rm LT}/F_{\rm FP})}{\delta_{\rm FP}(E) + 2/(\gamma+\gamma^2)}, \text{with } \tau_{\rm L} = E/\dot{E}_{\rm L}$$

and determine the mean and *turbulence scattering times*

$$\tau_{\rm scat} \simeq \tau_{\rm cross}^2/(T_{\rm esc} - \tau_{\rm cross}) \;\; \text{and} \;\; \tau_{\rm scat}^{\rm turb} \simeq \tau_{\rm scat}(1 + \tau_{\rm scat}/\tau_{\rm scat}^{\rm Coul})$$

(Petrosian & Chen, 2010 ApJ L, 712, 131)

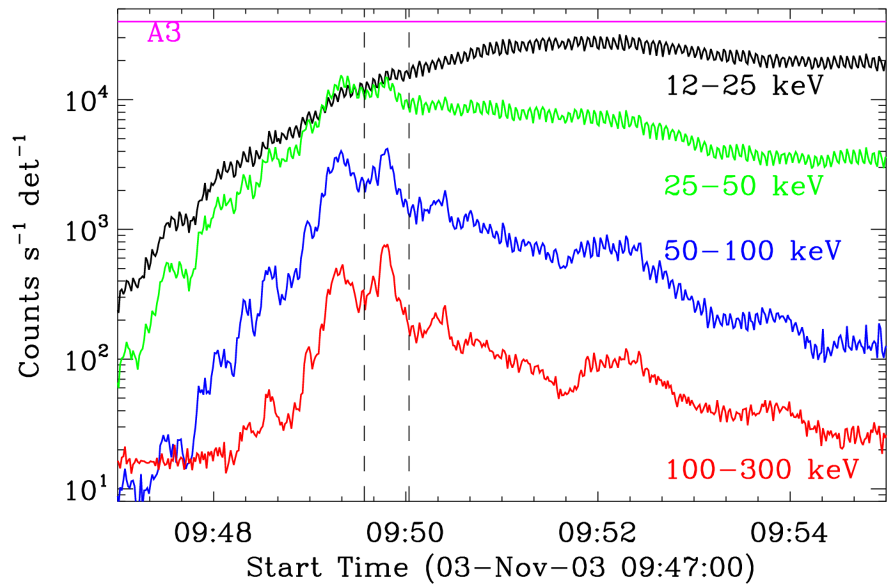
Applications and Results

We apply the inversion to images of two flares

1. 2003 November 3 Flare (X3.9 class)
2. 2005 September 8 Flare (M2.1 class)

and evaluate the escape and scattering times and compare with stochastic acceleration model parameters.

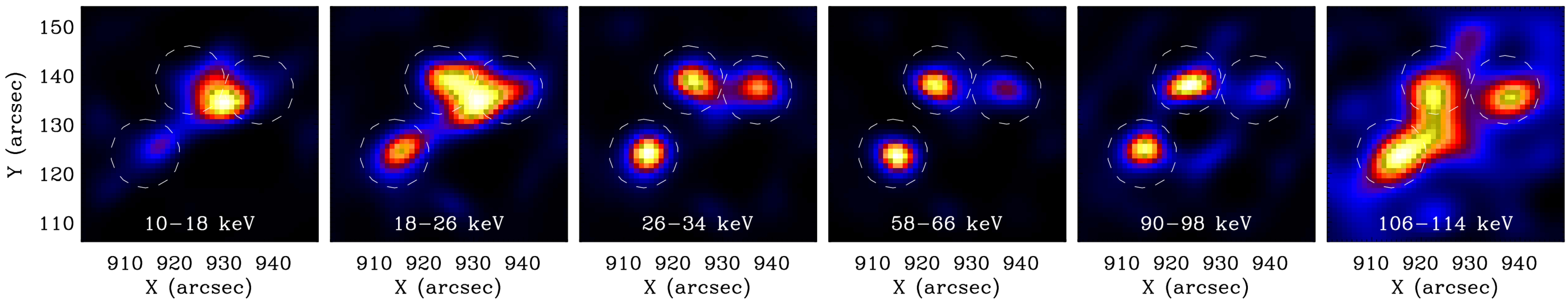
2003 Nov 3 Flare (X3.9 class)



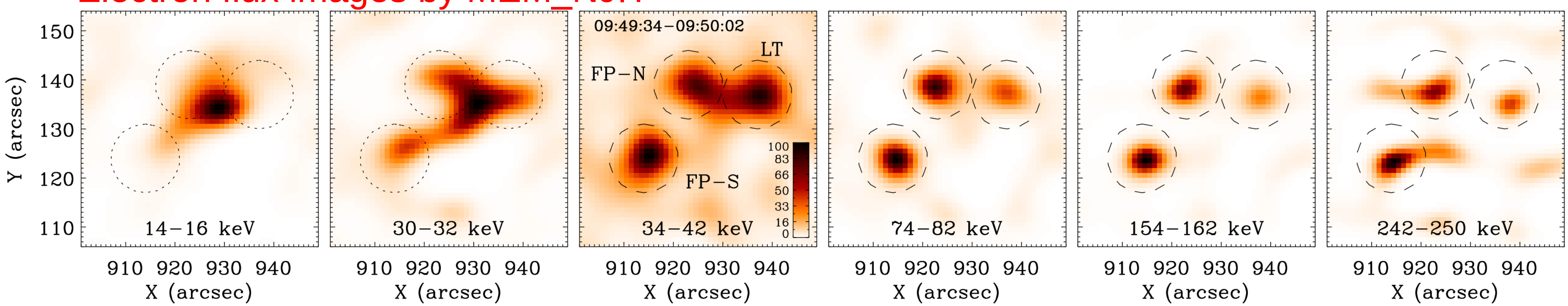
LT source detected up to 100-150 keV

(Chen & Petrosian, in preparation)

HXR images by MEM_NJIT

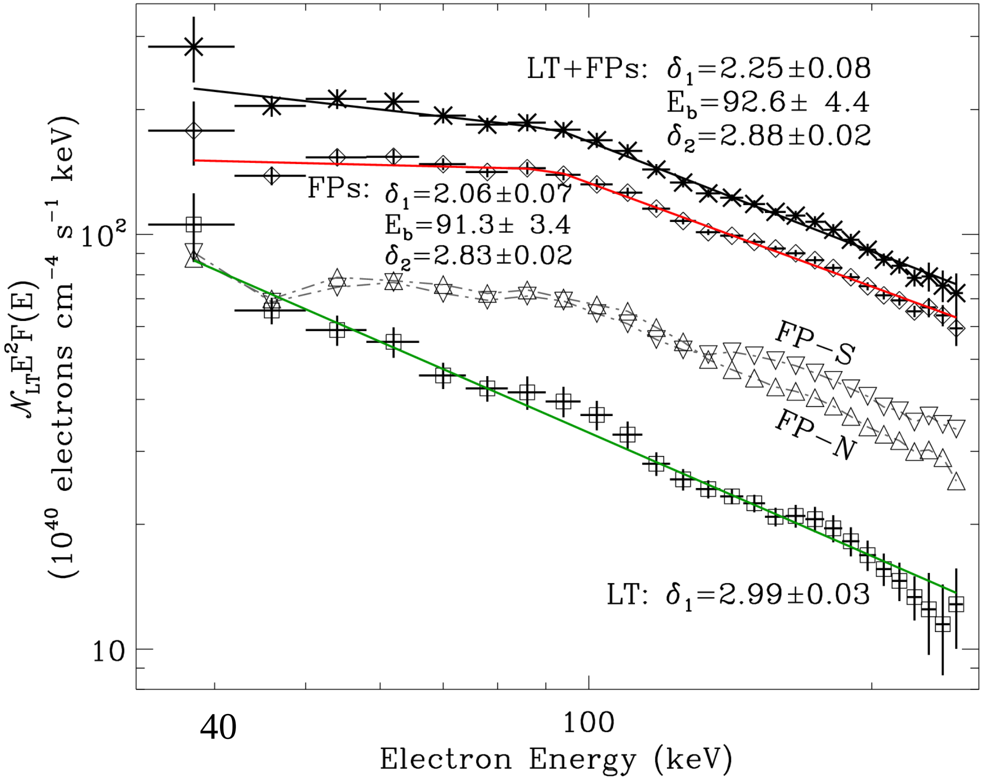


Electron flux images by MEM_NJIT

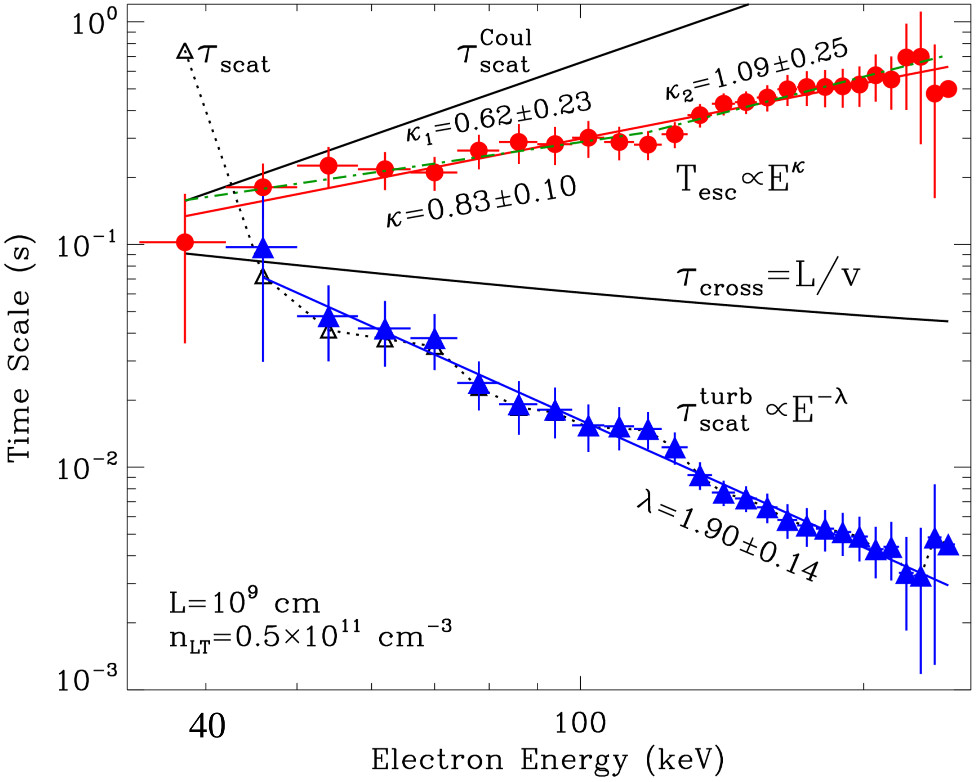


2003 Nov 3 Flare: Model Parameters

Electron Spectra

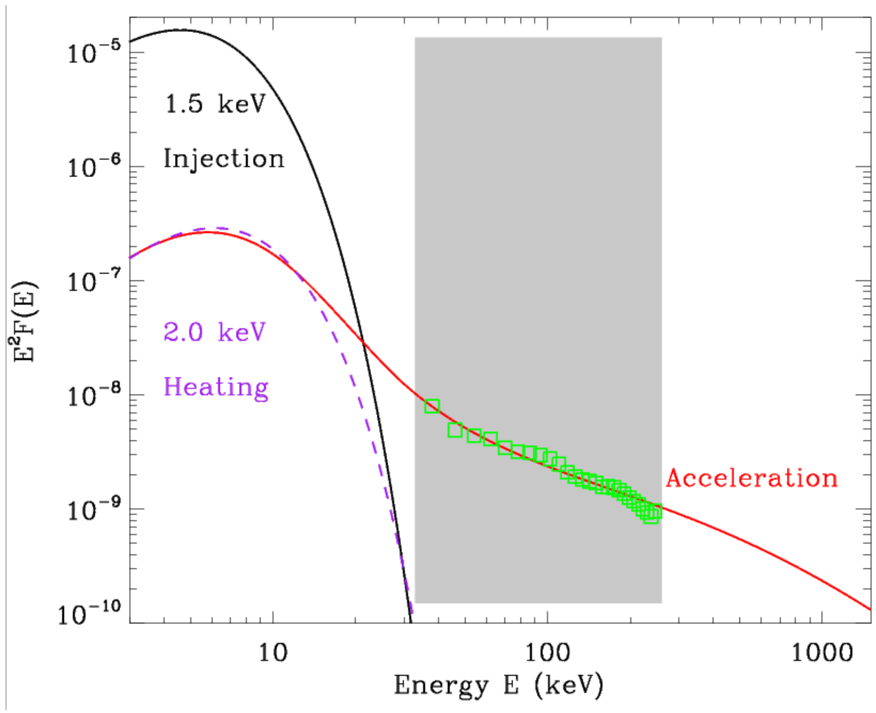


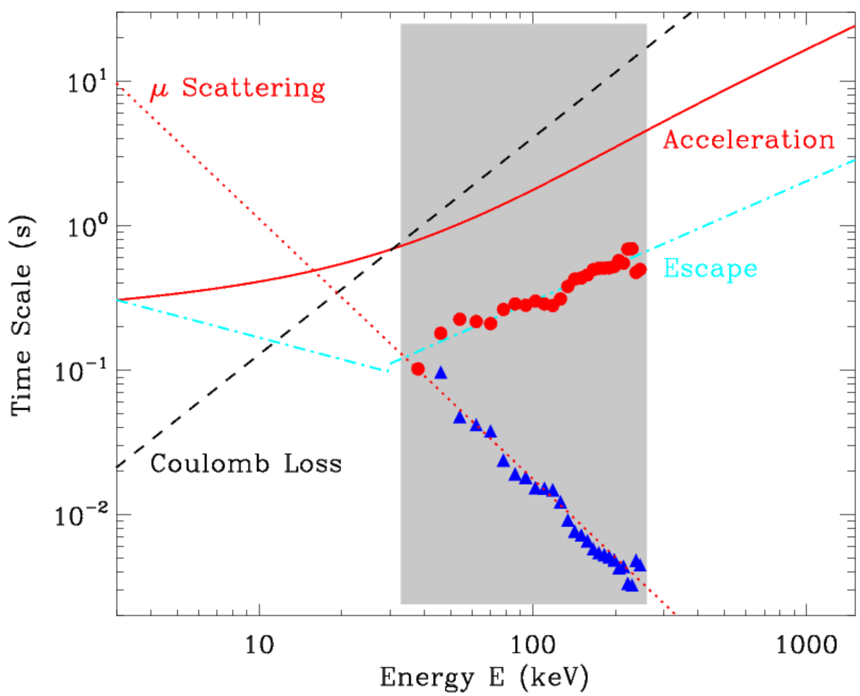
Time Scales



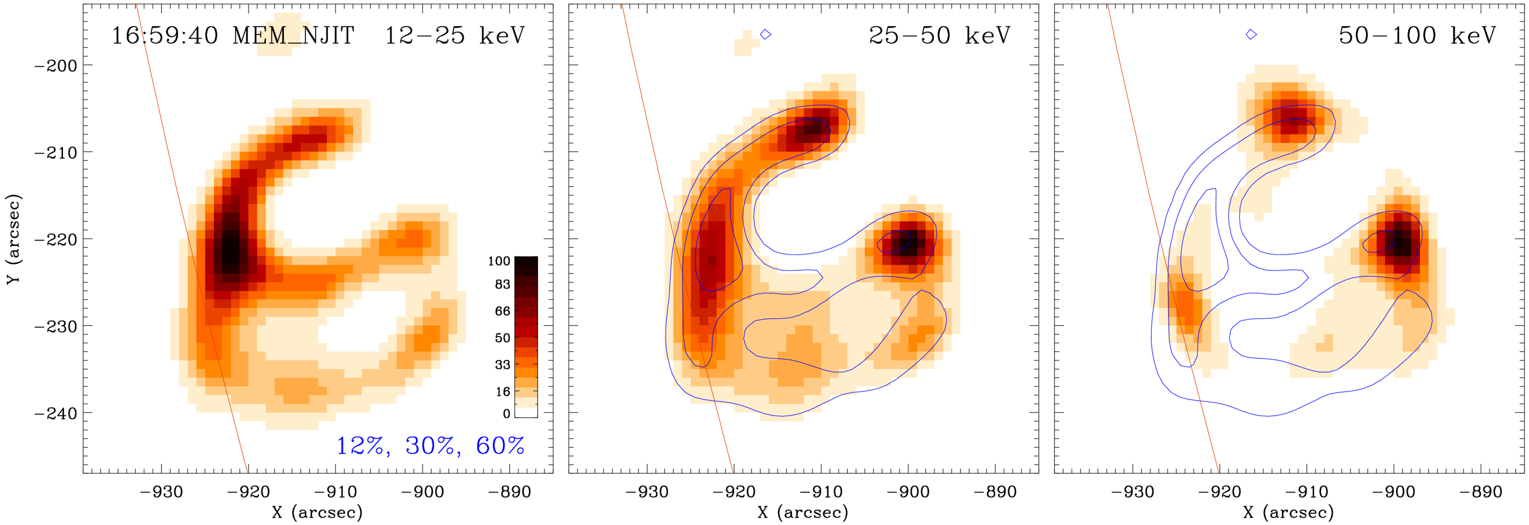
2003 Nov 3 Flare: A Complete Model

$$\frac{\partial N(E)}{\partial t} = \frac{\partial^2}{\partial E^2}\left[D_{\rm EE}N(E)\right] - \frac{\partial}{\partial E}\left[\left(A(E)-\dot{E}_{\rm L}(E)\right)N(E)\right] - \frac{N(E)}{T_{\rm esc}(E)} + \dot{Q}(E)$$



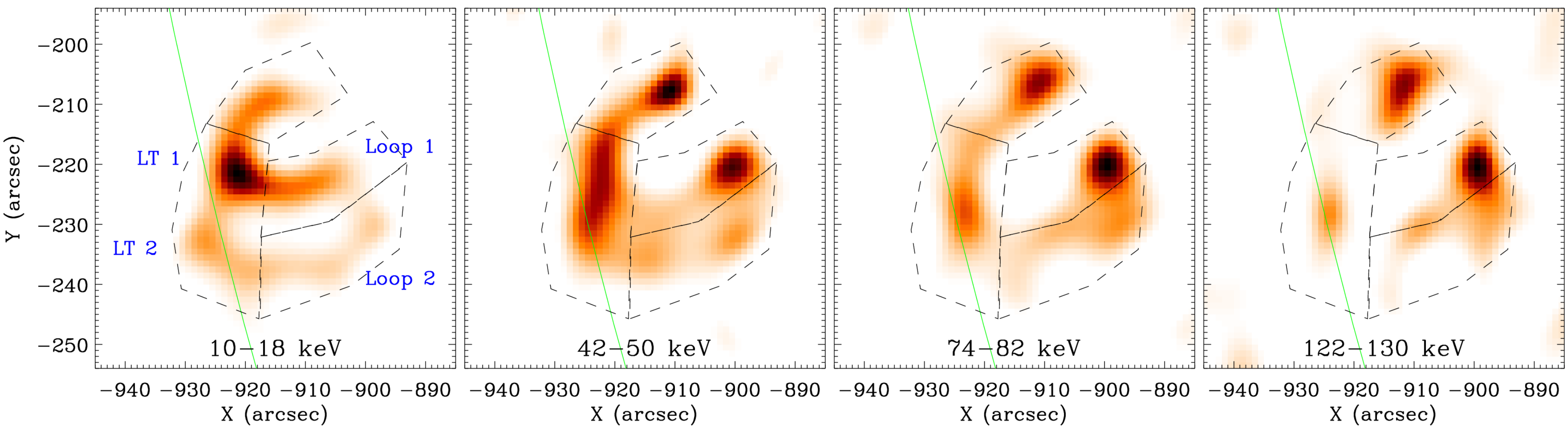


2005 Sep. 8 Flare: HXR/Electron Images



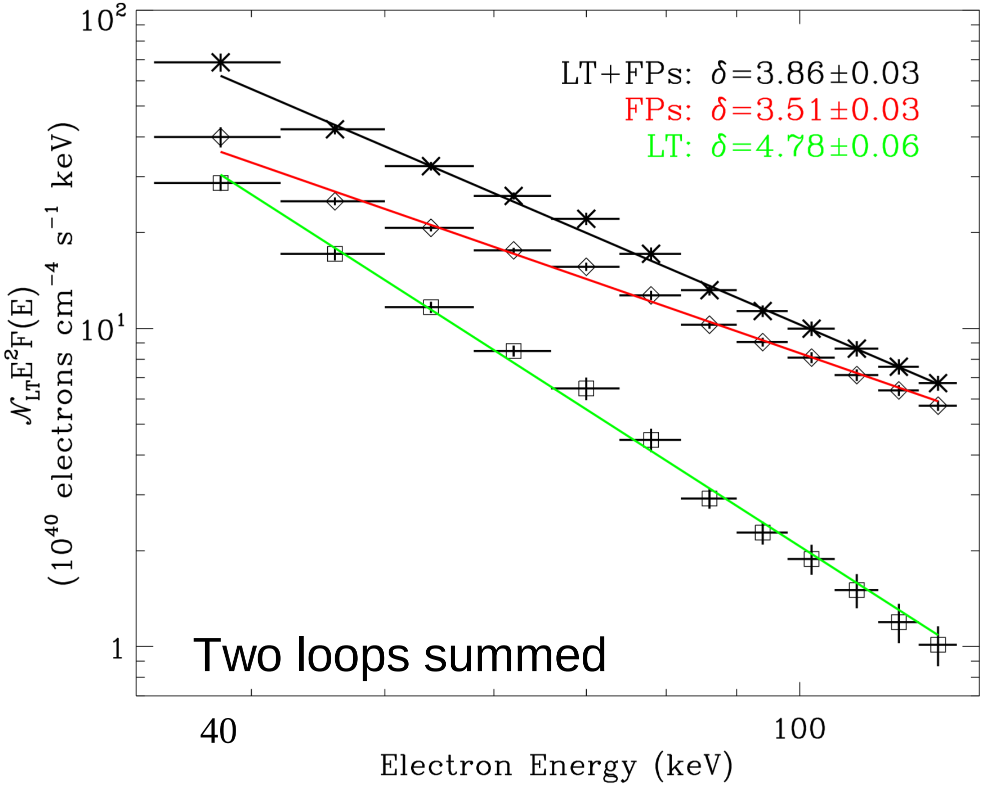
HXR:
2 loops

Electron

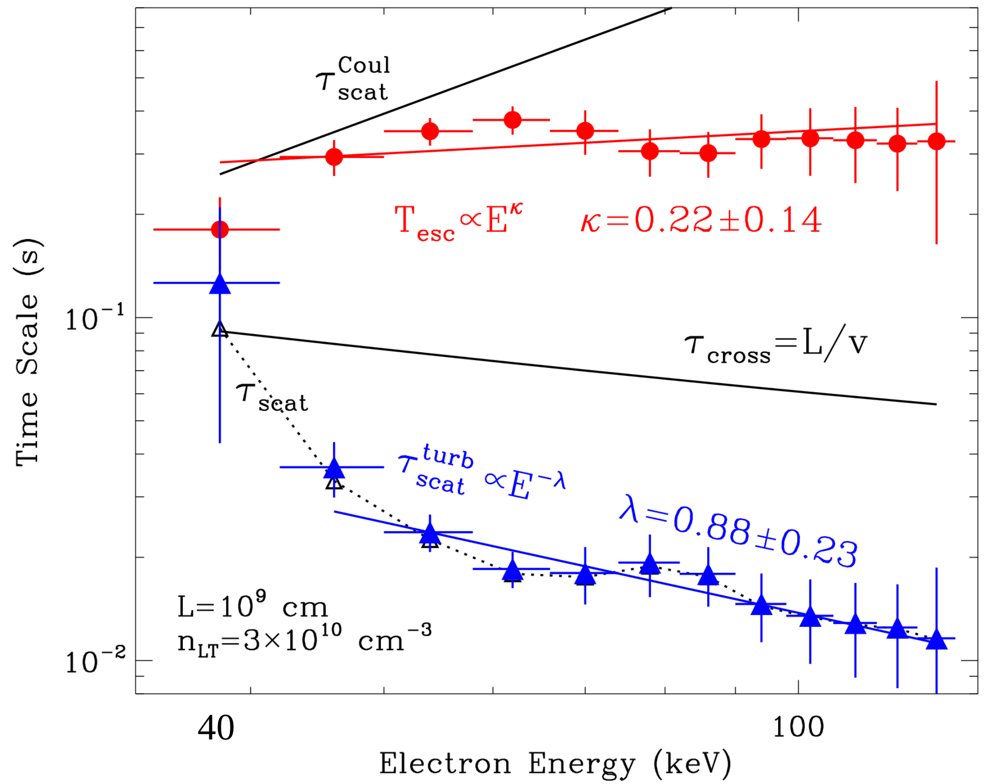


2005 Sep. 8 Flare: Model Parameters

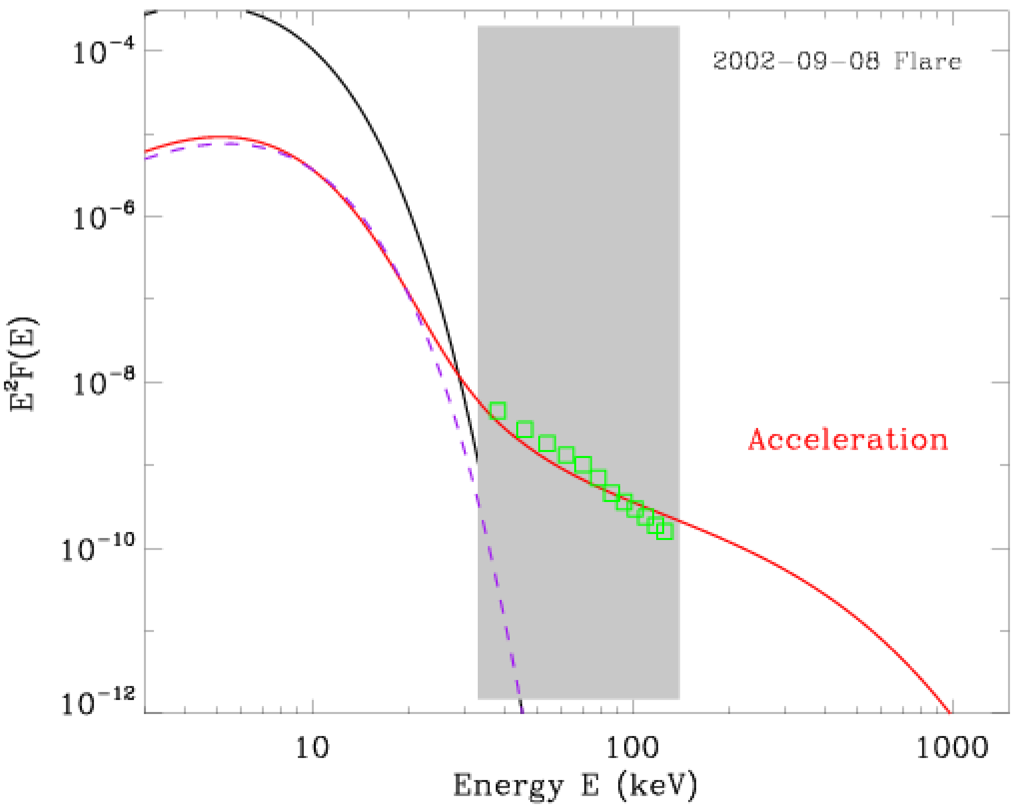
Electron Spectra

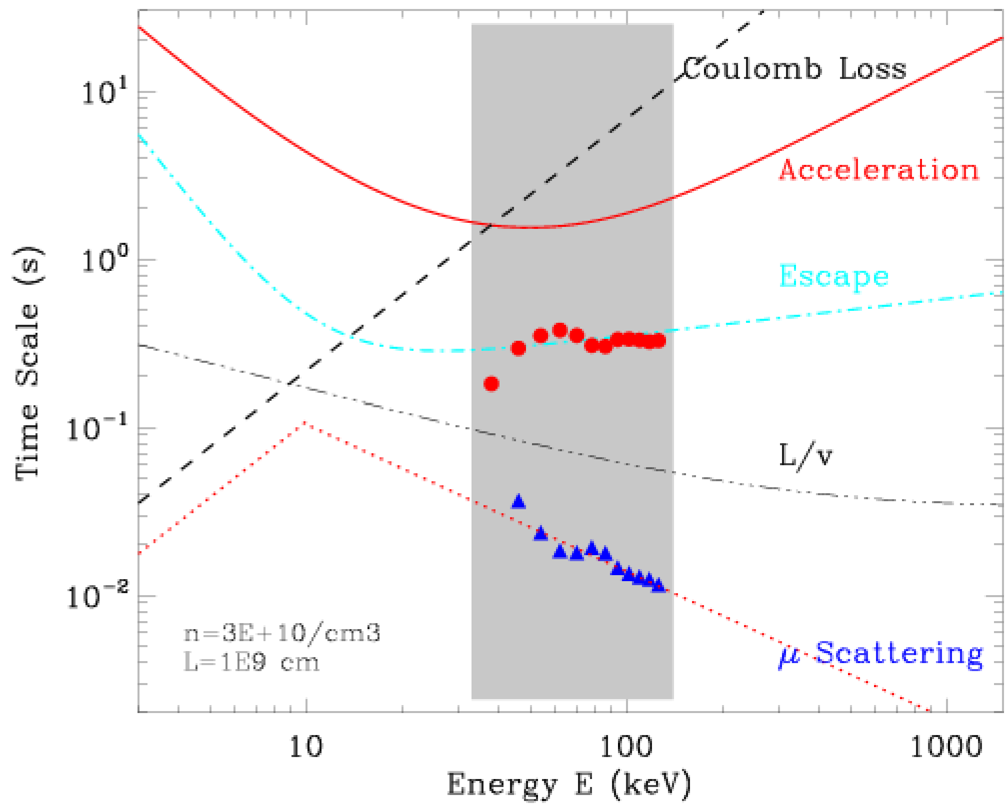


Time Scales

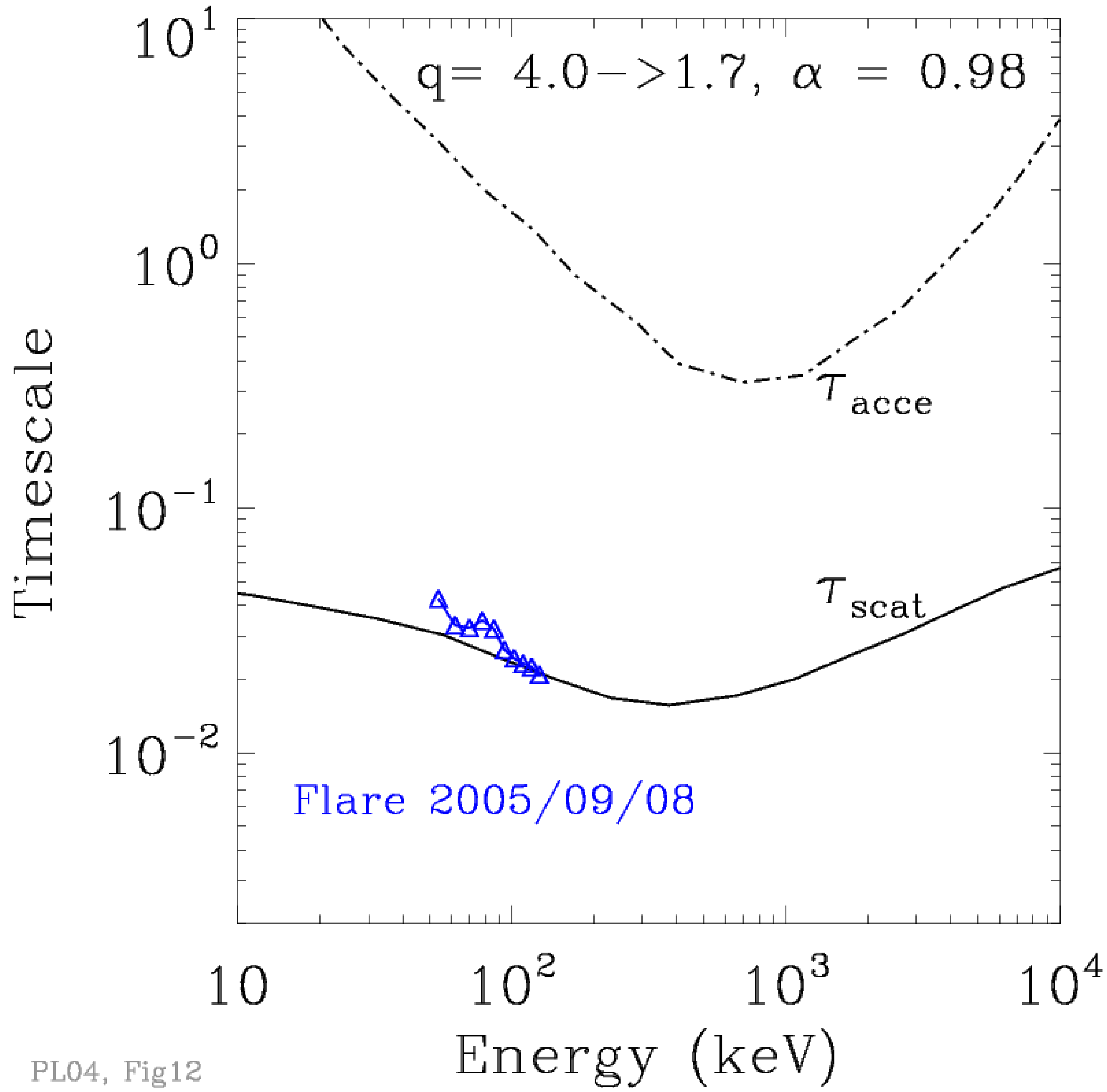


2006 Sep. 8 Flare: A Complete Model





Comparison With Stochastic Acceleration Model



PL04, Fig12

Some Required Values for the Parameters of the Stochastic Acceleration Model

$q = 3, \text{ and } \alpha \sim 1 (n \sim 10^{10} \text{ cm}^{-3}; b \sim 300 \text{ G}$

A scattering time of $\tau_{\text{scat}} \sim 2 \times 10^{-2} \text{ s}$

Requires $\tau_p^{-1} \sim 50 \text{ s}^{-1}$ and $\frac{u_{\text{turb}}}{B^2/8\pi} \left(\frac{ck_{\min}}{\Omega_e} \right)^2 \sim 3 \times 10^{-9}$

which means $u_{\text{turb}} \ll B^2/8\pi \text{ for } ck_{\min}/\Omega_e > 10^{-4}$

Thus, we need to have a small fraction of magnetic energy in form of sub-Alfvenic turbulence

SUMMARY and CONCLUSIONS

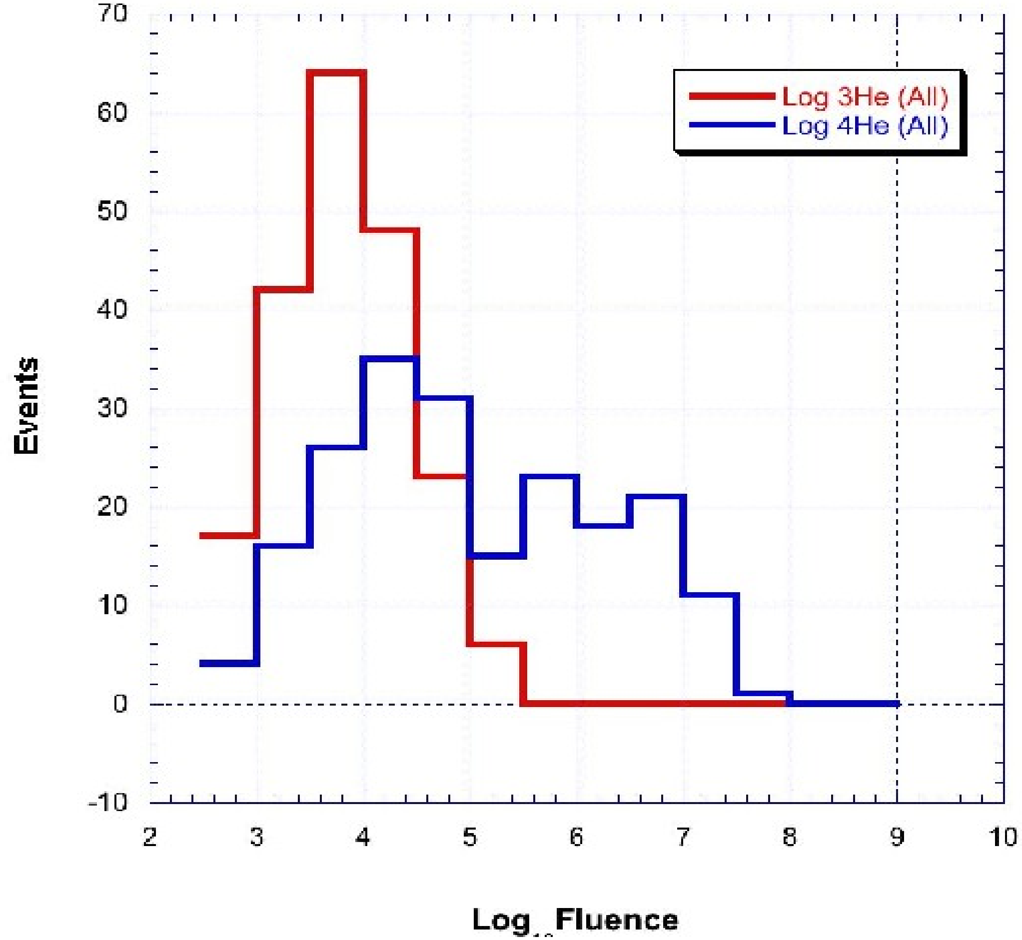
1. Several observations *require trapping* of the accelerated particles in the acceleration Site. The most likely candidate for this is *turbulence.*

2. This turbulence can in addition *accelerate* particles and this acceleration is the most efficient acceleration mechanism at *low energies and highly magnetized plasmas.*

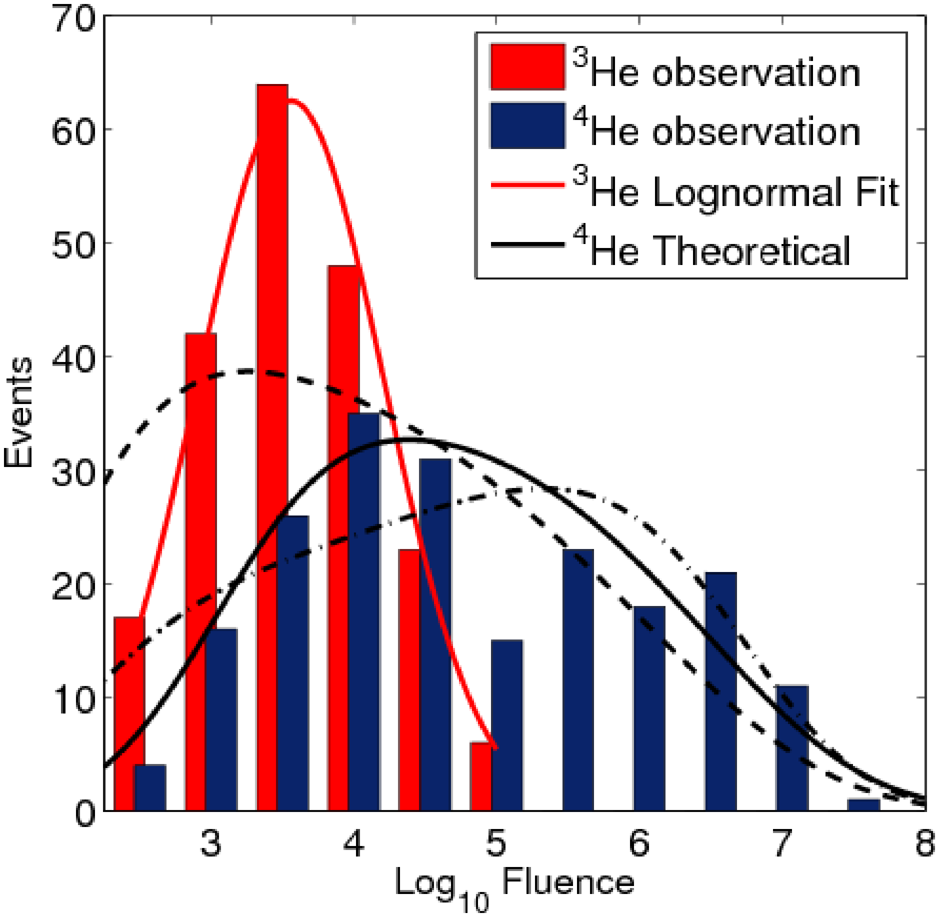
3. Relative acceleration of *electrons/protons; He3/He4* and relative spectra of *Looptop/Footpoint* sources can be reproduced by this process.

*4. T*he newly developed *regularized inversion technique* can be used to determine some of the acceleration parameters directly from the *RHESSI* X-ray data.

He3, He4 Fluence Distributions



Ho et al. 2004, ApJ



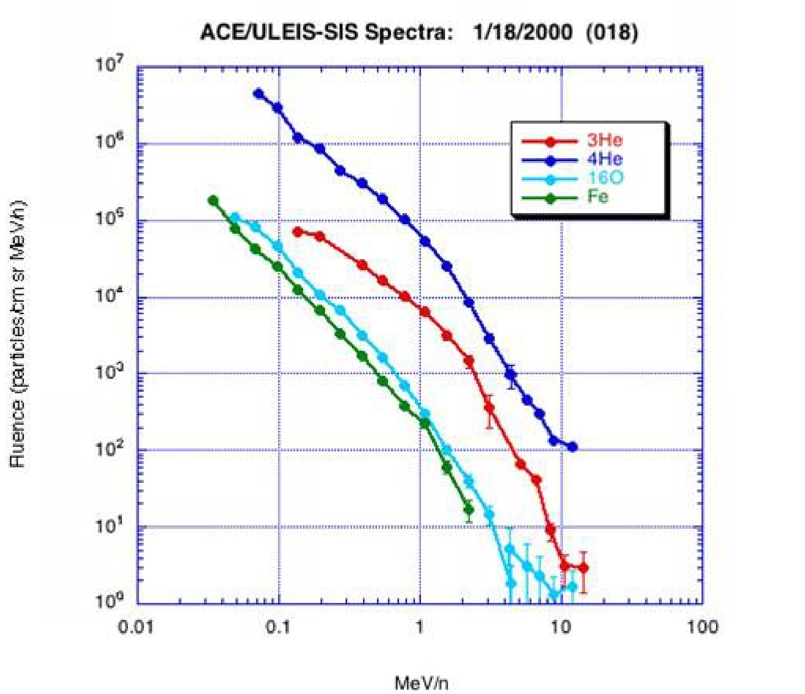
VP et al. 2010, ApJ

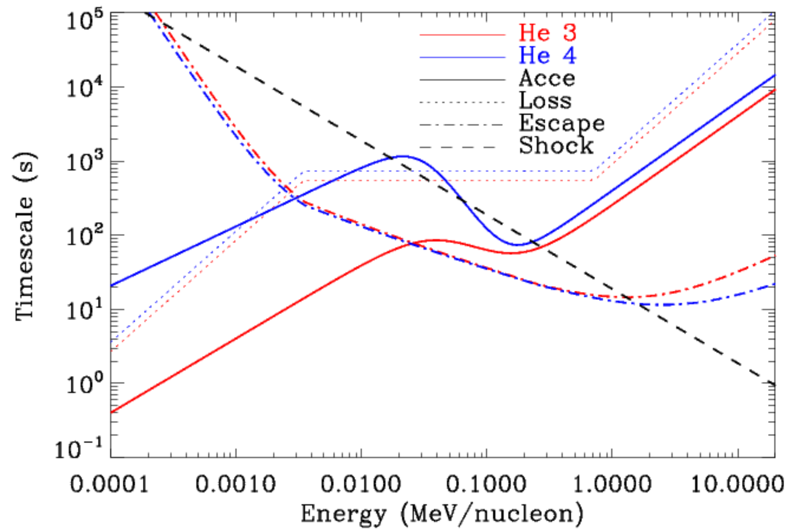
Variation of He3/He4 Spectra

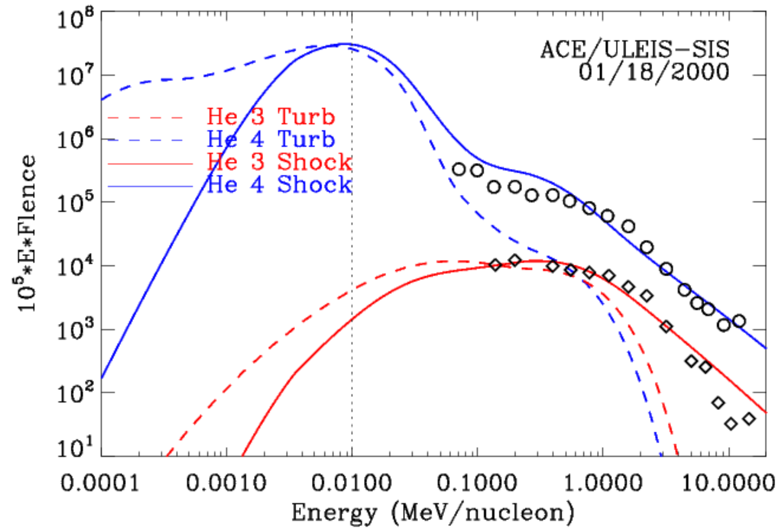
Spectra of “gradual” SEPs

Second Stage Acceleration

Event #11, Mason et al., ApJ, 574, 1039, 2002

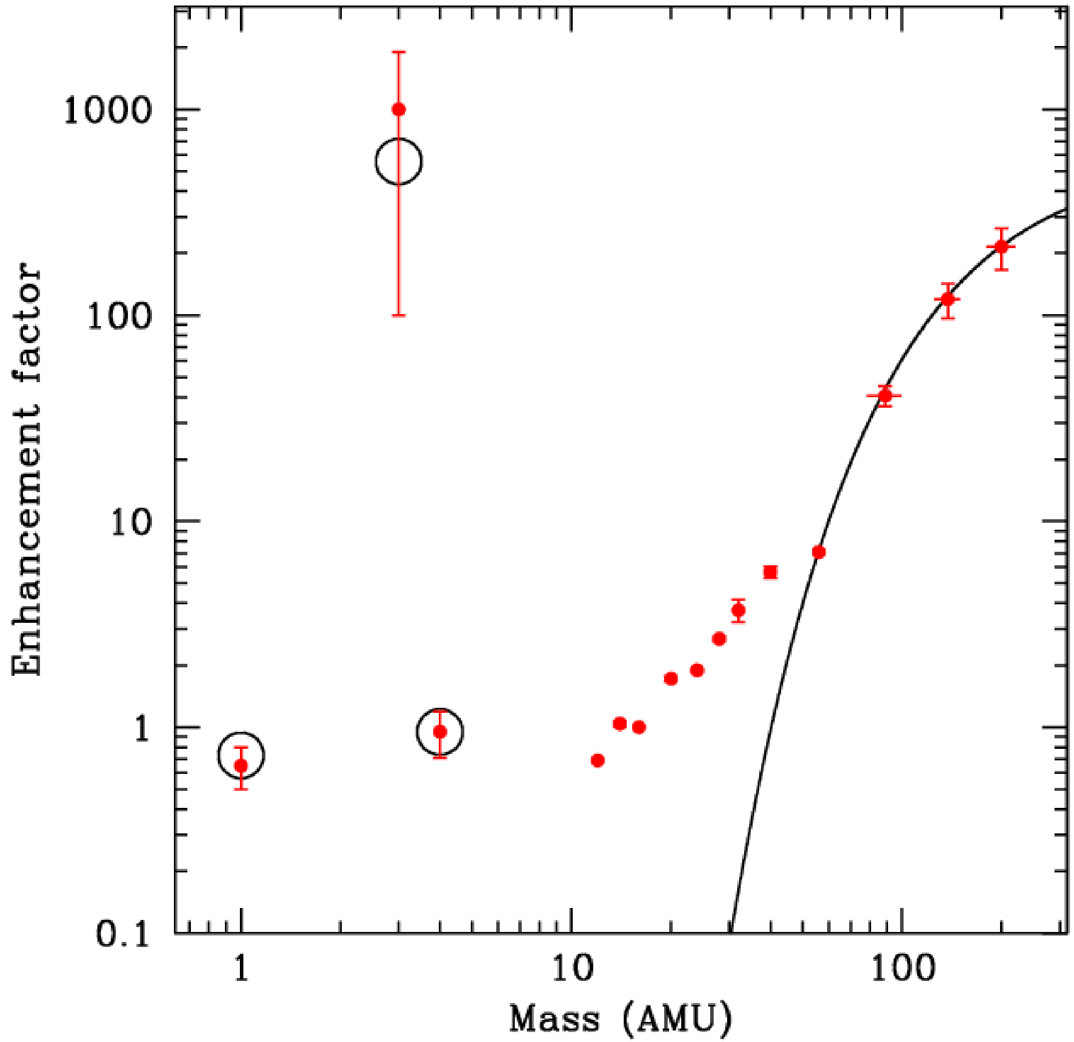






Chen, VP and , 2011

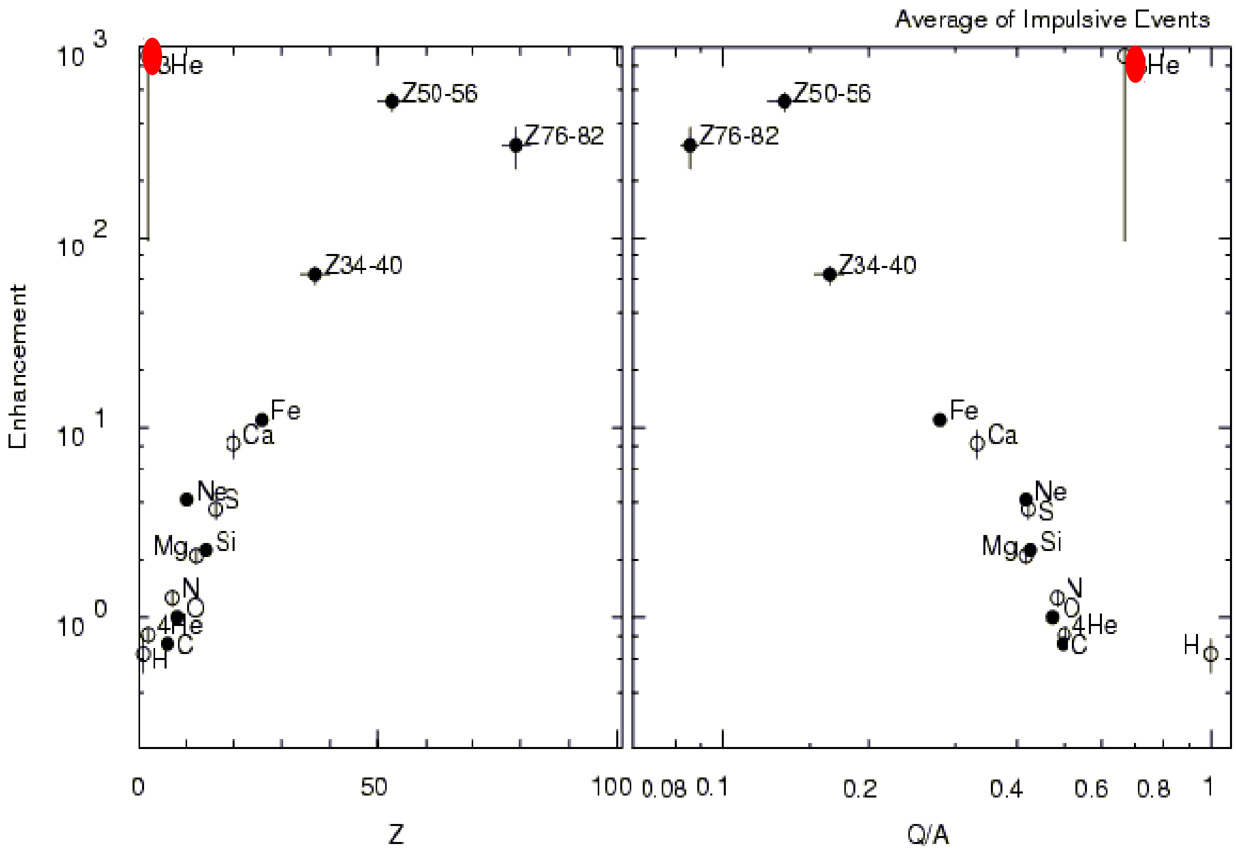
SEPs: P, He3, He4 and Heavy Ions



I. Observed Signatures of Acceleration

1. Solar Energetic Particles or ***SEPs*** Observed at 1 A.U.

Enhancements of 3He and Heavy Ions

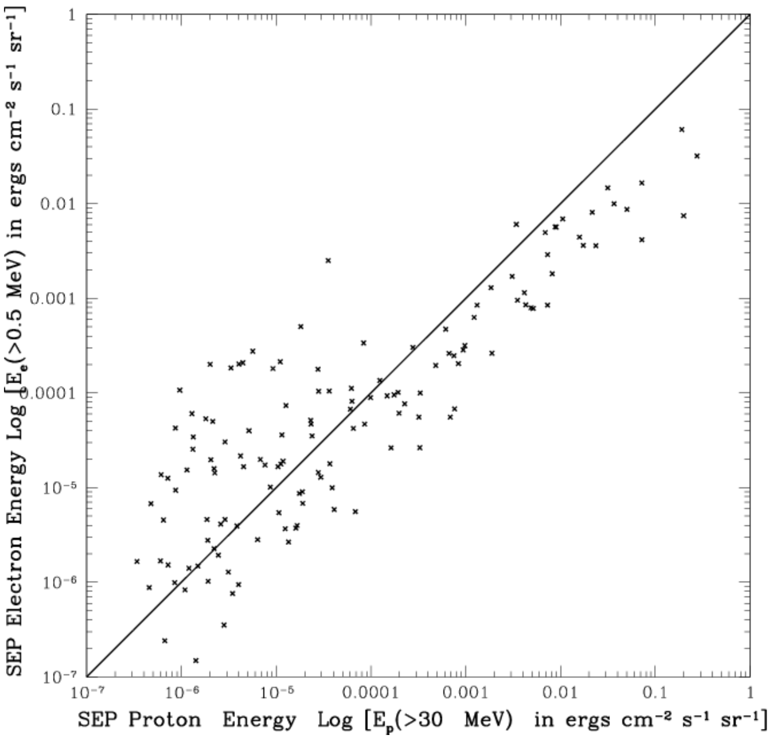


Reames et al

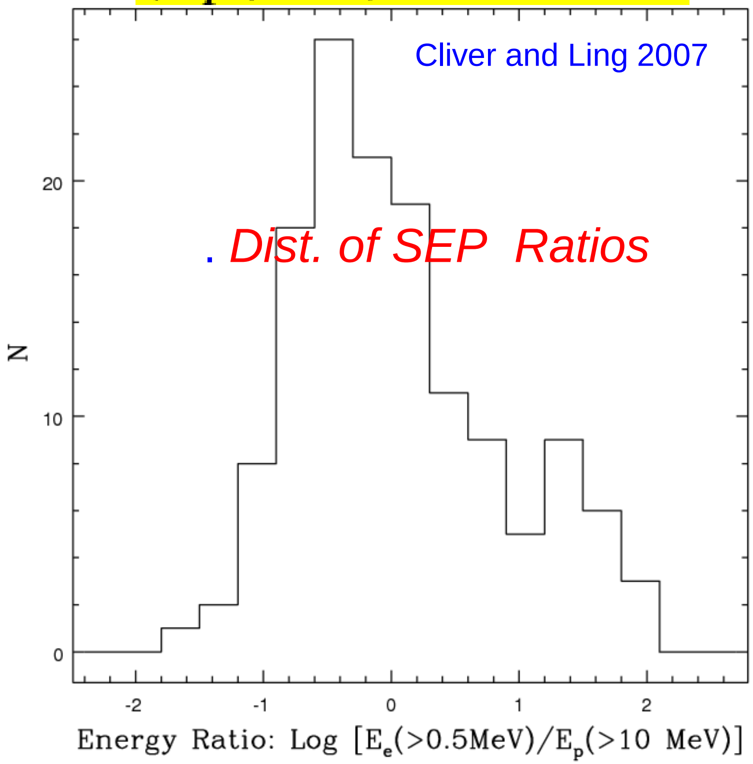
11/18/11

I. Observed Signatures of Acceleration

Electron vs Proton Acceleration Rates: SEPs



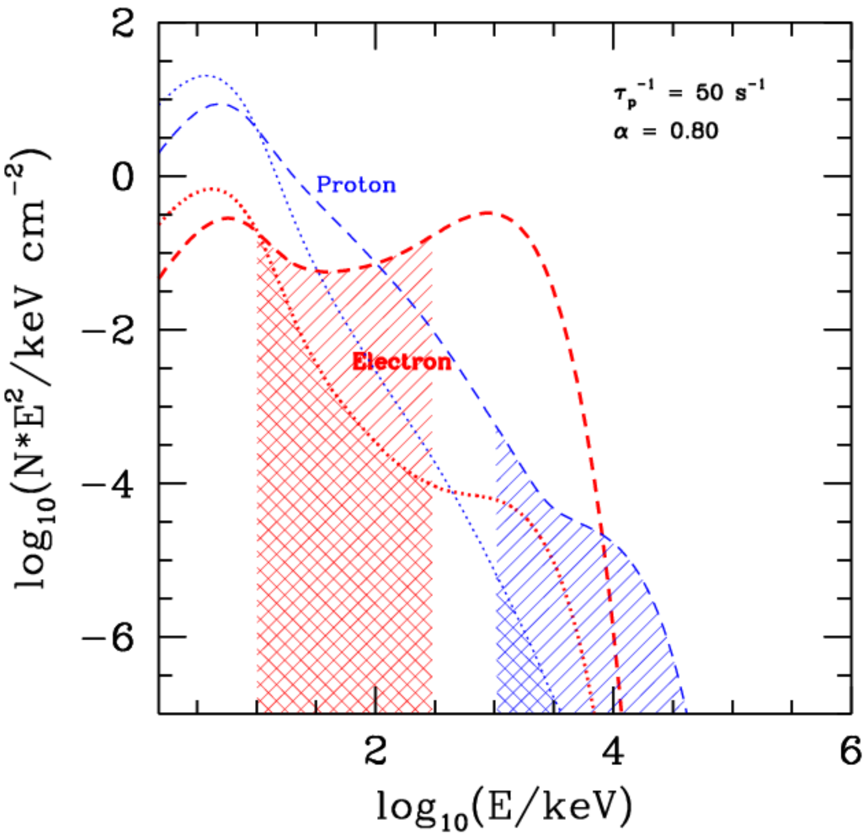
$$(\varepsilon_p/\varepsilon_e)_{\mathrm{SEPs}} \sim 3$$

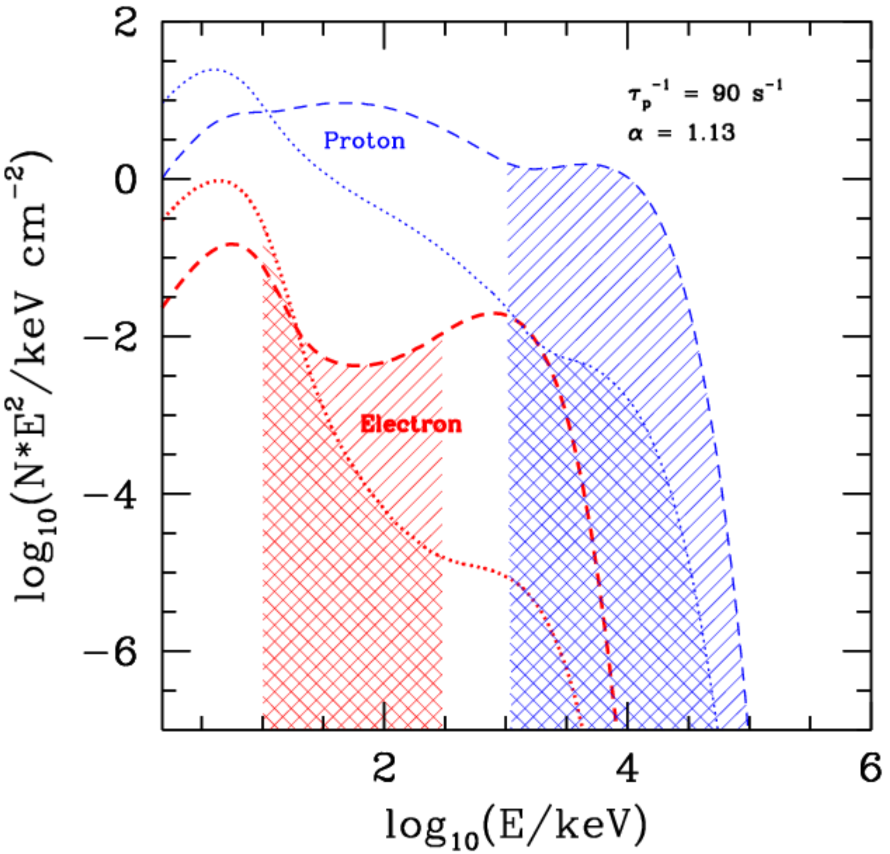


e vs p: Dependence on Magnetization

$$\alpha = (m_e/m_p)^{1/2}/\beta_A \propto n^{1/2}/B$$

$$\tau_p^{-1} = \left(\frac{\pi}{2}\right)\Omega_e\left(\frac{u_{\rm turb}}{B^2/8\pi}\right)(q-1)\left(\frac{ck_{\rm min}}{\Omega_e}\right)^{q-1}$$





Regularized Inversion of Photon Spectra to Electron Spectra

Start with the the detected photon count spectrum

$$J(\varepsilon') = \int I(\varepsilon) D(\varepsilon', \varepsilon) d\varepsilon = n \int_{\varepsilon}^{\infty} K(E, \varepsilon') F(E) dE; \text{ with } K(E, \varepsilon') = \int_{\varepsilon'}^{\infty} D(\varepsilon', \varepsilon) \sigma(E, \varepsilon) d\varepsilon$$

Invert this to get the electron flux by minimizing the matrix

$$||\mathbf{J} - \mathbf{K}.n\mathbf{F}||^2 + \lambda ||n\mathbf{F}||^2$$

Inversion of the total spectra *(Piana et al. 2003; Kontar et al. 2005)*

$$I_{\rm Tot} = I_{\rm LT} + I_{\rm FP} = \frac{nV}{4\pi R^2} \int_{\epsilon}^{\infty} v N_{\rm eff}(E) \sigma(\epsilon, E) {\rm d}E$$

Gives the *effective* electron flux or density spectrum $N_{\rm eff}(E)$

Regularized Inversion of Photons to Electrons

1. Inversion of total spectra *(Piana et al. 2003; Kontar et al. 2005)*

$$I_{\rm Tot} = I_{\rm LT} + I_{\rm FP} = \frac{nV}{4\pi R^2}\int_{\epsilon}^{\infty} v N_{\rm eff}(E)\sigma(\epsilon, E){\rm d}E$$

Gives the *total effective radiating* electron spectrum

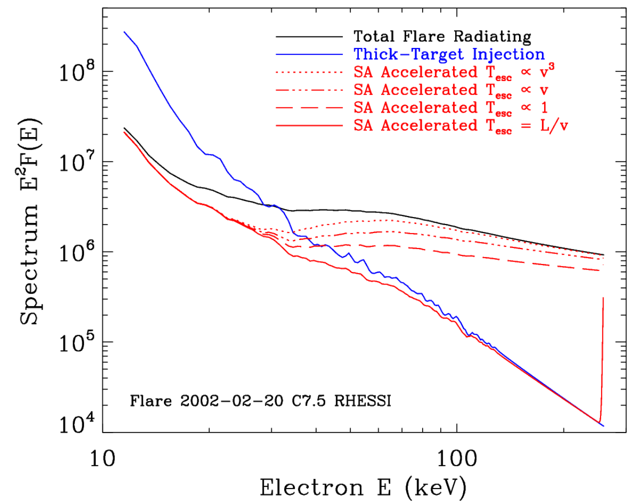
$$N_{\rm eff}(E) = N(E) + \frac{1}{\dot{E}_{\rm L}}\int_{E}^{\infty}\frac{N(E')}{T_{\rm esc}(E')}dE', \text{ or } \frac{{\rm d}(N/v)}{{\rm d}E} - \frac{N/v}{\dot{E}_{\rm L}T_{\rm esc}} = \frac{{\rm d}(N_{\rm eff}/v)}{{\rm d}E}$$

Solve for the *accelerated electron* spectrum

$$N(E) = N_{\rm eff}(E) - v(E)\int_{\eta(E)}^{\infty}\frac{N_{\rm eff}(E)}{v(E)}e^{\eta-\eta'}d\eta'$$

$$\text{with } d\eta = \frac{dE}{\dot{E}_{\rm L}T_{\rm esc}}$$

Requires a knowledge of escape time!



II. Transport Coefficients of Electrons and Ions

1 . Large Scale *B* field:

a. Adiabatic Invariance $\frac{\partial \ln B}{\partial s} \times \frac{\partial}{\partial \mu}[(1-\mu^2)f]$

b. Synchrotron Losses $\dot{E}_L = (4/9) r_0^2 c \beta^2 \gamma^2 B^2$ *Important only for electrons only*

2. Background Plasma Density *n*:

a. Inelastic Coulomb $\dot{E}_L = 4\pi r_0^2 nmc^3 \ln\Lambda/\beta$ *(e-e, e-p, p-p, p-e)*

b. Elastic Brem. And Nuclear Interactions (Lines and Pions)

3. Background Photons (Inverse Compton) *Important only for electrons only and at low B*

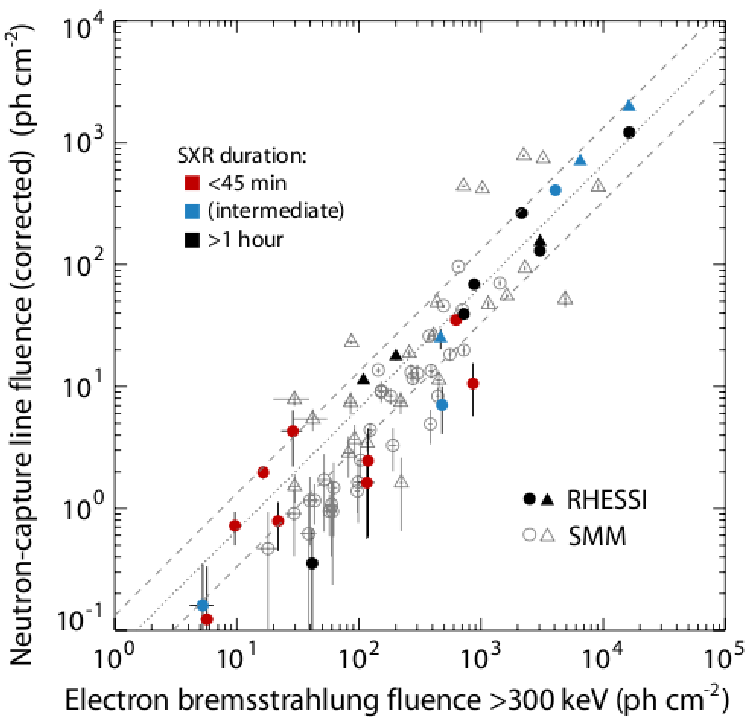
4. Plasma Waves or Turbulence: *Scattering and Acceleration Rates* $\propto \Omega(\varepsilon_{\rm turb}/B^2)$

Also Energy diffusion, Pitch angle changes and diffusion

Most Important Parameter $\alpha = \omega_p/\Omega \propto 1/\beta_A \propto \sqrt{n}/B$

I. Observed Signatures of Acceleration

Electron vs Proton Acceleration Rates: Flares



Shih et al.2009

$$(\varepsilon_p/\varepsilon_e)_{\text{Flares}} < 0.3$$