One dimensional model for confinement transition in magnetic confined plasmas

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Predator-prey model describing interaction between turbulence and zonal flow

drive

Reynolds stress

Feedback Loop

transport

Zonal flow

damping
Simple model – predator-prey model [Diamond ’94 PRL]

Self-consistence is essential.

Turbulence energy:
\[ \frac{\partial N}{\partial t} = \gamma_L N - \Delta \omega N^2 - \alpha U N \]

Zonal Flow energy:
\[ \frac{\partial U}{\partial t} = \alpha U N - \gamma_{damp} U \]

Constant variable
\[ H = \alpha (N + U) - \gamma_{damp} \ln N - \gamma_L \ln U \]

Fixed point: \( (0, 0), (N_L, 0), (N_H, U_H) \)
Why simple models?

- Simple does not mean “Easy”, but “Essential.”
- Of course, somehow INCORRECT
  - Correct in an adequate approximation or parameter regime.
  - Stability analysis gives us profound insights.
  - Time scale of phase transition
    - Time scale of LH transition?
- Comparison with experiments and/or simulation is essential.
Experimental motivation

- L-H threshold Power in low density region (typically lower than $3 \times 10^{19} \text{m}^{-3}$)
- I-phase as a transient phase between low and high confinement, i.e. L $\rightarrow$ I $\rightarrow$ H transition.
- Limit cycle oscillation in prior to the transition in TJ-II [Estrada ‘10 EPL], NSTX [Zweben ‘10 PoP], ASDEX Upgrade [Conway ‘11 PRL], EAST [Xu ‘11 PRL]
- Radial structure of mean flow shear in the I-phase limit-cycle oscillation
  - Dual shear layer in DIII-D [Schmitz, TTF ‘11]
- Poloidal rotation involving in the transition process in JT-60U [Kamiya ‘10 PRL]
Predator-prey system of Turbulence-Zonal flow-Mean flowPROFILE

Mean flow = ExB shear flow sustained by global equilibrium

[ E. Kim and Diamond '03 PRL ]
Predator-prey model -- with competition between ZF-mean flow (MF)

1 prey

Turbulence:

\[ \partial_t \mathcal{E} = \mathcal{E} \mathcal{N} - a_1 \mathcal{E}^2 - a_2 \mathcal{V}^2 \mathcal{E} - a_3 \mathcal{V}_{ZF}^2 \mathcal{E}, \]

2 predators

Zonal Flow (ZF):

\[ \partial_t \mathcal{V}_{ZF} = b_1 \frac{\mathcal{E} \mathcal{V}_{ZF}}{1 + b_2 \mathcal{V}^2} - b_3 \mathcal{V}_{ZF}, \]

\[ \partial_t \mathcal{N} = -c_1 \mathcal{E} \mathcal{N} - c_2 \mathcal{N} + Q. \]

Mean Flow equilibrium:

\[ \mathcal{V} = d \mathcal{N}^2 \]

- The intermediate limit-cycle oscillation describes a transition time-scale as zonal flow damping (~ion collision time).
Stability analysis of the 0D K.-D. model

Fixed points

- L-mode state
- Transient state
- H-mode state (unstable)
- QH-mode state (stable)

Hysteresis in power ramp up and down

- Mean flow
- Heat power

Stability analysis of the 0D K.-D. model formulated earlier in Refs. [Malkov and Diamond ‘09, PoP]

In this paper we have investigated a low order model of L-H transition...
The H-mode fixed point separates trajectories tending to the T-mode from those going to QH-mode.

**Bistability = initial value sensitivity!**
One-dimensional model for L-I-H transition
One-dimensional model for L-I-H transition

One dimensional feature gives new nonlinear dynamics, i.e. non-locality, which is characterized by non-diffusive turbulence transport, “turbulence spreading” [Hahm ‘02, PoP].

Turbulence intensity equation:

\[
\frac{\partial I}{\partial t} - \gamma_L I - \Delta \omega I^2 = \chi_N \frac{\partial}{\partial x} \left( I \frac{\partial I}{\partial x} \right)
\]

Local energy drive/dissipation

Non-local energy diffusion (nonlinear diffusion)

Fisher-KPP eq.

Ballistic Propagation velocity \( v \sim \sqrt{2\gamma D} \)

[ Gurcan ‘05, PoP]
We have developed a **1D model for I-phase**.

- Self-consistent 1D transport model for L-H transition
- Time-evolution of limit-cycle behavior triggered by zonal flow with fast time scale.
- Expansion of 0D Kim-Diamond model to 1D radial space.

**Remarks**

- Zonal flow / Mean flow competition, *a’la* Kim-Diamond’s
- Poloidal momentum spin-up at the edge transport barrier
1D transport model

\[ \partial_t p(x) + \partial_x \Gamma_p = \partial_x H \]
\[ \partial_t n(x) + \partial_x \Gamma_n = \partial_x S \]

\[ \Gamma_p = - (\chi_{\text{neo}} + \chi_o) \partial_x p \]
\[ \Gamma_n = -(D_{\text{neo}} + D_o) \partial_x n - Vn \]

**Pinch term**

TEP pinch

Thermoelectric pinch

\[ V = (v_{0,TEP} + v_{0,TE}) \]

Inward pinch

\[ V = \left( \frac{D}{R} - \frac{D}{L_T} \right) \quad (\propto I, \quad L_T < 0) \]

\[ n \sim \exp\left( -\frac{V}{D}r \right) \]

→ density peaking

**Neoclassical transport term**

Banana regime

\[ \chi_{\text{neo}} \sim \chi_{Ti} \sim \varepsilon_T^{-3/2} q^2 \rho_i^2 \nu_{ii} \]

\[ D_{\text{neo}} \sim (m_e / m_i)^{1/2} \chi_{Ti} \]

**Turbulent transport term**

\[ D_0 \sim \chi_0 \sim \tau_c c_s^2 I \]

→ Predator-prey model
Predator-prey model part
-- a la' Kim-Diamond’s

\[ \partial_t I = (\gamma_L - \Delta \omega I - \alpha_0 E_o - \alpha_V E_V) I + \chi_N \partial_x (I \partial_x I) \]

Driving term
Local dissipation
ZF shearing
MF shearing

\[ \partial_t E_0 = AE_0 \alpha_0 \left( I / (1 + \xi_0 E_V) - I_* \right) \]

Screening factor
Reynolds stress drive

Mean flow shearing:

\[ E_V = (\partial_x V_{ExB})^2 \]

ZF collisional damping
MF/ZF competition

\[ \chi_N \sim \chi_{N0} \chi_{GB} \sim \chi_{N0} \frac{\rho_i^2 c_s}{a} \]

Short time scale normalization \( \omega_* \sim c_s / a \) \( t \rightarrow t \)
Small spatial scale \( \rho_i \sim 0.01 a \)

Long time scale \( \tau_{ii} \sim 600 (a / c_s) \)
Long spatial scale normalization \( r/a \rightarrow r \)
Radial force balance equation:

\[
V'_{E\times B} = \frac{1}{eB} \left[ -\frac{1}{n^2} n' p' + \frac{1}{n} p'' \right] + \left( \frac{r}{qR} u_\parallel \right)' - u'_\theta
\]

\[
= \rho_i c_s L_p^{-1} (-L_n^{-1} + L_{dp}^{-1}) - u'_\theta
\]
Poloidal momentum spin-up

- Coupling radial and parallel momentum force balance equations, we obtain

\[-\frac{\partial u_\theta}{\partial t} = \frac{1}{nm} \left\langle \nabla \cdot (\vec{e}_y \tilde{\Pi}_{\text{turb}}) \right\rangle + \mu_{ii}^{(\text{neo})} (u_\theta - u_\theta^{(\text{neo})})\]

\[\sim \alpha_5 \frac{\gamma_L}{\omega_*} c_s^2 \partial_x I + \nu_{ii} q^2 R^2 \mu_{00} (u_\theta + 1.17 c_s \frac{\rho_i}{L_T})\]

Turbulence drive obtained from stress tensor [McDevitt, PoP ‘10]

Neoclassical effects

Eq. of poloidal rotation

Totally, time-evolving 5-fields \((n, p, I, E_0, \text{and } u_\theta)\) are solved numerically.
In power ramp up, above a threshold transition occurs, following ramp-up of core quantity of profiles.
I-phase is identified as an intermediate mode between L-H transition, with spatial structure.

ZF damping time dominant

$V_d \sim 10^{-4} \, c_s \sim 50\, \text{m/s}$

Inward propagation of ZF, turb, and MF from the edge, due to turbulence spreading.
As the power ramp up further, MF dominant region expands at the edge region.
Power ramp up/down, back transition, and hysteresis are seen.

Pressure at $r=0$, representing averaged gradient

Heat power

Power ramp up  Power ramp down
summary on this study in progress

- One dimensional extension of the Kim-Diamond model is introduced, including
  - Pressure/Density profile
  - 0D K-D model components (turbulence, ZF, MF)
  - Radial force balance, i.e. mean flow equilibrium
  - Poloidal rotation spin-up (neoclassical and turbulence drive)

- L-I-H-transitions with power ramp up are shown. Limit-cycle oscillation with ZF damping time scale is seen in the case of slow power ramp up.
  - Limit-cycle propagates outward and inward.
  - Hysteresis obtained in the power ramp up and down simulation.
Heat and Particle flux source

\[ S = \gamma_a \exp\left[-\frac{(a-r-d)^2}{2L_{dep}^2}\right] \]

\[ H = 2q_a \left(\frac{r}{a}\right) \left(1 - \left(\frac{r}{a}\right)^2\right) \]

Edge Boundary Condition: fixed for \( p, n \), free for \( I, E_0 \) and \( u_{\theta} \)