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One dimensional model for confinement transition in magnetic confined plasmas

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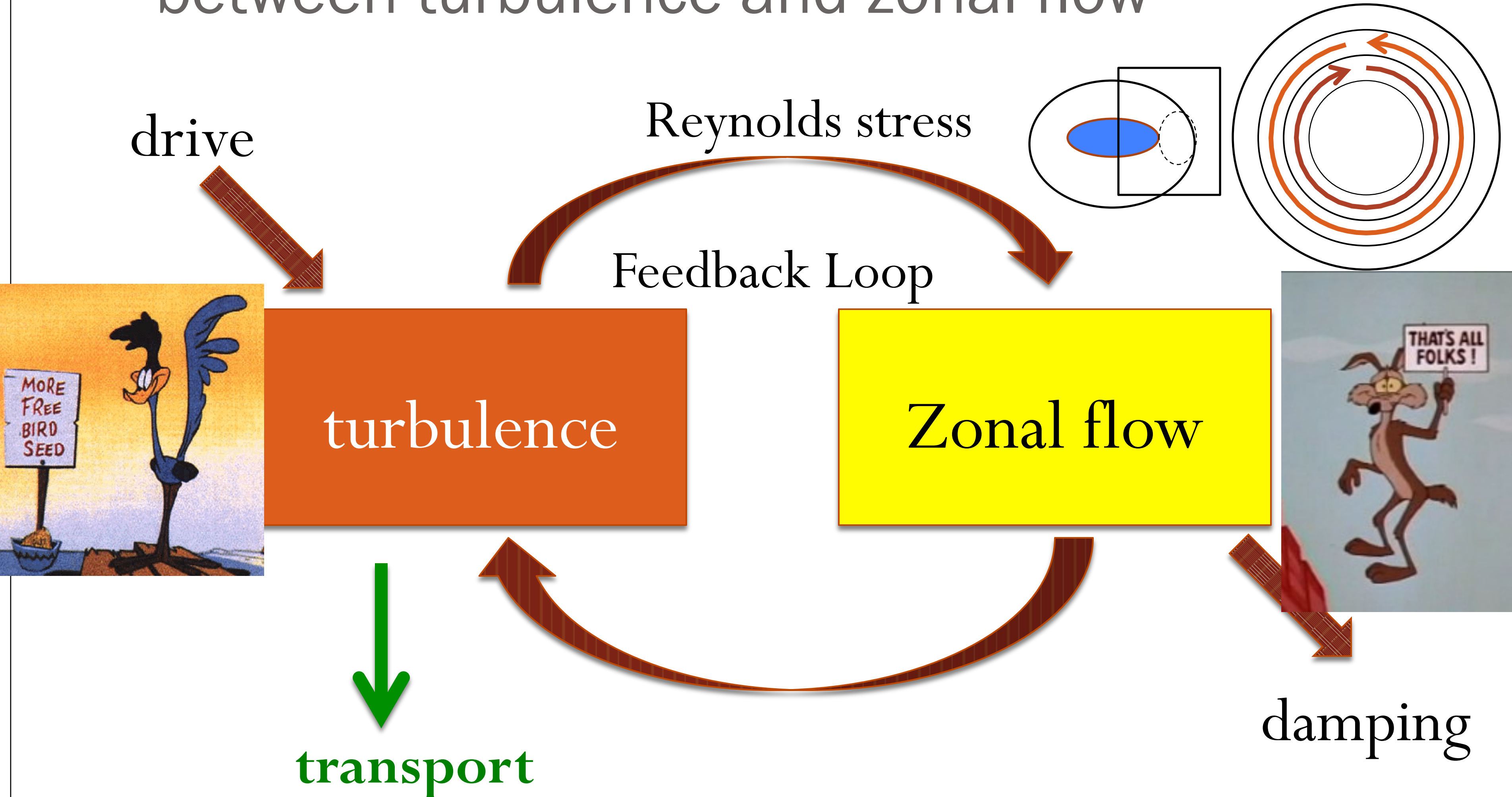
UCSD

Predator-prey model describing interaction between turbulence and zonal flow



Simple model – predator-prey model [Diamond '94 PRL]

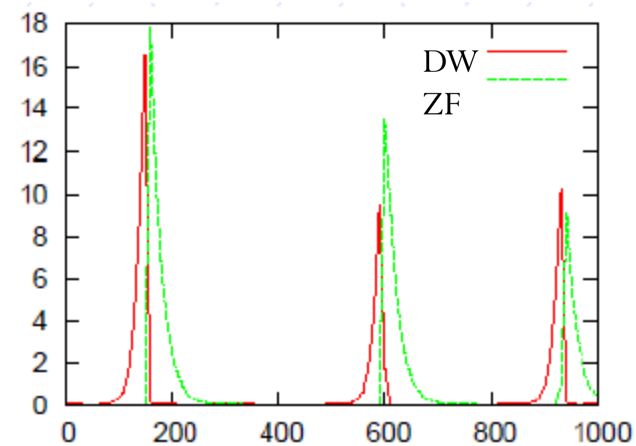
Self-consistence is essential.

Driving term — Nonlinear damping — Shearing

Turbulence energy: $$\frac{\partial N}{\partial t} = \gamma_L N - \Delta\omega N^2 - \alpha U N$$

Shearing — Damping of ZF (collisions)

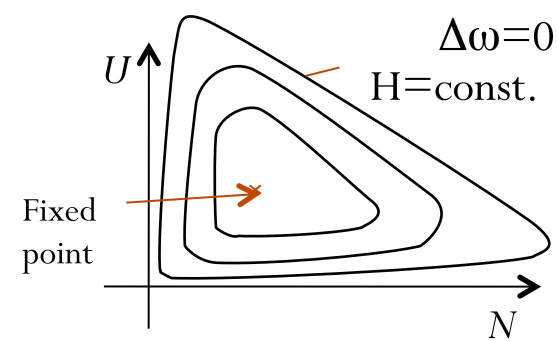
Zonal Flow energy: $$\frac{\partial U}{\partial t} = \alpha U N - \gamma_{damp} U$$



Constant variable

$$H = \alpha(N+U) - \gamma_{damp} \ln N - \gamma_L \ln U$$

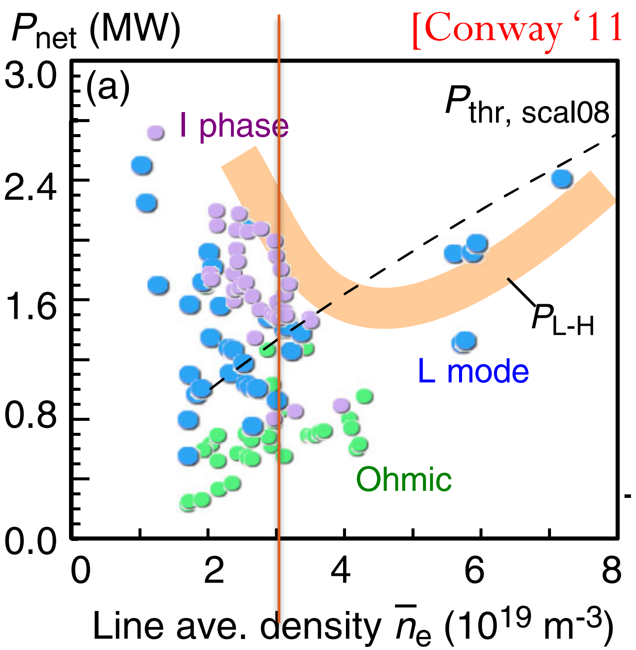
Fixed point: $(0,0), (N_L, 0), (N_H, U_H)$



Why simple models?

- Simple does not mean "Easy", but "**Essential**."
- Of course, somehow INCORRECT
  - Correct in an adequate approximation or param eter regime.
  - Stability analysis gives us profound insights.
  - Time scale of phase transition
    - Time scale of LH transition?
- Comparison with experiments and/or simulation is essential.

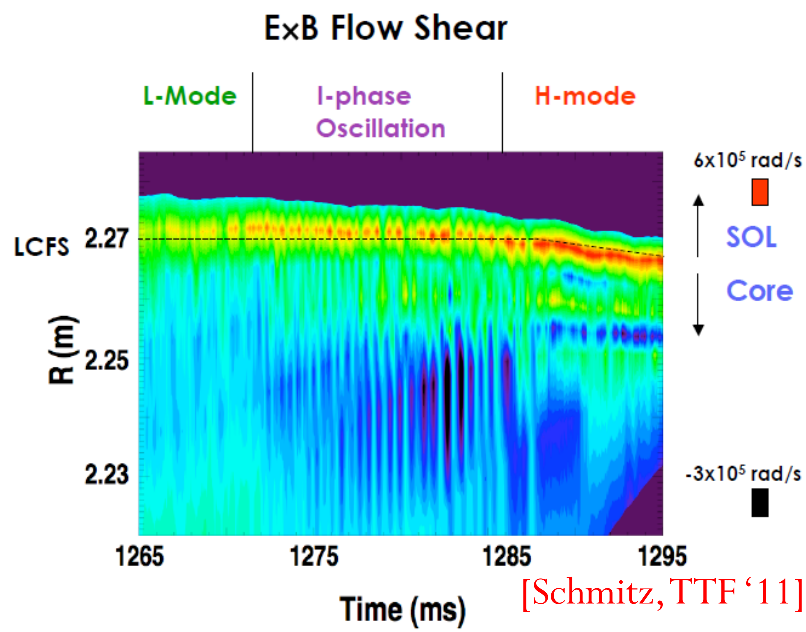
Experimental motivation



[Conway '11 PRL]

Narrow outer layer

Widening inner layer



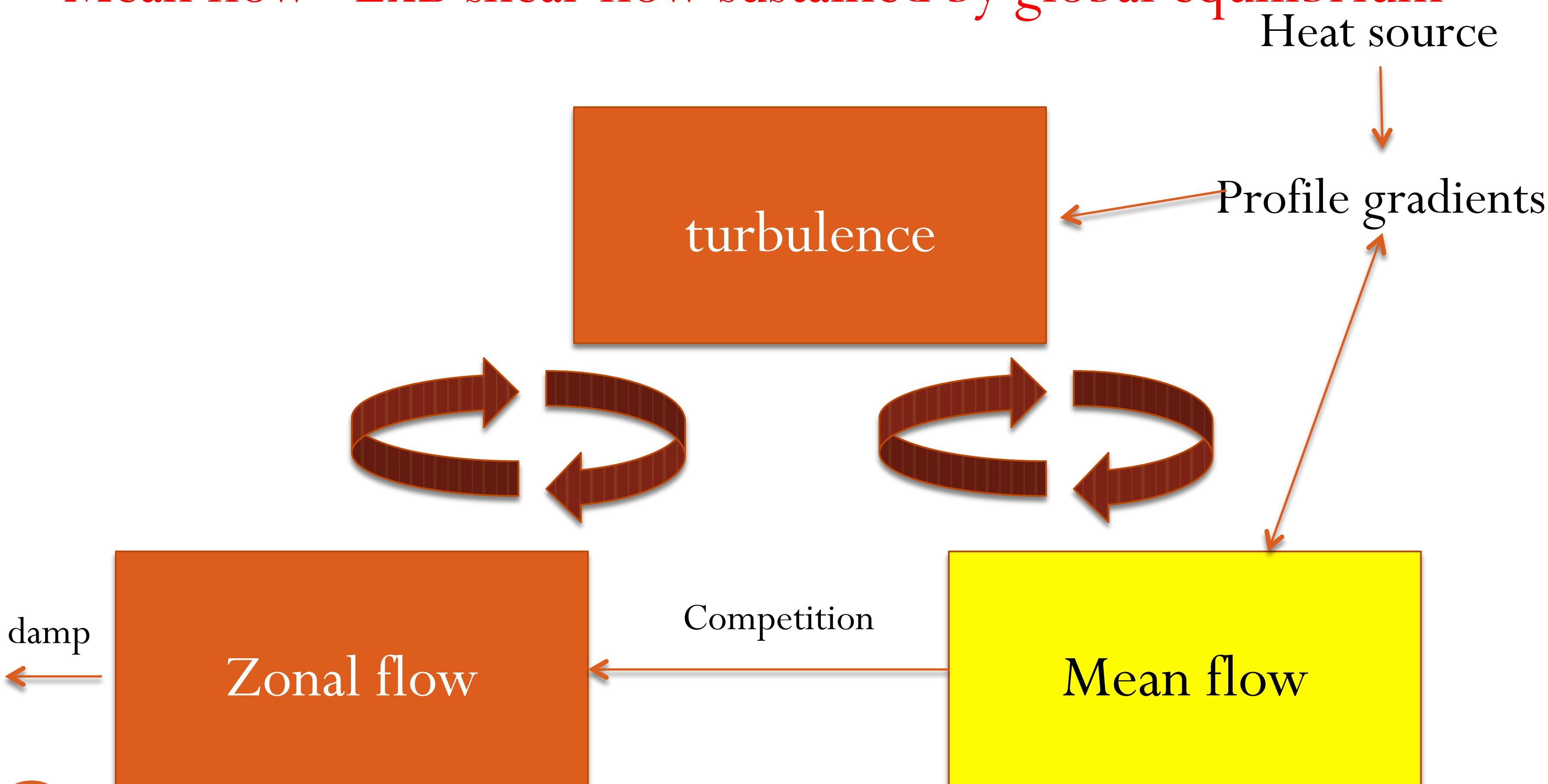
[Schmitz, TTF '11]

- L-H threshold Power in low density region (typically lower than $3\times10^{19}m^{-3}$)
- I-phase as a transient phase between low and high confinement, i.e. L➔I➔H transition.
- Limit cycle oscillation in prior to the transition in TJ-II[Estrada '10 EPL], NSTX[Zweben '10 PoP], ASDEX Upgrade[Conway '11 PRL], EAST[Xu '11 PRL]
- Radial structure of mean flow shear in the I-phase limit-cycle oscillation
  - Dual shear layer in DIII-D [Schmitz, TTF '11]
- Poloidal rotation involving in the transition process in JT-60U [Kamiya '10 PRL]

Predator-prey system of Turbulence-Zonal flow-Mean flow/profile gradient

[E. Kim and Diamond '03 PRL]

Mean flow=ExB shear flow sustained by global equilibrium



Predator-prey model
-- with competition between ZF-mean flow(MF)

1 prey

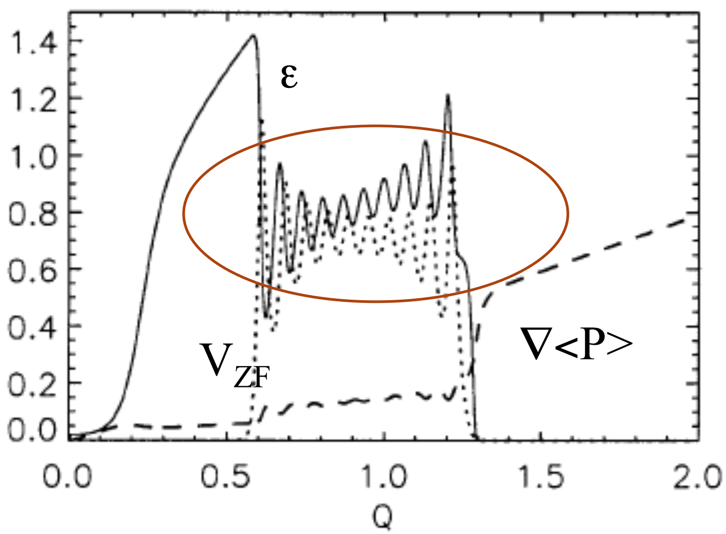
Turbulence: $$\partial_t \mathcal{E} = \mathcal{E}\mathcal{N} - a_1\mathcal{E}^2 - a_2 V^2 \mathcal{E} - a_3 V_{ZF}^2 \mathcal{E},$$

2 predators

Zonal Flow(ZF): $$\partial_t V_{ZF} = b_1 \frac{\mathcal{E} V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF},$$

pressure gradient ∇<P> $$\partial_t \mathcal{N} = -c_1 \mathcal{E}\mathcal{N} - c_2 \mathcal{N} + Q.$$

Mean Flow equilibrium: $$V = dN^2$$

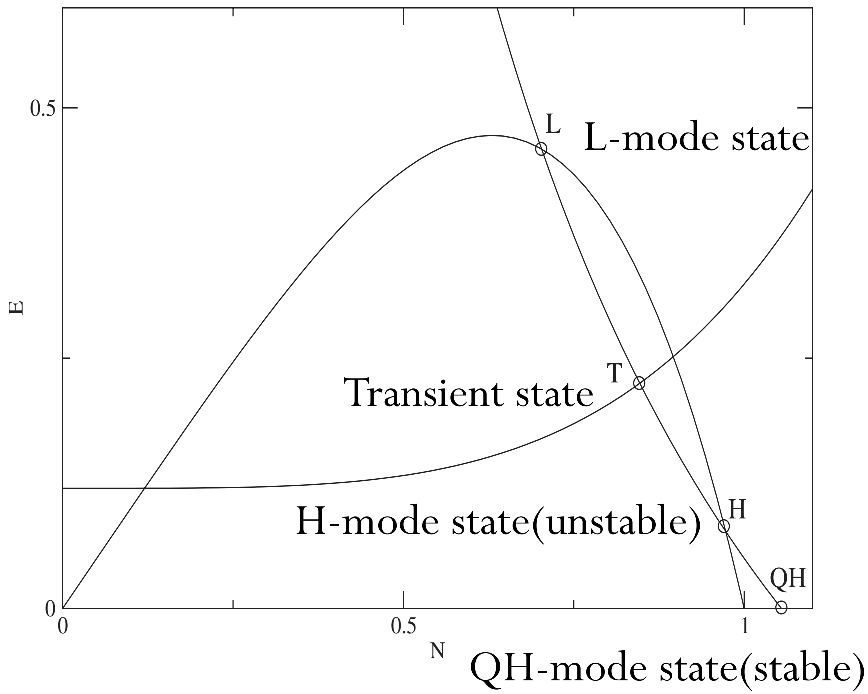


- The intermediate limit-cycle oscillation describes a transition time-scale as zonal flow damping (~ion collision time).

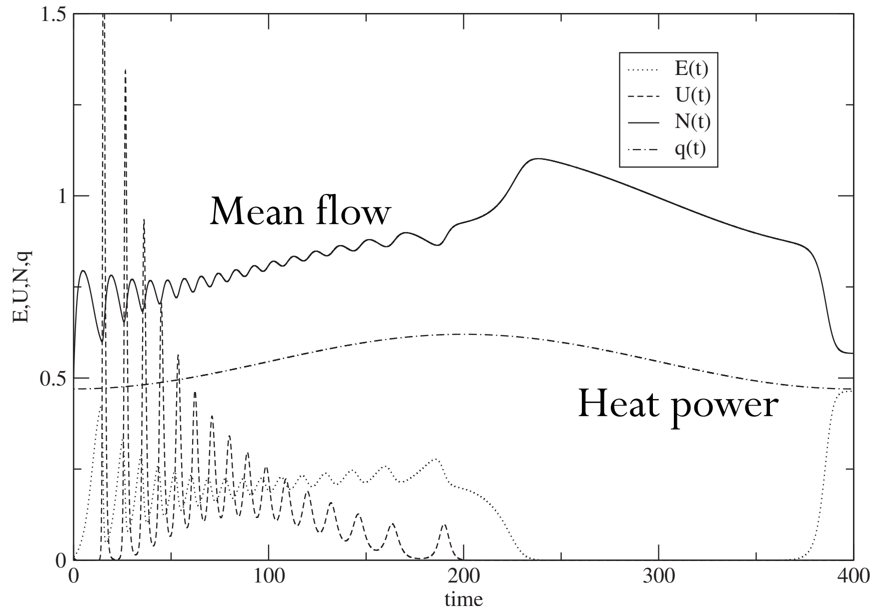
Stability analysis of the 0D K.-D. model

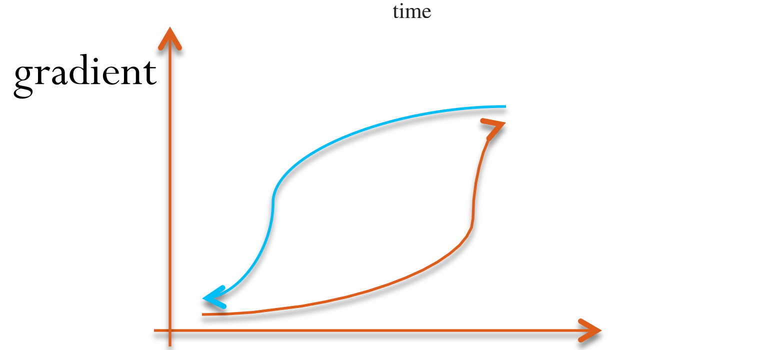
[Malkov and Diamond '09, PoP]

Fixed points



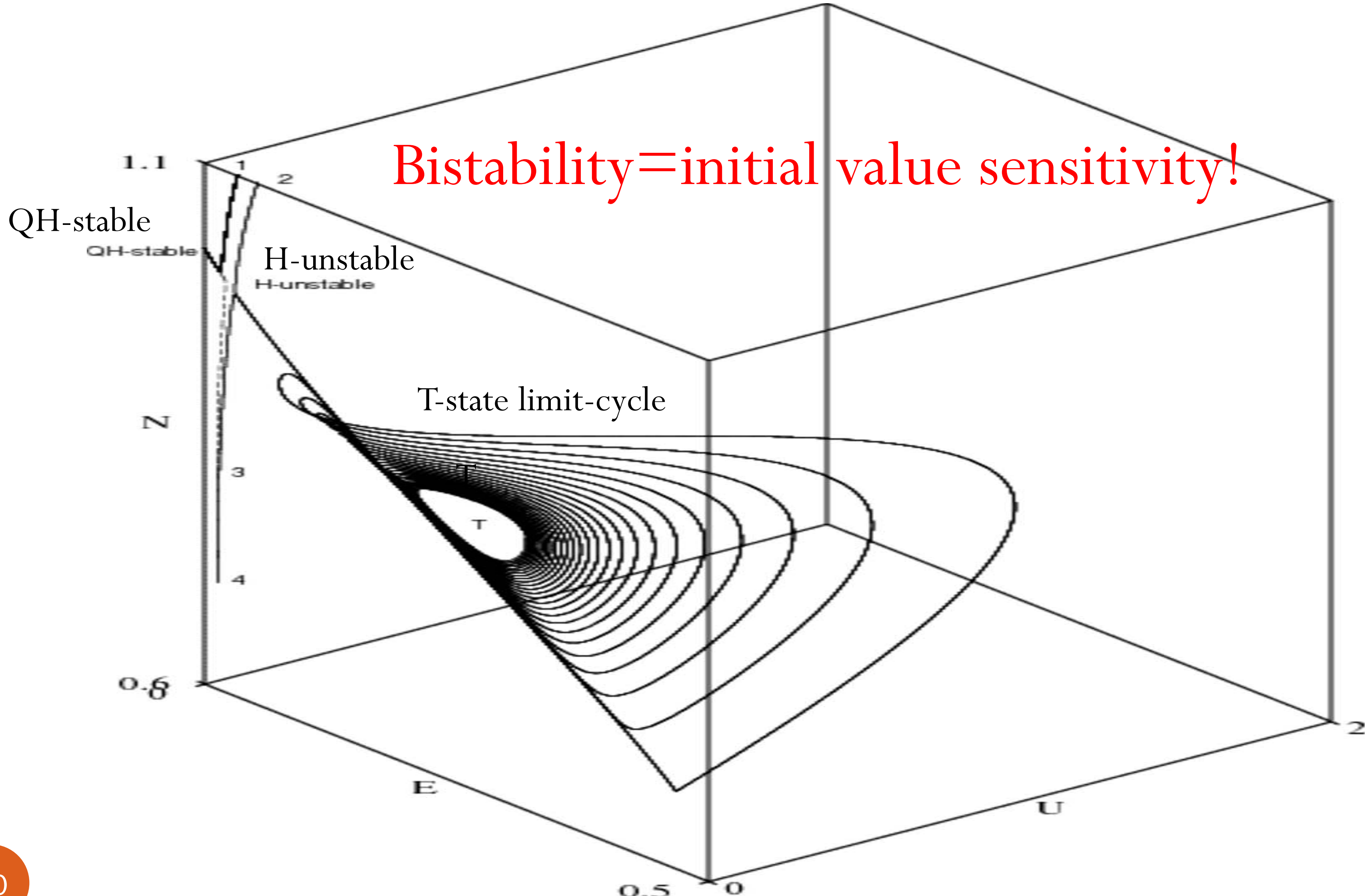
Hysteresis in power ramp up and down





→ Bistability

The H-mode fixed point separates trajectories tending to the T-mode from those going to QH-mode



One-dimensional model for L-I-H transition

One-dimensional model for L-I-H transition

One dimensional feature gives new nonlinear dynamics, i.e. non-locality, which is characterized by non-diffusive turbulence transport, "turbulence spreading" **[Hahm '02, PoP]**.

Turbulence intensity equation:

$$\frac{\partial I}{\partial t} - \gamma_L I - \Delta\omega I^2 = \chi_N \frac{\partial}{\partial x}\left(I \frac{\partial I}{\partial x} \right)$$

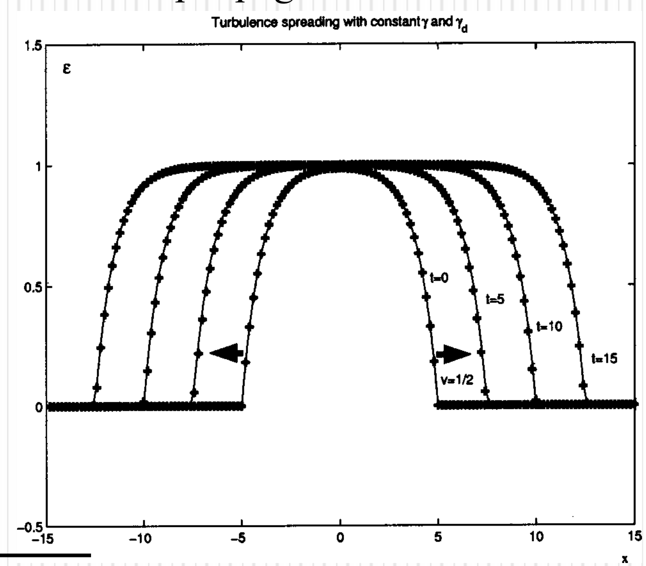
Local energy drive/dissipation

Non-local energy diffusion (nonlinear diffusion)

→ Fisher-KPP eq.

Ballistic Propagation velocity $v \sim \sqrt{2\gamma D}$

Front propagation



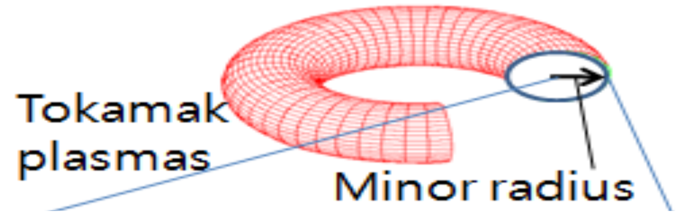
[Gurcan '05, PoP]

We have developed a 1D model for I-phase.

- Self-consistent 1D transport model for L-H transition
  - Time-evolution of limit-cycle behavior triggered by zonal flow with fast time scale.
  - Expansion of 0D Kim-Diamond model to 1D radial space.
- Remarks
  - Zonal flow / Mean flow competition , *a' la* Kim-Diamond's
  - Poloidal momentum spin-up at the edge transport barrier

1D transport model

x: radial direction



pressure $$\partial_t p(x) + \partial_x \Gamma_p = \partial_x H$$

density $$\partial_t n(x) + \partial_x \Gamma_n = \partial_x S$$

$$\Gamma_p = -(\chi_{neo} + \chi_o)\partial_x p$$

$$\Gamma_n = -(D_{neo} + D_o)\partial_x n - Vn$$

Neoclassical transport term

Banana regime

$$\chi_{neo} \sim \chi_{Ti} \sim \varepsilon_T^{-3/2} q^2 \rho_i^2 \nu_{ii}$$

$$D_{neo} \sim (m_e / m_i)^{1/2} \chi_{Ti}$$

Turbulent transport term

$$D_0 \sim \chi_0 \sim \tau_c c_s^2 I$$

→***Predator-prey model***

Pinch term

TEP pinch

Thermoelectric pinch

$$V = (v_{0,TEP} + v_{0,TE})$$ Inward pinch

$$\cong \left(\frac{D}{R} - \frac{D}{L_T} \right) \qquad (\propto I,\ L_T < 0)$$

$$n \sim \exp(-\frac{V}{D} r)$$

→density peaking

Predator-prey model part -- a la' Kim-Diamond's

Turbulence intensity: $\gamma_L \sim \gamma_{L0} \frac{c_s}{R} \sqrt{\frac{R}{L_T} - \left(\frac{R}{L_T}\right)_{crit}}$ ← gained from p,n profile

$$\partial_t I = (\gamma_L - \Delta\omega I - \alpha_0 E_o - \alpha_V E_V) I + \chi_N \partial_x (I \partial_x I)$$

Driving term; Local dissipation; ZF shearing; MF shearing; Turbulence spreading

Zonal flow energy: $E_0 = V_{ZF}^2$

$\alpha_0 \sim \alpha_V \sim \tau_{ac0} \frac{\sqrt{a\rho_i}}{c_s}$ $(\tau_{ac} << 1)$

$$\partial_t E_0 = A E_0 \alpha_0 (I / (1 + \zeta_0 E_V) - I_*)$$

$\chi_N \sim \chi_{N0} \chi_{GB} \sim \chi_{N0} \frac{\rho_i^2 c_s}{a}$

Screening factor; Reynolds stress drive; MF/ZF competition; ZF collisional damping

$I_* = \gamma_{damp} / \alpha_0$

$\gamma_{damp} \sim \nu_{ii} / R$

Mean flow shearing:

$$E_V = (\partial_x V_{E\times B})^2$$

→ gained by radial force balance

Short time scale normalization $\omega_* (\sim c_s / a) t \rightarrow t$

Long time scale τ_{ii} $(=1/\nu_{ii}) \sim 600 (a/c_s)$

Small spatial scale $\rho_i \sim 0.01a$

Long spatial scale normalization $r/a \rightarrow r$

Radial force balance equation:

Gained from global profiles

$$V'_{E\times B} = \frac{1}{eB}\left[-\frac{1}{n^2}n'p' + \frac{1}{n}p''\right] + \left(\left[\frac{r}{qR}u_{\parallel}\right]' - u'_{\theta}\right)$$

Density gradient

Pressure curvature

Diamagnetic drift term

Parallel flow (not considered here)

Poloidal flow driven by neoclassical and turbulent drives

$$= \rho_i c_s L_p^{-1}(-L_n^{-1} + L_{\frac{dp}{dx}}^{-1}) - u'_{\theta}$$

Poloidal momentum spin-up

- Coupling radial and parallel momentum force balance equations, we obtain

Turbulence drive obtained from stress tensor **[McDevitt, PoP '10]**

Neoclassical effects

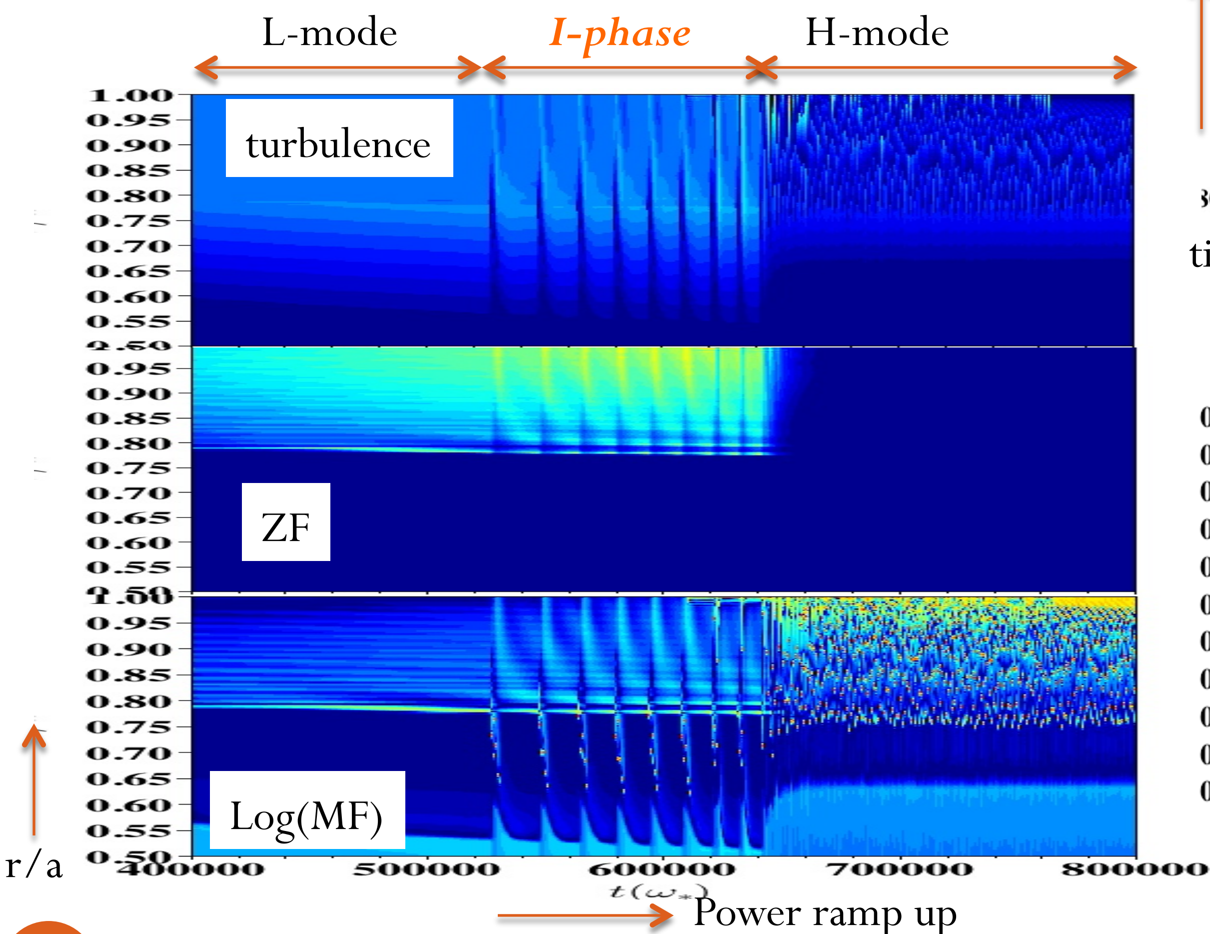
Eq. of poloidal rotation

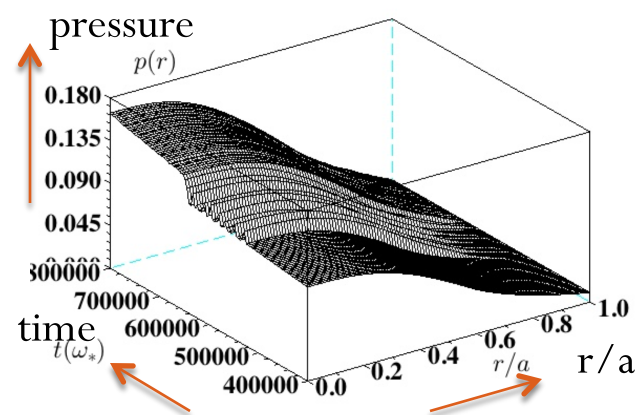
$$-\frac{\partial u_\theta}{\partial t} = \frac{1}{nm}\left\langle \nabla \cdot (\hat{e}_y \overleftrightarrow{\Pi}_{turb}) \right\rangle + \mu_{ii}^{(neo)}(u_\theta - u_\theta^{(neo)})$$

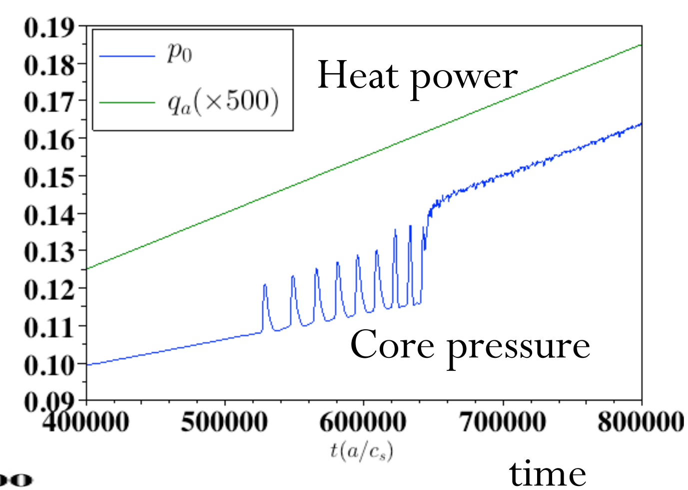
$$\sim \alpha_5 \frac{\gamma_L}{\omega_*} c_s^2 \partial_x I + \nu_{ii} q^2 R^2 \mu_{00}(u_\theta + 1.17 c_s \frac{\rho_i}{L_T})$$

Totally, time-evolving 5-fields (n, p, I, E_0, and u_θ) are solved numerically.

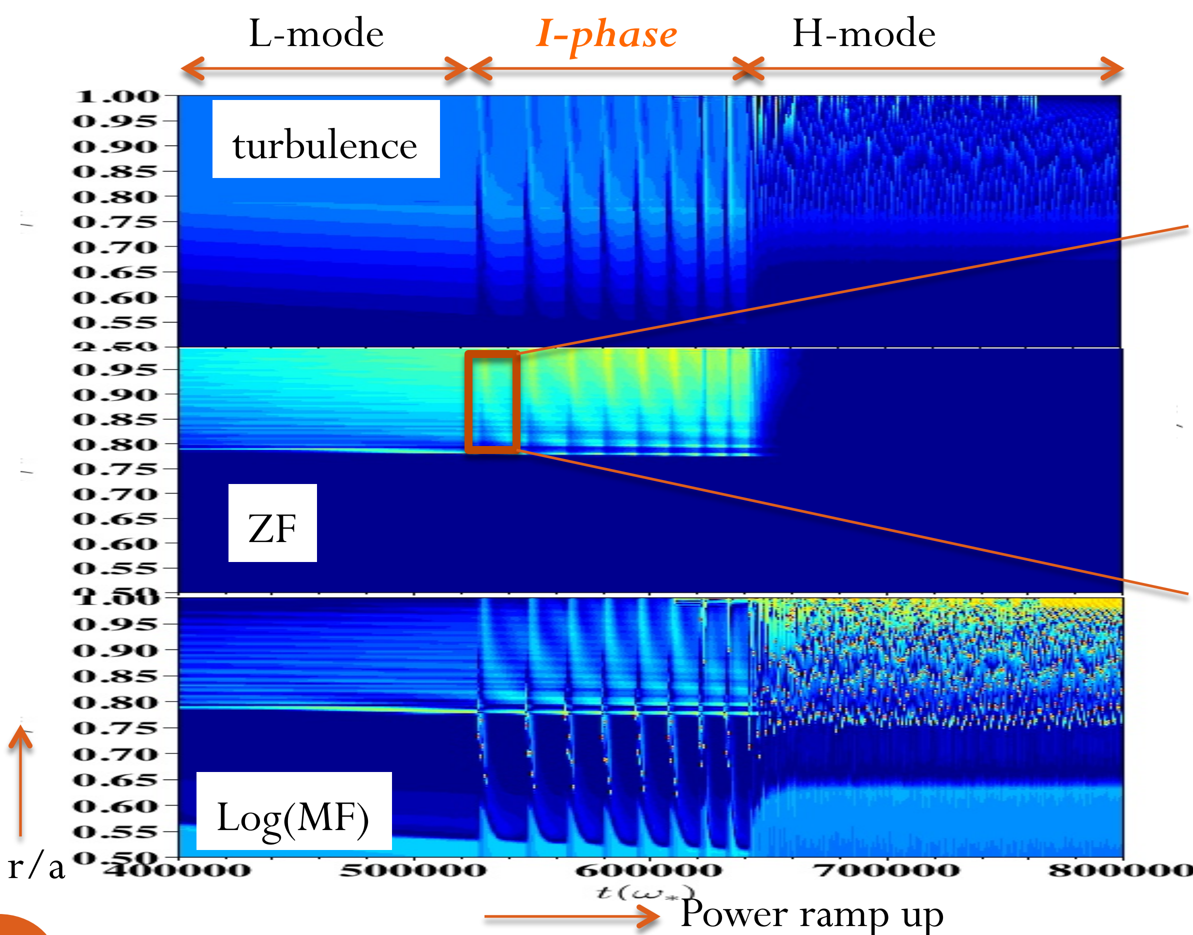
In power ramp up, above a threshold transition occurs, following ramp-up of core quantity of profiles



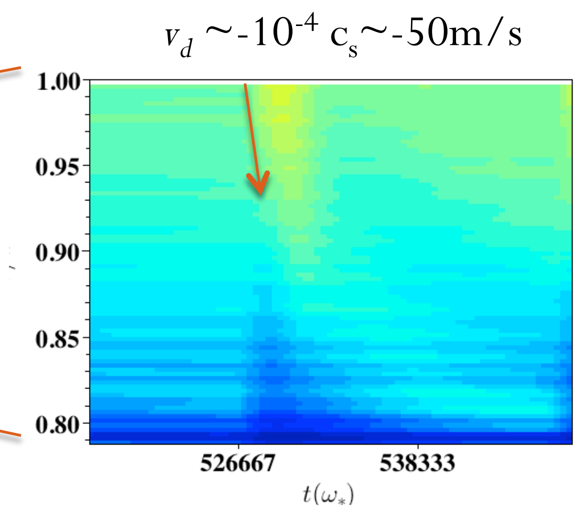




I-phase is identified as an intermediate mode between L-H transition, with spatial structure.

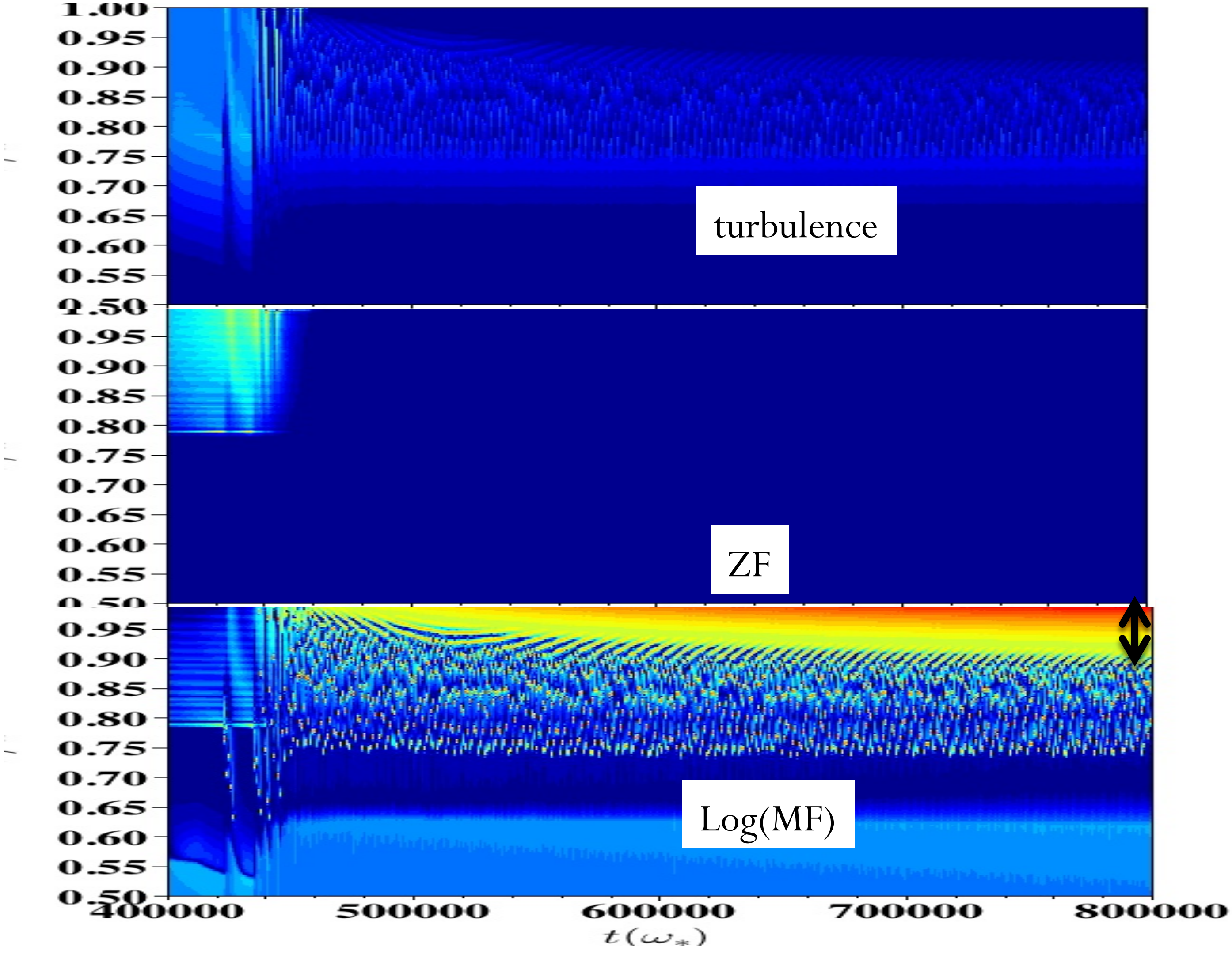


ZF damping time dominant



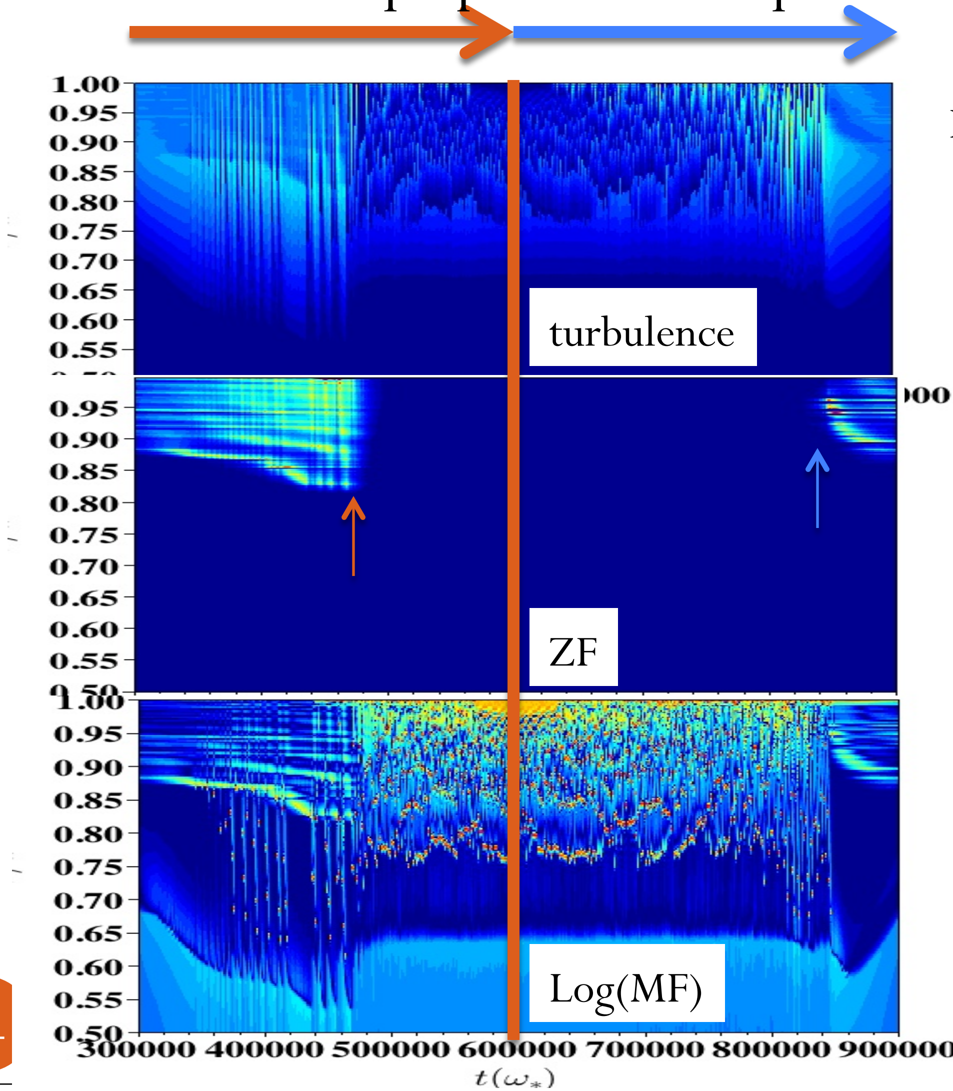
Inward propagation of ZF, turb, and MF from the edge, due to turbulence spreading.

As the power ramp up further, MF dominant region expands at the edge region.

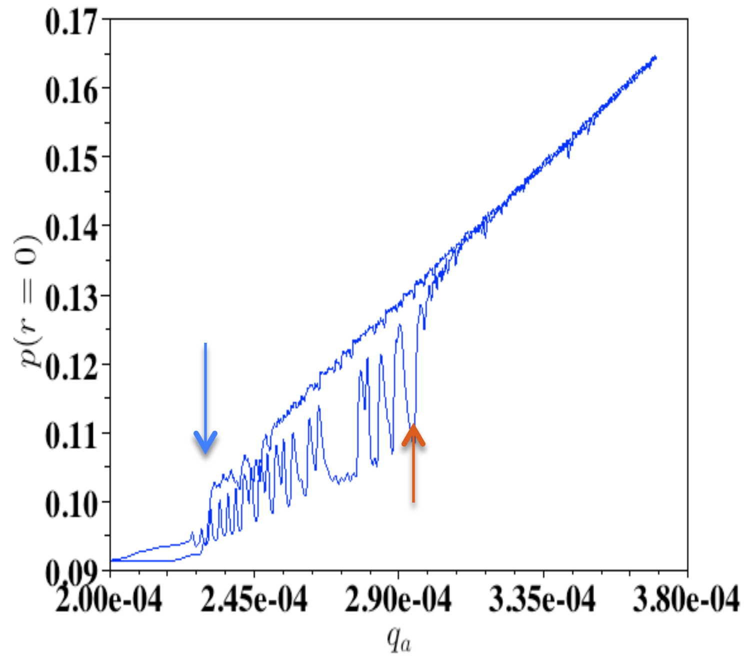


Power ramp up/down, back transition, and hysteresis are seen.

Power ramp up Power ramp down



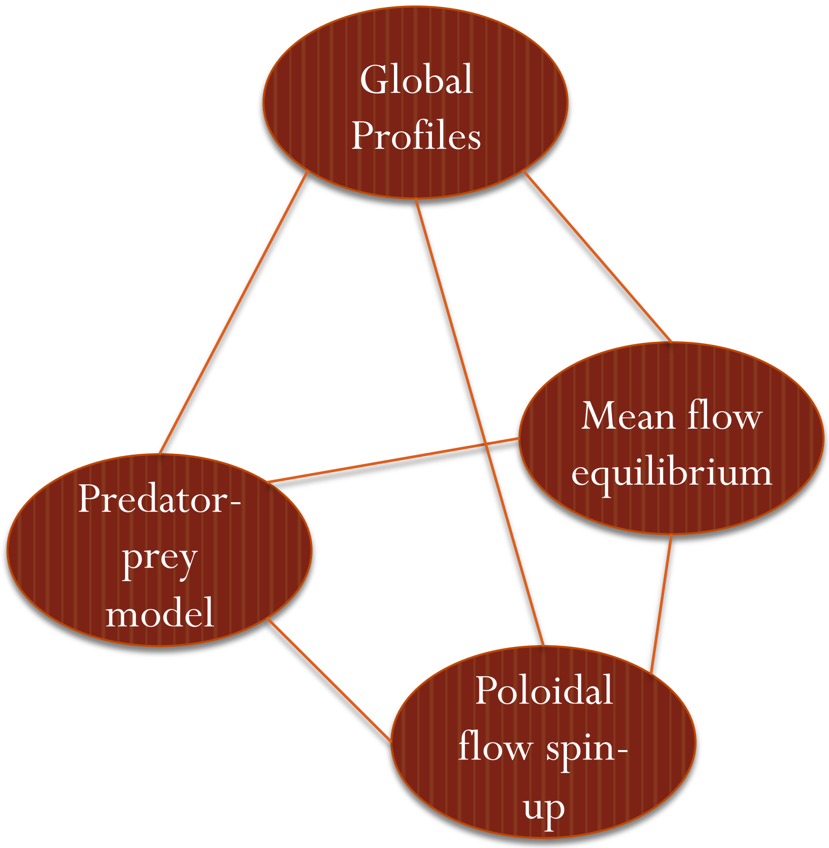
Pressure at r=0, representing averaged gradient



Heat power

summary on this study in progress

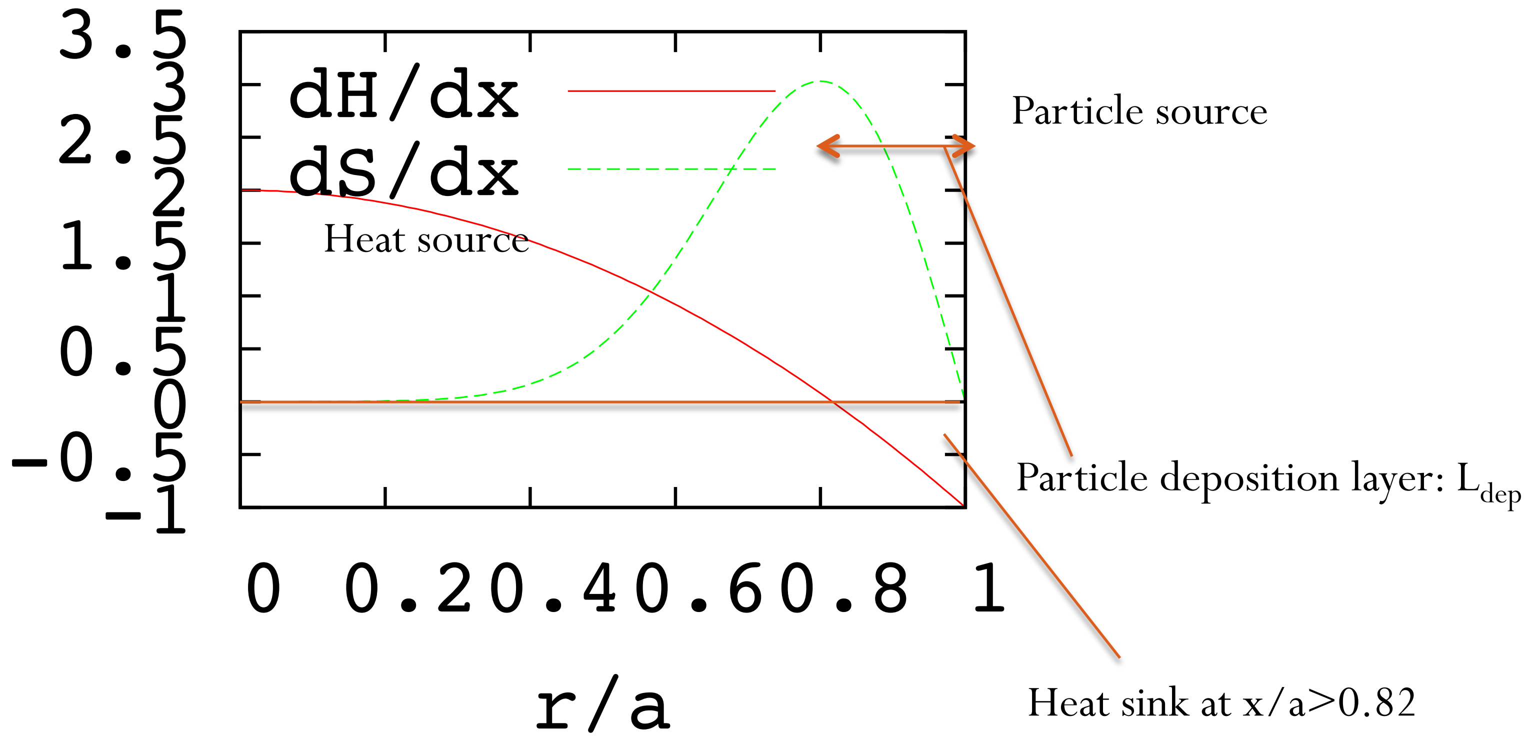
- One dimensional extension of the Kim-Diamond model is introduced, including
  - Pressure/Density profile
  - 0D K-D model components (turbulence, ZF, MF)
  - Radial force balance, i.e. mean flow equilibrium
  - Poloidal rotation spin-up (neoclassical and turbulence drive)
- L-I-H-transitions with power ramp up are shown. Limit-cycle oscillation with ZF damping time scale is seen in the case of slow power ramp up.
  - Limit-cycle propagates outward and inward.
  - Hysteresis obtained in the power ramp up and down simulation.



Heat and Particle flux source

$$S = \gamma_a \exp\left[-\frac{(a-r-d)^2}{2L_{dep}^2} \right]$$

$$H = 2q_a \left(\frac{r}{a}\right)\left(1-\left(\frac{r}{a}\right)^2\right)$$



Edge Boundary Condition: fixed for p, n, free for I, E_0 and u_θ