Hydrodynamical instabilities of a supernova blast wave at the Sedov stage

A. Zankovich, I. Kovalenko, V. Korolev
Volgograd State University

Fundamental Processes of Astrophysical Turbulence
Nov 16-19, 2011, APCTP, Pohang, Korea

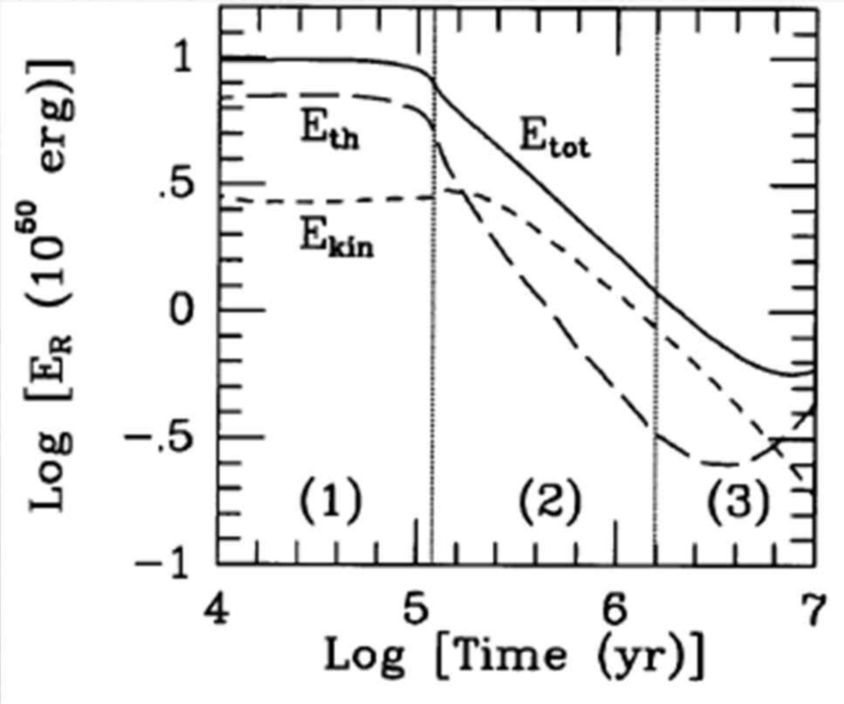
Purpose of research

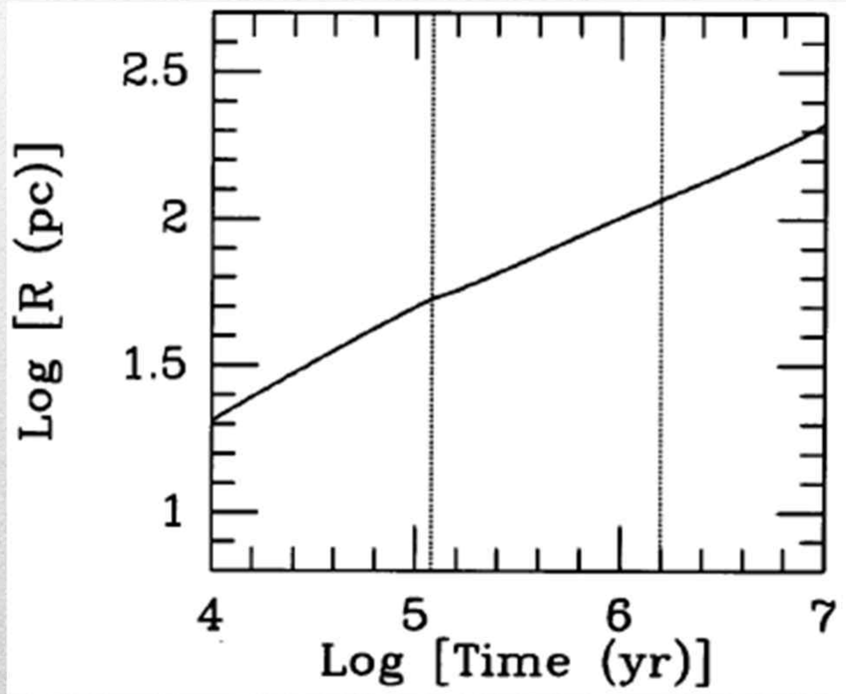
New mechanisms of hydrodynamic instabilities, which can contribute to the turbulence development in interstellar medium.

Layout

- history of problem
- Stability of adiabatic shock wave generated by a strong point explosion against internal perturbations (self oscillations) .
- Stability of a shock against external (forced) perturbations
  - linear analysis
  - 2-d numerical modeling.

The evolution of a supernova remnant. Key stages

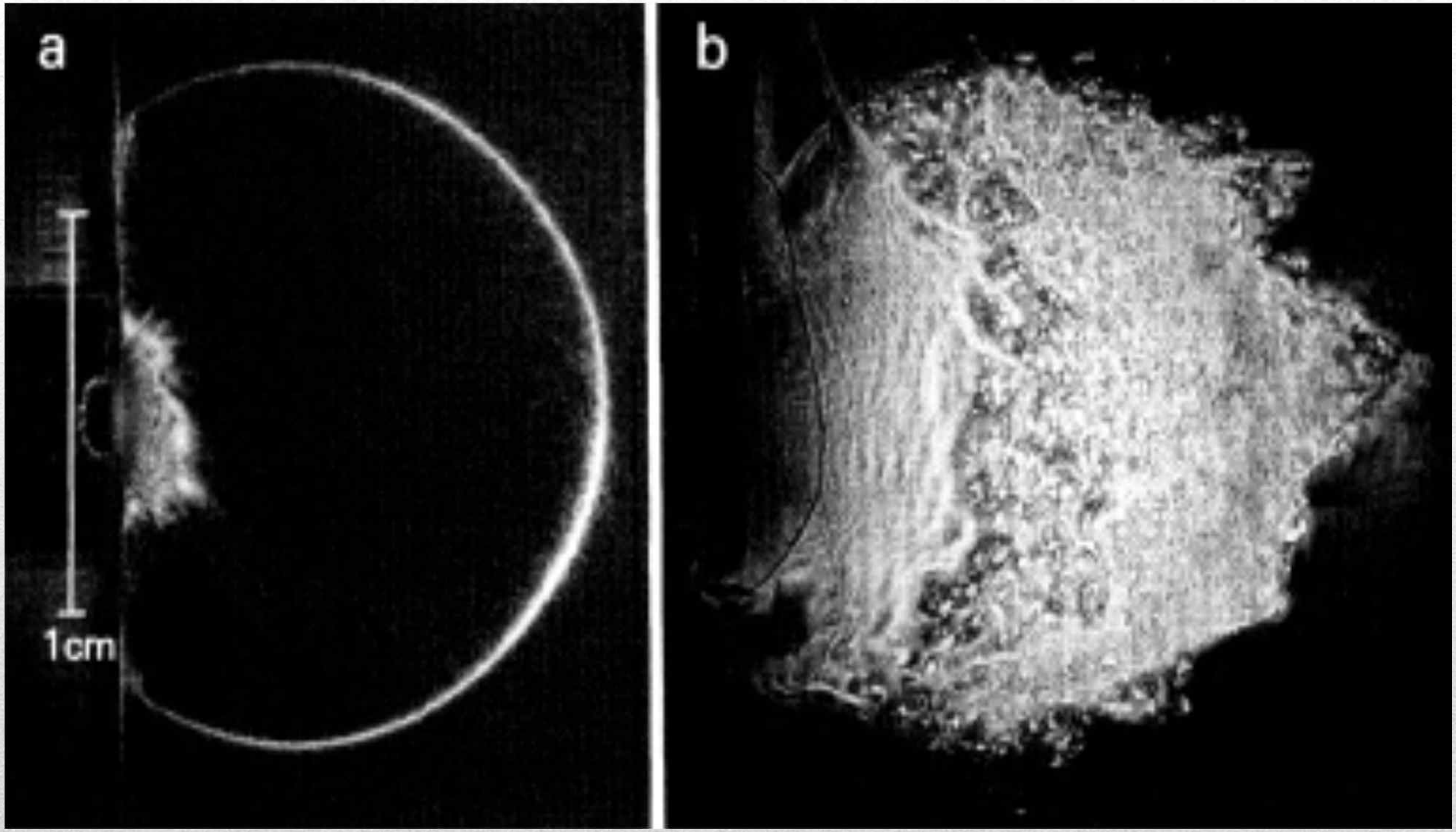




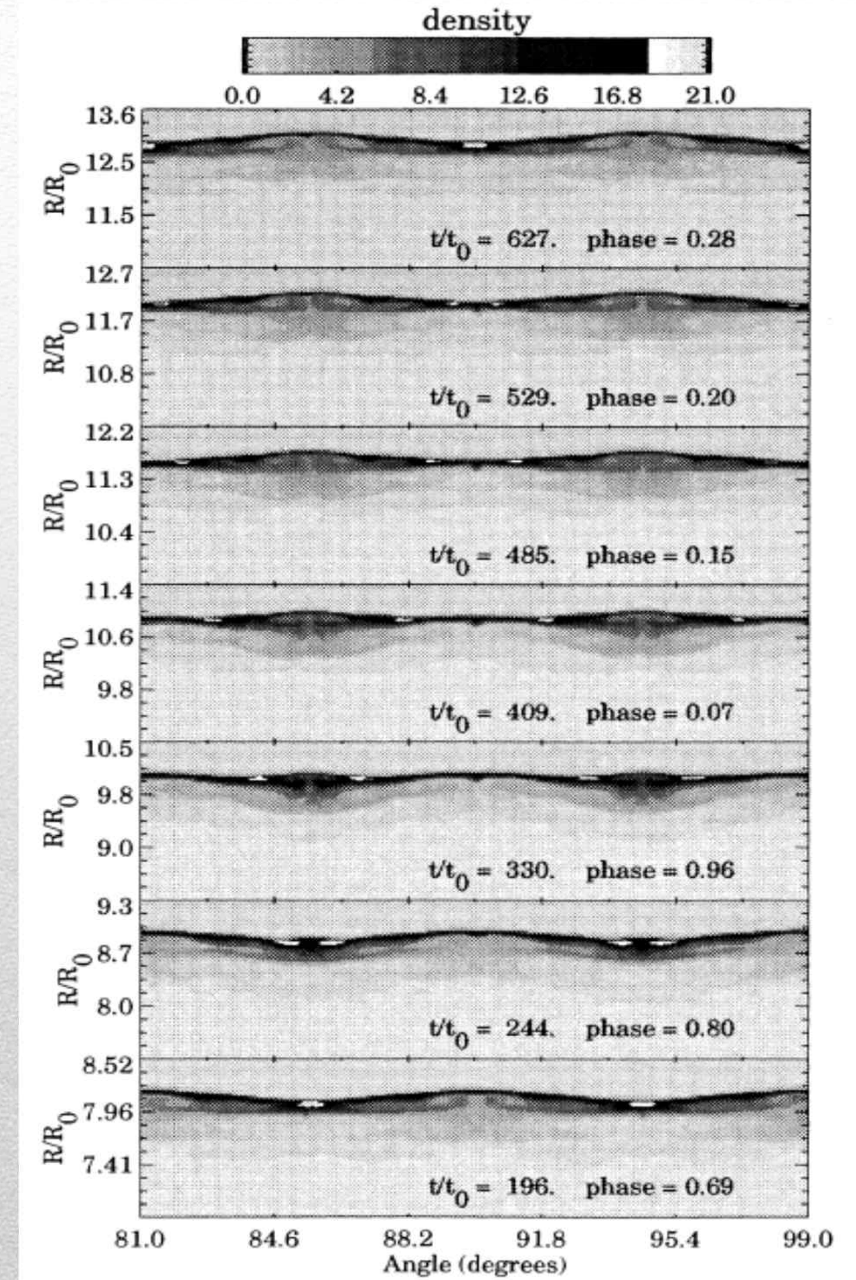
(Thornton, Gaudlitz, Janka. 1998 Phys.Rev.Lett)

Hall of Fame

Isenberg	1977
Cheng	1979
Bernstein & Book	1980
Vishniac	1983
Gaffet	1984
Gaffet	1984-II
Book	1986
Ryu & Vishniac	1987
Vishniac & Ryu	1988
Vishniac & Ryu	1989
Goodman	1990
Ryu & Vishniac	1991



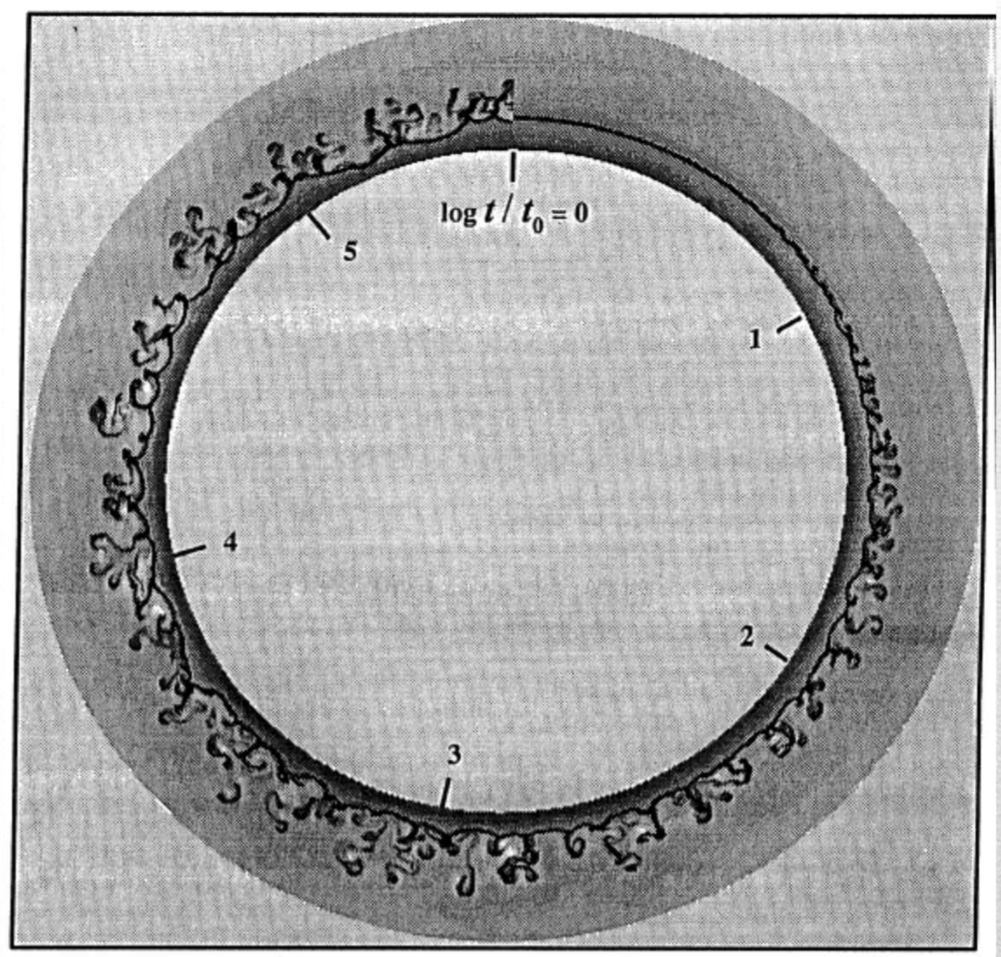
Experimental simulations.
Grun et al. 1991, Phys.Rev.Lett.



Instability of adiabatic shocks:

Numerical 2d simulations

MacLow & Norman, 1993

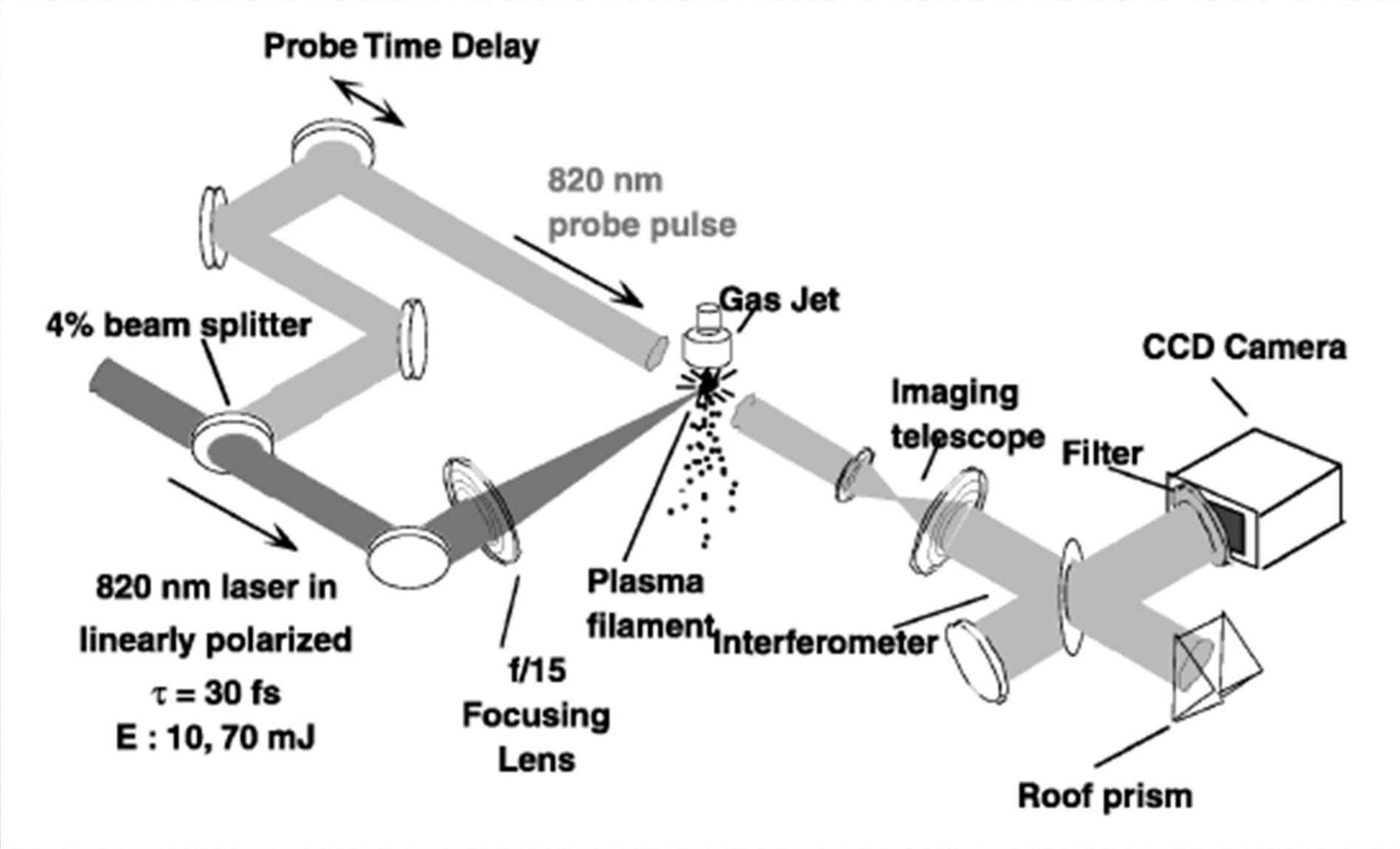


Instability of adiabatic shocks

Numerical 2d simulations

hollow blast wave

Chevalier, Blondin 1995



Experimental simulations 2.x
Hansen et al 2002; Laming & Grun 2002;
Moore, Symes, Smith. 2004.

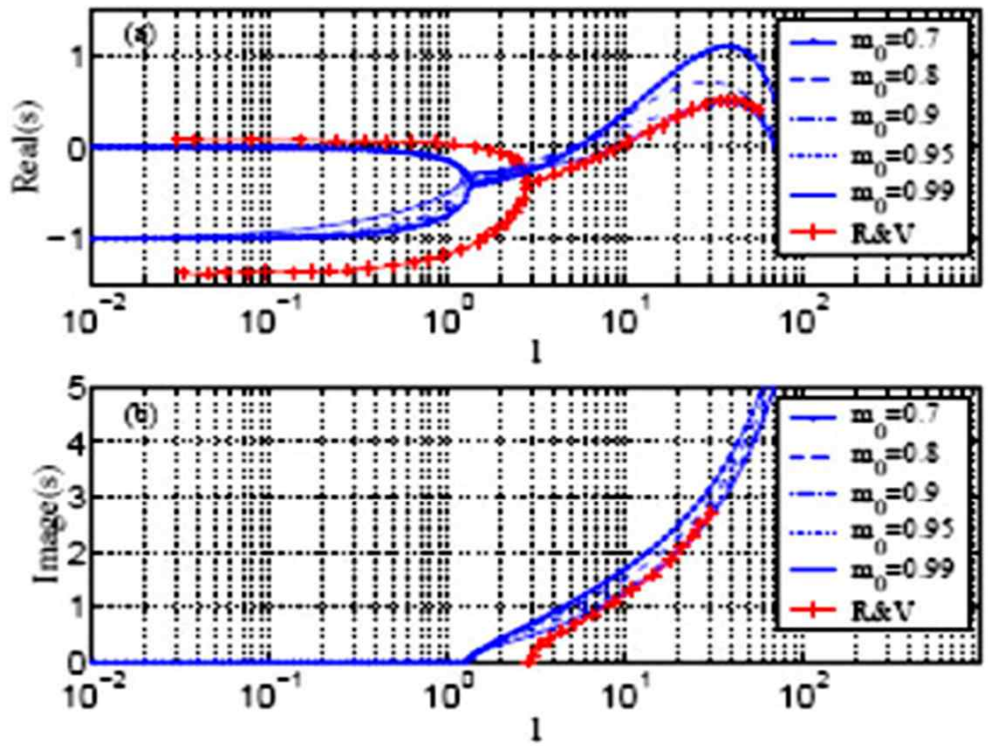
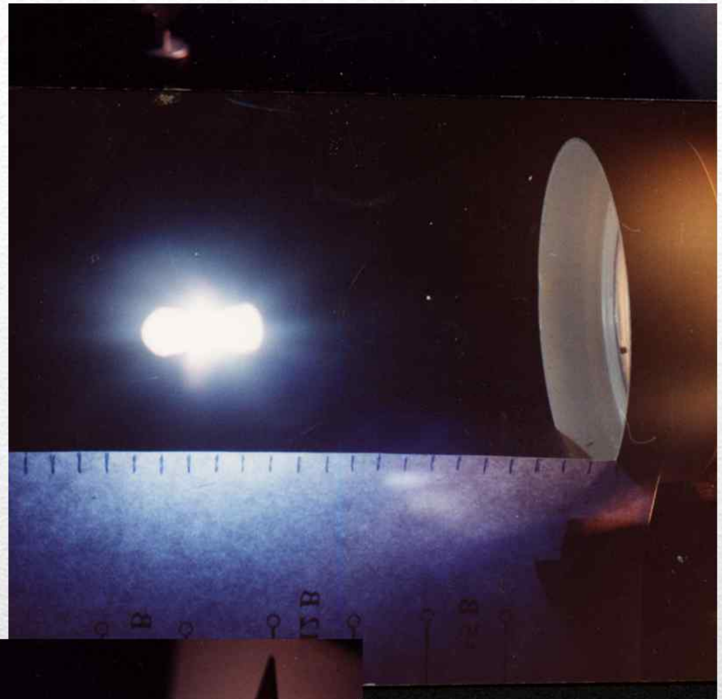
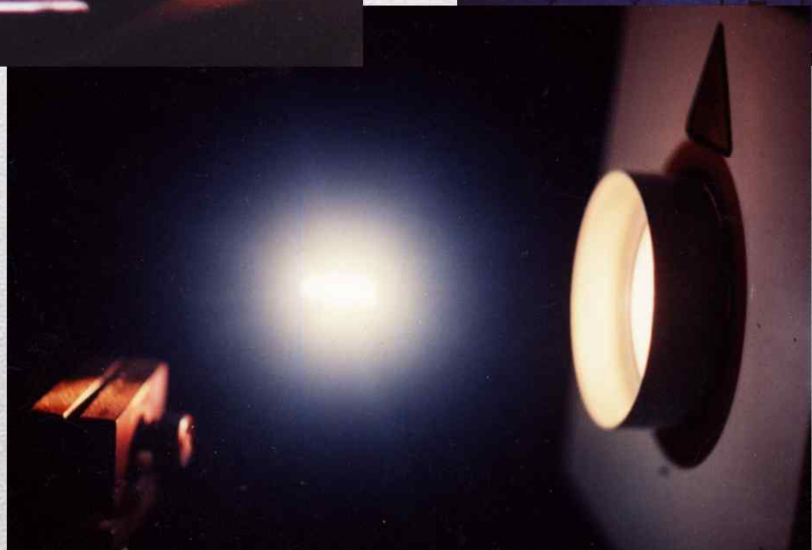


FIG. 2.— Perturbation growth rate, s, as function of wave number for various values of m_0, the fraction of self-similar solution mass contained in the self-similar part of the flow, for $\omega = 0, \gamma = 1.1$. The lines denoted "R&V" show the results of the analysis of Ryu & Vishniac (1987). The perturbation amplitude evolves as $f \propto t^{\mathrm{Re}(s)} \exp(i\mathrm{Im}(s) \ln t)$.

Linear analysis:
Modified inner boundary conditions
Kushnir, Waxman, Shvarts 2006





We decided to conduct its own investigation.

2. Instability of adiabatic blast wave (self oscillations)

Supernova remnant model:

- Continuous medium
- Hydrodynamic approximation
- Inhomogeneous background
- Point explosion: instant local injection of energy
- Adiabatic flow

Environment:

- Ideal, polytropic, inviscid gas with adiabatic index γ. Initial distribution of gas density is a power-law in radius with index ω.
- Unperturbed medium in steady state:

  1. Velocity of gas: $V_0=0$

  2. Density distribution: $\rho_0 = Ar^{(-\omega)}$

  3. Gas pressure $P_0 = 0$

- Explosion - instantaneous injection of energy E_0 in the origin at $t_0=0$.

Basic equations:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0 \\ \dfrac{\partial (\rho \vec{v})}{\partial t} + div\left(\rho \vec{v}\vec{v} + P \hat{\mathrm{I}}\right) = 0 \\ \dfrac{\partial}{\partial t}\left(\rho\left(\dfrac{v^2}{2} + \dfrac{P}{(\gamma - 1)\rho}\right)\right) + div\left(\rho \vec{v}\left(\dfrac{v^2}{2} + \dfrac{\gamma P}{(\gamma - 1)\rho}\right)\right) = 0 \end{cases}$$

Outer boundary conditions:

$$\rho_1(v_1 - V_s) = \rho_2(v_2 - V_s)$$

$$\rho_1(v_1 - V_s)^2 + P_1 = \rho_2(v_2 - V_s)^2 + P_2$$

$$\frac{1}{2}(v_1 - V_s)^2 + \frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} = \frac{1}{2}(v_2 - V_s)^2 + \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2}$$

$$v_{\tau 1} = v_{\tau 2}$$

Normalization of variables

$$R_s \propto (E_0)^{\frac{\delta}{2}} t^{\delta}$$

$$\delta = \frac{2}{5-\omega}$$

$$V_s = \dot{R}_s = \delta \frac{R_s}{t}$$

$$v_s = \frac{2V_s}{\gamma+1}$$

$$P_s = \frac{2\rho_0 V_s^2}{\gamma+1}$$

$$\rho_s = \rho_0 \frac{\gamma+1}{\gamma-1}$$

$$\tilde{v}(\xi) = \frac{v(r)}{v_s}$$

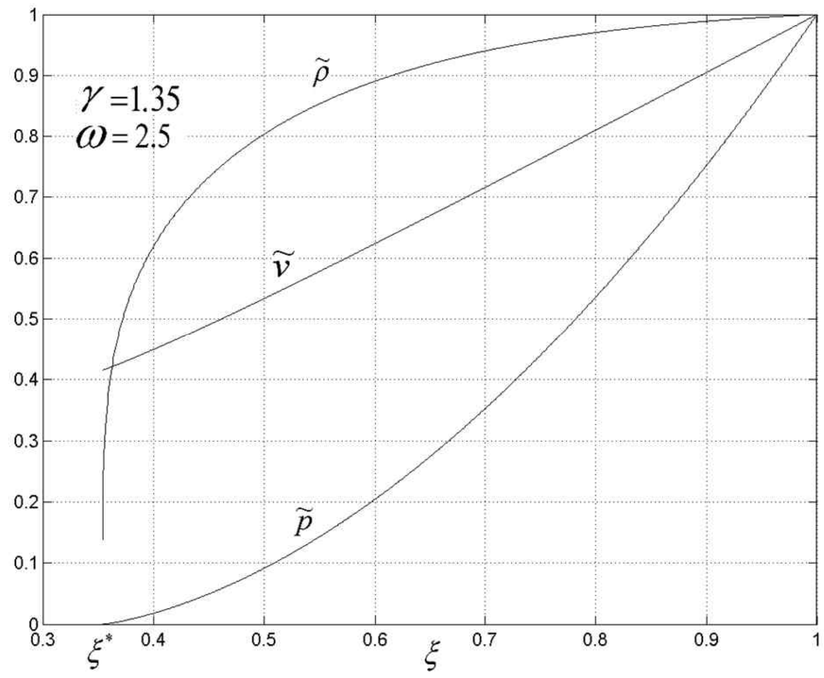
$$\tilde{p}(\xi) = \frac{P(r)}{P_s}$$

$$\tilde{\rho}(\xi) = \frac{\rho(r)}{\rho_s}$$

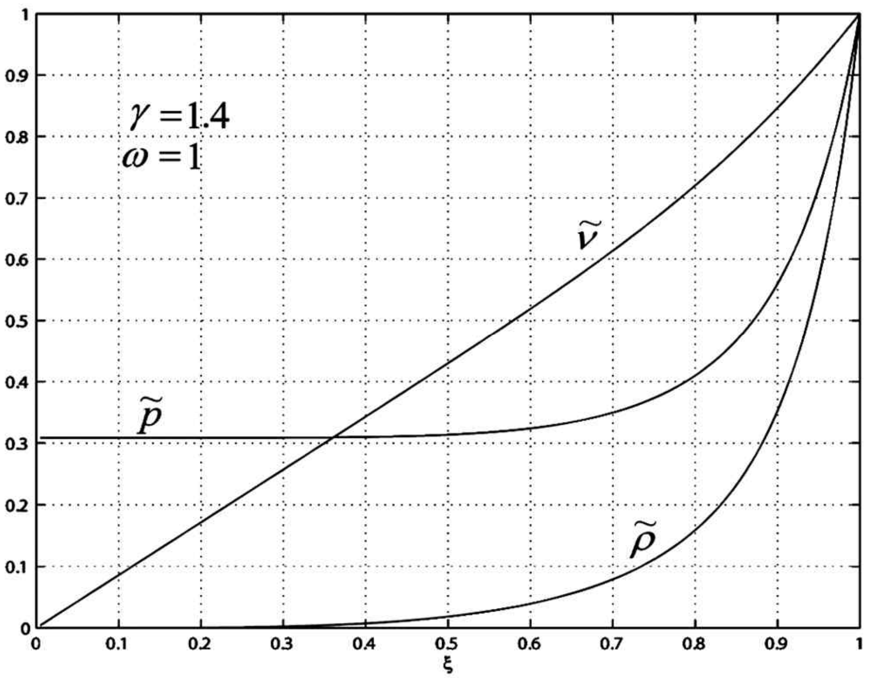
$$\xi = \hat{r} = \frac{r}{R_s}$$

$$\rho_0 = Ar^{(-\omega)}$$

Exact solution (Sedov, 1946)



Flow with a shell (hollow)



Flow, extended to the center of symmetry

Decomposition of perturbations by spherical harmonics

$$\xi = \frac{r}{R_s}; \;\; \tau = \ln(t/t_0); \quad e^{-is\tau} \propto t^{-is}.$$

$$F(\xi,\theta,\varphi,t) = F_0(\xi) + \delta F(\xi,\theta,\varphi)\cdot e^{-is\tau},$$

where $F = (P,\rho,V_r,V_\theta,V_\varphi),\;\; F_0 = (P_0,\rho_0,V_{r_0},0,0),\;\; \delta F = (\delta P,\delta\rho,\delta V_r,\delta V_\theta,\delta V_\varphi).$

$$f_i = \frac{\delta F_i}{F_{i_s}}, i = 1...3;$$

Decompose the perturbations into spherical harmonics :

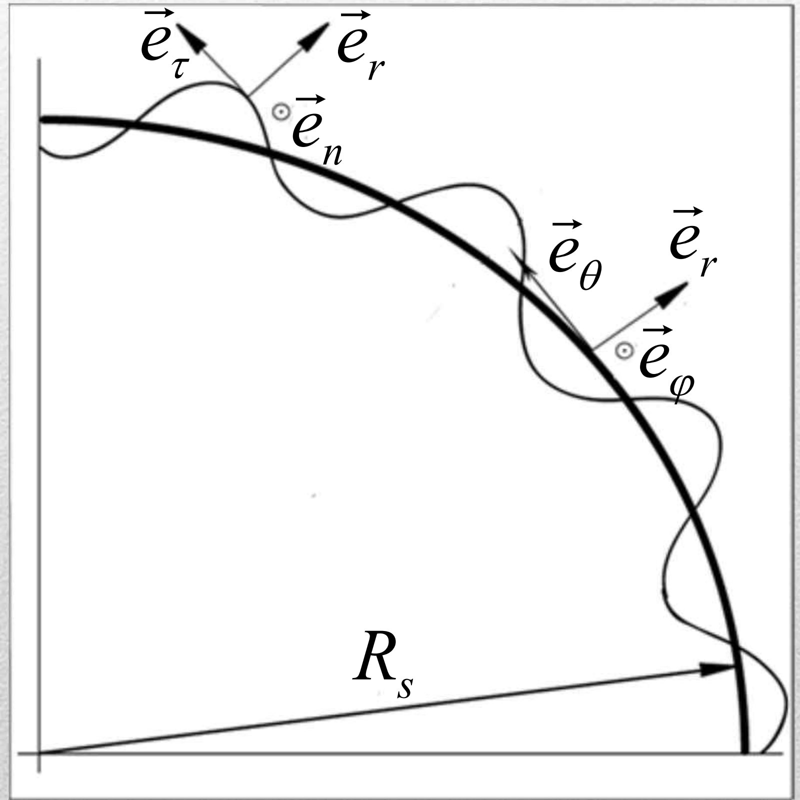
$$f_i(\xi,\theta,\varphi) = \sum_{l,m} C_{l,m} f_{i\,l,m}(\xi)\cdot Y_{l,m}(\theta,\varphi)\cdot e^{-is\ln(t)} \propto t^{-is};$$

$$f_{V\theta}(\xi,\theta,\varphi) = \sum_{l,m} C_{l,m}\left(f_{V\,T\,l,m}(\xi)\frac{dY_{l,m}(\theta,\varphi)}{d\theta} - f_{V\perp l,m}(\xi)\frac{dY_{l,m}(\theta,\varphi)}{\sin(\theta)d\varphi}\right)t^{-is}$$

$$f_{V\varphi}(\xi,\theta,\varphi) = \sum_{l,m} C_{l,m}\left(f_{V\,T\,l,m}(\xi)\frac{dY_{l,m}(\theta,\varphi)}{\sin(\theta)d\varphi} + f_{V\perp l,m}(\xi)\frac{dY_{l,m}(\theta,\varphi)}{d\theta}\right)t^{-is}$$

Natural coordinate system

$$\vec{e}_r, \qquad \vec{e}_\tau = \nabla_{\theta\varphi} Y_{lm}, \qquad \vec{e}_n = \vec{e}_r \times \vec{e}_\tau .$$



$$\nabla_{\theta\varphi} = \left(0, \frac{\partial}{\partial\theta}, \frac{\partial}{\sin\theta\partial\varphi}\right)$$

$$a_r(\vec{r}) = \sum C_{lm} a_{rlm}(r) Y_{lm}(\theta,\varphi),$$

$$a_\theta(\vec{r}) = \sum C_{lm}\left(a_{\tau lm}\frac{\partial Y_{lm}}{\partial\theta} - a_{nlm}\frac{\partial Y_{lm}}{\sin\theta\partial\varphi}\right)$$

$$a_\varphi(\vec{r}) = \sum C_{lm}\left(a_{\tau lm}\frac{\partial Y_{lm}}{\sin\theta\partial\varphi} + a_{nlm}\frac{\partial Y_{lm}}{\partial\theta}\right)$$

System of linearized equations

$$\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{df\rho}{d\xi} + \tilde{\rho}\,\frac{dfv_r}{d\xi} + \left(\frac{d\tilde{v}}{d\xi} + 2\,\frac{\tilde{v}}{\xi} - \frac{\gamma+1}{2}\,\omega - \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right)f\rho + \left(\frac{d\tilde{\rho}}{d\xi} + 2\,\frac{\tilde{\rho}}{\xi}\right)fv_r - l(l+1)\,\frac{\tilde{\rho}}{\xi}\,fv_T = 0\,,$$

$$\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\tilde{\rho}\,\frac{dfv_r}{d\xi} + \frac{\gamma-1}{2}\,\frac{dfp}{d\xi} - \frac{\gamma-1}{2}\,\frac{1}{\tilde{\rho}}\,\frac{d\tilde{p}}{d\xi}\,f\rho + \left(\frac{d\tilde{v}}{d\xi} - \frac{\gamma+1}{2}\,\frac{3-\omega}{2} - \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right)\tilde{\rho} fv_r = 0\,,$$

$$\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\tilde{\rho}\,\frac{dfv_T}{d\xi} + \left(\frac{\tilde{v}}{\xi} - \frac{\gamma+1}{2}\,\frac{3-\omega}{2} - \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right)\tilde{\rho} fv_T + \frac{\gamma-1}{2}\,\frac{1}{\xi}\,fp = 0\,,$$

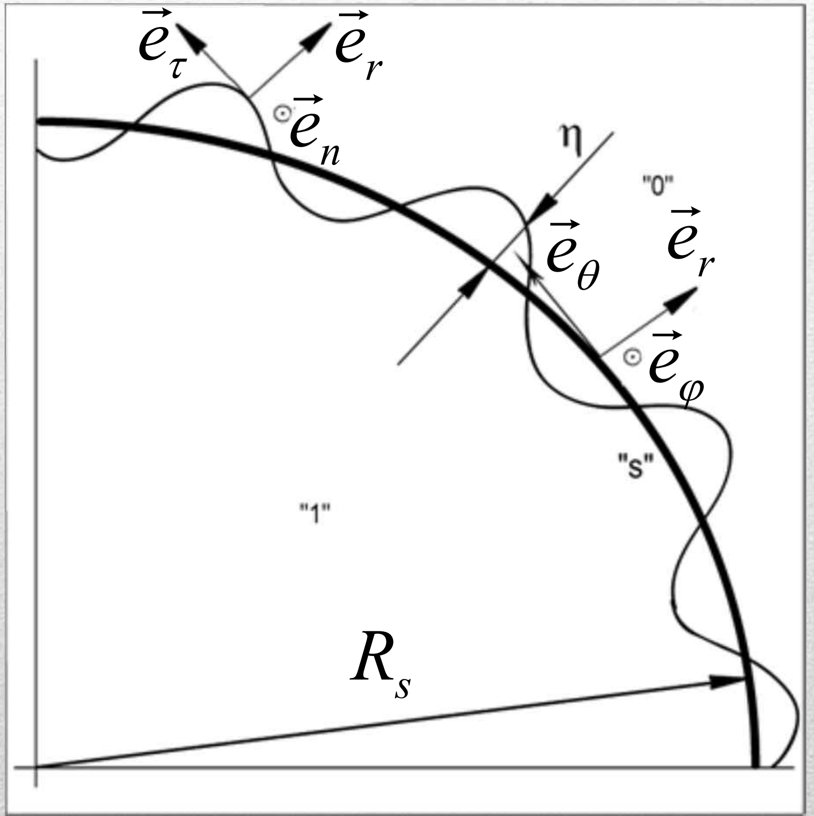
$$-\gamma\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{df\rho}{d\xi} + \left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{\tilde{\rho}}{\tilde{p}}\,\frac{dfp}{d\xi} + \left[-\gamma\,\frac{1}{\tilde{\rho}}\,\frac{d\tilde{\rho}}{d\xi} + \frac{1}{\tilde{p}}\,\frac{d\tilde{p}}{d\xi}\right]\tilde{\rho} fv_r +$$

$$+\gamma\left[\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{1}{\tilde{\rho}}\,\frac{d\tilde{\rho}}{d\xi} + \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right]f\rho - \left[\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{1}{\tilde{p}}\,\frac{d\tilde{p}}{d\xi} + \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right]\frac{\tilde{\rho}}{\tilde{p}}\,fp = 0\,,$$

$$\left(\tilde{v} - \frac{\gamma+1}{2}\xi\right)\frac{dfv_n}{d\xi} + \left(\frac{\tilde{v}}{\xi} - \frac{\gamma+1}{2}\,\frac{3-\omega}{2} - \frac{\gamma+1}{2}\,\frac{5-\omega}{2}is\right)fv_n = 0$$

(subscripts l,m are dropped for simplicity)

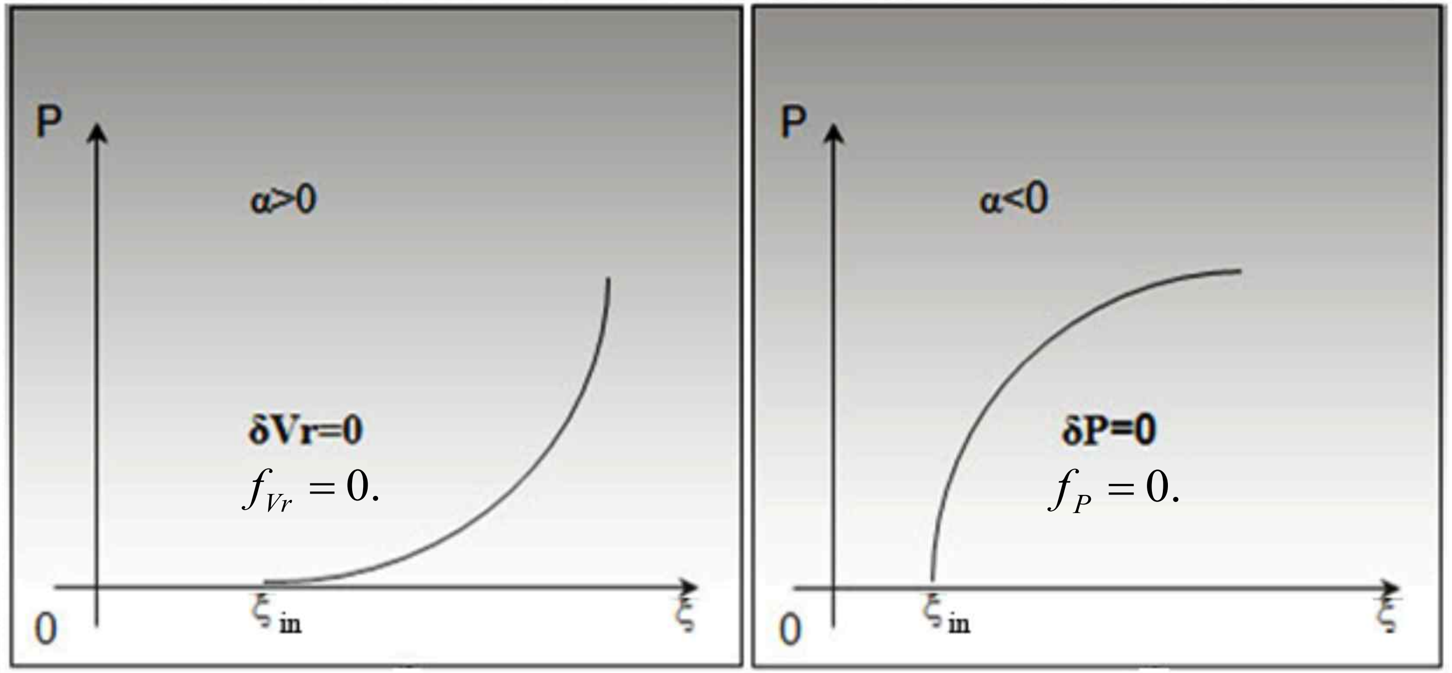
Conditions on a perturbed shock front



External boundary condition:

$$
\begin{cases}
f_{\rho_s} = -\omega\eta - \left(\dfrac{\partial \rho}{\partial \xi}\right)\eta; \\
f_{Vr_s} = \eta - \dfrac{1}{2} I(5-\omega) s\,\eta - \left(\dfrac{\partial V_r}{\partial \xi}\right)\eta; \\
f_{P_s} = 2\eta - \omega\eta - \left(\dfrac{\partial P}{\partial \xi}\right)\eta - I(5-\omega) s\,\eta; \\
f_{V\tau_s} = -\eta \\
f_{Vn_s} = 0.
\end{cases}
$$

Conditions at inner boundary. Hollow case.



General condition: no sources at inner boundary

In the case of solution with shell, inner boundary condition depends on the asymptotic behavior of the unperturbed pressure:

$$\underset{\xi \to \xi_{in}}{P} \propto \left(\xi - \xi_{in}\right)^{\alpha + 1}, \; here \;\; \alpha = \frac{\omega(1+\gamma) - 6}{(6 - 3\gamma - \omega)};$$

Conditions at inner boundary. No hollow.

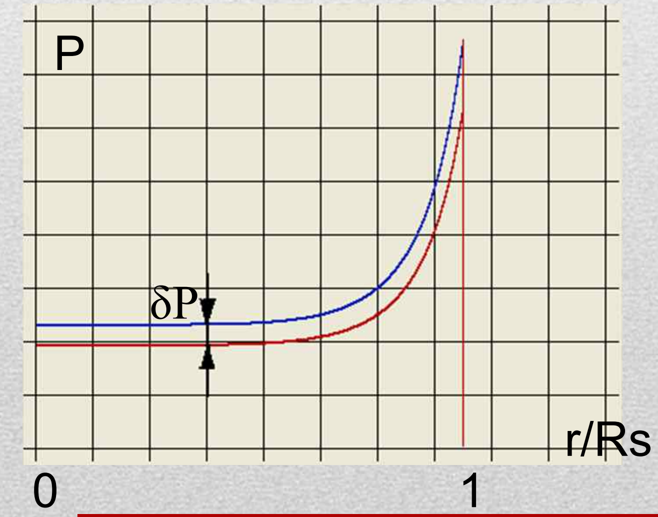
no sources => condition of impermeability:

both velocity and perturbation of velocity vanish

$$\delta V_r = 0 \;\; => \;\; f_{Vr} = 0.$$

Ryu & Vishniac (1991) use another internal boundary condition:

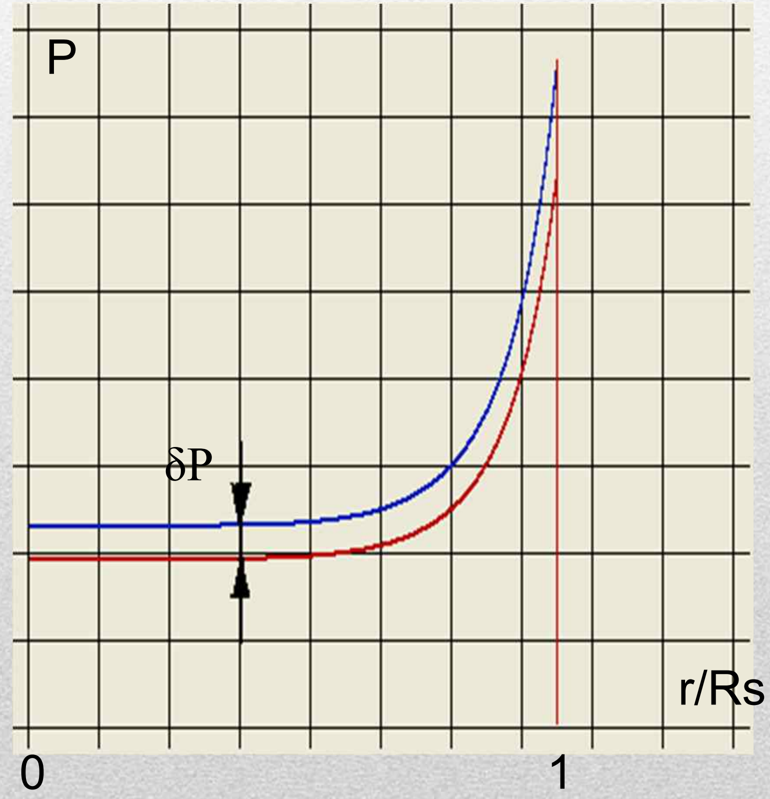
$$\delta P = 0$$



Counterexample: consider two flows at the same moment $t > 0$ from explosions with energies E_0 and $E' = E_0 + \delta E$. More energetic flow has greater $P'(0)$ than $P(0)$, hence, $\delta P \neq 0$.

Eigenfrequency found analytically

$$E_0 \to E_0 + \delta E; \qquad R_s \propto (E_0)^{\frac{1}{5-\omega}} \cdot (t)^{\frac{2}{5-\omega}}$$



$$\frac{\delta R_s}{R_s} \equiv \frac{R_s(E_0 + \delta E, t) - R_s(E_0, t)}{R_s(E_0, t)} \propto$$

$$\propto \frac{\left((E_0 + \delta E)^{\frac{1}{5-\omega}} - (E_0)^{\frac{1}{5-\omega}} \right) \cdot t^{\frac{2}{5-\omega}}}{(E_0)^{\frac{1}{5-\omega}} \cdot t^{\frac{2}{5-\omega}}} \propto t^0$$

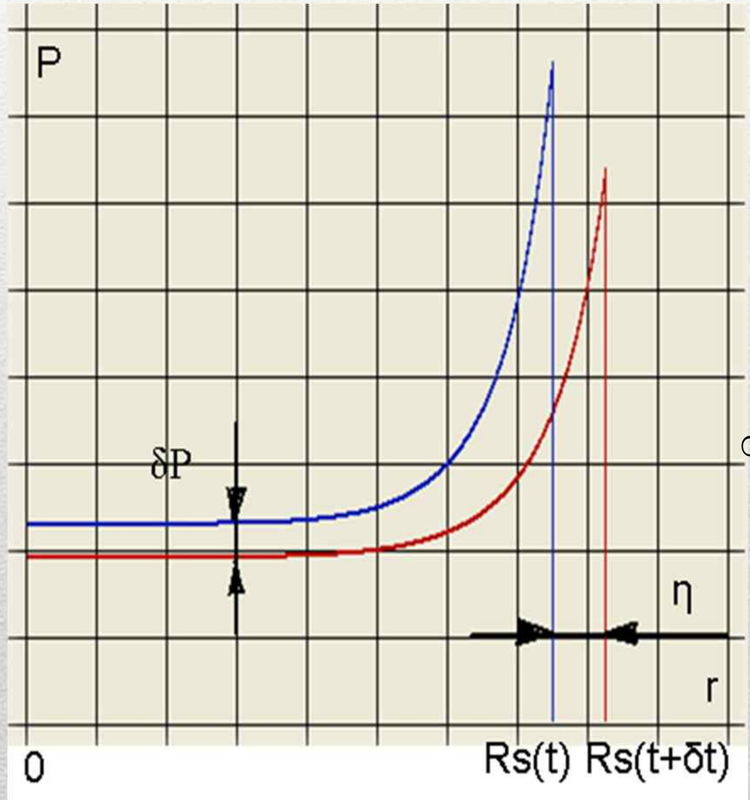
=> $s = 0$ =>

$(l = 0, \mathrm{Im}(s) = 0)$

– the first point on the spectrum

Another eigenfrequency found analytically

$$t_0 \to t_0 + \delta t; \qquad R_s \propto (E_0)^{\frac{1}{5-\omega}} \cdot (t)^{\frac{2}{5-\omega}}$$



$$\frac{\delta R_s}{R_s} \equiv \frac{R_s(E_0, t_0+\delta t) - R_s(E_0, t_0)}{R_s} \propto$$

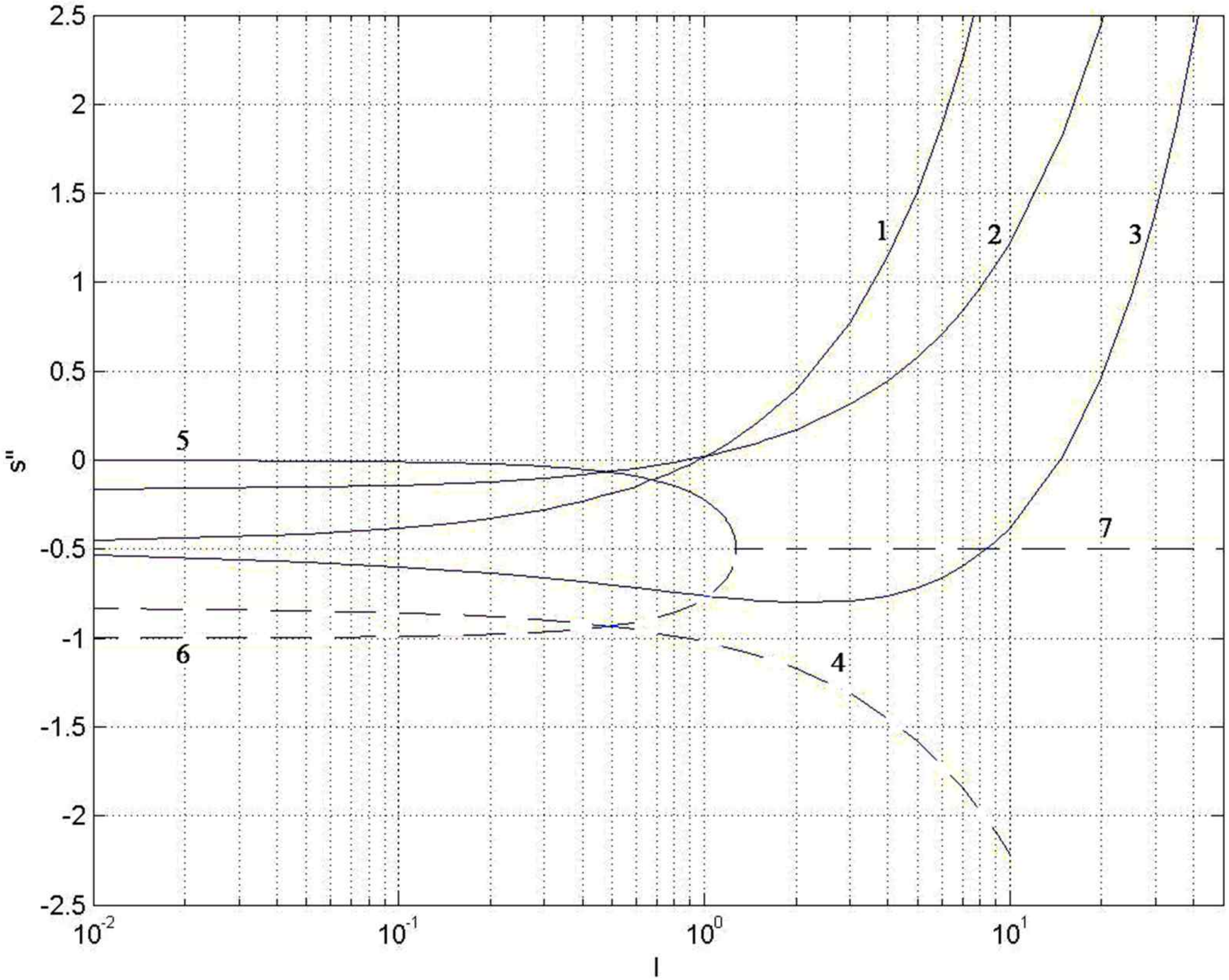
$$\propto \frac{\left((t - t_0 + \delta t)^{\frac{2}{5-\omega}} - (t-t_0)^{\frac{2}{5-\omega}} \right)}{(t-t_0)^{\frac{2}{5-\omega}}} \propto \frac{2}{5}\frac{(\delta t)}{t-t_0} \propto t^{-1}$$

$$\Rightarrow s = -i \Rightarrow$$

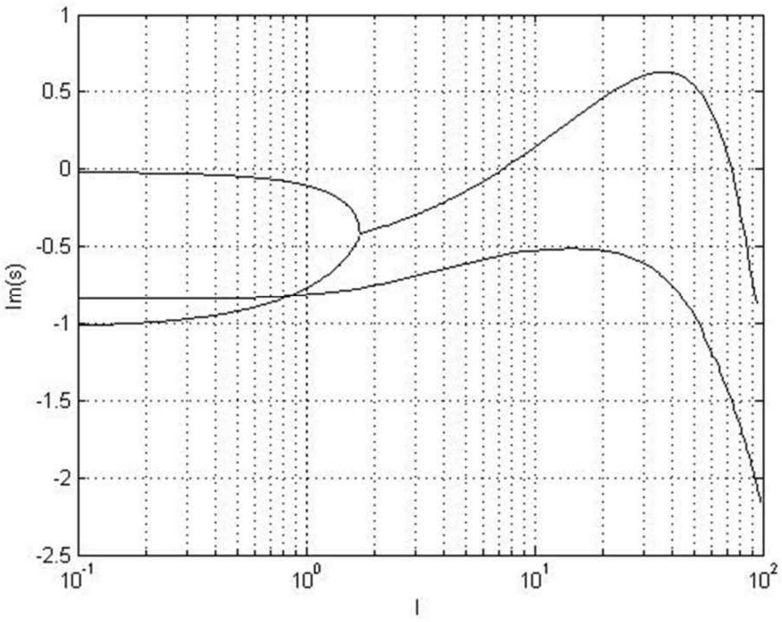
$$(l = 0, \mathrm{Im}(s) = -1)$$

– the second point on the spectrum

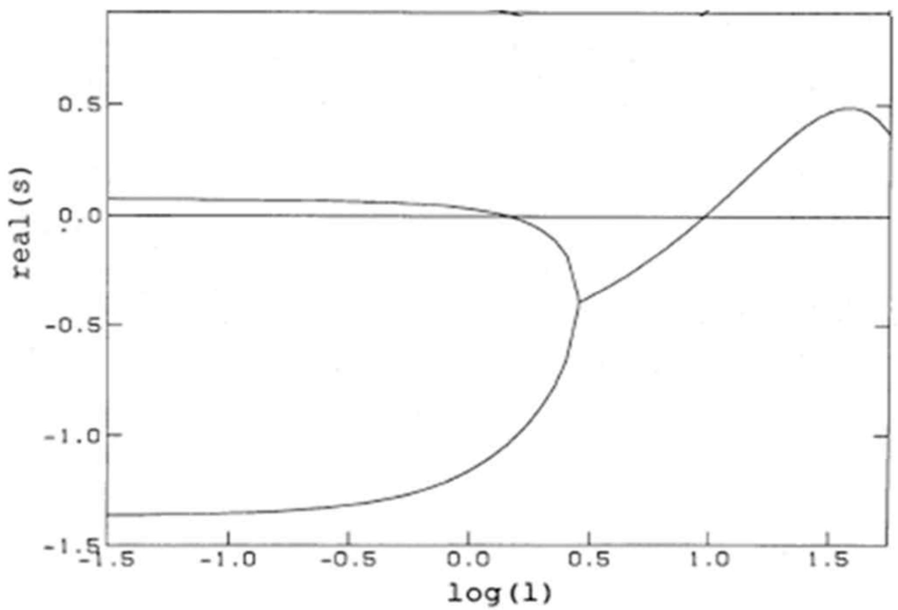
Analytically found spectrum for γ = 4/3, ω=(7-γ)/(1+ γ)≈2.43



The homogeneous atmosphere
ω=0, γ=1.1

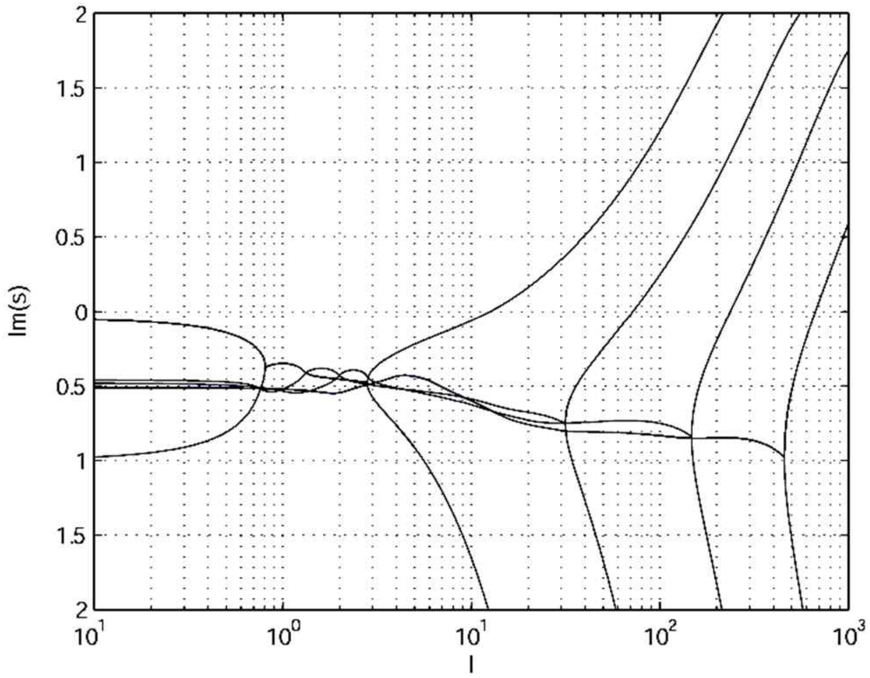


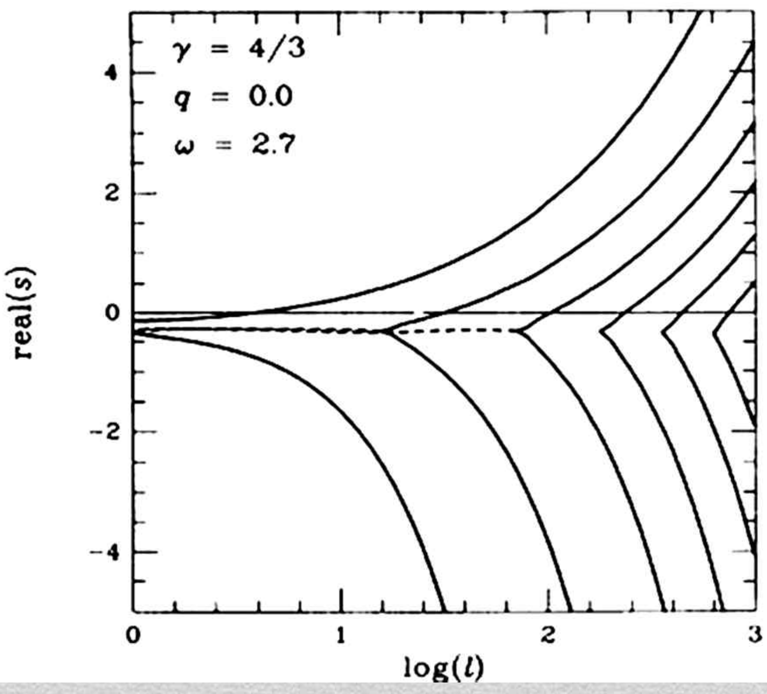
The homogeneous atmosphere
ω=0, γ=1.1 (Ryu&Vishniac, 1987г.)



The inhomogeneous medium
ω=2.7, γ=4/3

The power-law distribution of density
(Ryu&Vishniac, 1991г.)





The potential and vortex components of the perturbed velocities field

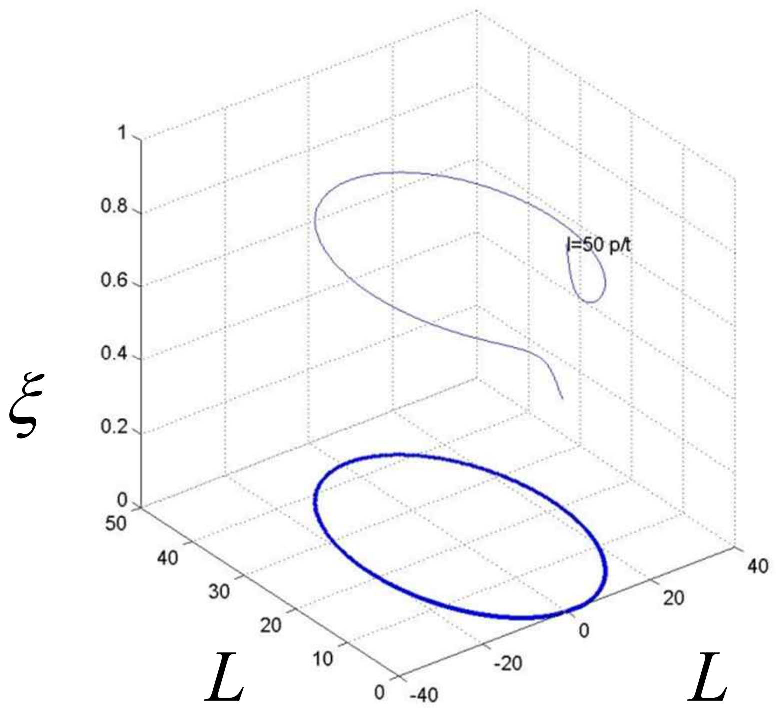
$$\delta\vec{v} = \delta\vec{v}_{\mathrm{pot}} + \delta\vec{v}_{\mathrm{vort}} = \nabla\phi + \mathrm{rot}\,\vec{A}$$

$$L^2 = \frac{\left\langle \left|\nabla\phi\right|^2 \right\rangle}{\left\langle \left|\mathrm{rot}\,\vec{A}\right|^2 \right\rangle}, \qquad \cos\alpha = \frac{\left(\nabla\phi, \mathrm{rot}\,\vec{A}\right)}{\sqrt{\left\langle \left|\nabla\phi\right|^2 \right\rangle \left\langle \left|\mathrm{rot}\,\vec{A}\right|^2 \right\rangle}}$$

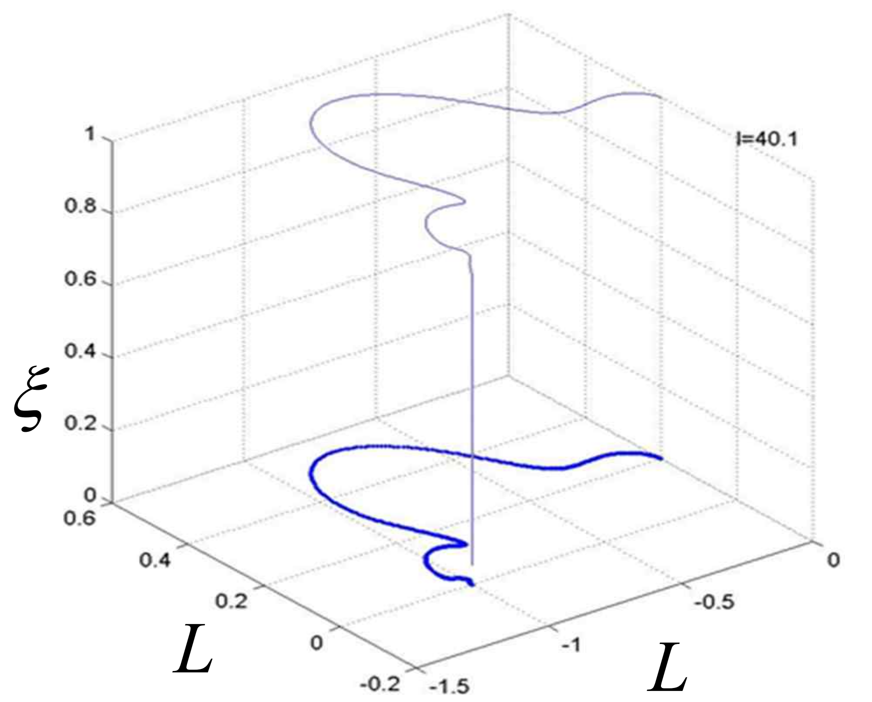
$$\left\langle f(\vec{r}) \right\rangle = \frac{1}{4\pi}\iint f \sin\theta\, d\theta\, d\varphi, \qquad \left(\vec{a}, \vec{b}\right) = \frac{1}{2}\left(\left\langle \vec{a}^{*} \cdot \vec{b} \right\rangle + \left\langle \vec{a} \cdot \vec{b}^{*} \right\rangle\right)$$

Perturbation velocity hodograph

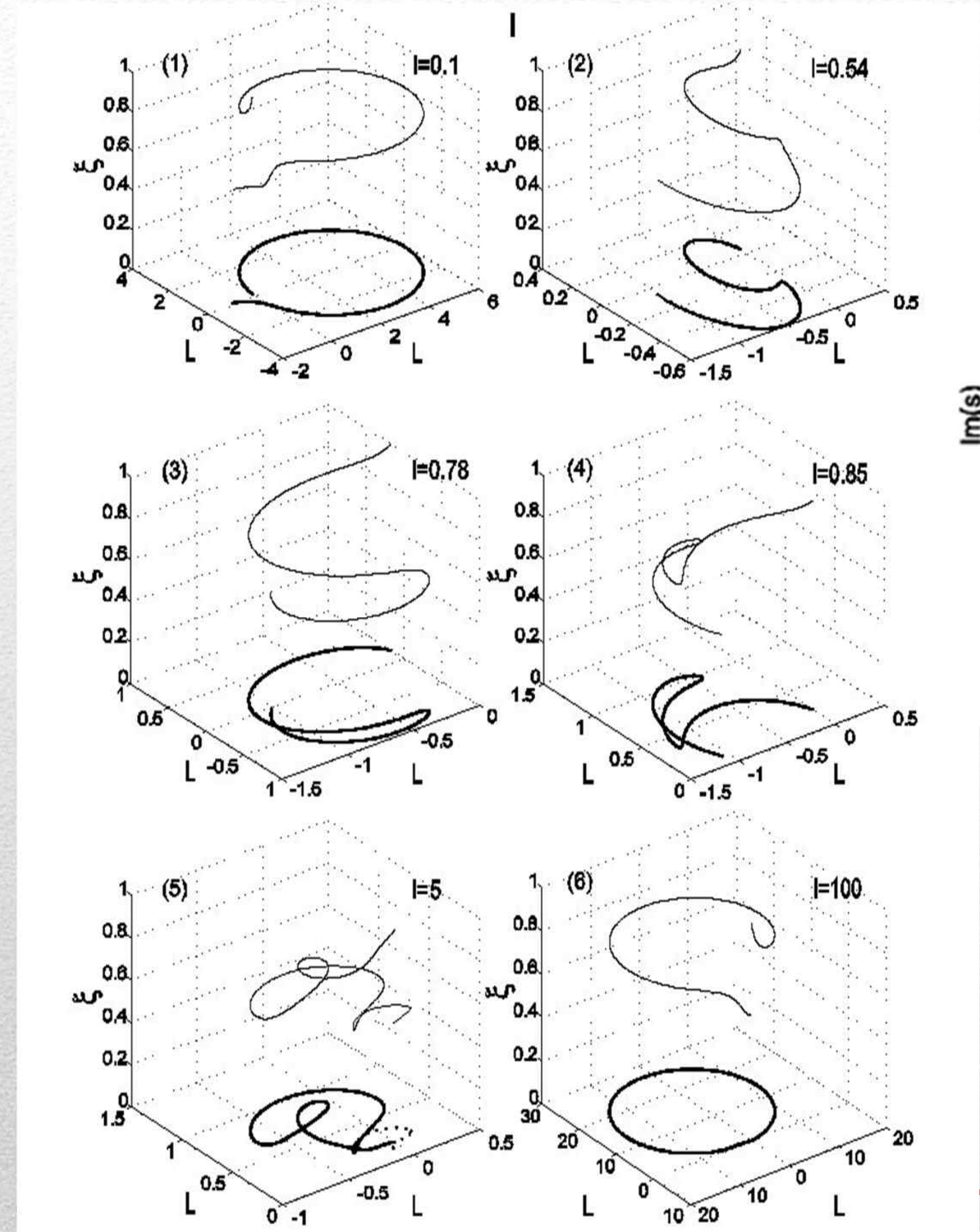
The flow with a shell

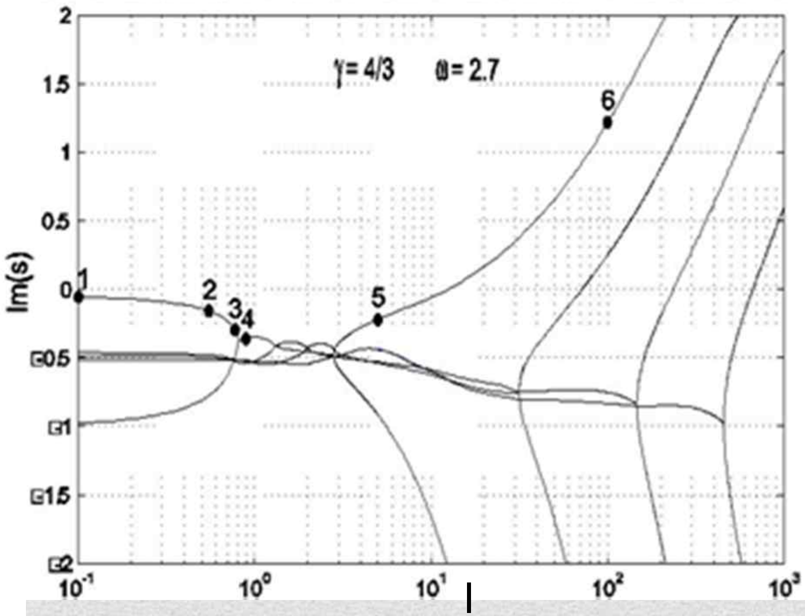


The flow, extended to the center of symmetry



Analysis of disturbed flow in the case with the shell.



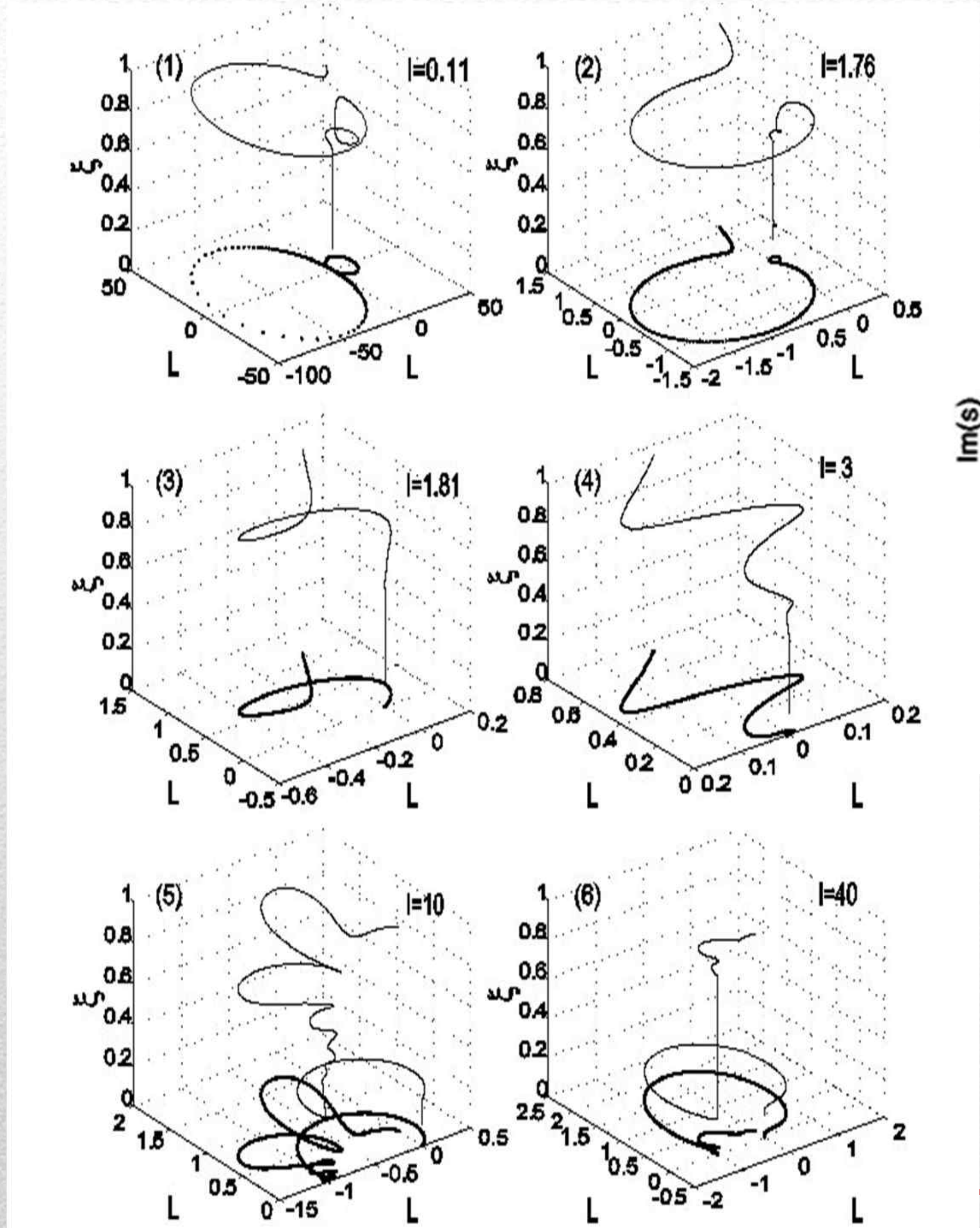


$l -$ number of spherical harmonic

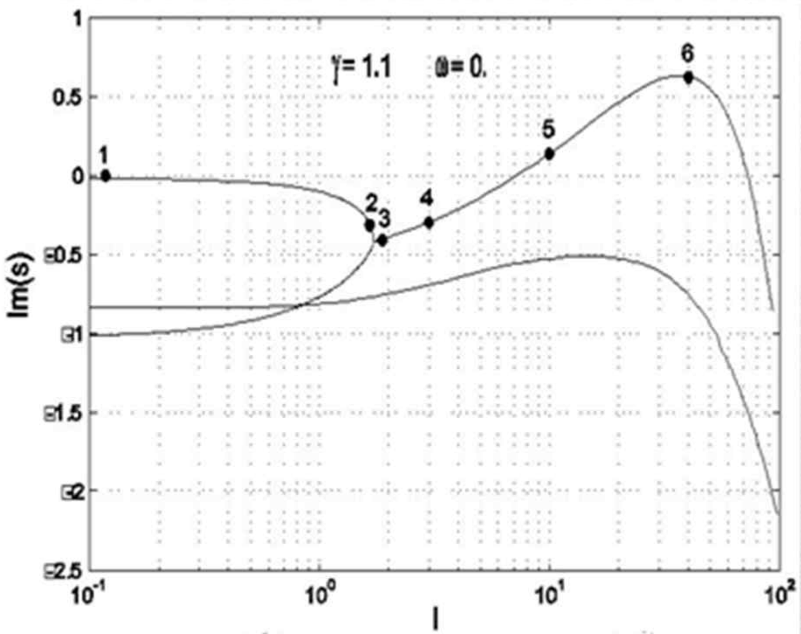
$s -$ increment of the harmonic with number l

$L = \frac{|\delta \vec{v}_{\rm pot}|}{|\delta \vec{v}_{\rm vort}|} -$ ratio of the potential part of perturbation to the vortex part of perturbation

$\xi \in (\xi_{in} ... 1) -$ normalized radius



Analysis of disturbed flow in the case without the shell.



$l-$ number of spherical harmonic

$s-$ increment of the harmonic with number l

$L = \frac{|\delta \vec{v}_{pot}|}{|\delta \vec{v}_{vort}|} -$ ratio of the potential part of perturbation to the vortex part of perturbation

$\xi \in (0...1) -$ normalized radius

3. Instability of adiabatic blast waves (forced oscillations)

New insight

- Analysis of shock response to external perturbations
- Search of possible resonance.

Environment:

- Ambient medium is perturbed now. Consider steady state perturbations :

  1. Vortex perturbation $V_0 = 0 + \delta V$

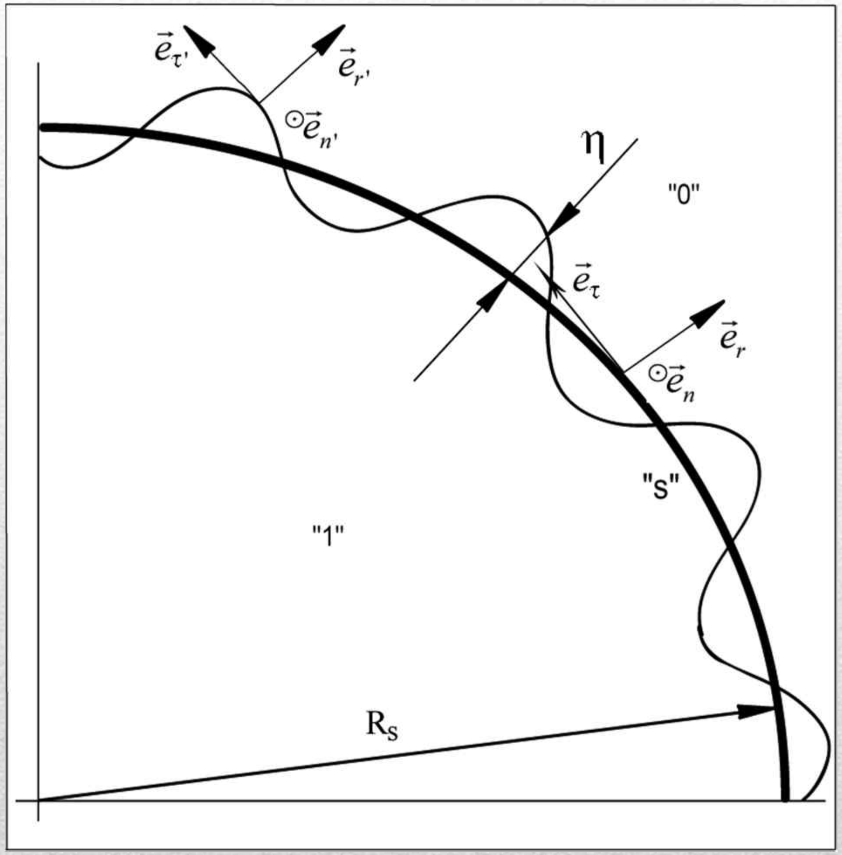
  2. Entropy perturbation $\rho_0 = Ar^{(-\omega)} + \delta\rho$

  3. Acoustic perturbations are prohibited in this consideration since

     $P_0 = 0$, otherwise $P = \delta P$ can be < 0 => unphysical.

- Explosion - injection of energy $E0$ in the origin, at the initial time $t0 = 0$.

The conditions on the perturbed shock front



External boundary condition:

$$
\begin{cases}
f_{\rho_s} = -\omega\eta - \left(\dfrac{\partial\rho}{\partial\xi}\right)\eta + f_{\rho_0}; \\
f_{Vr_s} = \eta - \dfrac{1}{2} I(5-\omega) s\eta - \left(\dfrac{\partial V_r}{\partial\xi}\right)\eta + \dfrac{f_{Vr_0}(\gamma-1)}{\gamma+1}; \\
f_{P_s} = 2\eta - \omega\eta - \left(\dfrac{\partial P}{\partial\xi}\right)\eta - I(5-\omega)s\eta + \dfrac{f_{\rho_0}(\gamma+1)}{\gamma-1} - \dfrac{4 f_{Vr_0}}{\gamma+1}; \\
f_{V\tau_s} = -\eta + f_{V\tau_0}; \\
f_{Vn_s} = f_{Vn_0}.
\end{cases}
$$

Eigenvalues:

Self oscillation: s - frequency

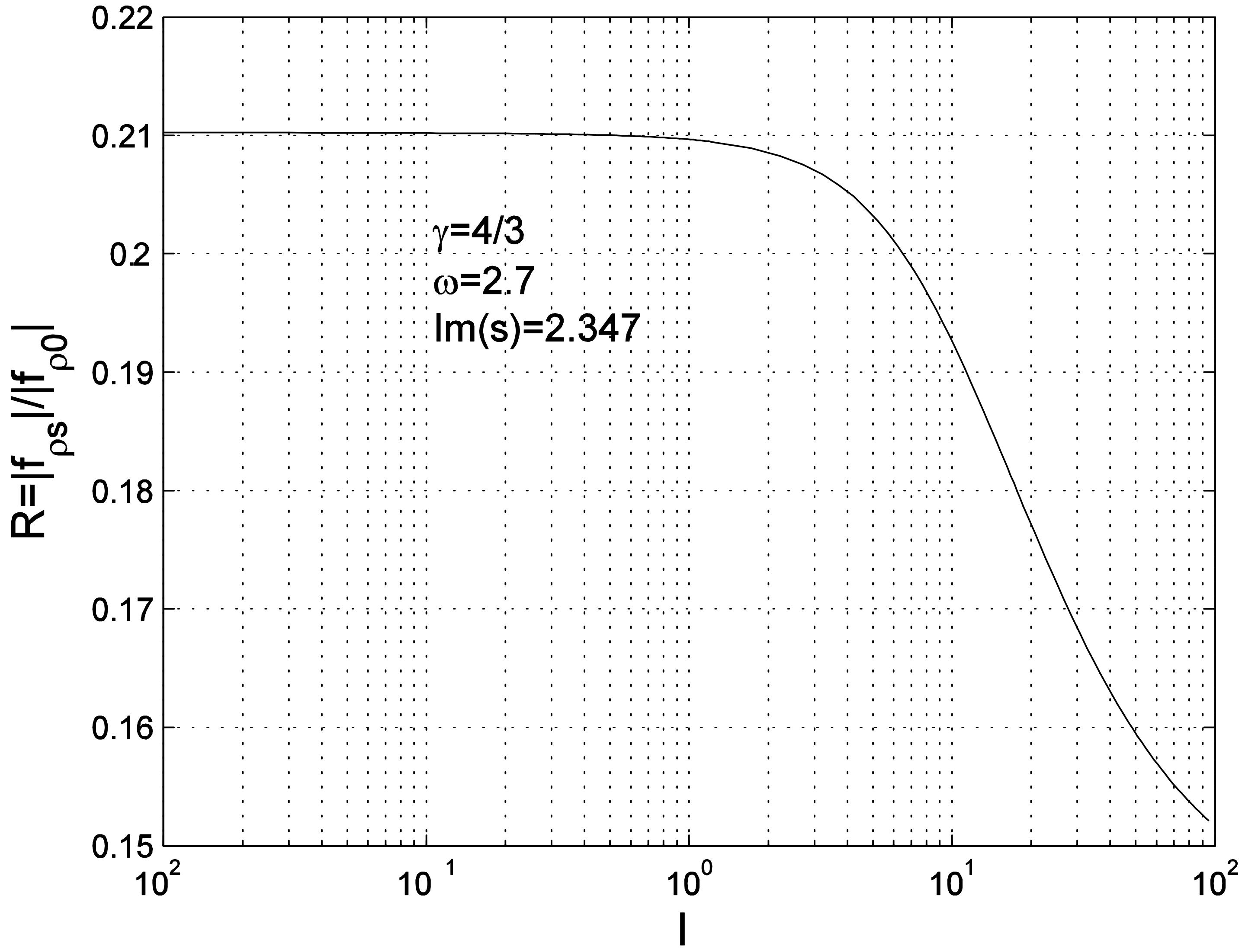
forced oscillation: η - front displacement amplification factor for density perturbations :

$$R = \frac{|f_{\rho_S}|}{|f_{\rho_0}|};$$

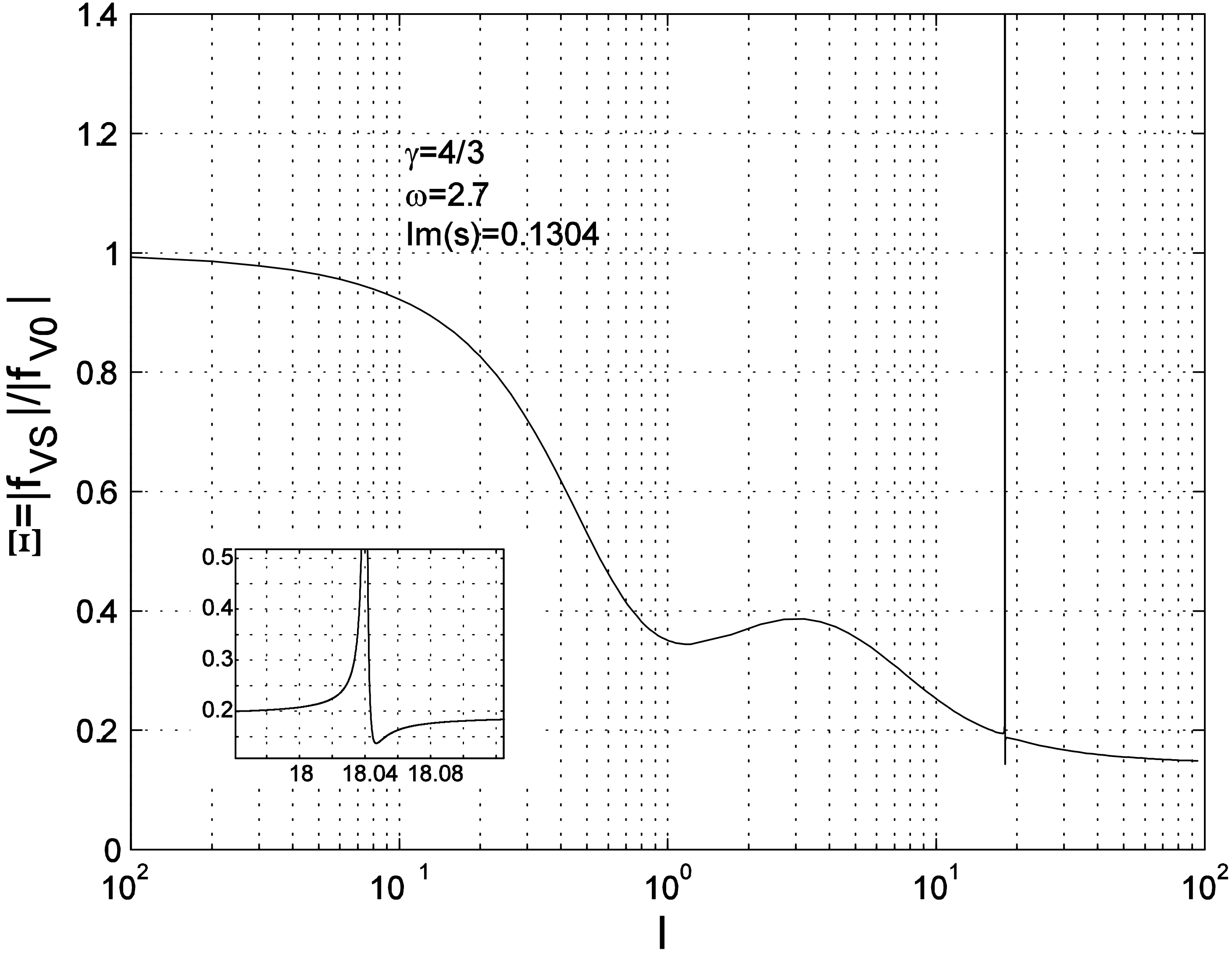
amplification factor for velocity perturbations :

$$\Xi = \frac{|\vec{f}_{v_S}|}{|\vec{f}_{v_0}|};$$

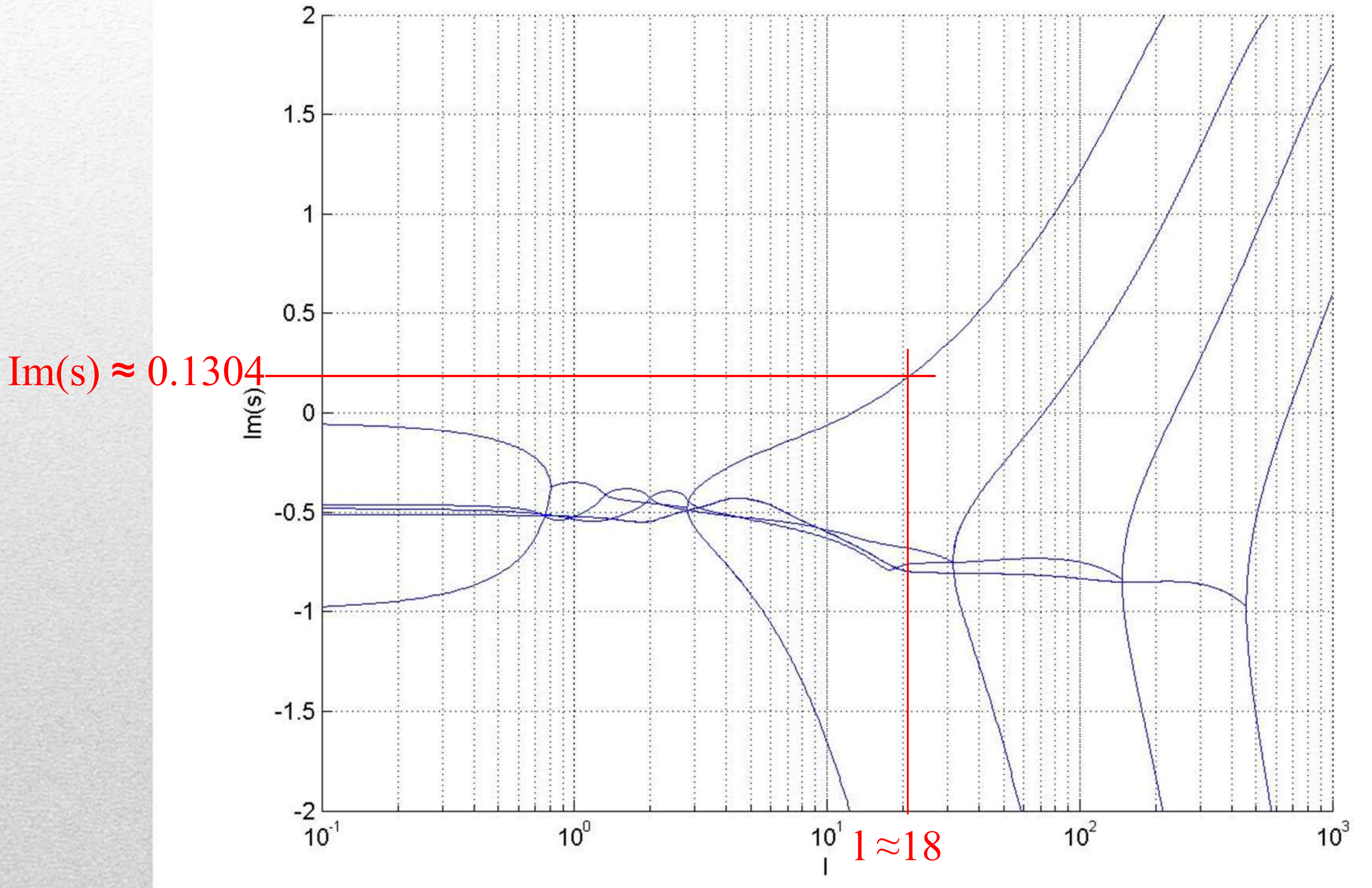
Amplification density factor for entropy perturbations, hollow case



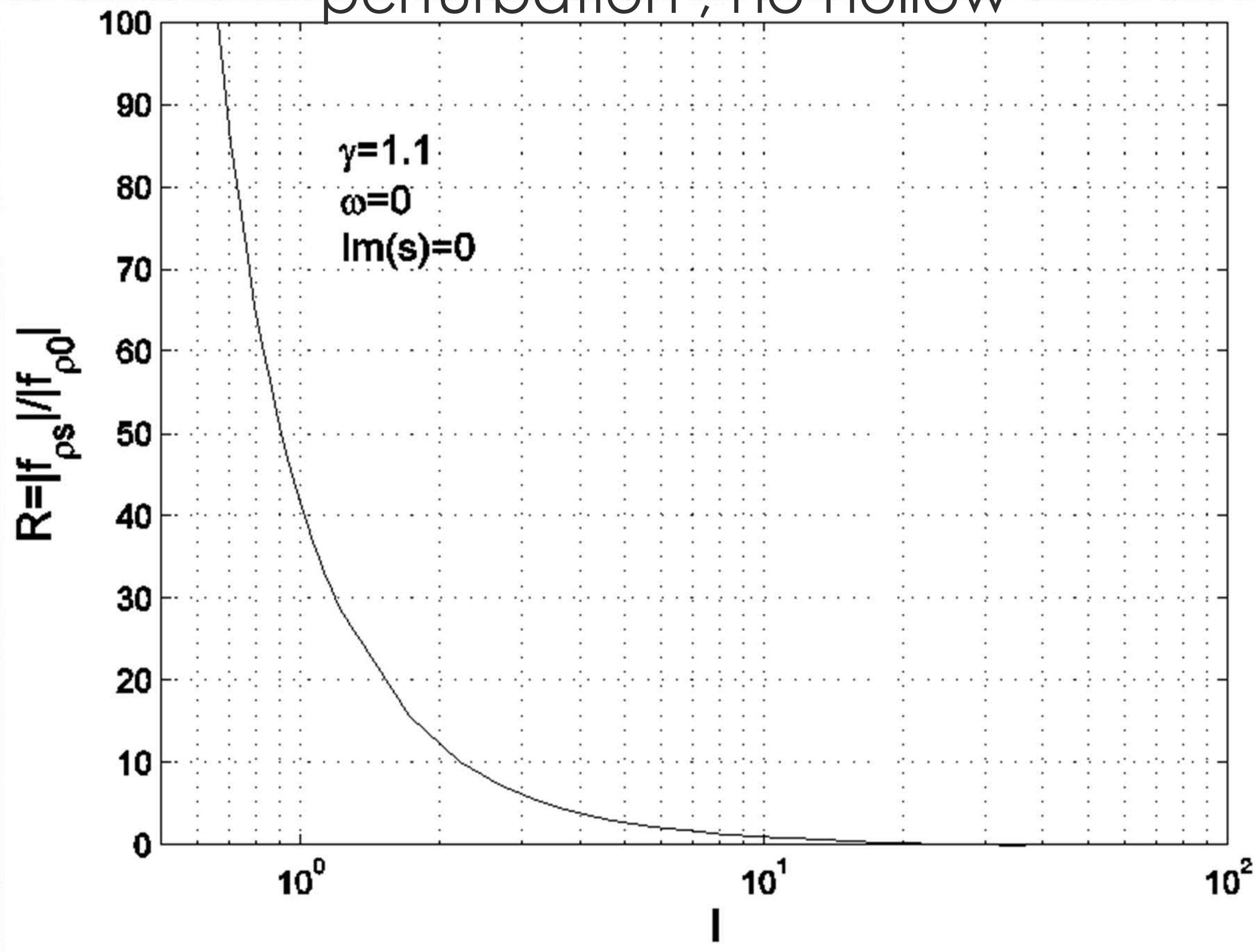
Amplification velocity factor for vortex perturbation, hollow case



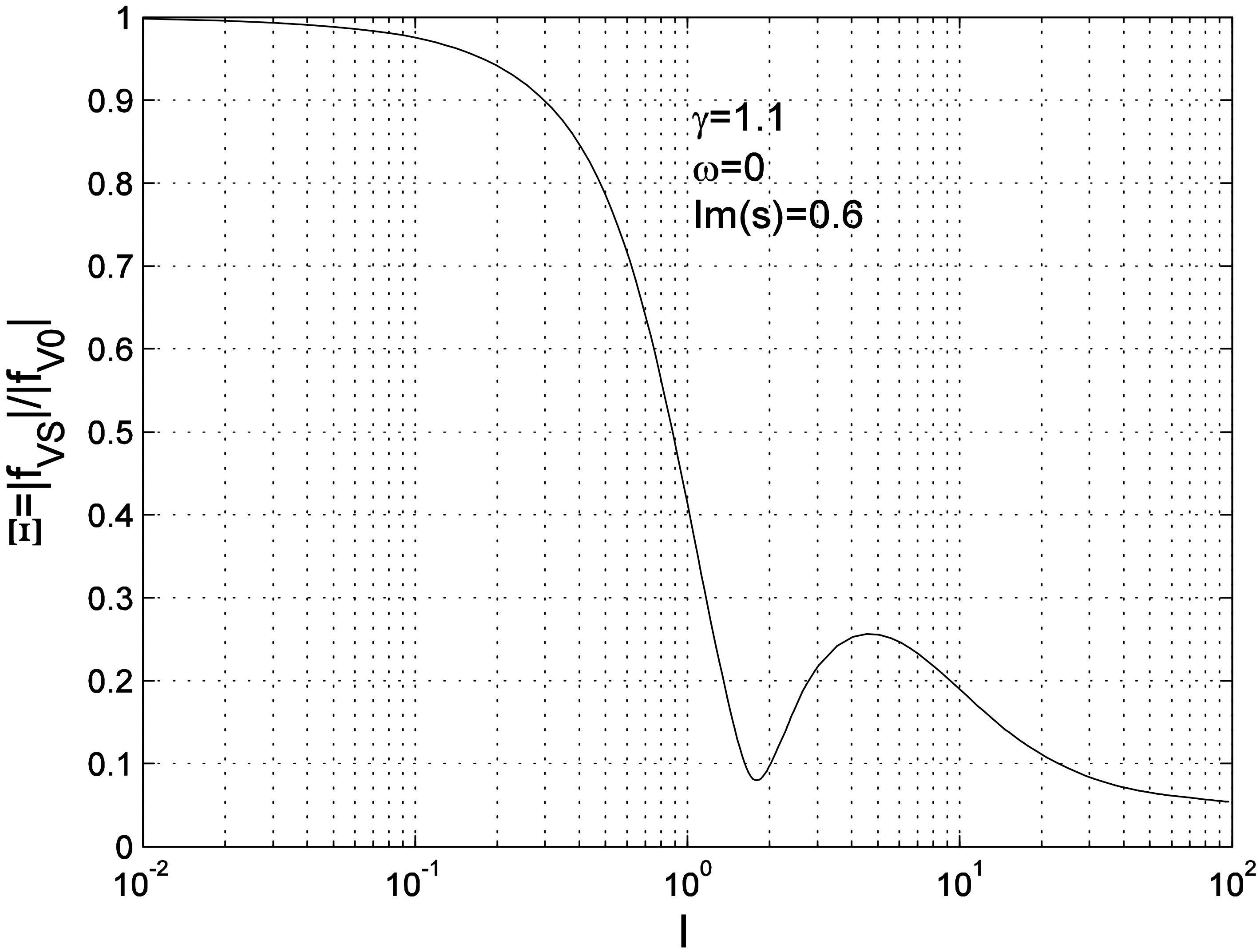
Inhomogeneous medium ω=2.7, γ=4/3, hollow case



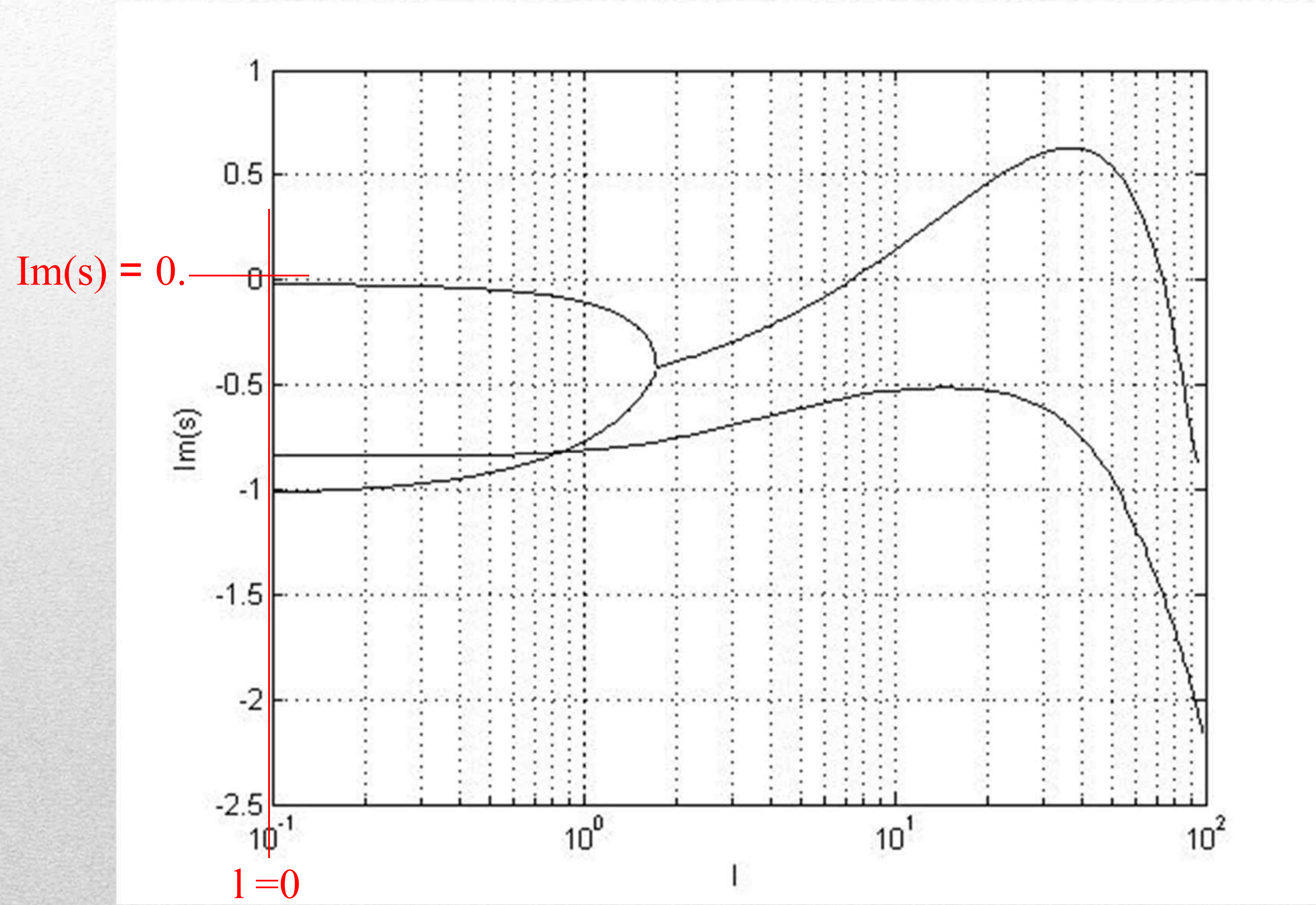
Amplification density factor for entropy perturbation , no hollow

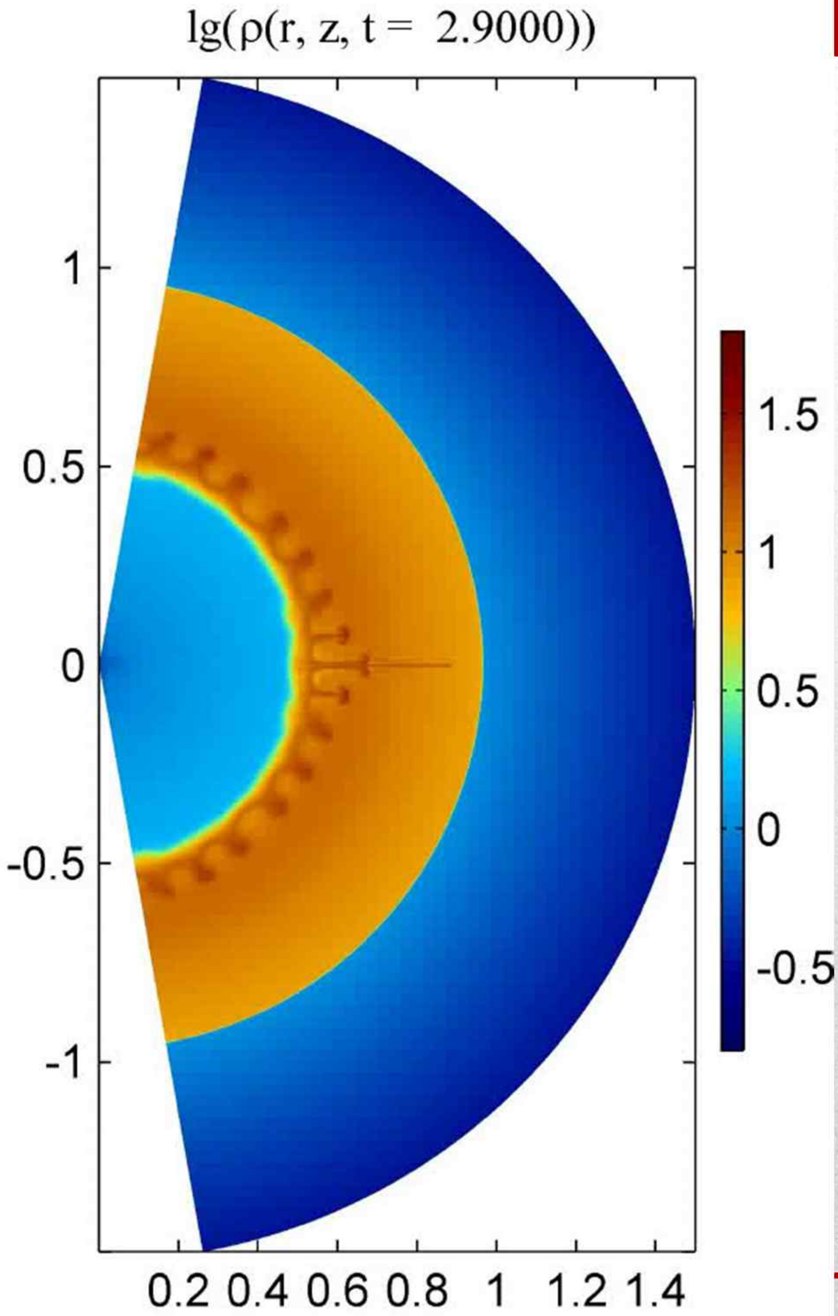


Amplification velocity factor for vortex perturbation, no hollow



Homogeneous medium ω=0, γ=1.1, no hollow





2-d numerical model

Hollow case
Inhomogeneous medium:
power-law distribution of density with index ω=2.7,
adiabatic index γ=4/3.
vortex perturbation:
harmonic number l=18
perturbation amplitude = 1%

Conclusion

- This work presents in a sense a sequel of research by Ryu and Vishniac (1987-1991) who studied stability of adiabatic spherical shock against self-excited oscillations.
- We instead analyze stability of adiabatic spherical shock against forced oscillations when shock propagates into an ambient medium with small inhomogeneities of density and velocity. These mimic the influence of turbulent motion or clouds segregation in the interstellar medium on the dynamics of supernova remnant at the adiabatic Sedov stage.
- We show by linear analysis and 2D hydrodynamical modeling that the shock wave is subject to resonant amplification of certain spherical harmonics in both cases. The instability found can bring to both rapid star formation and turbulization of interstellar gas.