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Nonlinear drive of linearly damped modes by phase-space structures.

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Motivation

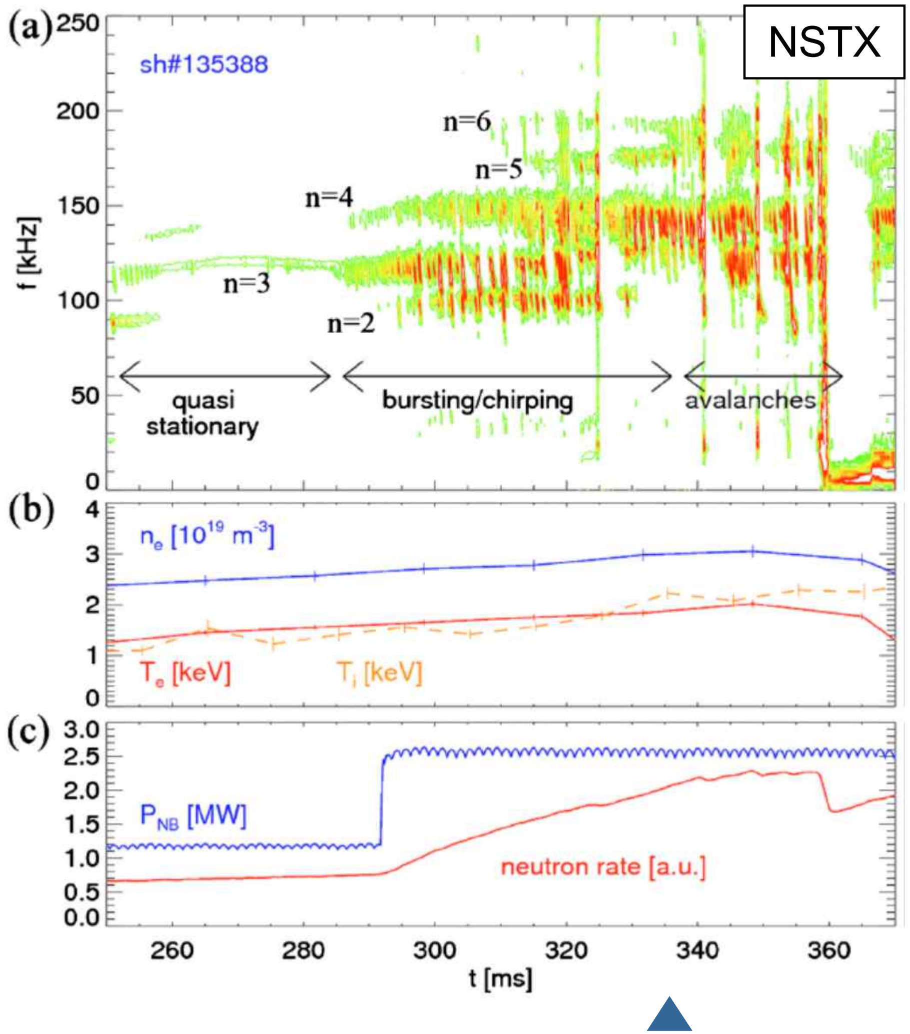
- **High-energy ions** drive Alfvén Eigenmodes, which may lead to their premature ejection.

- Fast particles transport and loss depend on the generation and evolution of **phase-space structures** (holes, clumps, blobs, granulations…)

  Dupree, et al., PF (82)
  Berman, et al., PF (85)
  Berk, et al., PoP (99)
  Kosuga, et al., PoP (10)

- Linearly damped modes can be driven by these structures (**subcritical instabilities**)

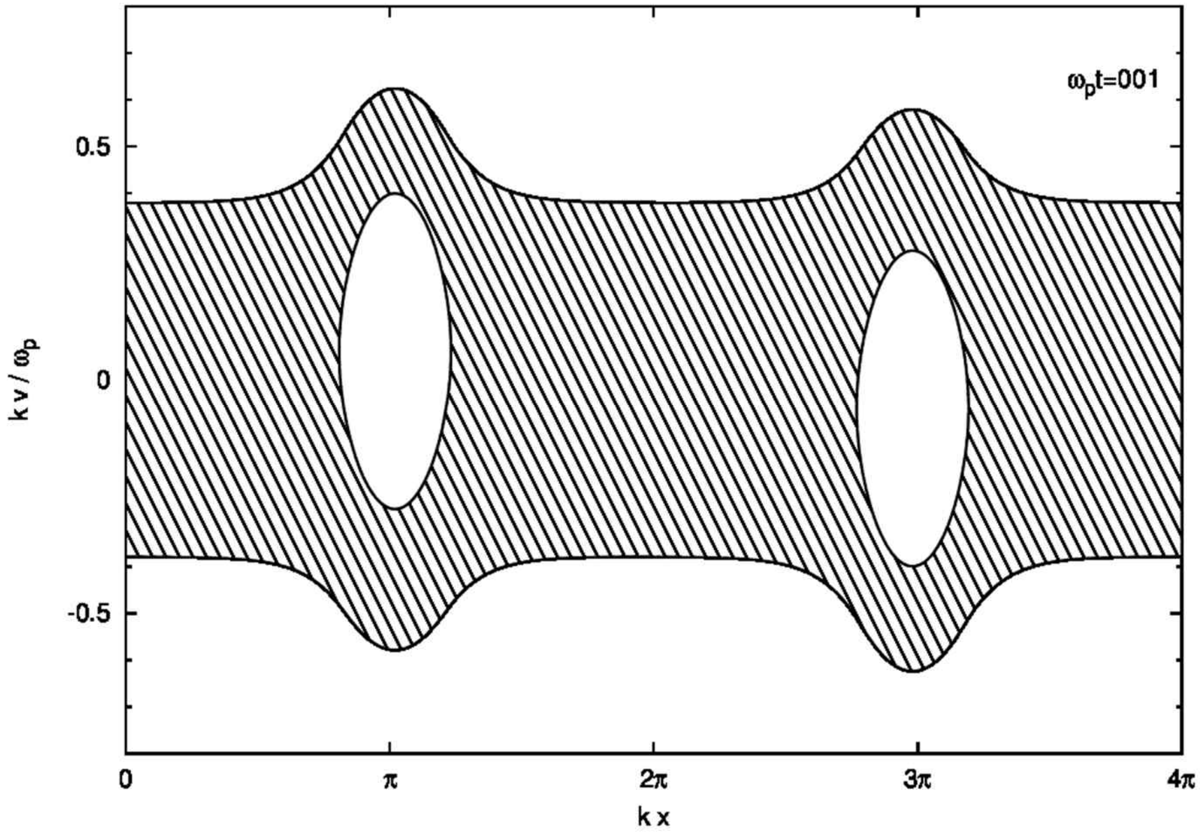
  Berk, Breizman, et al., PoP (99)



Podestà, et al., PoP (10)

Approach

- **First step:** single resonance, which features isolated, long-lived coherent structures (Kubo >> 1)



Self-binding structures (water-bag model with 2 BGK holes).

Roberts and Berk, PRL(67)
Berk, PF(70)

↔ 2D fluid in a gravitational field

↔ negative-mass instability

↔ 

- **BB model** as a tractable model with key ingredients, with qualitative and quantitative similarities with experiments

Berk, Breizman, et al., PFB(90), PRL(96), PoP(99)

Outline

I) Berk-Breizman model
II) Momentum exchange
III) Phasestrophy growth

Summary

Perspectives

The BB model

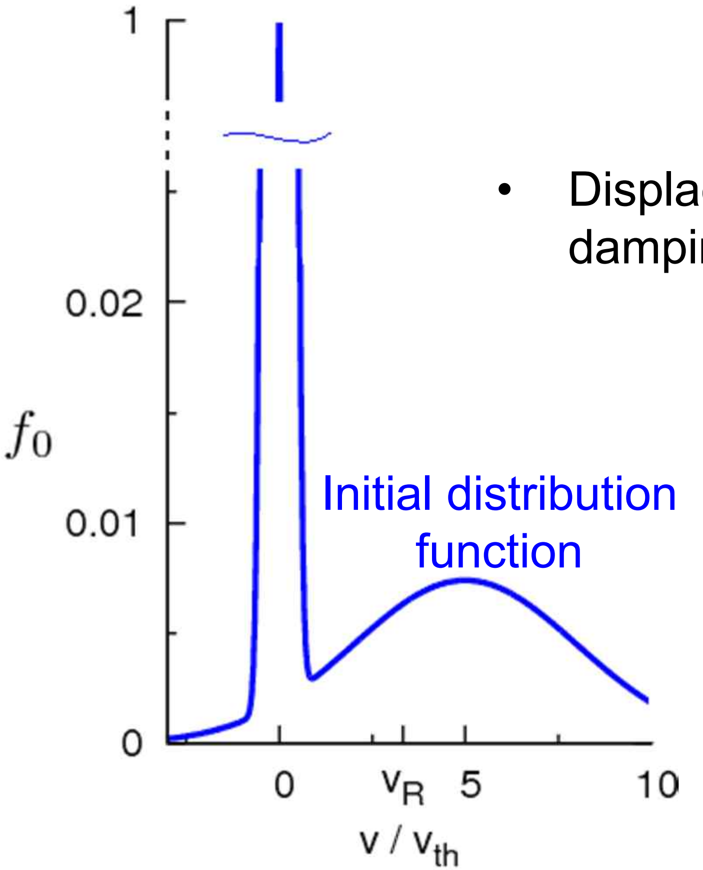
Berk, et al., PoP(95)

Classic "bump-on-tail" instability, with collisions and external damping.

- 1D kinetic equation with a collision operator including dynamical friction (drag), and velocity-space diffusion,

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial v} = C(f - f_0)$$

This work: collisionless case $C(f - f_0) = 0$



- Displacement Current Equation with an external wave damping accounting for background dissipative mechanisms,

$$\frac{\partial E}{\partial t} = -4\pi q \int v(f - \bar{f})\mathrm{d}v - 2\gamma_d E$$

- Single electrostatic wave, $E(x,t) = \hat{E}_k(t)e^{\imath kx} + \text{c.c.}$

COBBLES

COnservative **B**erk-**B**reizman semi-**L**agrangian **E**xtended **S**olver

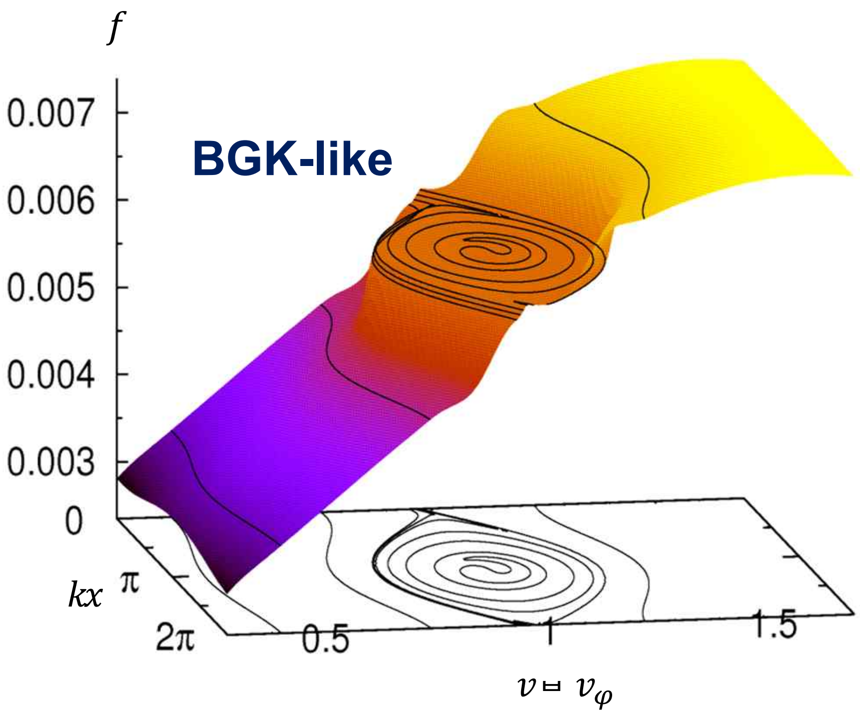
Lesur, et al., JAEA-R(07)

Lesur, et al., PoP(09)

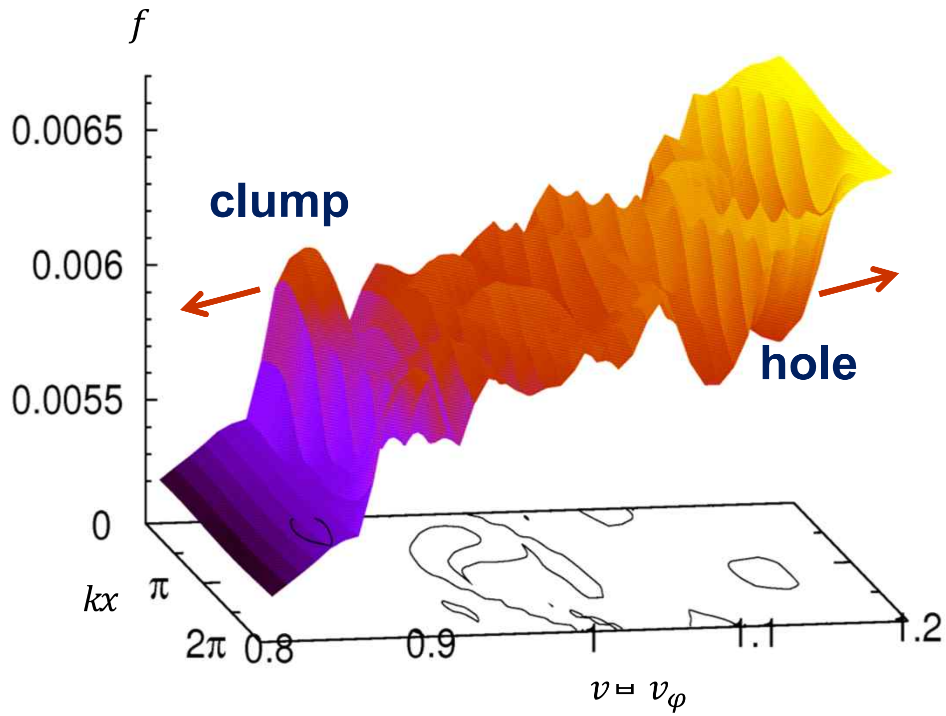
Holes and clumps

The presence of enough damping leads to the creation and evolution of phase-space structures. This self-organisation process allows to tap much more free energy to compensate for γ_d.

Berk, Breizman, Petviashvili, PLA(97)



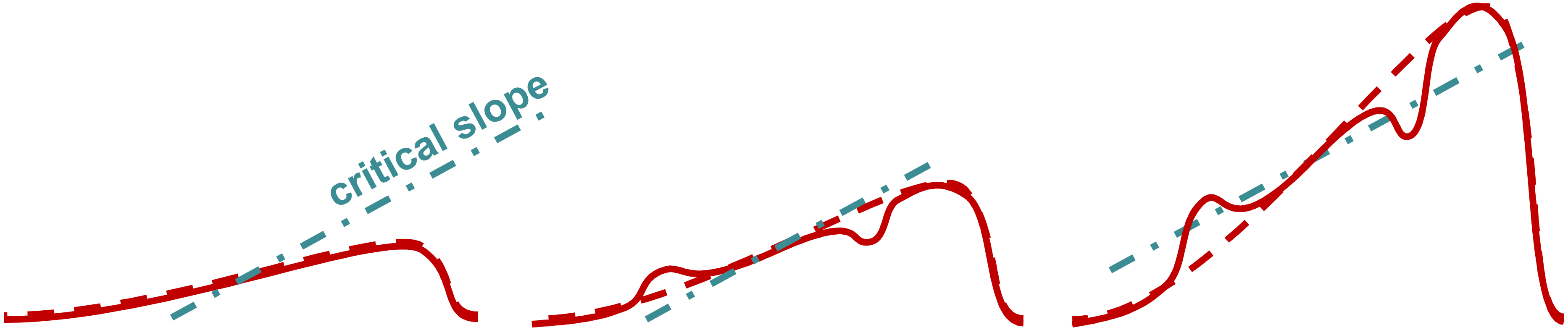
No damping ($\gamma_d = 0$)



Finite damping ($\gamma_d \sim \gamma_L$)

Subcritical instability

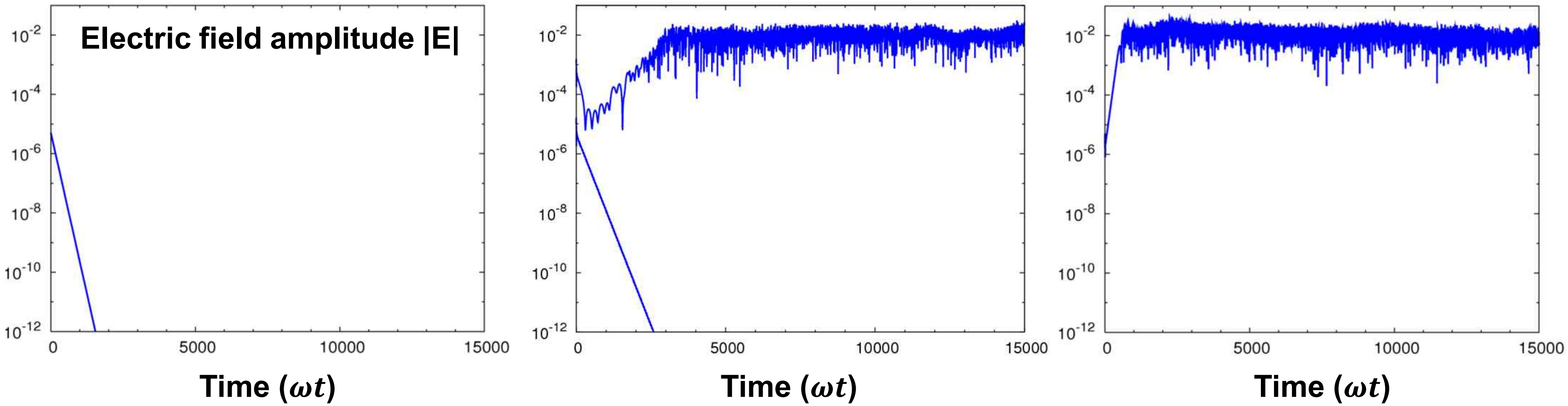
Linear growth rate $\gamma \approx \gamma_L - \gamma_d \quad \Rightarrow \quad$ Critical slope $\gamma_L = \gamma_d$



Stable, $\gamma < 0$

Nonlinearly unstable, $\gamma < 0$ (Subcritical instability)

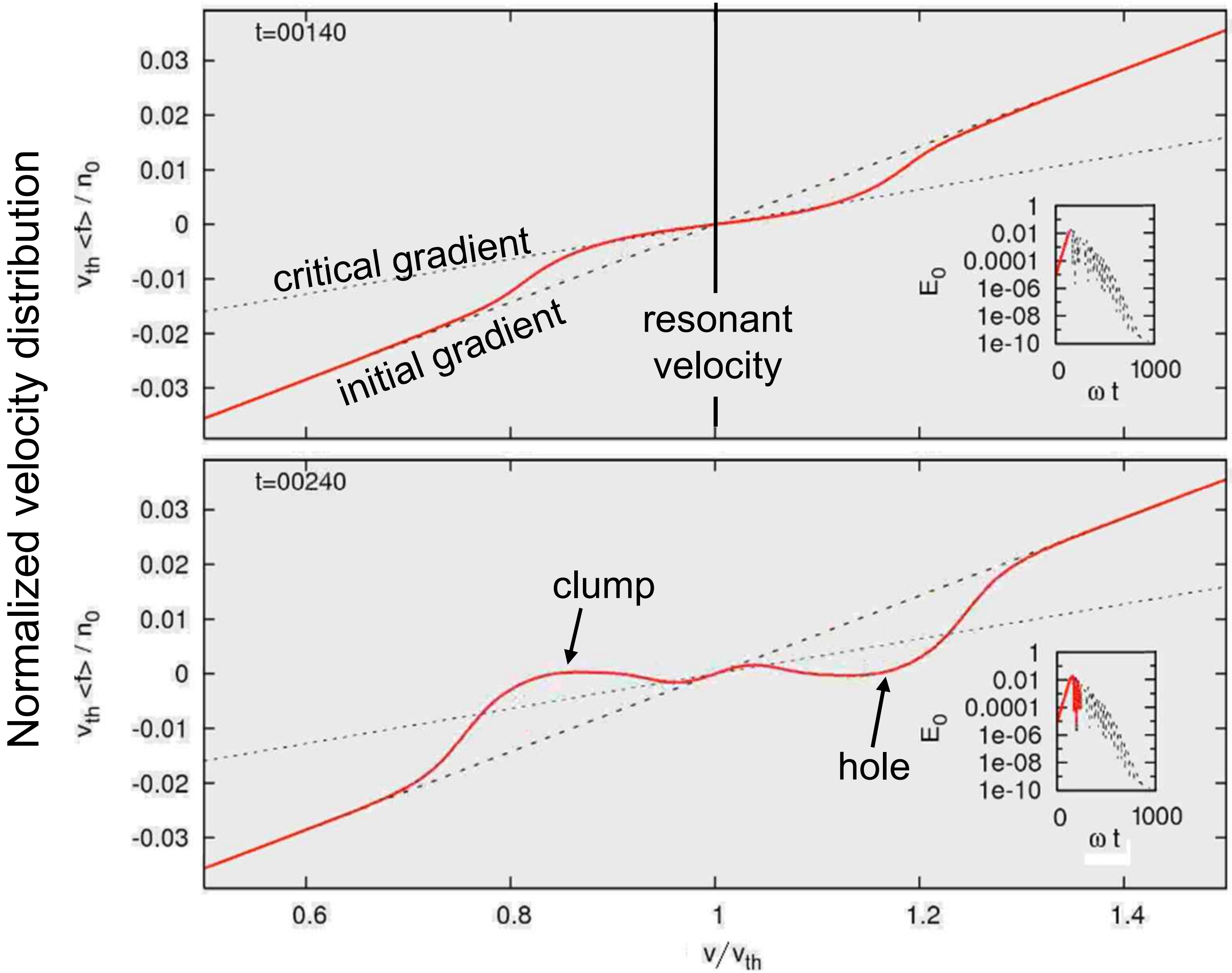
Unstable, $\gamma > 0$



Navigation

Symmetric case

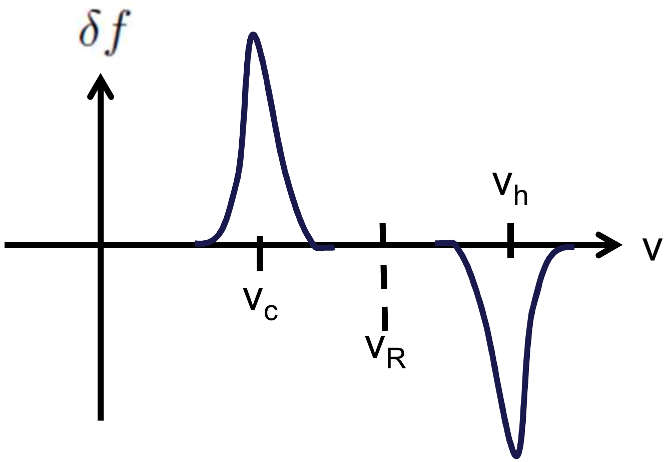
In *δf* simulations with $f_0(v)$=const., we observe symmetric hole/clump pairs



Momentum exchange

$$f = f_0(v) + \delta f(x, v, t)$$

$$\langle p \rangle \equiv \int v \, \delta f \, \mathrm{d}v$$



Without dissipation, $\frac{\mathrm{d} \langle p \rangle}{\mathrm{d}t} = 0$

hole momentum must balance clump momentum

Incompatible with symmetric hole/clump pairs, where $v_h \langle p \rangle_c = v_c \langle p \rangle_h$

With dissipation, $\frac{\mathrm{d} \langle p \rangle}{\mathrm{d}t} = \int \langle E \delta f \rangle \, \mathrm{d}v$

holes and clumps can exchange momentum with the wave

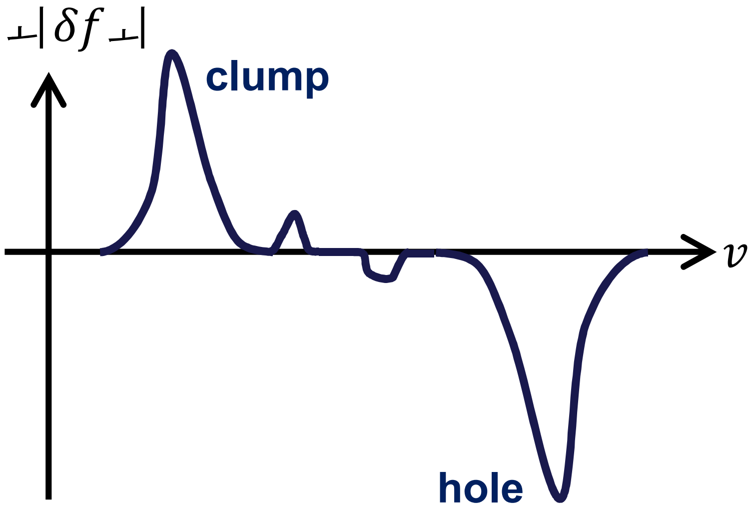
$\Rightarrow$ Wave/particles momentum exchange is a mechanism of hole/clump creation and growth

Conditions/threshold for existence of phase-space structures?

Navigation

Phasestrophy

$$f = f_0(v) + \delta f(x, v, t)$$



Phasestrophy is the phase-space density auto-correllation,

$$\Psi \equiv \int \langle \delta f^2 \rangle \, \mathrm{d}v$$

Diamond, Itoh^2, Modern Plasma Physics
Kosuga, Diamond, to be published

In parallel with potential enstrophy in quasi-geostrophic systems,

$$\mathcal{E} \equiv \langle \delta q^2 \rangle$$

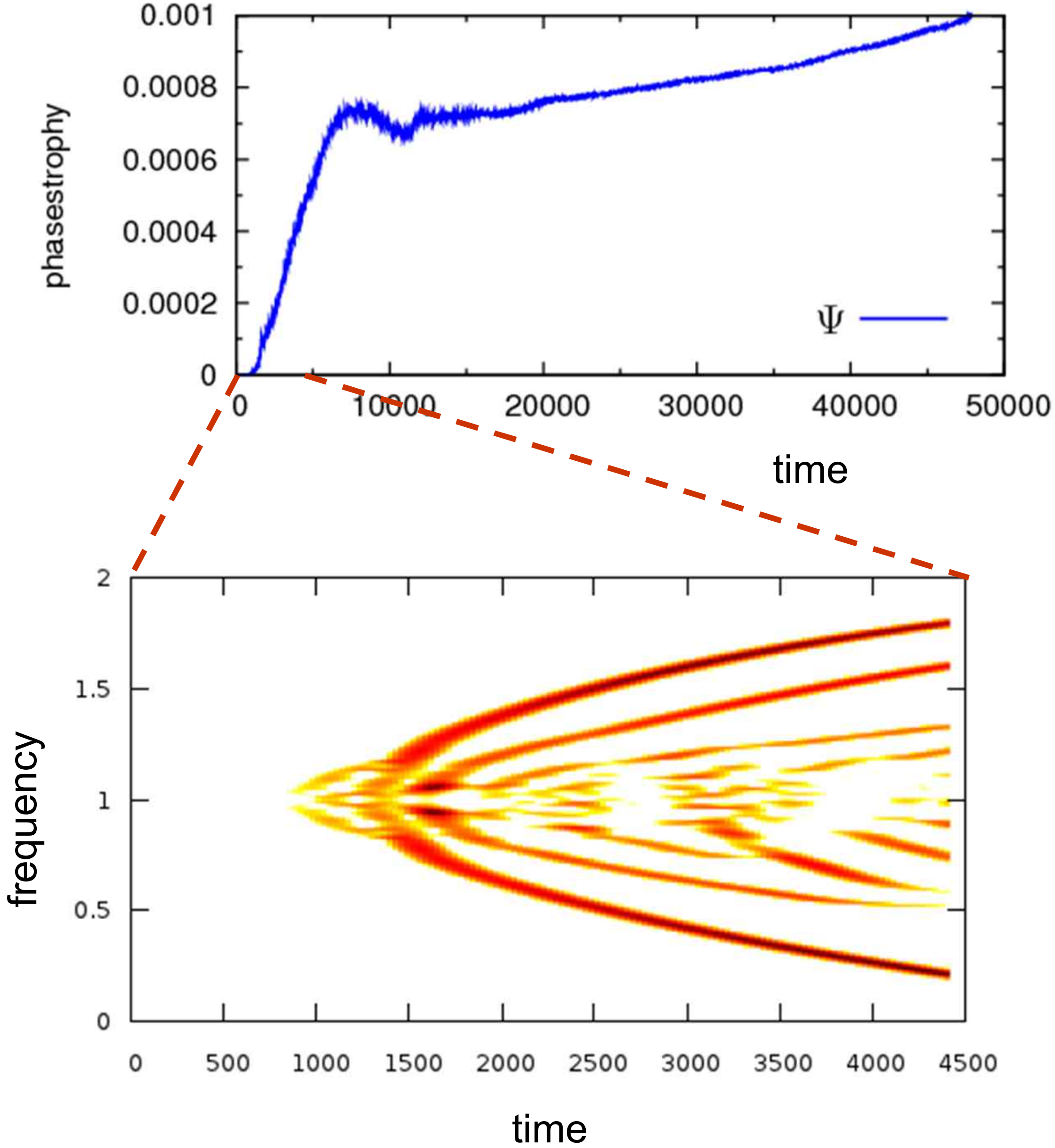
(q is the potential vorticity)

Relation with relative entropy:

If $f_0(v)$=const., $\Psi = 2 f_0 \, \delta S + O\left[\left(\frac{\delta f}{f_0}\right)^3\right]$

with $\delta S \equiv \int \left\langle f \log \frac{f}{f_0} \right\rangle \mathrm{d}v$

Phasestrophy growth



Total phasestrophy grows in time as holes and clumps are continuously created.

The growth is nonlinear.

Phasestrophy growth is proportional to E-f correlation

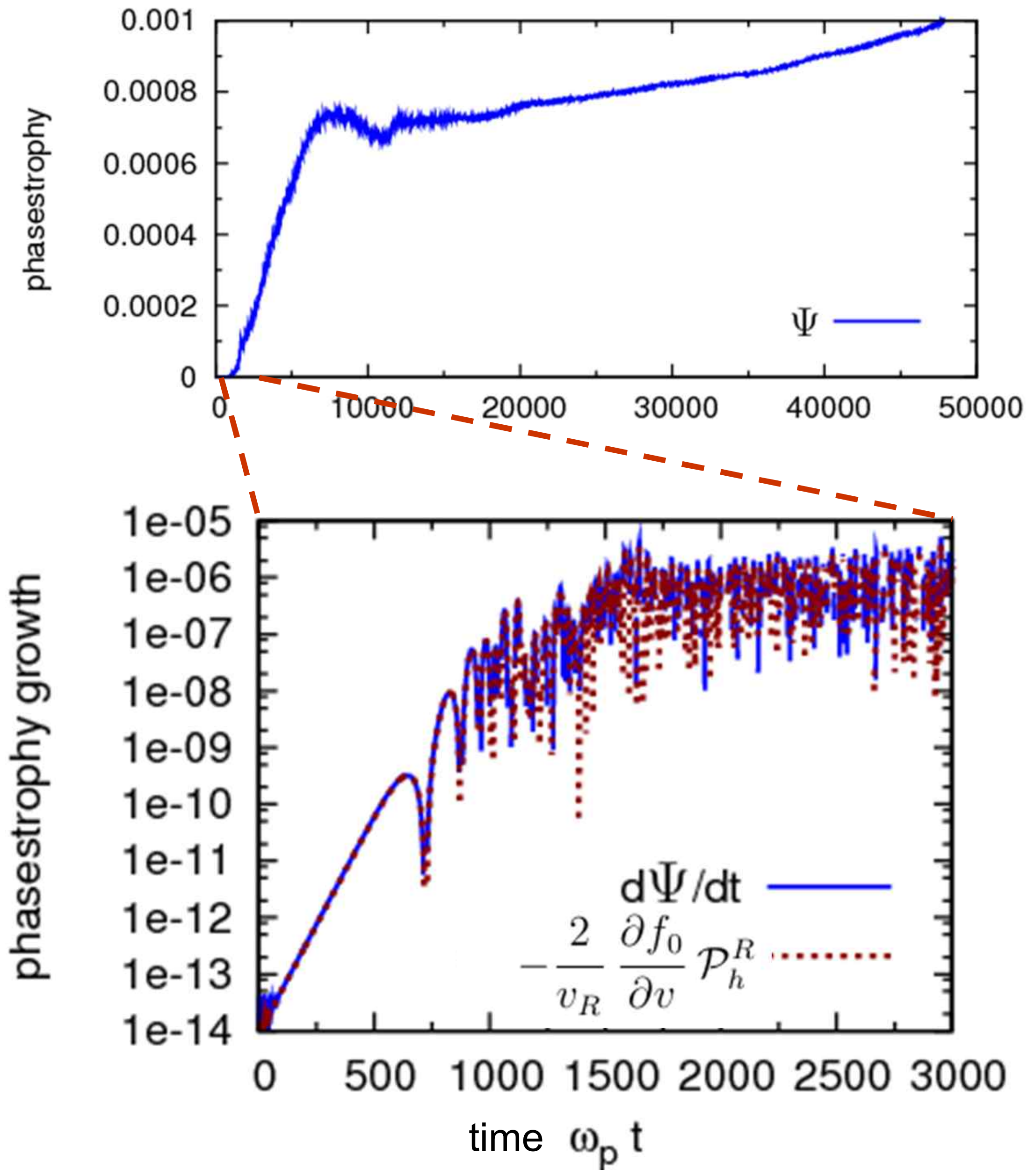
$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -2\frac{\partial f_0}{\partial v}\int \langle E\delta f\rangle\,\mathrm{d}v$$

(assuming a constant slope for f_0)

Also proportional to change in momentum associated with h/c evolution

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -2\frac{\partial f_0}{\partial v}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

Drive by dissipation



Resonant part of particle/wave energy transfer

$$
\begin{aligned}
\frac{\mathrm{d}\Psi}{\mathrm{d}t} &= -\frac{2}{v_R}\frac{\partial f_0}{\partial v}\mathcal{P}_h^R \\
&= \frac{2}{v_R}\frac{\partial f_0}{\partial v}\left(\frac{1}{2}\frac{\mathrm{d}E_0^2}{\mathrm{d}t} + \gamma_d E_0^2\right) \\
&= \frac{2}{v_R}\frac{\partial f_0}{\partial v}\left(\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}t} + \mathcal{P}_d\right)
\end{aligned}
$$

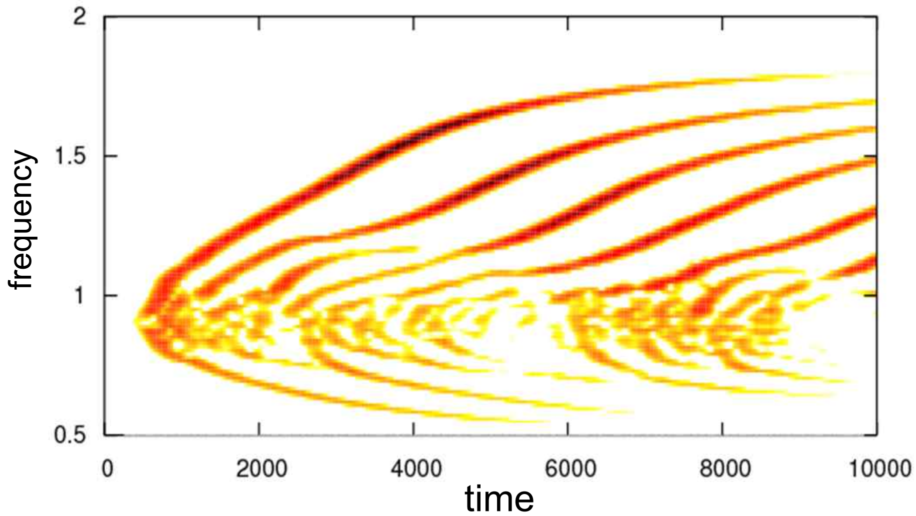
Variation of total wave energy, which includes sloshing energy.

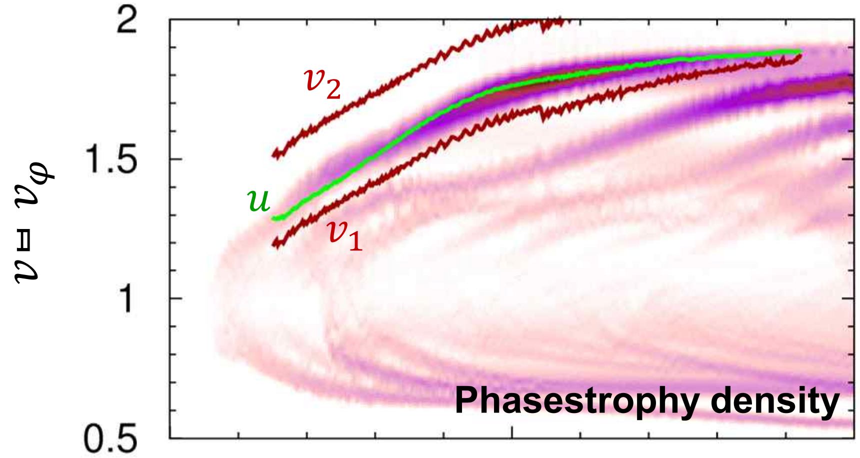
Power due to dissipation

$$\Rightarrow \quad \frac{\mathrm{d}\Psi}{\mathrm{d}t} \sim \mathcal{P}_d$$

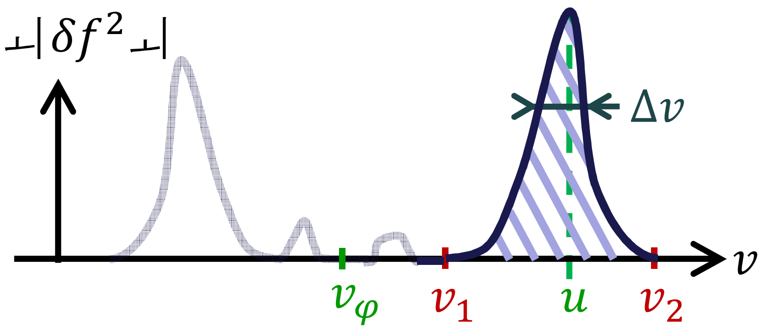
in quasi steady-state

Single structure

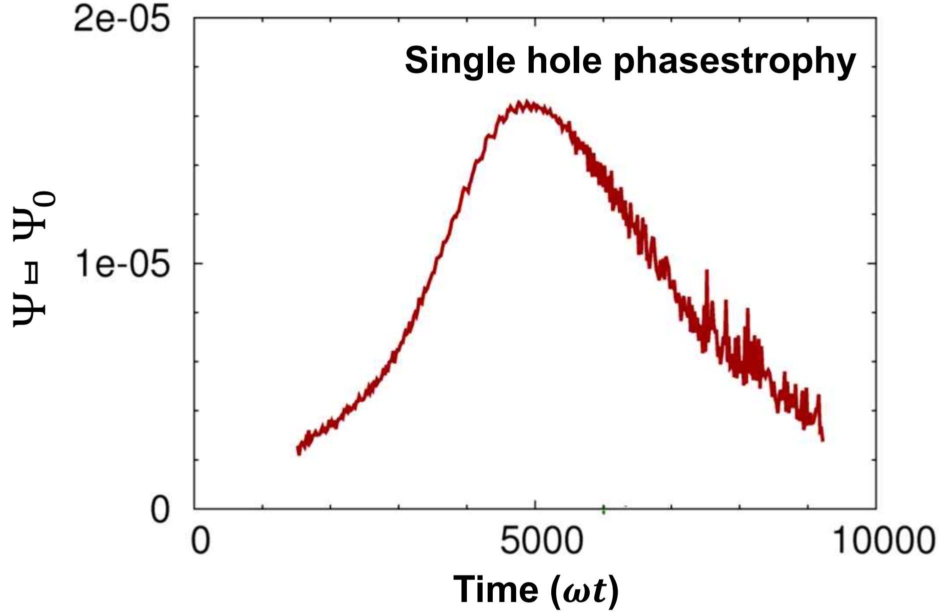




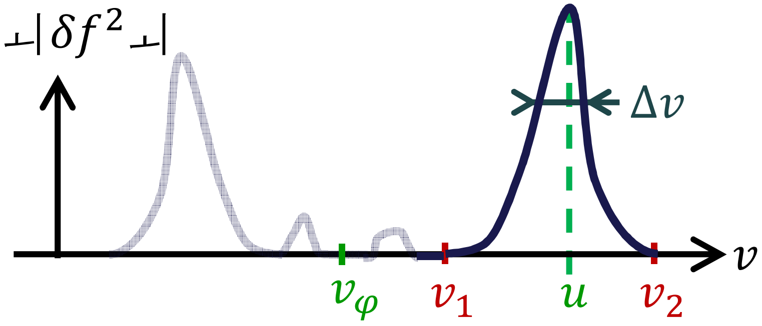
We track a single phase-space structure



$$\Psi \equiv \int_{v_1}^{v_2} \langle|\delta f^2|\rangle \, dv$$



Hole/clump growth rate



In the spirit of Dupree's theory of phase-space density holes,

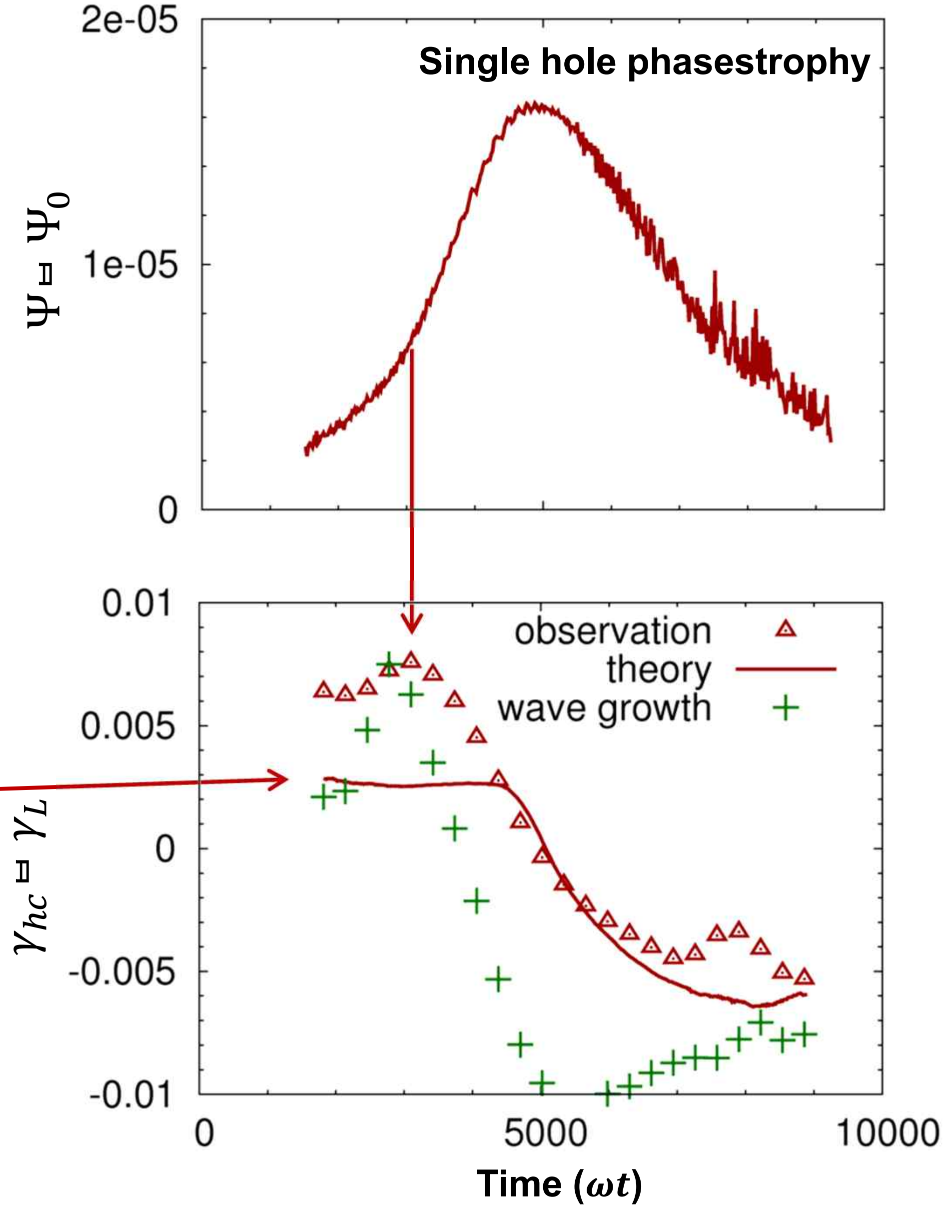
Dupree, PF(82)

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = 2\gamma_d \frac{\partial f_0}{\partial v} \int dv' \int dv \frac{v' \langle \delta f(v') \delta f(v) \rangle}{(ku)^2 + (2\gamma_d)^2}$$

$$\gamma_{hc} \equiv \Delta v \left.\frac{\partial f_0}{\partial v}\right|_u \frac{2\,\gamma_d\, u}{(ku)^2 + (2\gamma_d)^2}$$

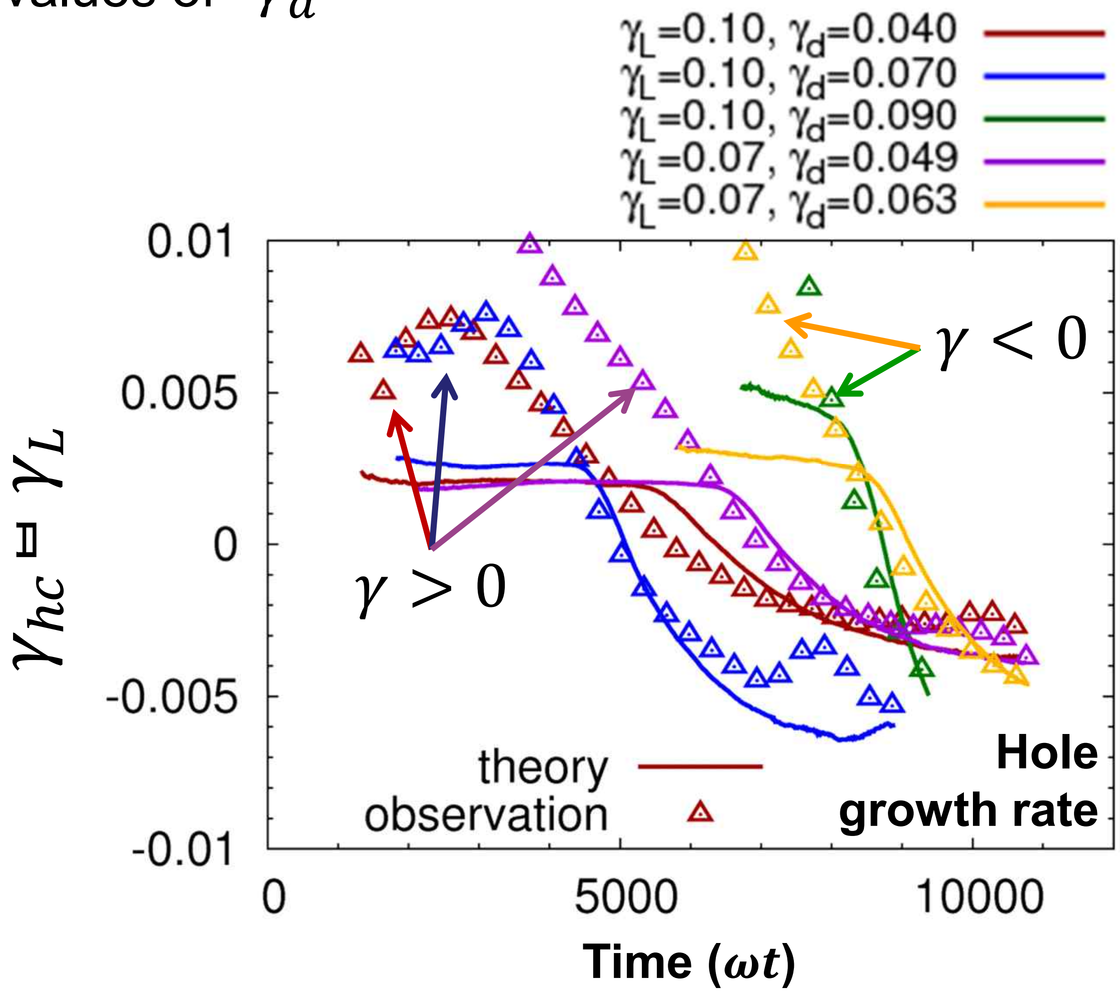
Diamond, Kosuga, Lesur,
Festival de Théorie 2009, Aix

Our simulations show a qualitative agreement with theory



Scaling with damping rate

We repeat the analysis for different distribution functions, and different values of γ_d



γ_{hc} is independent of the sign of γ

$\Rightarrow$ New understanding of subcritical instability

Nonlinear growth only requires $\gamma_d > 0$

(and a large enough seed)

In a nutshell

- Growth of phase-space structures, quantified by phasestrophy growth, can be related equivalently to:
    - Momentum exchange between the structure and the wave,
    - Damping power.
- For a single hole or clump, a new theory gives a simple expression for the structure growth rate, $\gamma_{hc} \sim \gamma_L \, \gamma_d$. Qualitative agreement with simulations. Some discrepancy at the beginning of chirping.
- New interpretation for nonlinear drive of instability

Take-out message

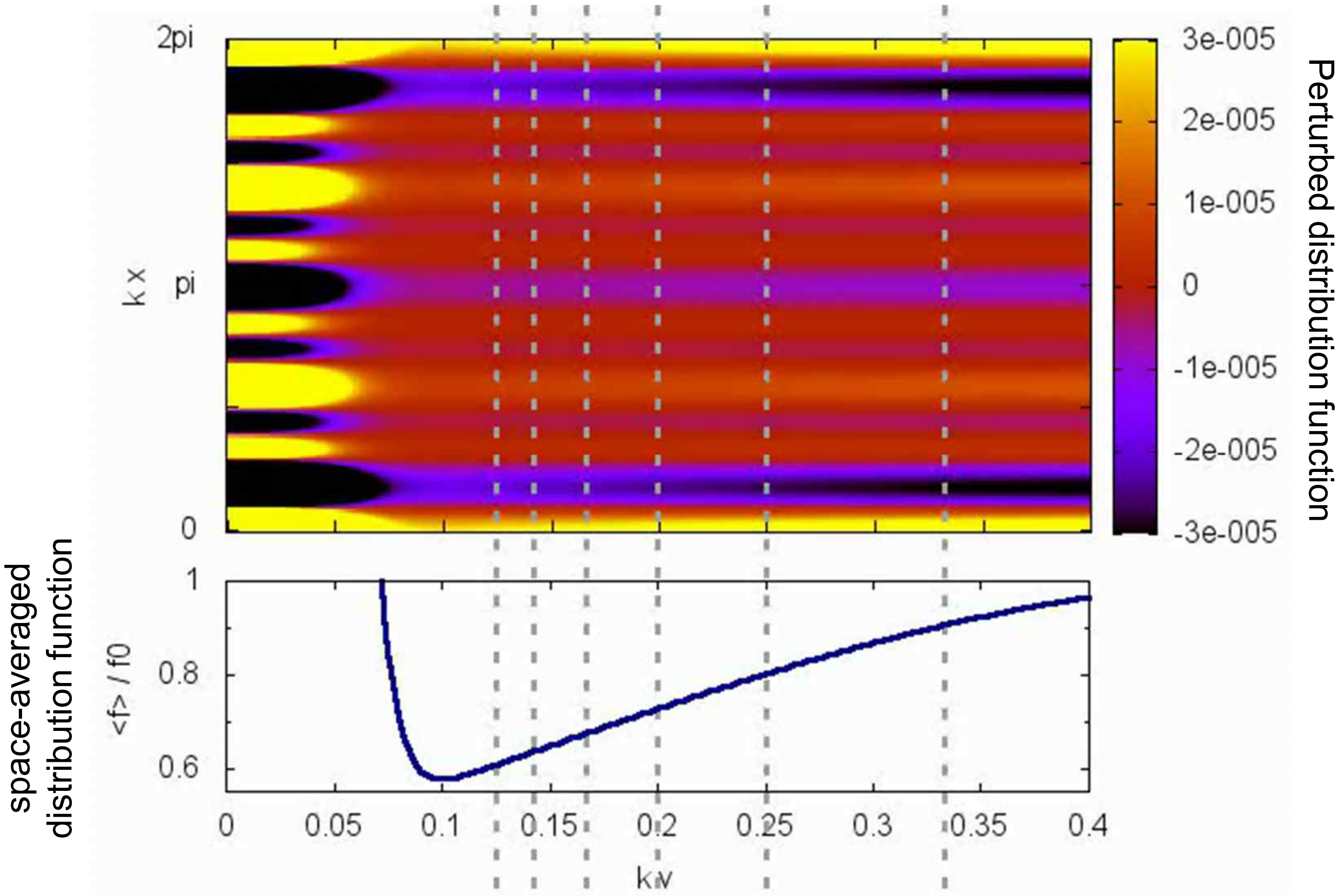
Importance of phase-space structures: can drive instabilities, transport, …

Perspectives

- Improvements on this work (minimum seed, validity range, effect of collisions).
- Application to drift wave holes and their interaction with zonal flow.
- Multiple resonances (next slide)

Phase-space turbulence

- Extension of quasi-linear theory with interacting phase-space structures.
  $\Rightarrow$ **“phase-space turbulence”**



Multiple-resonances simulation showing coherent phase-space structures, which disappear on a collisional diffusion time (not on a quasi-linear diffusion time).