Statistical properties of supersonic turbulence in the Lagrangian and Eulerian frameworks[1]

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Overview

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  - Forcing module
  - Tracer particles
- Results
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  - Lagrangian and Eulerian structure functions
    - Statistical theory of the large-scale velocity increments
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Simulation and Methods

- FLASH3
- Grid-Resolution: 1024^3
- Periodic boundary condition
- Isothermal gas,
  i.e. the pressure $P = \rho c_S^2$
- Initial condition:
  - Uniform density: $< \rho >_V = 1$
  - Velocity field: $v(t = 0) = 0$

Hydrodynamical equations

$$\frac{\partial s}{\partial t} + (\mathbf{v} \circ \nabla)s = -\nabla \circ \mathbf{v} \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \circ \nabla)\mathbf{v} = -c_\mathrm{s}^2 \nabla s + \mathbf{F}\,, \quad (2)$$

Forcing module

The random forcing term $\mathbf{F}$ is derived from a Ornstein-Uhlenbeck process

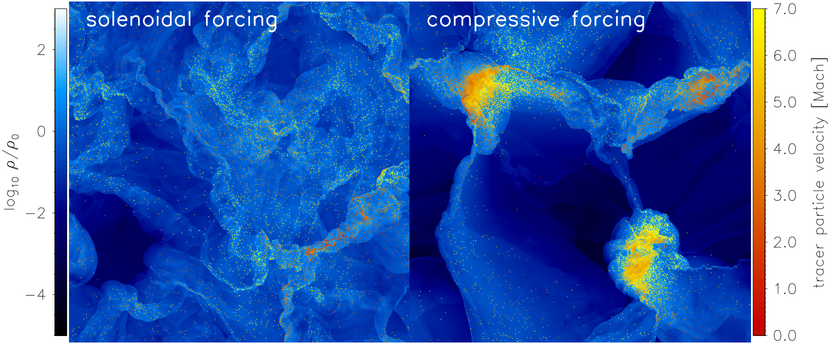
$$\mathrm{d}\widehat{\mathbf{F}}(\mathbf{k},t) = F_0(\mathbf{k}, T_{\mathrm{ac}})\mathcal{P}^{\zeta}(\mathbf{k})\frac{\mathrm{d}\mathbf{W}(t)}{T_{\mathrm{ac}}} - \widehat{\mathbf{F}}(\mathbf{k},t)\frac{\mathrm{d}t}{T_{\mathrm{ac}}} \quad (3)$$

- $\mathrm{d}\mathbf{W}(t)$ is a three-dimensional Gaussian random increment
- $\mathcal{P}^{\zeta}(\mathbf{k})$ is a projection tensor in Fourier space.
  $\zeta = 1$, $\mathbf{F}$ is purely solenoidal (i.e., $\nabla \circ \mathbf{F} = 0$),
  $\zeta = 0$, $\mathbf{F}$ is purely compressive (i.e., $\nabla \times \mathbf{F} = 0$)
- Forcing amplitude $F_0(\mathbf{k}, T)$, with
  $F_0(\mathbf{k}, T) \neq 0 \Leftrightarrow 1 <| \mathbf{k} |< 3$

Tracer particles

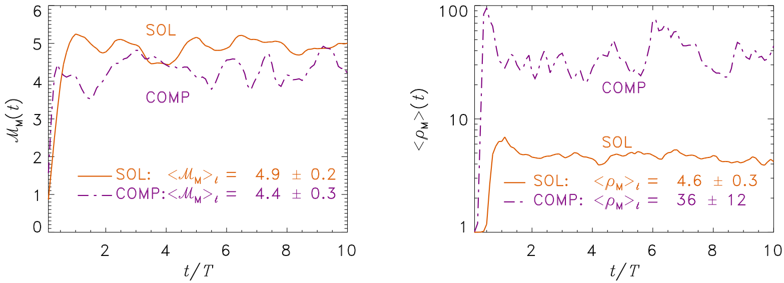
- Number of tracer particles: 512^3
- Tracer particles start uniformly distributed and at rest
- Cloud-in-cell interpolation for the velocity and density
- Euler method in times
- Passive tracers of the fluid motion

Results

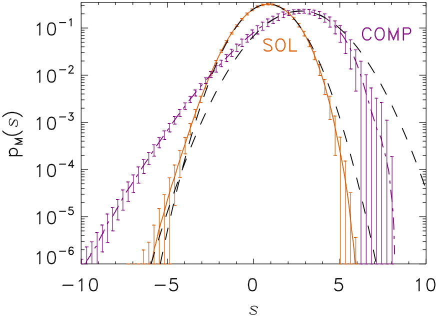


solenoidal forcing
compressive forcing
$\log_{10} \rho/\rho_0$
2
0
−2
−4
tracer particle velocity [Mach]
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0

RMS Mach number and the density



Probability density function of the gas density



- $s = ln(\rho/\rho_0)$
- sol :
  - $\sigma_s = 1.23$
  - Gaussian distributed
- comp:
  - $\sigma_s = 1.77$
  - non-Gaussian wings

$$\sigma \rho/\rho_0 = b\mathcal{M}$$

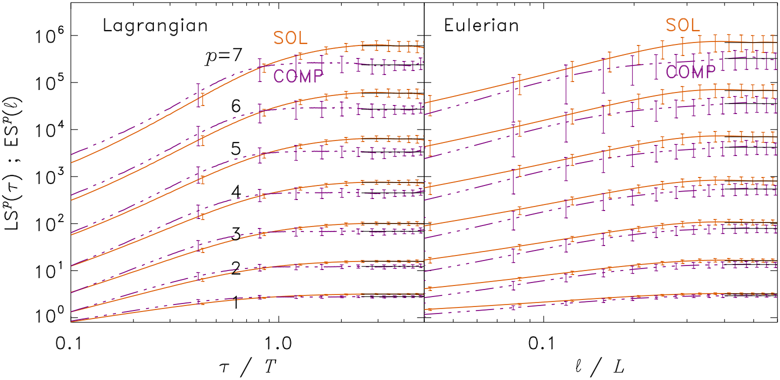
[Padoan et al. (1997), Passot & Vazquez-Semadeni 1998]

sol : $b = 0.36$

comp : $b = 1.08$

[Federrath et al. (2008, 2010)]

Lagrangian and Eulerian structure functions



Statistical theory of the large-scale velocity increments

Assumption:

- Autocorrelation function of the velocity vanishes for $\ell \to \infty$
- δv are Gaussian distributed for $\ell \to \infty$

$$S^p(\ell) = \langle |\delta v(\ell)|^p \rangle = \int |\delta v(\ell)|^p \, P(\delta v, \ell) \mathrm{d}(\delta v) \tag{4}$$

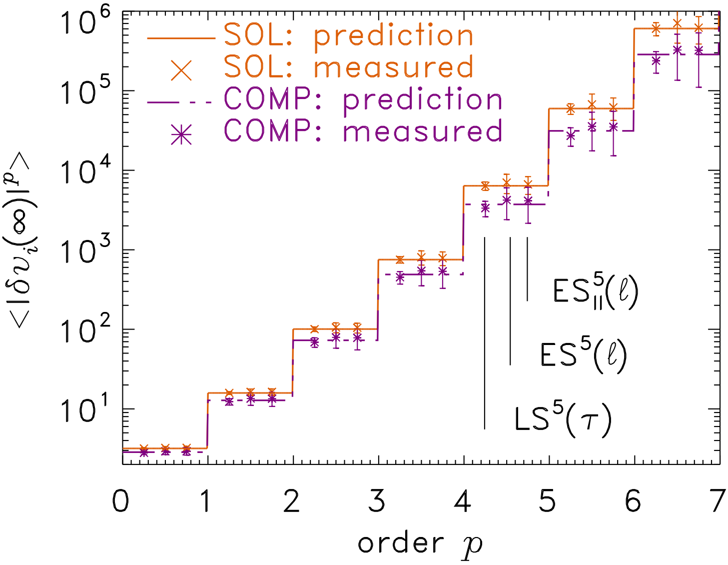
$$S^p(\ell \to \infty) = \frac{2}{\sigma\sqrt{2\pi}} \int_0^\infty (\delta v)^p e^{-\frac{(\delta v)^2}{2\sigma^2}} \,\mathrm{d}\,(\delta v) = \frac{\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}} (\sqrt{2}\sigma)^p \tag{5}$$

Statistical theory of the large-scale velocity increments

$$S^2(\ell) = \langle |\delta v(\ell)|^2 \rangle = \langle v(r+\ell)^2 \rangle + \langle v(r)^2 \rangle - 2\langle v(r+\ell) v(r) \rangle \quad (6)$$

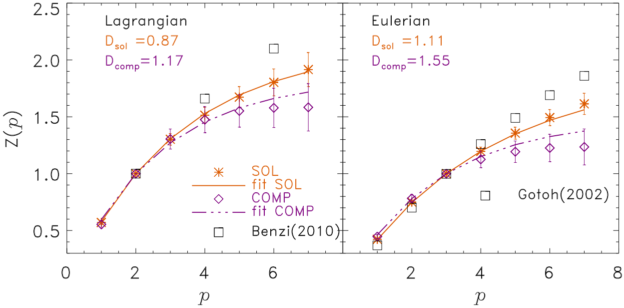
$$S^2(\ell \to \infty) = 2\mathcal{M}^2 c_{\mathrm{s}}^2 = \sigma^2 \quad (7)$$

$$\Rightarrow S^p(\ell \to \infty) = \frac{\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}} (\sqrt{8}\mathcal{M})^p \quad (8)$$



SOL: prediction
SOL: measured
COMP: prediction
COMP: measured
10^6
10^5
10^4
10^3
10^2
10^1
$<|\delta v_i(\infty)|^p>$
$ES_{\parallel}^5(\ell)$
$ES^5(\ell)$
$LS^5(\tau)$
0
1
2
3
4
5
6
7
order p

Scaling behaviour and intermittency



$$Z_E(p) = (1-\Delta_E)\frac{p}{3} + \frac{\Delta_E}{1-\beta_E}(1-\beta_E^{p/3})$$

$$D = 3 - \Delta/(1-\beta)$$

[She Leveque (1994)]

$\Delta = 2/3$ [She & Leveque 1994]

$\Delta = 1$ [Schmidt et al. 2008]

Summary and conclusion

We analysed hydrodynamical grid simulations (1024^3) with Lagrangain tracer particles (512^3) and examine the effects of purely solenoidal ($\nabla \circ F = 0$) and purely compressive ($\nabla \times F = 0$) forcing on the statistical properties of supersonic turbulence.
Results:

- $S^p(\ell \to \infty) = \frac{\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}} (\sqrt{8}\mathcal{M})^p$
- The Lagrangian framework exhibits a more intermittent behaviour than the Eulerian
- Compressive forcing yields a more intermittent behaviour than solenoidal

Thank you for your attention.
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arXiv.org : 1111.2748