6th Korean Astrophysics Workshop

Structures in MHD Turbulence

David H. Porter
University of Minnesota

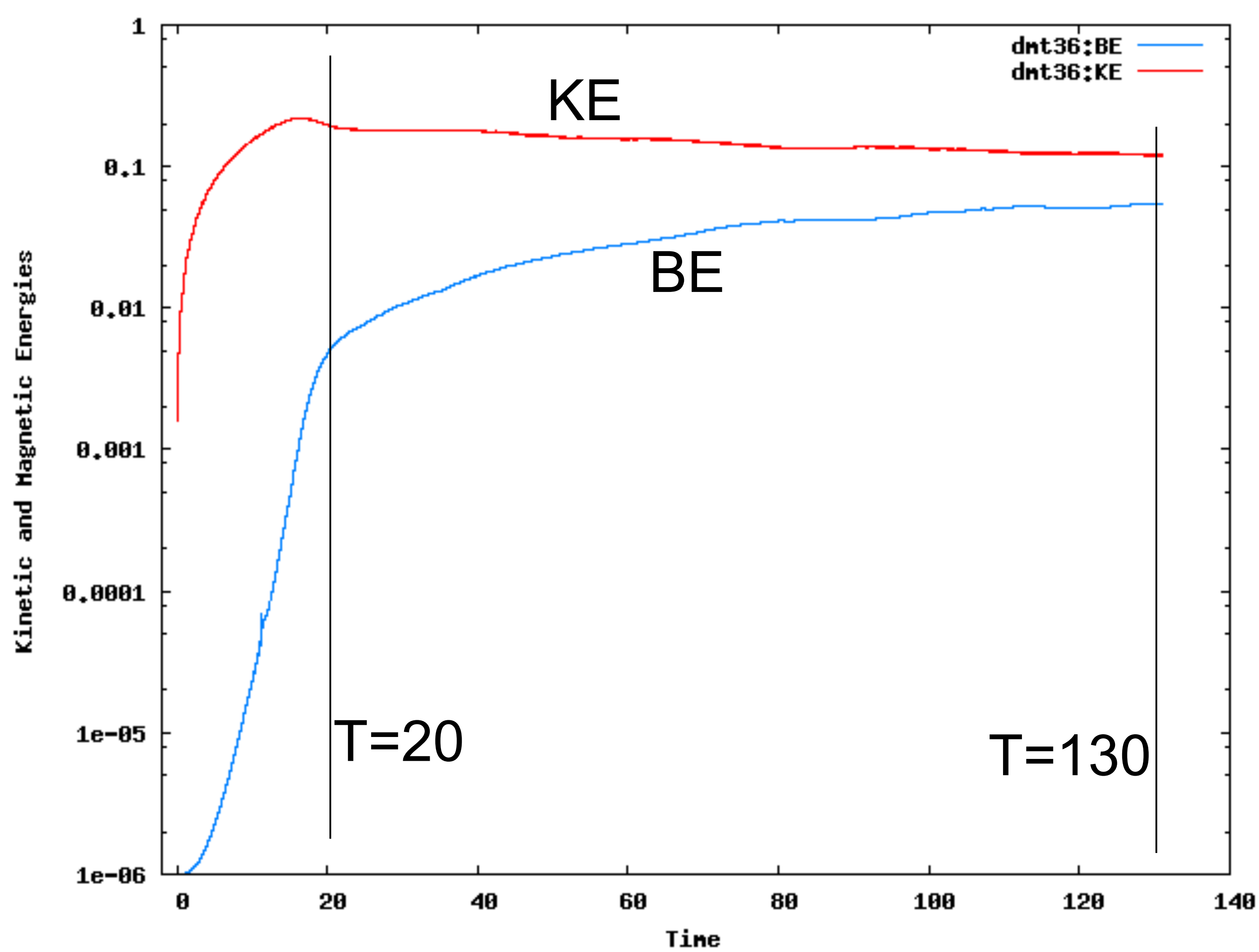
Collaborators: Tom Jones
Dongsu Ryu
Jungyeon Cho

Outline

- Differences between Neutral fluid and MHD Turbulence
  – Use simulation with initial beta = 10^6, solenoidal driving, 2048^3 mesh
  – T=20: high beta & developed velocity spectrum
  – T=130: B-field near saturation
- Structures in strong field MHD turbulence
  – Motivation: visualizations
  – Measures
    - Characterize
    - Automatically identify
    - Evaluate statistical significance
- Development structures
  – rate of strain field
  – time scale for development

Compare structures at T=20 & 130

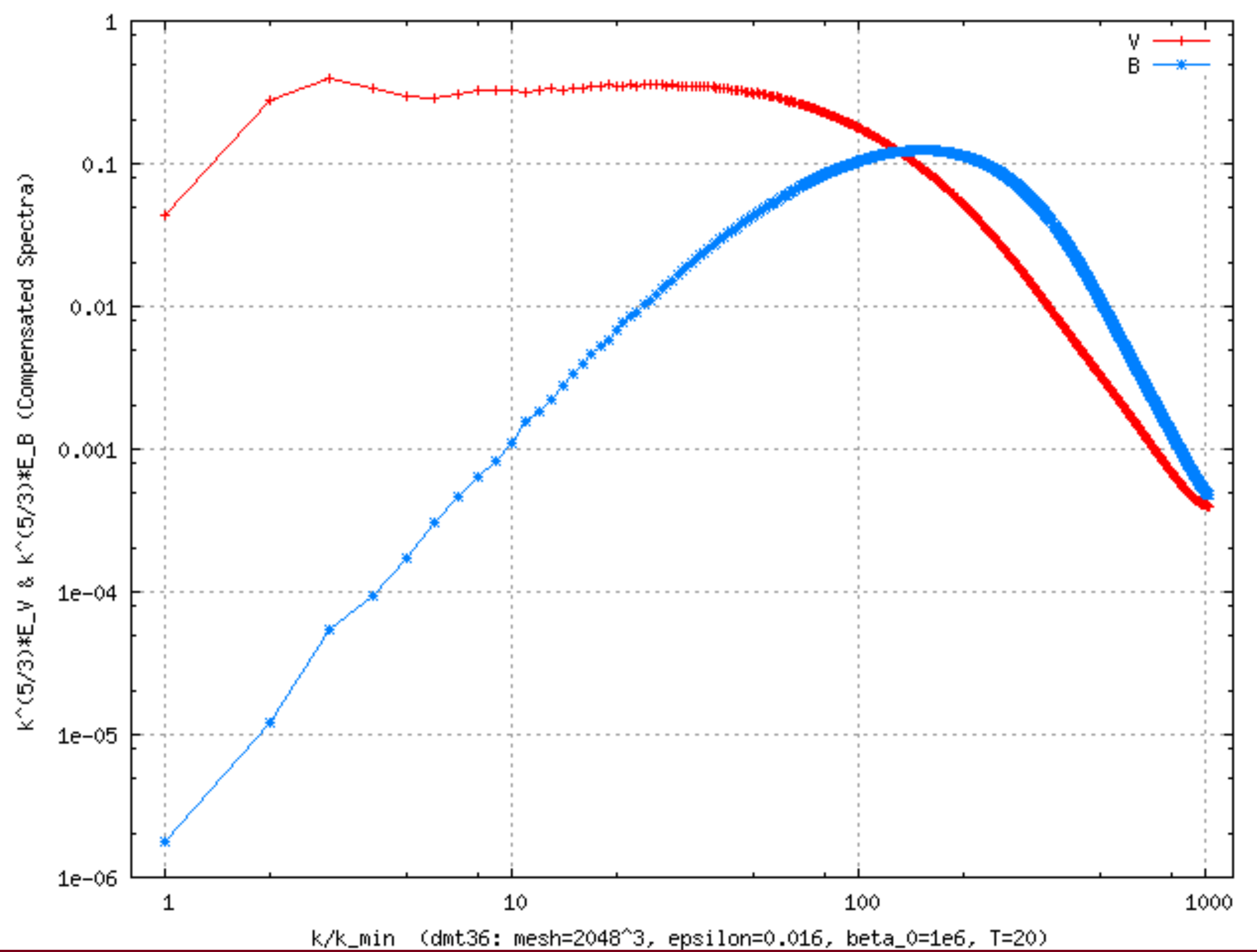
- T=20: Fluid Turbulence is fully developed & B-fields is weak
- T=130: B-field strong enough to dominate flow



Compensated V & B Spectra: T=20

V-field is fully developed.

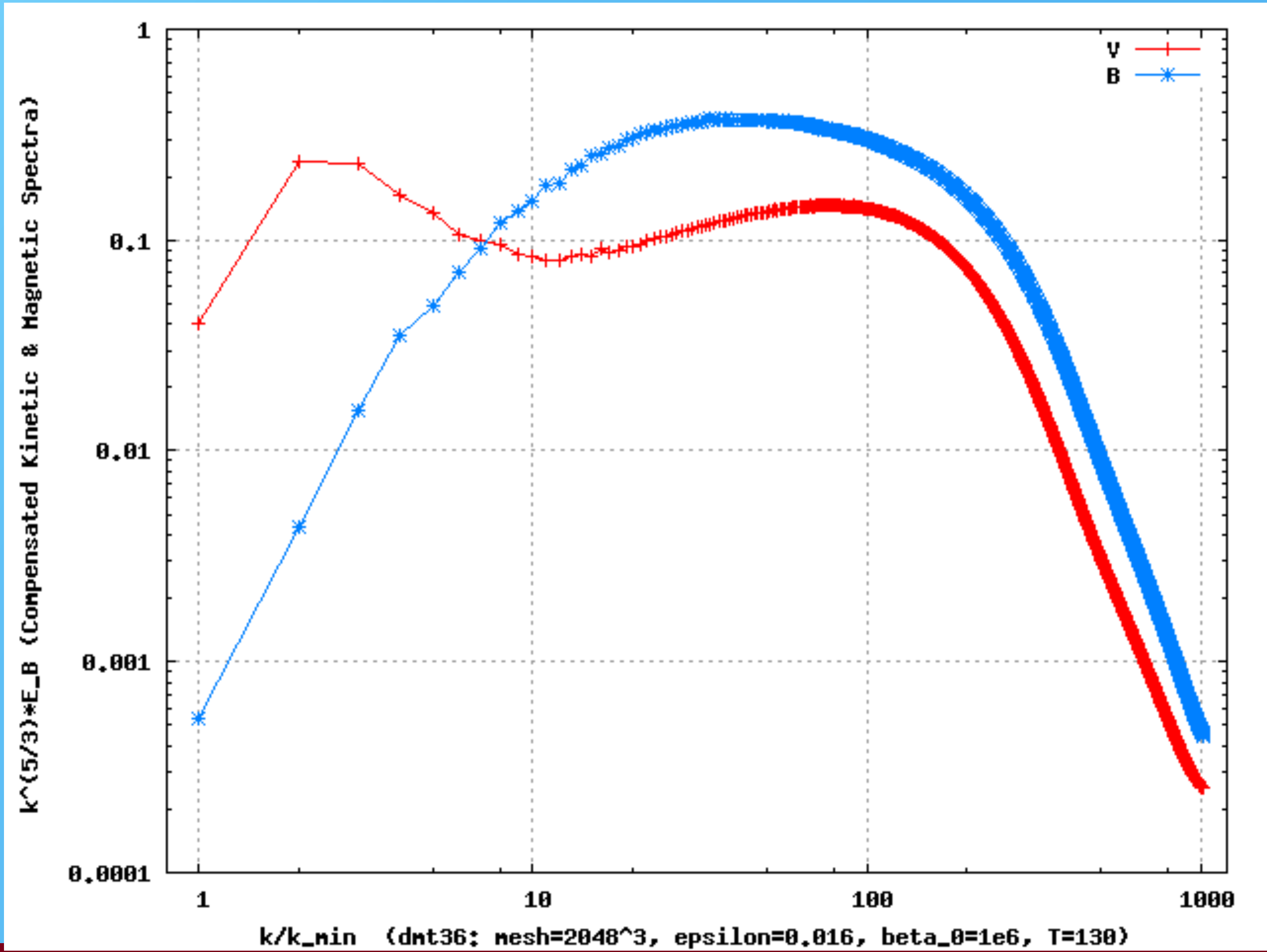
B-field still very weak.



Compensated V & B spectra: T=130

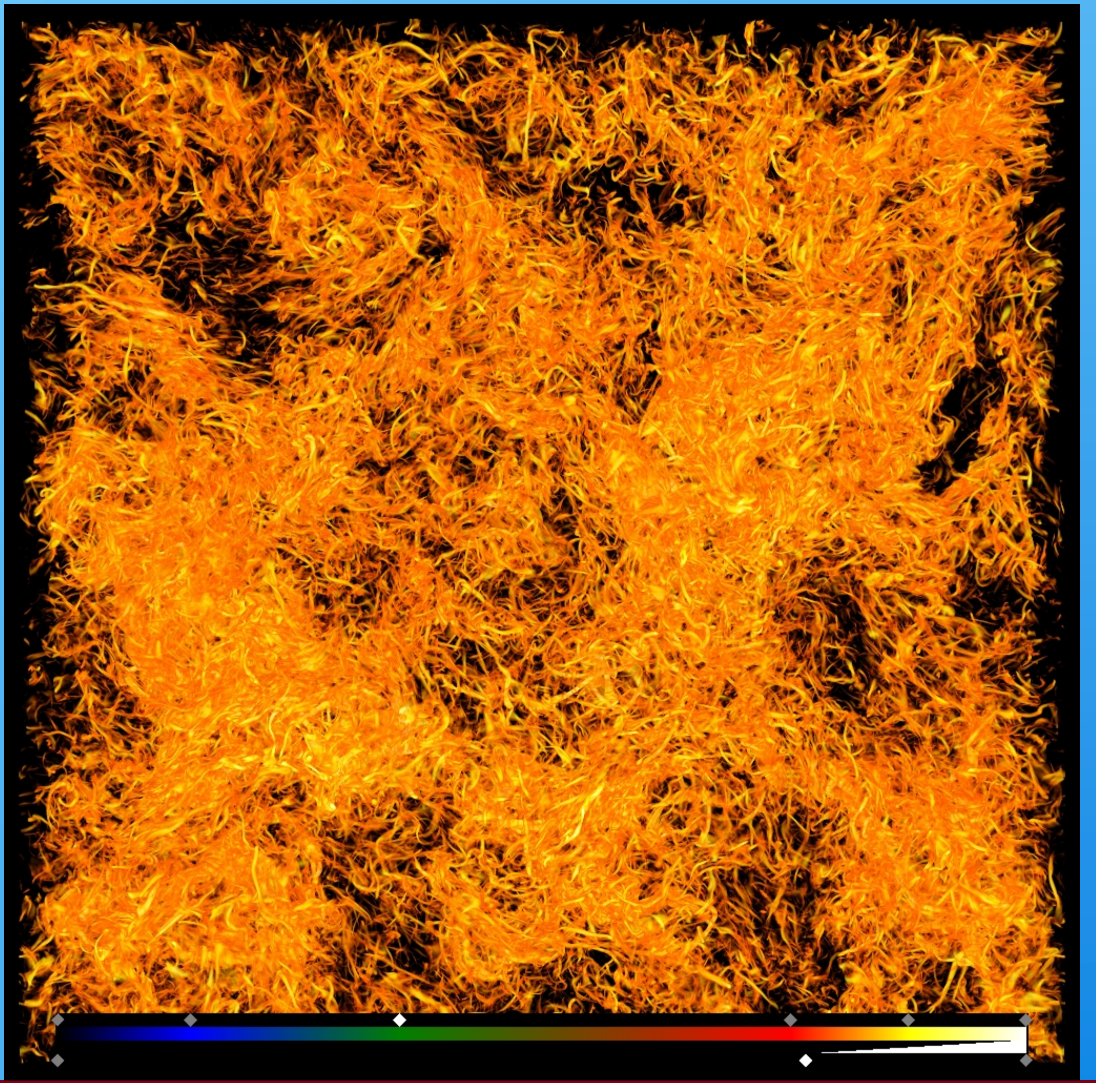
B-field now dominant for k>10.

V-field strongly influenced.



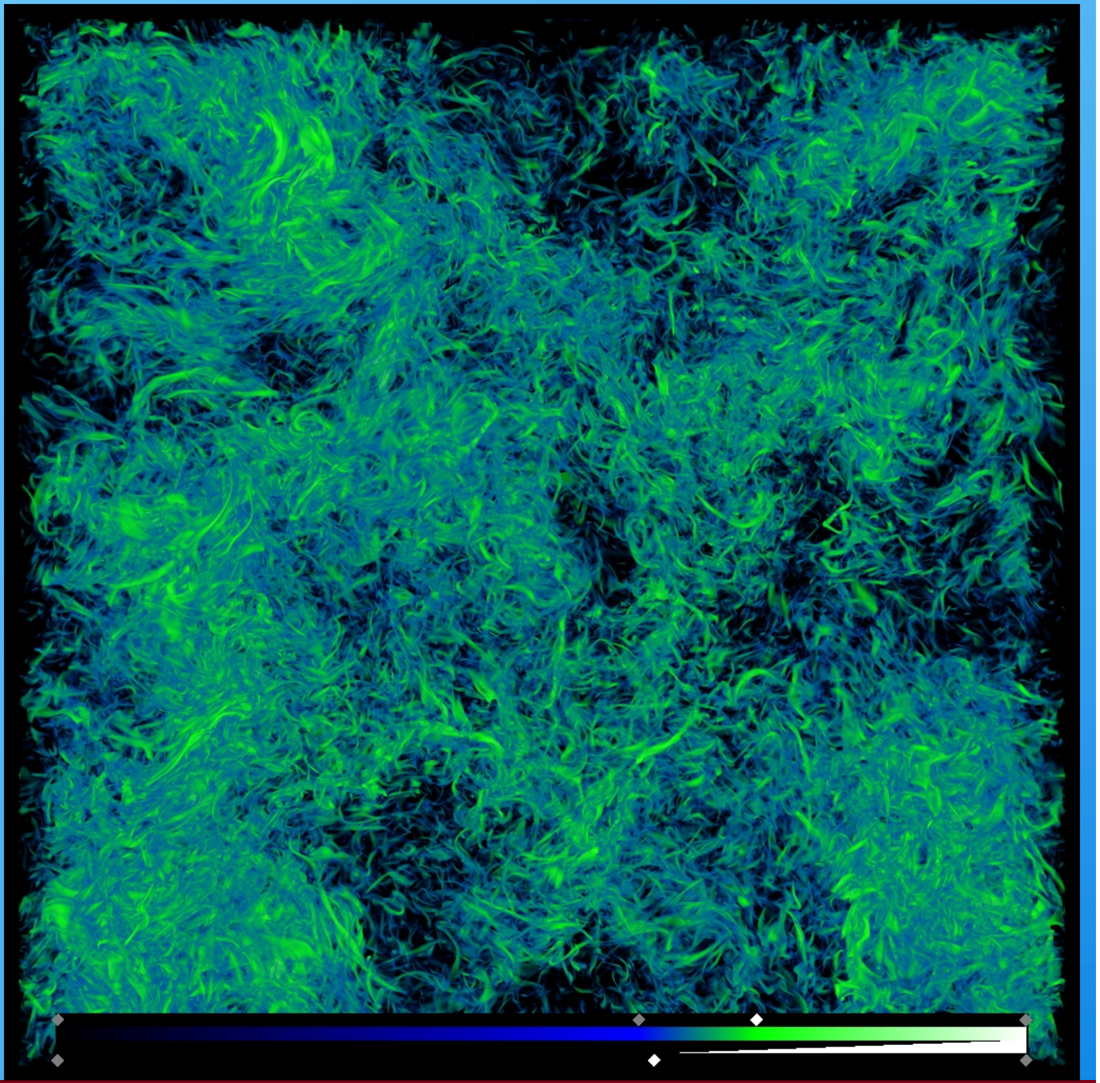
Vorticity T=20 (in slab)

- B-field is weak and does not back-react on flow.
- Flow behaves essentially like a neutral fluid.
- Flow on smallest (dissipation) scale dominated by vortex tubes.



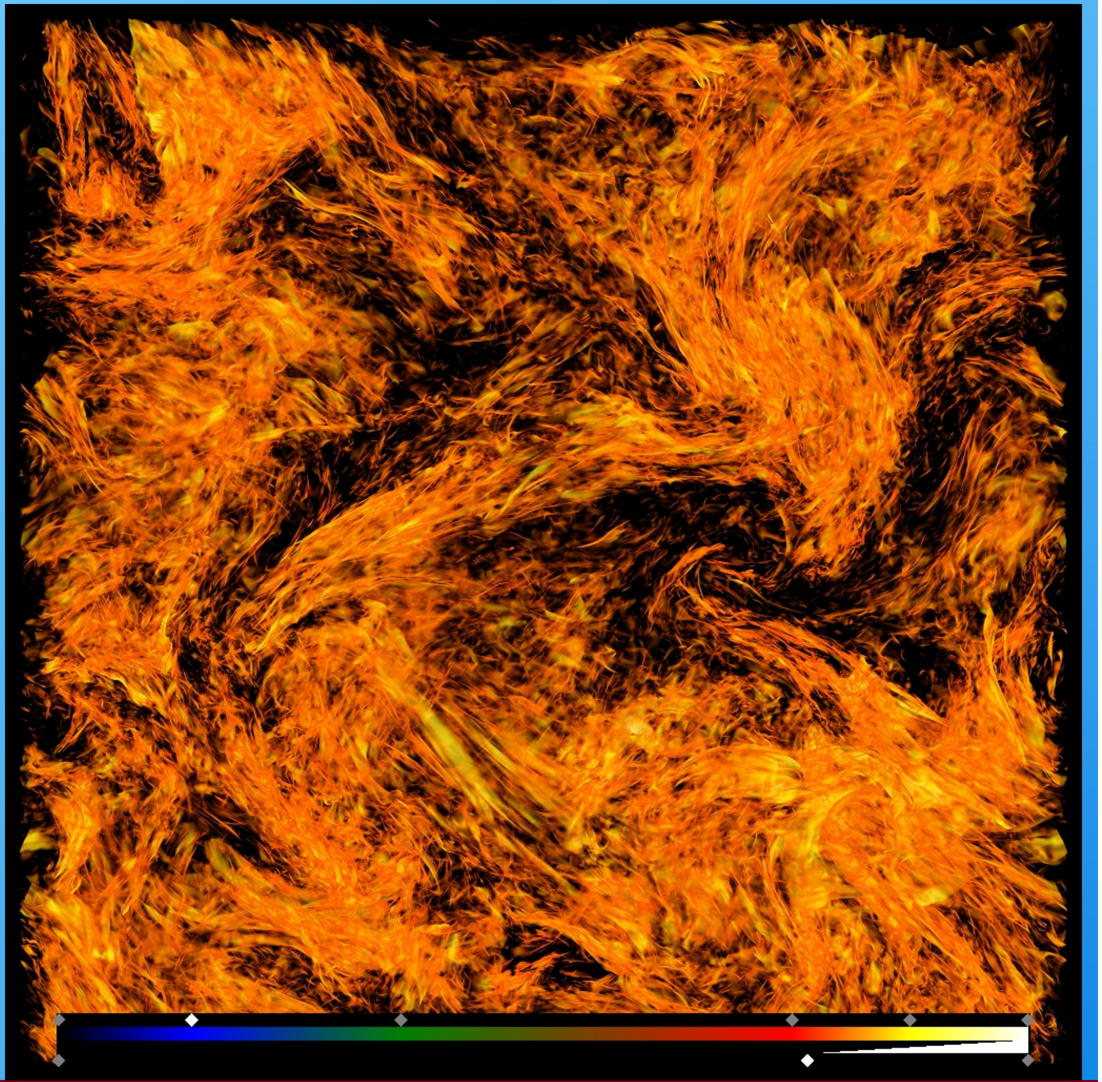
Magnetic Energy: T=20 (in slab)

- B-field grows fastest in where rate of strain is largest,
- Rate of stain is larger and persistent in vortex tubes.
- Weak B-field tents to trace vortex tubes.



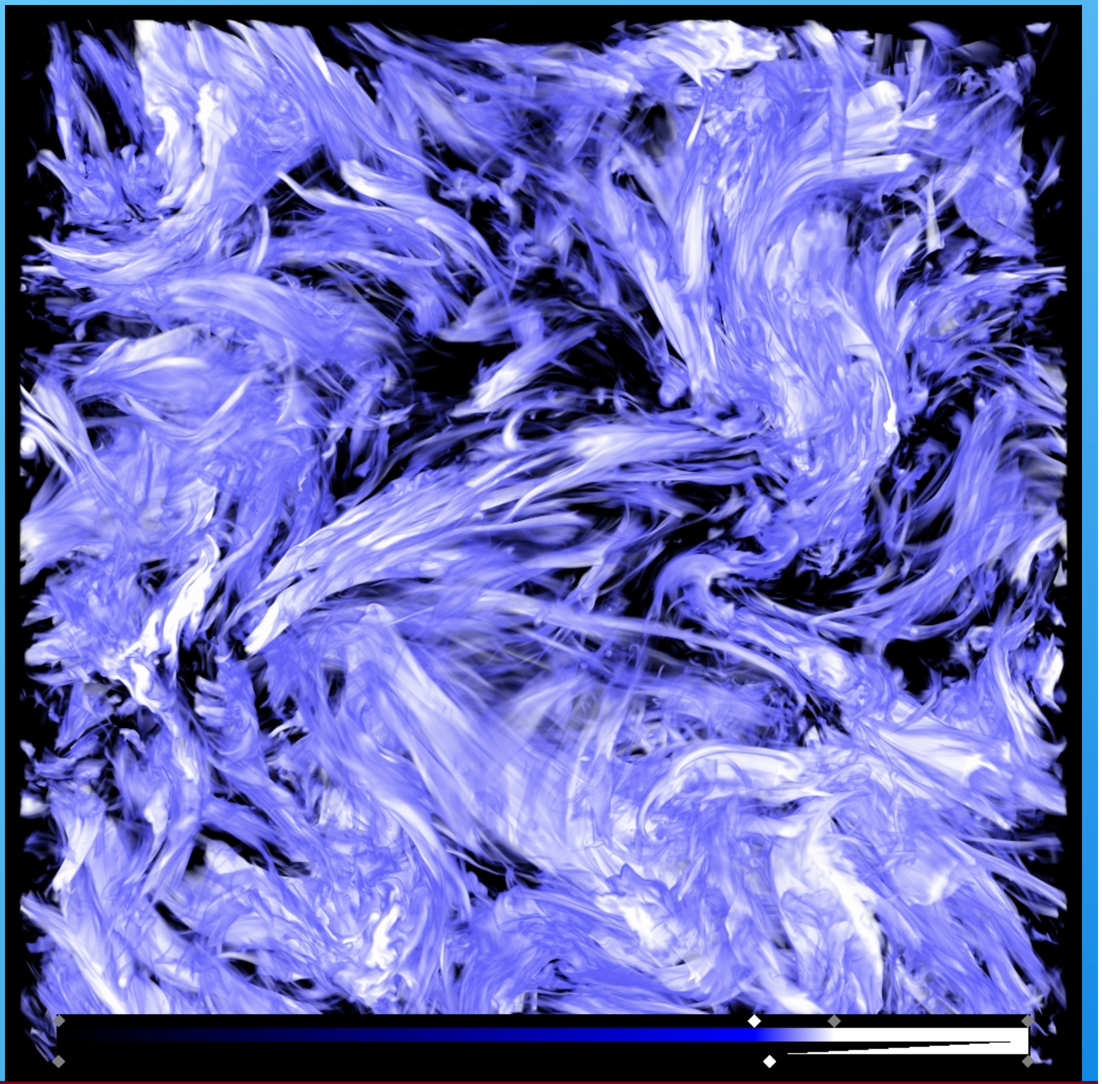
Vorticity T=130 (in slab)

- Flow now strongly influence by B-field.
- Vortex tubes are gone
- Velocity fluctuations on small scales now greatly inhibited by B



B Field Energy T=130 (in slab)

- B-field has strong filamentary structures.
- B-field appears to have many thin, parallel, and closely packed structures.
- Adjectives:
  - Fibers
  - Layers



The Point

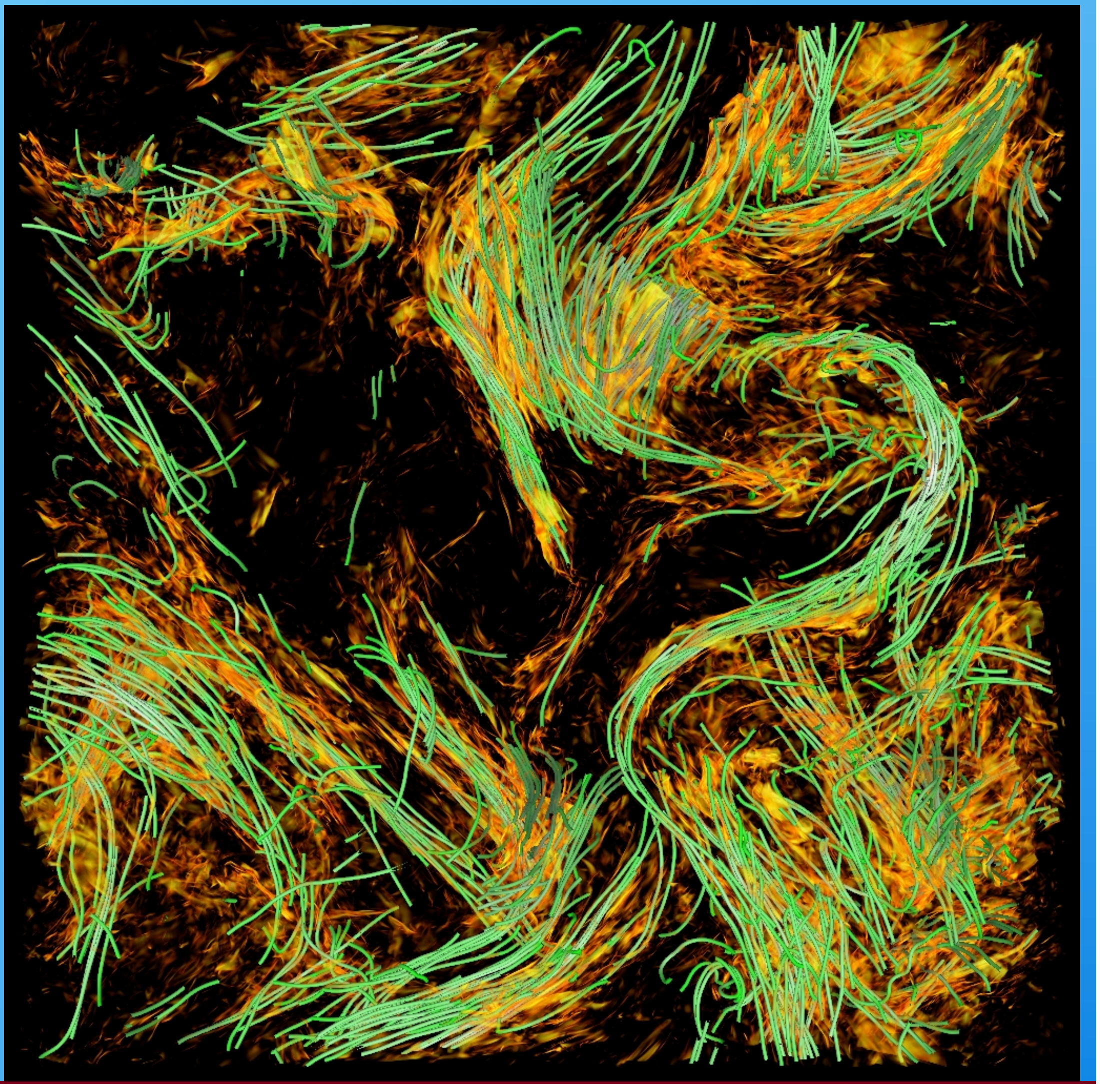
- Weak B-field: very much like neutral fluid turbulence
  - Small scales dominated by vortex tubes
  - Velocity spectra Kolmogorov like at intermediate scales
  - B-field follows Velocity structures
    - Fastest development in vortex tubes
  - **NO SUPRIZES**

- Strong (nearly saturated) B field: very different
  - B-field back reacting on velocity
  - Vortex tubes gone
  - Strongest B organized in fibers or layers

CHARACTERIZE/QUANTIFY SATURATED STATE

Vorcity & B-field at Saturation

- Vorticity and B-field appear to be co-local on large scales
- Details
  - T=130
  - Tubes show B-field
  - Volume rendered vorticity
  - Structures span range of driving (1/2 of box)
  - B more coherent than Vorticity.



Correlation Coefficient

- Correlation coefficient between magnitude of vorticity and magnetic field strength.

$$r(P,Q) = \frac{<PQ> - <P><Q>}{[(<P^2> - <P>^2)(<Q^2> - <Q>^2)]^{1/2}}$$

$$P = \log(|\omega|)$$

$$Q = \log(|B|)$$

- Evaluate on raw data to measure if fields are correlated on smallest scale

- Evaluate on blended data (2x2x2, 4x4x4, …) so measure correlation as a function of scale

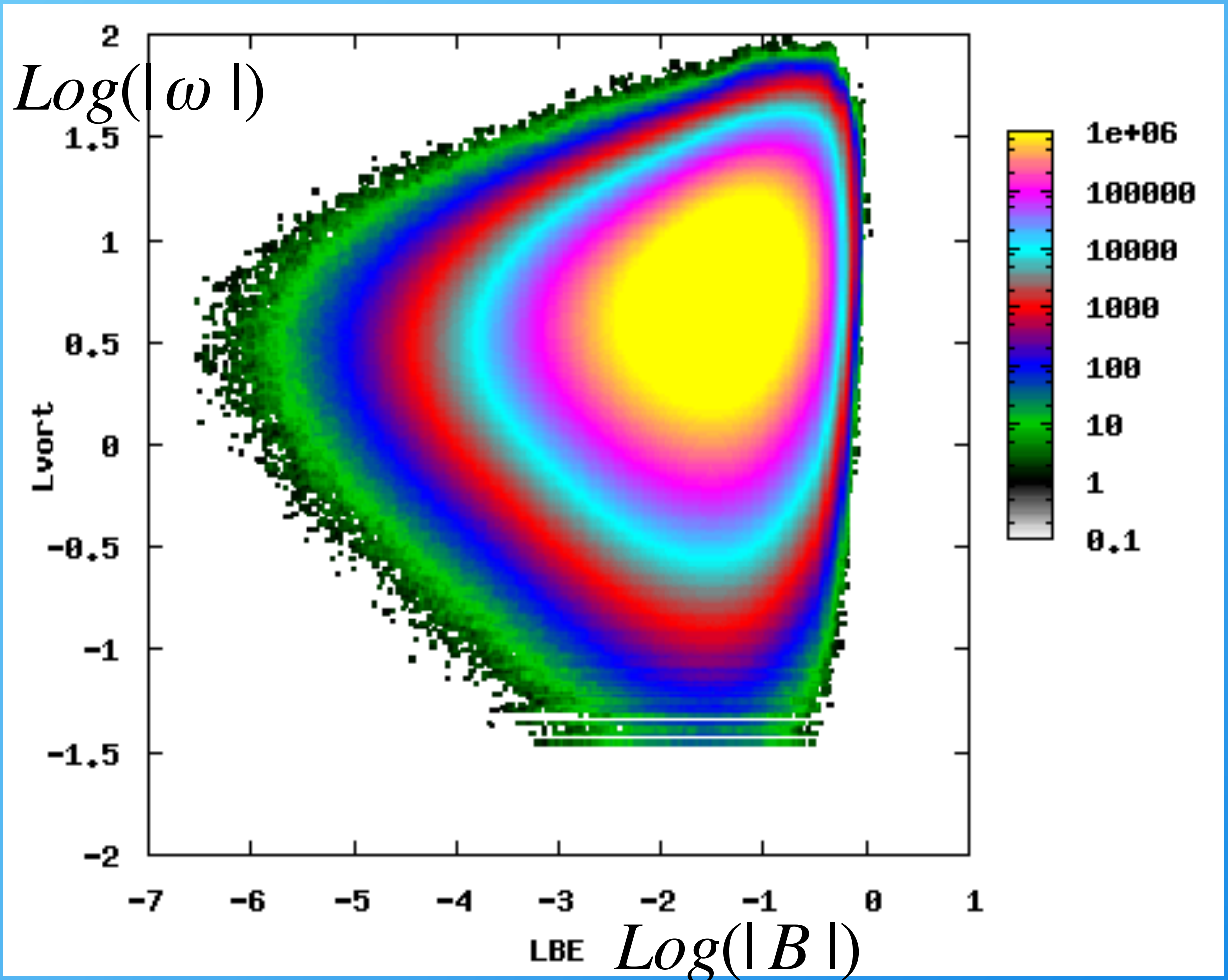
Log Vorticity Vs. Log B

Blend Factor = 1

T=130

Run: dmt36

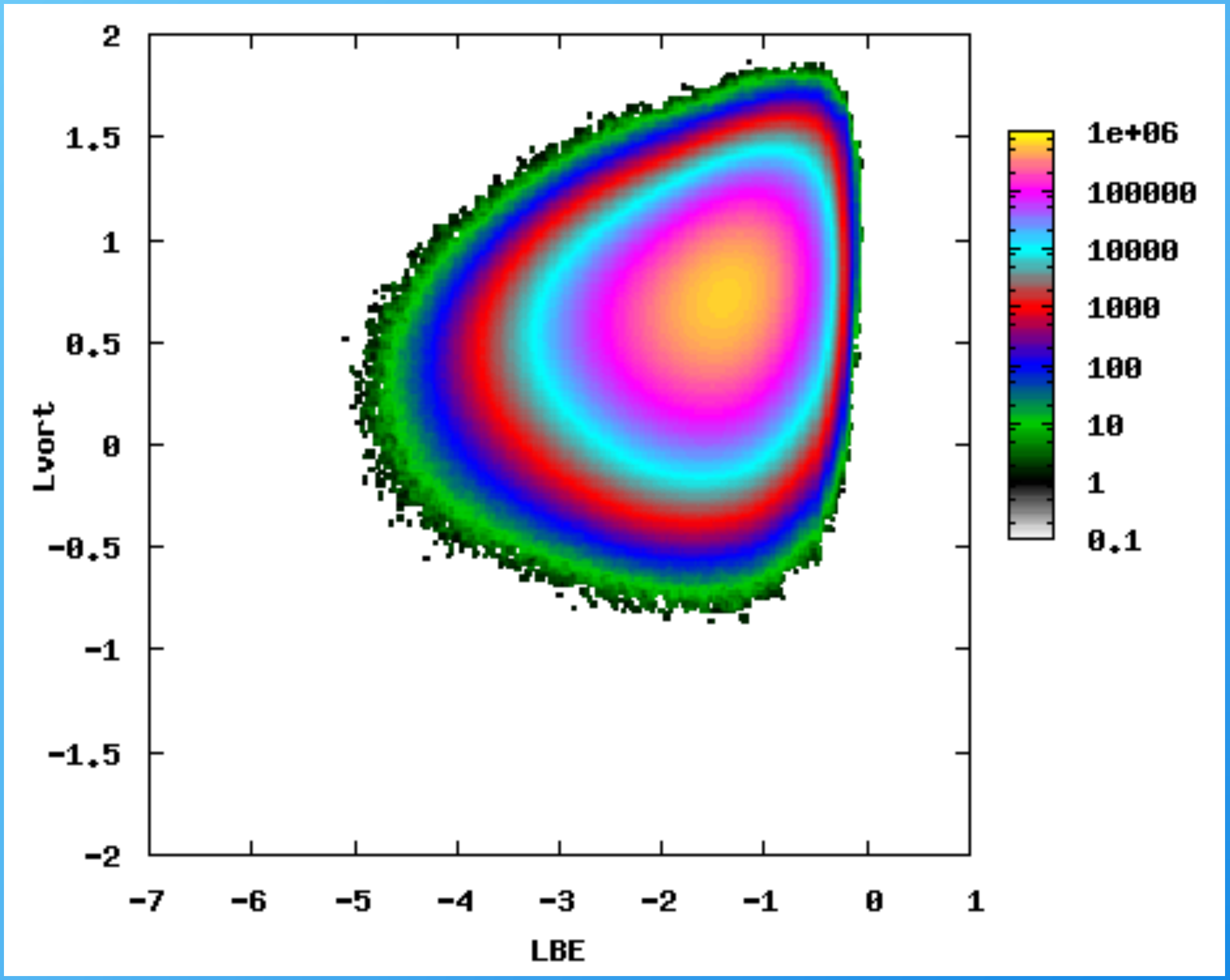
Correlation Coef
r=0.219



Log Voricity Vs. Log B (blended)

Blend Factor = 2

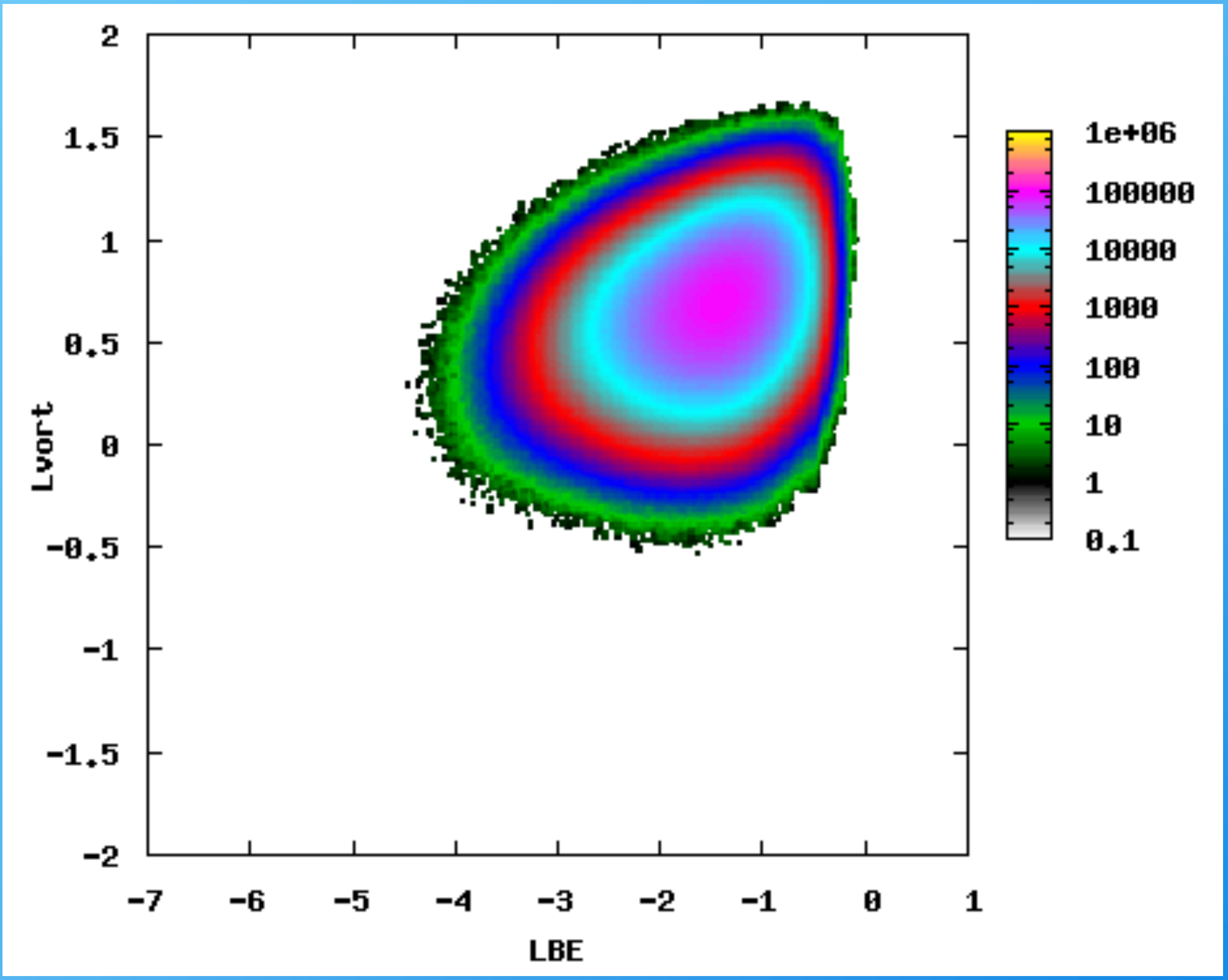
Correlation Coef
r=0.233



Log Voricity Vs. Log B (blended)

Blend Factor = 4

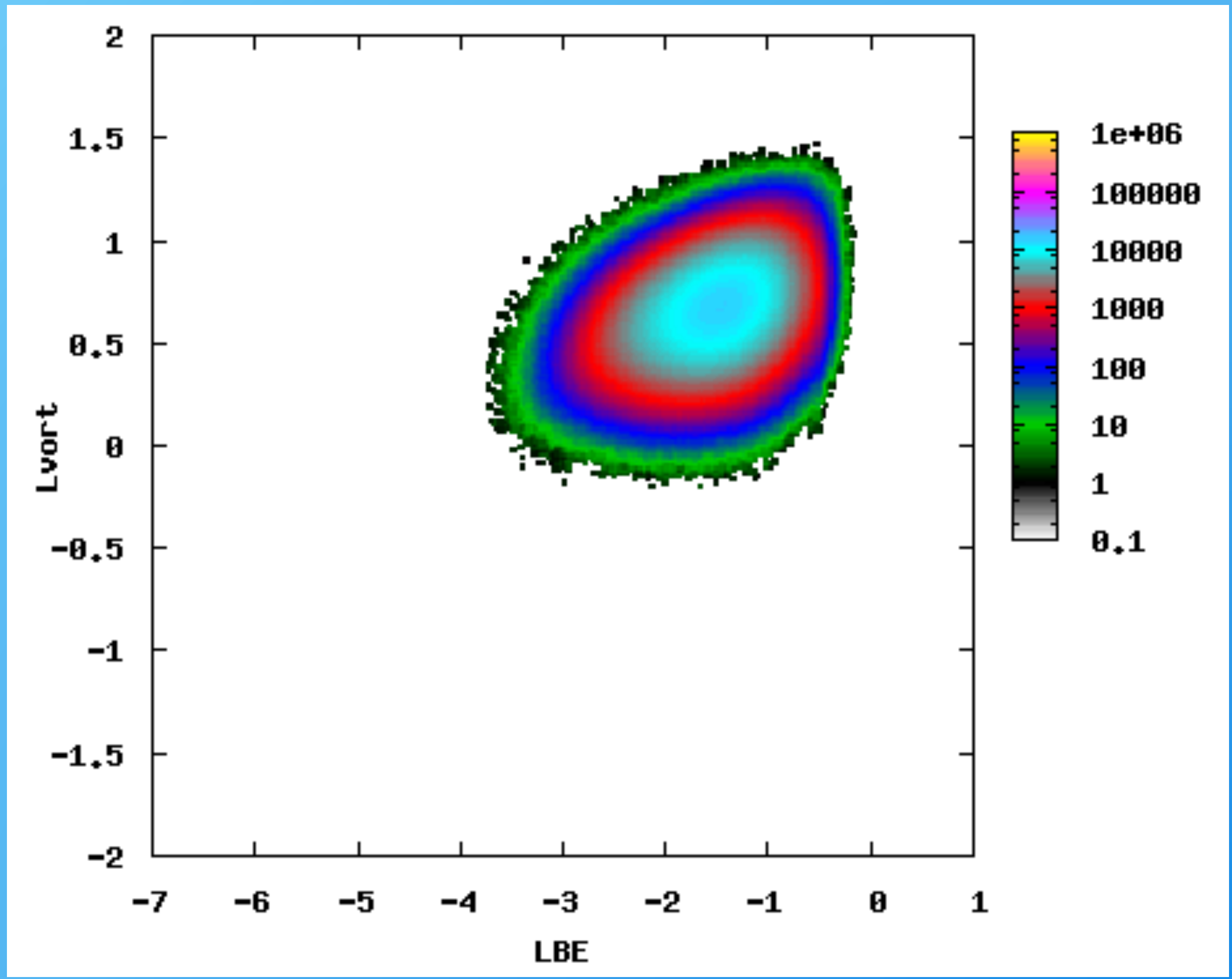
Correlation Coef
r=0.265



Log Voricity Vs. Log B (blended)

Blend Factor = 8

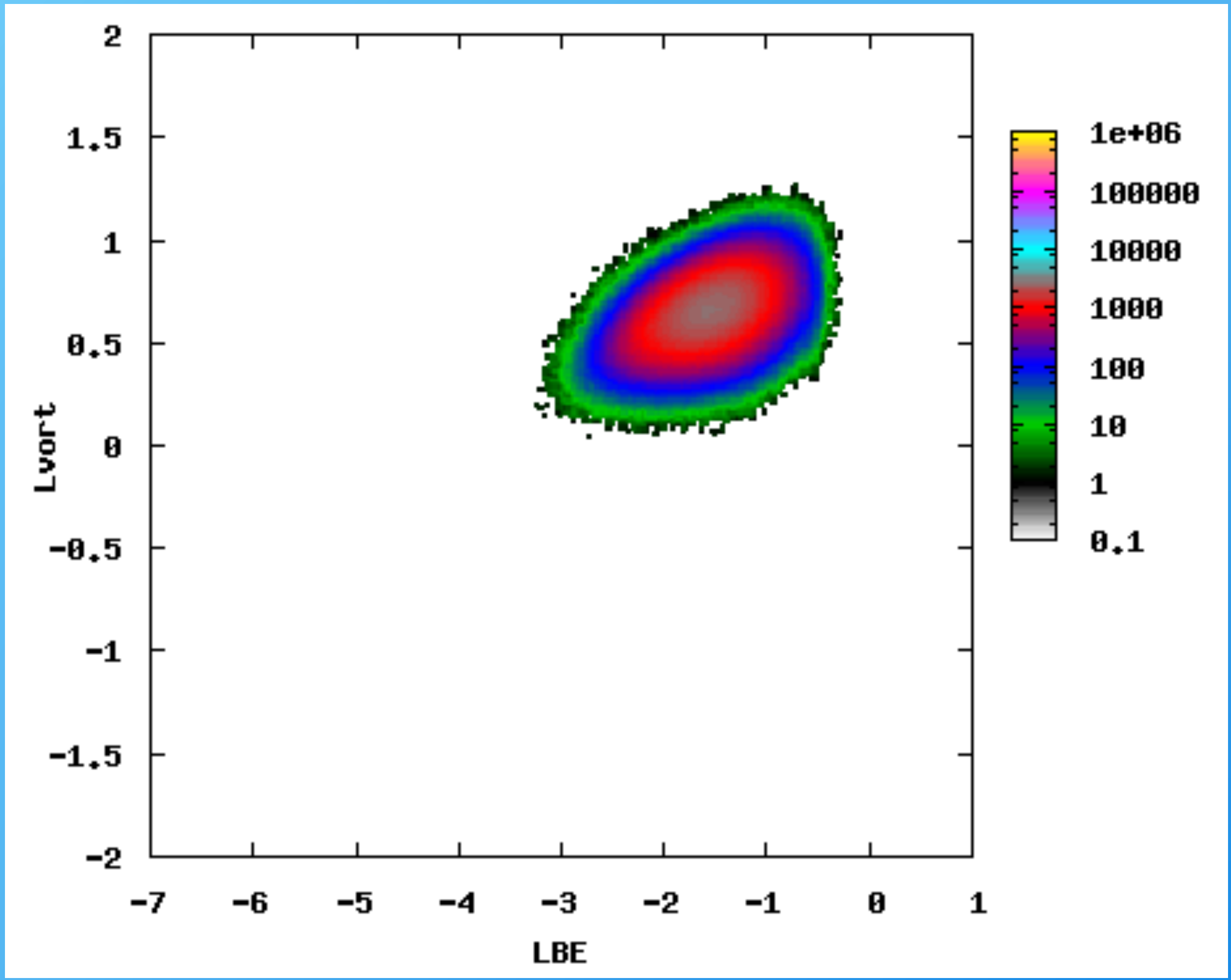
Correlation Coef
r=0.326



Log Voricity Vs. Log B (blended)

Blend Factor = 16

Correlation Coef
r=0.419



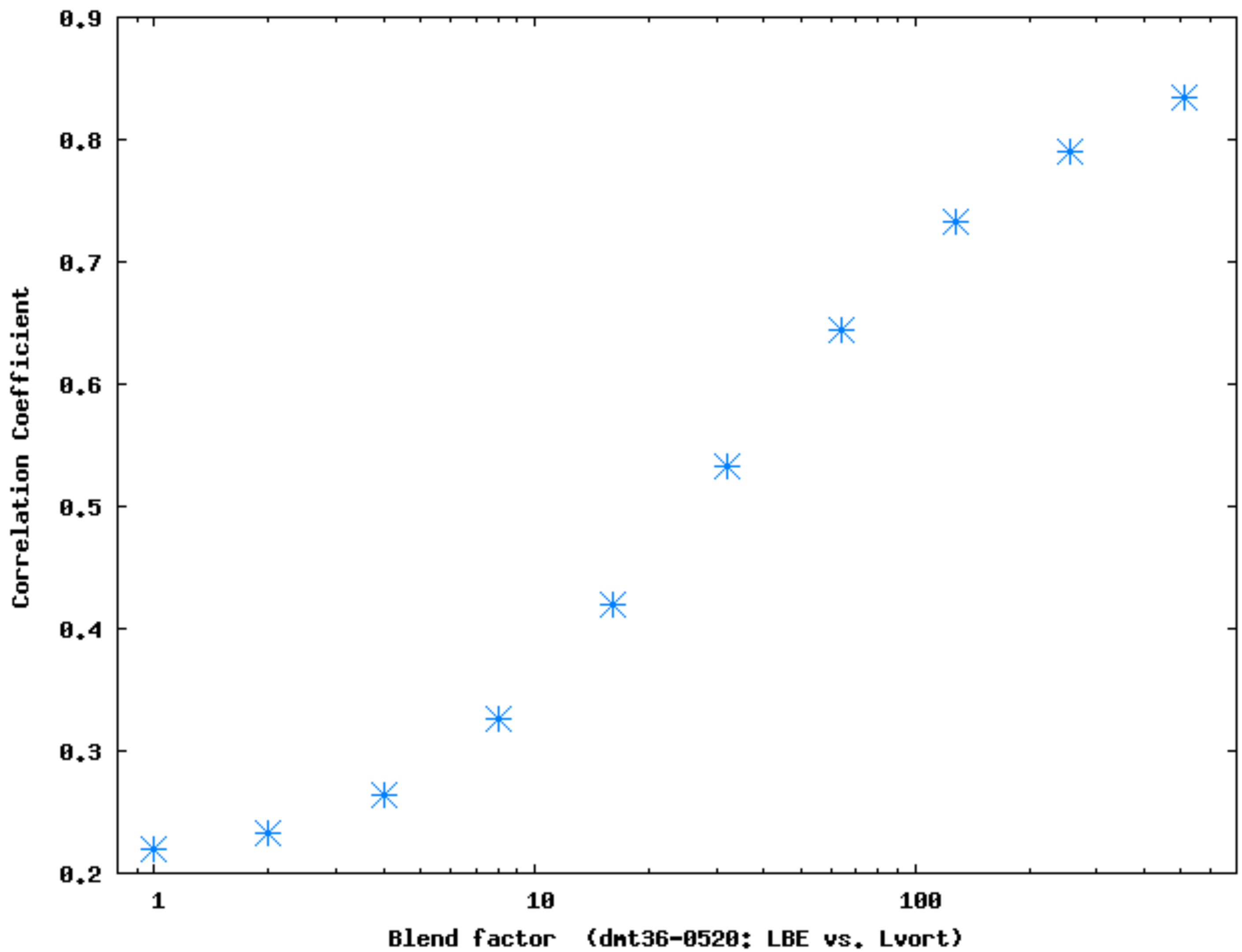
Correlation of B with Voricty at Saturtion

Correlation fn. vs. blending factor

BF=1: no blending
Correlation~0.2

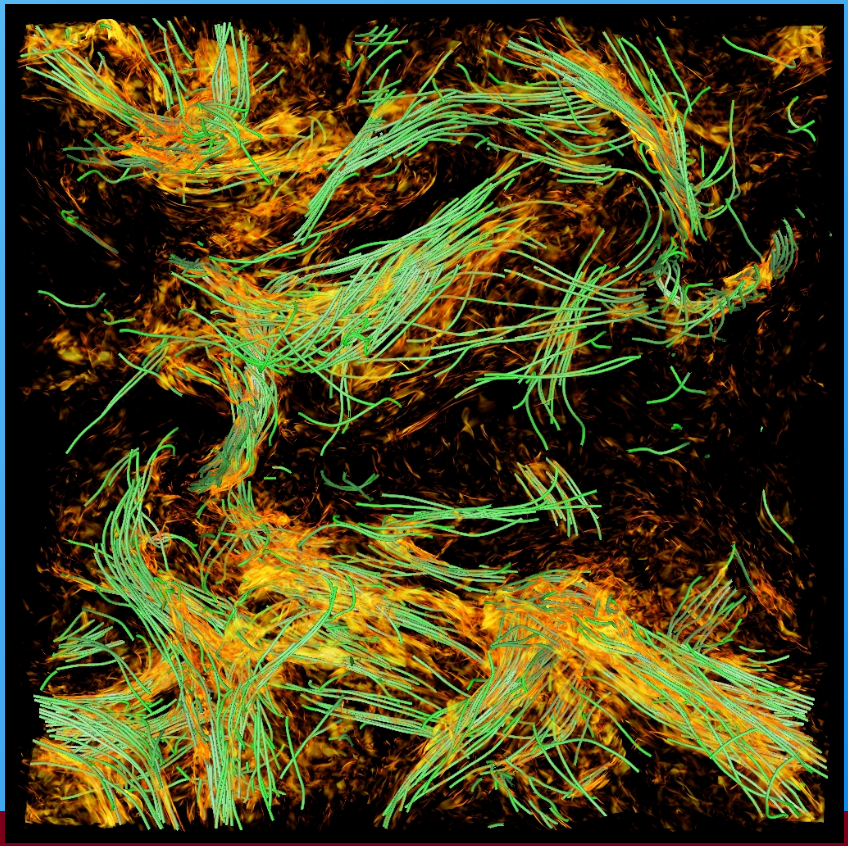
BF=512: ¼ of box
Correlation~0.8

B field & Vorticity structures are in near each other, but not at the same points.



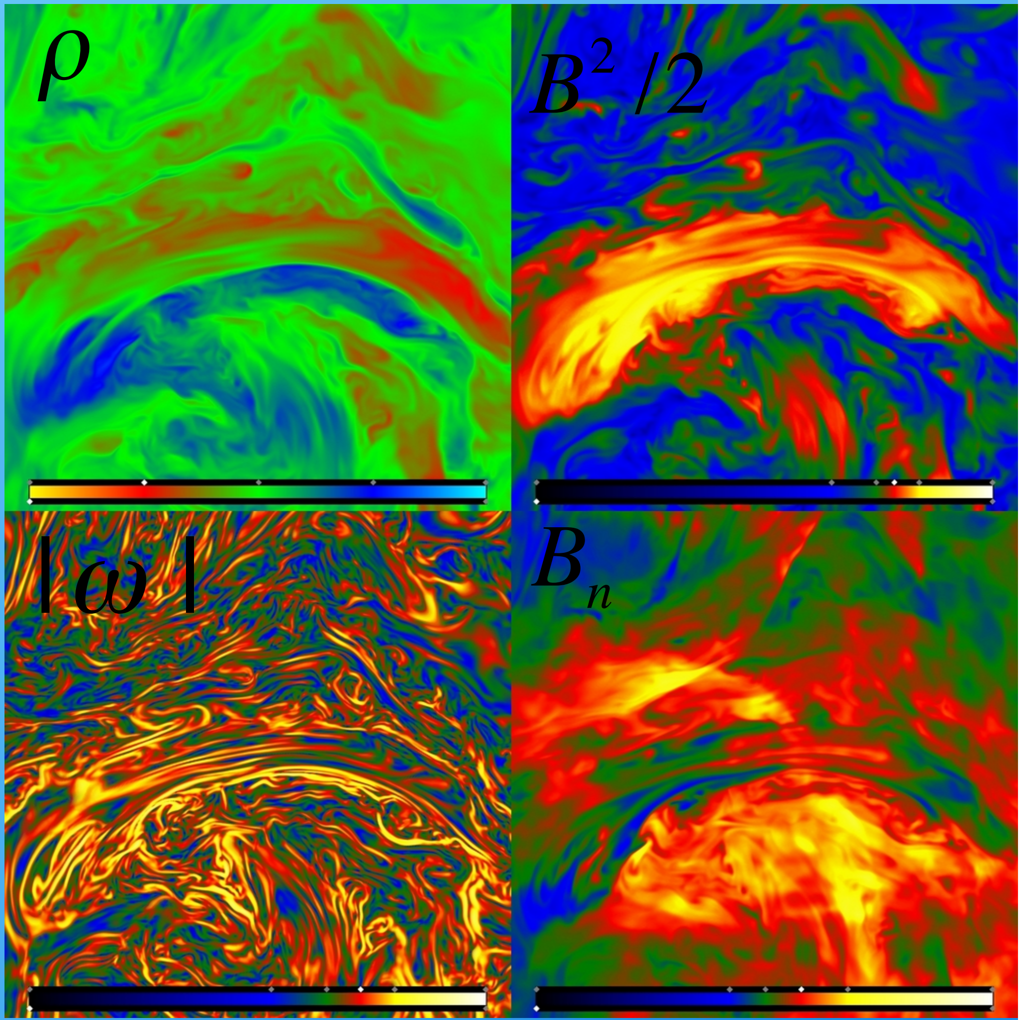
Motivation & Goals

- Both visualizations and cross-correlation indicate
  - B-field and vorticity are not co-local on smallest scales
  - B-field and vorticity are co-local on larger scales
- Visually: B & vorticity
  - Aligned
  - Interleaved
  - Part of larger structures
- Goals:
  - Characterize structures
  - Measure statistical significance
  - Identify generation mechanism



4 fiels in cross-section of a strip

- Fields shown:
  - Density
  - Magnetic energy
  - Vorticity
  - Normal B (out of plane)
- Multiple, closely spaced nearly parallel layers
- All 4 fields influence by (participate in) the structure



Premis: Fields Organized in Layers

- Layers are thin
- All fields show multiple stacked layers
- Visually well defined direction of most rapid variation

- **How well can field variation be characterized as functions of just one variable?**

- Field gradients locally indicate direction of variation
- However, volume averages of gradients will tend to 0

Principle Directions of Variation

Gradient matrix: tensor product of gradient vector with itself

Q in {rho,Vx,Vy,Vz,Bx,By,Bz}

$$a_{ij} = \left\langle \frac{\partial Q}{\partial x_i} \frac{\partial Q}{\partial x_j} \right\rangle_V$$

Real & symmetric, which means:

- Eigenvectors are orthogonal
- Eigen values are real

Order eigenvalues by value

$$\lambda_{\max} \geq \lambda_{,mid} \geq \lambda_{mid}$$

Label eigenvectors correspondingly

$$\{\hat{e}_{\max}, \hat{e}_{mid}, \hat{e}_{\min}\}$$

In eigenvector coordinates, gradient matrix is diagonal .

Eigenvalues are non-negative, and generically positive

$$a_{ii} = \lambda_i = \left\langle (\hat{e}_i \bullet \vec{\nabla} Q)^2 \right\rangle_V \geq 0$$

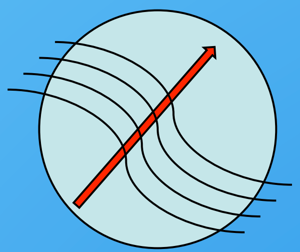
Measure of Allignment

Ratio of max to mid to min eigenvalues

Eige values will vary with field

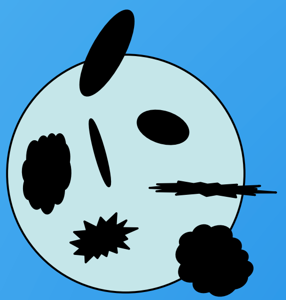
Strong alignment

$$\lambda_{max} / \lambda_{min} >> 1$$



Weak alignment

$$\lambda_{max} / \lambda_{min} \sim 1$$



Samples Taken in Spherical Volumes

Gradient matrix $a_{ij} = \left\langle \frac{\partial Q}{\partial x_i} \frac{\partial Q}{\partial x_j} \right\rangle_V$

Evaluated in spherical volumes

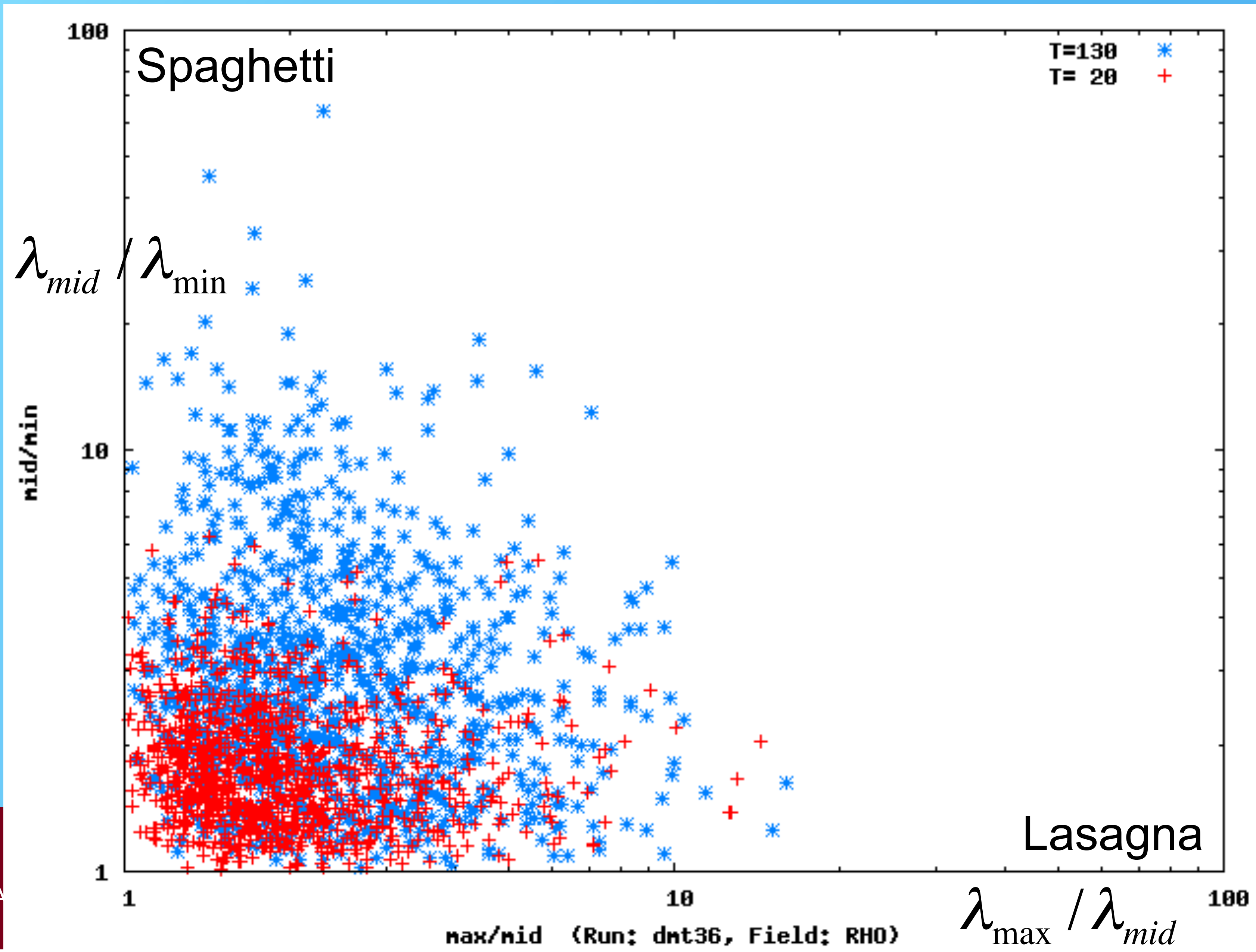
1000 spherical volumes

10x10x10 array

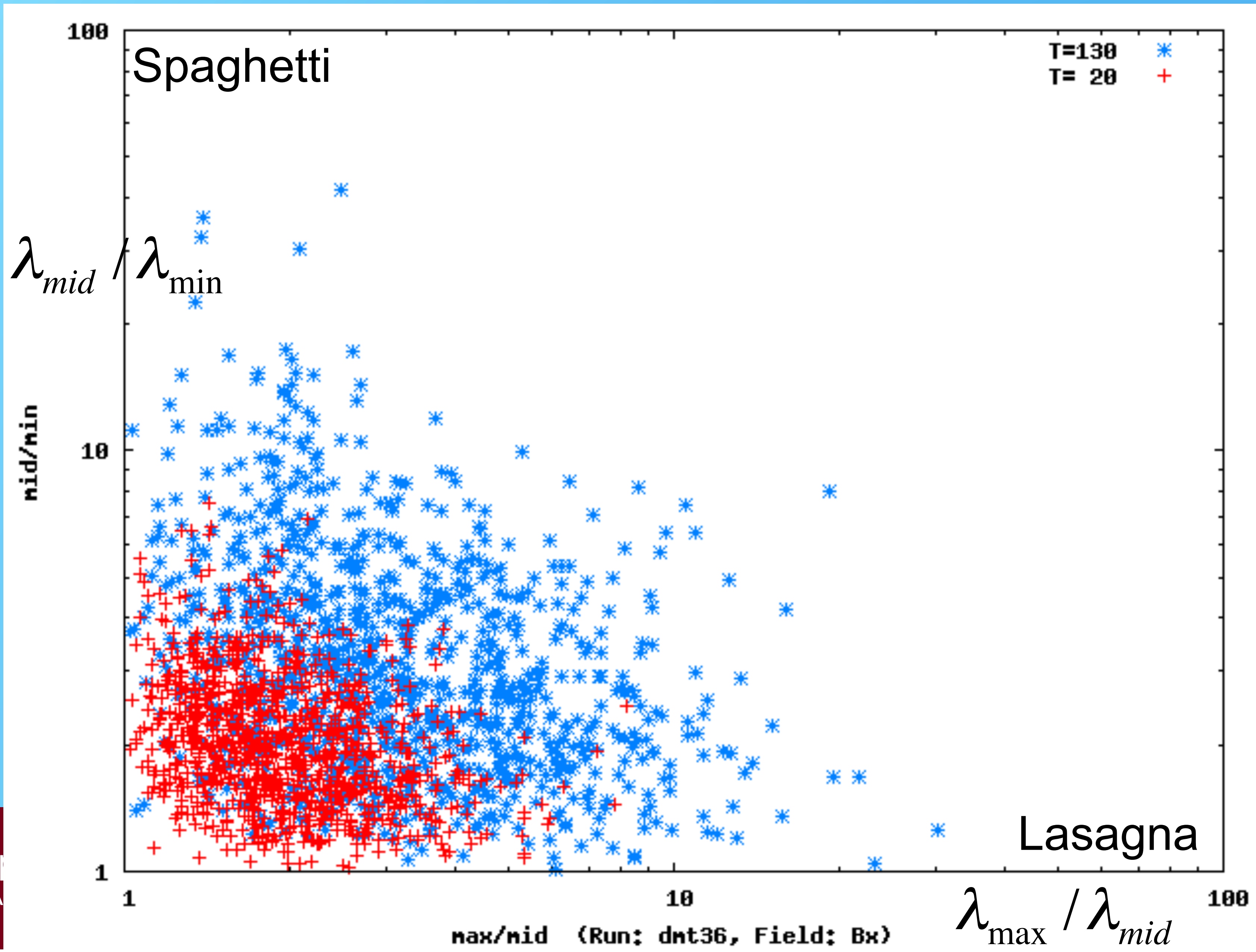
Radius of each sphere R=0.4

Full domain is 10 on a side

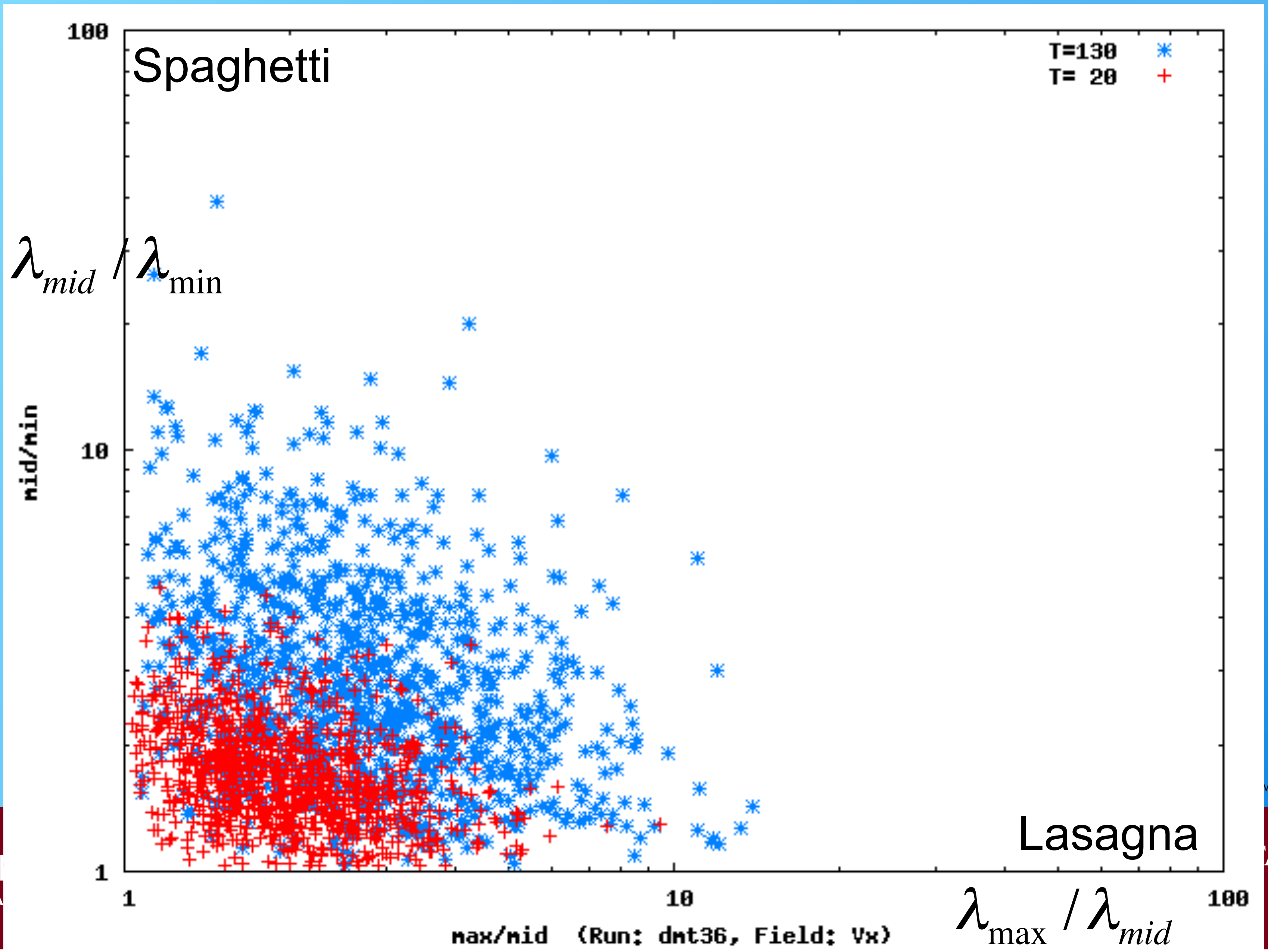
Shapes of structures: RHO



Lambda Ratios for Bx



Lambda Ratios for Vx

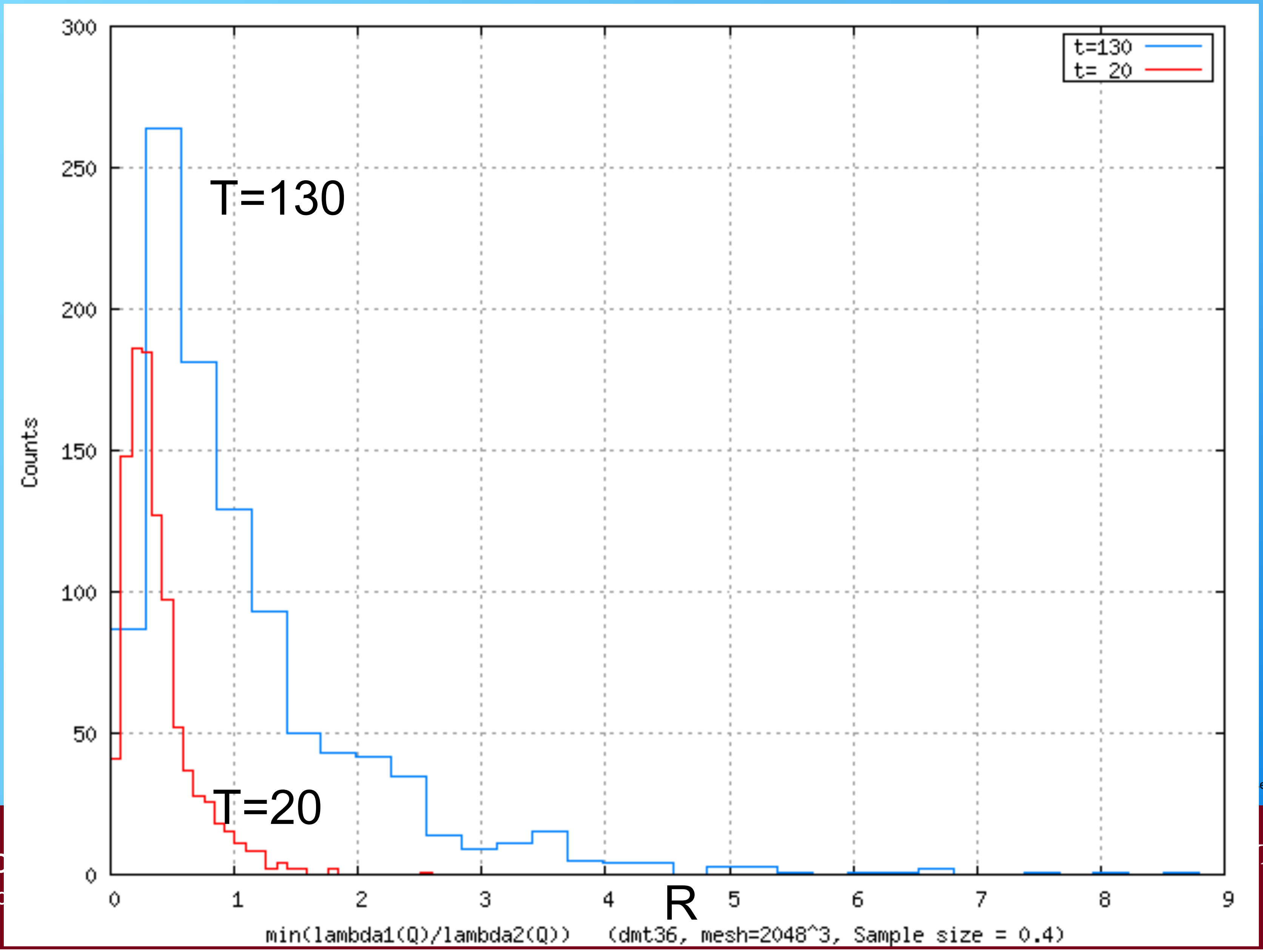


Measure of how strongly all variations are in just one direction

$$R \equiv \min(\lambda_{\max}(Q) / \lambda_{mid}(Q), \quad Q \in \{\rho, u_i, b_i\})$$

- Take minimum of ratio over all field variables as a conservative measure of alignment.
- Strongest variation in just one direction (lasagna)
- If R is large, then ALL fields have just one direction in which the primarily vary (i.e., all variations orthogonal directions are small by comparison in the mean-square).

Alignment Strength: Distributions



Sup
for Ac
TA

Alignment Between Fields

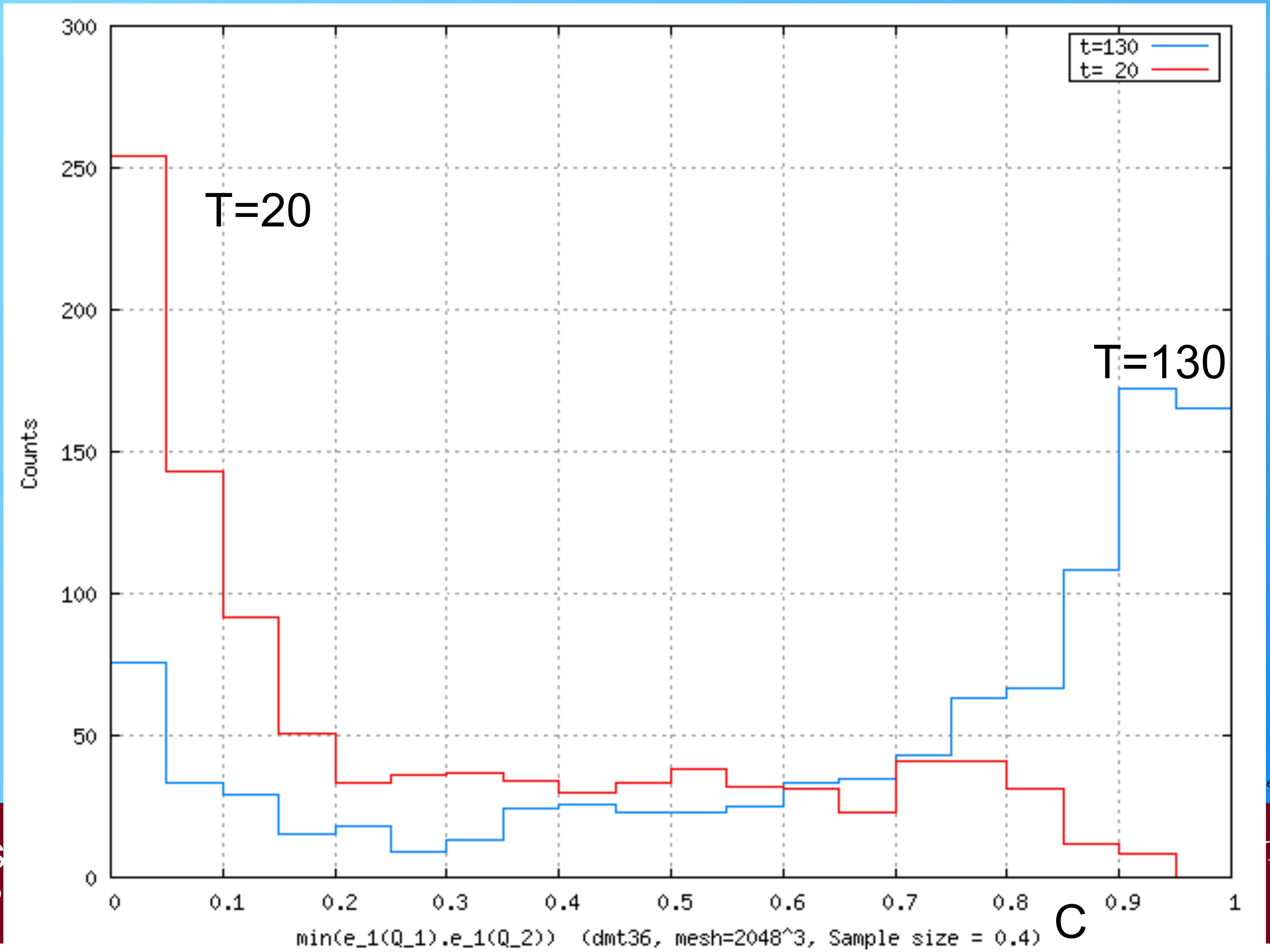
$$C \equiv \min(|\, \hat{e}_{\max}(Q) \bullet \hat{e}_{\max}(P) \,|, \quad P \,\&\, Q \in \{\rho, u_i, b_i\})$$

Are the directions of variation of different fields aligned?

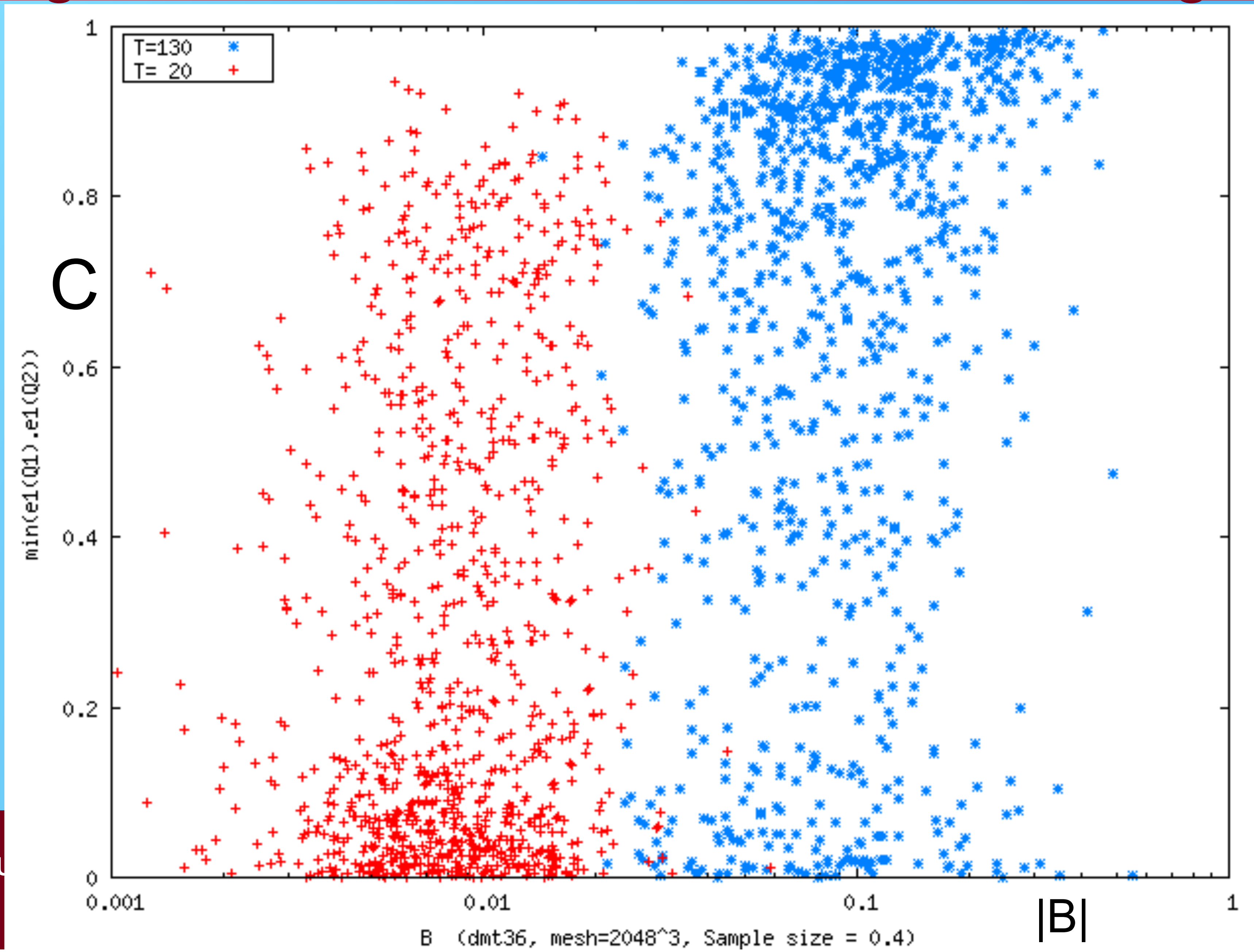
C is cosine of maximum angle between strongest direction of variation between any two fields

If C is large, then directions of variation of all fields are nearly aligned

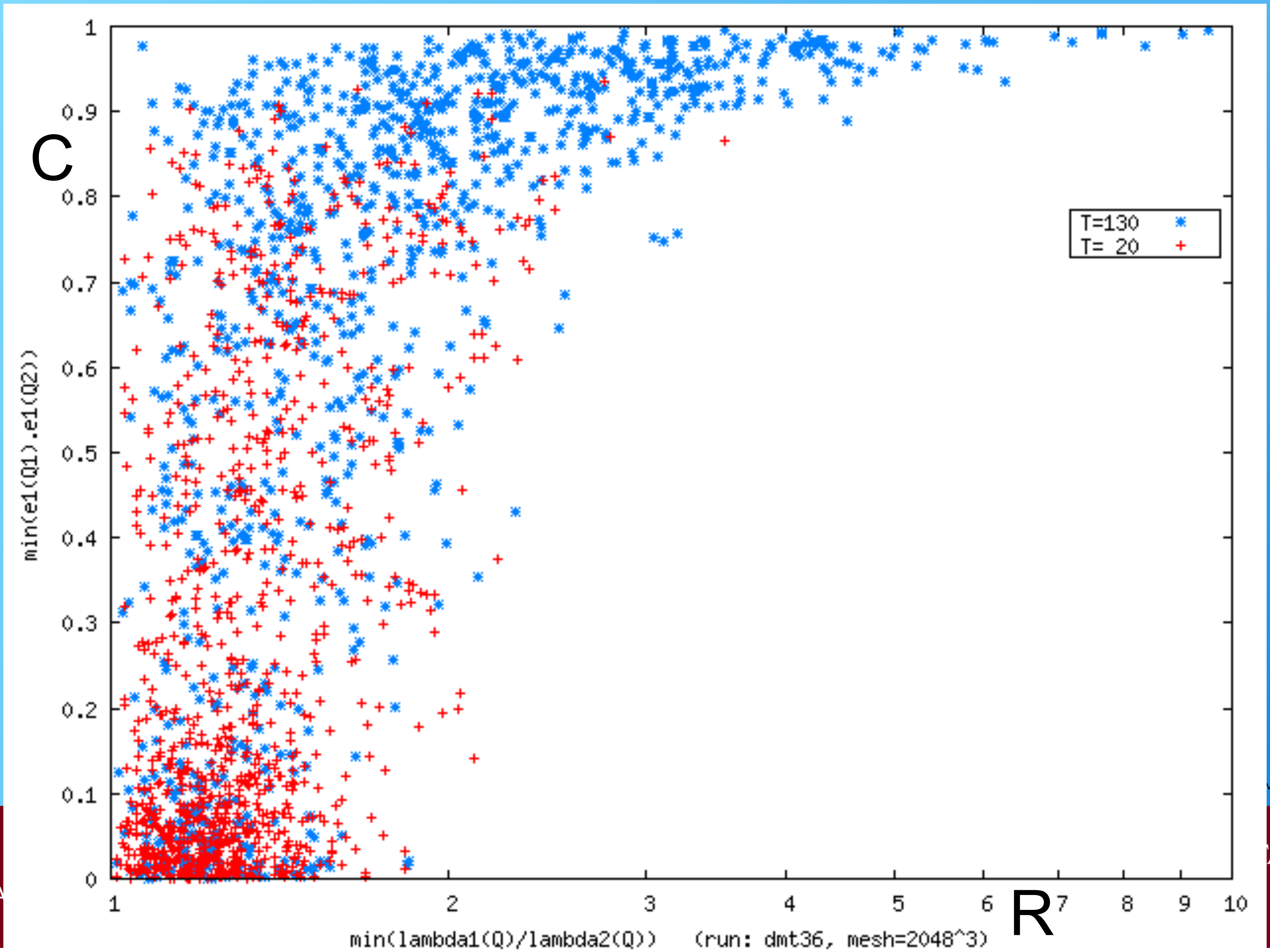
Alignment Between Fields



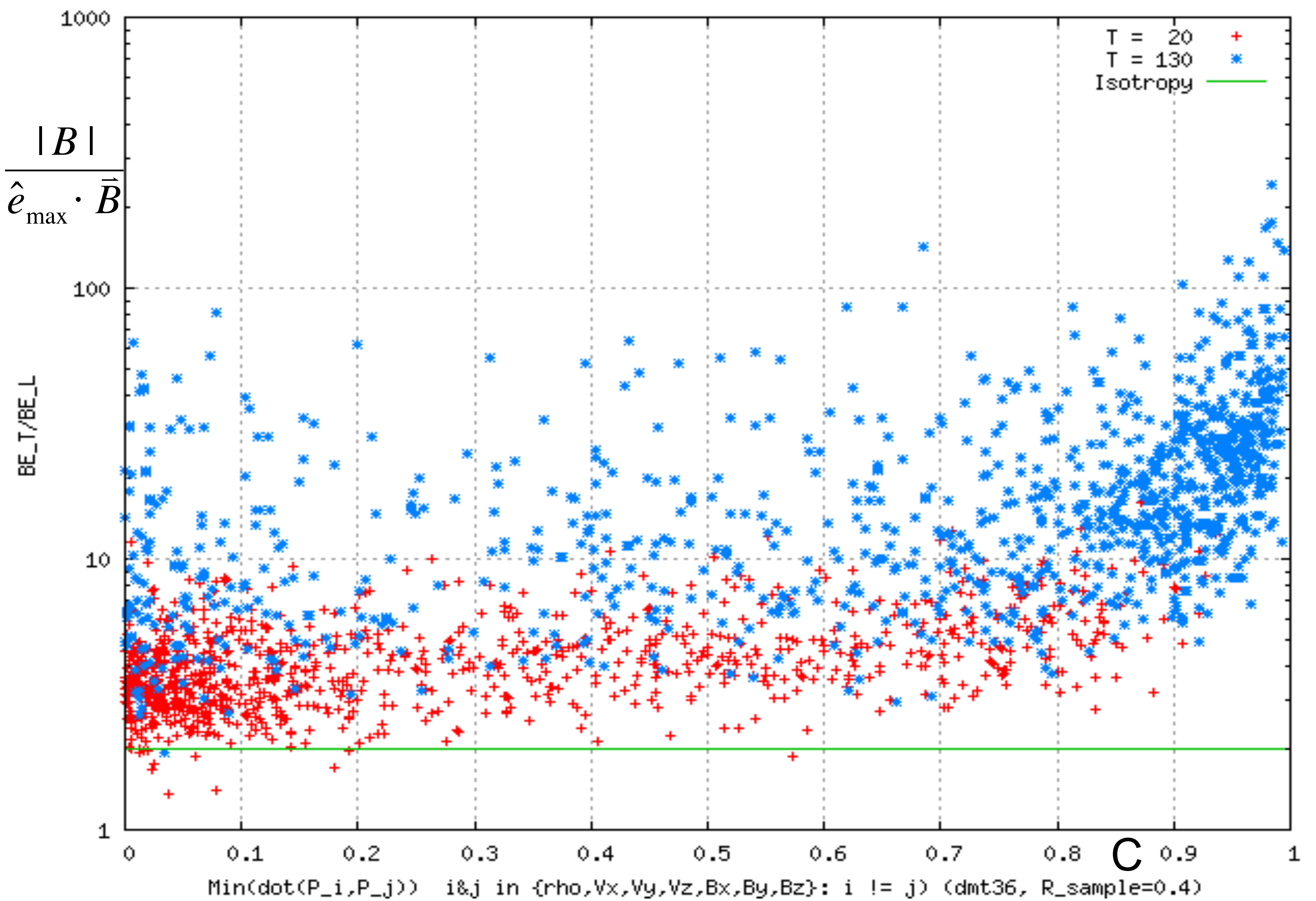
Alignment of Variation Vs. B-Field strength



Min_L2/L1 vs. Min_dot



Alignment Vs. Anisotropy of B

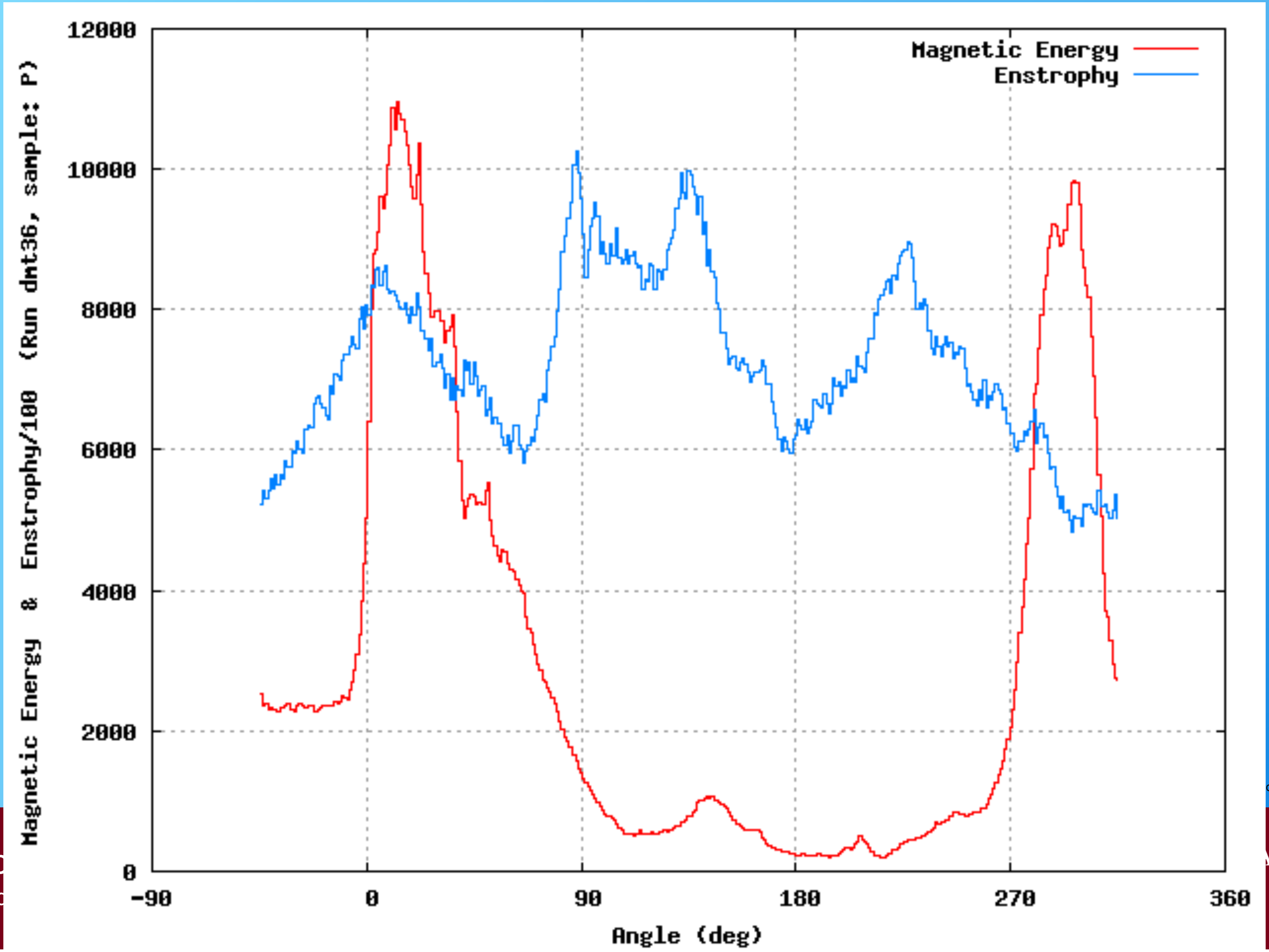


Alignment of B within a Layer

In regions containing well defined layers (C>>1), B is the plane of the layer.

How is B lined up with the direction of most rapid variation?

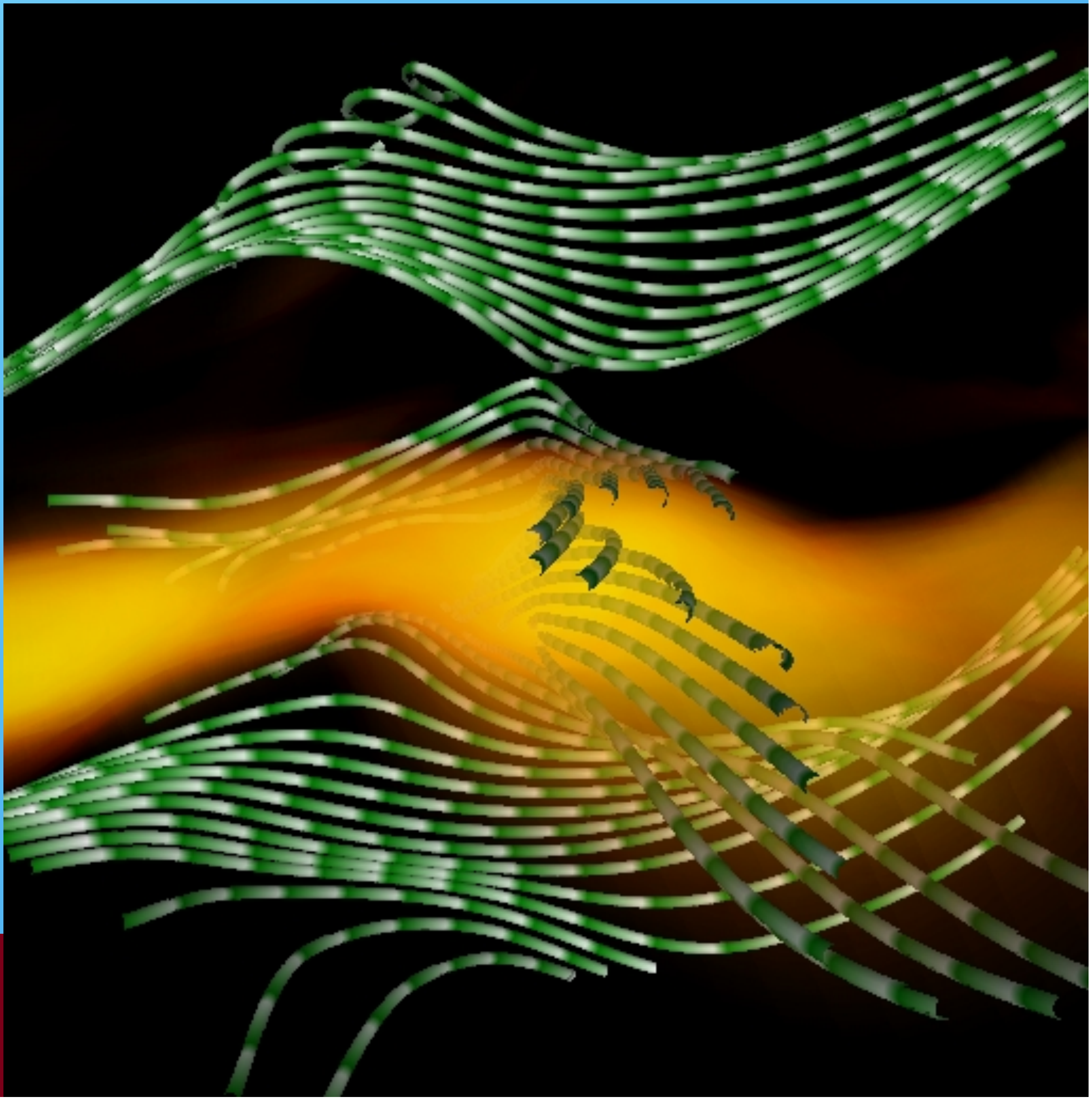
Angular B In A Layer



Sup
for Ac

B-Field Sampled Across Slip Surface

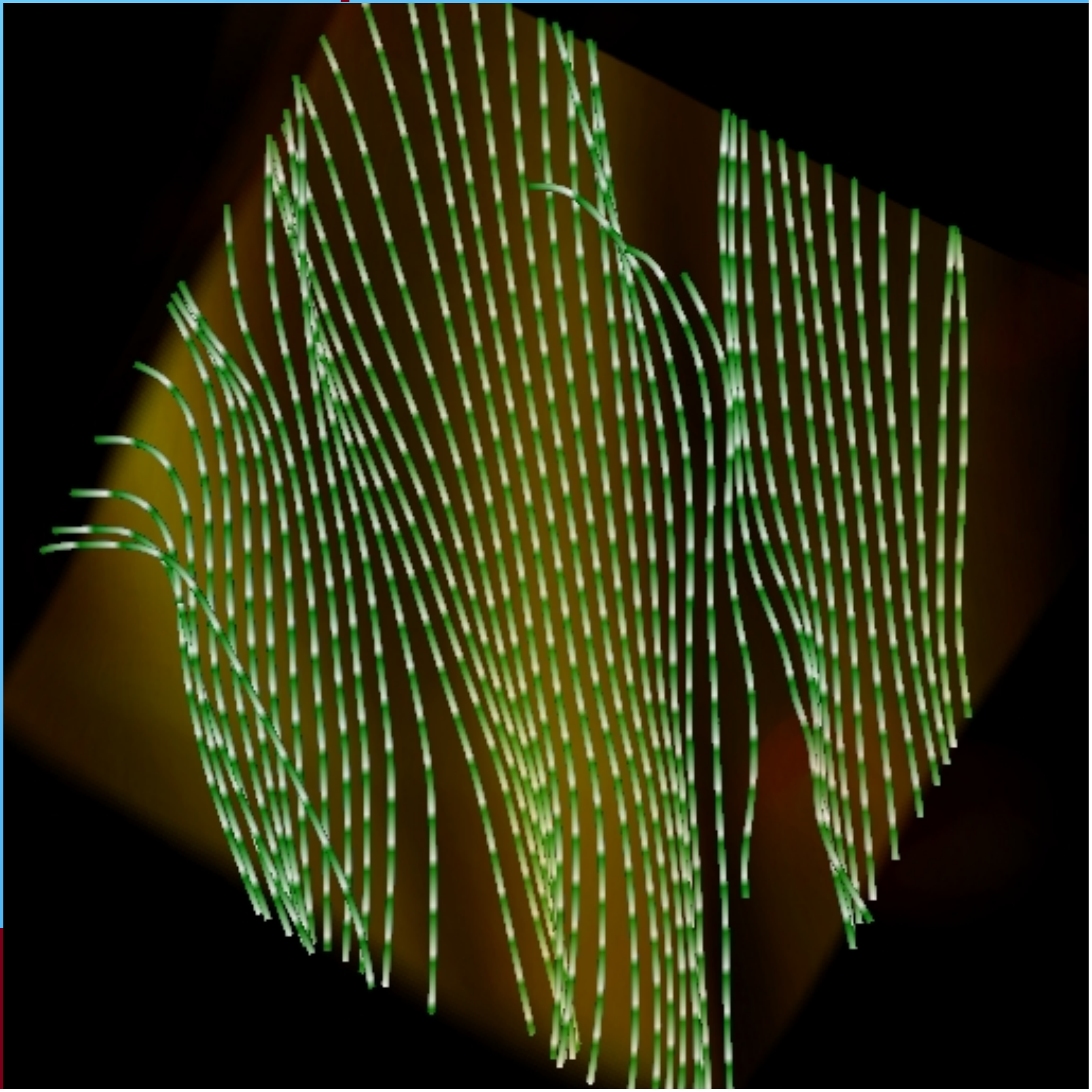
- T=130
- View in plane of slip surface (along curl(V))
- B trace foot points sampled on line normal to slip surface
- B field:
  - To left above
  - To view inside
  - To right below



Supercomputing Institute
for Advanced Computational Research

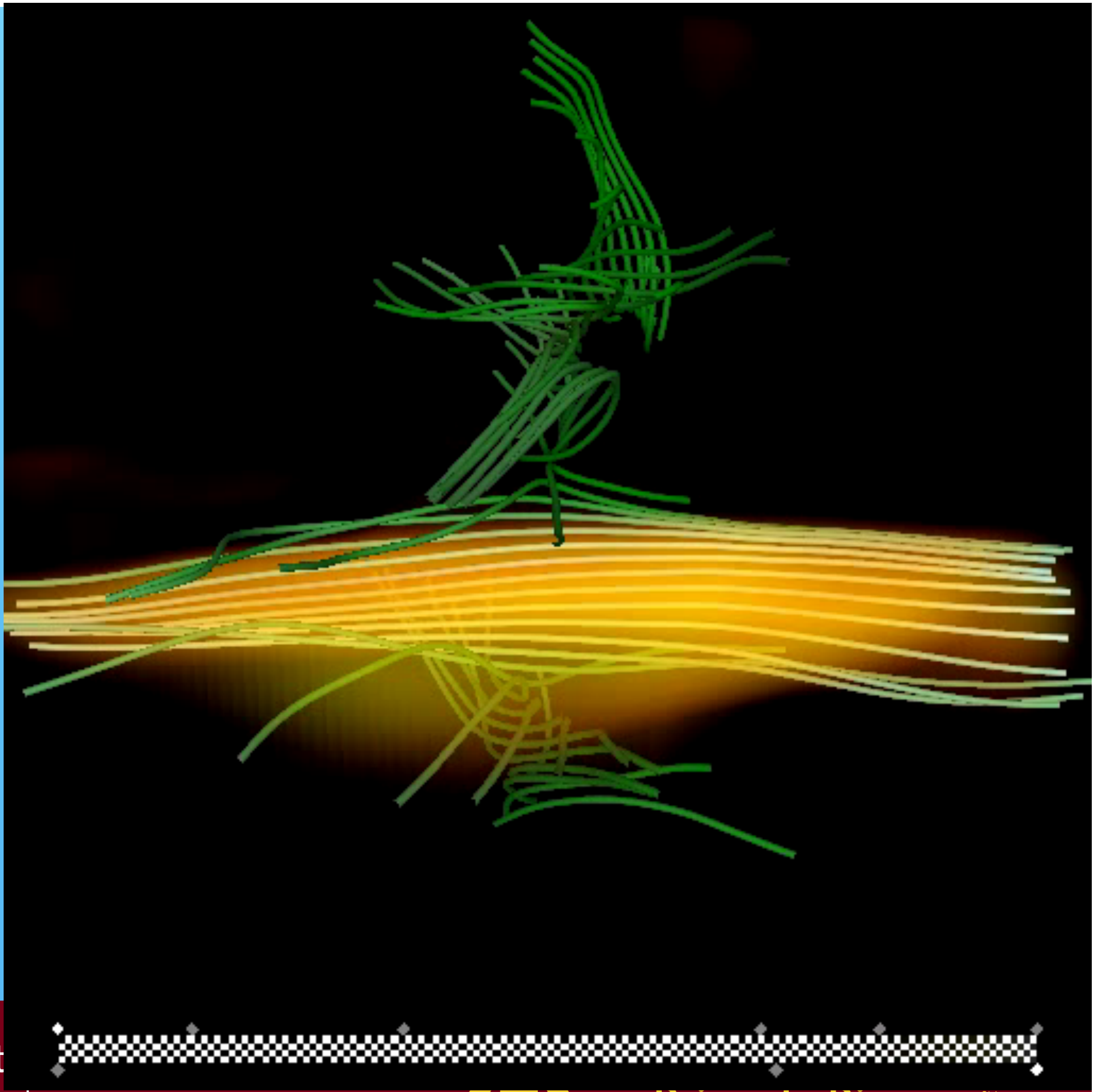
B-field in Slip Surface

- T=130
- View normal to slip sufrace
- Grean to white ramps show direction of B



Rotation of structure

Color of tubes shows relative B field strength



Development of Laminated Sturcutres

- Can measure rate of strain in principle directions of variation.
- Is rate of strain sufficient to produce thin layers?

Rate of Strain in Direction of Max Variation

- 7 variables lead to 7 directions

$$\hat{e}_{\max}(Q), \quad Q \in \{\rho, V_X, V_Y, V_Z, B_X, B_Y, B_Z\}$$

- Principle direction of average tensor product

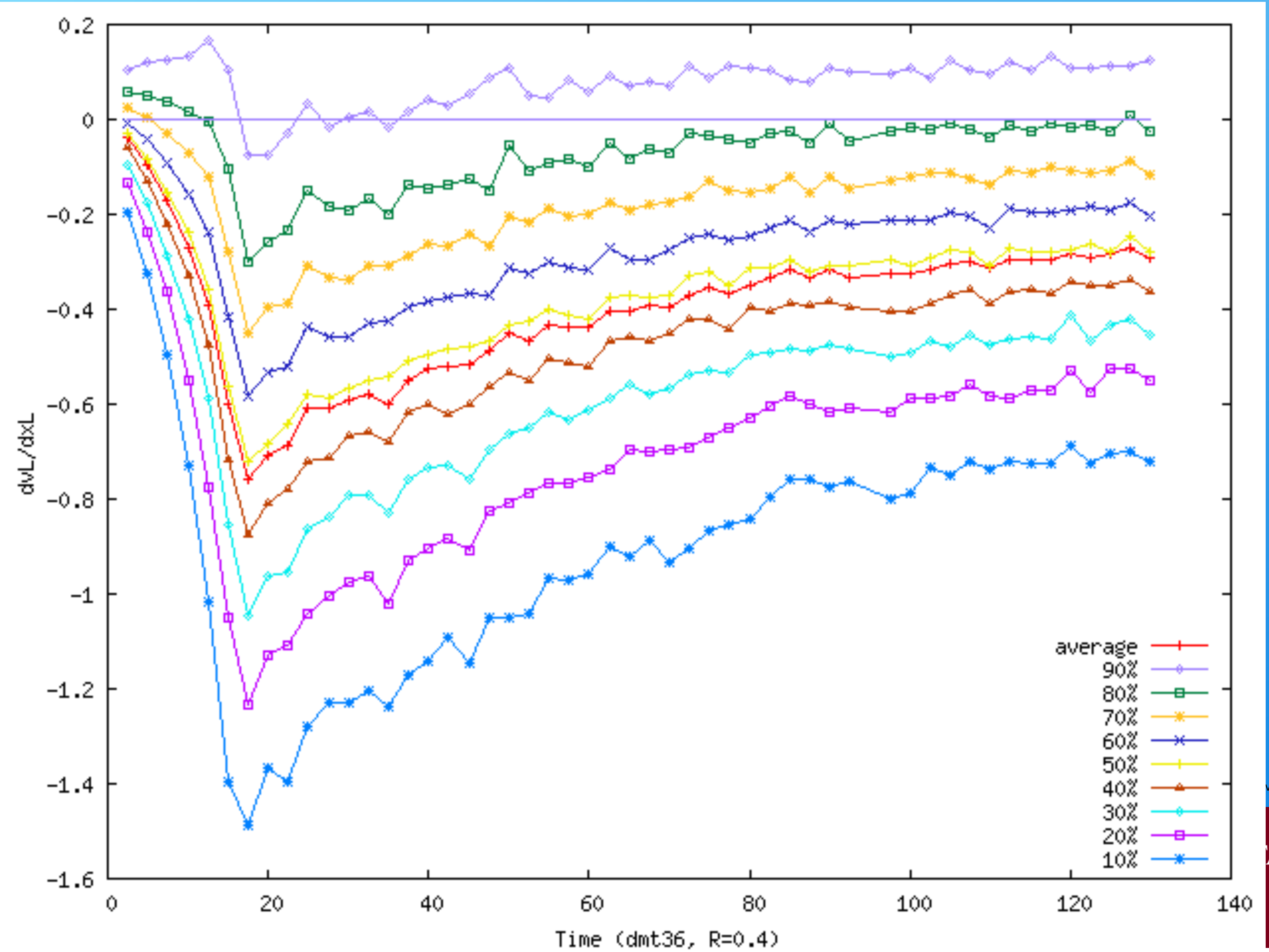
$$\left\langle \hat{e}_{\max}(Q)\hat{e}_{\max}(Q) \right\rangle_Q \Rightarrow \hat{e}_L$$

- V_L,L is rate of strain in common direction of Max variation across all 7 variables

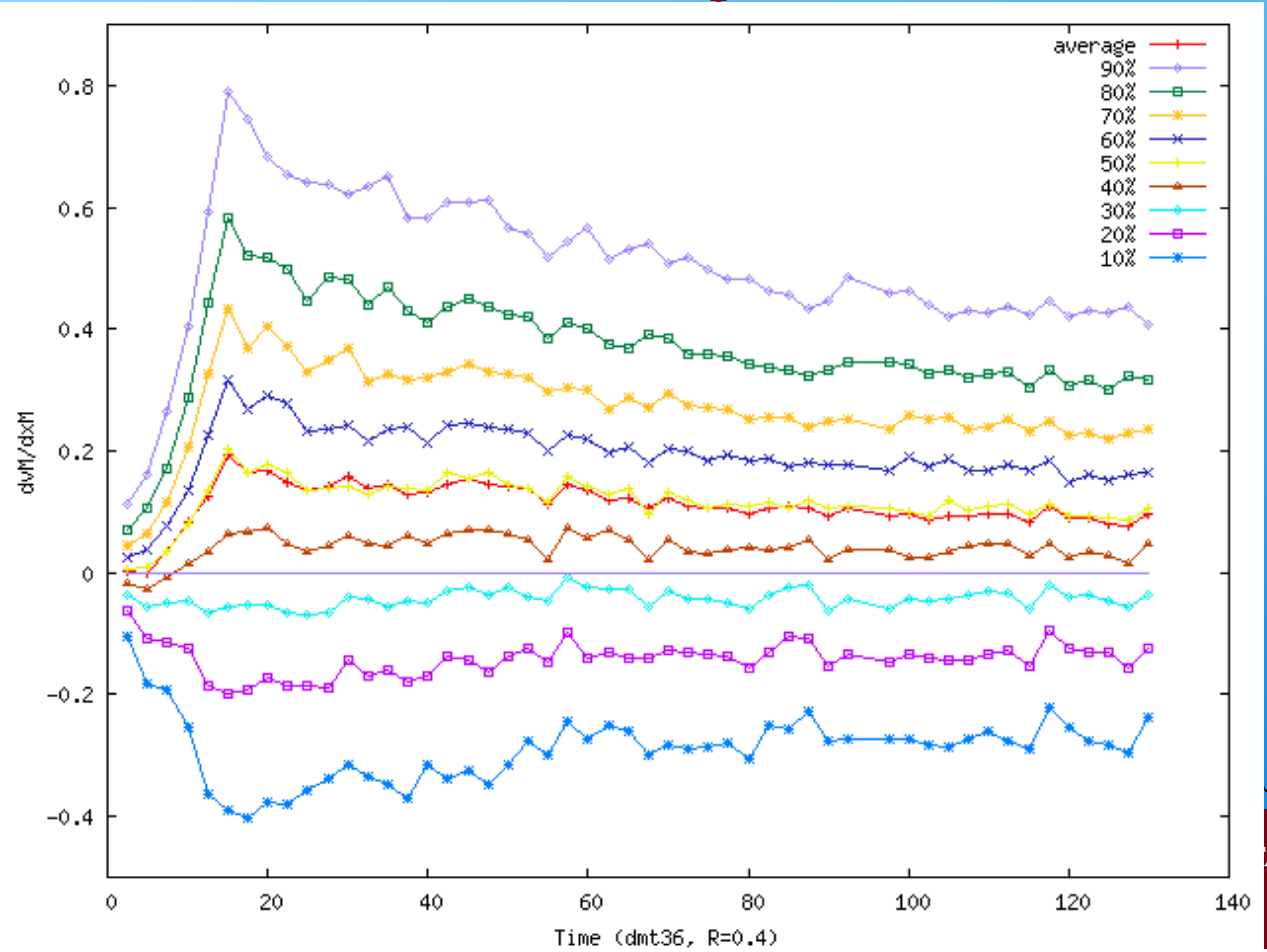
$$V_{L,L} = \left\langle \hat{e}_L \cdot \frac{\partial \vec{V}}{\partial \vec{x}} \cdot \hat{e}_L \right\rangle_{ball}$$

Can measure rate of strain in MID and MIN directions also

Rate of Strain along MAX Variation

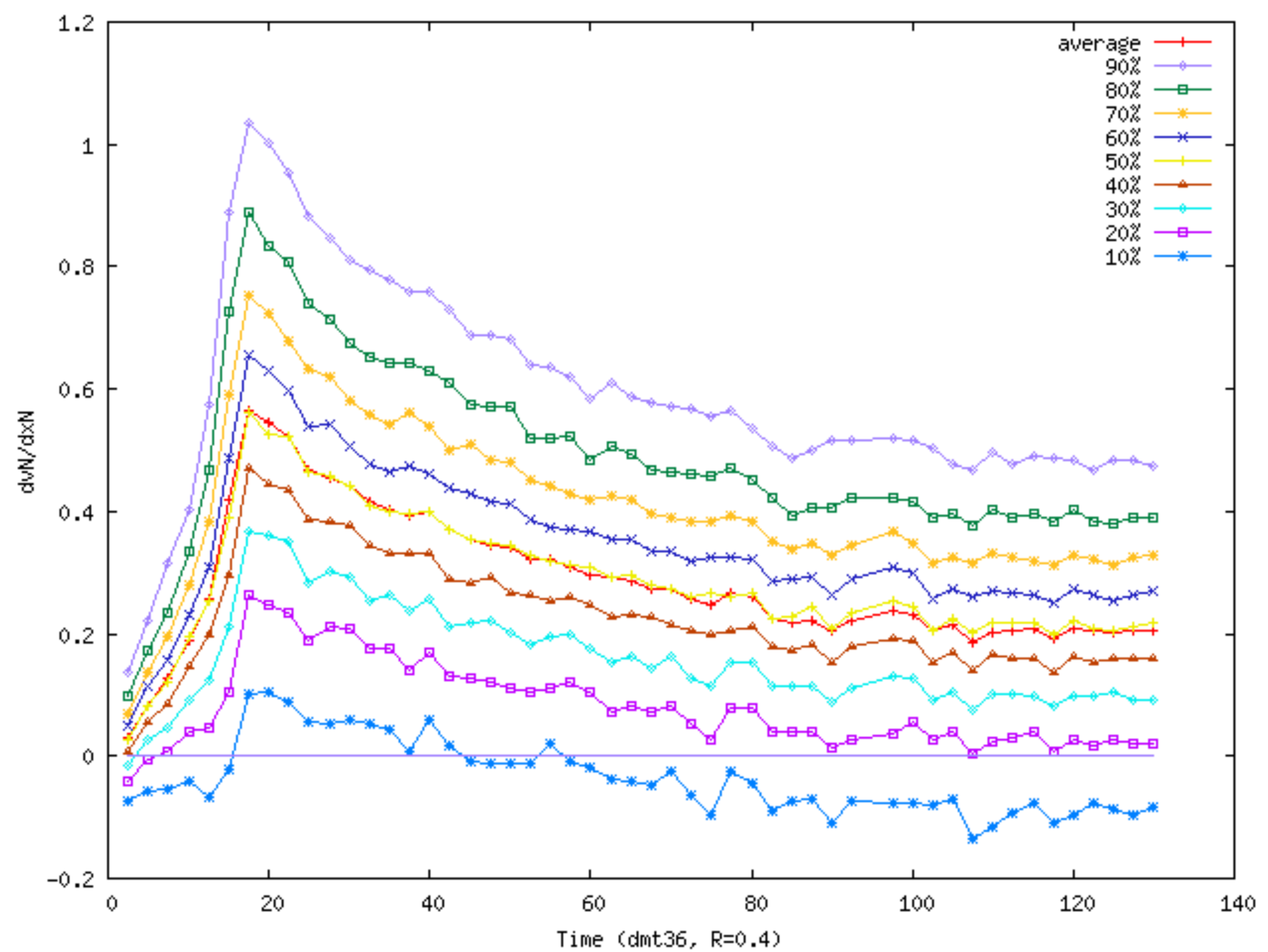


Rate of Strain Along MID Variation



Su
for
ved.
A

Rate of Strain Along MIN Variation



Su
for

ved.

A

Rate of Strain → Layers

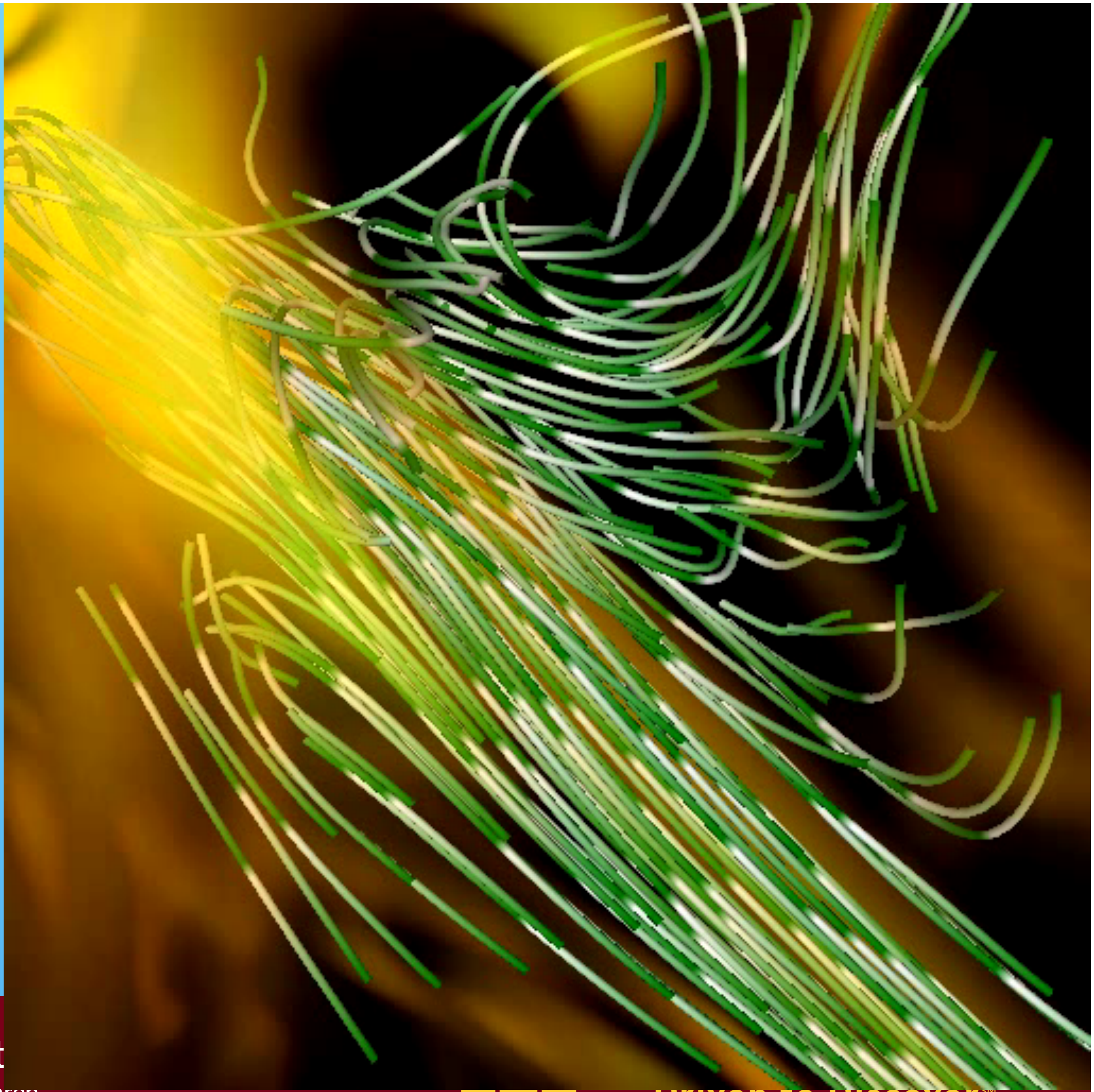
- Rate of Strain in direction of most rapid variation is predominantly negative
  - Median is always negative → most elements are compressing (squeezing) in the direction of most rapid variation
- Amplitude of strain rate enough to explain filaments: In simulation units:
  - Median e-folding times < 3 over last 100 time units
  - 30 e-folding times

Movie of squeeze

From d6a52

50% compressive

Move starts with a strong filament and follows back in time



Conclusions

Weak B-field turbulence

- Very much like neutral fluid turbulence
- B-field follows the flow
- B develops fastest on small scales in vortex tubes

Distinctive features of saturated B-field turbulence

- Layers
- Interleaved fields
- Multiple criss-crossed laminates
- Rate of strain field produces and maintains laminated structures

Thank You