Weak Field Amplification in Moderately Compressible MHD Turbulence

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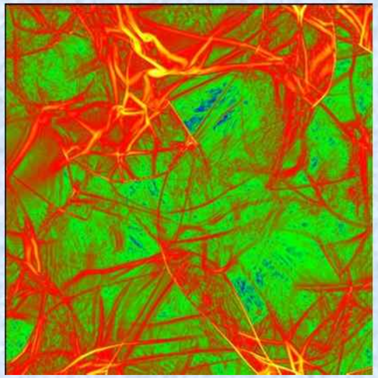

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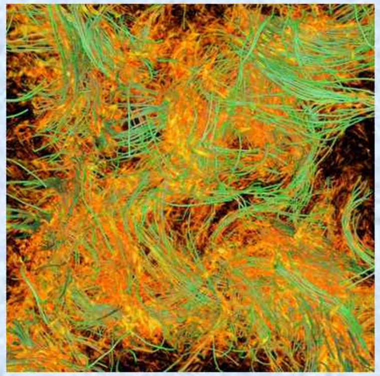


Outline

- Motivating Astrophysics – Galaxy Clusters
  weak large scale fields
- A couple of relevant MHD issues
- Ideal MHD simulations of two limiting cases

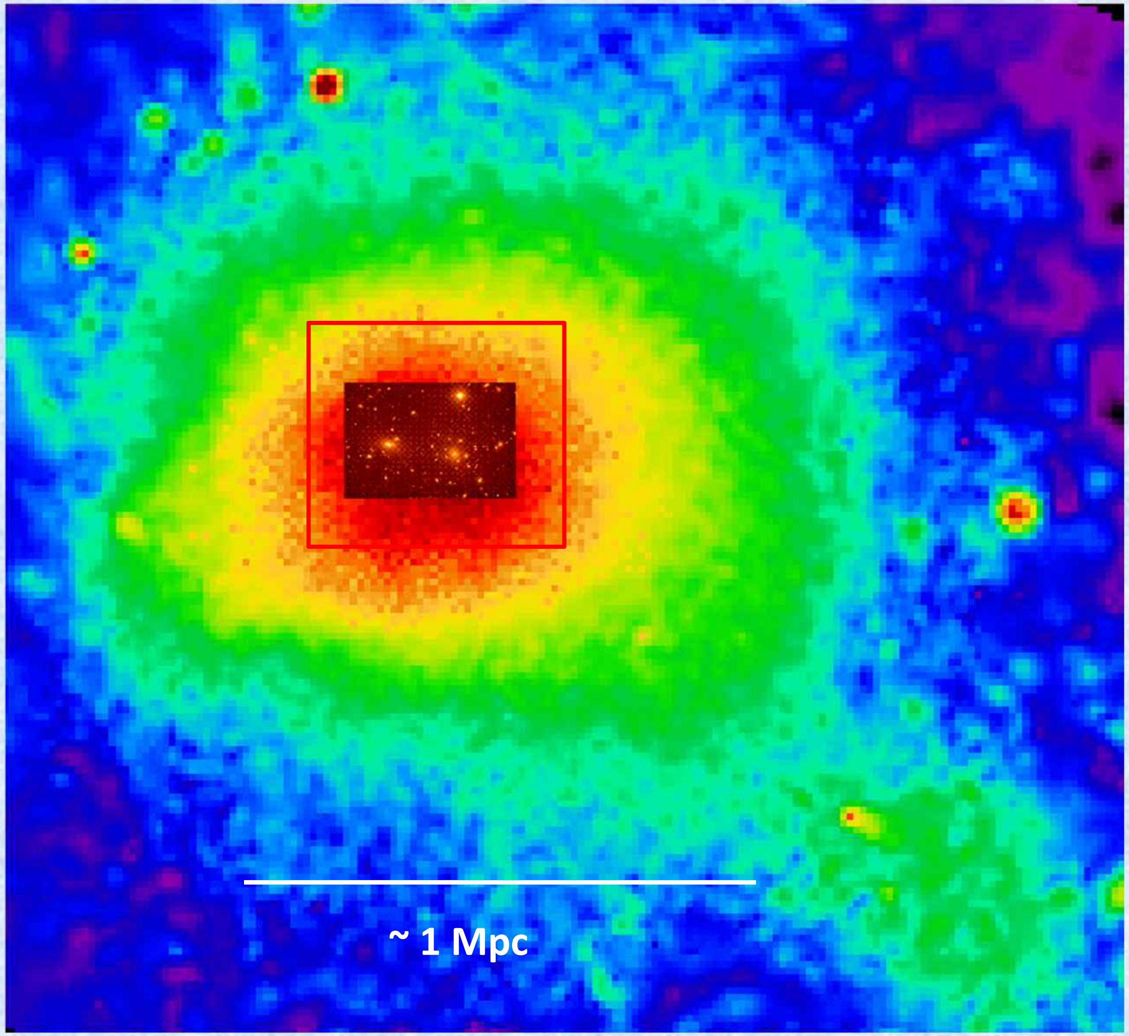
Coma Cluster: Diffuse, ICM Thermal X-rays

ICM

kT ~ 8 keV

v_{th} ~ 1500 km /s
c_s ~ 1100 km/s

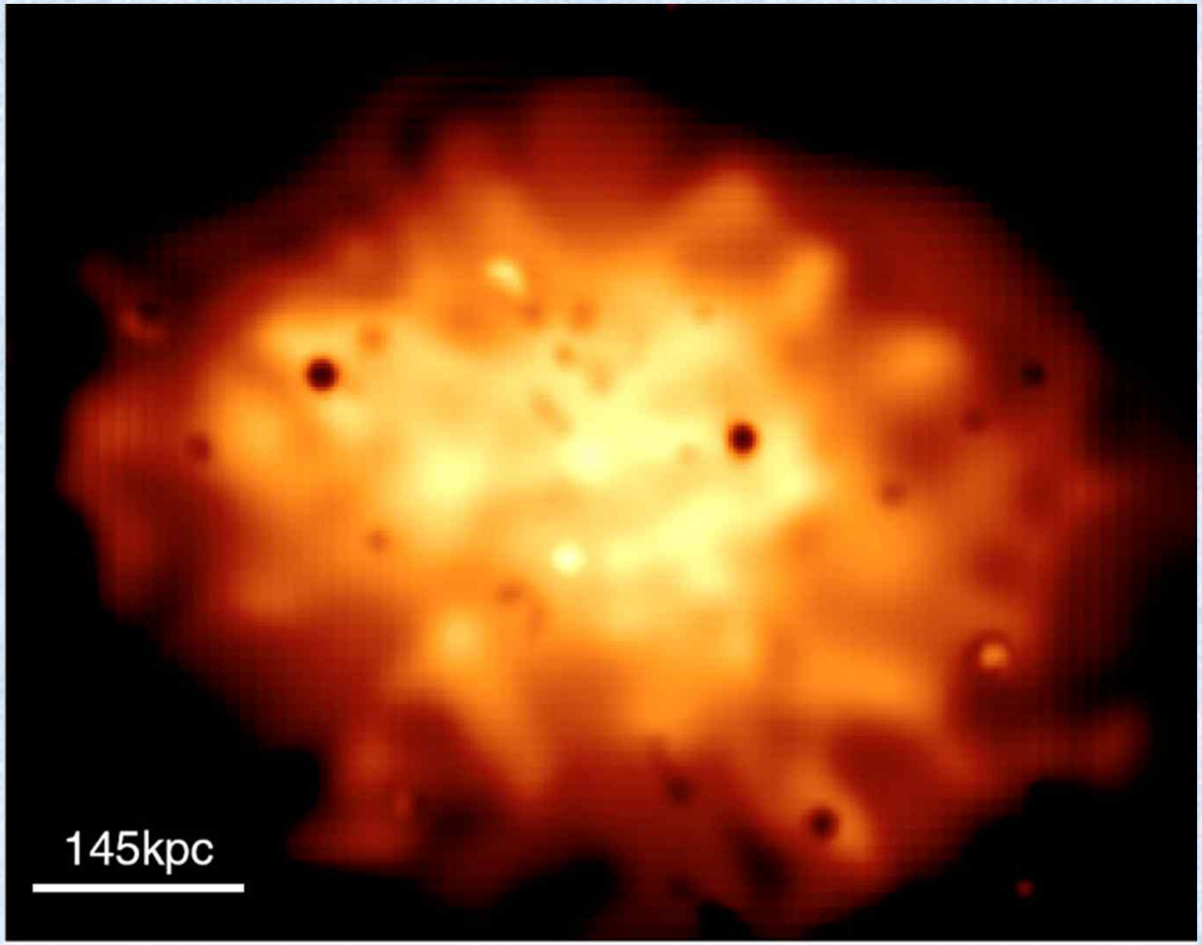
$N_{e,core}$ ~10^{-2}/cm^3
P_{core} ~ 10^{-10} dyne/cm^2



ROSAT

Note: 1000 km/sec ≈ 1 kpc/Myr = 1 Mpc/Gyr

Coma Cluster: Turbulence
Projected ICM Pressure Distribution (X-rays --XMM)



Roughly consistent with $P(k) \propto k^{-7/3}$ spec

100 kpc < L_{max} < R_{core}

Inferred:

v_{turb} ~250 km/s

P_{turb} ~10% P_{therm}

Schuecker + 04

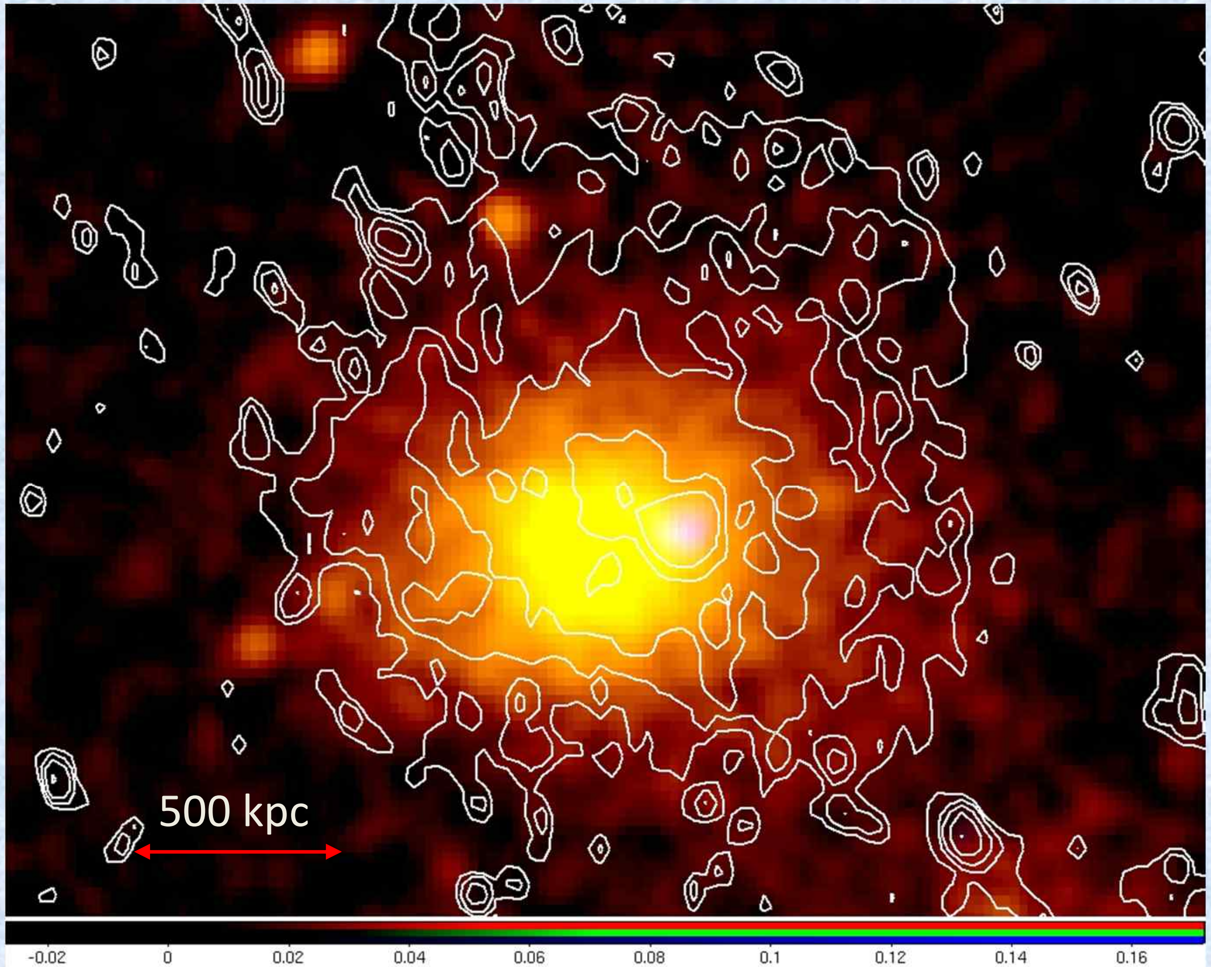
Coma Cluster: Diffuse, unpolarized radio halo
=> >GeV electrons in > μG magnetic field (β~ 100)

ROSAT image

375 MHz radio contours (WSRT)

Mostly unpolarized,

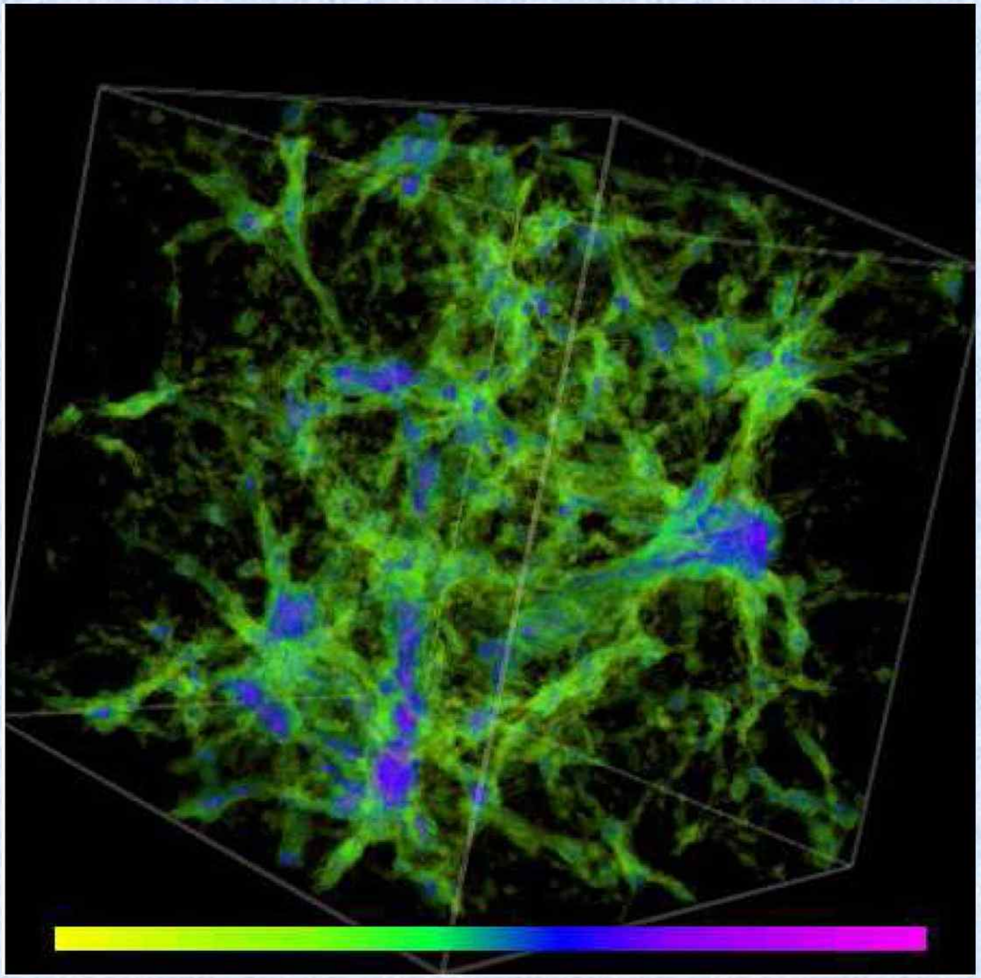
Absence of strong mean field (?)



Brown & Rudnick 10

Cosmological Simulations Suggest Turbulence Likely
=> Amplification of very weak seed fields

Distribution of Turbulently Amplified Magnetic Fields
in such a simulation

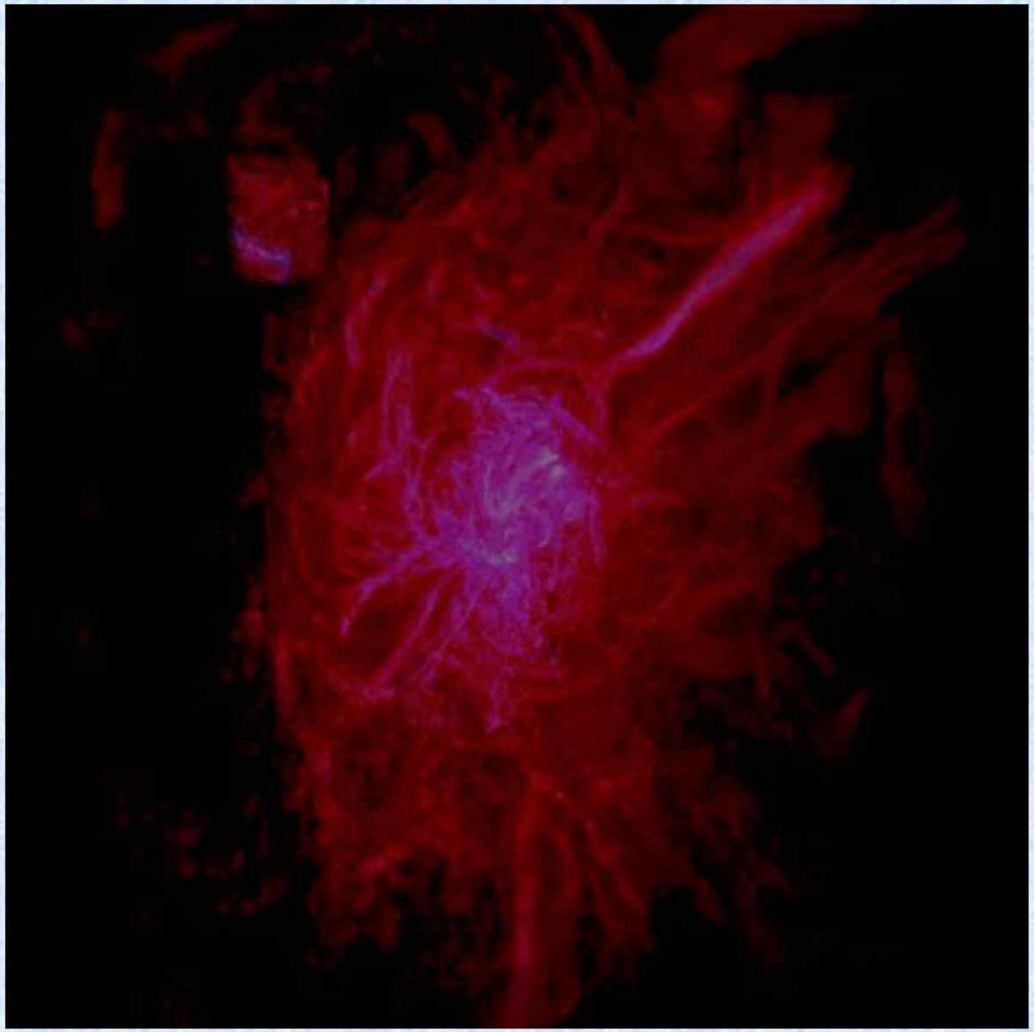


100 h^{-1} Mpc^3 box

Ryu+ 08

Magnetic Field in Simulation Cluster

Volume rendering
Log(B)



1 h^{-1} Mpc^3 box

MHD Cosmology Simulation by K. Dolag (Pete Mendygral)

Creation & Evolution of Vorticity

$$\frac{\partial \omega}{\partial t} = \nabla \times (u \times \omega) + \nu \nabla^2 \omega = \frac{1}{\rho^2} \nabla \rho \times \nabla P$$ + driving terms

a) Baroclinic Source Term
(e.g., in complex post shock flows)

b) Crocco's Theorem: Vorticity Generation at Steady, Oblique, Curved Shocks

$$\omega_{in} = 0$$

$$\Rightarrow \omega_{out} = \frac{(\rho_2 - \rho_1)^2}{\rho_2 \rho_1} \mathrm{K} u_1 \times \widehat{n} \sim \frac{(\sigma - 1)^2}{\sigma} \frac{u_{shock}}{R_{shock}}$$

$$\sigma = \frac{\rho_2}{\rho_1}$$

Then Vortex Stretching to Amplify

$$\frac{\partial \omega}{\partial t} = \nabla \times (u \times \omega) + \nu \nabla^2 \omega = \frac{1}{\rho^2} \nabla \rho \times \nabla P$$

$$\frac{d \ln\left(|\omega| \big/ \rho \right)}{dt} = \frac{\omega \cdot [(\omega \cdot \nabla) u]}{\omega^2} \sim \frac{dl / dt}{l},$$

Similar Magnetic Field Generation and Amplification :

The Magnetic Induction Equation using Generalized Ohm's Law

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \eta \nabla^2 B - \frac{1}{e n_e^2} \nabla n_e \times \nabla P_e$$

η is resistivity

Mathematical structure same as the Vorticity Equation:

- Source Term (Biermann Battery) when $\nabla n_e \times \nabla P_e \neq 0$
  (e.g., at curved shocks)
- Field intensity, B(t) $\propto$ l(t) --stretch and fold amplification)
- Dissipation, diffusion measure, $R_M = uL/\eta = P_r R_e$
  Where $P_r = \nu/\eta$ is the Prandtl number

Generation of Magnetic Fields in Simulations of Driven, Compressible MHD Turbulence

- 3D periodic box $L_x = L_y = L_z = 10$ (up to 2048^3 zones)
  MHD TVD -- 2^{nd} order Eulerian, compressible MHD
  Constrained Transport to maintain $\nabla \cdot B = 0$

- Isothermal, $c_s = 1$, so box <u>sound crossing time = 10</u>
  (also roughly largest eddy turnover time, t_{eddy})
- $\langle\rho\rangle=1$

- "Ideal" MHD, so $P_r = \nu/\eta \sim 1$

- Initially very weak, mean field, $\beta=10^6$

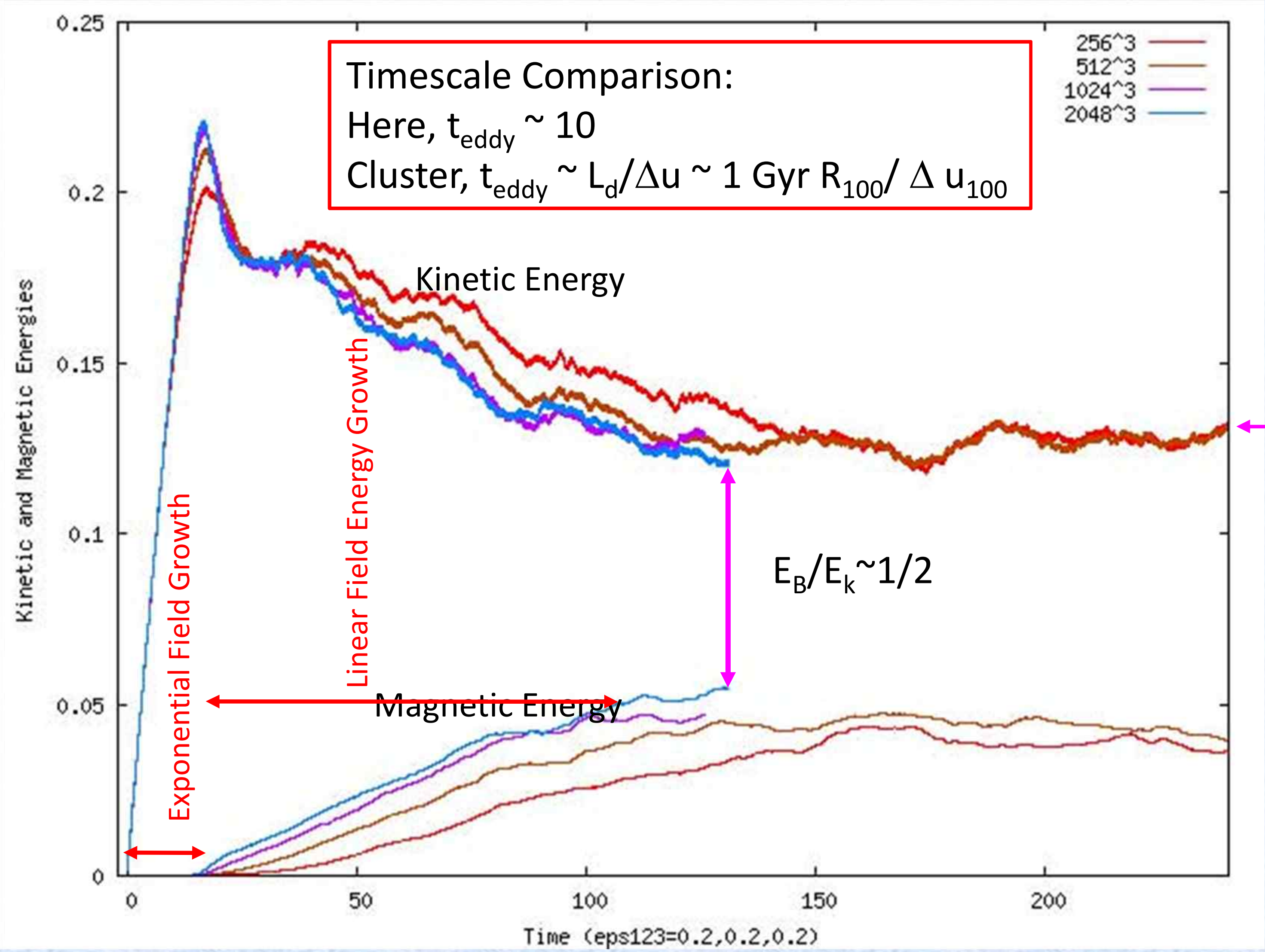
- Random Driving Power, $P_k \propto k^6 \exp(-8k/k_p)$, $k_p=4\pi/L_x$,
  peaks ~ $L_d = 2/3\ L_x$,
  => $u_{RMS} \sim 1/2\ c_s$

- Driving form ranges between purely solenoidal ($\nabla \cdot \delta u = 0$)
  to purely compressional ($\nabla \times \delta u = 0$)

Illustrate Two Extremes of Forcing

- Case 1: Purely Solenoidal ($\nabla \cdot \delta u = 0$)
- Case 2: Purely Compressional ($\nabla \times \delta u = 0$)

Case 1, Purely Solenoidal Forcing: Turbulent Energy Evolution

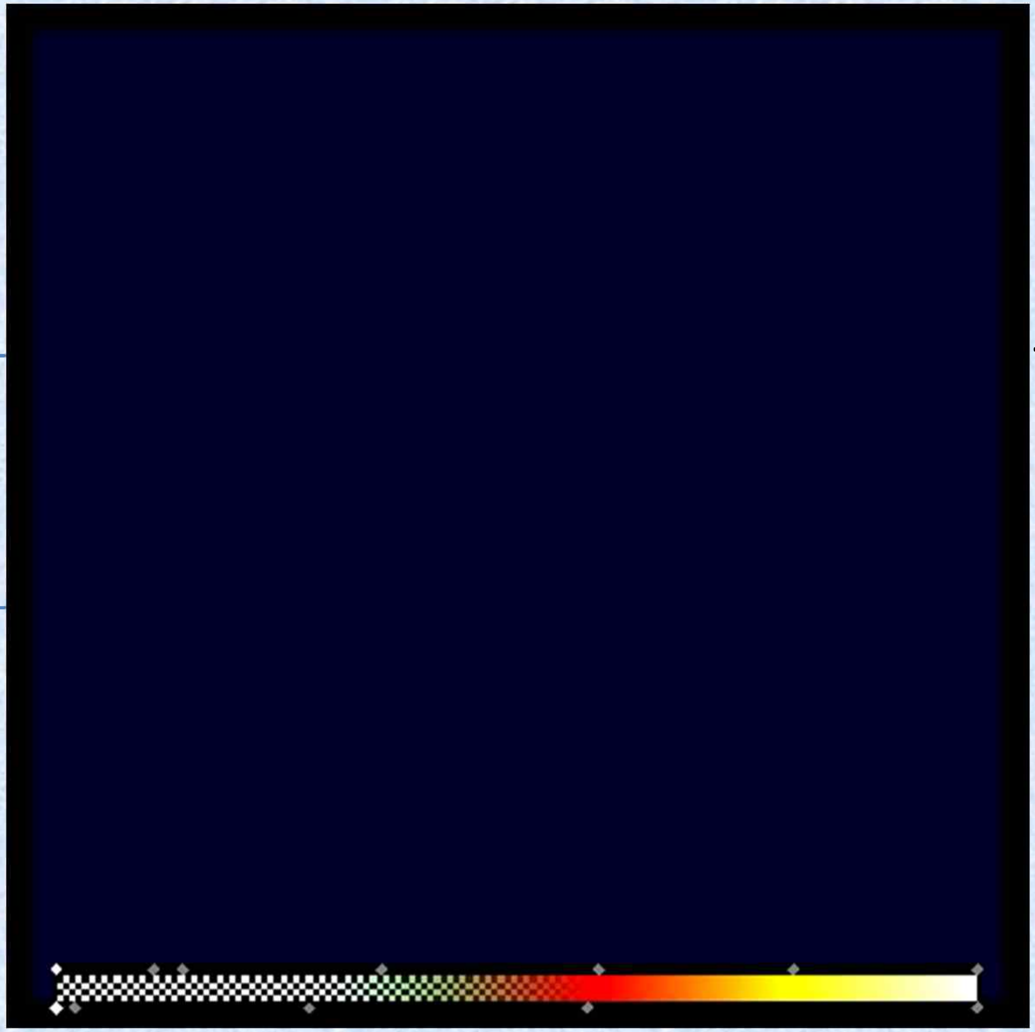


u_{rms}~0.5 c_s

Porter+

Case 1:
Magnetic Field Evolution

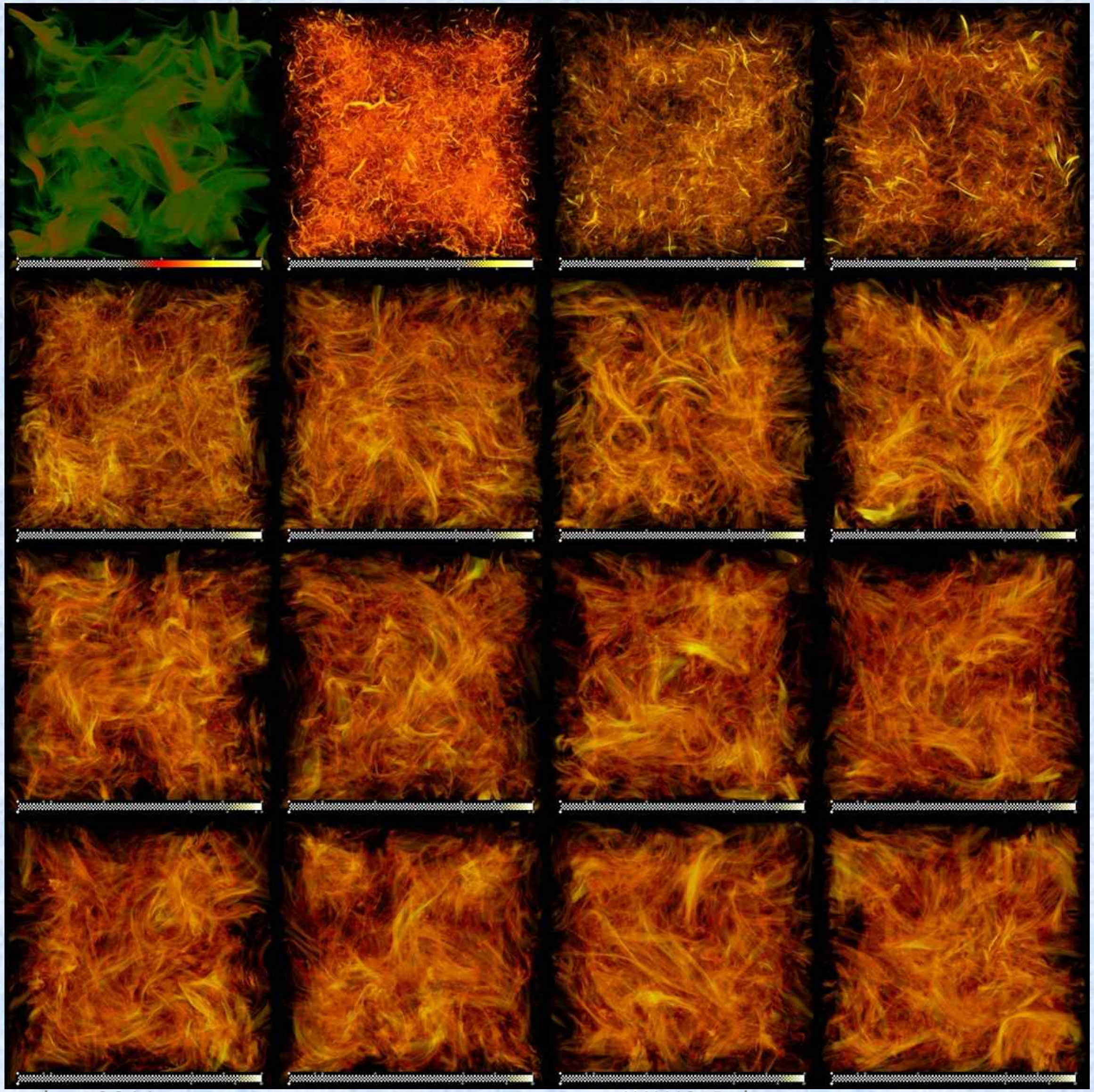
Note evolution of scales and transition from 'tubes' to 'ribbons'



t = 0 to t = 250

1024^3 simulation

Case 1:
Magnetic Flux Structure Summary



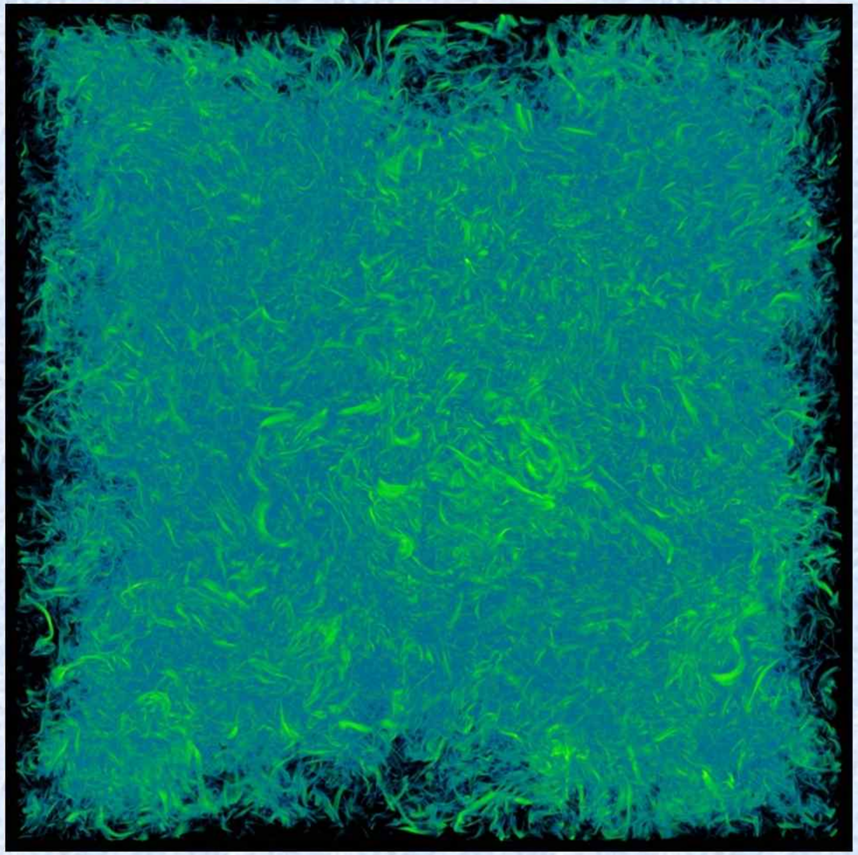
t=10, 20, ..., 160

1024^3

Log B

Case 1:
Magnetic Flux Structures
2048^3 simulation

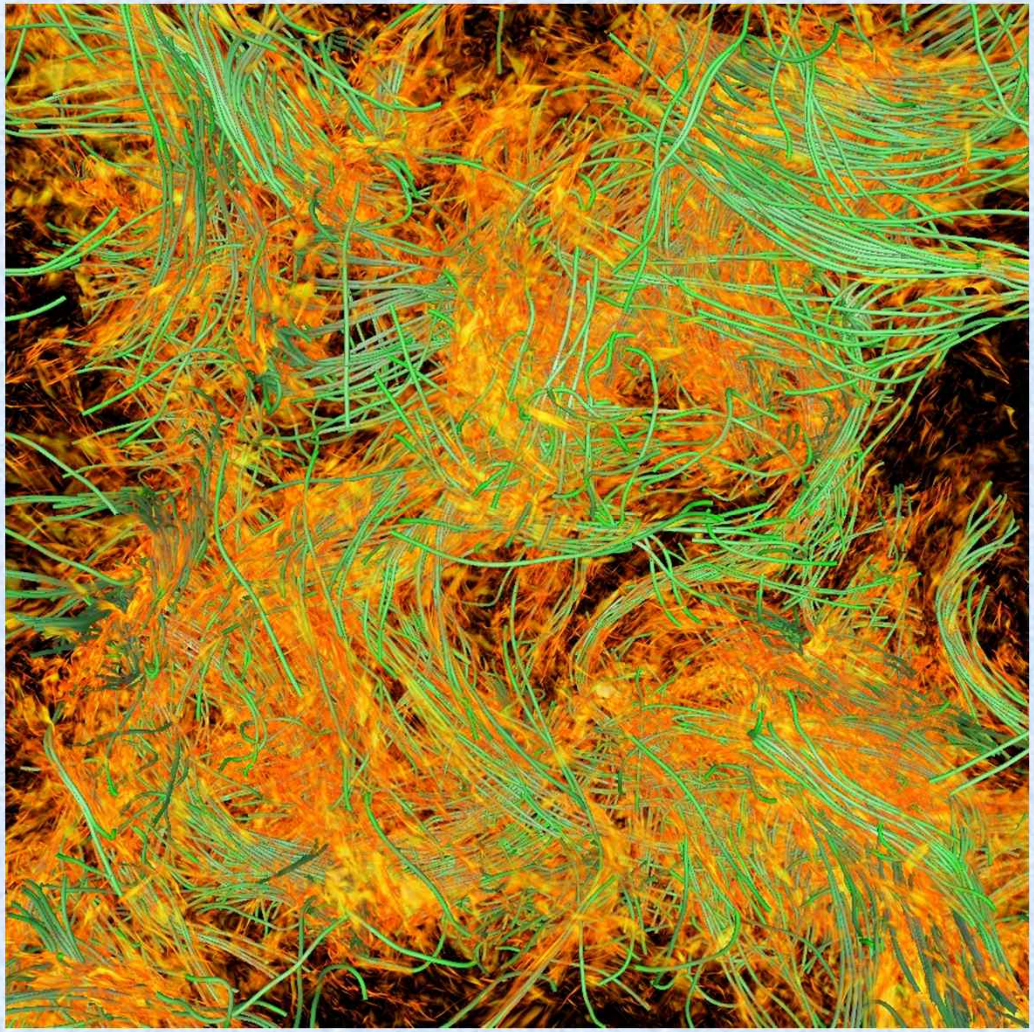
t = 20
end of exponential phase



t = 130
early 'saturation'

Case 1:
Magnetic Flux Lines & Vorticity

t = 130
2048^3
simulation



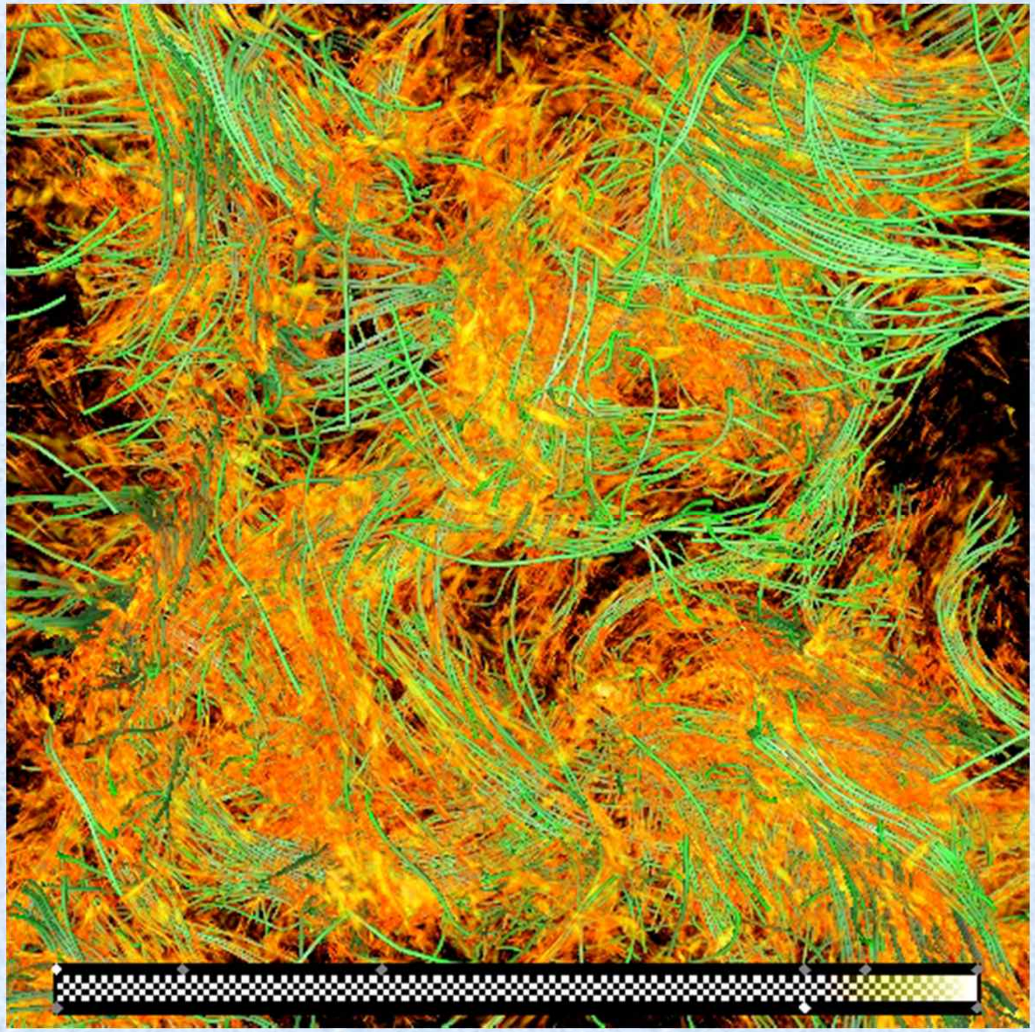
Flux tubes
&
$|\omega|=|\nabla\times u|$
rendering

Porter, Ryu,
Cho & Jones

Case 1:
Magnetic Flux Lines & Vorticity

t = 130
2048^3
simulation

Rotation
animation



Flux tubes
&
$|\omega|=|\nabla\times u|$
rendering

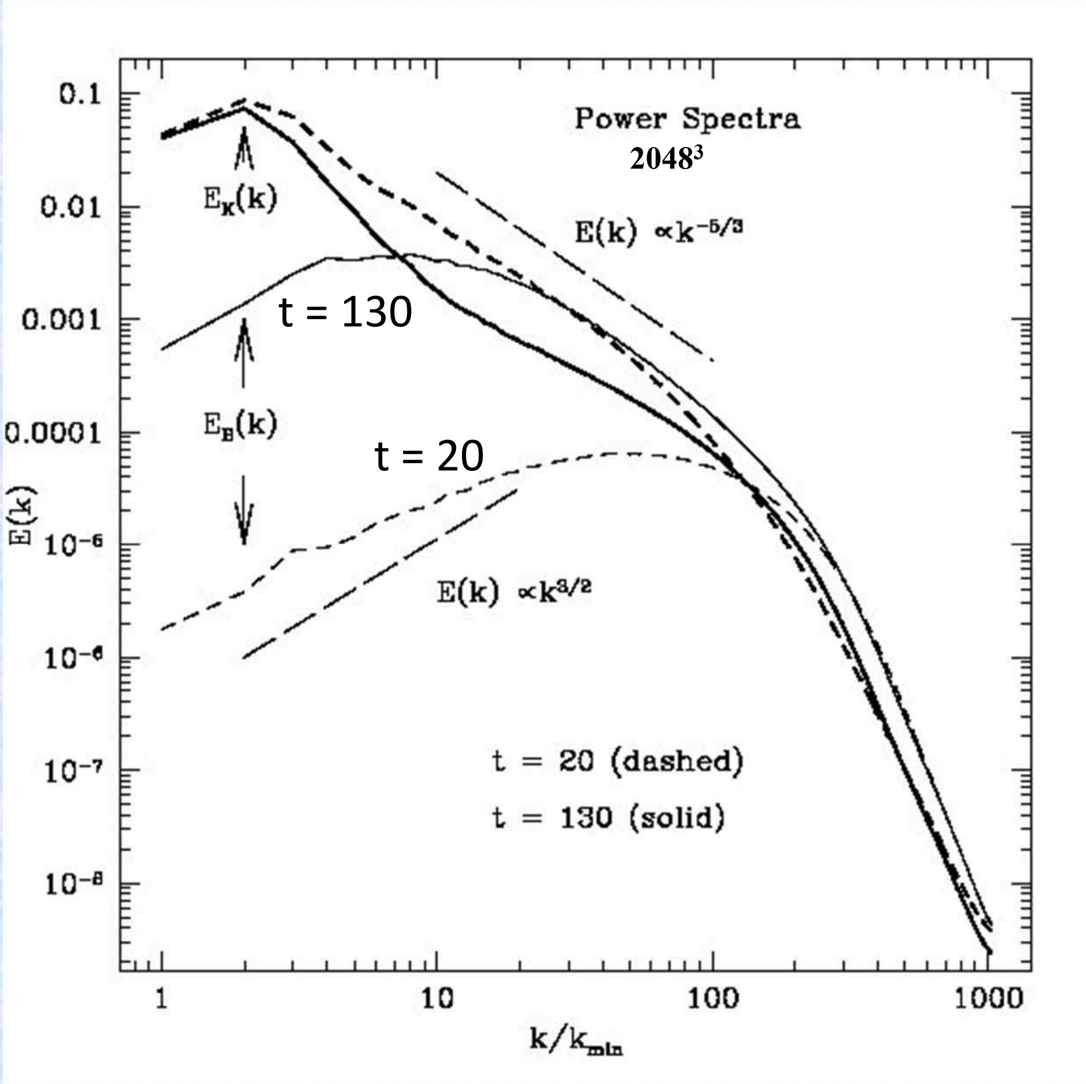
Porter, Ryu,
Cho & Jones

Case 1: Comparison incompressible simulation Magnetic Field Structures

Spectral code
Cho & Ryu

512^3 simulation

Case 1:
Energy Spectra

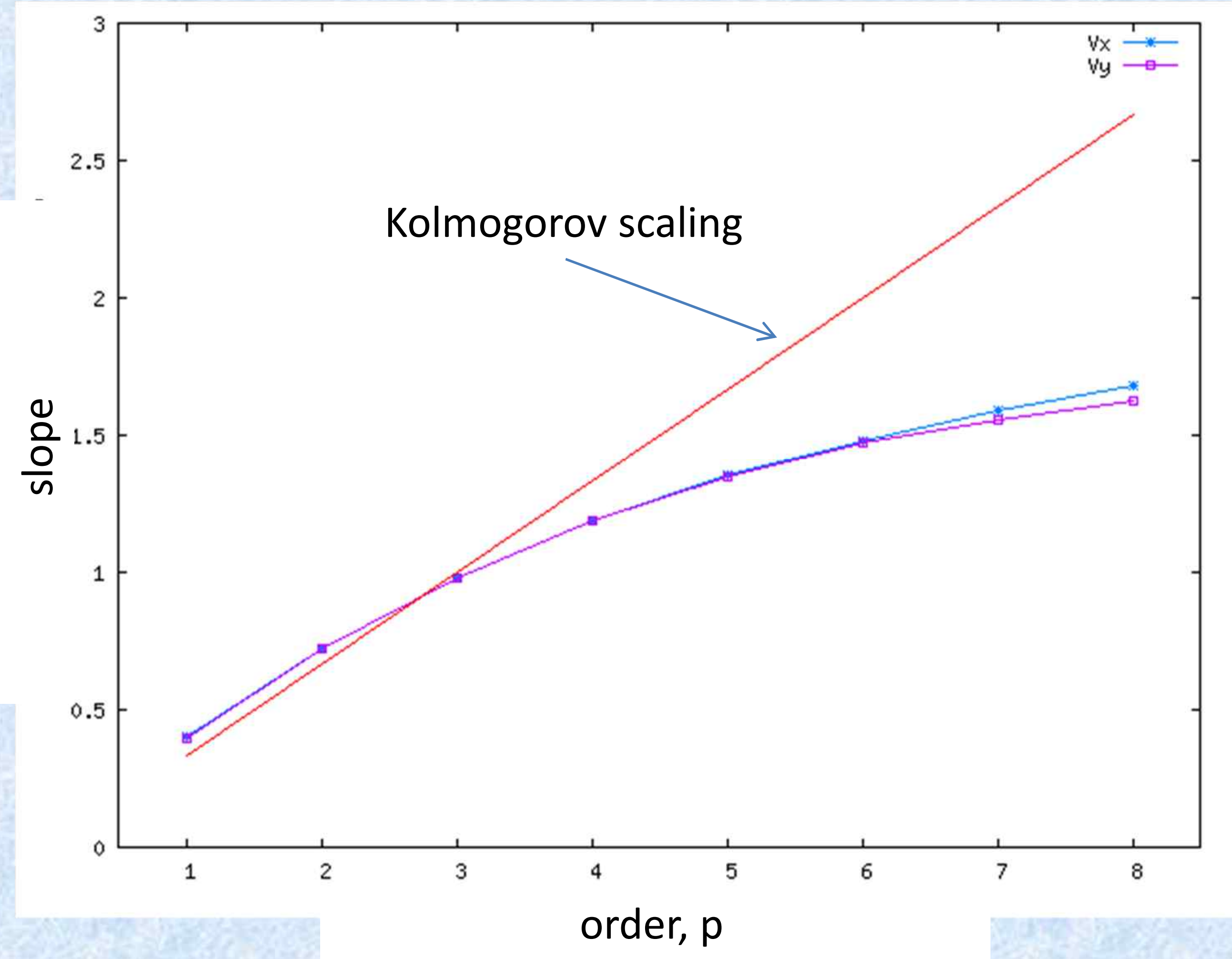


$k/k_{min} = 1$ is full box

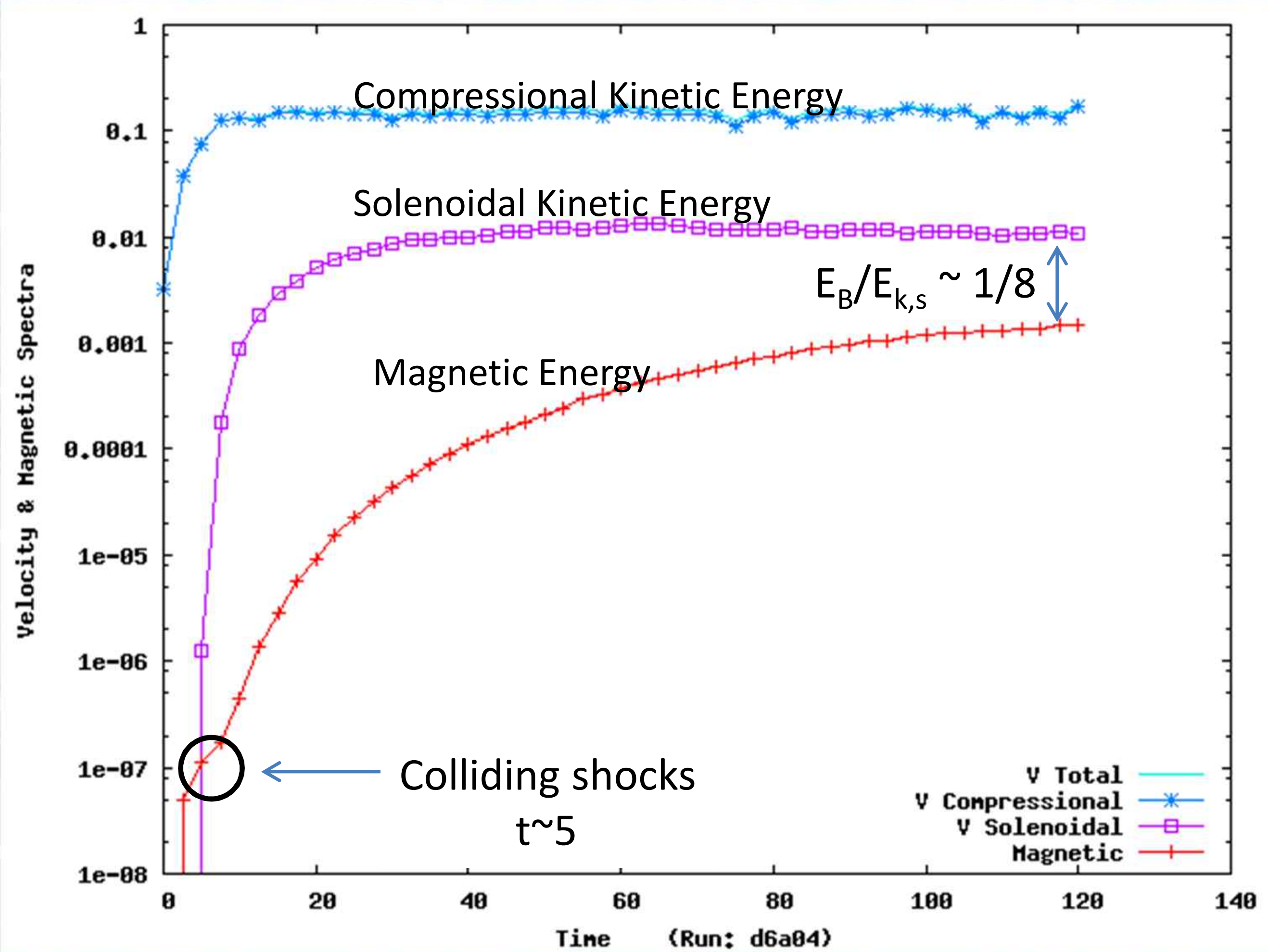
2048^3 compressible

Kinetic energy is mostly solenoidal

Case 1: Scaling Relations
Structure Function slopes



Case 2: Purely Compressional Forcing Turbulent, Spectral Energy Evolution



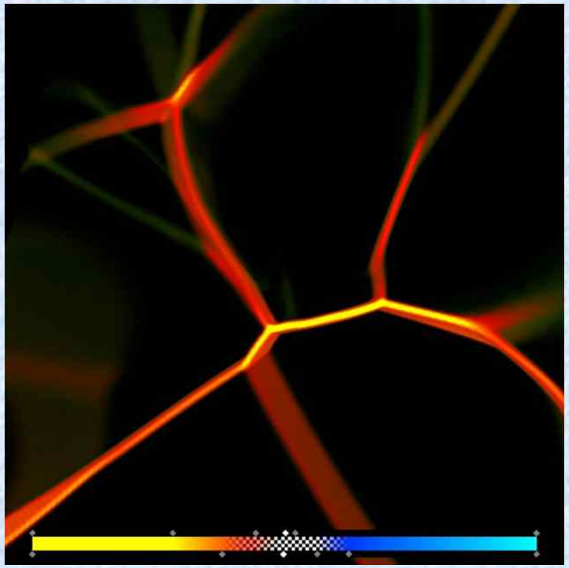
1024^3 simulation

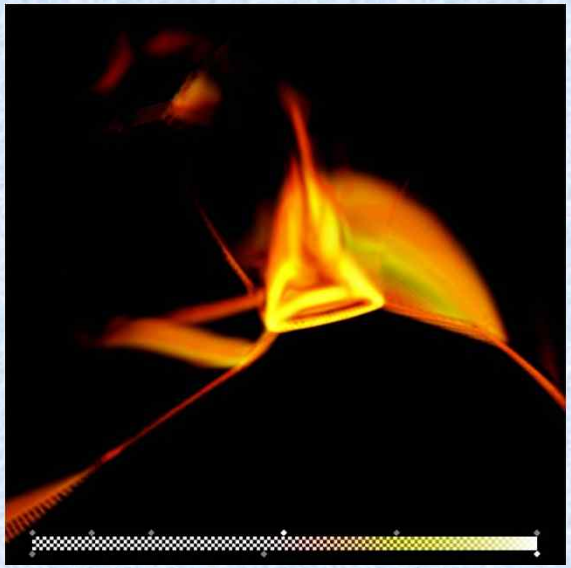
Case 2: Purely “Compressional” Driving

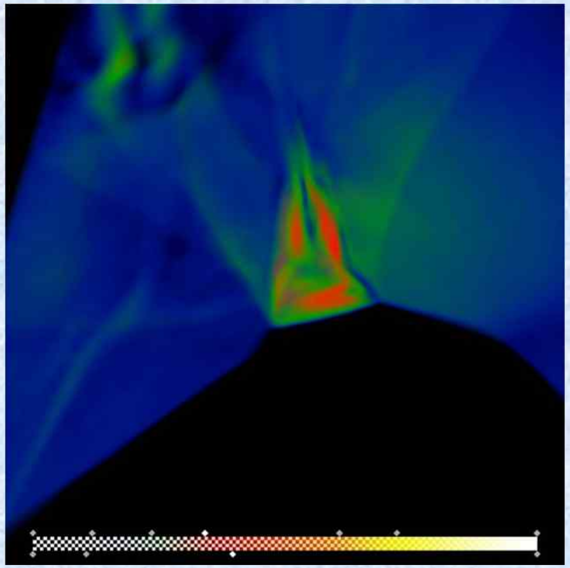
Generation of vorticity and magnetic field

Zoomed in slice at t = 5
showing relationships

Intersecting shocks => Vorticity => Magnetic field strength



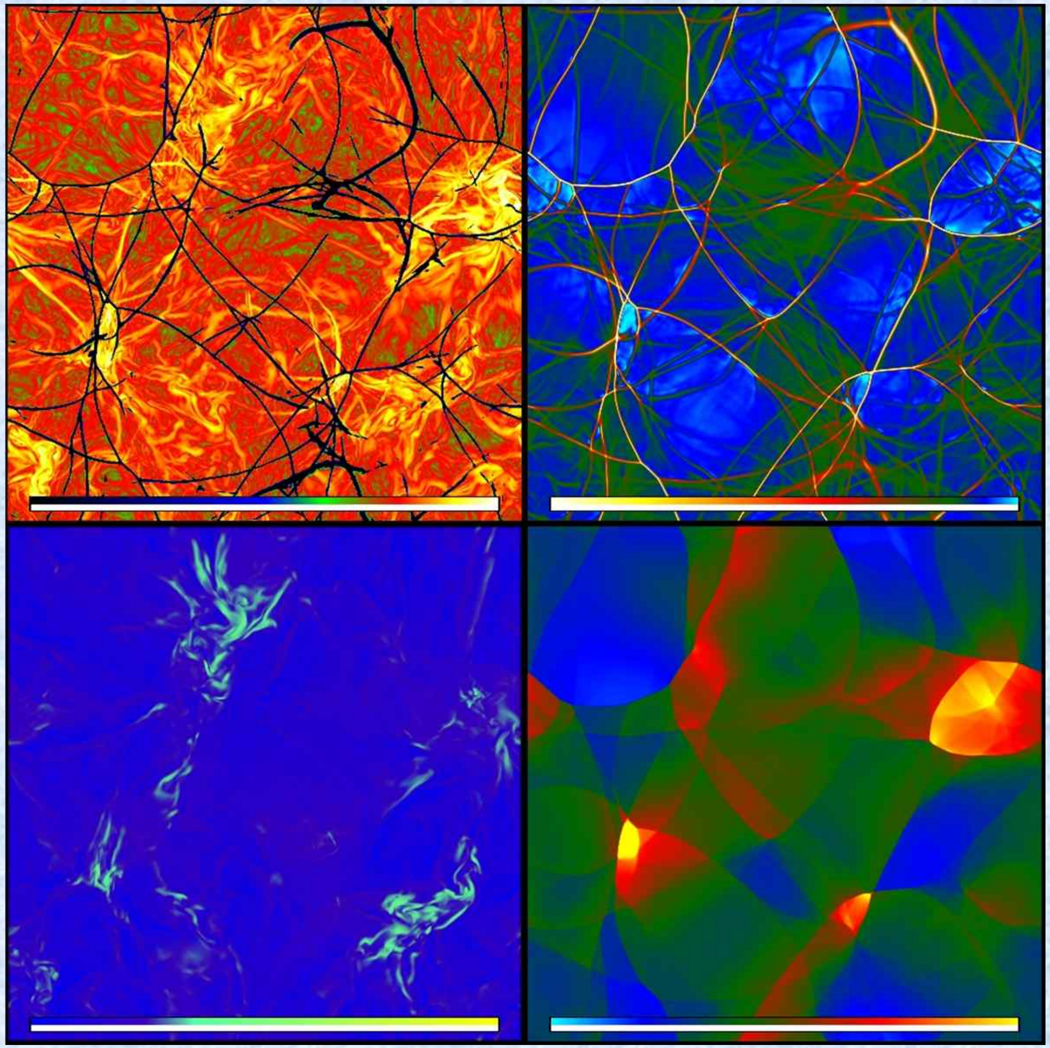




512^3 simulation

Case 2: Generation of vorticity and magnetic field

Slice at t = 20



$|\omega|$

$\nabla \cdot u$

B

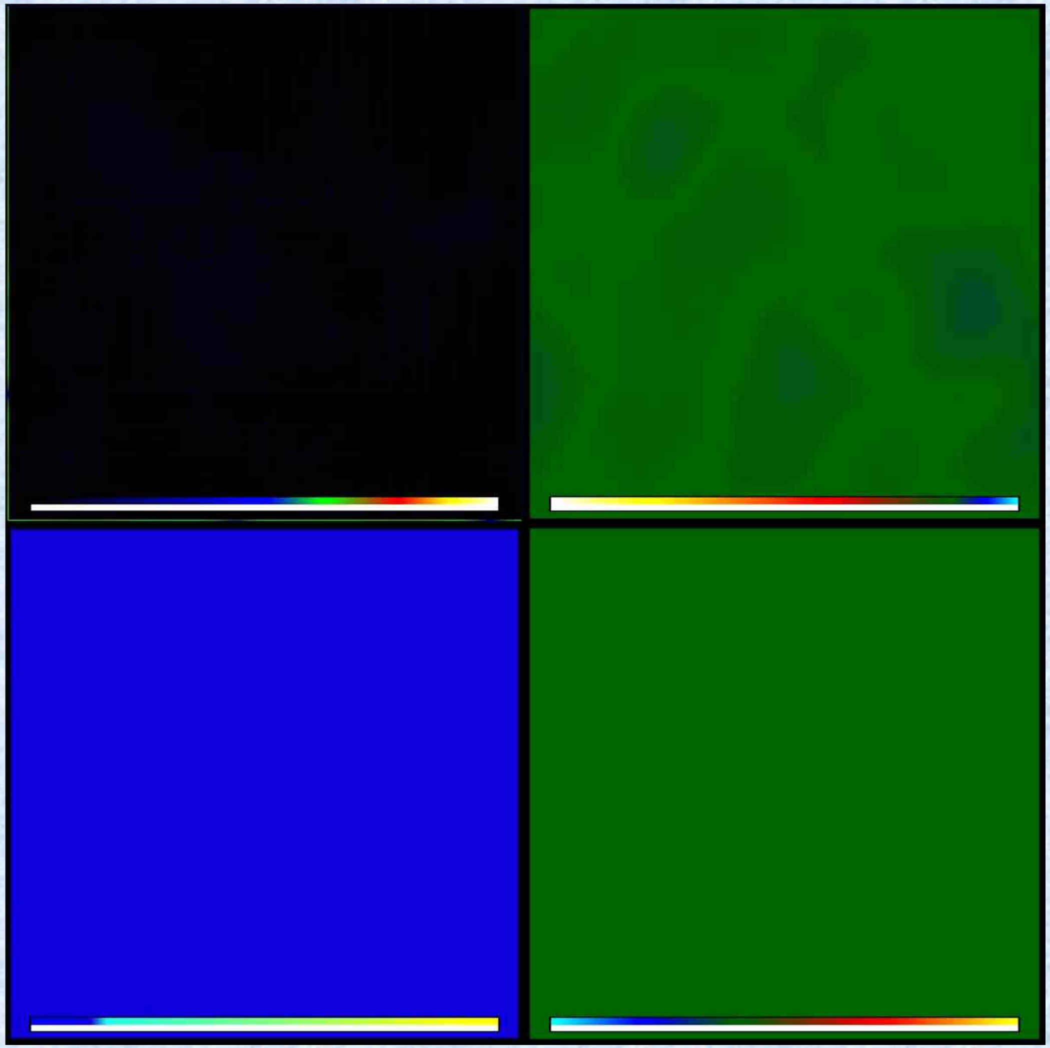
ρ

1024^3 simulation

Case 2: Generation of vorticity and magnetic field

$|\omega|$

$\nabla \cdot u$

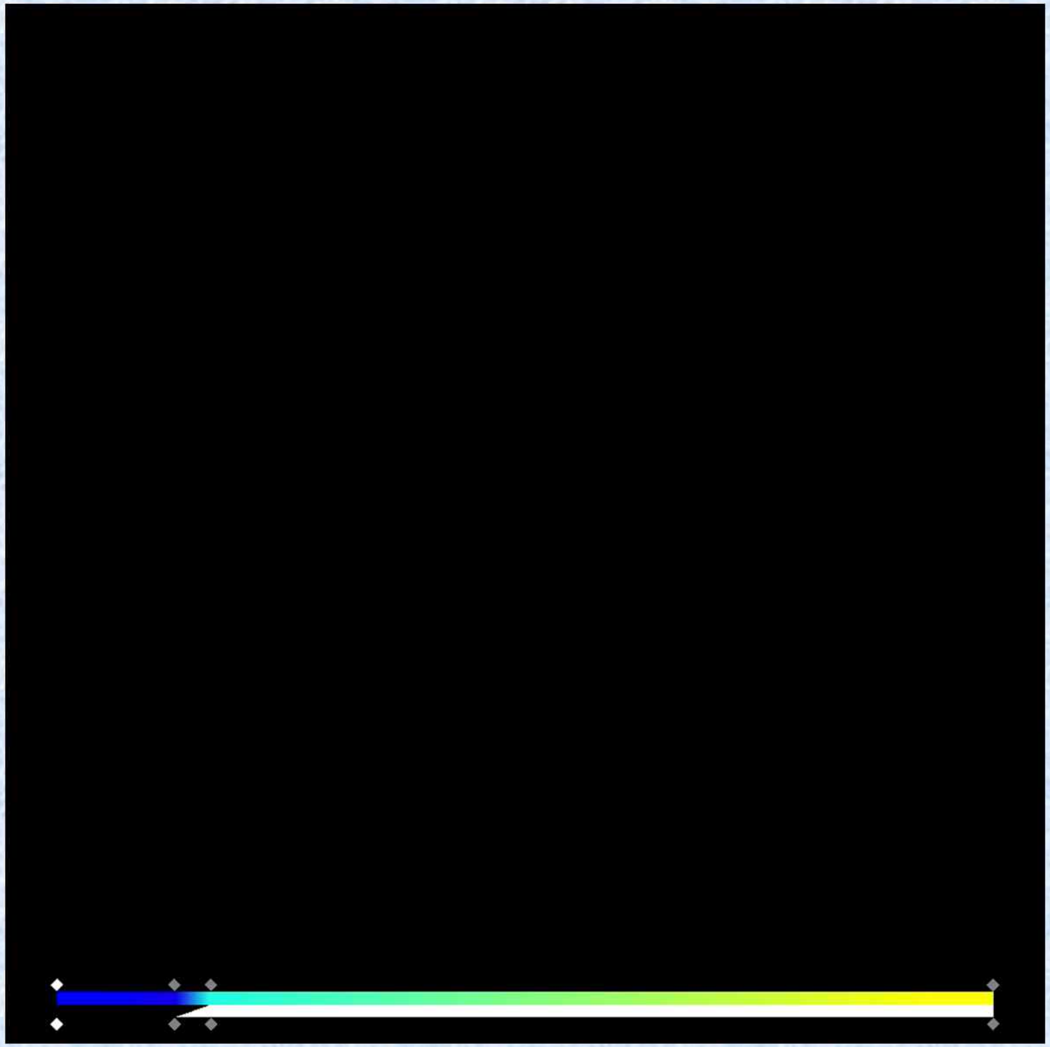


B

ρ

1024^3 simulation

Case 2:
Magnetic Field Evolution



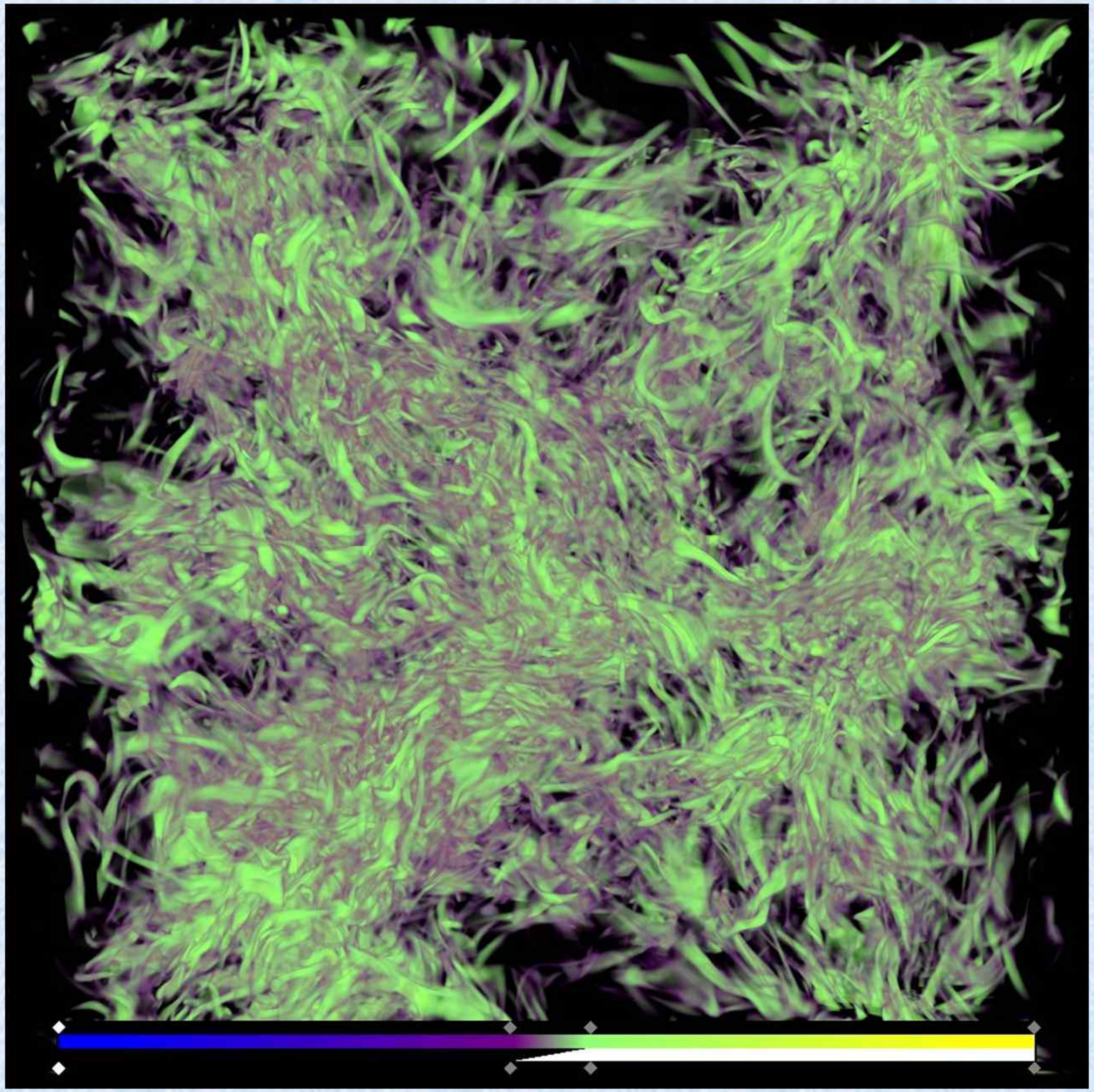
t = 0 to t = 120

1024^3 simulation

Note slow development of filaments and propagating patterns following shocks

Sliding color scale

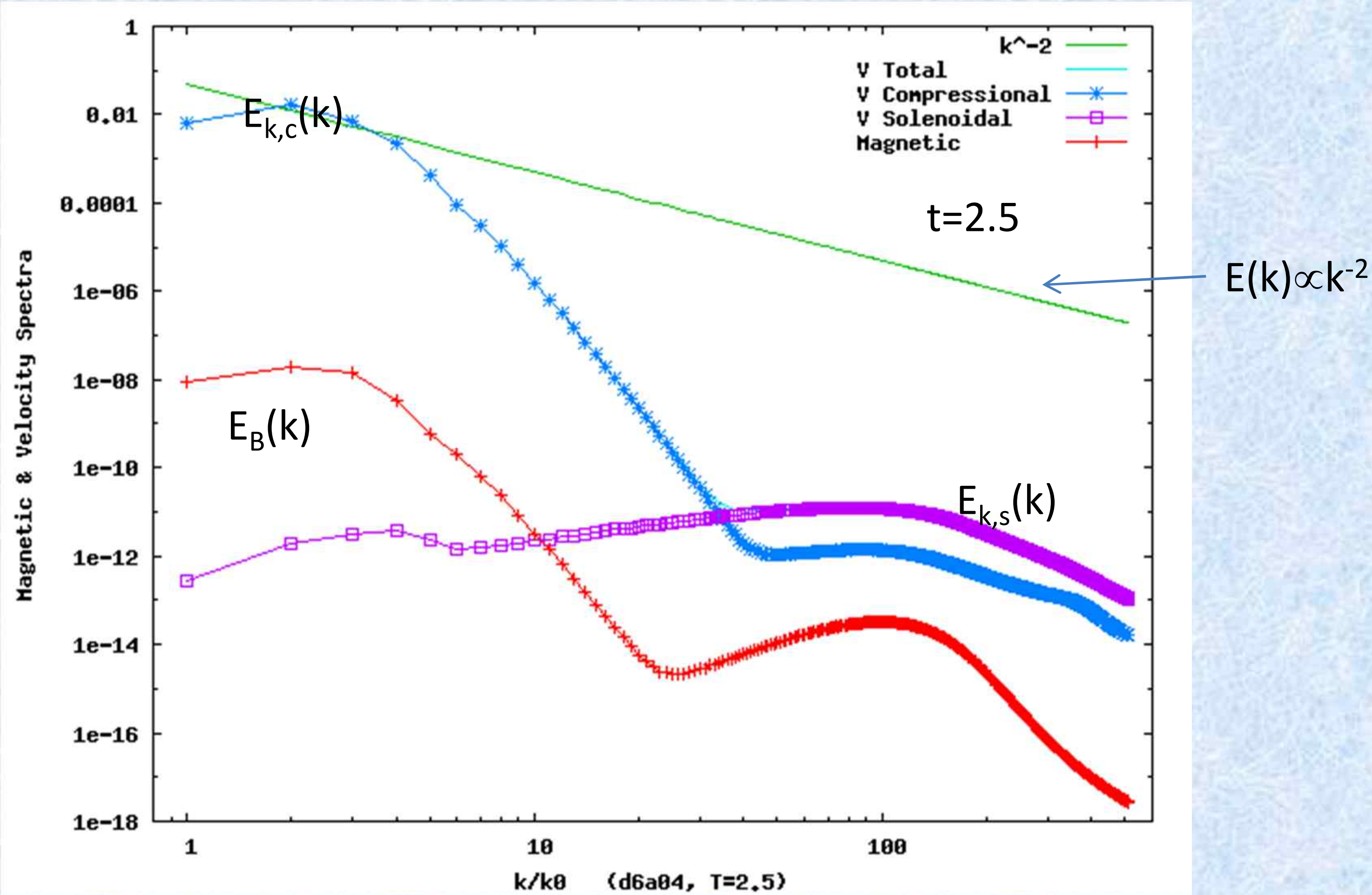
Case 2: Magnetic Field Structures



t = 120

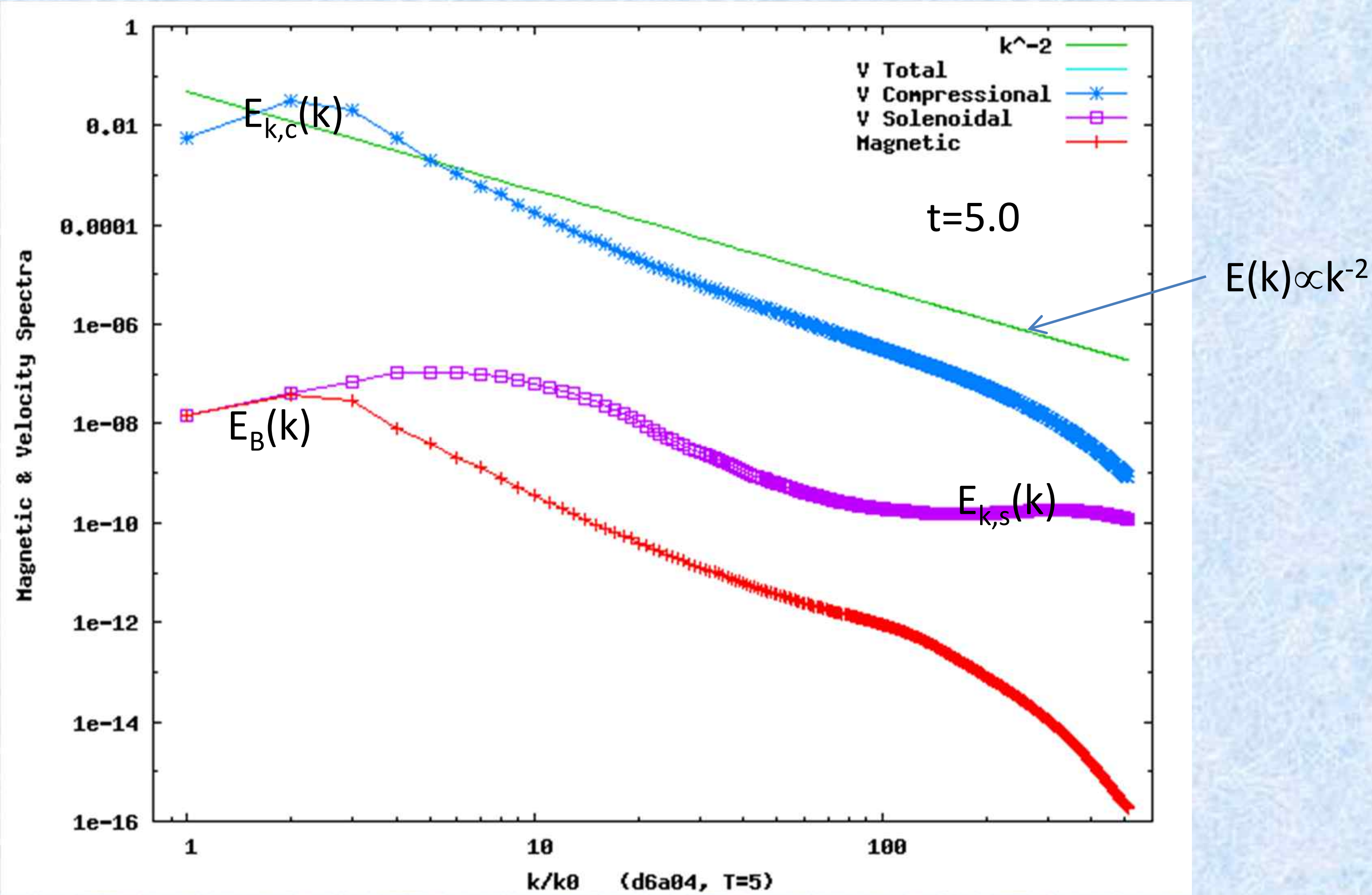
1024^3 simulation

Case 2:
Power Spectrum Evolution



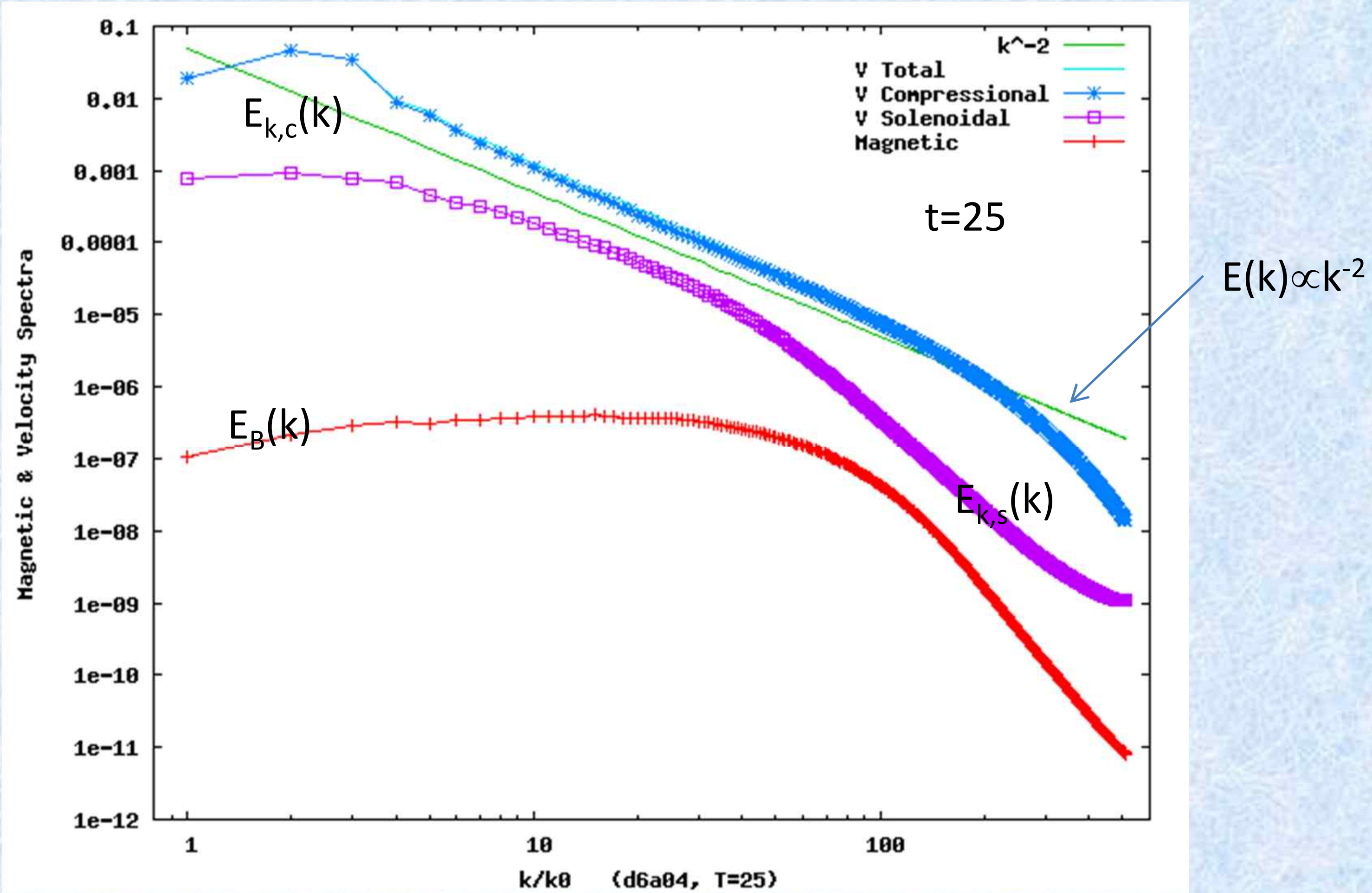
Note: Green, k^{-2}, line is the same in all frames

Case 2:
Power Spectrum Evolution



Note: Green, k^{-2}, line is the same in all frames

Case 2:
Power Spectrum Evolution



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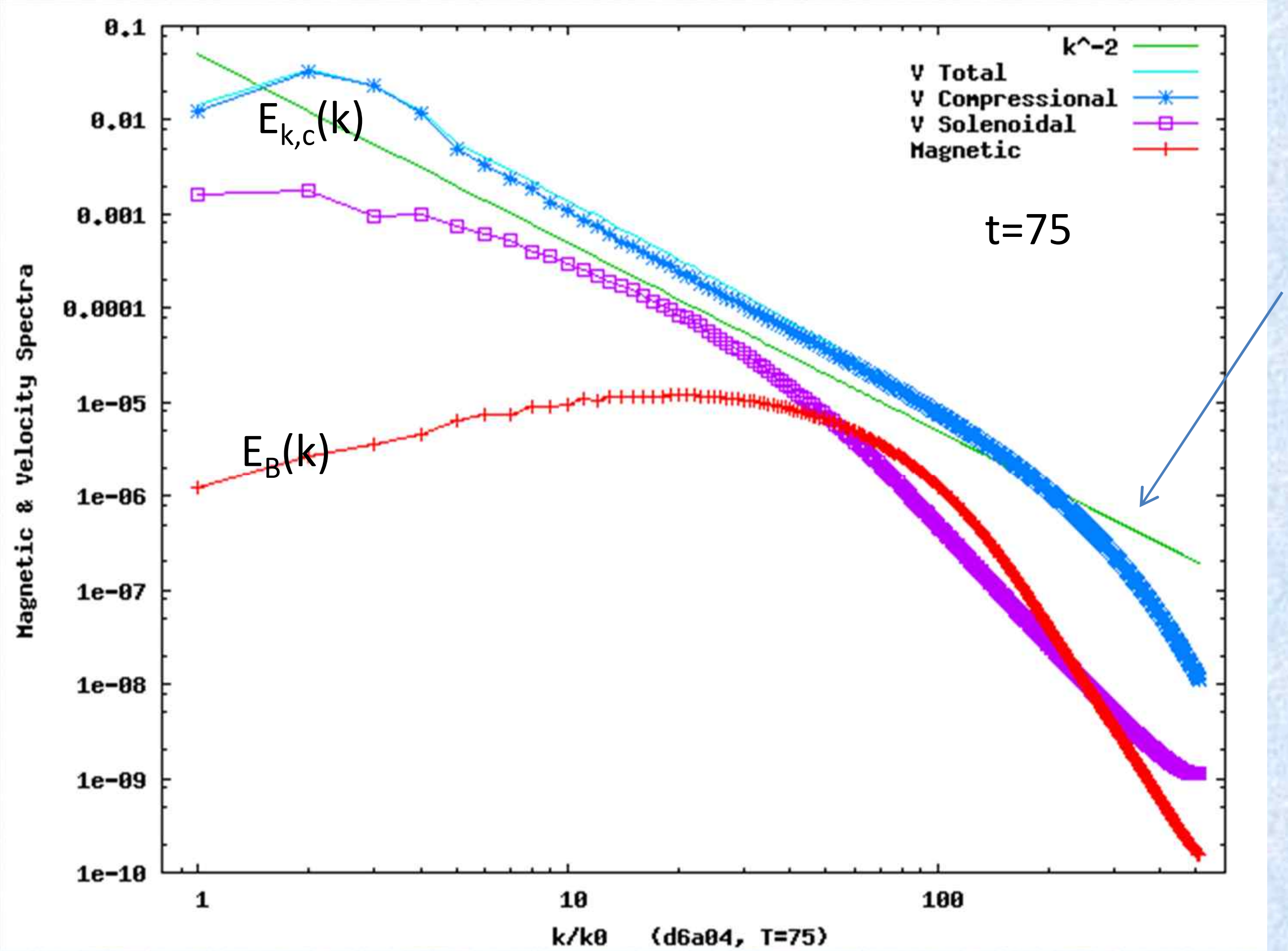
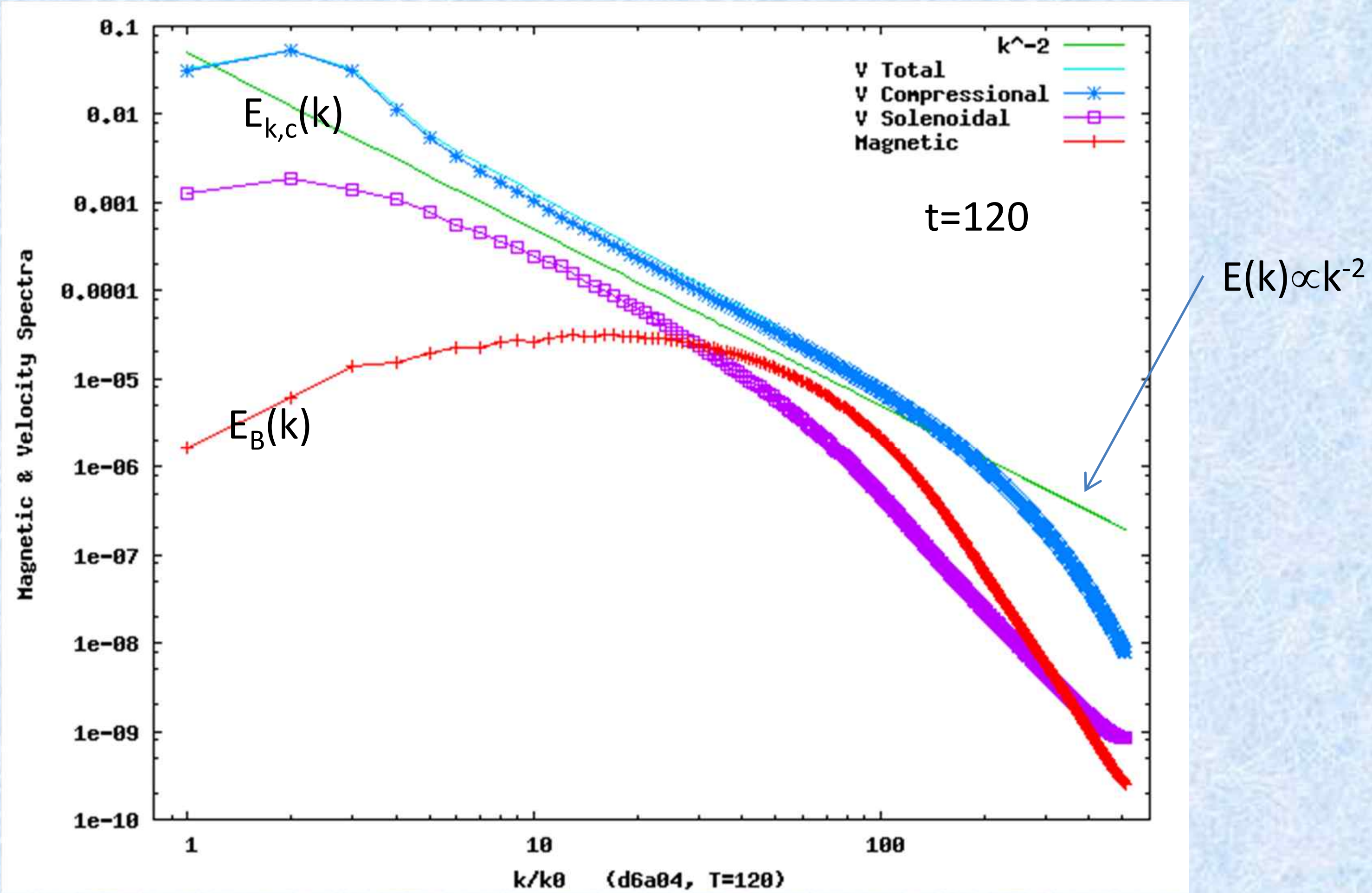
Case 2:
Power Spectrum Evolution



$E(k) \propto k^{-2}$

Note: Green, k^{-2}, line is the same in all frames

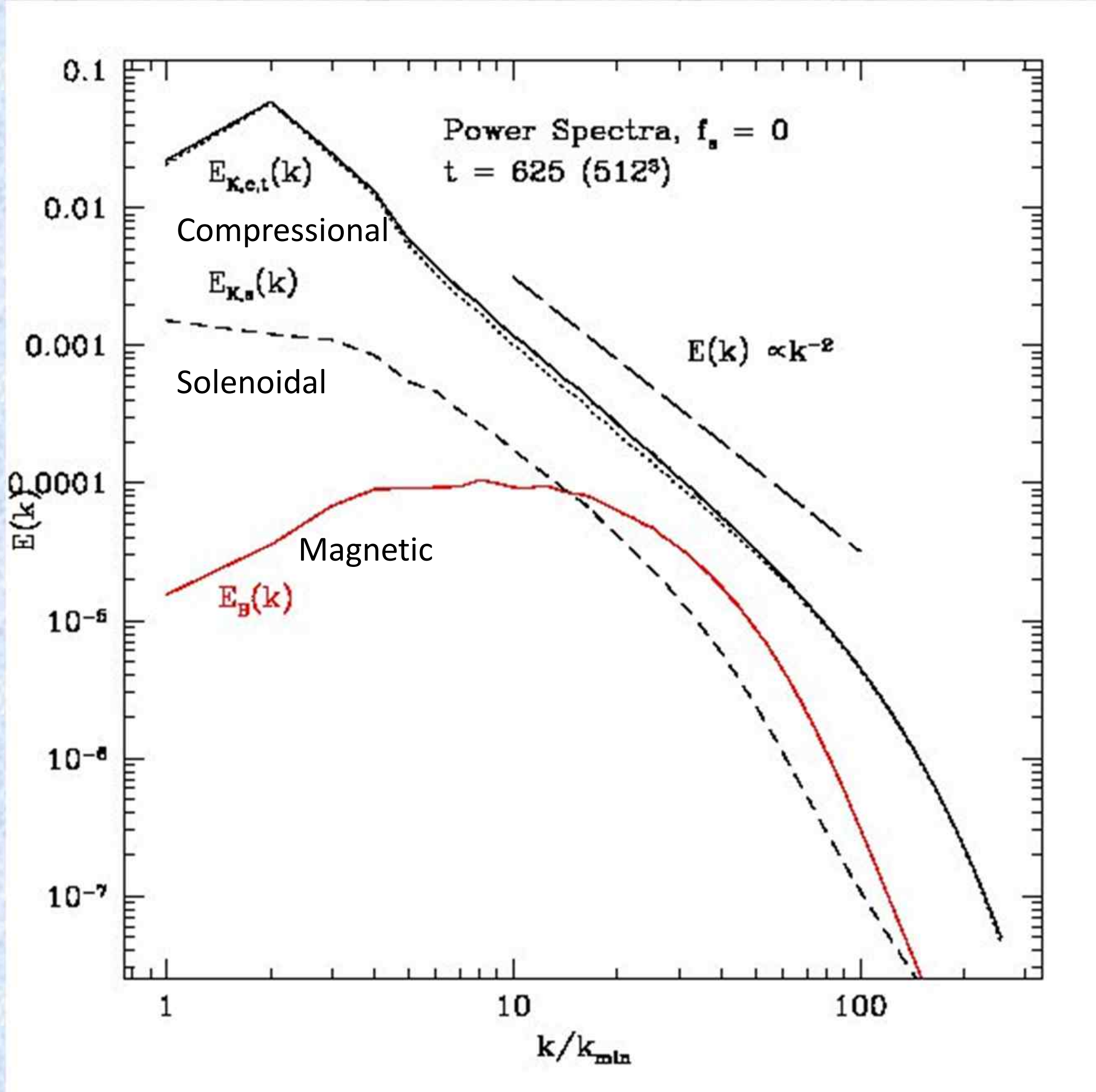
Case 2:
Power Spectrum Evolution



Note: Green, k^{-2}, line is the same in all frames

Case 2: Power Spectra, t = 625

512^3 simulation



Summary

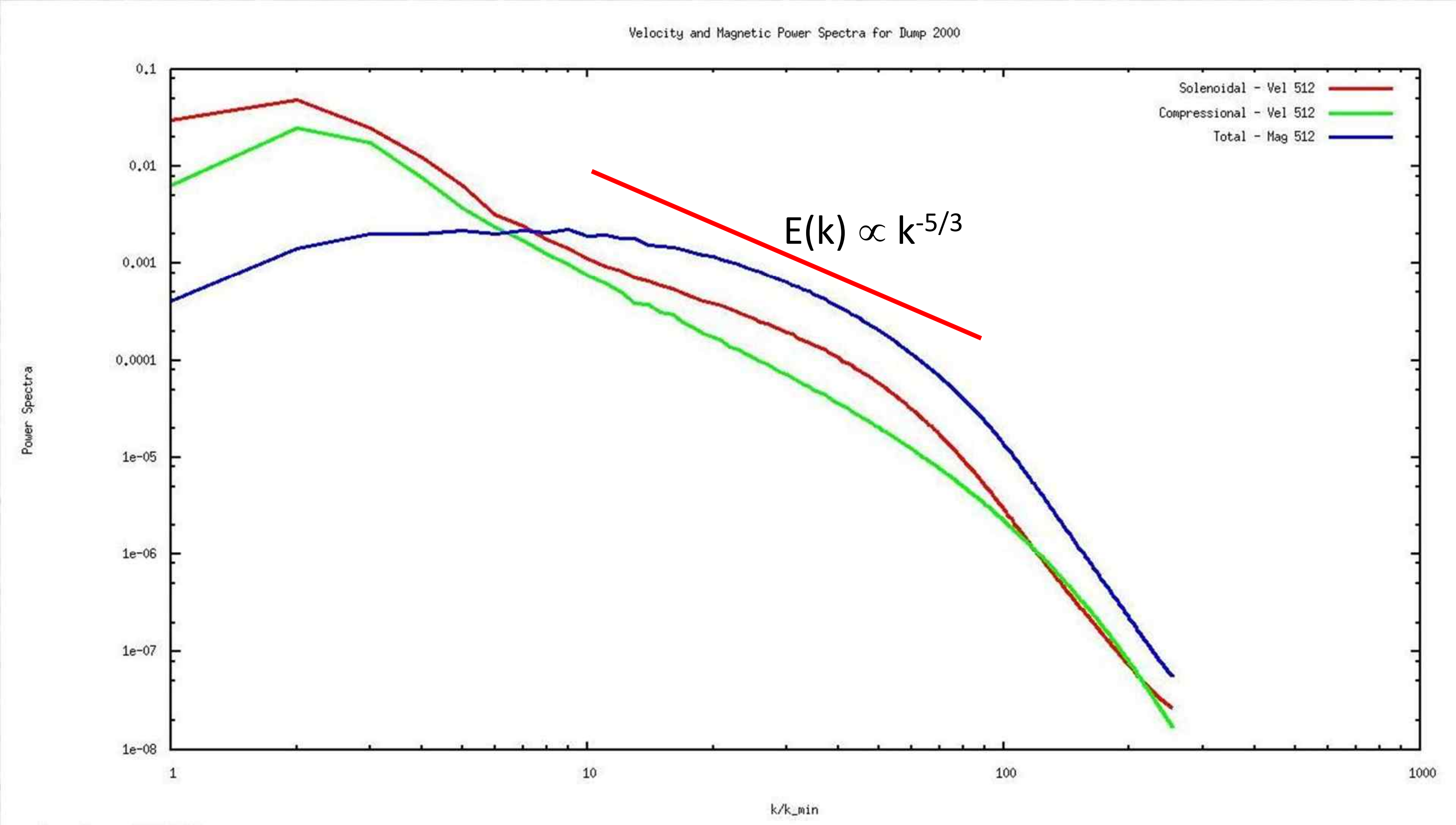
- Turbulence driven by structure formation (& other processes) may be very significant in galaxy clusters, transferring a significant energy to magnetic fields by the small scale dynamo

- If the magnetic field reaches saturation levels it may be highly intermittent
- If the magnetic field reaches saturation the field topology may evolve from flux tubes to flux ribbons (laminated)
- Compressive forcing to generate vorticity modifies turbulence properties significantly

- In the cluster context, there may not be enough time available to reach a fully saturated state (consistent with observed large β)

Thanks!

Case 3
Energy Spectra



Some Background Physics:
I – Generation & Evolution of Vorticity

Shear (vorticity) is a basic element of 3D turbulence

$$\omega = \nabla \times u \sim \frac{u}{r_{eddy}} \sim 1/\tau_{eddy}$$

Curl of Navier Stokes Equation* => Vorticity Equation:

$$\frac{\partial \omega}{\partial t} = \nabla \times (u \times \omega) + \nu \nabla^2 \omega = \frac{1}{\rho^2} \nabla \rho \times \nabla P$$

ν is viscosity

*(Ignoring MHD, Maxwell stresses for the moment)

Case 1: Magnetic Field Structure in Saturated Flow:

-u & B fields intermittent

-striated on scales $l<l_A$ ($v_A(l_A) = u(l_A)$)

ribbon-like magnetic flux structures

t = 130

2048^3

simulation

$l_A \sim 0.2L$

Porter, Ryu, Cho & Jones