Mach number dependence of the turbulent dynamo: solenoidal vs. compressive flows

...or „The forcing matters“

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Monash **C**entre for **A**strophysics

ZENTRUM FÜR ASTRONOMIE

Turbulence forcing

Purely hydrodynamical, supersonic turbulence

MHD turbulent dynamo (subsonic versus supersonic)

Polaris Flare: Bensch, Stutzki, Ossenkopf 2001 distance ~ 150 pc

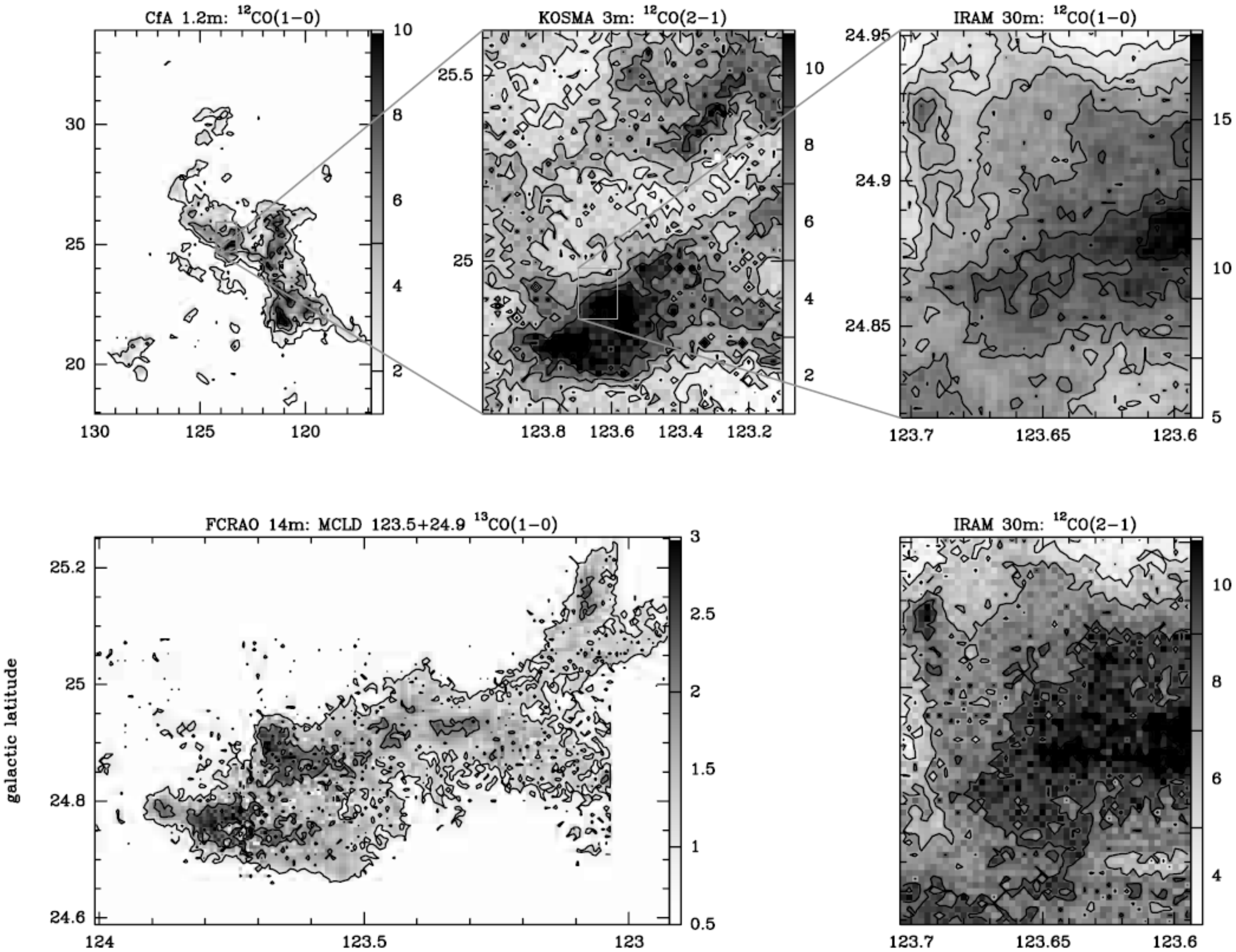
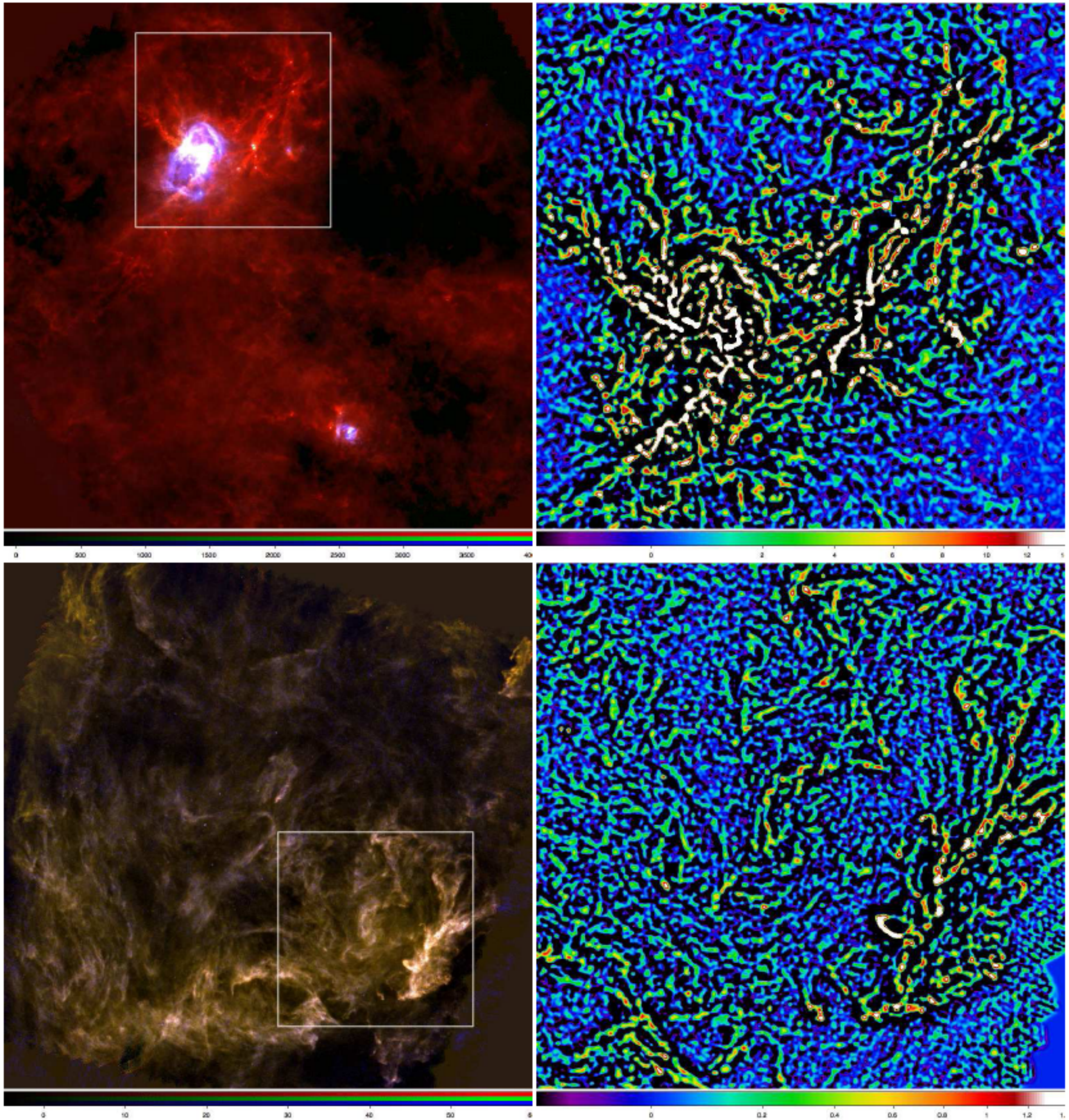


Fig. 8. Velocity integrated spectral line maps of the rotational transition ^{12}CO $J = 1 \rightarrow 0$, ^{12}CO $J = 2 \rightarrow 1$ and ^{13}CO $J = 1 \rightarrow 0$, observed towards the Polaris Flare, and one of its cores, MCLD 123.5+24.9. The transition and the telescope are indicated at the top of each panel. The line intensity is given in main beam brightness temperature, $T_{\rm mb}$. Iso-intensity levels are shown from 2 to 8 in steps of 2 (CfA map), 1 to 11 by 2 (KOSMA), 1 to 4 by 1 (FCRAO), 5 to 17 by 2 (IRAM, ^{12}CO $J = 1 \rightarrow 0$), 3 to 11 by 2 (IRAM, ^{12}CO $J = 2 \rightarrow 1$), in units of K km s^{-1}

Aquila and Polaris:
Men'shchikov et. al. 2010

Aquila
FOV 15pc
(star-forming)

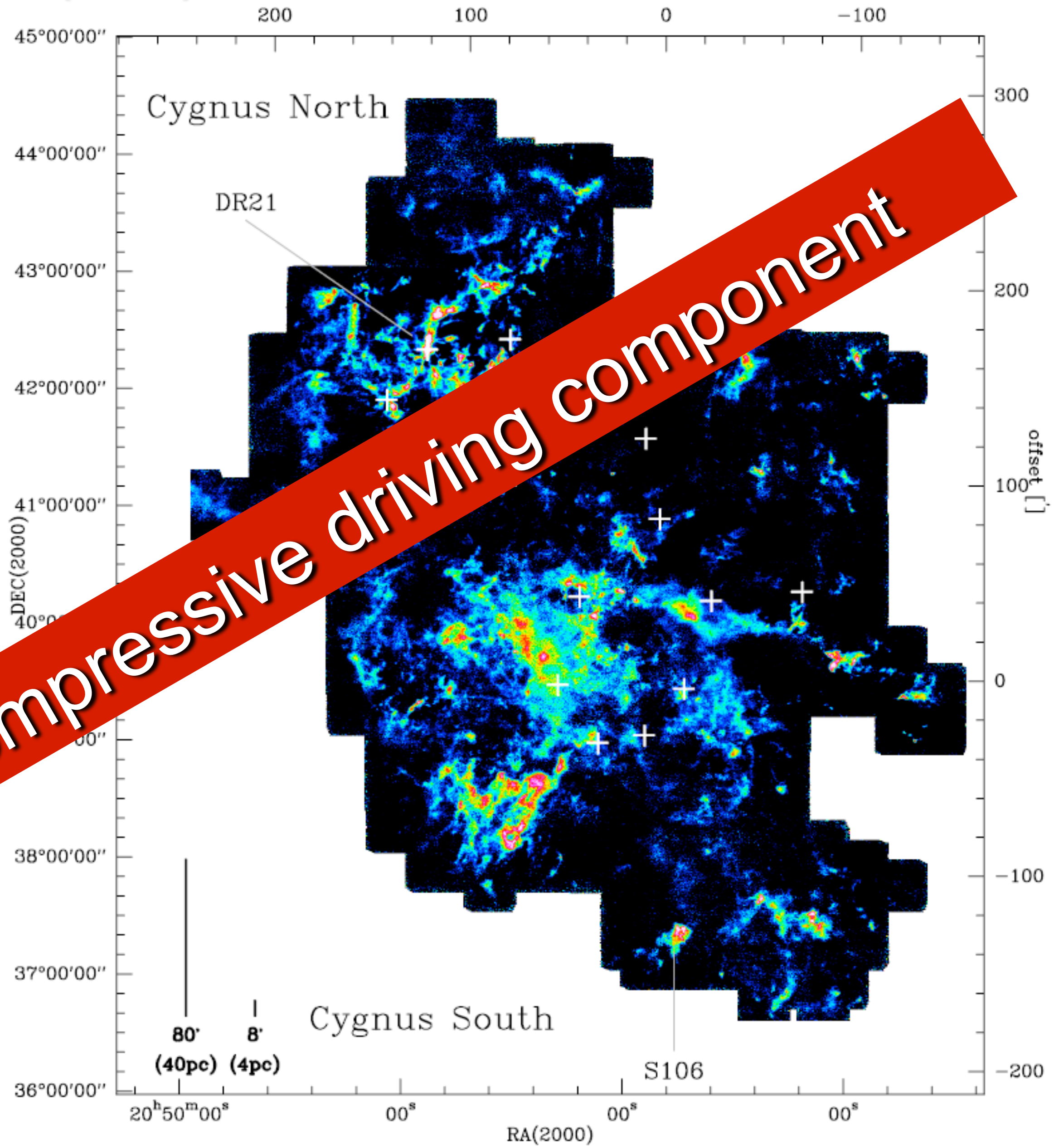
Polaris Flare
FOV 9pc
(quiescent)



Cygnus X: Schneider et al. (2011)

Giant molecular cloud complex

Turbulence driven by
Supernova explosions,
Ionization fronts,
Protostellar jets/winds,
Gravitational collapse,
Galactic rotation, MRI



Significant compressive driving component

Turbulence driving experiments

Purely hydrodynamical, supersonic turbulence

MHD turbulent dynamo (subsonic versus supersonic)

Turbulence forcing – solenoidal versus compressive

Typical setup for forced turbulence simulations:

e.g., Vazquez 1994, Padoan+ 1997, Passot+ 1998, Stone+ 1998, Mac Low 1999, Klessen+ 2000, Heitsch+ 2001, Cho+ 2002, Boldyrev+ 2002, Li+ 2003, Padoan+2004, Jappsen+ 2005, Ballesteros+ 2006, Kritsuk+ 2007, Dib+ 2008, Offner+ 2008, Kowal+ 2008, Schmidt+ 2009, Cho+ 2009

"Turbulence in a box"

- Periodic boundary conditions
- Isothermal EOS: $P = c_s^2 \rho$
- Neglect self-gravity
- Driven to supersonic speeds (Mach 5-10)
- Large-scale **Forcing term *f***

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\, \boldsymbol{v} = -\frac{1}{\rho} \nabla P - \cancel{\nabla \Phi} + \boldsymbol{f}$$

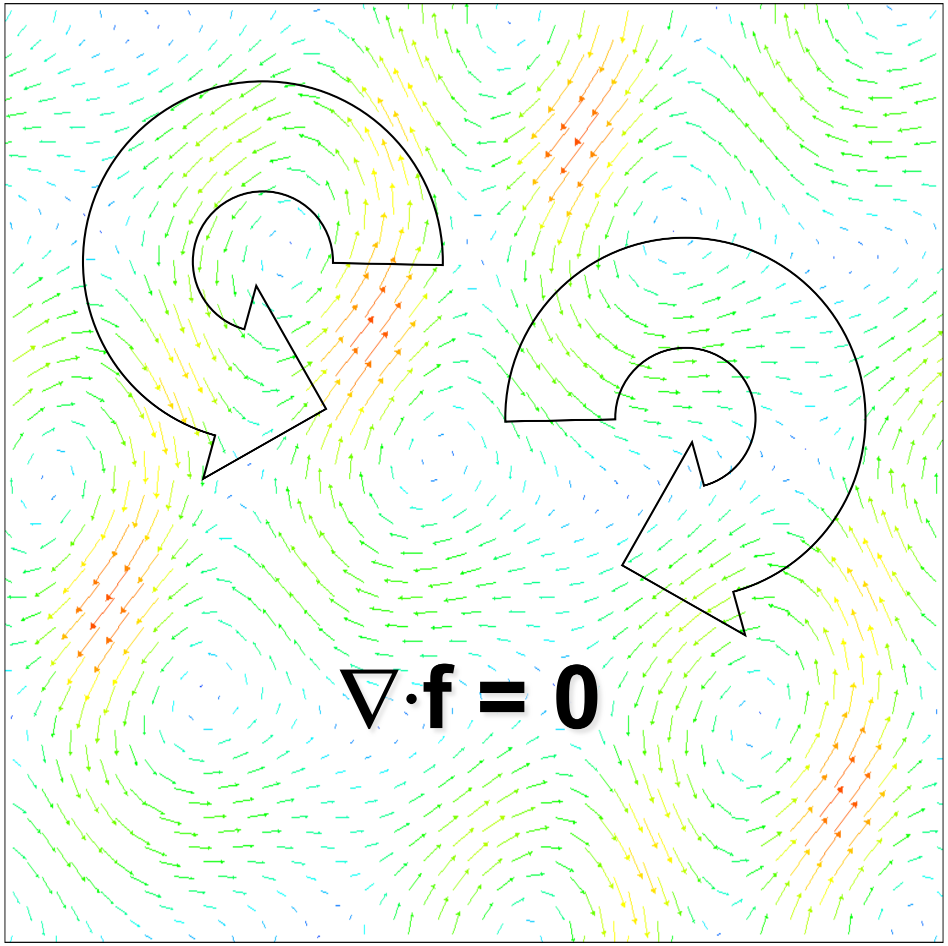
$$\sout{\frac{\partial}{\partial t}(\rho e) + \nabla \cdot [\boldsymbol{v}(\rho e + P)] = \rho \boldsymbol{v} \nabla \Phi + \rho \boldsymbol{v} \cdot \boldsymbol{f}}$$

$$\sout{\Delta \Phi = 4\pi G \rho}$$

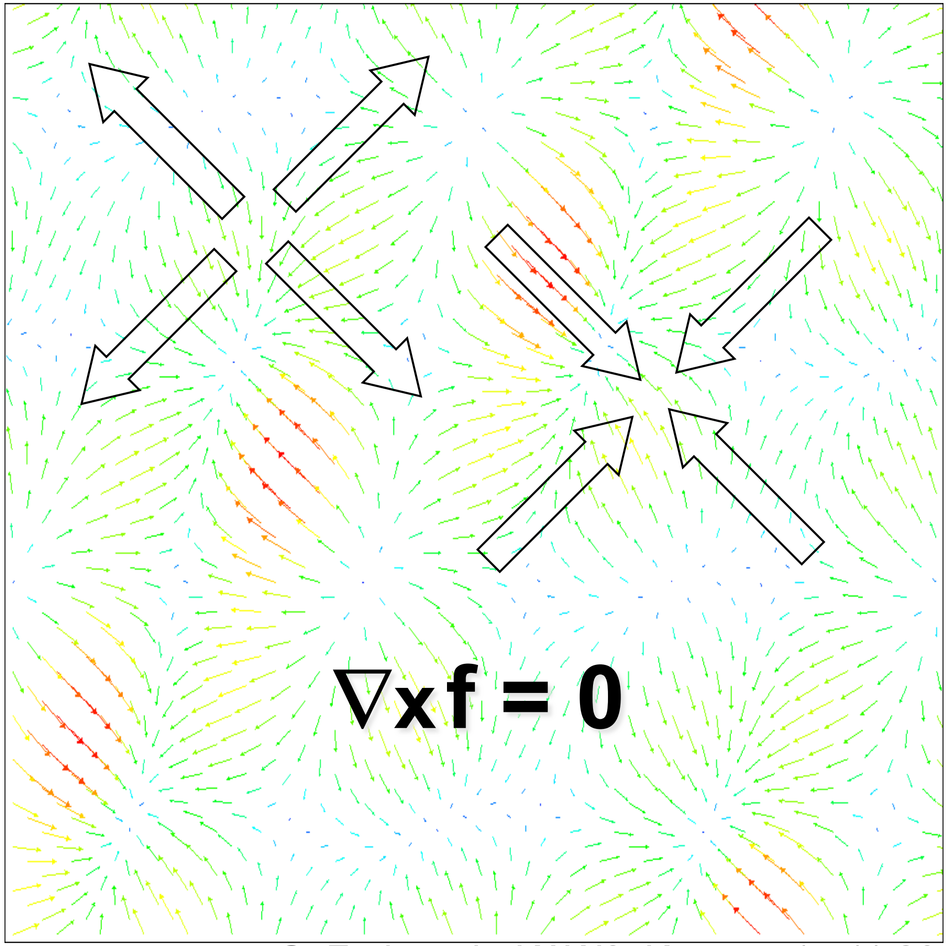
Turbulence forcing – solenoidal versus compressive

Ornstein-Uhlenbeck process (stochastic process with autocorrelation time)
→ **forcing varies smoothly in space and time**
following a well defined random process

Solenoidal forcing



Compressive forcing



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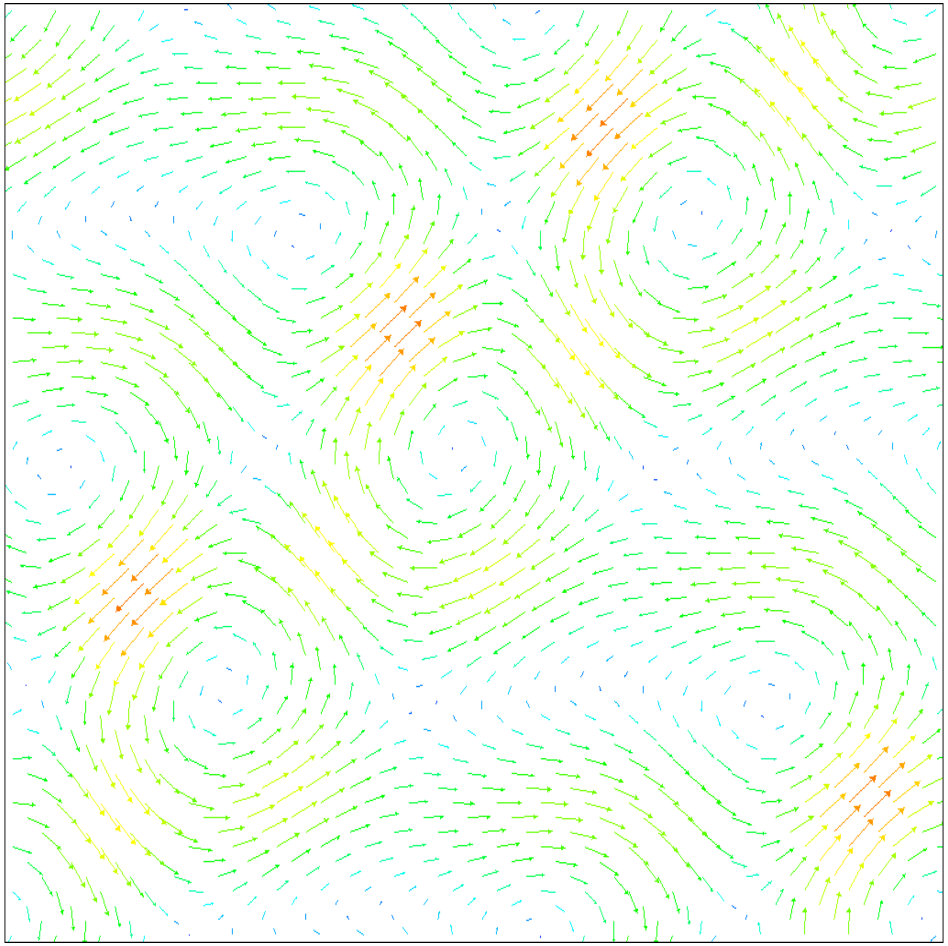
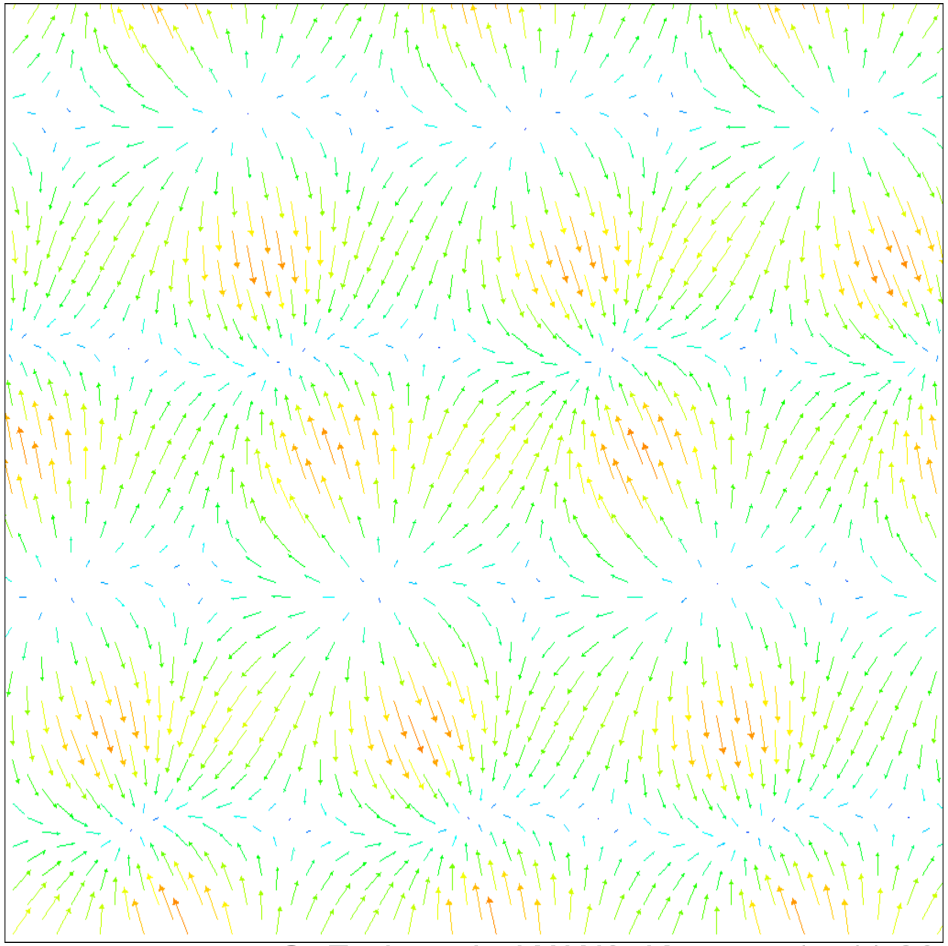


Compressive forcing



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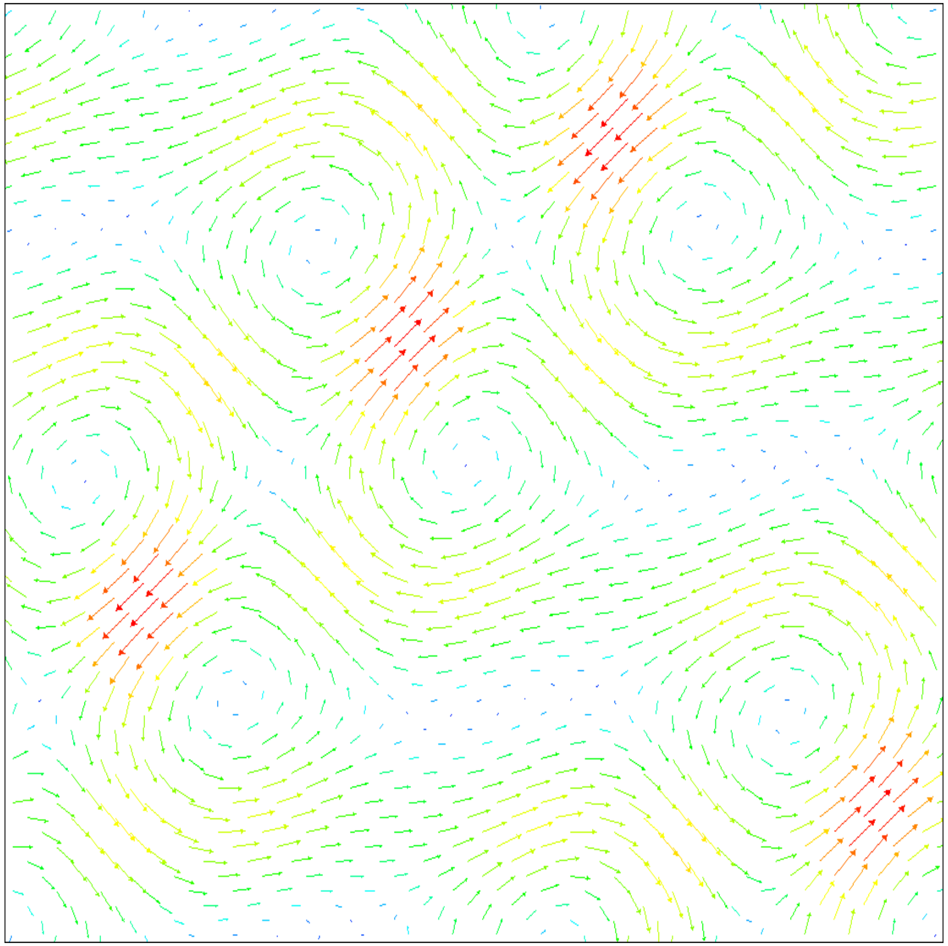
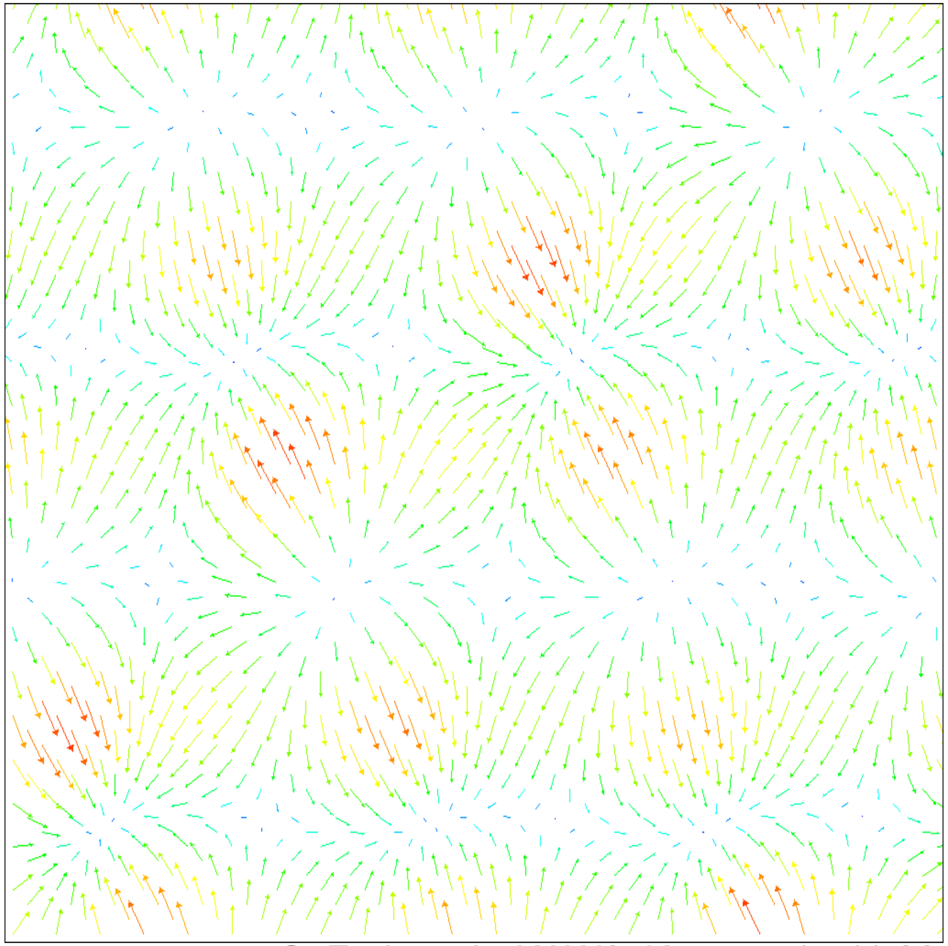


Compressive forcing



Turbulence forcing – solenoidal versus compressive

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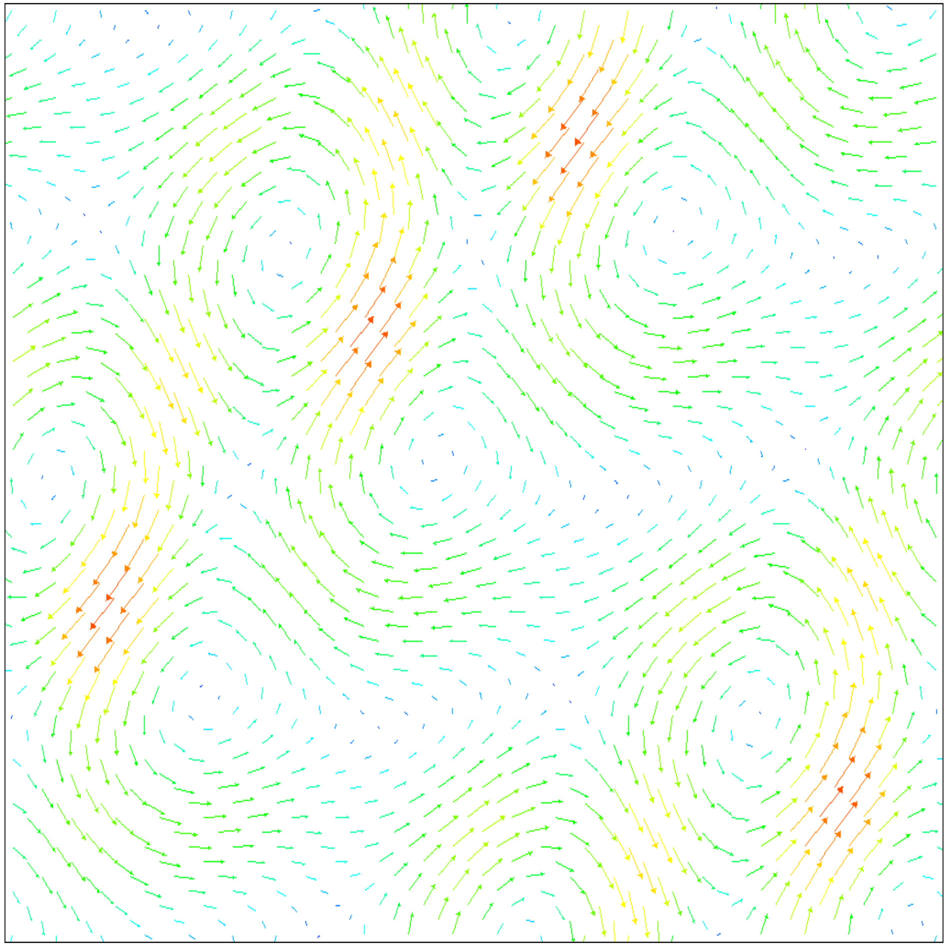
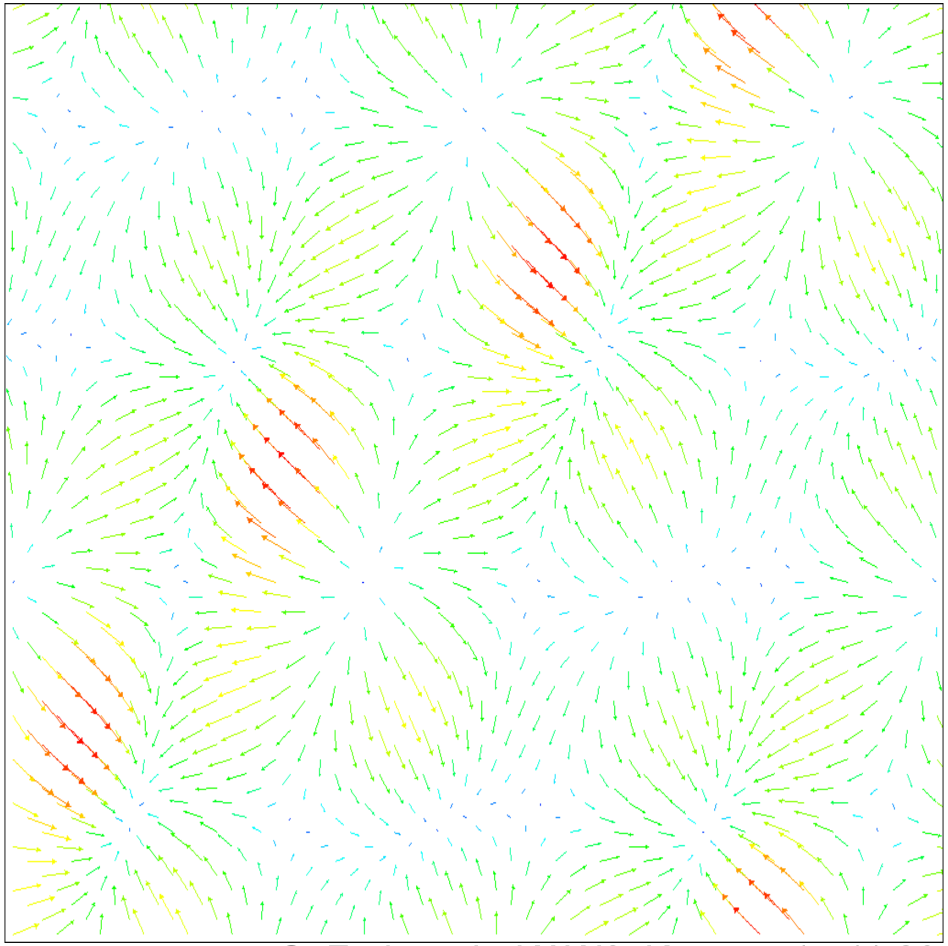
Solenoidal forcing



Compressive forcing



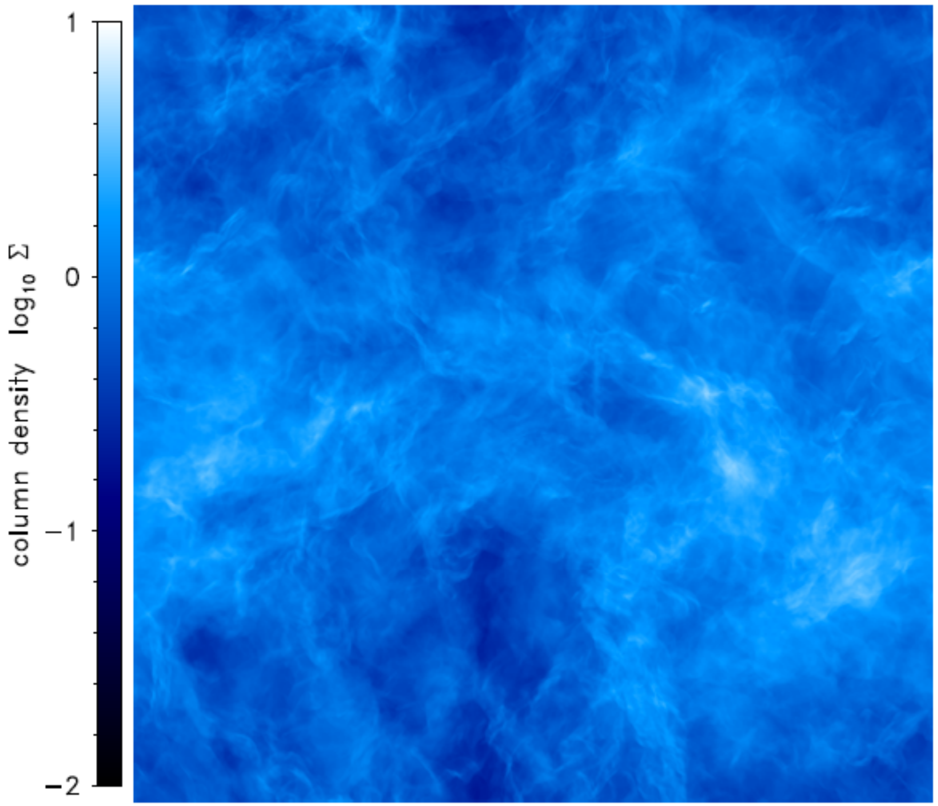
Turbulence driving experiments

Purely hydrodynamical, supersonic turbulence

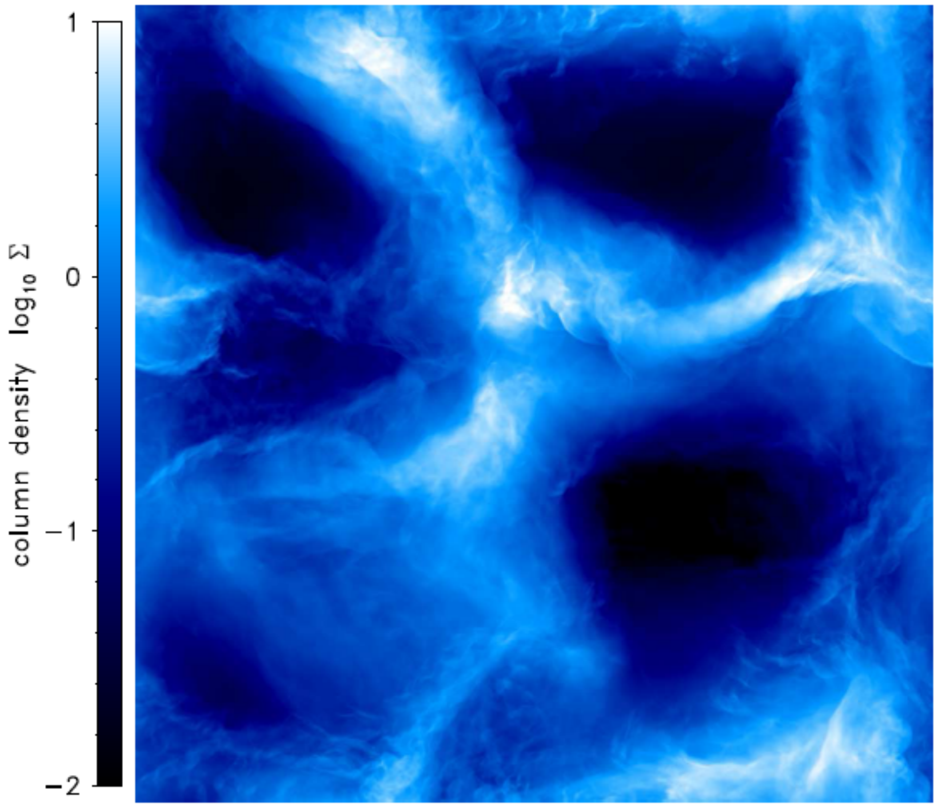
MHD turbulent dynamo (subsonic versus supersonic)

Turbulence forcing – solenoidal versus compressive

Solenoidal forcing



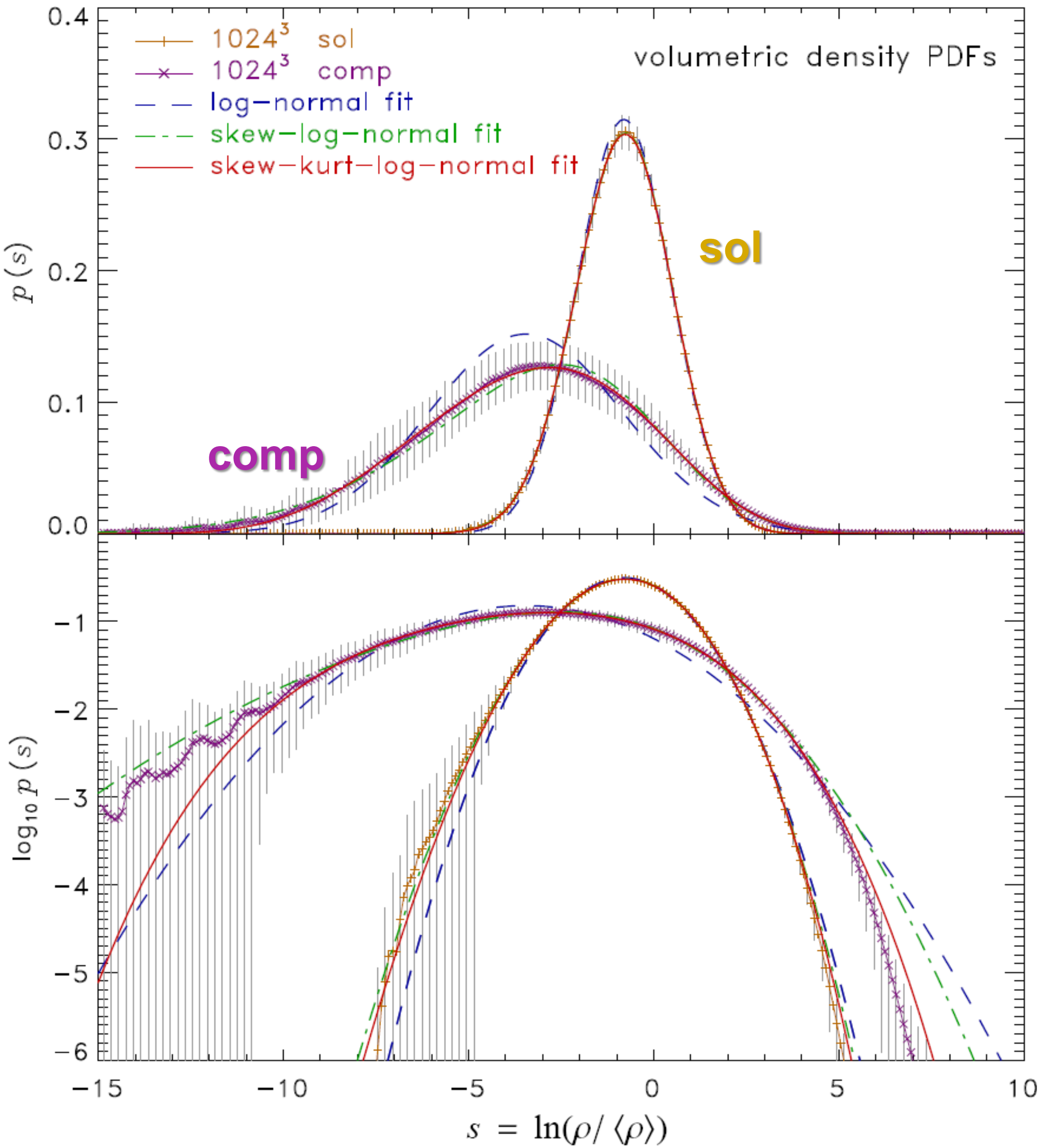
Compressive forcing



Compressive forcing yields 3 times larger density dispersion for the same Mach number

Federrath et al. (2008, 2009, 2010)

Turbulence forcing – solenoidal versus compressive



gas density PDF

PDFs are close to log-normals:

$$p_s \, \mathrm{d}s = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left[-\frac{(s - \langle s \rangle)^2}{2\sigma_s^2}\right] \mathrm{d}s$$

$$s = \ln(\rho / \langle \rho \rangle)$$

$$\sigma_\rho / \rho_0 = b\mathcal{M}$$

b = 1/3 (sol)

b = 1 (comp)

Federrath et al.(2008, 2010)

Turbulence forcing – solenoidal versus compressive

Density PDF is key for star formation

- **CMF / IMF** (Padoan & Nordlund 02, Hennebelle & Chabrier 08,09)
- **Star formation efficiency** (Elmegreen 08)
- **Kennicutt-Schmidt relation** (Elmegreen 02, Krumholz & McKee 05, Tassis 07)
- **Star formation rate** (Krumholz & McKee 05, Padoan & Nordlund 11)

All rely on integrals over the turbulent density PDF !

$$\mathrm{SFR_{ff}} = \frac{\epsilon_{\mathrm{core}}}{\phi_t} \int_{x_{\mathrm{crit}}}^{\infty} x p(x)\, dx$$

e.g., Krumholz & McKee 2005, Hennebelle & Chabrier 2011

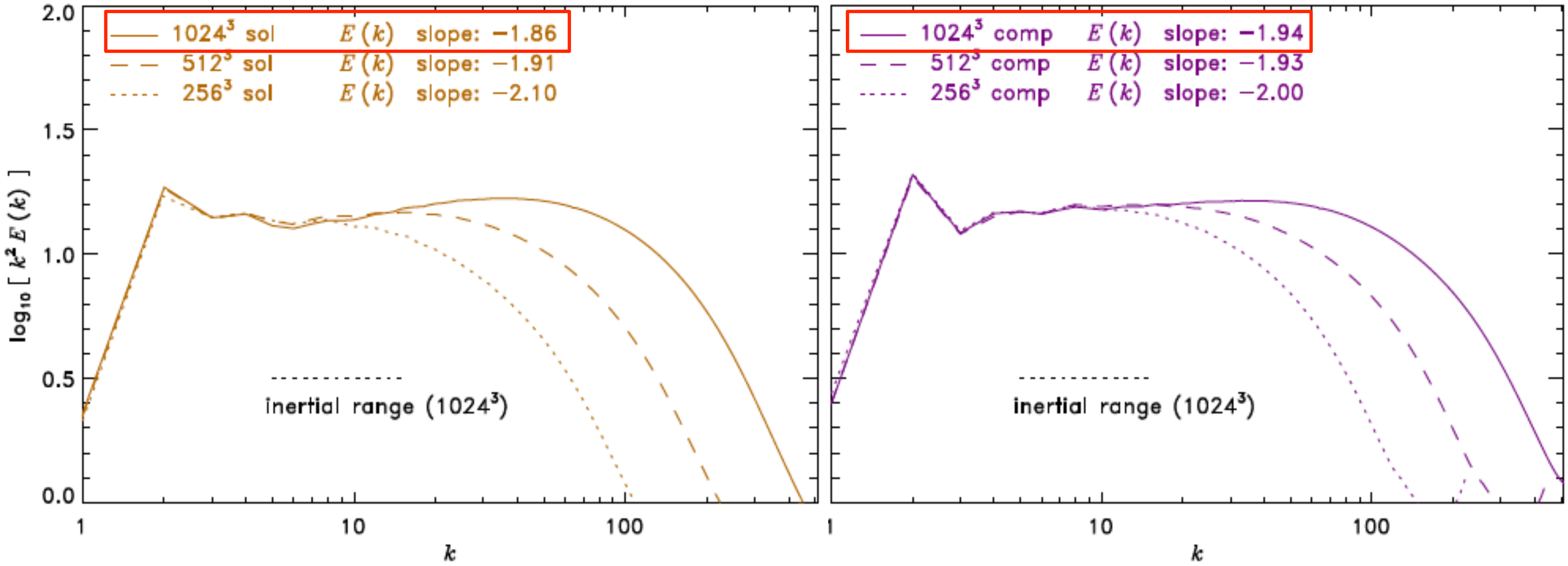
Turbulence forcing – solenoidal versus compressive

scaling relations,

e.g., Larson law of turbulent clouds in the ISM: $P(k) \sim k^{-1.8-2.0}$

(see e.g., Heyer & Brunt 2004)

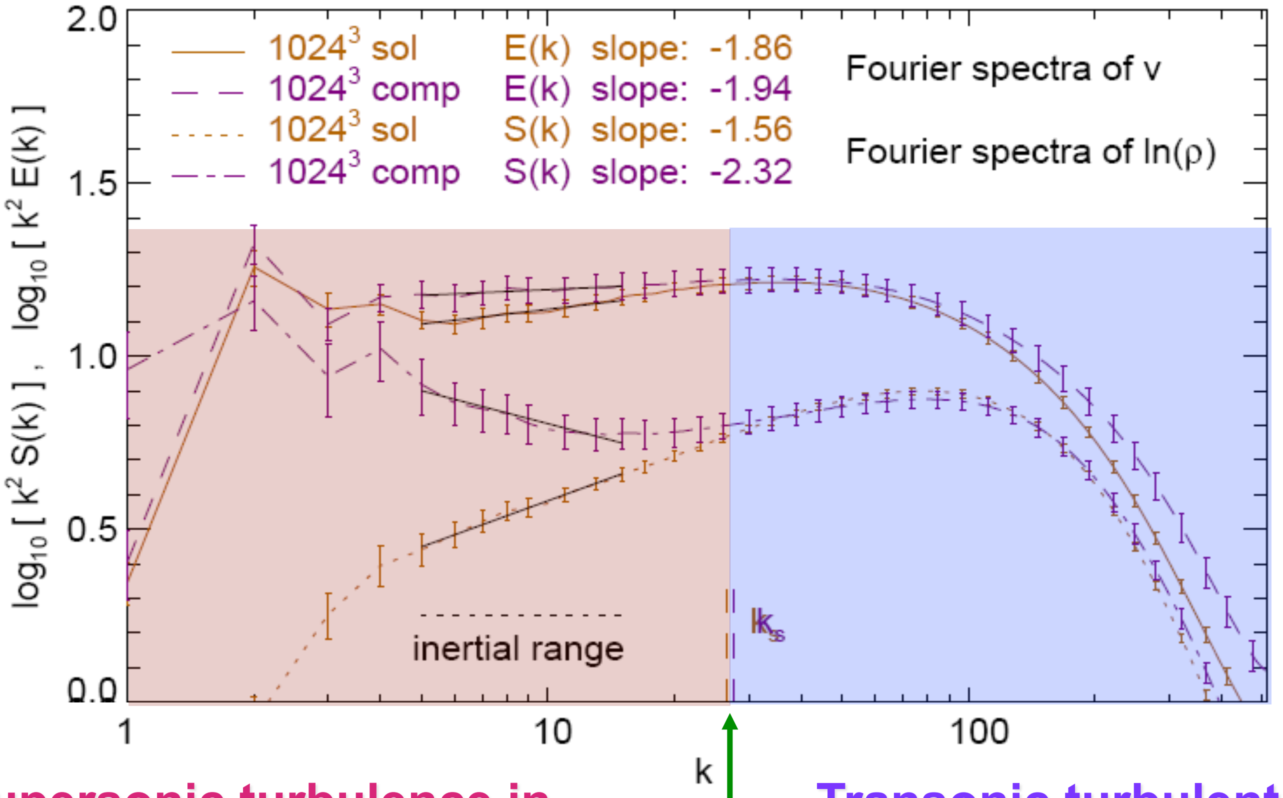
Solenoidal forcing | **Compressive forcing**



Federrath, Roman-Duval, Klessen, Schmidt & Mac Low, 2010, A&A 512, A81

Turbulence forcing – solenoidal versus compressive

Spatial correlations in the turbulent field (FTs, SFs, Δ-var, PCA) are consistent with observations (e.g., Larson 1981, Solomon+ 1987, Falgarone 1992, Heyer+ 2004)



Supersonic turbulence in molecular clouds (~10…0.1pc)

Transonic turbulent clumps and cores (~0.1…0.01pc)

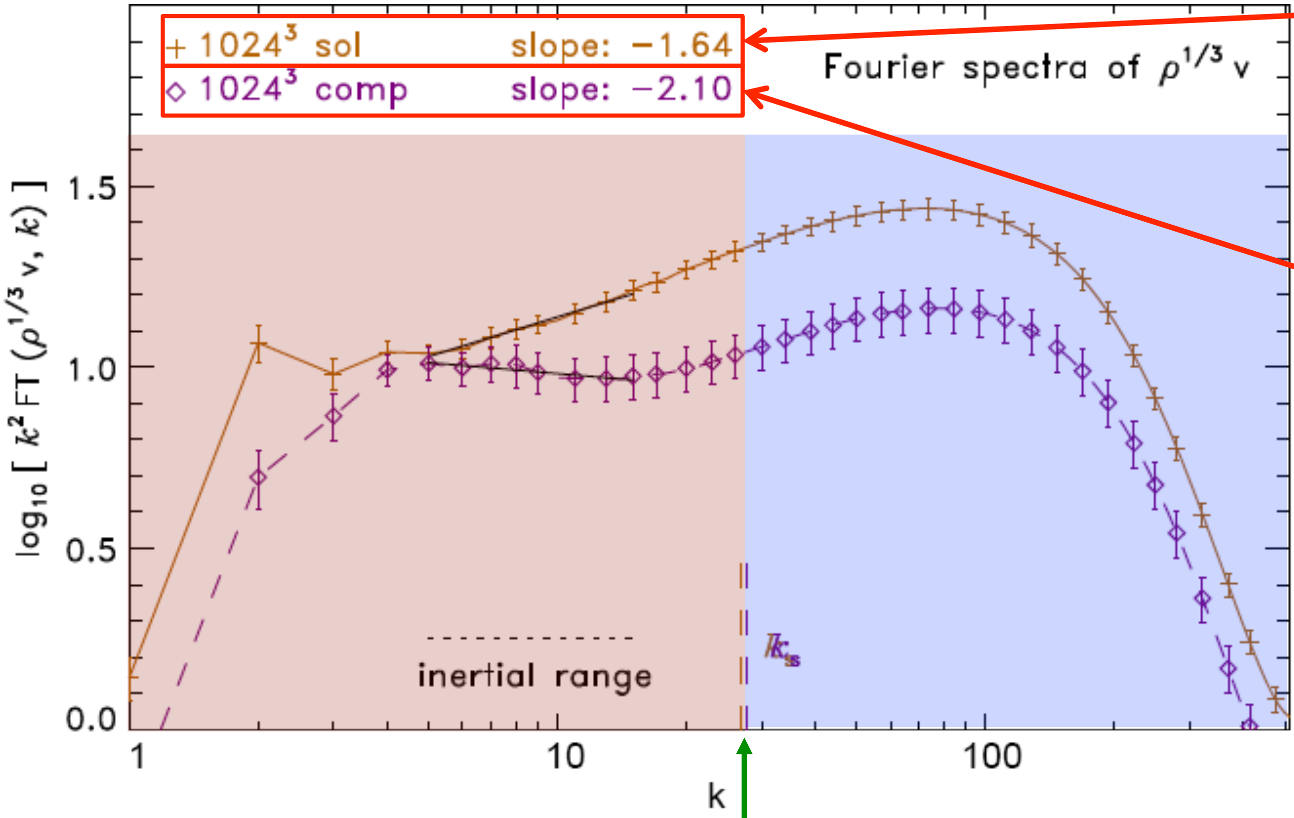
Sonic scale (~ 0.1pc)

Federrath, Roman-Duval, Klessen, Schmidt, Mac Low (2010, A&A 512, A81)

Turbulence forcing – solenoidal versus compressive

$\rho^{1/3}v$ - spectrum

(suggested by Kritsuk et al. 2007;
see also Kowal et al. 2008; Schmidt et al. 2008; Aluie 2011)



~ k^-5/3
K41

~ k^-19/9
predicted by
Galtier &
Banerjee
2011, PRL

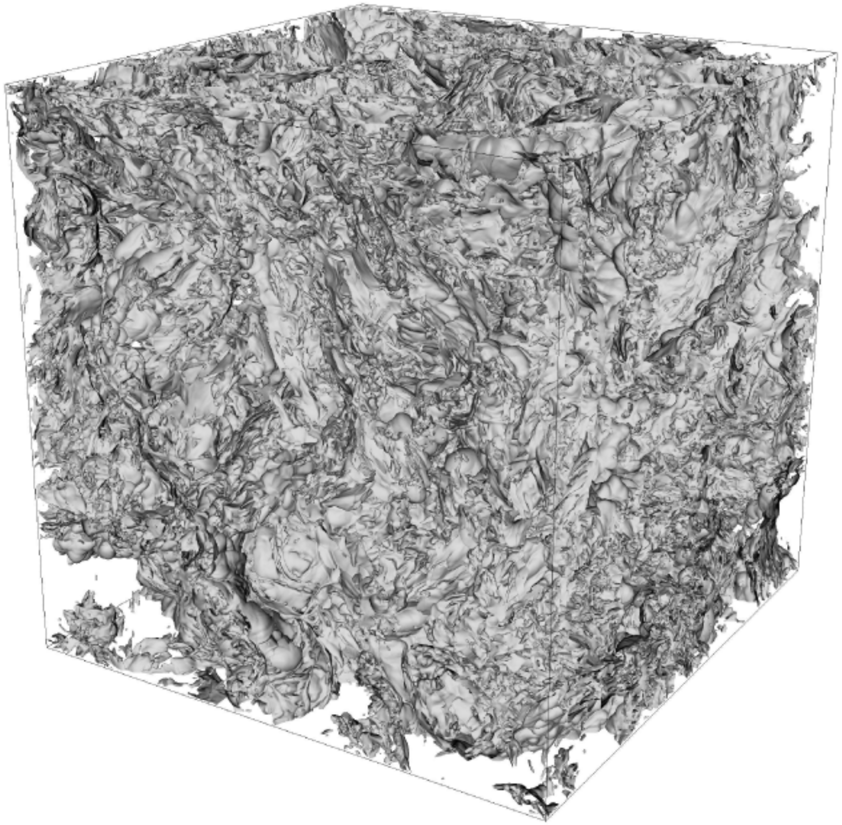
Sonic scale

Federrath, Roman-Duval, Klessen, Schmidt, Mac Low (2010, A&A 512, A81)

On the fractal dimension of supersonic turbulence

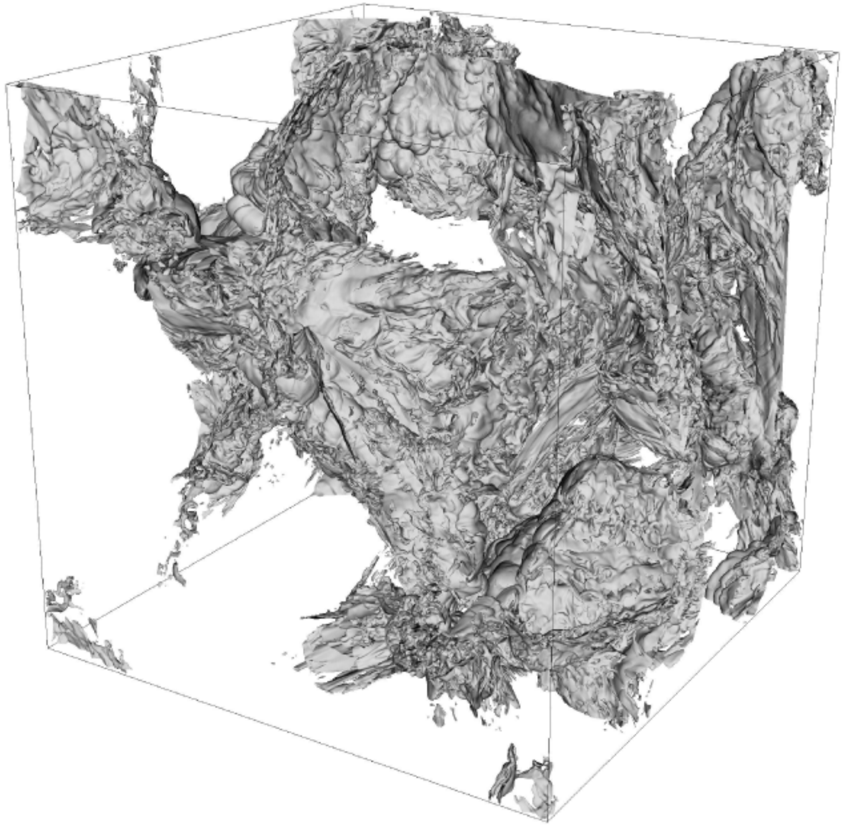
Fractal structures by box-counting, Δ-variance, perimeter-area methods:

Solenoidal forcing



D = 2.6±0.1

Compressive forcing



D = 2.3±0.1

Federrath, Klessen & Schmidt 2009, ApJ 692, 364

Turbulence driving experiments

Purely hydrodynamical, supersonic turbulence

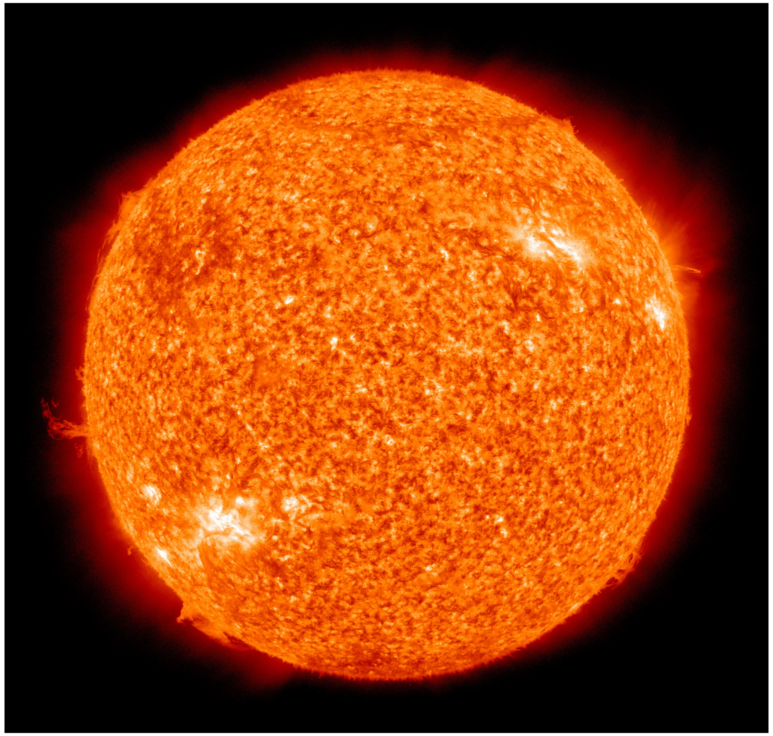
MHD turbulent dynamo (subsonic versus supersonic)

Turbulent dynamo – solenoidal versus compressive

Motivation:

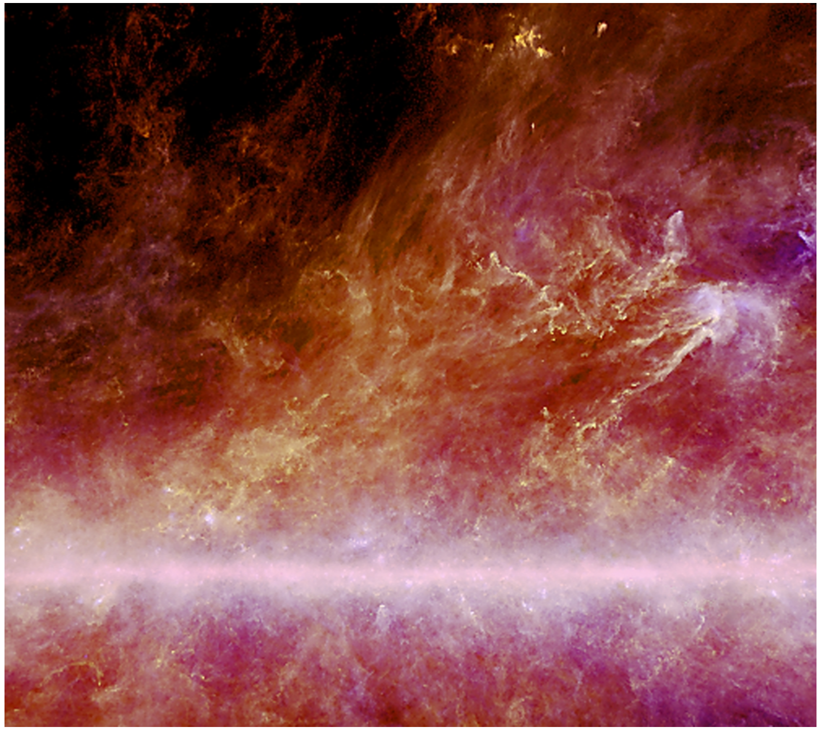
- Mach number dependence of turbulent dynamo unknown
- Effects of compression unclear
- Fundamental difference between

subsonic flows



NASA/SDO (AIA)

supersonic flows



ESA/HFI, IRAS 2010

Turbulent dynamo – solenoidal versus compressive

- Use idealized controllable turbulence box simulations
- purely solenoidal & purely compressive forcing
- Turbulence with Mach numbers in the range 0.02 – 20

Turbulent dynamo – solenoidal versus compressive

MHD equations solved on 128^3 – 512^3 grid cells (FLASH v2.5, v4)

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$

$$\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} - \mathbf{B} \otimes \mathbf{B}) + \nabla p_\star = \nabla \cdot (2 \nu \rho \boldsymbol{\mathcal{S}}) + \rho \mathbf{F},$$

$$\partial_t E + \nabla \cdot [(E + p_\star)\, \mathbf{u} - (\mathbf{B} \cdot \mathbf{u})\, \mathbf{B}] = \nabla \cdot [2 \nu \rho \mathbf{u} \cdot \boldsymbol{\mathcal{S}} + \mathbf{B} \times (\eta \nabla \times \mathbf{B})],$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\nabla \cdot \mathbf{B} = 0,$$

Reynolds number, magnetic Prandtl number:

$$\mathrm{Re} \approx 1500 \qquad \mathrm{Pm} \approx 2$$

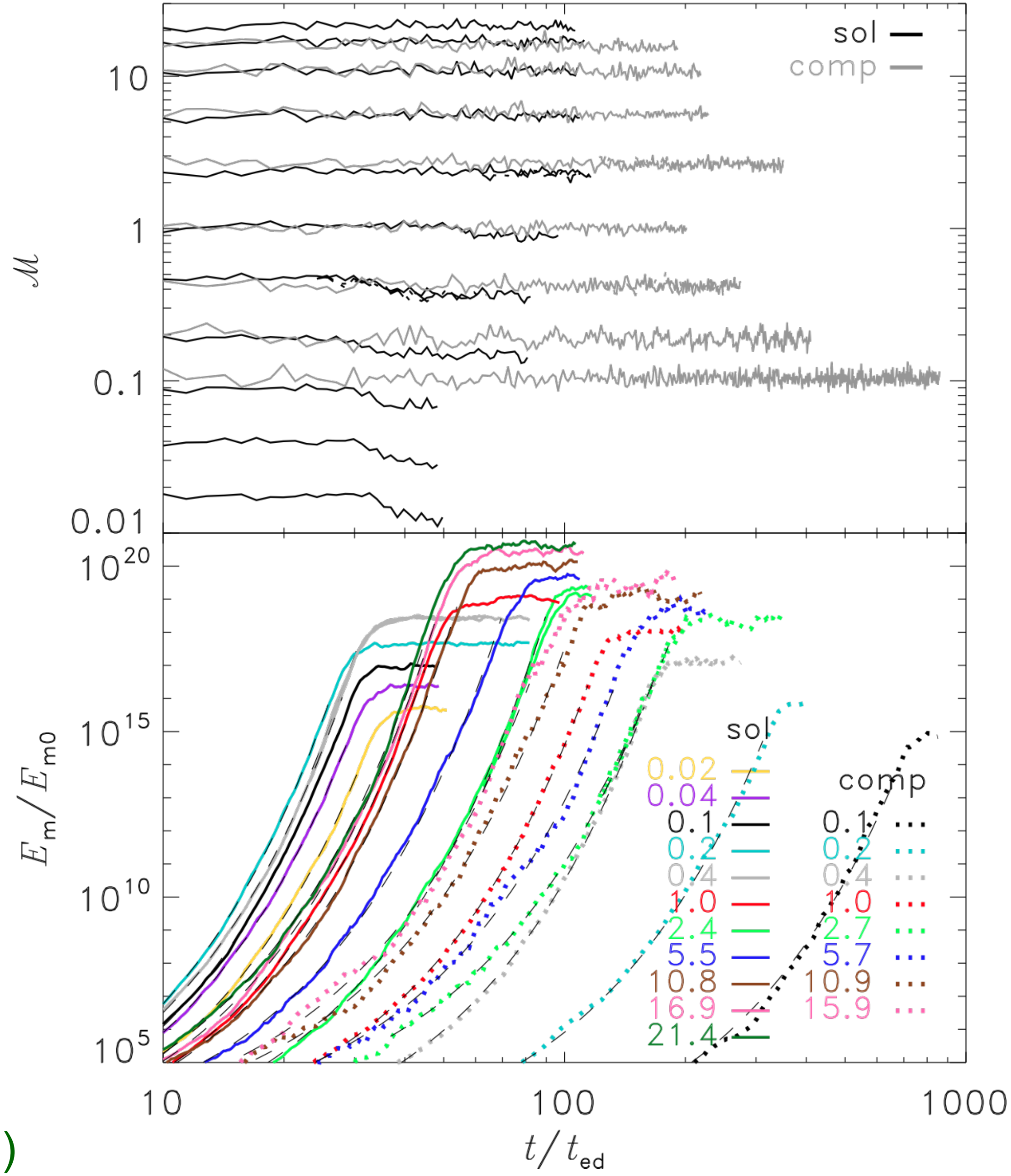
Federrath et al. (2011, PRL 107, 4504)

Turbulent dynamo – solenoidal versus compressive

Time evolution

Mach number:

Magnetic energy:



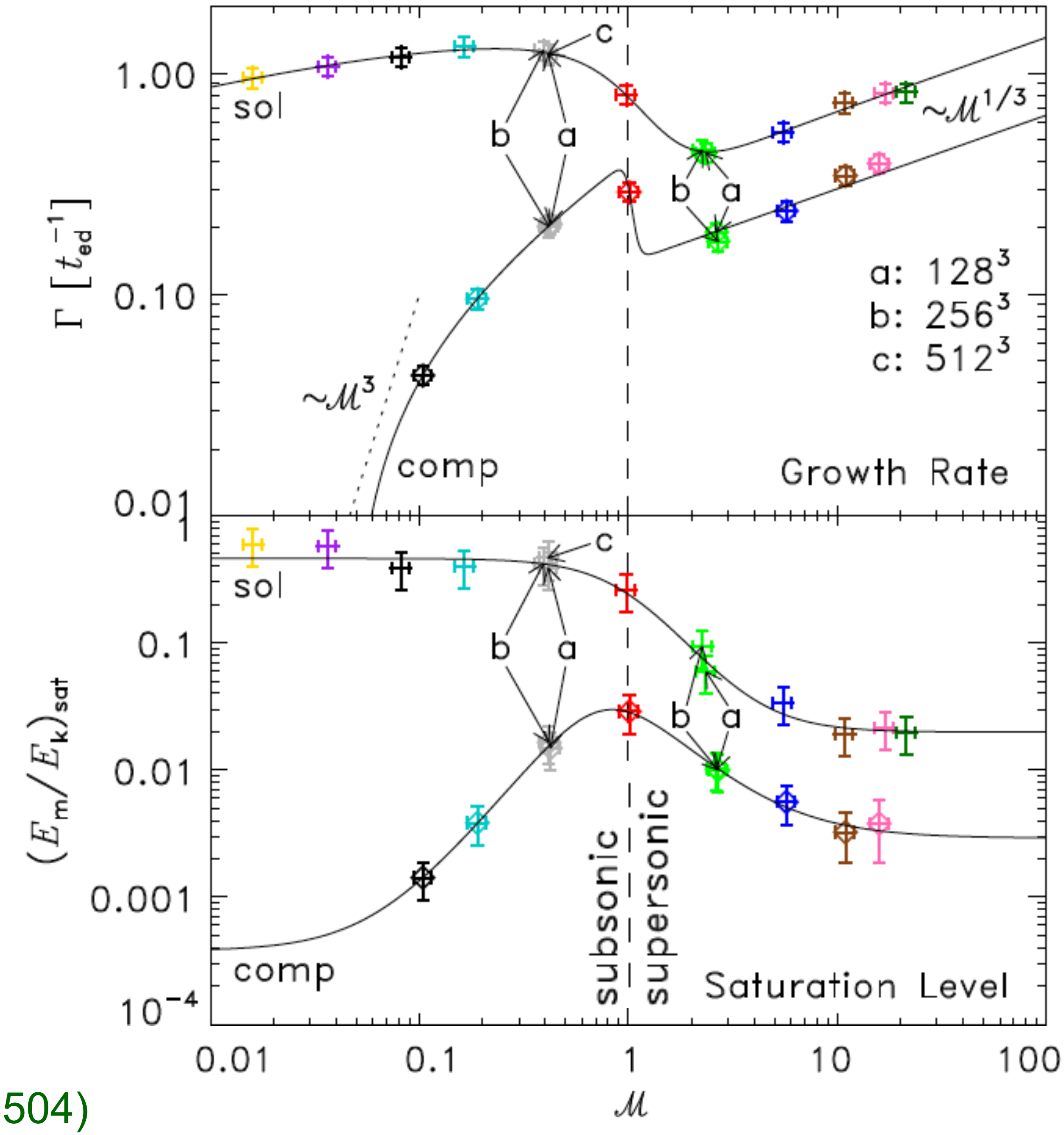
Federrath et al. (2011, PRL 107, 4504)

Turbulent dynamo – solenoidal versus compressive

Growth rates & saturation levels as function of Mach

Growth rate:

Saturation level: (E_{mag}/E_{kin})



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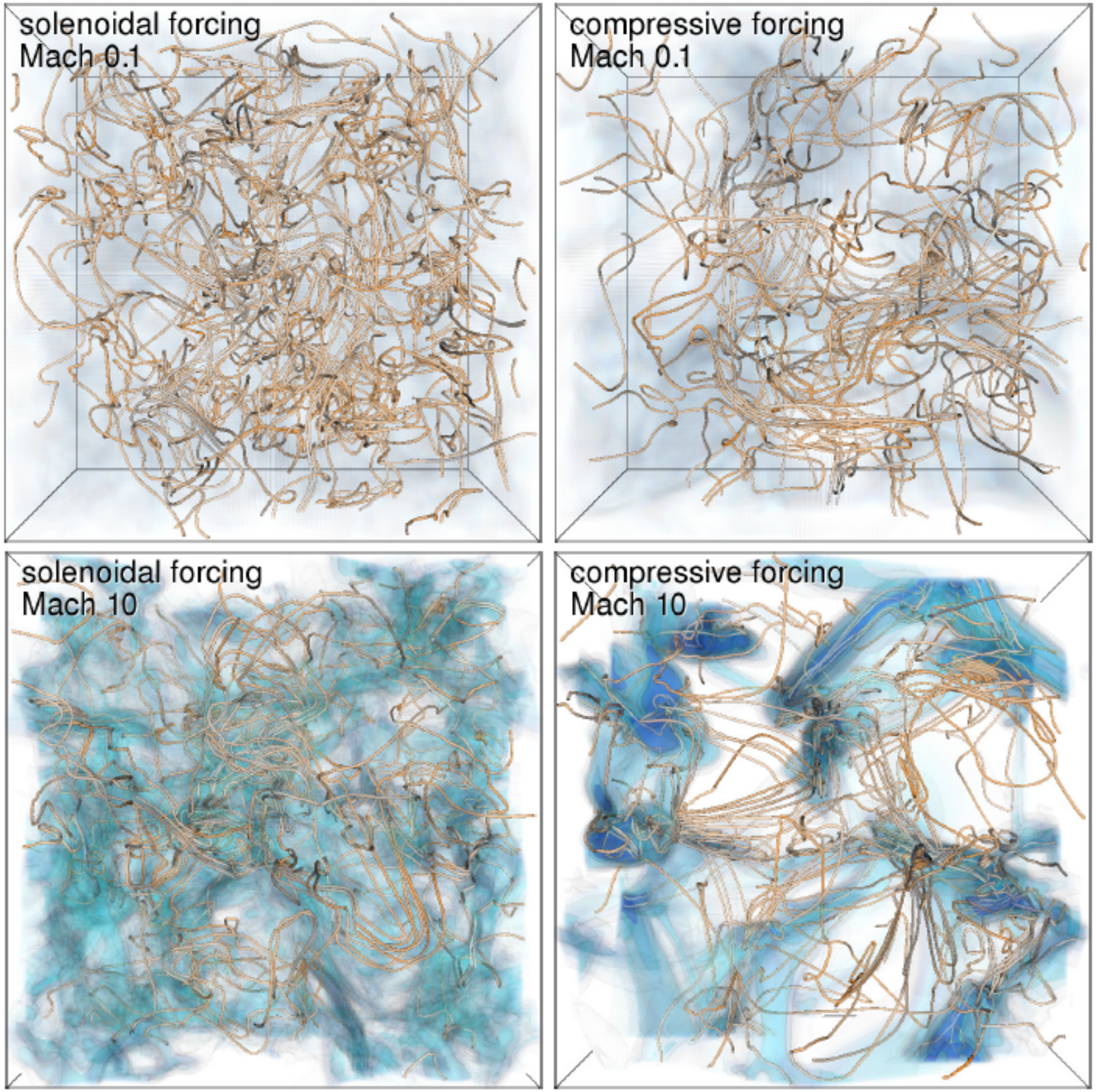
Turbulent dynamo – solenoidal versus compressive

Mach 0.1 sol

Mach 0.1 comp



Mach 10 sol

Mach 10 comp

Turbulent dynamo – solenoidal versus compressive

Generation of vorticity (see also Mee & Brandenburg 2006; Del Sordo & Brandenburg 2011):

$$\partial_t \boldsymbol{\omega} = \boxed{\nabla \times (\mathbf{u} \times \boldsymbol{\omega})} + \boxed{\nu \nabla^2 \boldsymbol{\omega}} + \boxed{\frac{1}{\rho^2} \nabla \rho \times \nabla p} + \boxed{2\nu \nabla \times (\boldsymbol{\mathcal{S}} \nabla \ln \rho)} + \boxed{\nabla \times \mathbf{F}}$$

- non-linear term (amplification!)
- diffusion
- baroclinic term (zero for isothermal)
- „anti-diffusion“ (only for compressible turbulence)
- forcing term (zero for compressive forcing)

Strain tensor:

$$\mathcal{S}_{ij} = (1/2)(\partial_i u_j + \partial_j u_i) - (1/3)\delta_{ij} \nabla \cdot \mathbf{u}$$

Reynolds number in our simulations:

$$\mathrm{Re} \approx 1500$$

Enough for the non-linear term to amplify small seeds generated by „anti-diffusion“ term

Federrath et al. (2011, PRL 107, 4504)

Turbulent dynamo – solenoidal versus compressive

Extending small-scale dynamo theory to compressive flows (solving Kazantsev 1967 equation; see Subramanian 1999):

$$\frac{\partial M_\mathrm{L}}{\partial t} = 2\kappa_\mathrm{diff} M_\mathrm{L}'' + 2\left(\frac{4\kappa_\mathrm{diff}}{r} + \kappa_\mathrm{diff}'\right) M_\mathrm{L}' + \frac{4}{r}\left(\frac{T_\mathrm{N}}{r} - \frac{T_\mathrm{L}}{r} - T_\mathrm{N}' - T_\mathrm{L}'\right) M_\mathrm{L}$$

$$\kappa_\mathrm{diff}(r) = \eta + T_\mathrm{L}(0) - T_\mathrm{L}(r).$$

ansatz for solution:

$$M_\mathrm{L}(r,t) \equiv \frac{1}{r^2\sqrt{\kappa_\mathrm{diff}}}\psi(r)\mathrm{e}^{2\Gamma t}.$$

ansatz for velocity correlations:

$$v(\ell) \propto \ell^\vartheta$$

$$T_\mathrm{L}(r) = \frac{VL}{3}\left(1 - (r/L)^{\vartheta+1}\right)$$

Kolmogorov (theta=1/3): $$T_\mathrm{N}^\mathrm{K}(r) = \frac{VL}{3}\left(1 - \frac{5}{3}\left(\frac{r}{L}\right)^{4/3}\right)$$

Burgers (theta=1/2): $$T_\mathrm{N}^\mathrm{B}(r) = \frac{VL}{3}\left(1 - \frac{2}{5}\left(\frac{r}{L}\right)^{3/2}\right)$$

Schober et al. (2011, PRE submitted, arXiv: 1109.4571)

Turbulent dynamo – solenoidal versus compressive

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Kolmogorov (theta=1/3):

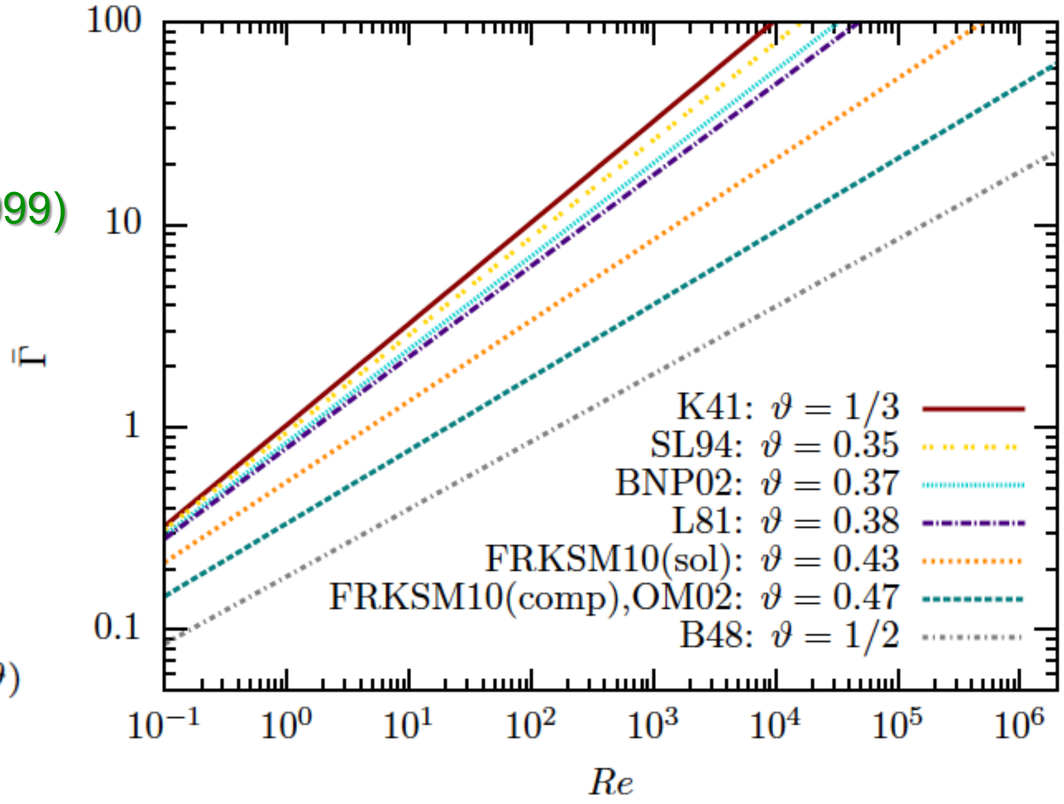
$$\Gamma^{\mathrm{K}} \propto Re^{1/2}$$ (see Subramanian 1999)

Burgers (theta=1/2):

$$\Gamma^{\mathrm{B}} \propto Re^{1/3}$$

Growth rate as function of theta:

$$\Gamma = \frac{(163 - 304\vartheta)\ V}{60\ L} Re^{(1-\vartheta)/(1+\vartheta)}$$



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Conclusions

Compared turbulence statistics
for solenoidal (divergence-free) and compressive (curl-free) forcing

- Strong influence on gas density statistics (PDF ➔ star formation)
- Spectra steeper than Kolmogorov
- Sonic scale as characteristic scale in compressibe, supersonic turbulence

- Dynamo growth rate and saturation level depend strongly on Mach and forcing
- Vorticity generation is extremely different, in particular for subsonic flows

⟹ **The forcing matters!**