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Mean-structure–turbulence interaction in magnetic reconnection

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Topics

- Introduction
- Cross helicity effects
- Flow–turbulence interaction in magnetic reconnection
- Summary

How I feel about turbulence

Mean–turbulence interaction

Mean-flow energy

$$\left(\frac{\partial}{\partial t}+U_a\frac{\partial}{\partial x_a}\right)\frac{1}{2}\mathbf{U}^2 = +\langle u'_a u'_b\rangle \frac{\partial U_a}{\partial x_b} - \nu\left\langle \frac{\partial U_b}{\partial x_a}\frac{\partial U_b}{\partial x_a}\right\rangle - (\mathbf{U}\cdot\nabla)P + \cdots$$

drain

pressure gradient

Turbulent energy

production

$$\left(\frac{\partial}{\partial t}+U_a\frac{\partial}{\partial x_a}\right)\left\langle\frac{1}{2}\mathbf{u}'^2\right\rangle = -\langle u'_a u'_b\rangle\frac{\partial U_a}{\partial x_b} - \nu\left\langle\frac{\partial u'_b}{\partial x_a}\frac{\partial u'_b}{\partial x_a}\right\rangle + \nabla\cdot\left(-\left\langle\left(\frac{\mathbf{u}'^2}{2}+p'\right)\mathbf{u}'\right\rangle + \nu\nabla\left\langle\frac{\mathbf{u}'^2}{2}\right\rangle\right)$$

Without the mean velocity (shear), there is no production of turbulent energy. Hence, turbulence only decays.

Enhancement of transport

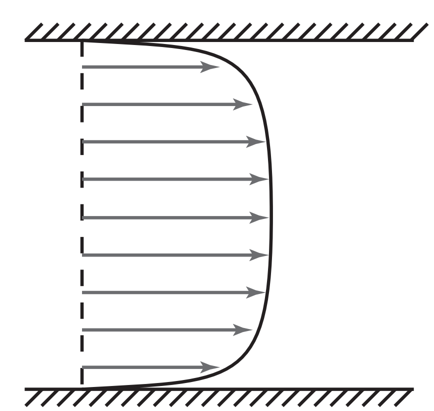
$$\frac{DU_\alpha}{Dt} \equiv \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x_a}\right) U_\alpha = -\frac{\partial P}{\partial x_\alpha} - \frac{\partial}{\partial x_a} \underline{\langle u'_a u'_\alpha \rangle} + \nu \frac{\partial^2 U_\alpha}{\partial x_a^2}$$

Reynolds stress $\quad \langle u'_\alpha u'_\beta \rangle = \frac{2}{3} K \delta_{\alpha\beta} - \nu_{\mathrm{T}} \left(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha} \right) \quad$ (Model)

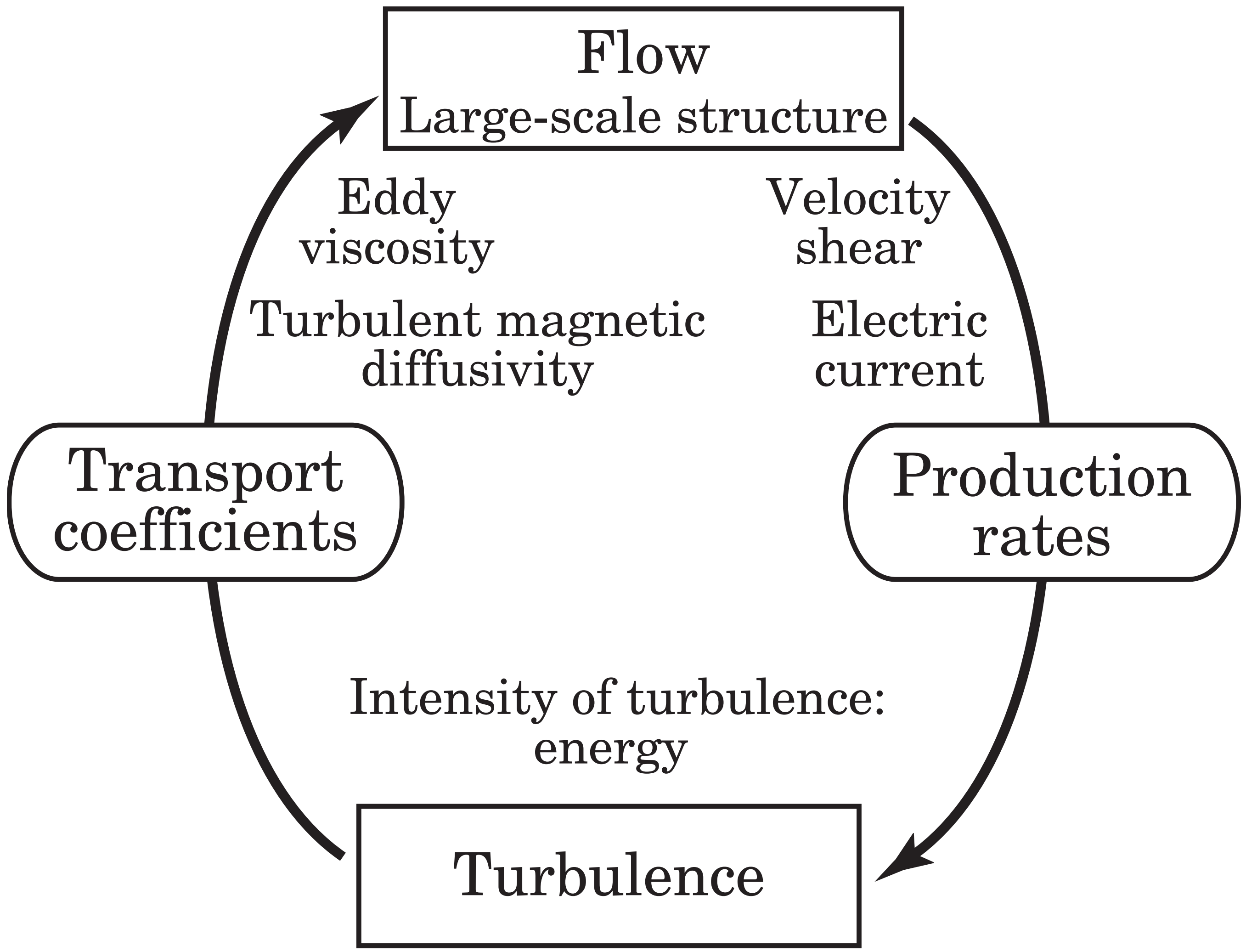
ν_{T} : eddy viscosity (turbulent viscosity) (Boussinesq, 1877)

$$\rightarrow \quad \frac{\partial U_\alpha}{\partial t} + U_a \frac{\partial U_\alpha}{\partial x_a} = -\frac{\partial P}{\partial x_\alpha} + \frac{\partial}{\partial x_a} \left[(\nu + \underline{\nu_{\mathrm{T}}}) \left(\frac{\partial U_\alpha}{\partial x_a} + \frac{\partial U_a}{\partial x_\alpha} \right) \right]$$

- enhancing transport
- spatial and temporal dependence



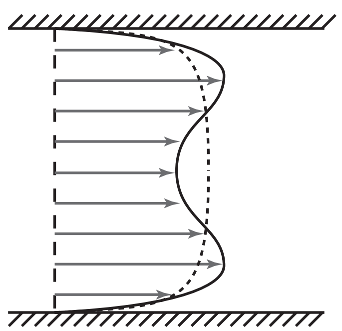
Mean-field structures determine
the properties of turbulence
through production rates

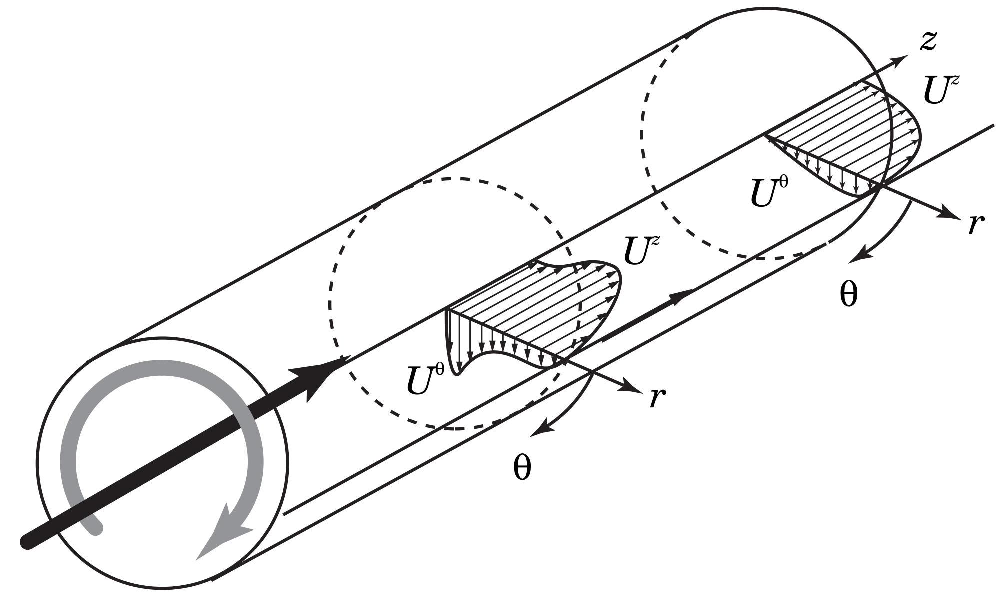


Turbulence properties determine
the mean-field structures
through transport coefficients

Suppression of transport

Turbulent swirling pipe flow





Large-scale structure again! ⟵ Additional symmetry breakage

$$\mathcal{R}_{\alpha\beta} \equiv \langle u'_\alpha u'_\beta \rangle$$
$$= \frac{2}{3} K \delta_{\alpha\beta} - \nu_{\mathrm{T}} \left(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha} \right)$$
$$+ \eta \left[\Omega_\alpha \frac{\partial H}{\partial x_\beta} + \Omega_\beta \frac{\partial H}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \left(\mathbf{\Omega} \cdot \nabla \right) H \right]$$

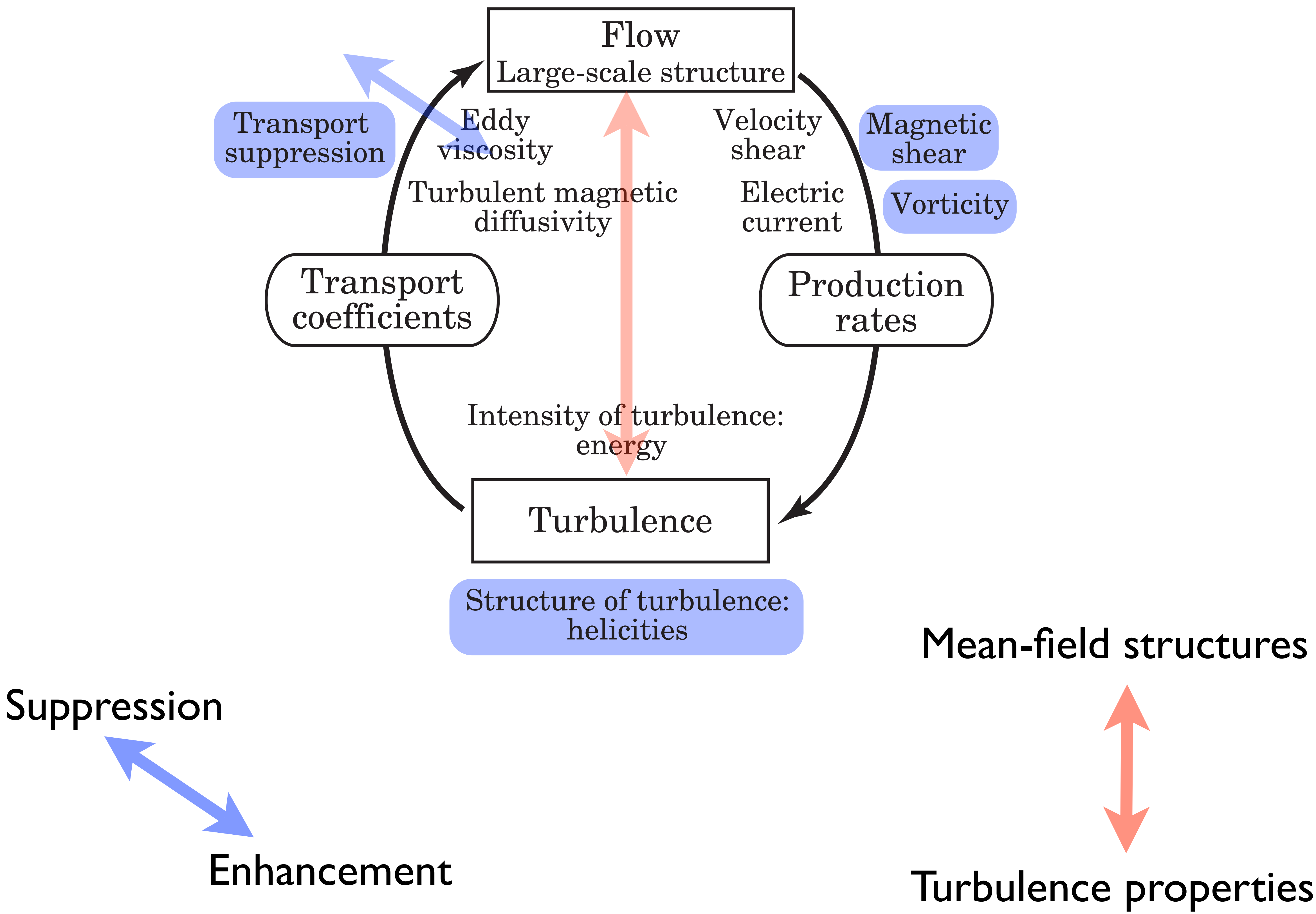
Kinetic helicity

$$H = \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$$

Mean vorticity

$$\mathbf{\Omega} = \nabla \times \mathbf{U}$$

Transport suppression due to helicity effect (Yokoi & Yoshizawa, 1993)


Flow
Large-scale structure
Transport suppression
Eddy viscosity
Turbulent magnetic diffusivity
Velocity shear
Magnetic shear
Electric current
Vorticity
Transport coefficients
Production rates
Intensity of turbulence: energy
Turbulence
Structure of turbulence: helicities
Mean-field structures
Turbulence properties
Suppression
Enhancement

What is
cross helicity?

Definition

Cross correlation between the velocity and magnetic fields

Total amount of the cross helicity $\int_V \mathbf{u} \cdot \mathbf{b} dV$

Turbulent cross helicity (density) $\langle \mathbf{u}' \cdot \mathbf{b}' \rangle \equiv W$

Mean(-field) cross helicity (density) $\mathbf{U} \cdot \mathbf{B} \equiv W_{\mathrm{M}}$

Properties

- Inviscid invariant
- Geometrical interpretation
- Pseudoscalar
- Boundedness
- Alfvén wave
- Transport suppression

What is
cross helicity good for?

Turbulence dynamo

Induction equation $$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \nabla^2 \mathbf{b}$$

$$\mathbf{u} = \mathbf{U} + \mathbf{u}', \quad \mathbf{b} = \mathbf{B} + \mathbf{b}', \quad \cdots$$

Mean induction equation $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \mathbf{E}_{\mathrm{M}} + \eta \nabla^2 \mathbf{B}$$

turbulent electromotive force

$$\begin{aligned} \mathbf{E}_{\mathrm{M}} &\equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle \\ &= \alpha \mathbf{B} - \beta \nabla \times \mathbf{B} + \gamma \mathbf{\Omega} \end{aligned}$$

Mean vorticity

$$\mathbf{\Omega} = \nabla \times \mathbf{U}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \underline{\beta}) \nabla \times \mathbf{B}] + \nabla \times (\underline{\alpha} \mathbf{B} + \underline{\gamma} \mathbf{\Omega})$$

Enhanced resistivity

Generation due to pseudoscalars

Transport coefficients are determined by the turbulence properties

turbulent magnetic diffusivity

$$\mathbf{E}_{\rm M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle = \alpha \mathbf{B} - \beta \nabla \times \mathbf{B} + \gamma \nabla \times \mathbf{U}$$

helicity effect   cross-helicity effect

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \beta) \nabla \times \mathbf{B}] + \nabla \times (\alpha \mathbf{B} + \gamma \mathbf{\Omega})$$

$$\alpha = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[-H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + H_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

kinetic helicity   current helicity

$$\beta = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

kinetic energy   magnetic energy

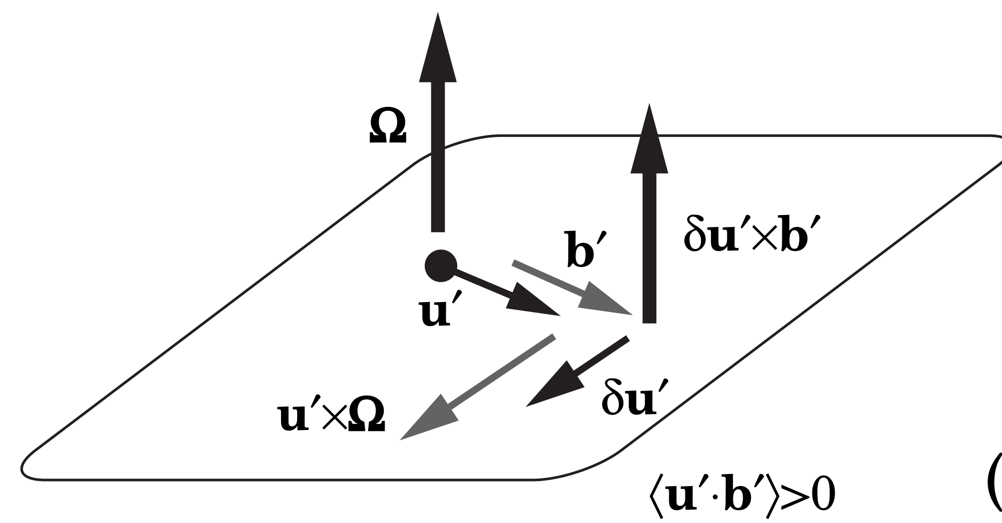
$$\gamma = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{ub}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bu}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

cross helicity

Cross-helicity (γ) effect

(Yoshizawa, 1990; Yoshizawa & Yokoi, 1993)

$$\delta \mathbf{u}' = \tau \mathbf{u}' \times \mathbf{\Omega}$$



(Yokoi, 1999)

- Correlation between **u**′ and **b**′
- Local angular-momentum conservation

$$[\mathbf{E}_{\rm M}]_\gamma = \langle \delta \mathbf{u}' \times \mathbf{b}' \rangle = +\tau_\gamma \langle \mathbf{u}' \cdot \mathbf{b}' \rangle \mathbf{\Omega}$$

→ Turbulent electromotive force contribution parallel to the mean vorticity

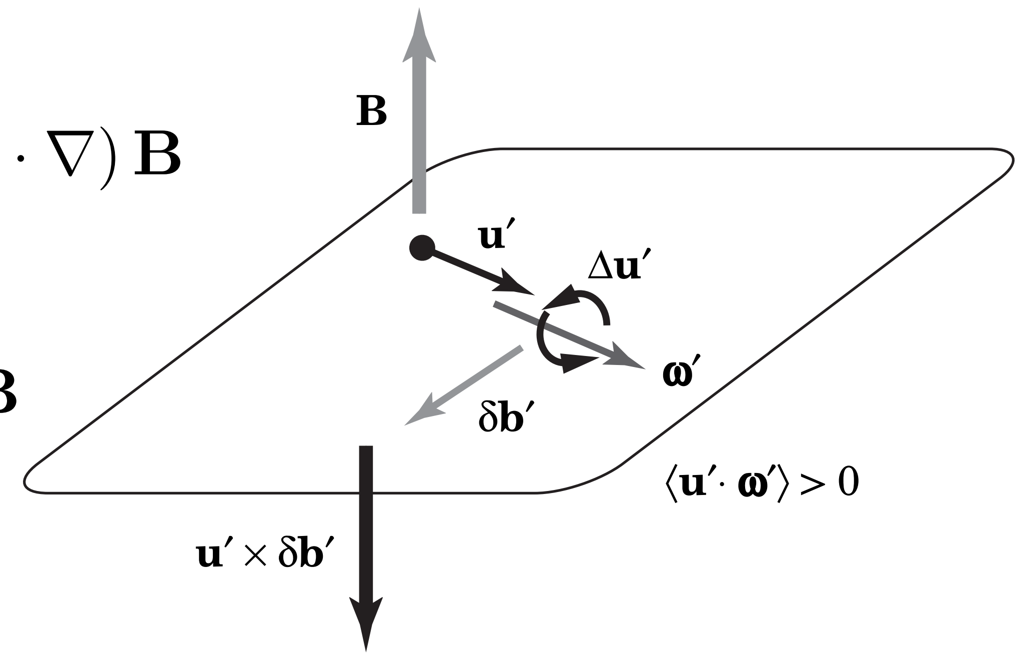
Helicity (α) effect

Kinetic helicity

(Parker, 1955; Steenbeck, Krause & Rädler, 1966)

$$\delta \mathbf{b}' = -(\nabla \cdot \boldsymbol{\xi}_\perp)\,\mathbf{B} + \underline{(\mathbf{B}\cdot\nabla)\,\boldsymbol{\xi}_\perp} - (\boldsymbol{\xi}_\perp \cdot \nabla)\,\mathbf{B}$$

$$[\mathbf{E}_\mathrm{M}]_{\alpha\mathrm{K}} = \langle \mathbf{u}' \times \delta\mathbf{b}' \rangle = -\tau_{\alpha\mathrm{K}} \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle\, \mathbf{B}$$

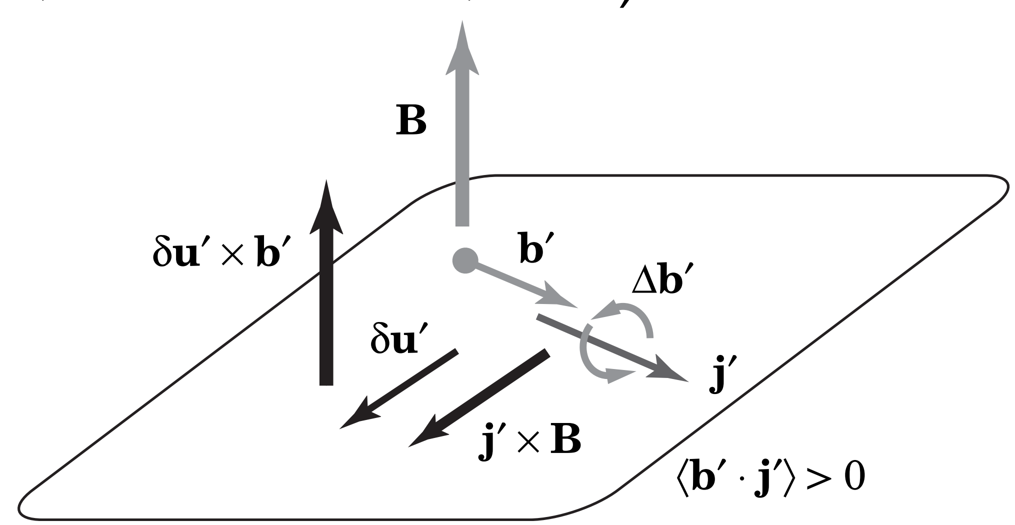


Current helicity

(Pouquet, Frisch & Léorat, 1976)

$$\delta \mathbf{u}' = \tau \mathbf{j}' \times \mathbf{B}$$

$$[\mathbf{E}_\mathrm{M}]_{\alpha\mathrm{M}} = \langle \delta\mathbf{u}' \times \mathbf{b}' \rangle = +\tau_{\alpha\mathrm{M}} \langle \mathbf{b}' \cdot \mathbf{j}' \rangle\, \mathbf{B}$$



→ Turbulent electromotive force contribution parallel and antiparallel to the mean magnetic field

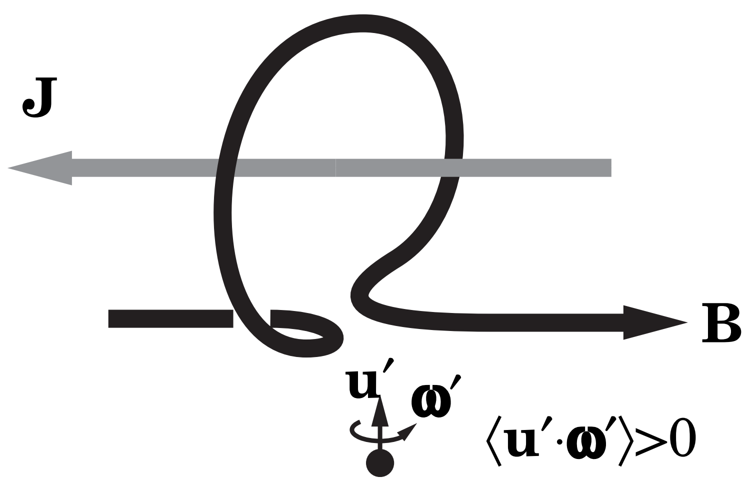
Helicity and cross-helicity dynamos

helicity dynamo

$$\mathbf{E}_{\mathrm{M}} = \alpha\mathbf{B} - \beta\nabla\times\mathbf{B} + \gamma\nabla\times\mathbf{U}$$

cross-helicity dynamo

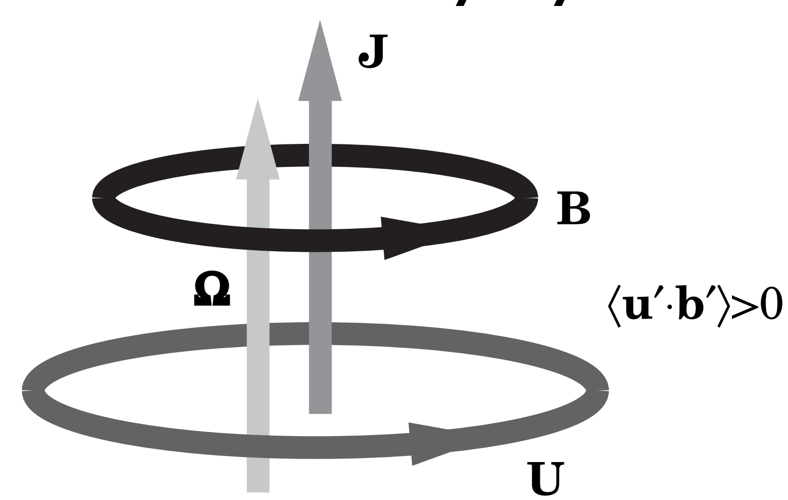
Helicity or α dynamo



(Krause & Rädler, 1980)

$\mathbf{J} \parallel \mathbf{B}$

Cross-helicity dynamo



(Yoshizawa & Yokoi, 1993)

$\mathbf{J} \parallel \mathbf{\Omega}$

$\mathbf{B} \parallel \mathbf{U}$

Solar Dynamo

(Yoshizawa, Kato, & Yokoi, ApJ 2000)

Positive cross helicity

↓ $\mathbf{B}_0 = \frac{\gamma}{\beta}\mathbf{U}$

Toroidal magnetic field B_0

↓ $\mathbf{J}_1 = \frac{\alpha}{\beta}\mathbf{B}_0 = \frac{\alpha}{\beta}\frac{\gamma}{\beta}\mathbf{U}$

Poloidal magnetic field B_1

↓ $P_{W1} = -\alpha \mathbf{B}_1 \cdot \mathbf{\Omega}$

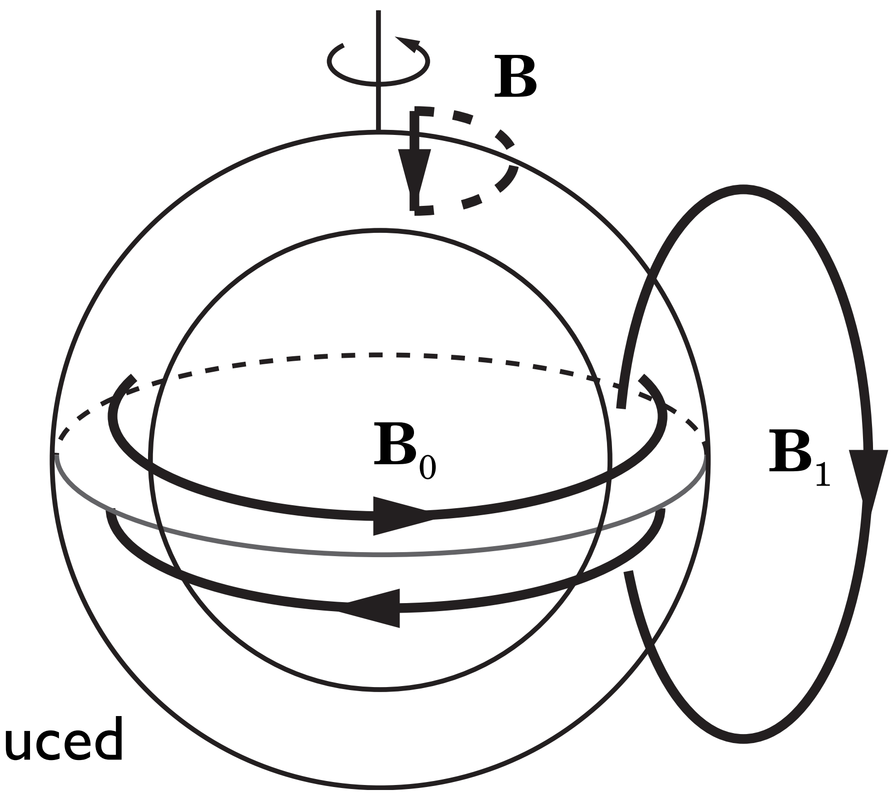
Negative cross helicity

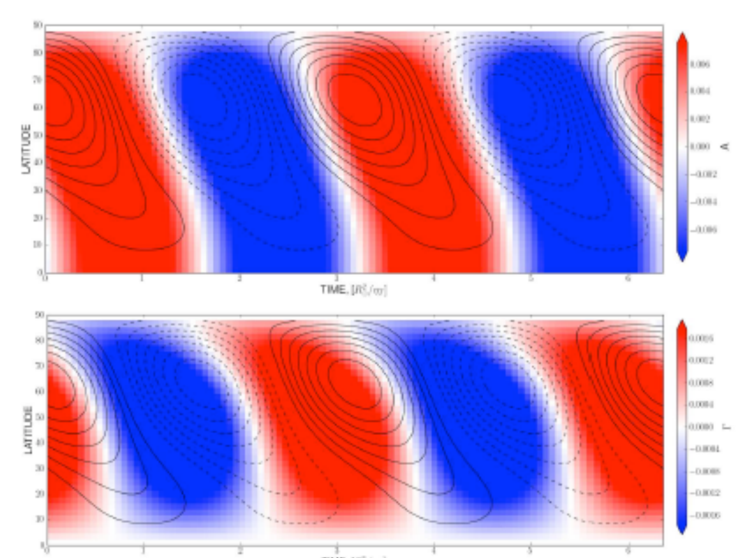
└→ Periodic reversal

Generation of the toroidal field due to the cross-helicity (γ) effect

Generation of the poloidal field due to the helicity (α) effect

Negative cross-helicity generation due to the induced poloidal magnetic field B_1





Butterfly diagram is generated without resort to the Ω effect

with Valery Pipin (2011)

Question:
How and how much cross helicity can exist in turbulence?

What makes cross helicity?

$$\mathcal{W}_{\rm tot} = \int_V \mathbf{u} \cdot \mathbf{b}_* dV$$

$$\frac{d\mathcal{W}_{\rm tot}}{dt} = \int_S \left[\left(\frac{1}{2}\mathbf{u}^2 - \frac{\Gamma}{\Gamma - 1}\frac{p}{\rho} \right) \mathbf{b}_* - (\mathbf{u} \cdot \mathbf{b}_*)\, \mathbf{u} \right] \cdot \mathbf{n}\ dS \qquad p = \rho^\Gamma$$

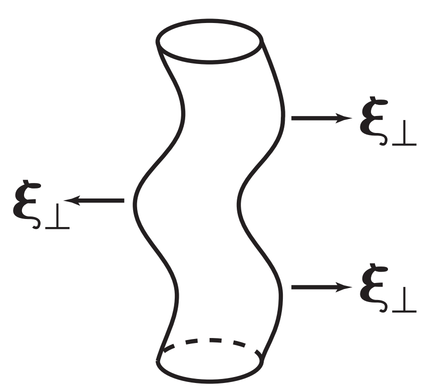
$$\int_S (\mathbf{u} \cdot \mathbf{b}_*)\, \mathbf{u} \cdot (-\mathbf{n}) dS$$

cross-helicity influx
through the boundary

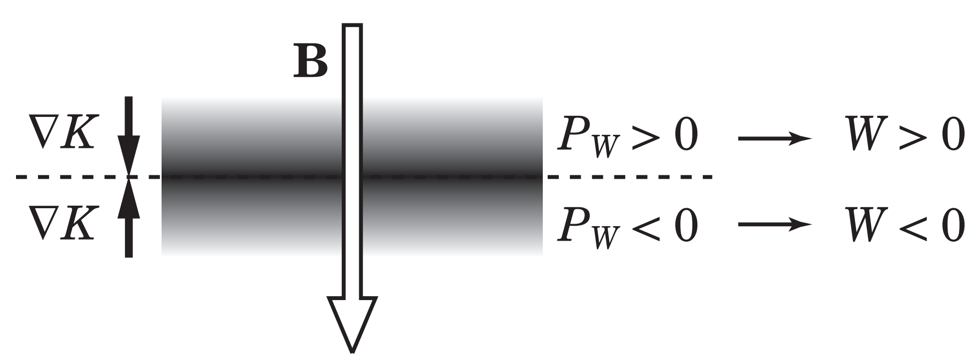
$$\int_S \left(\frac{1}{2}\mathbf{u}^2 - \frac{\Gamma}{\Gamma - 1}\frac{p}{\rho} \right) \mathbf{b}_* \cdot \mathbf{n}\ dS = \int_V \mathbf{b}_* \cdot \nabla \left(\frac{1}{2}\mathbf{u}^2 - \frac{\Gamma}{\Gamma - 1}\frac{p}{\rho} \right)\ dV$$

energy inhomogeneity
along the magnetic field

If we have a sort of energy inhomogeneity along the magnetic field, the cross helicity can be supplied to the system.



Alfvén wave



Magnetic field threading a turbulent disk

Large-scale production mechanism

$$W_\mathrm{c} = \langle \mathbf{u}' \cdot \mathbf{b}'_\mathrm{c} \rangle$$

$$\frac{DW_\mathrm{c}}{Dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) W_\mathrm{c}$$

$$= -\frac{1}{2} \left\langle u'^a u'^b - \frac{1}{4\pi\overline{\rho}} b'^{\,a}_\mathrm{c} b'^{\,b}_\mathrm{c} \right\rangle \left(\frac{\partial B^b_\mathrm{c}}{\partial x^a} + \frac{\partial B^a_\mathrm{c}}{\partial x^b} \right)$$

$$- \langle \mathbf{u}' \times \mathbf{b}'_\mathrm{c} \rangle \cdot \mathbf{\Omega}$$

$$- \frac{\Gamma - 1}{\overline{\rho}} \langle q' \mathbf{b}'_\mathrm{c} \rangle \cdot \nabla \overline{\rho}$$

$$- W_\mathrm{c} \nabla \cdot \mathbf{U}$$

$$+ \mathbf{B}_\mathrm{c} \cdot \nabla \left\langle \frac{1}{2} \mathbf{u}'^2 \right\rangle$$

$$- \varepsilon_{W_\mathrm{c}} + T_{W_\mathrm{c}}$$

internal energy $q = C_V(\theta)\theta$

C_V : specific heat at constant volume

θ : temperature

$q = Q + q'$

plasma pressure $p = R\rho\theta = (\Gamma - 1)\rho q$

R : gas constant

$\Gamma = C_P / C_V$ ratio of specific heats

C_P : specific heat at constant pressure

$$\nabla \cdot (K\mathbf{B}) - \langle \mathbf{b}' \cdot \nabla p'_\mathrm{M} \rangle$$

$$\mathbf{B} \cdot \nabla K - \langle \mathbf{b}' \cdot \nabla p'_\mathrm{M} \rangle + K \nabla \cdot \mathbf{B}$$

$$= B^a \frac{\partial}{\partial x^a} \left\langle \frac{1}{2} u'^b u'^b \right\rangle - \frac{1}{4} \langle u'^b u'^b + b'^b b'^b \rangle \frac{B^a}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x^a} + \cdots$$

Cross-helicity generation mechanism

Evolution equation of the turbulent cross helicity $W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$

$$\frac{DW}{Dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) W = P_W - \varepsilon_W + \nabla \cdot \mathbf{T}_W$$

where
$$P_W = -\mathcal{R}^{ab} \frac{\partial B^a}{\partial x^b} - \mathbf{E}_\mathrm{M} \cdot \boldsymbol{\Omega}$$ production rate

$$\varepsilon_W = (\nu + \eta) \left\langle \frac{\partial u'^a}{\partial x^b} \frac{\partial b'^a}{\partial x^b} \right\rangle$$ dissipation rate

$$\mathbf{T}_W = K\mathbf{B} - \left\langle (\mathbf{u}' \cdot \mathbf{b}')\, \mathbf{u}' - \left(\frac{\mathbf{u}'^2 + \mathbf{b}'^2}{2} - p'_\mathrm{M} \right) \mathbf{b}' \right\rangle$$ transport rate

with
$$\mathcal{R}^{\alpha\beta} = \langle u'^\alpha u'^\beta - b'^\alpha b'^\beta \rangle$$ Reynolds stress

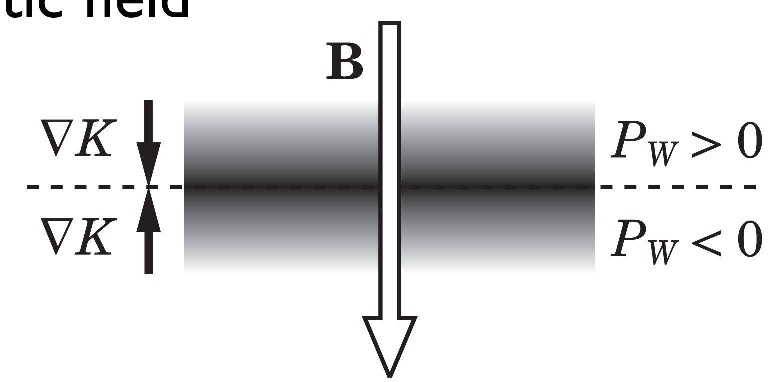
$$\mathbf{E}_\mathrm{M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle$$ Turbulent electromotive force

- Generation due to vorticity

$$-\mathbf{E}_\mathrm{M} \cdot \boldsymbol{\Omega} = -\alpha \mathbf{B} \cdot \boldsymbol{\Omega} + \beta \mathbf{J} \cdot \boldsymbol{\Omega} - \gamma \boldsymbol{\Omega}^2$$

- Generation due to inhomogeneity along the magnetic field

$$\nabla \cdot (\mathbf{B}K) = \mathbf{B} \cdot (\nabla K)$$



$P_W > 0 \longrightarrow W > 0$

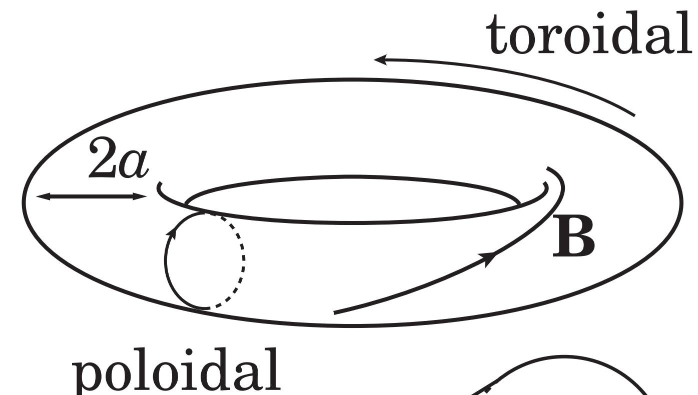
$P_W < 0 \longrightarrow W < 0$

Generation due to vorticity

(Yokoi, 1999)

$$P_{W2} = -\mathbf{E}_{\mathrm{M}} \cdot \mathbf{\Omega} = -\alpha \mathbf{B} \cdot \mathbf{\Omega} + \underline{\beta \mathbf{J} \cdot \mathbf{\Omega}} - \gamma \mathbf{\Omega}^2$$

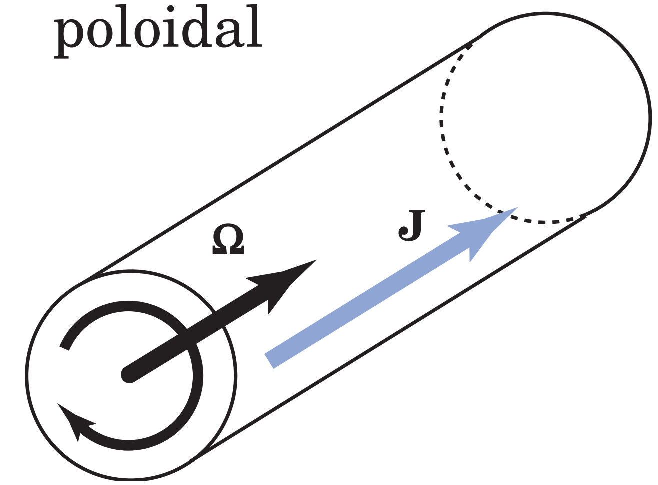
with $\quad \mathbf{\Omega} = \nabla \times \mathbf{U} \qquad \mathbf{J} = \nabla \times \mathbf{B}$

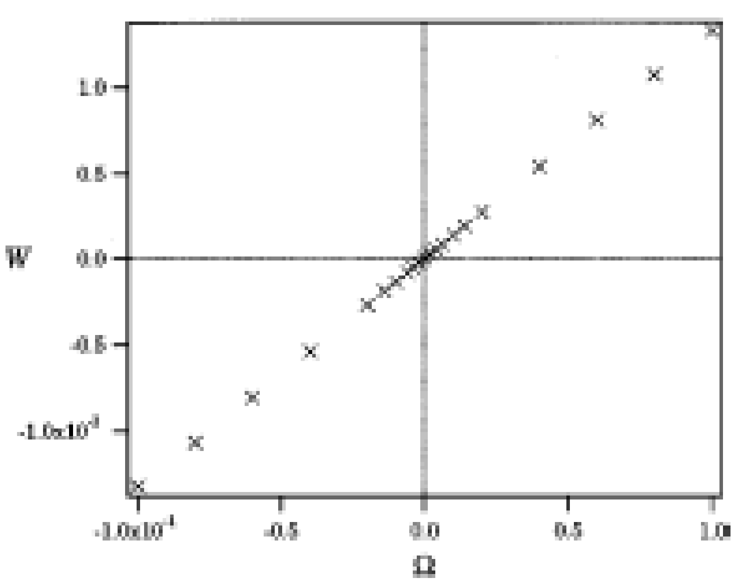


Plasma current + poloidal rotation

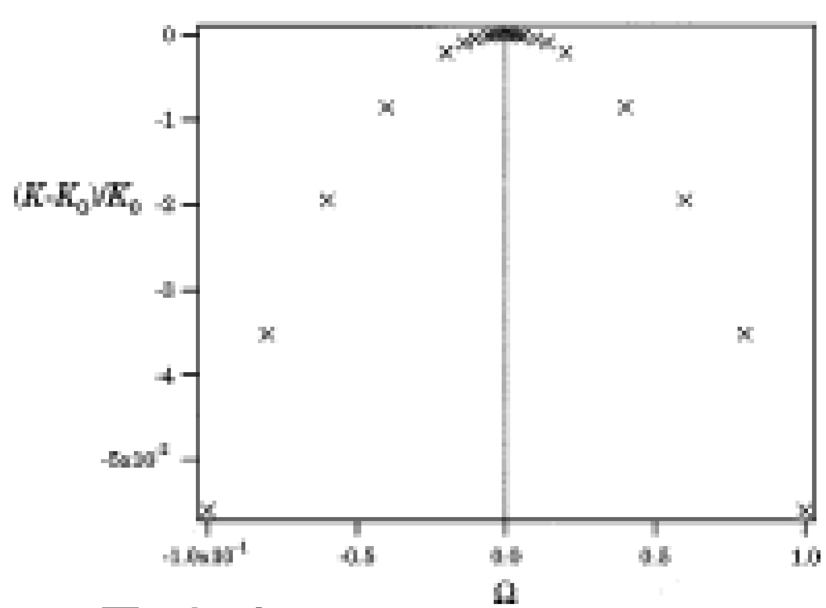
$$\mathbf{J} \cdot \mathbf{\Omega} > 0 \quad \rightarrow \quad P_{W2} > 0 \quad \rightarrow \quad W > 0$$

$$\mathbf{J} \cdot \mathbf{\Omega} < 0 \quad \rightarrow \quad P_{W2} < 0 \quad \rightarrow \quad W < 0$$

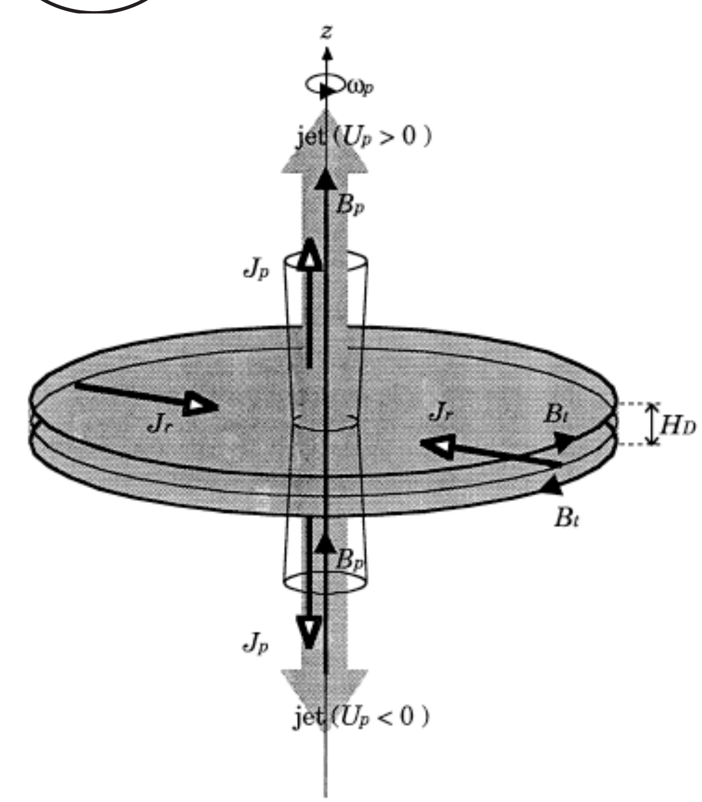




Generated cross helicity against the imposed rotation



Turbulence suppression rate against the imposed rotation

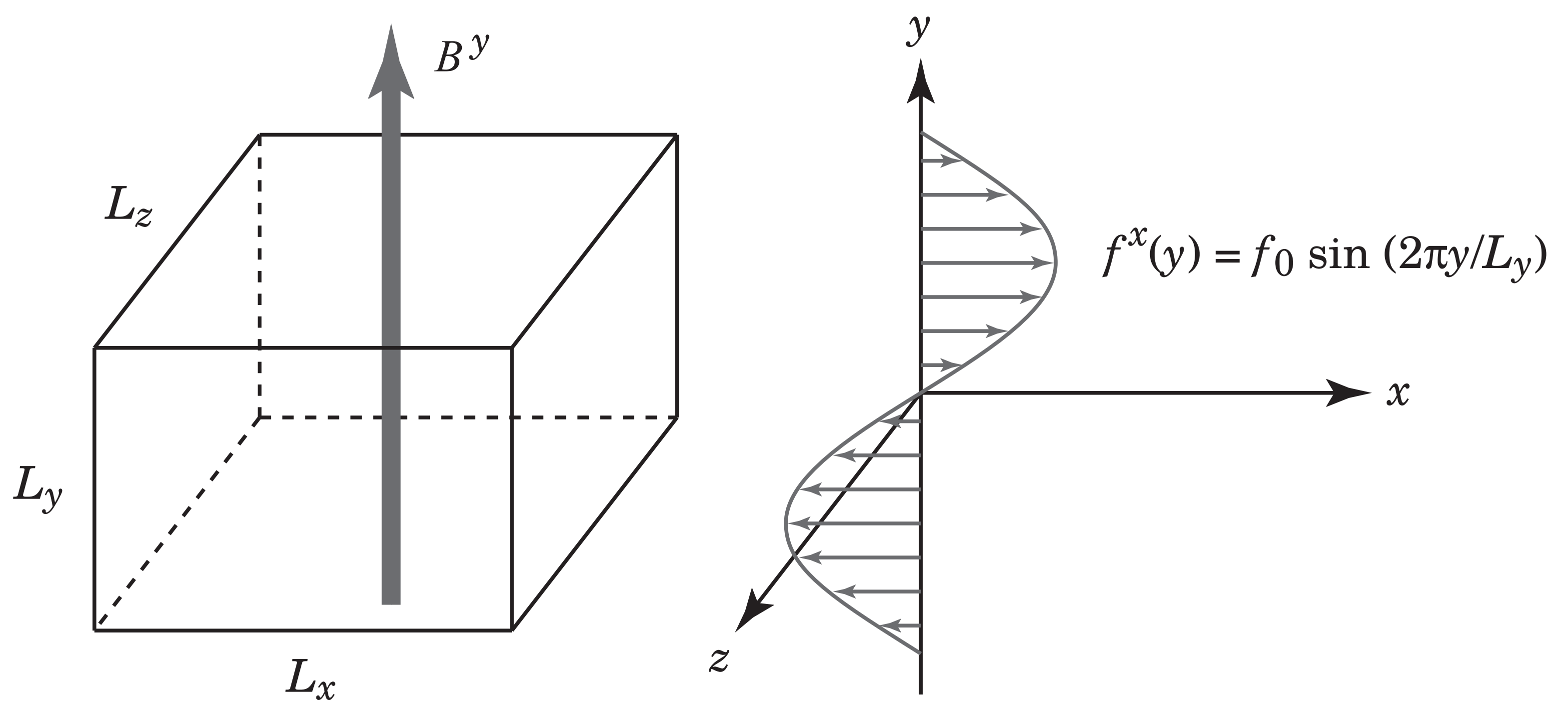


How relevant?

DNS of Kolmogorov flow

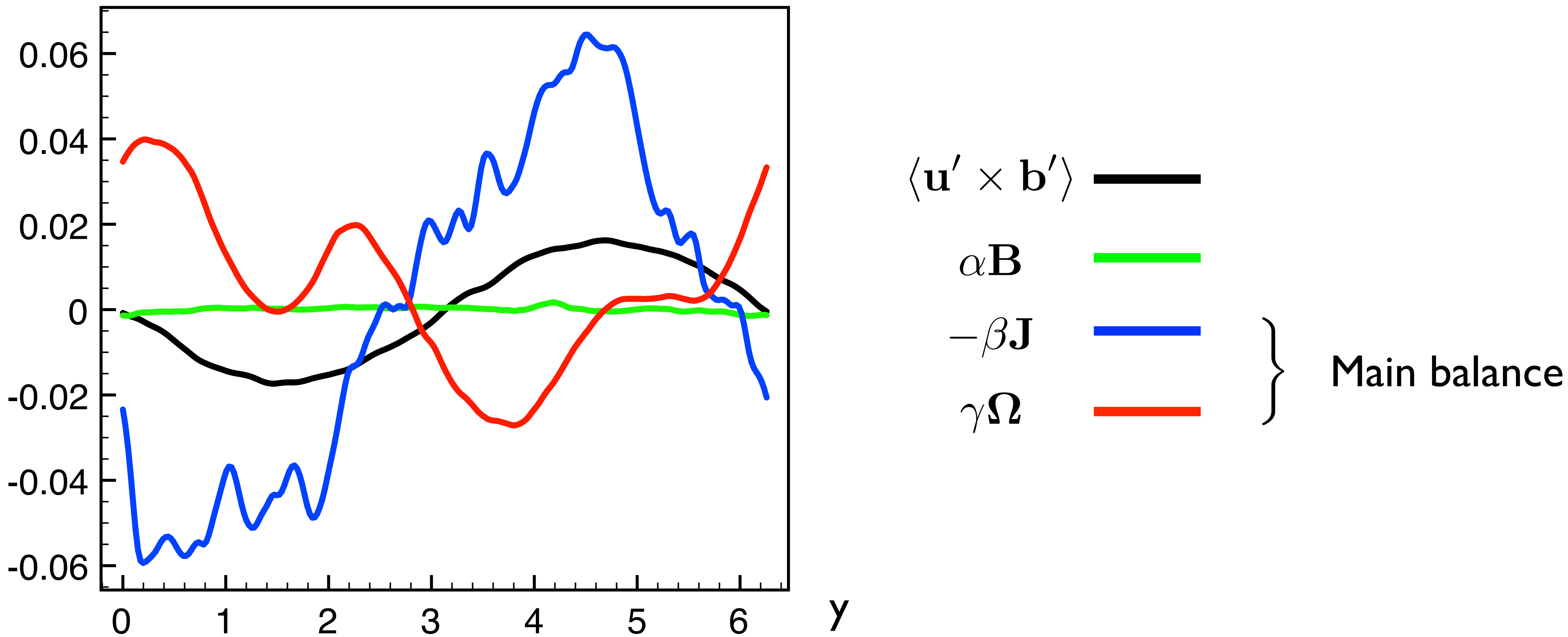
(Yokoi & Balarac, 2011)

- 3D (256^3) periodic flow with external forcing $f^x(y) = f_0 \sin(2\pi y/L_y)$
- Mean shear velocity
- Constant magnetic field imposed [y (inhomogeneous) direction]
- Homogeneous in x and z directions



cf. Archontis flow, a generalization of the Arnold–Beltrami–Childress flow (Sur & Brandenburg, 2009)

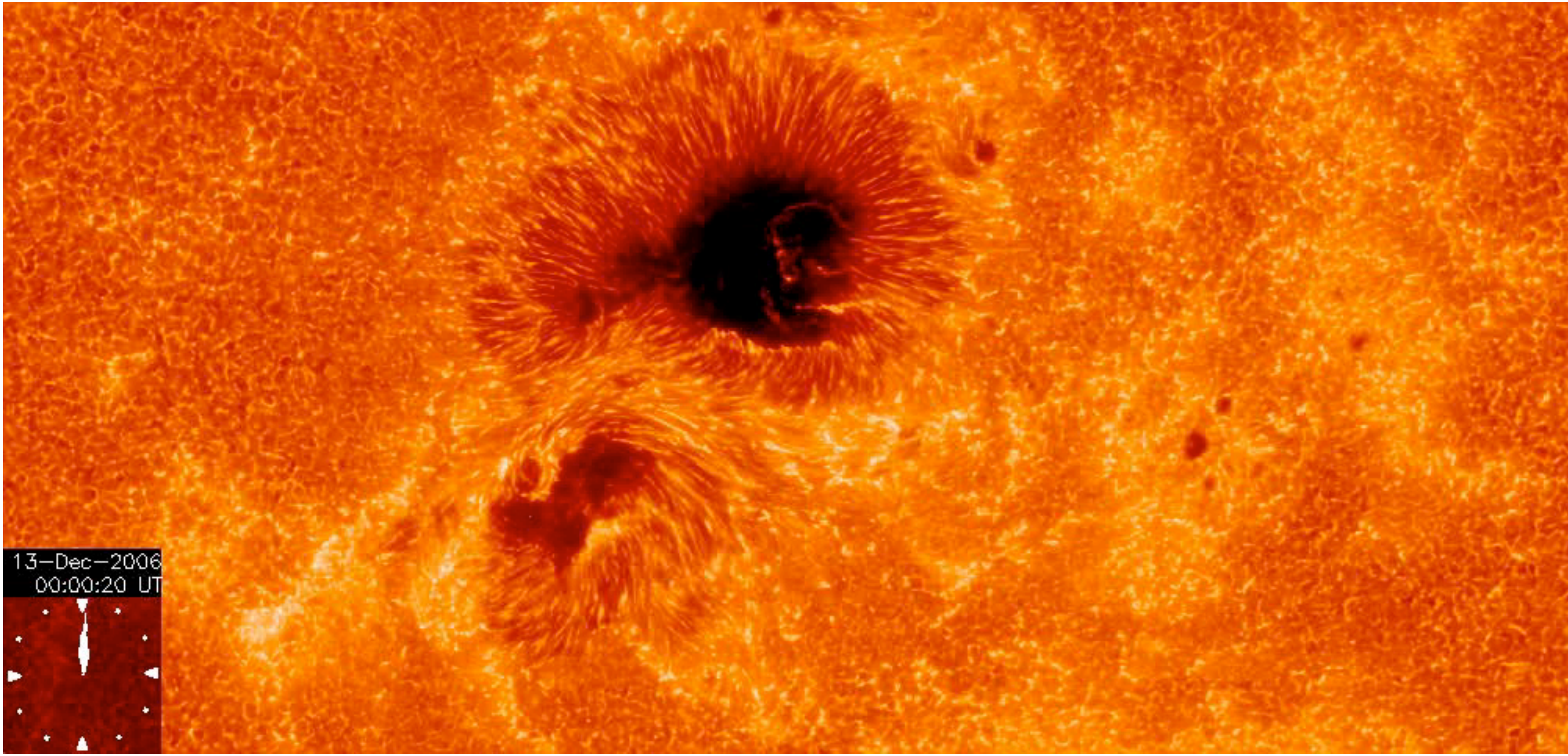
Turbulent electromotive force



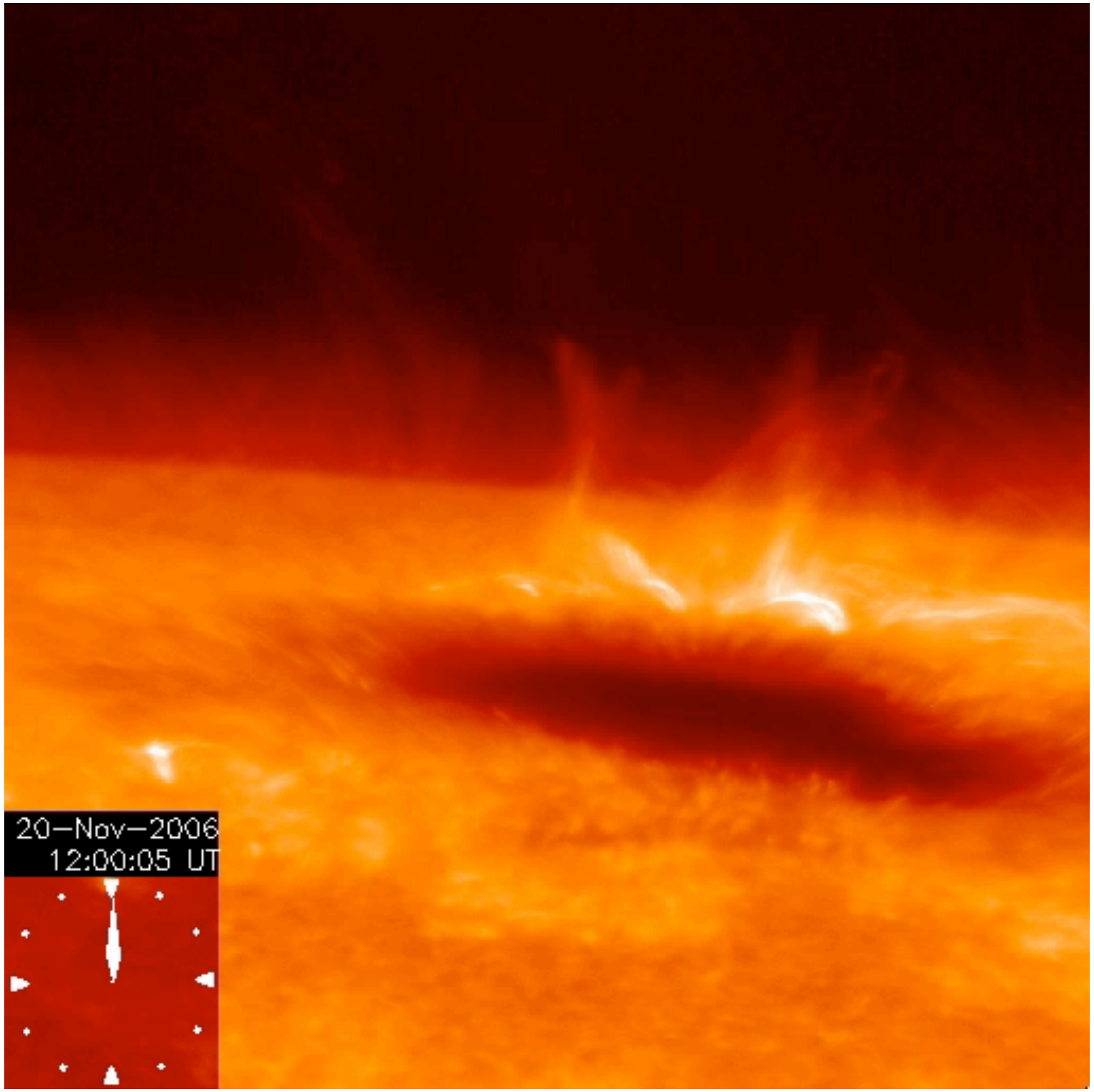
The cross-helicity effect, rather than the helicity or α effect, plays a dominant role in balancing the turbulent magnetic diffusivity effect

Flow–turbulence interaction in magnetic reconnection

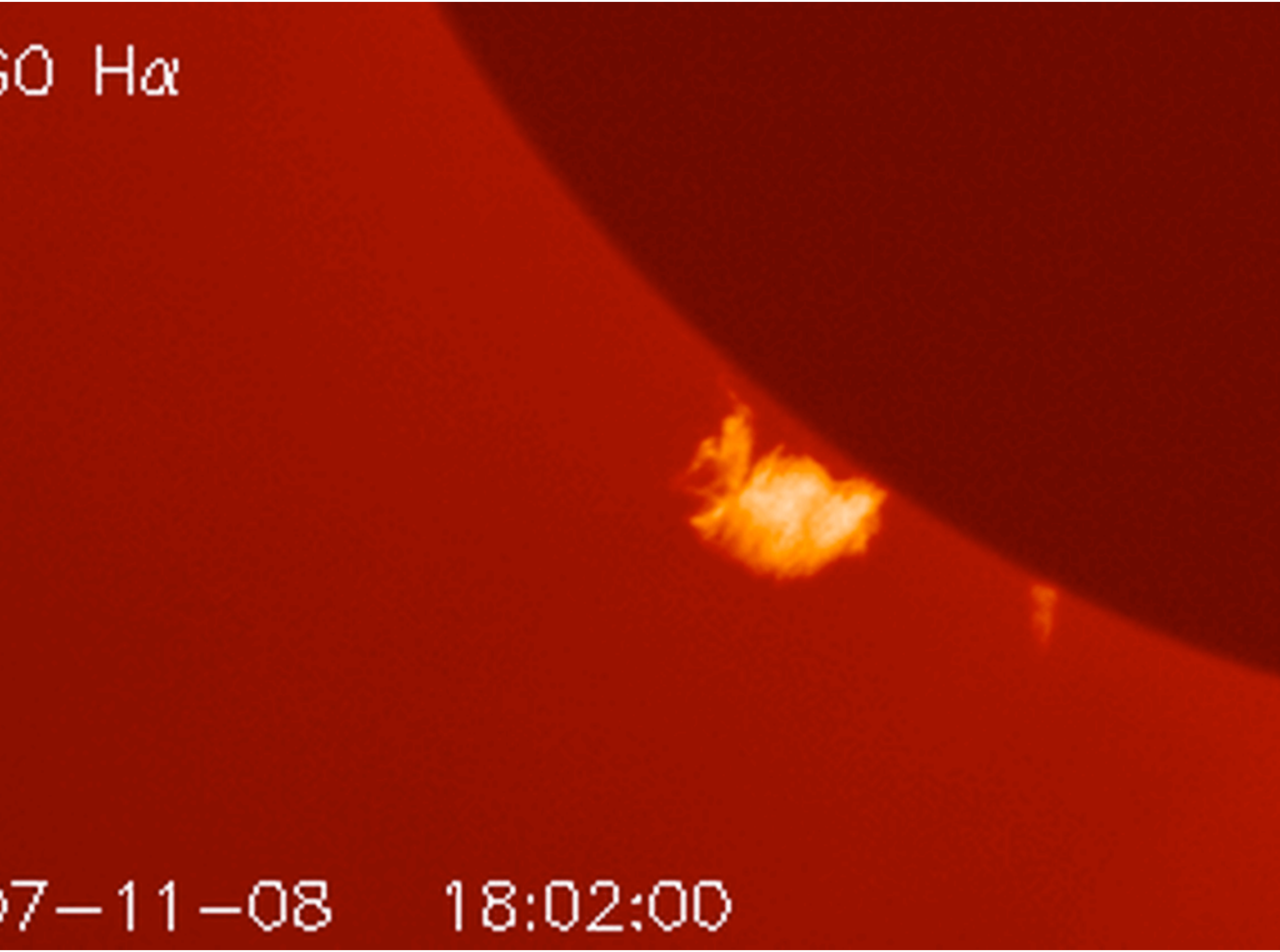
Yokoi & Hoshino (2011) Phys. Plasmas **18**, 111208



Courtesy of the HINODE Group, NAOJ



Courtesy of the HINODE Group, NAOJ


GO Hα
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Too slow

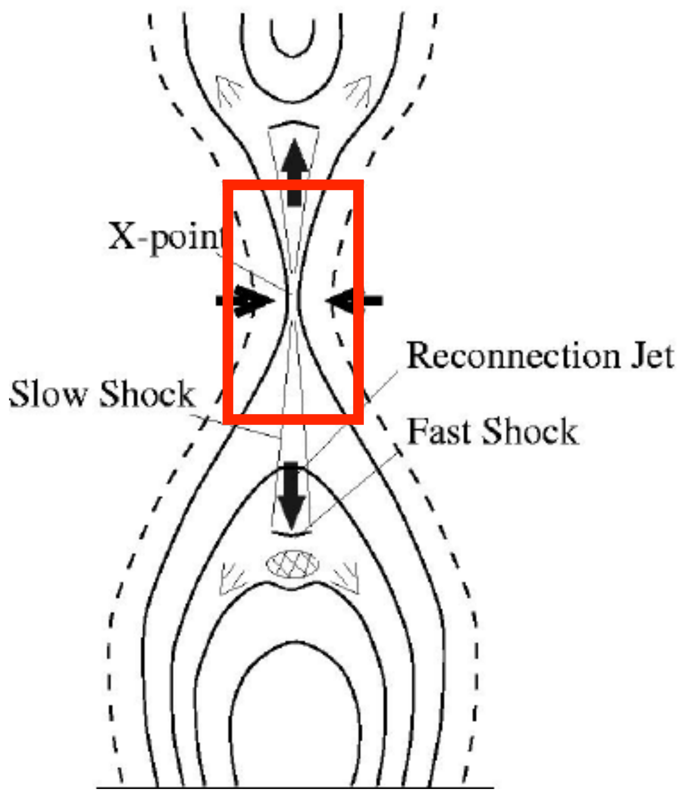
$$M_{\rm in} = \frac{U_{\rm in}}{V_{\rm Ain}} = \frac{\delta}{L} = S^{-1/2}$$

Lundquist number $S = \frac{\mu_0 L V_{\rm A}}{\eta}$

astrophysical and space plasmas $S \gg 10^6$

⟶ **Fast reconnection**

Gap of scales



Thickness of current sheet

$$\delta = \rho_{\rm i} \sim 10 \text{ m}$$

Ion Larmor radius $\rho_{\rm i}$

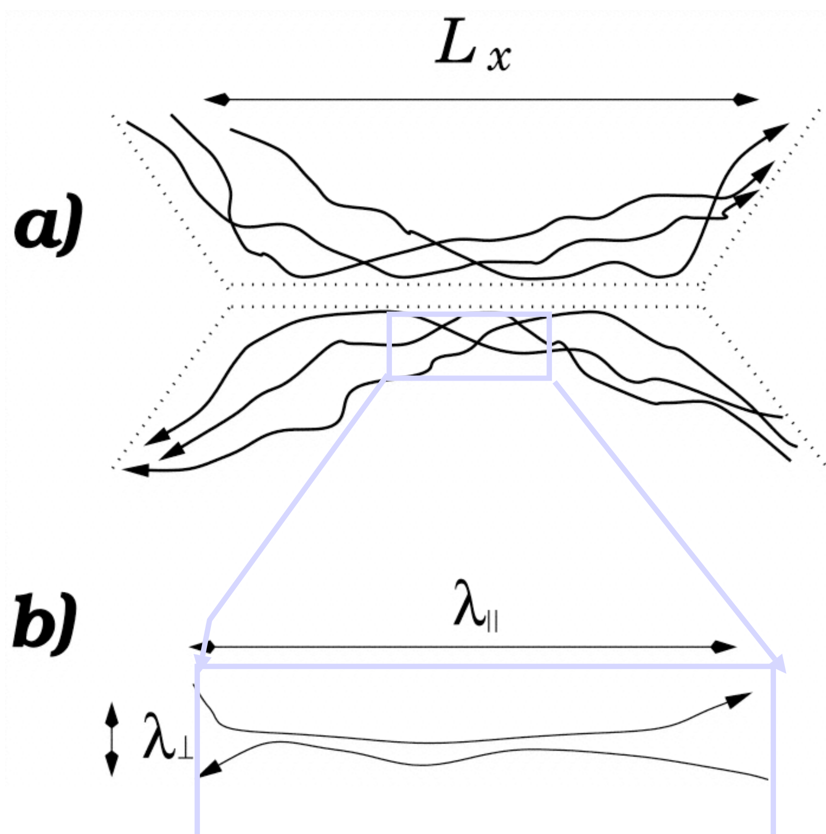
Flare scale 10^4 km

⟶ **Localized resistivity**

⟶ **Matching of scales**

Turbulent reconnection

Lazarian & Vishniac (1999)



$$M_{\rm in} = \frac{U_{\rm in}}{V_{\rm Ain}} \leq M_{\rm turb}^2$$

$M_{\rm turb}$: large-scale magnetic Mach number of turbulence

Fractal current sheet

Tajima & Shibata (1997)

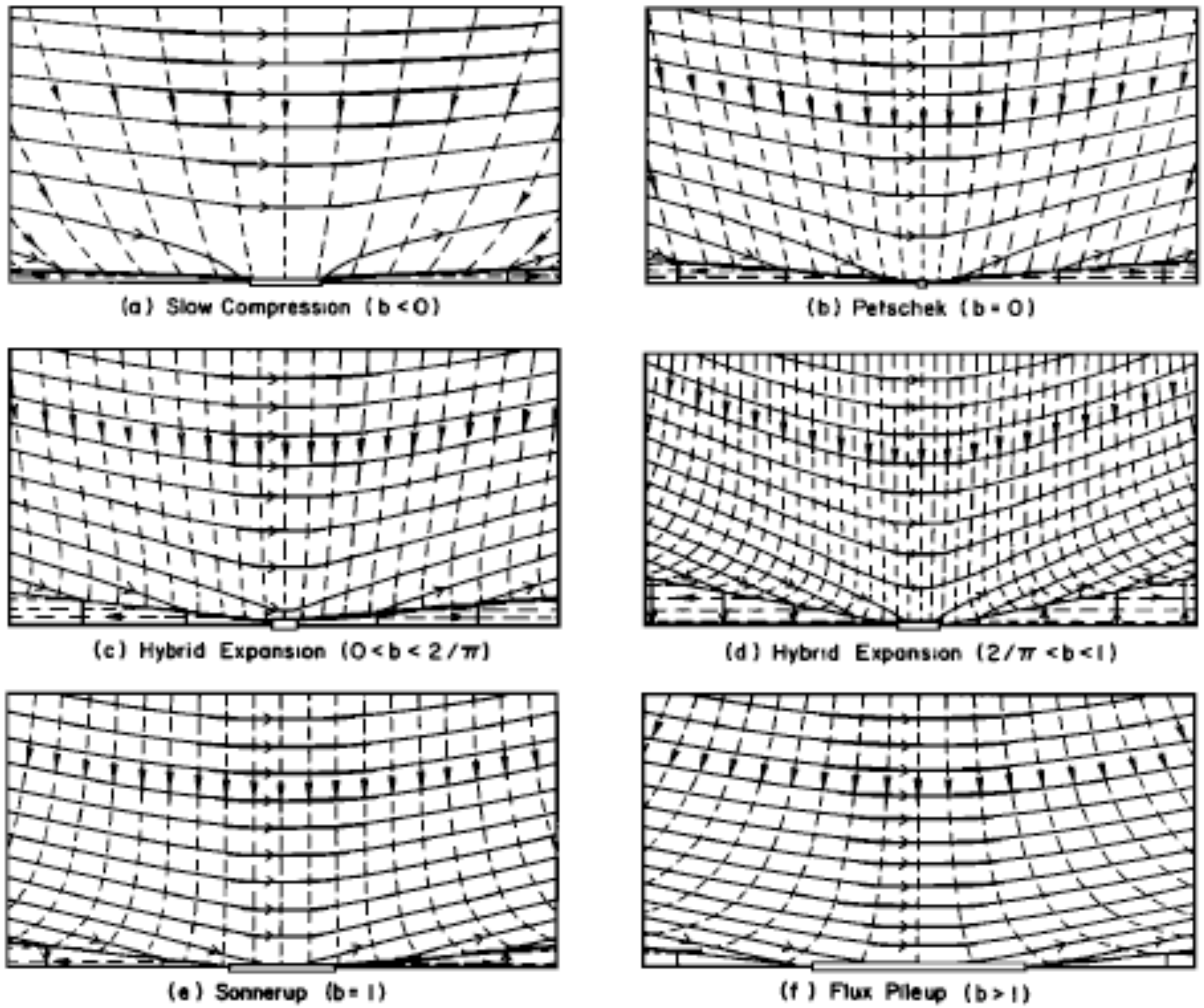


Fig. 5. Magnetic field lines (solid lines) and streamlines (dashed lines) for different classes of solution with external magnetic Reynolds number $R_{me} = 500$. The open rectangular boxes indicate the lengths of the diffusion regions: (*a*) $b = -2.0$, $M_e = 0.043$; (*b*) $b = 0$, $M_e = 0.091$; (*c*) $b = 0.3$, $M_e = 0.100$; (*d*) $b = 0.8$, $M_e = 0.200$; (*e*) $b = 1.0$, $M_e = 0.100$; (*f*) $b = 2.0$, $M_e = 0.100$. Only every third streamline in the outflow jets is shown [from *Priest and Forbes*, 1986].

Forbes & Priest, 1987

Turbulence effects

Reynolds (+ turbulent Maxwell) stress

$$\mathcal{R}^{\alpha\beta} \equiv \left\langle u'^{\alpha} u'^{\beta} - b'^{\alpha} b'^{\beta} \right\rangle$$
$$= \frac{2}{3} K_{\mathrm{R}} \delta^{\alpha\beta} - \nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} + \left[\Gamma^{\alpha} \Omega^{\beta} + \Gamma^{\beta} \Omega^{\alpha} \right]_{\mathrm{D}}$$

Enhancement Suppression

Turbulent electromotive force

$$\mathbf{E}_{\mathrm{M}} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle$$
$$= -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}$$

Enhancement Suppression

$$\alpha = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[-H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + H_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

$$\beta = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\mathrm{K}}$$

$$\gamma = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{ub}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bu}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\mathrm{M}}$$

$$\boldsymbol{\Gamma} = \frac{1}{15} \int d\mathbf{k}\ k^{-2} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \nabla H_{uu}(k, \mathbf{x}; \tau, \tau_1, t)$$

What should be solved ...

$$\frac{\partial \mathbf{U}}{\partial t} = \cdots + \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\mathcal{R}} + \cdots$$

$$\frac{\partial \mathbf{B}}{\partial t} = \cdots + \nabla \times \mathbf{E}_{\mathrm{M}} + \cdots$$

$$\begin{aligned} \mathcal{R}^{\alpha\beta} &\equiv \left\langle u'^{\alpha} u'^{\beta} - b'^{\alpha} b'^{\beta} \right\rangle \\ &= \frac{2}{3} K_{\mathrm{R}} \delta^{\alpha\beta} - \nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} + \left[\Gamma^{\alpha} \Omega^{\beta} + \Gamma^{\beta} \Omega^{\alpha} \right]_{\mathrm{D}} \end{aligned}$$

$$\begin{aligned} \mathbf{E}_{\mathrm{M}} &\equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle \\ &= -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega} \end{aligned}$$

$\frac{\partial \beta}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

$\frac{\partial \gamma}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

$\frac{\partial \alpha}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

Mean induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} + \mathbf{E}_{\mathrm{M}}) + \eta \nabla^2 \mathbf{B}$$

$$\mathbf{E}_{\mathrm{M}} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \mathbf{\Omega}$$

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, \quad \mathbf{J} = \mathbf{J}_0 + \delta \mathbf{J}$$

Reference
$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}_0) + \nabla \times (\alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0)$$

Modulation
$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \delta \mathbf{B}) - \nabla \times (\beta \nabla \times \delta \mathbf{B} - \gamma \nabla \times \mathbf{U})$$

$$\longrightarrow \quad \delta \mathbf{B} = \frac{\gamma}{\beta} \mathbf{U} = C_{\mathrm{W}} \frac{W}{K} \mathbf{U}$$

$$\frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \leq 1$$

Mean momentum equation

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \boldsymbol{\Omega} + \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\mathcal{R}} + \mathbf{F} - \nabla \left(P + \frac{1}{2}\mathbf{U}^2 + \left\langle \frac{1}{2}\mathbf{b}'^2 \right\rangle \right)$$

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{U} \times \mathbf{B} + \mathbf{E}_{\rm M} \right)$$

$$\begin{cases} \mathcal{R}^{\alpha\beta} = \frac{2}{3} K_{\rm R} \delta^{\alpha\beta} - \nu_{\rm K} \mathcal{S}^{\alpha\beta} + \nu_{\rm M} \mathcal{M}^{\alpha\beta} \\ \mathbf{E}_{\rm M} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega} \end{cases}$$

Mean Lorentz force $\mathbf{J} \times \mathbf{B} = \frac{1}{\beta} (\mathbf{U} \times \mathbf{B}) \times \mathbf{B} + \frac{\gamma}{\beta} \boldsymbol{\Omega} \times \mathbf{B} - \frac{1}{\beta} \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{B}$

$$\mathbf{U} = \mathbf{U}_0 + \delta \mathbf{U}, \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \delta \boldsymbol{\Omega}$$

Reference $\frac{\partial \boldsymbol{\Omega}_0}{\partial t} = \nabla \times \left(\mathbf{U}_0 \times \boldsymbol{\Omega}_0 + \nu_{\rm K} \nabla^2 \mathbf{U_0} + \mathbf{F} \right)$

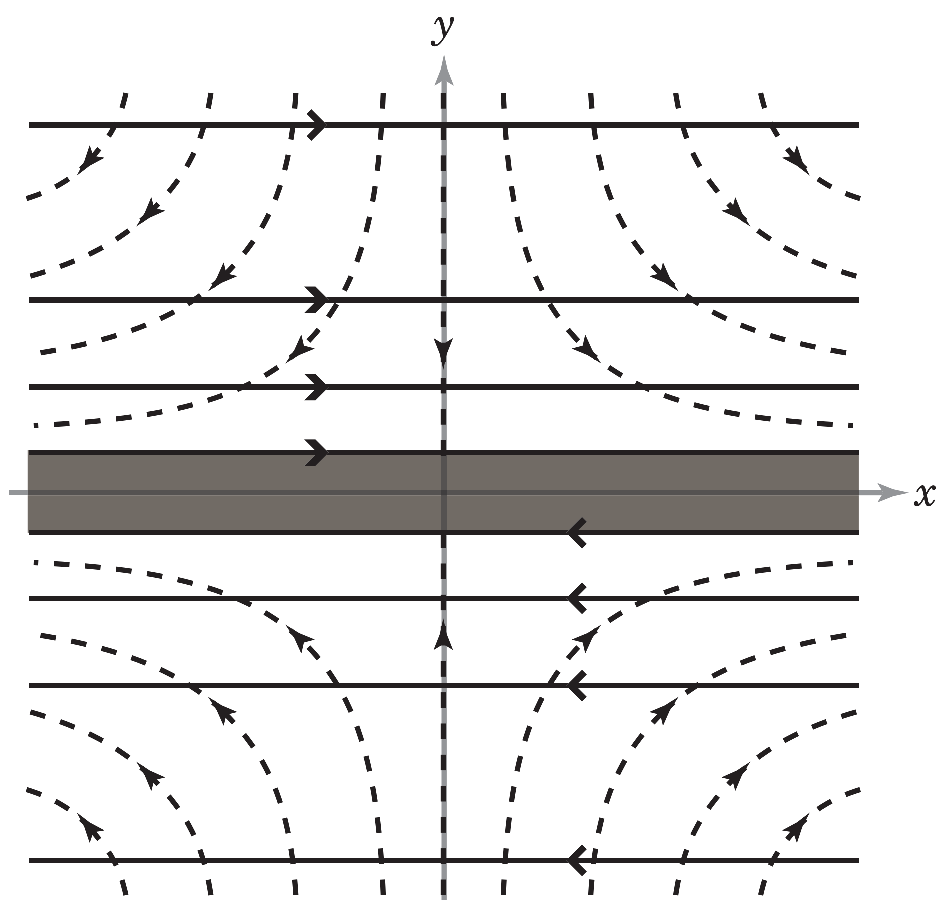
Modulation $\frac{\partial \delta \boldsymbol{\Omega}}{\partial t} = \nabla \times \left[\left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \times \boldsymbol{\Omega}_0 + \nu_{\rm K} \nabla^2 \left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \right]$

→ $$\delta \mathbf{U} = \frac{\gamma}{\beta} \mathbf{B} = C_{\rm W} \frac{W}{K} \mathbf{B}$$

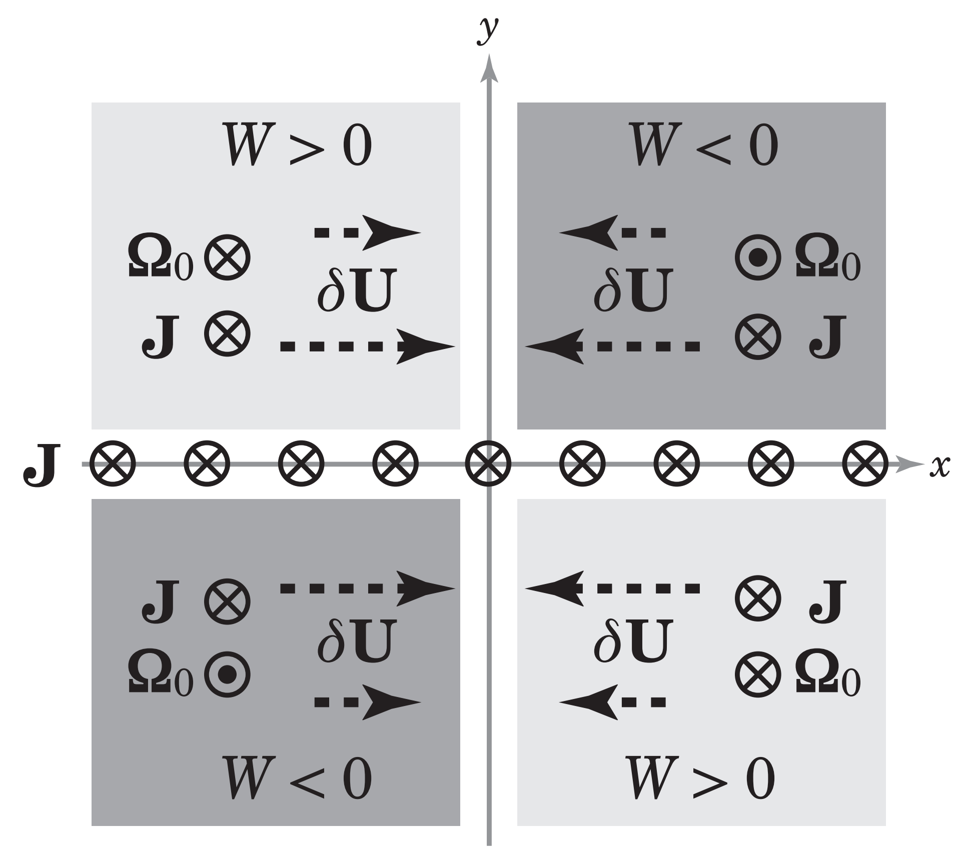
$$\frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \le 1$$

Mean-field configuration and turbulence

Inflow and outflow

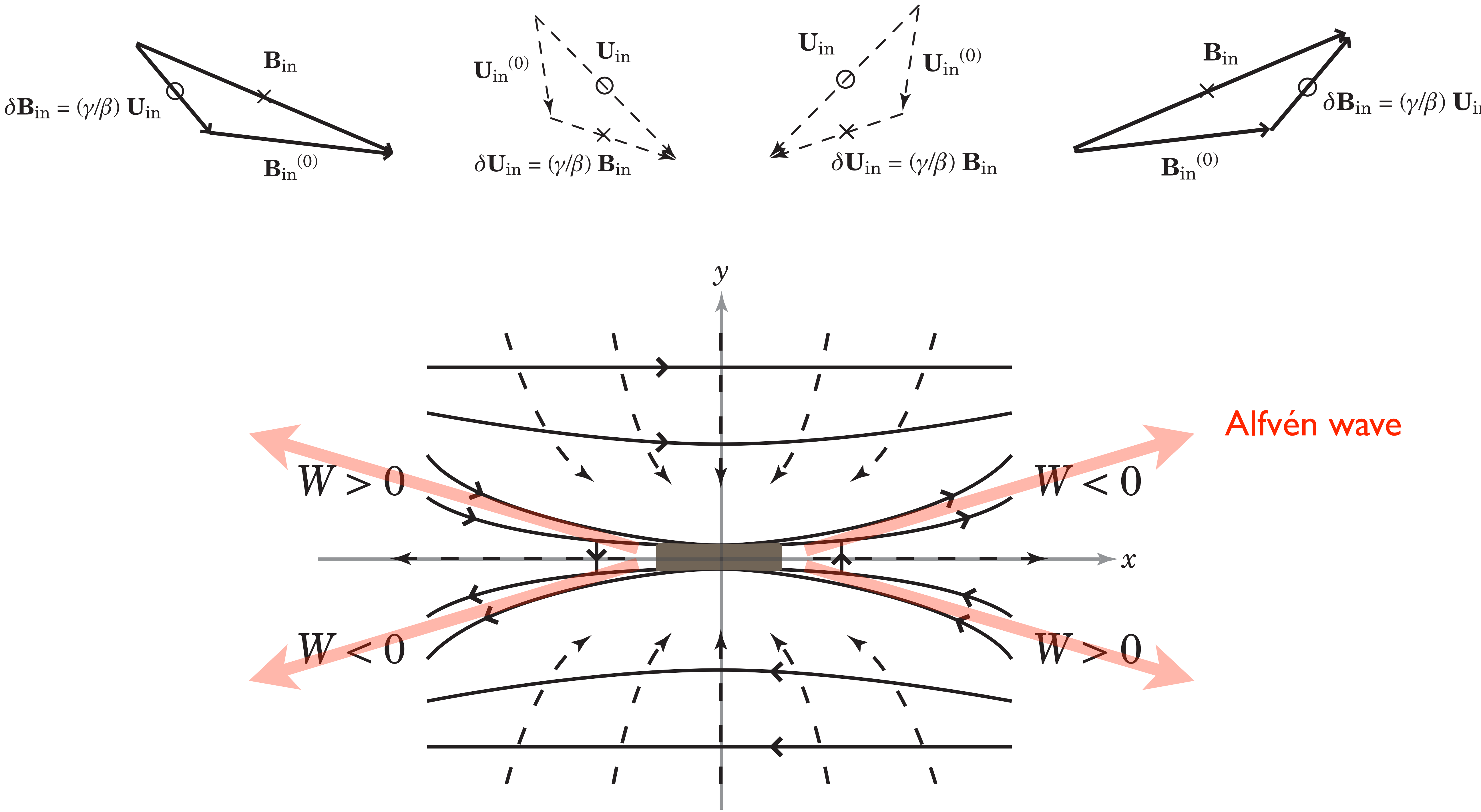


Spatial distribution of turbulent cross helicity



$$\mathbf{J} \cdot \mathbf{\Omega} > 0 \quad \rightarrow \quad P_{W2} > 0 \quad \rightarrow \quad W > 0$$

$$\mathbf{J} \cdot \mathbf{\Omega} < 0 \quad \rightarrow \quad P_{W2} < 0 \quad \rightarrow \quad W < 0$$



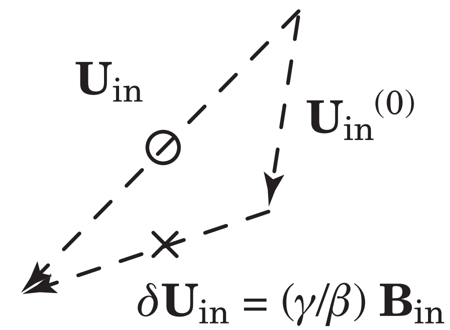
$\mathbf{B}_{in}$
$\delta\mathbf{B}_{in} = (\gamma/\beta)\,\mathbf{U}_{in}$
$\mathbf{B}_{in}{}^{(0)}$
$\mathbf{U}_{in}$
$\mathbf{U}_{in}{}^{(0)}$
$\delta\mathbf{U}_{in} = (\gamma/\beta)\,\mathbf{B}_{in}$
$\mathbf{U}_{in}$
$\mathbf{U}_{in}{}^{(0)}$
$\delta\mathbf{U}_{in} = (\gamma/\beta)\,\mathbf{B}_{in}$
$\mathbf{B}_{in}$
$\delta\mathbf{B}_{in} = (\gamma/\beta)\,\mathbf{U}_{in}$
$\mathbf{B}_{in}{}^{(0)}$
y
x
Alfvén wave
$W > 0$
$W < 0$
$W < 0$
$W > 0$

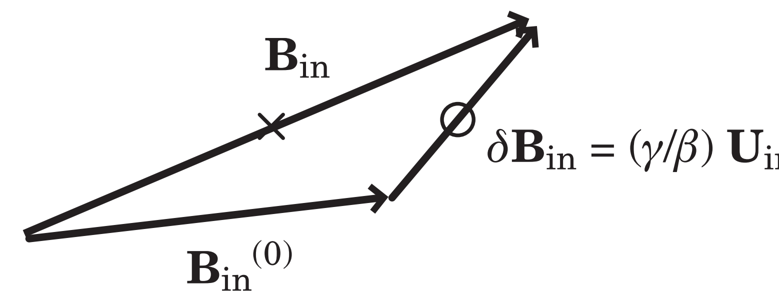
Reconnection rate

$$M_{\rm in} \equiv \frac{U_{\rm in}}{V_{\rm Ain}} = \frac{U_{\rm in}}{B_{\rm in}}$$

$$\mathbf{U}_{\rm in} = \mathbf{U}_{\rm in}^{(0)} + \delta \mathbf{U}_{\rm in} = \mathbf{U}_{\rm in}^{(0)} + \frac{\gamma}{\beta}\mathbf{B}_{\rm in}$$

$$\mathbf{B}_{\rm in} = \mathbf{B}_{\rm in}^{(0)} + \delta \mathbf{B}_{\rm in} = \mathbf{B}_{\rm in}^{(0)} + \frac{\gamma}{\beta}\mathbf{U}_{\rm in}$$





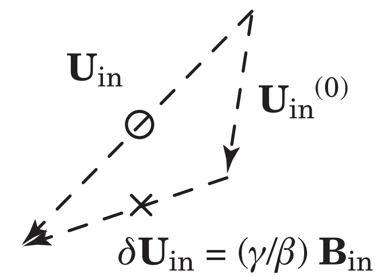
$$\begin{pmatrix} 1 & -\frac{\gamma}{\beta} \\ -\frac{\gamma}{\beta} & 1 \end{pmatrix} \begin{pmatrix} {}^t\mathbf{U}_{\rm in} \\ {}^t\mathbf{B}_{\rm in} \end{pmatrix} = \begin{pmatrix} {}^t\mathbf{U}_{\rm in}^{(0)} \\ {}^t\mathbf{B}_{\rm in}^{(0)} \end{pmatrix}$$

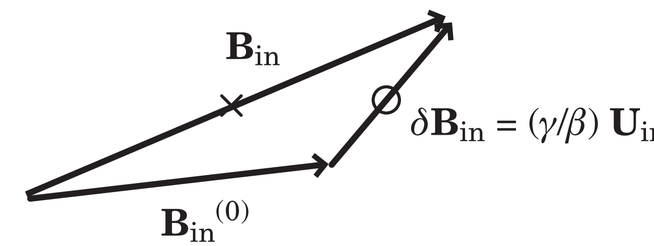
$$\begin{pmatrix} {}^t\mathbf{U}_{\rm in} \\ {}^t\mathbf{B}_{\rm in} \end{pmatrix} = \left[1 - \left(\frac{\gamma}{\beta}\right)^2\right]^{-1} \begin{pmatrix} 1 & \frac{\gamma}{\beta} \\ \frac{\gamma}{\beta} & 1 \end{pmatrix} \begin{pmatrix} {}^t\mathbf{U}_{\rm in}^{(0)} \\ {}^t\mathbf{B}_{\rm in}^{(0)} \end{pmatrix}$$

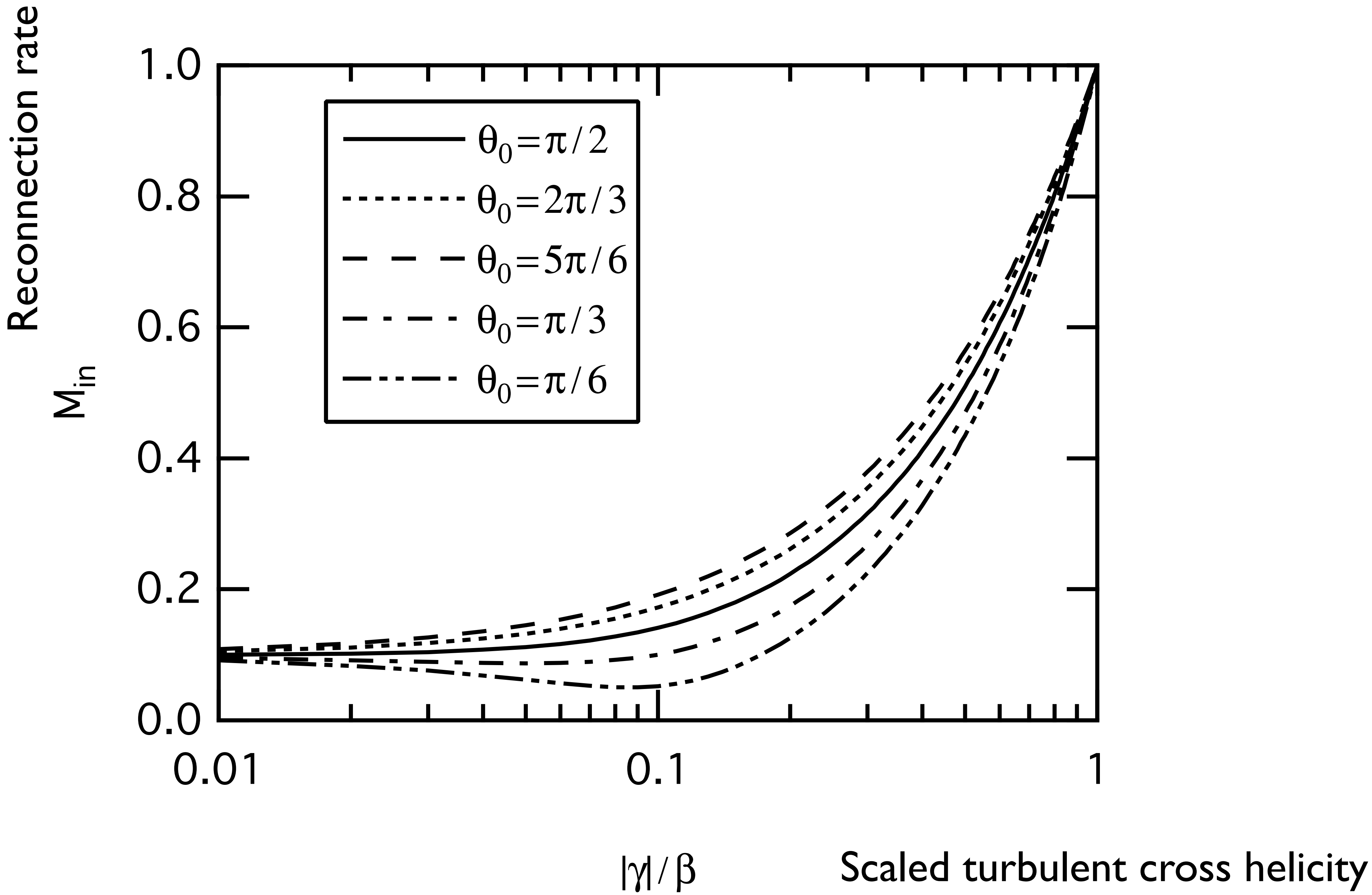
$$M_{\rm in} = \left[\frac{{M_{\rm in}^{(0)}}^2 + (\gamma/\beta)^2 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}{(\gamma/\beta)^2 {M_{\rm in}^{(0)}}^2 + 1 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}\right]^{1/2}$$

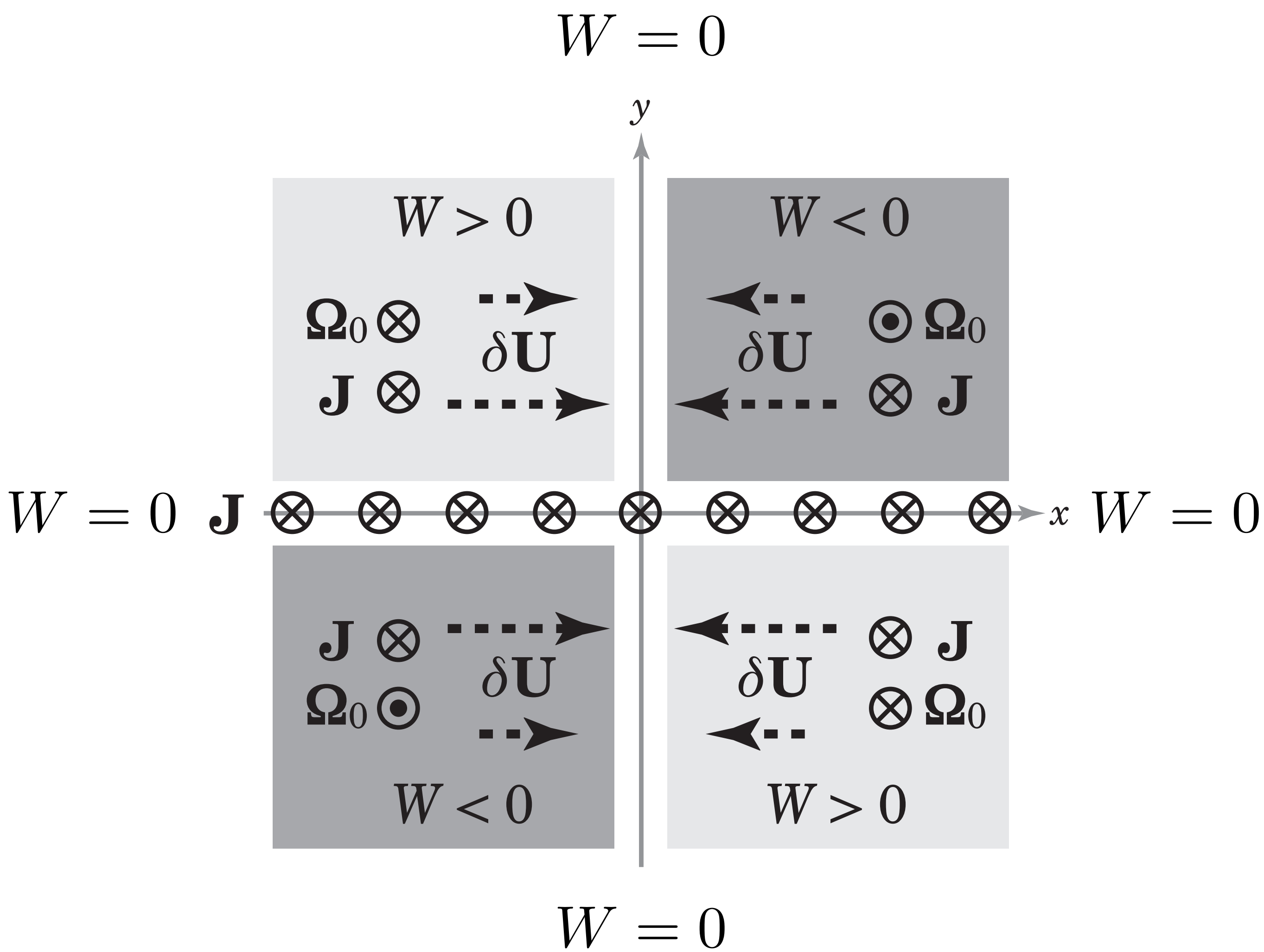
$$\frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \le 1$$

$$M_{\rm in} = \left[\frac{{M_{\rm in}^{(0)}}^2 + (\gamma/\beta)^2 + (2\gamma/\beta) M_{\rm in}^{(0)} \cos\theta_0}{(\gamma/\beta)^2 {M_{\rm in}^{(0)}}^2 + 1 + (2\gamma/\beta) M_{\rm in}^{(0)} \cos\theta_0} \right]^{1/2}$$

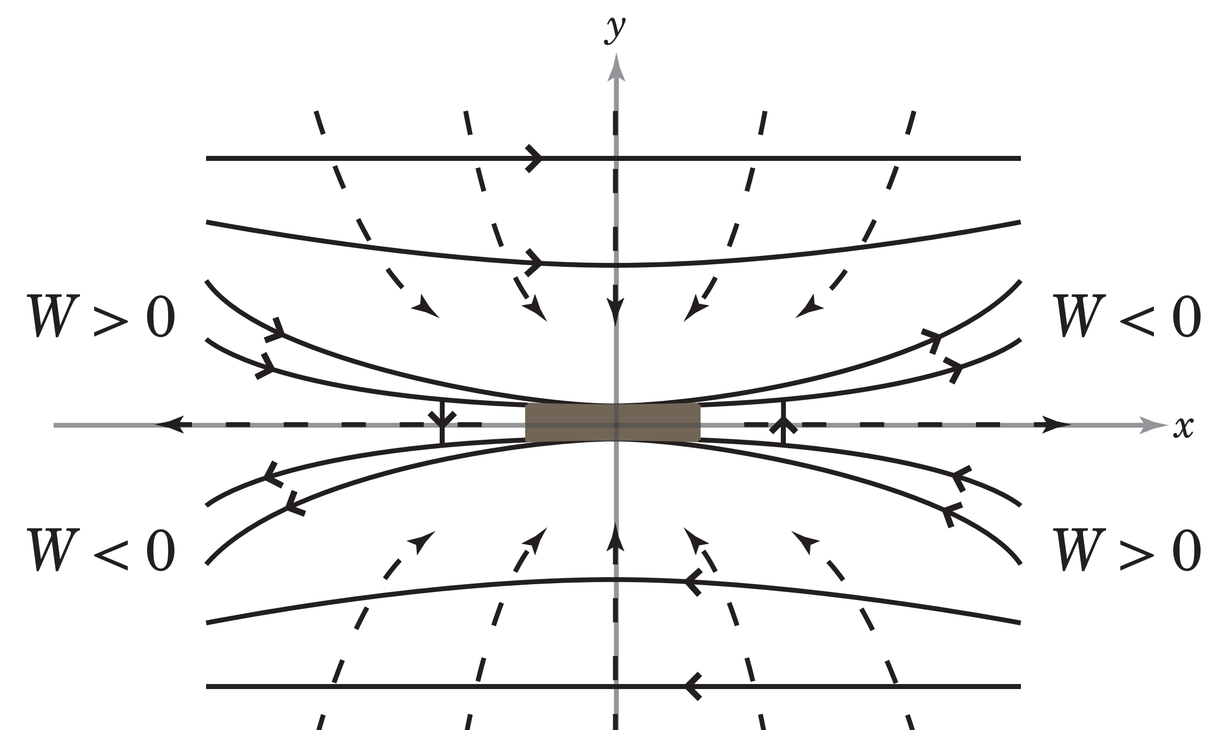








$W = 0$
y
$W > 0$
$W < 0$
$\mathbf{\Omega}_0$
$\delta\mathbf{U}$
$\mathbf{J}$
$W = 0$
$\mathbf{J}$
x
$W = 0$
$\mathbf{J}$
$\mathbf{\Omega}_0$
$\delta\mathbf{U}$
$W < 0$
$W > 0$
$W = 0$

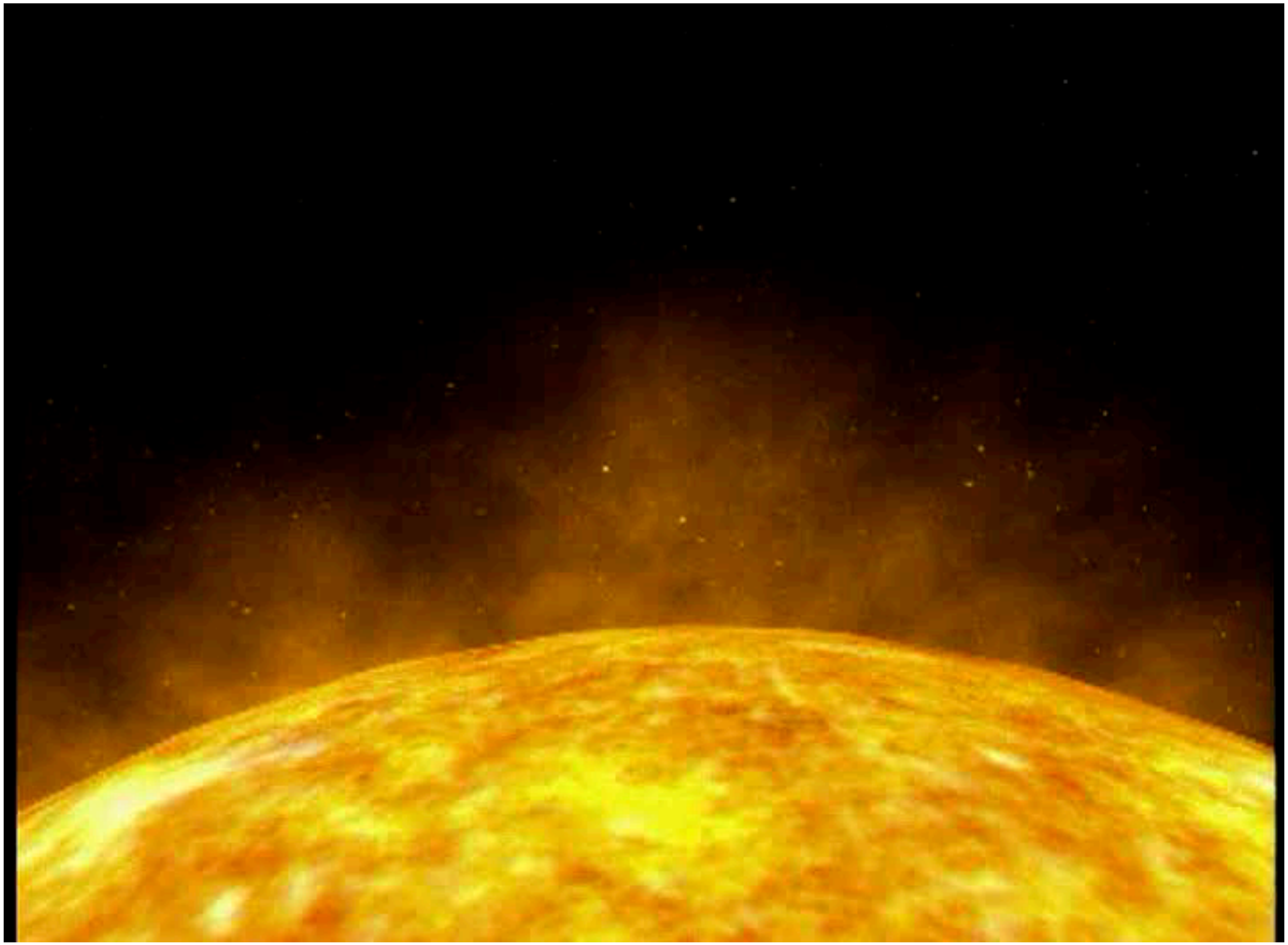


y
$W > 0$
$W < 0$
x
$W < 0$
$W > 0$

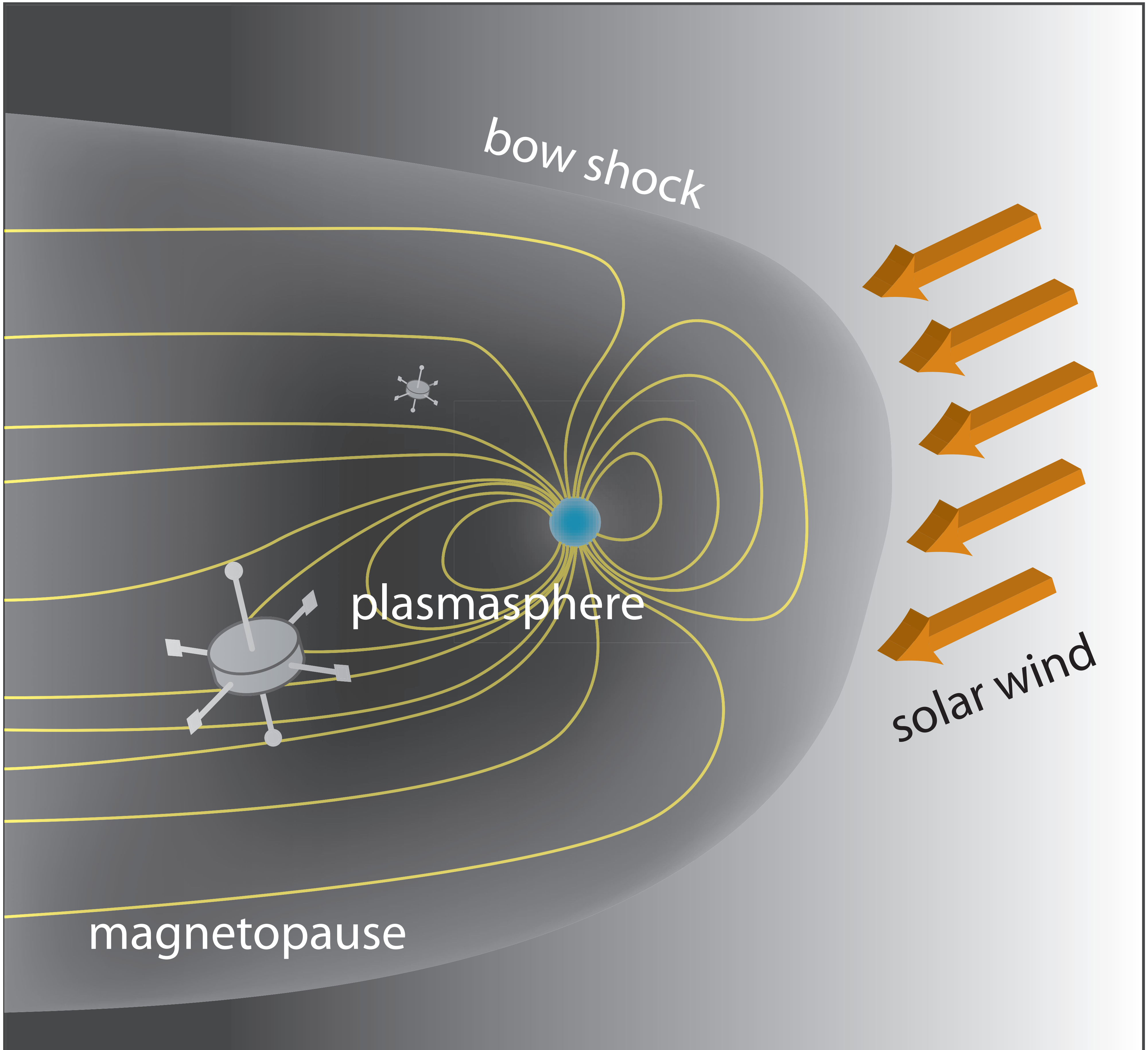
Reconnection

- Turbulent magnetic diffusivity enhances the reconnection rate
- Turbulent cross helicity work for suppressing transport, changing mean-field configuration
- Configuration favorable for fast reconnection
- Combination of the spatial distributions of turbulent magnetic diffusivity and cross helicity
- Turbulent reconnection is confined to a very narrow region where only the turbulent magnetic diffusivity is dominant

Reconnection
in solar flare and geomagnetic substorm



(SOHO animation)


bow shock
plasmasphere
solar wind
magnetopause

Summary

- Large-scale inhomogeneity
- Breakage of symmetry in turbulence
- Dynamo:
  Large-scale vorticity + turbulent cross helicity
- Momentum transport suppression:
  Large-scale magnetic shear + turbulent cross helicity
- Cross-helicity generation mechanisms
- DNS of Kolmogorov flow
- Numerical simulations and observations in progress