ISM Turbulence: Effects of Compressibility and Gravity

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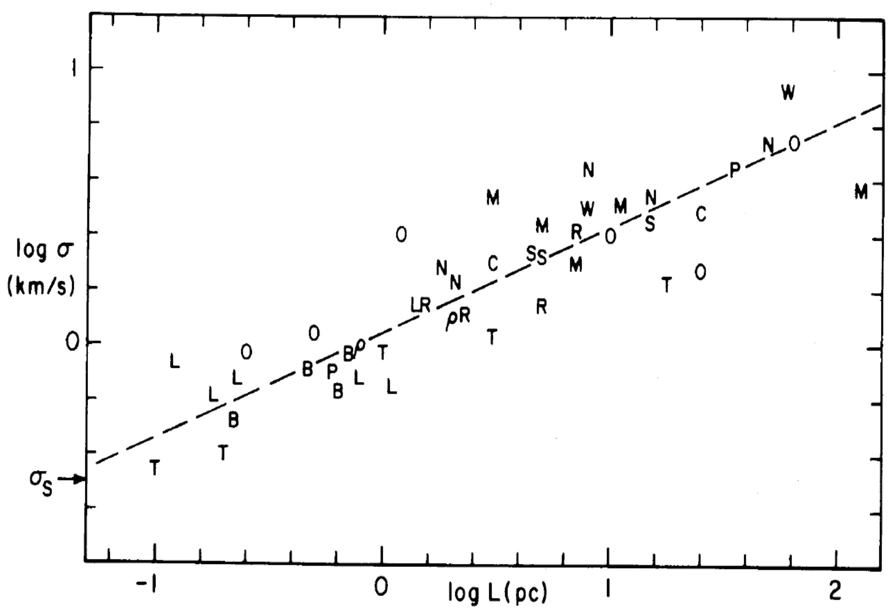
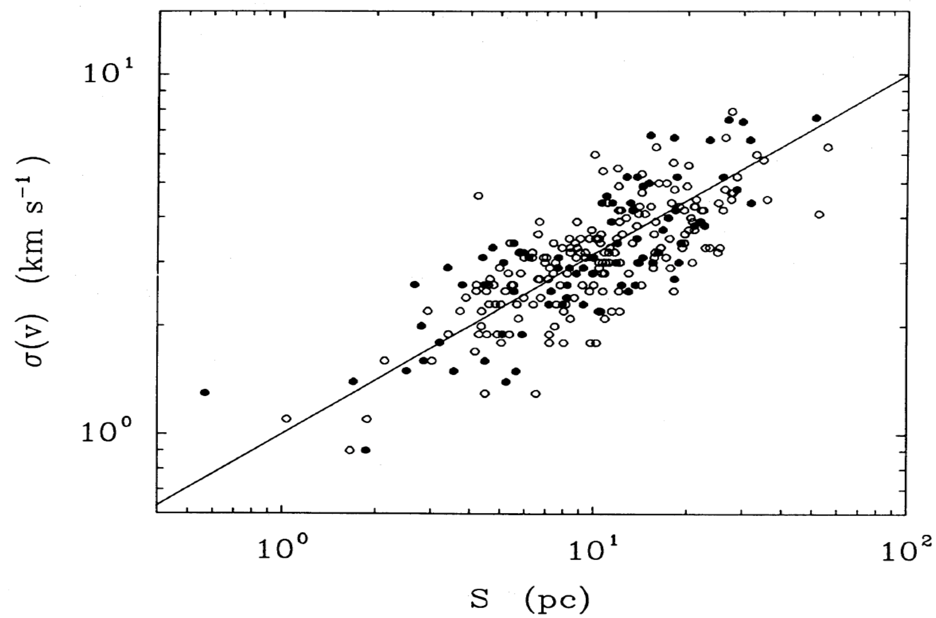
Outline

- Motivation: Do we interpret Larson's scaling laws correctly?
- Observations of molecular clouds vs. supersonic turbulence models.
- Star-forming molecular clouds vs. adaptive mesh refinement simulations with self-gravity.
- Further reading: ApJ 727, L20 (2011); arXiv:1111.2827 (2011).

Motivation: Interpretations differ

Linewidth–size relation for molecular clouds



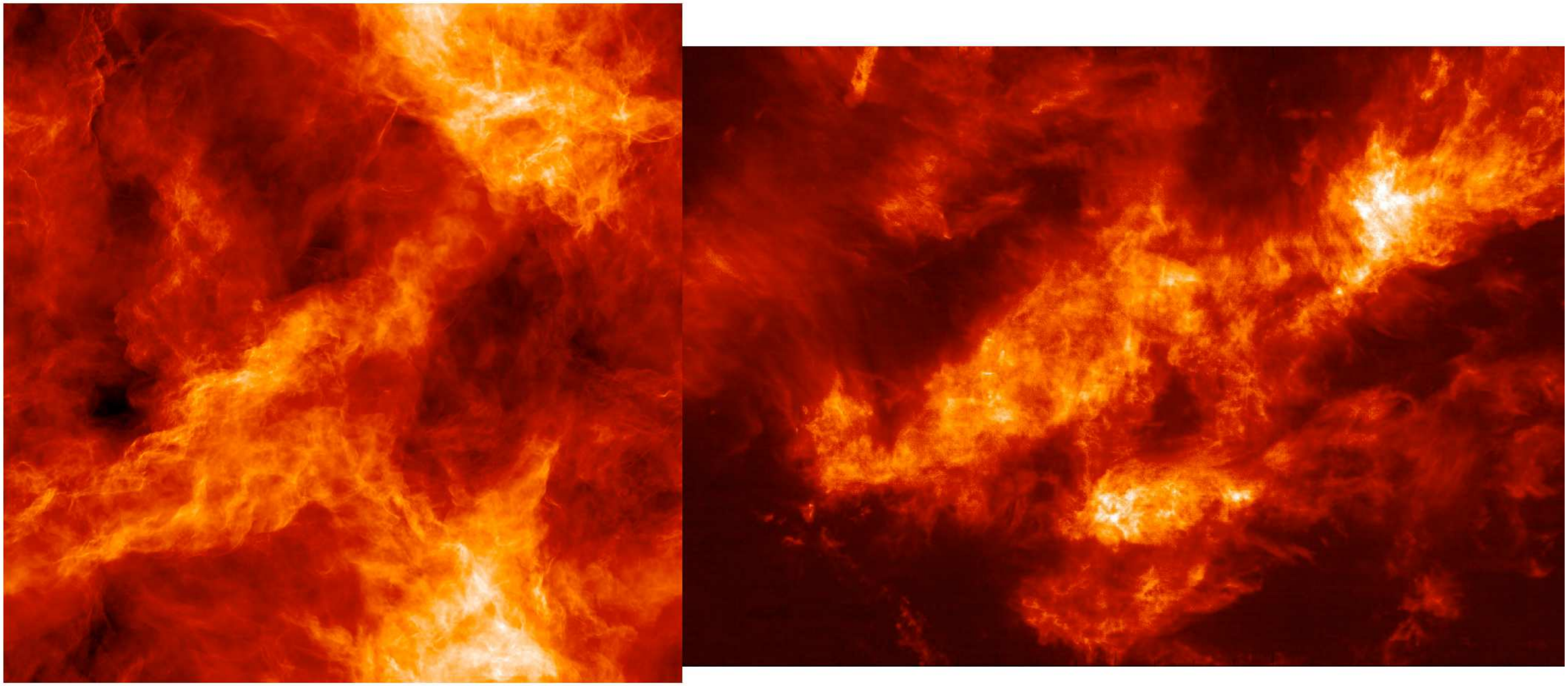


- **Left:** Larson (1981): $\sigma_u = 1.10 L^{0.38}$ km s^{-1}.
- Observed nonthermal linewidths originate from a **common hierarchy of interstellar turbulent motions**. Structures cannot have formed by simple gravitational collapse.
- **Right:** Solomon et al. (1987): $\sigma_u = (1.0 \pm 0.1) S^{0.5 \pm 0.05}$ km s^{-1}.
- The size-linewidth relation arises from **virial equilibrium**, $\sigma_u = (\pi G \Sigma)^{1/2} R^{1/2}$. MCs are in or near virial equilibrium since their mass determined dynamically agrees with other independent measurements. MCs are not in pressure equilibrium with warm/hot ISM.

I. Supersonic turbulence, no gravity

Column density maps

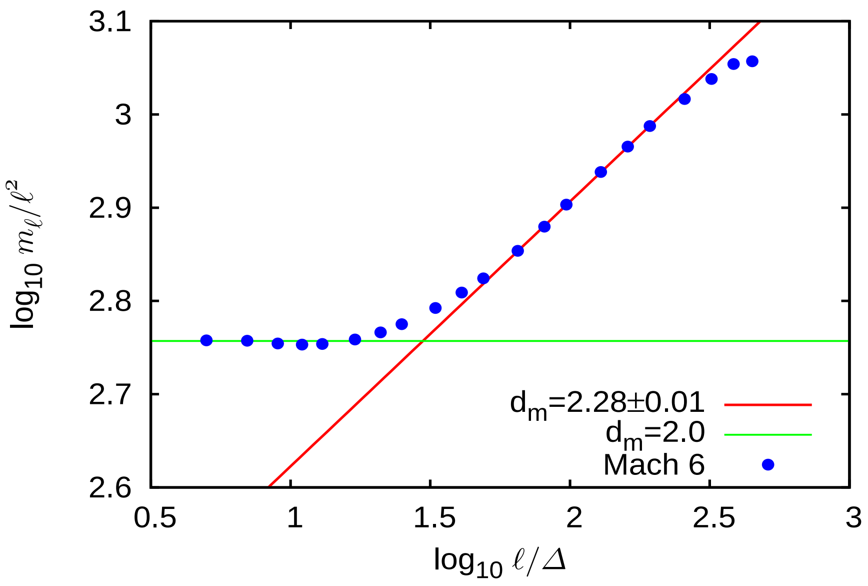
Density structures are overall morphologically similar, but...

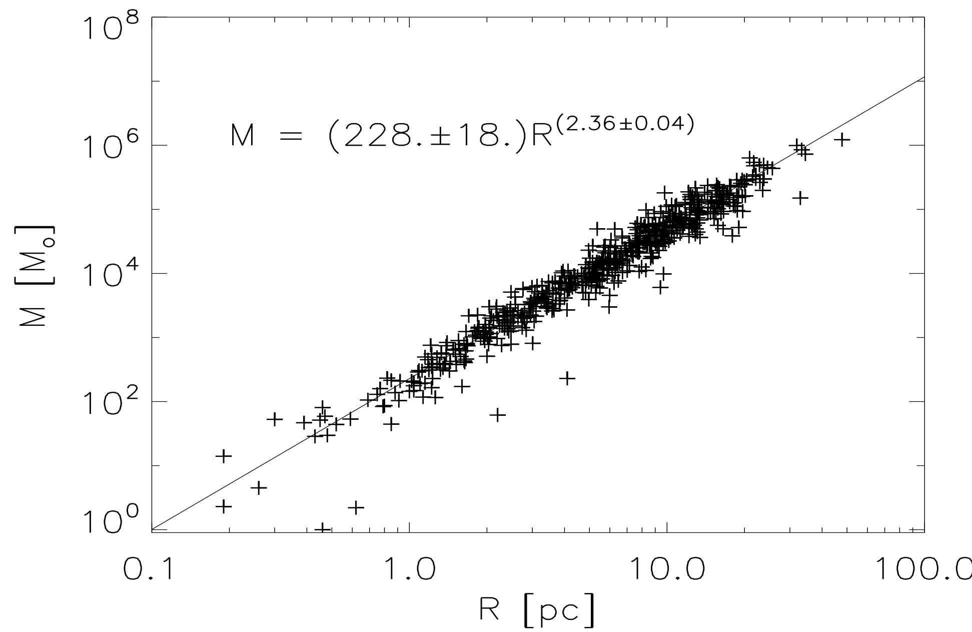


- **Left:** 2048^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2009].
- **Right:** ^{12}CO$(1-0)$ map of Taurus MC [Goldsmith et al. 2008].
- "Striations" are missing in HD simulations. Trans-Alfvénic turbulence in low-density gas.
- ^{12}CO is "blind" with respect to dense star-forming filaments, where gravity plays a role.

Mass–size relation

$m_\ell = m_0(\ell/\ell_0)^{d_\mathrm{m}}$, where d_m is the mass dimension

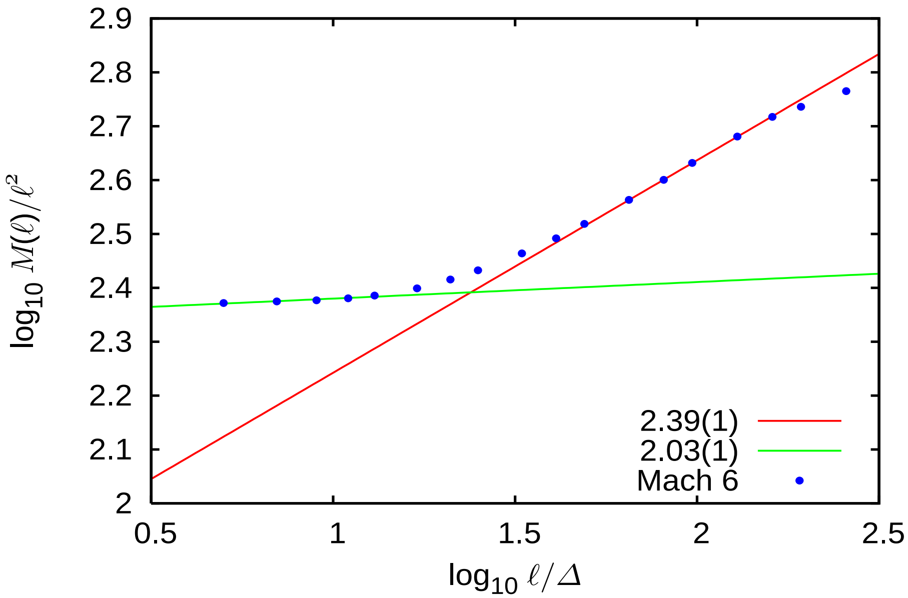


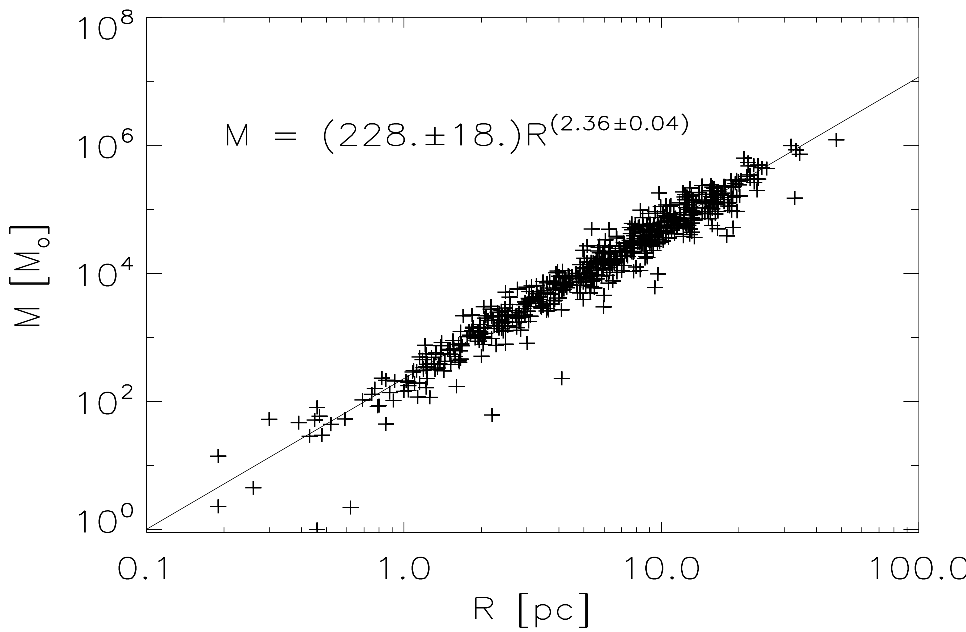


- **Left:** 2048^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2009].
- Solenoidal forcing: $d_\mathrm{m} = 2.28 \pm 0.01$ in the inertial range.
- On small scales: $d_\mathrm{m} \approx 2$ – shock fronts.
- **Right:** Mass–size relation for 580 MCs: $d_\mathrm{m} = 2.36 \pm 0.04$ [Roman-Duval et al., 2010], see also earlier papers by Falgarone & Phillips (1991); Elmegreen & Falgarone (1996).
- **Mass dimensions are similar.**

Mass–size relation

$m_\ell = m_0(\ell/\ell_0)^{d_\mathrm{m}}$, where d_m is the mass dimension



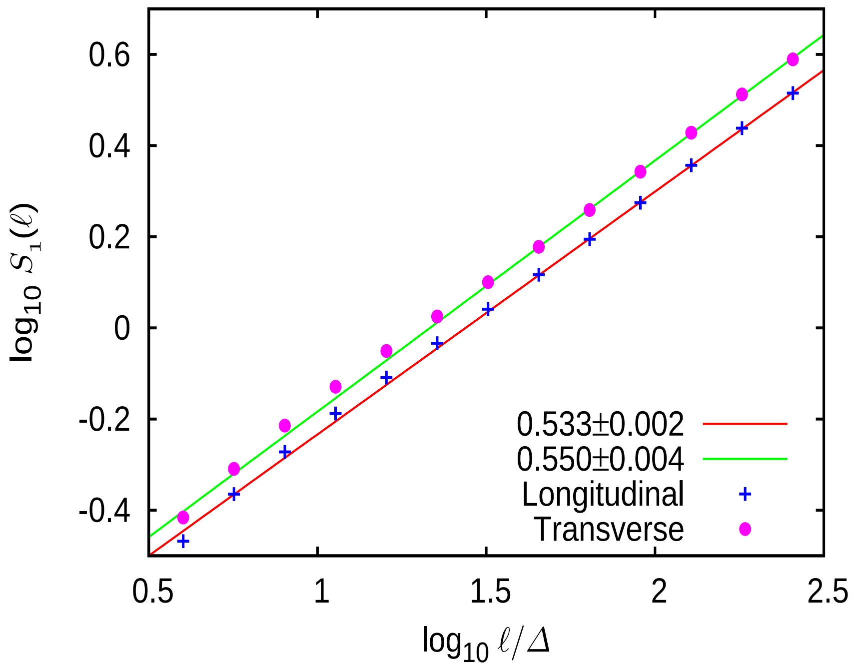


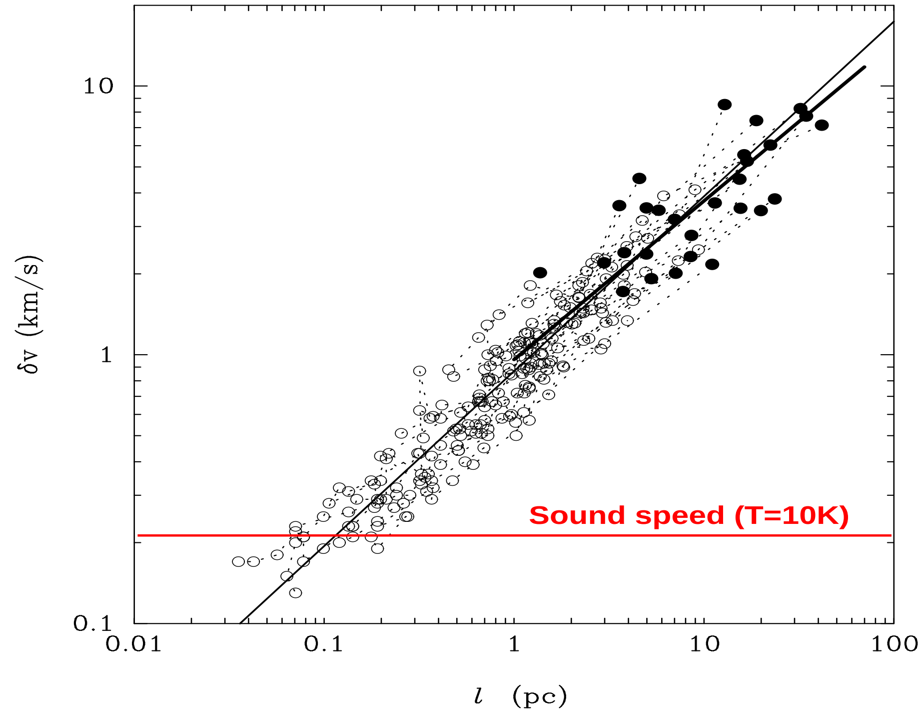
- **Left:** 1024^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2007].
- Natural forcing: $d_\mathrm{m} = 2.39 \pm 0.01$ in the inertial range.
- On small scales: $d_\mathrm{m} \approx 2$ – shock fronts.
- **Right:** Mass–size relation for 580 MCs: $d_\mathrm{m} = 2.36 \pm 0.04$ [Roman-Duval et al., 2010], see also earlier papers by Falgarone & Phillips (1991); Elmegreen & Falgarone (1996).
- **Mass dimensions are similar.**

Linewidth–size relation

First-order structure functions of velocity:

$$S_1(u,\ell) \equiv \langle|\delta u|\rangle = u_0 \ell^{\zeta_1}$$



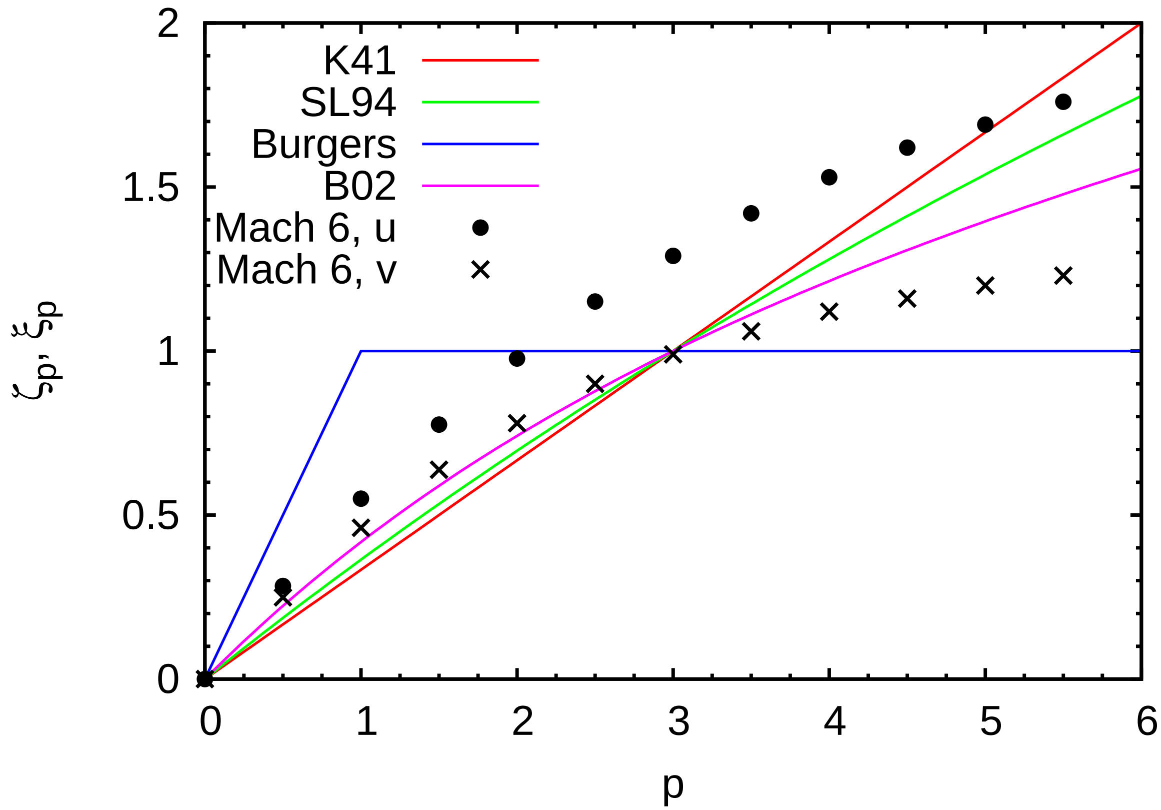


- **Left:** 1024^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2007].
- Simulation: $\zeta_1 = 0.54 \pm 0.01$.
- **Right:** A sample of 27 GMCs (including substructure) [Heyer & Brunt, 2004].
- Observation: $\zeta_1 = 0.56 \pm 0.02$; u_0 is "universal."
- **First-order velocity SFs have similar slopes.**

Universality in compressible turbulence?

Third-order structure functions of velocity do not scale linearly:

$$S_3(u,\ell) \propto \ell^{1.3}$$



- 1024^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2007].
- Density-weighted velocity: $v \equiv \rho^{1/3} u$ Total energy is conserved: $E = \rho u^2/2 + c_s^2 \rho \ln \rho$
- Linear scaling: $S_3(v,\ell) \propto \ell^1$ independent of the Mach number.
- **Density-weighted velocity is a good candidate for universal behavior.**

Two ways to determine $\Sigma_\ell \propto \ell^?$ based on dimensional analysis:

1. Using the mass-size relation:
   - Assume $d_{\rm m} = 2.36 \pm 0.04$ [Roman-Duval et al., 2010]
   - Then $\Sigma_\ell \propto m_\ell \ell^{-2} \propto \ell^{d_{\rm m}-2} \propto \ell^{0.36 \pm 0.04}$
2. Using the cascade concept:
   - Assume $\rho_\ell (\delta u_\ell)^3 \ell^{-1} \propto \Sigma_\ell (\delta u_\ell)^3 \ell^{-2} \propto \Sigma_\ell \ell^{3\zeta_1 - 2} \propto const$
   - Assume $\zeta_1 = 0.56 \pm 0.02$ [Heyer & Brunt, 2004]
   - Then $\Sigma_\ell \propto \ell^{2-3\zeta_1} \propto \ell^{0.32 \pm 0.06}$

Linewidth–size and mass–size relations both give $\Sigma_\ell \propto \ell^{1/3}$

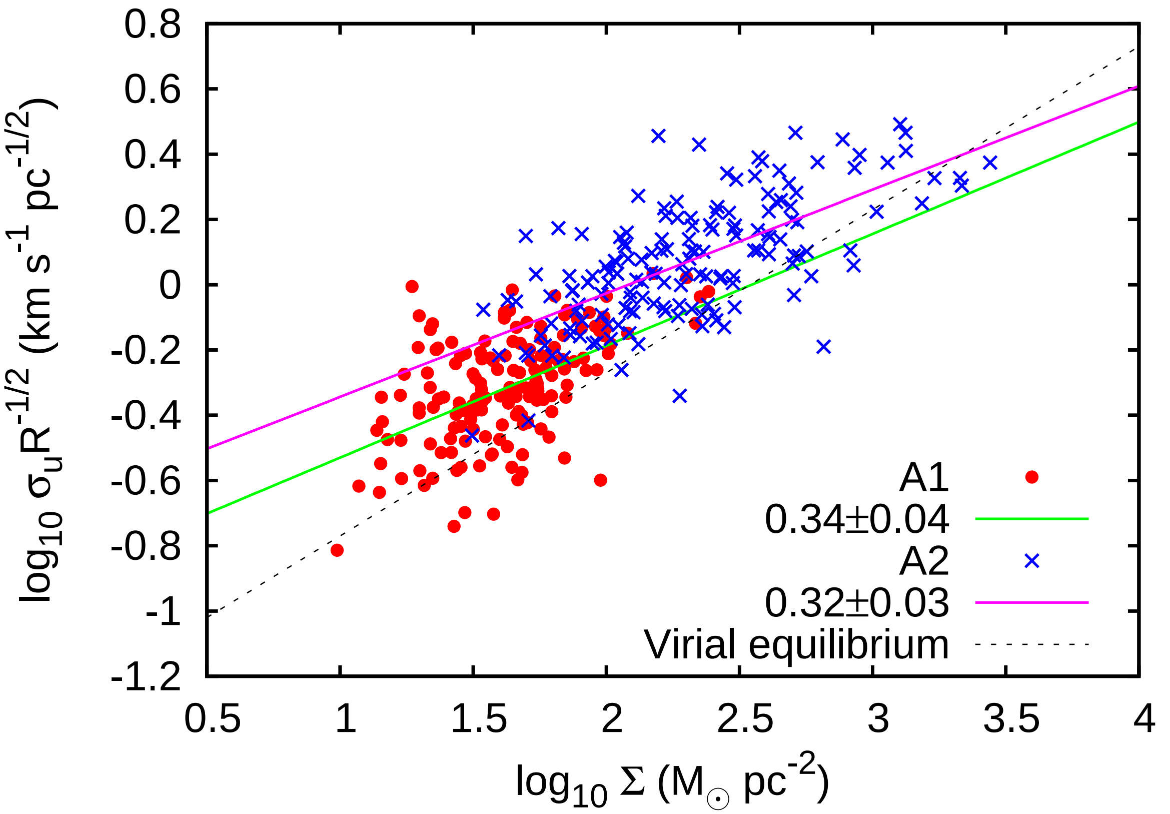
Linewidth–size scaling coefficient vs. column density

Find $u_0' \equiv \delta u_\ell \ell^{-1/2} \propto \Sigma_\ell^?$ based on dimensional analysis:

- Assume $S_1(v, \ell) = \langle|\delta v_\ell|\rangle \sim \langle\epsilon_\ell^{1/3}\rangle \ell^{1/3}$, where $v \equiv \rho^{1/3} u$
- Intermittency $\langle\epsilon_\ell^{1/3}\rangle \sim \ell^{\tau_{1/3}}$, where $\tau_{1/3} \approx 0.055$ [Pan et al. 2009]
- Then $\delta u_\ell \ell^{-1/2} \propto \rho_\ell^{-1/3} \ell^{-1/6+\tau_{1/3}} \propto \Sigma_\ell^{-1/3} \ell^{1/6+\tau_{1/3}}$
- We know that $\Sigma_\ell \propto \ell^{1/3}$
- Therefore $\delta u_\ell \ell^{-1/2} \propto \Sigma^{1/6+3\tau_{1/3}} \propto \Sigma^{0.33}$

Linewidth–size scaling coefficient $u_0' \propto \Sigma^{1/3}$

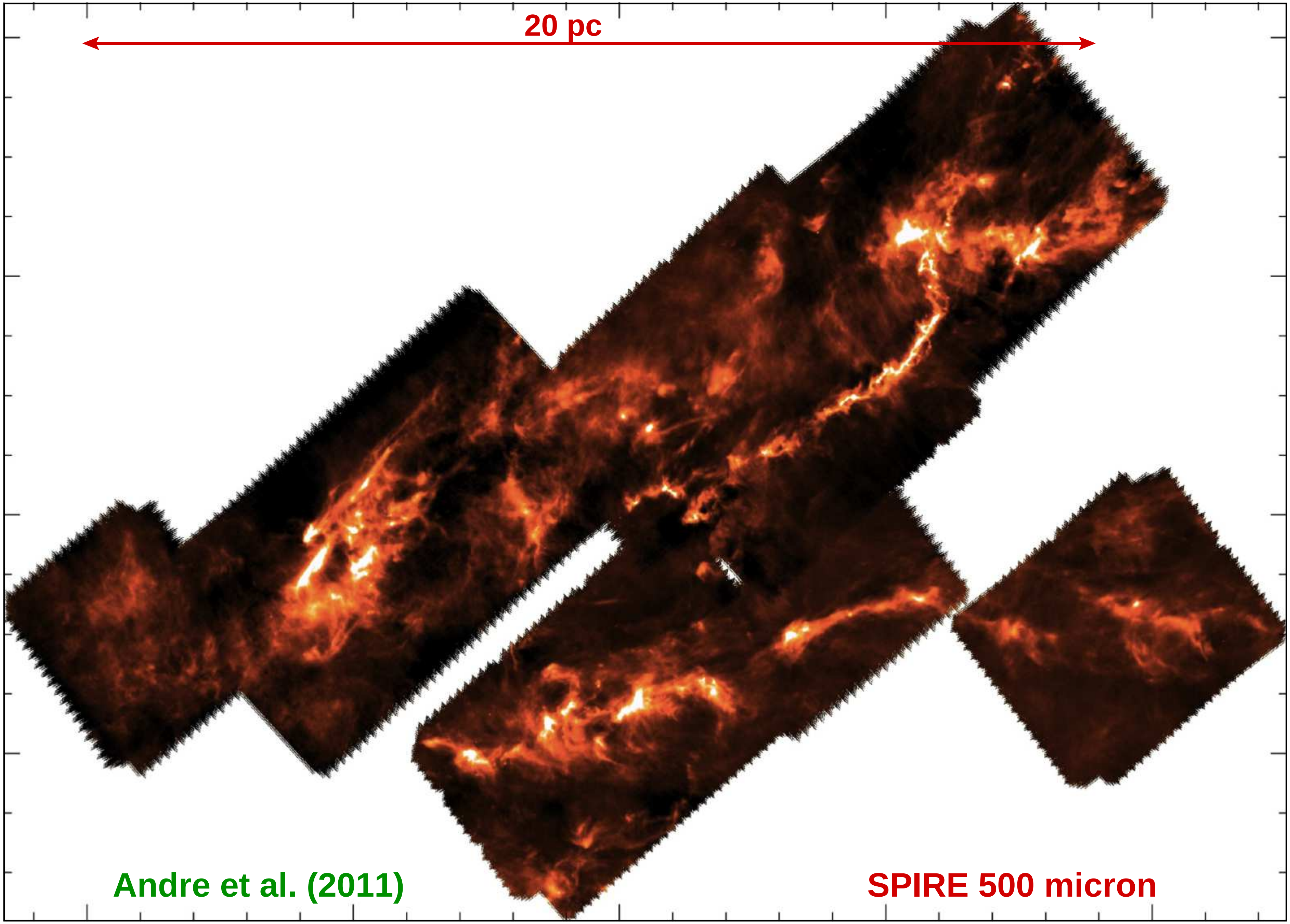
The observed $u_0' - \Sigma$ relation



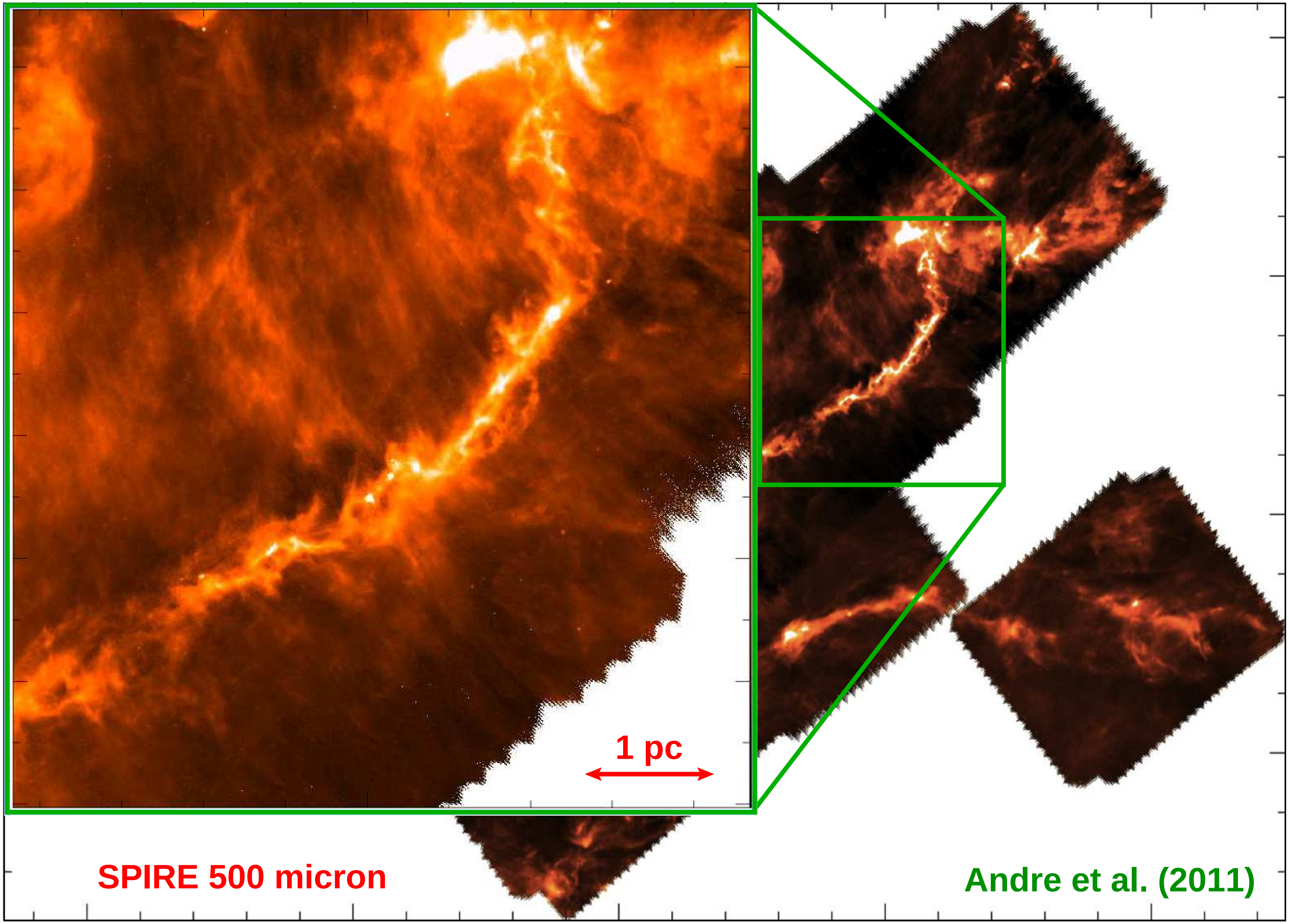
- ^{13}CO observations of 162 MCs with improved angular/spectral resolution [Heyer et al. 2009].
- Solid lines w. slopes 0.34 ± 0.04 and 0.32 ± 0.03 show least square fits to A1 and A2 subsets.
- Dashed line shows the correlation expected for clouds in virial equilibrium, $\sigma_u R^{-1/2} \propto \Sigma^{1/2}$.
- **Observed scaling is reasonably close to our prediction.**

II. Compressible turbulence + gravity

Herschel Gould Belt Survey: Taurus

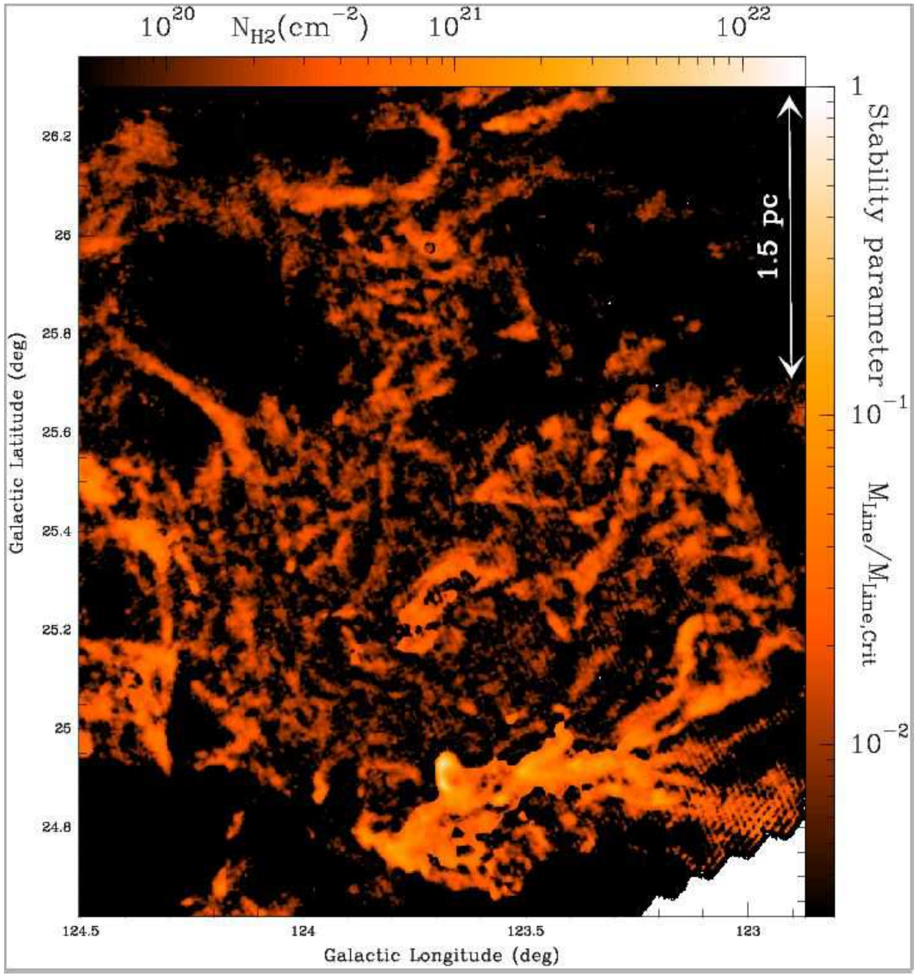


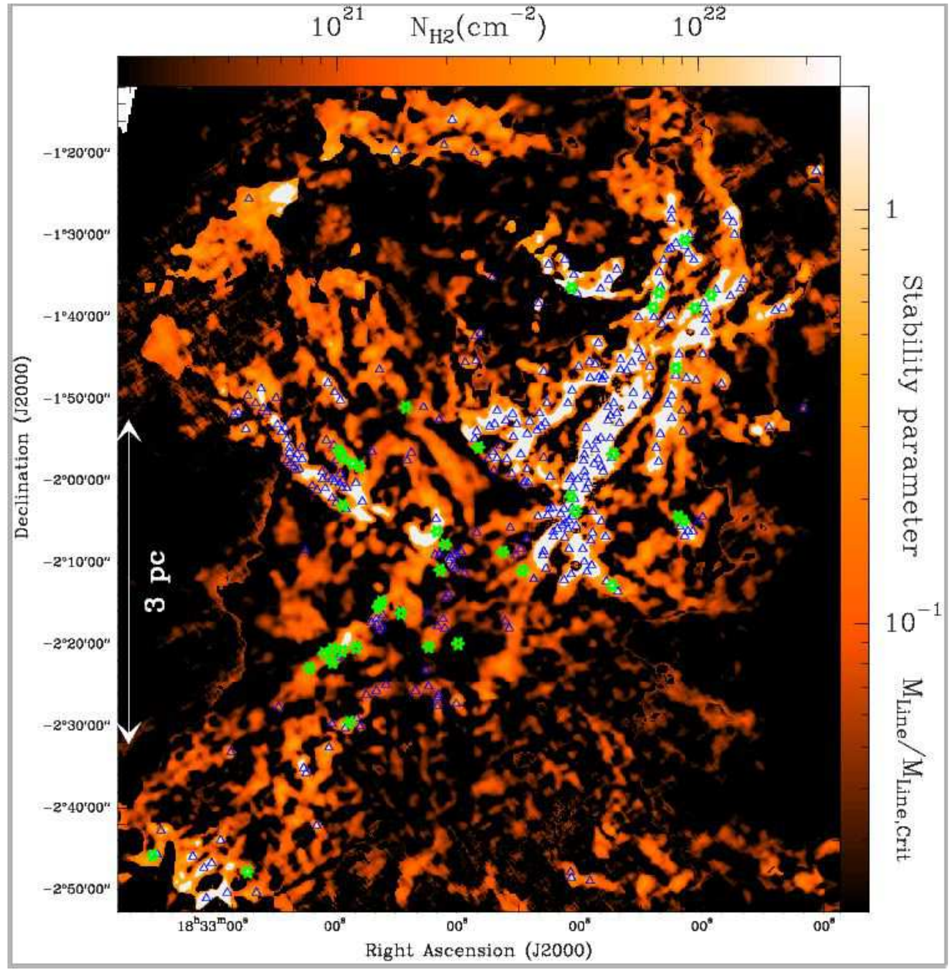
Herschel: Taurus B213 filament



Column density maps of fields in Polaris and Aquila

Herschel-SPIRE/PACS: 70, 160, 250, 500μm [André et al. 2010]



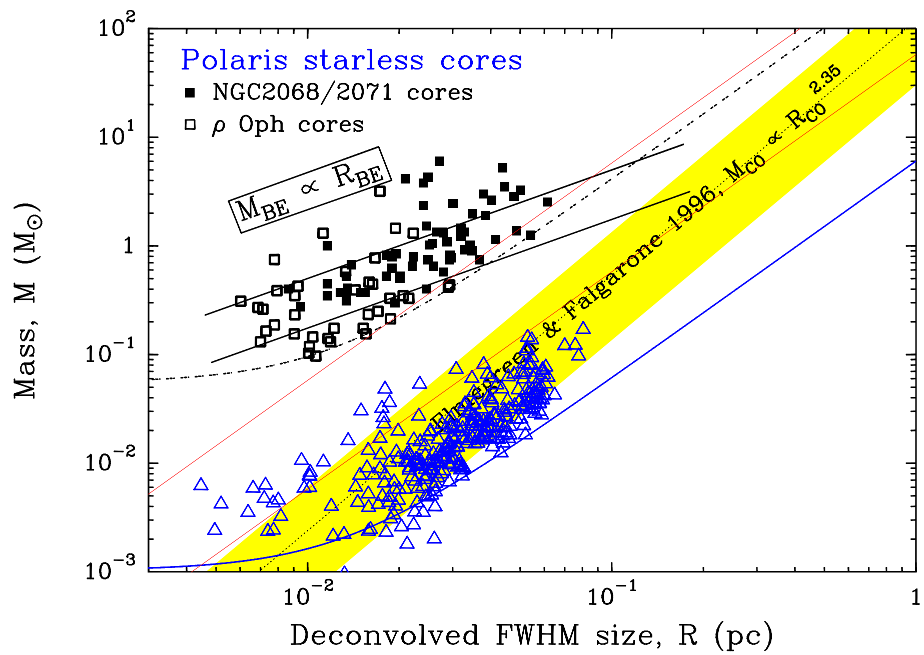


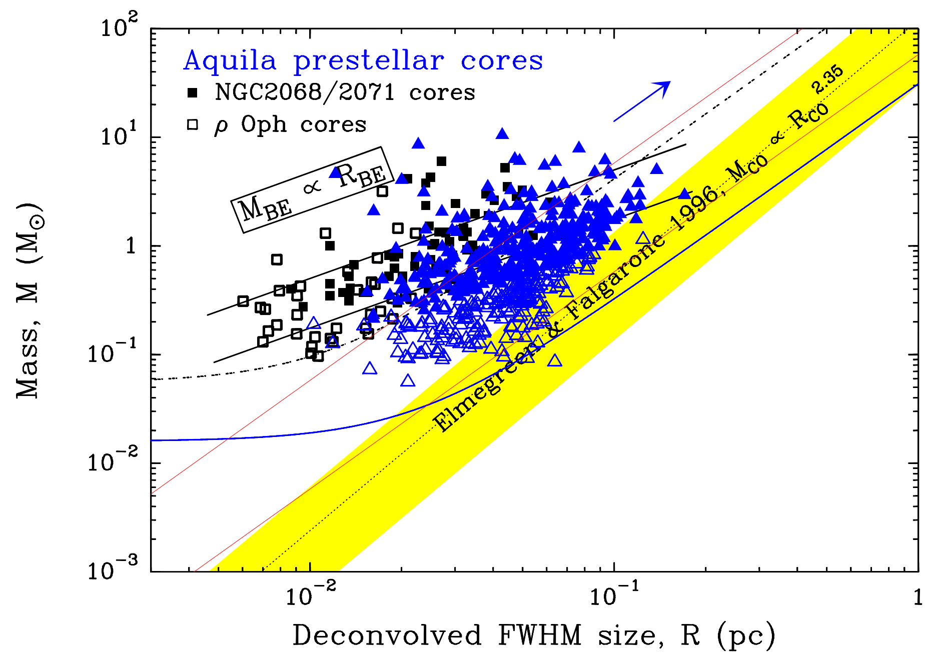
- **Left:** A subfield in **Polaris Flare**. Curvlet transform used to enhance contrast.
- **Right:** A subfield in **Aquila**: ★ – Class0 protostars; △ – prestellar cores.
- Stability analysis by Inutsuka & Miyama (1997) ⇒ **Polaris** filments are stable.

Mass–size diagrams for starless cores

Herschel-SPIRE/PACS: 70, 160, 250, 500μm

André et al. (2010); Könyves et al. (2010); Men'shchikov et al. (2010)



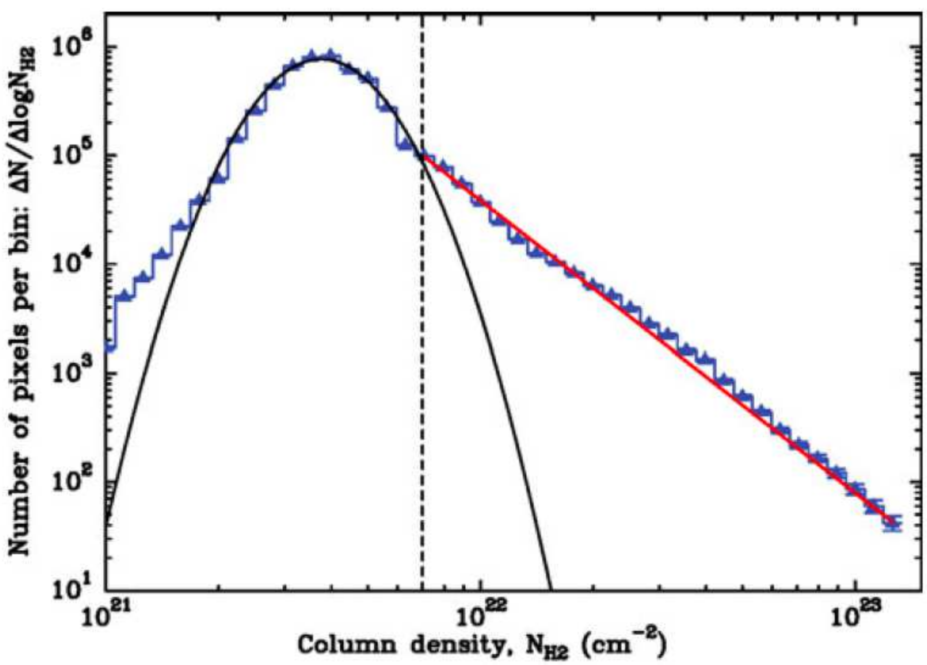


- **Left:** 302 starless cores of **Polaris** are unbound.
- **Right:** Out of 541 starless cores in **Aquila** 341 were classified as prestellar (▲).

Column density PDF & starless cores in Aquila

If nonlinear gravitational instability indeed settles on stable attractive similarity solutions for gravitational collapse, then the high-end density PDF and the mass–size correlation for prestellar cores are related.

- Density distribution: $\rho(r) \propto r^{-n} \Rightarrow$ Column density: $\Sigma(R) \propto R^{1-n}$
- Density PDF: $dV(\rho)/d\rho \propto \rho^m \Rightarrow$ Column density PDF: $dS(\Sigma)/d\Sigma \propto \Sigma^p$
- $n = 12/7$ for the PF solution [Penston 1969] $\Rightarrow m = -3/n = 7/4 = 1.75$ and $p = -2.8$
- $n = 2$ for the LP solution [Larson-Penston 1969] $\Rightarrow m = -1.5$ and $p = -2/(n-1) = -2$



High-end PDF power index:

$p = -2.7 \pm 0.1$

[André et al. 2011]

Mass–size relation based on the PDF:

$d_{\rm m} = 3 - n = 2(1 + 1/p) \approx 1.26$

Mass–size relation for 541 starless cores:

$m_\ell \propto \ell^{1.13 \pm 0.07}$

[André et al. 2010]

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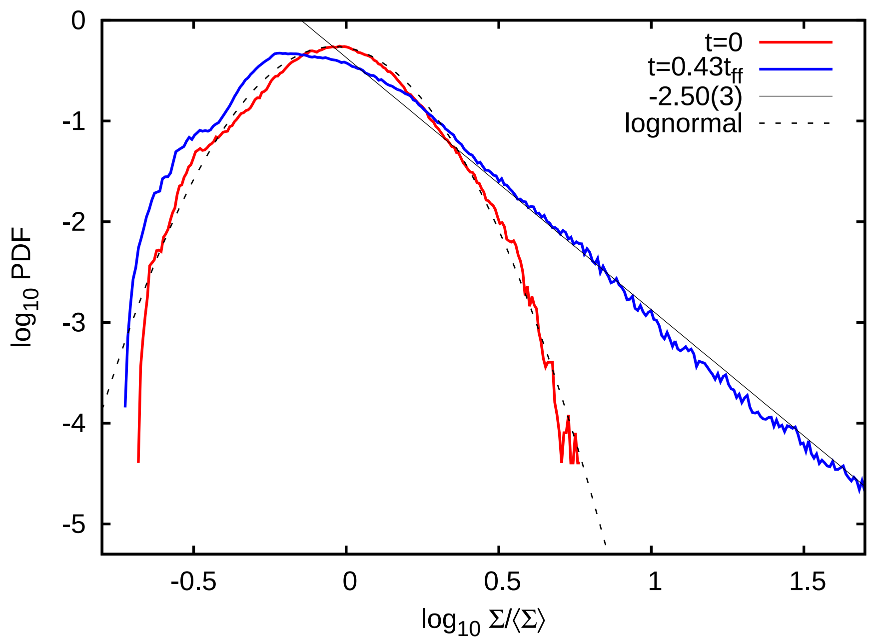


High-end PDF power index:

$p = -2.50 \pm 0.03$

[Kritsuk et al. 2011]

Mass–size relation based on the PDF:

$d_{\rm m} = 2(1 + 1/p) \approx 1.20 \pm 0.01$

Mass–size relation for 541 starless cores:

$m_\ell \propto \ell^{1.13 \pm 0.07}$

[André et al. 2010]

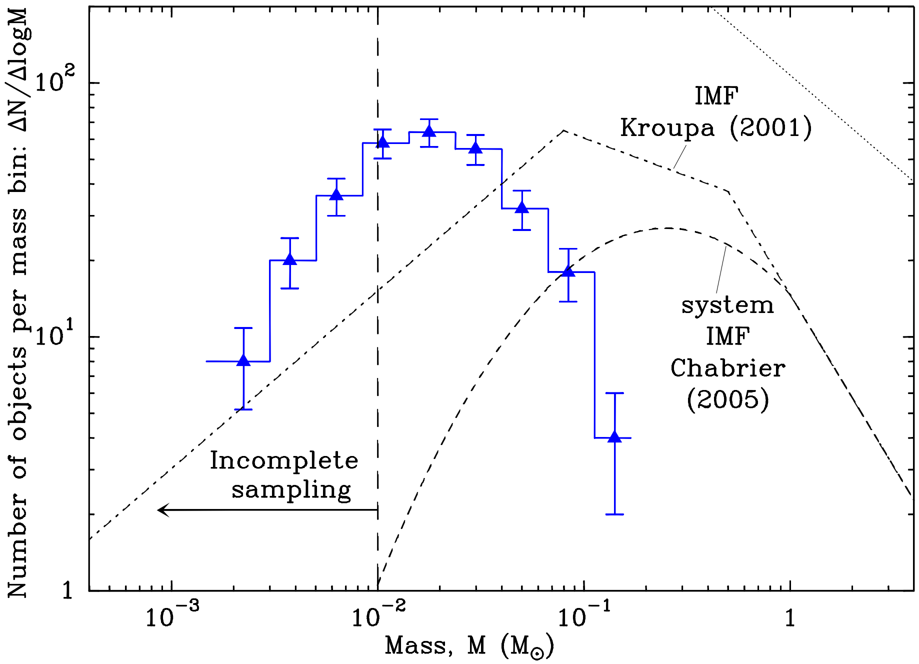
Jeans mass as a function of scale

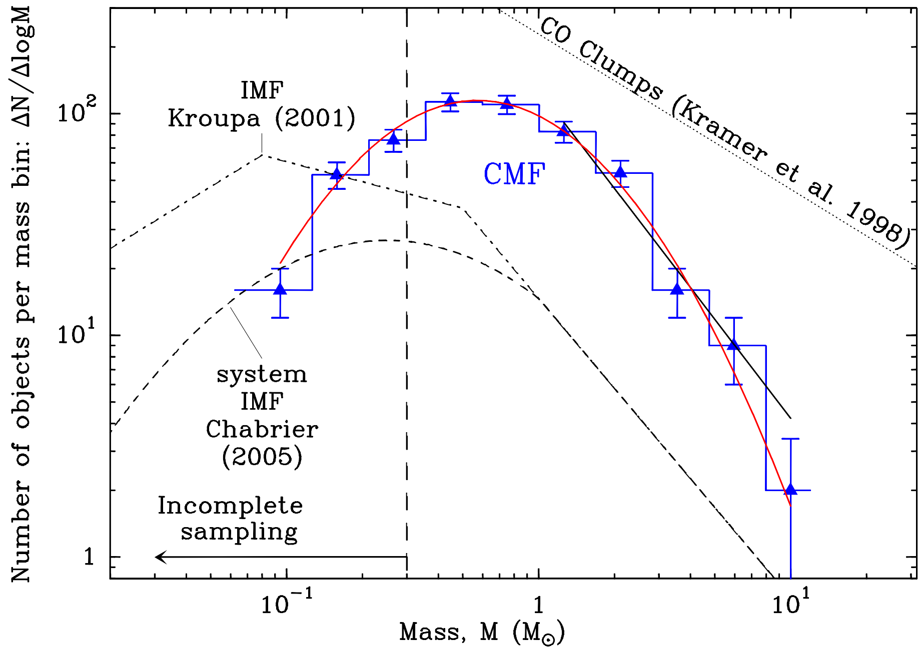
- Jeans mass $m^J_\ell \propto \sigma^3_\ell \rho^{-1/2}_\ell$, where $\sigma^2_\ell \equiv \delta u^2_\ell + c^2_{\rm s}$ [Chandrasekhar 1951].
- Sonic scale $\ell_{\rm s}$ such that $\delta u_{\ell_{\rm s}} = c_{\rm s}$.
- If $\ell \lesssim \ell_{\rm s}$ then $\sigma^2_\ell \approx c^2_{\rm s}$ and $m^J_\ell \propto \rho^{-1/2}_\ell \propto \ell^{(3-d_{\rm m})/2} \propto \ell^{0.32}$.
- If $\ell \gtrsim \ell_{\rm s}$ then $\sigma^2_\ell \approx \delta u^2_\ell$ and
  $m^J_\ell \propto \delta u^3_\ell \rho^{-1/2}_\ell \propto \delta v^3_\ell \rho^{-3/2}_\ell \propto \ell^{1+3(3-d_{\rm m})/2} \propto \ell^{1.96}$.
- A small change in σ_ℓ scaling creates a big break in Jeans mass scaling!
- Because of that, the ratio $\mu_\ell \equiv m_\ell / m^J_\ell$ changes its slope at $\ell_{\rm s}$:
- $\mu_\ell \propto \ell^{3(d_{\rm m}-1)/2} \propto \ell^{2.04}$ for $\ell \lesssim \ell_{\rm s}$;
- $\mu_\ell \propto \ell^{(5d_{\rm m}-11)/2} \propto \ell^{0.4}$ for $\ell \gtrsim \ell_{\rm s}$.
- This creates a range of Jeans-unstable structures ($\mu_\ell > 1$) above $\ell_{\rm s}$.
- $\ell_{\rm s} \sim 0.1$ pc sets the characteristic mass for the CMF, if $\mu_{\ell_{\rm s}} > 1$.

Core Mass Function for Polaris & Aquila

Herschel-SPIRE/PACS: 70, 160, 250, 500μm

André et al. (2010)





- **Left:** CMF for 302 starless cores of **Polaris**.
- **Right:** CMF for 541 starless cores in **Aquila**.
- A lognormal fit and a power law fit with a slope of -1.5 ± 0.2.
- Compare to Salpeter slope of -1.35 in the $\mathrm{d}N/\mathrm{d}\log M$ format.

Summary

- Larson's scaling relations are controlled by interstellar turbulence on scales $0.1-50$ pc.
- Gravity can be important in GMCs with masses in excess of 10^4 $M_\odot$.
- In translucent clouds, self-similar turbulent scaling is preserved down to 10^{-3} pc.
- In overdense regions, self-gravity breaks the turbulence-induced scaling at the sonic scale.
- Prestellar cores form in a range of gravitational instability right above the sonic scale ~ 0.1 pc.

Acknowledgments

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