Magnetic Helicity and the Rapid Growth of Large Scale Magnetic Fields

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The Large Scale Dynamo

- The accumulation of magnetic energy in the largest scale modes of a system and their subsequent evolution

- In astrophysics we are concerned with the limit where the resistivity is very small. (The difference between zero resistivity and infinitesimal resistivity can be large!)

Dynamos

Consider the limit of small resistivity, and ignore plasma effects (a dense medium, or scales large enough that these effects are ignorable).

$$\partial_t \vec{B} = \nabla \times \left(\vec{V} \times \vec{B} - \eta \vec{J} \right)$$

$$D_t \bar{B} = \nabla \times \overline{\left(u \times b \right)} + \left(\bar{B} \square \nabla \right) \bar{V}$$

α ---- Ω   Dynamo

$$\overline{\langle u \times b \rangle} = \alpha_{ij} \bar{B}_j + \beta_{ijk} \partial_j \bar{B}_k + \dots$$

Topology 1 – Reconnection

- If the resistivity is exactly zero, then it can be shown that the topology of the magnetic field lines is invariant. "Large Scale" magnetic fields are intermittent and full of reversals on all scales.

- This difficulty goes away if reconnection is fast (occurs on dynamical time scales) even for infinitesimal resistivity.

- This effect is analogous to the way an infinitesimal viscosity in a turbulent medium destroys energy

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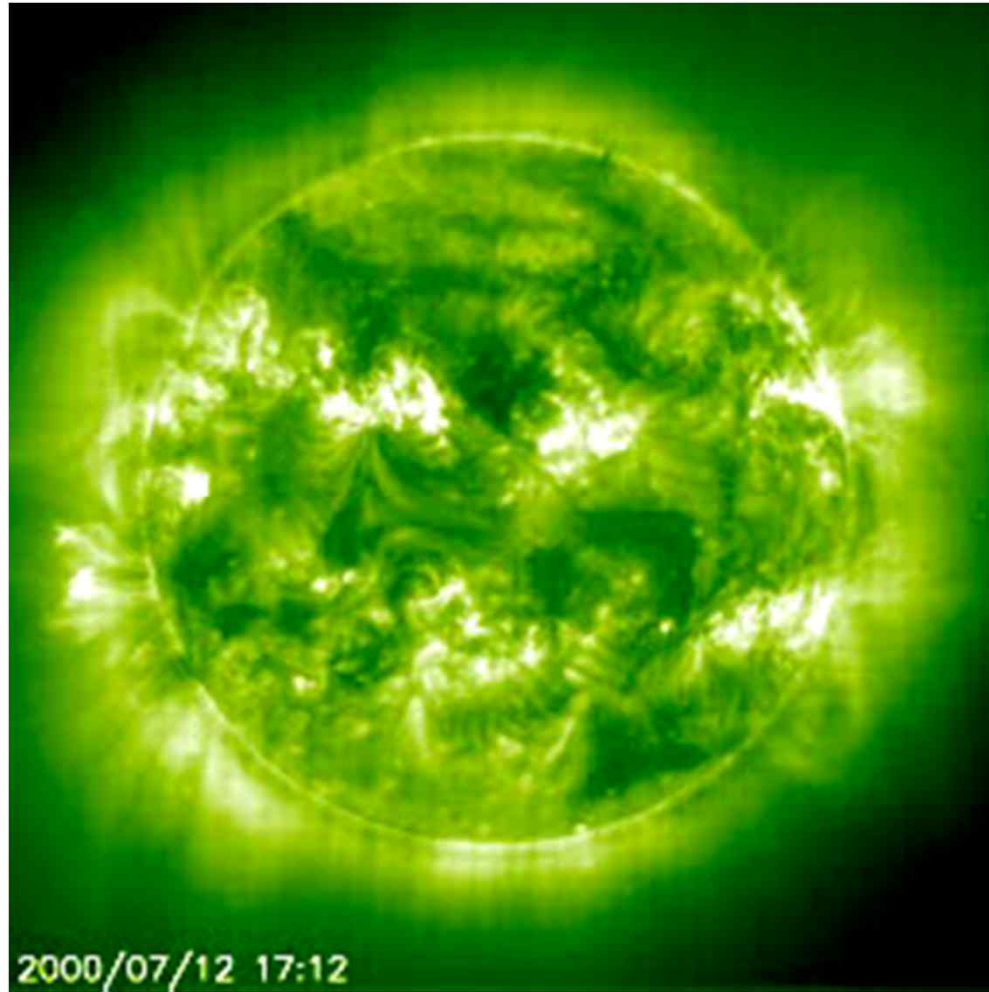
3D Turbulence and Current Sheets

- Without turbulence reconnection speeds are limited by current sheet thickness.
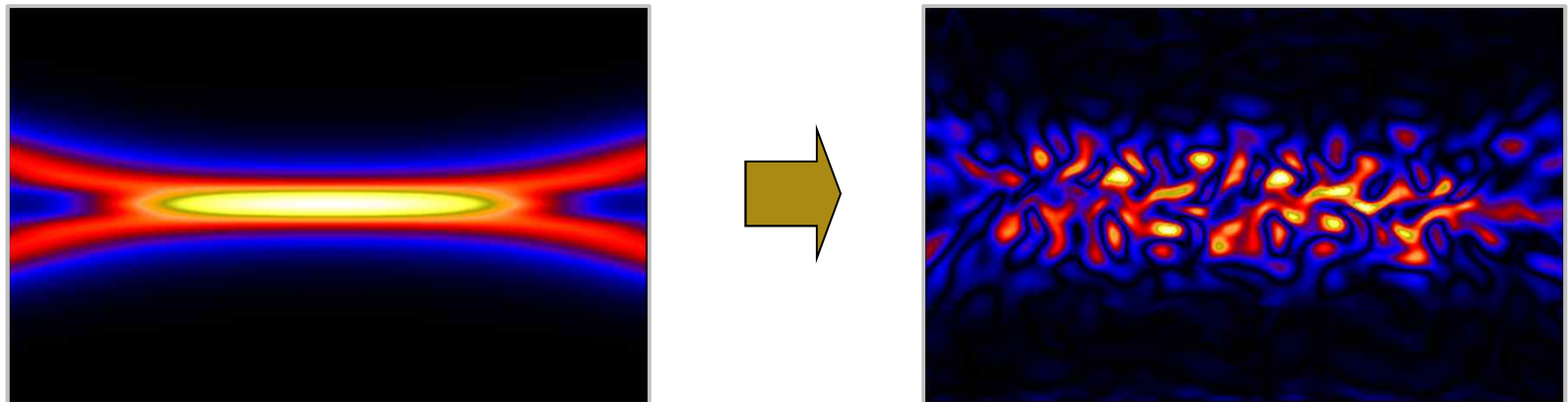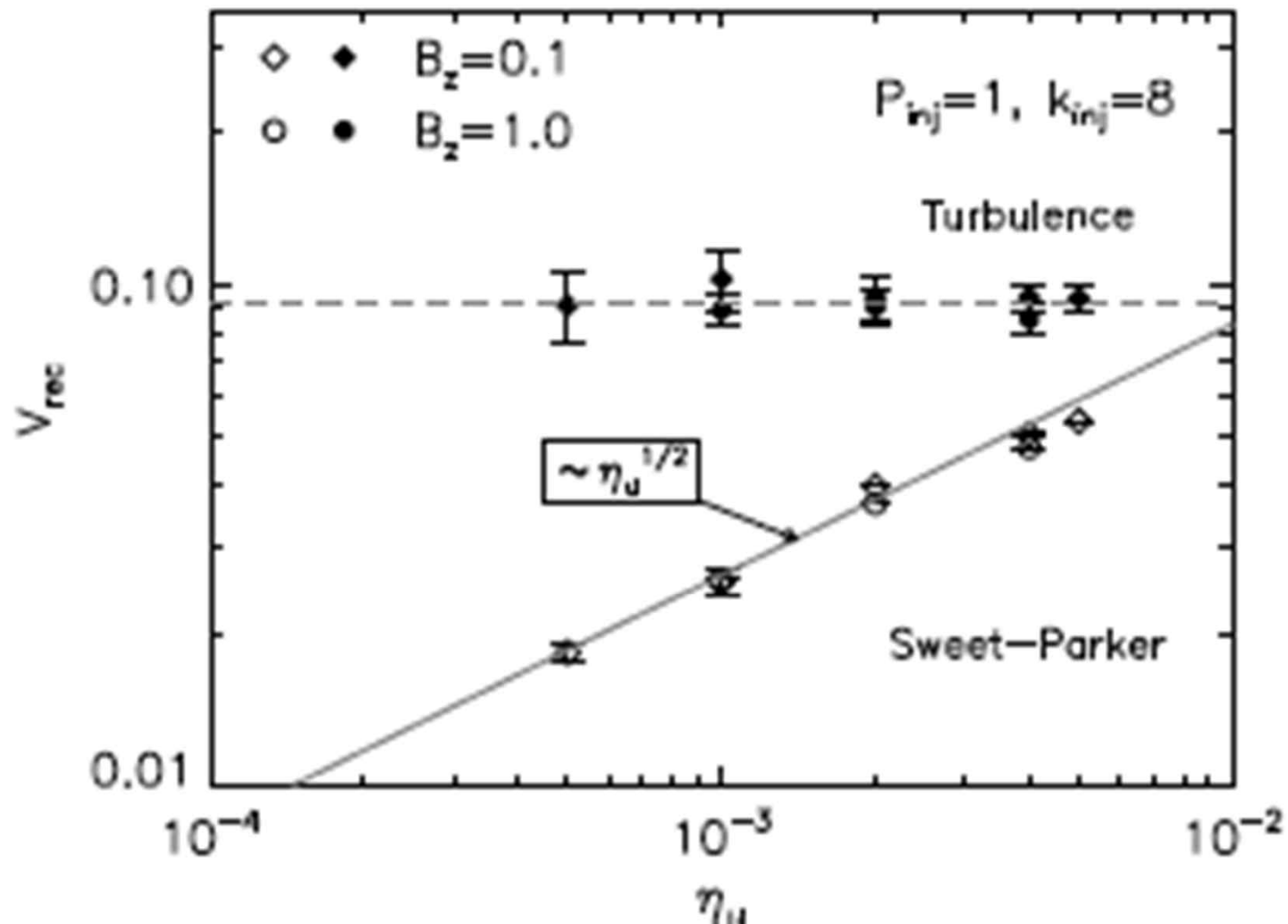- With turbulence …. (LV99, LEV2011)

Figure from Kowal et al. 2009

Reconnection speed at fixed injection power and scale with strong and weak guide fields

Topology II – What is the large scale electric field?

- Random fluctuations from individual eddies causes a random walk in large scale modes. This generates a long wavelength tail with a Poisson spectrum. (VB97)

- If shear is present then the field component in the direction of the large scale flow is amplified by a factor $S\tau_{diffusion}$

- This does not produce a very strong large scale field, but it provides the seed field in galactic disk evolution (at

What about systematic effects? How can we evaluate $\left\langle \vec{u} \times \vec{b} \right\rangle$?

Take the time derivative, use the induction and force equations and multiply by the correlation time τ. This is a Taylor series in time (convergence?).

$$\alpha_{ij} \approx \varepsilon_{ikl} \left(\overline{u_k \partial_j u_l} - \overline{b_k \partial_j b_l} \right) \tau$$

Or

(current helicity−kinetic helicity) x correlation time

$$h_k \equiv \overline{u \cdot \nabla \times u}$$

$$h_j \equiv \overline{b \cdot j}$$

Magnetic Helicity Conservation

$$\partial_t H_B = -\nabla \cdot \left[H_B \vec{V} + \vec{B}(\Phi - \vec{A} \cdot \vec{V} \right]$$

$$H_B \equiv \vec{A} \cdot \vec{B}$$

H_B depends on our choice of gauge. Does this conservation law have any physical significance?

$$\vec{A} = \int \frac{\vec{j}\left(\vec{r}\,' + \vec{r}\right)}{4\pi r'} d^3 r'$$

If we take   then the current

$$\overline{\vec{j} \cdot \vec{b}} \approx k^2 \overline{\vec{a} \cdot \vec{b}}$$

helicity and the magnetic helicity are closely related.

"α" suppression

If we only consider h, the eddy scale contribution to the magnetic helicity, the conservation equation becomes:

$$\partial_t h + 2\vec{B}\cdot\overline{(u \times b)} = -\nabla\cdot\vec{j}_h$$

$$h \equiv \overline{\vec{a}\cdot\vec{b}}$$

If the RHS is zero, then the electromotive force drives an accumulation of h, which turns off the electromotive force. **This is not really α suppression**. This is suppression of the electromotive force, regardless of its origin. (Gruzinov and Diamond)

To drive a dynamo beyond this point we need

$$\vec{j}_h \neq 0$$

Vishniac, Cho 2001

This suggests a fairly drastic reordering of causality for dynamos, at least for fields strong enough that the magnetic helicity on small scales becomes saturated.

The turbulence drives a magnetic helicity flux, which then determines the parallel component of the electromotive force. On dimensional grounds

$$\delta_h D_{Turb} B^2 (\Omega \tau)$$

aligned with the spin of the system. It can be estimated more precisely following the same procedure we used to get the electromotive force.

In the nonlinear limit

$$\left(\overline{u \times b}\right)_{\parallel} = \frac{-1}{2B} \nabla \square \vec{j}_h$$

This gives an electromotive force proportional to the derivative of the magnetic field, i.e. a "β− Ω" dynamo.

Predicted features of this dynamo:

1. The magnetic field has to exceed a critical fraction of the rms turbulent velocity, $\sim(\Omega\tau)$

2. The growth rate will be roughly

$$\Gamma \sim \frac{u_{rms}}{L}\Omega\tau$$

3. Saturation due to the stiffness of the field lines once the field energy exceeds the turbulent energy density. Suggests limit around

$$B_{sat} \sim L\Omega$$

4. Works fine in a periodic box. No ejection of h necessary.

How (what) did we do?
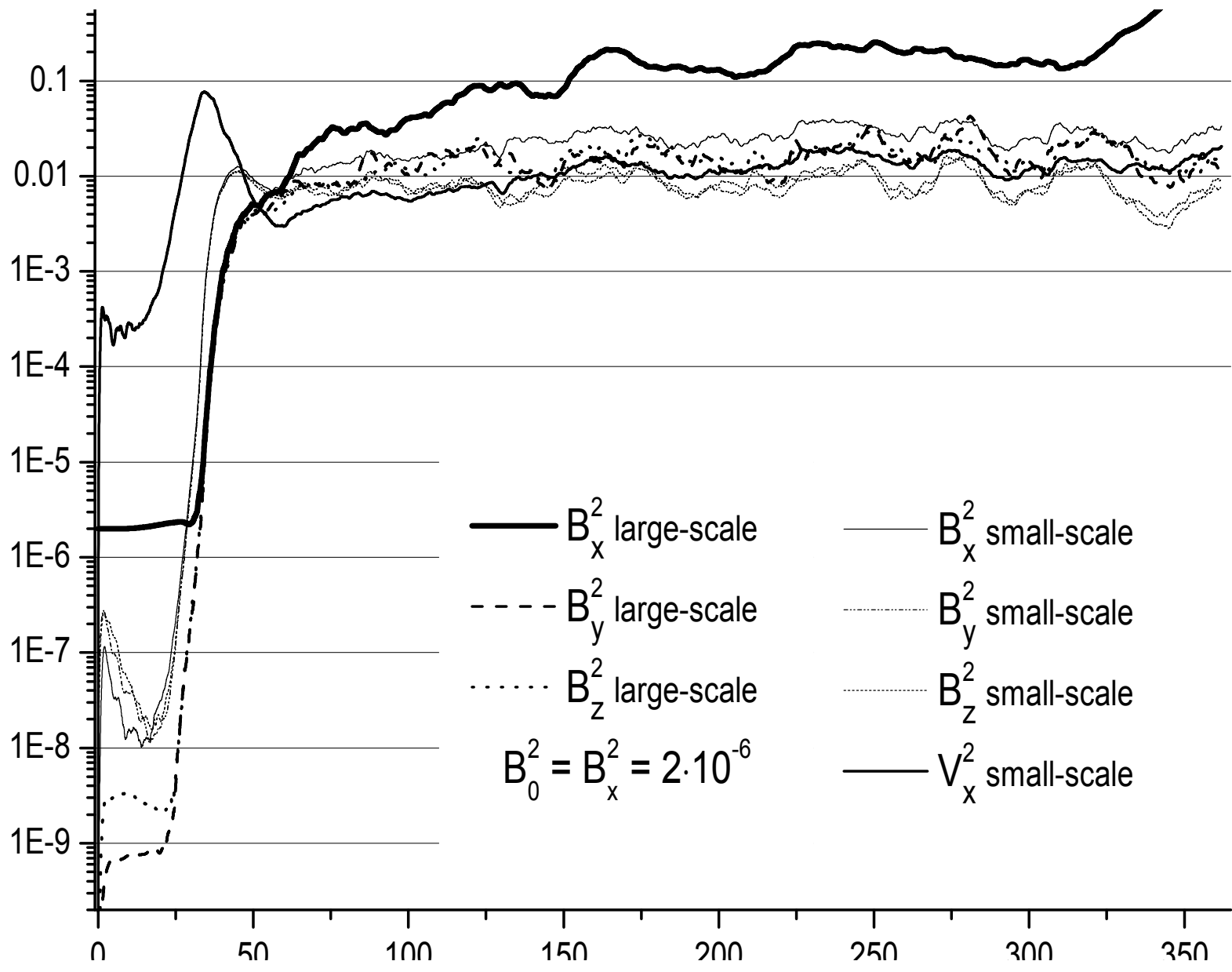## (Shapovalov & Vishniac, ApJ 2011)

- Periodic box simulations (256 and 512 cubed)

- Explicit viscosity and resistivity, tuned to be just above the level of the grid effects

- Large scale sinusoidal forcing to create periodic shear at $k_x=1$

- Small scale forcing (varied, but typically k~25) with a typical eddy turn over rate close to the large scale shear. (Non-helical forcing)
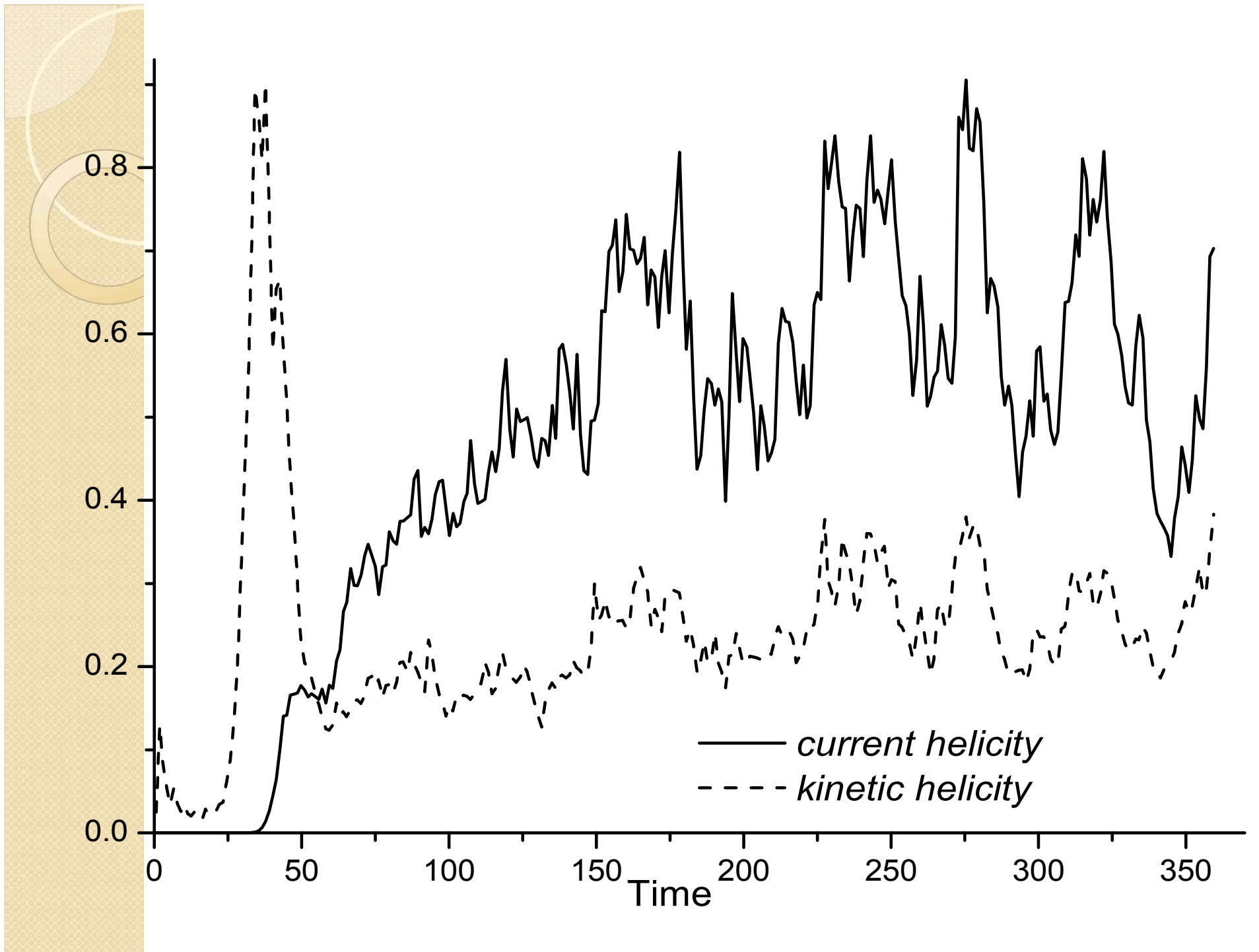
What did we see?

- Good (enough) magnetic helicity conservation. Dissipative losses were a bit more than an order of magnitude lower than all the other terms in the conservation equation including the div(j) term.
- Sufficiently anisotropic small scale turbulence would produce a strong dynamo.
- This was not a particularly good MHD turbulence simulation. Not enough dynamic range on small scales, but the flow was chaotic. (Doubling the resolution produced similar results, i.e.

What did we see?

- The large scale field was not steady, but moved in space and in intensity within the box.

- The kinetic helicity showed a strong peak early on in our "typical" case, but relaxed thereafter to a fraction of the current helicity.

- The field strength saturated close to the level of the shearing velocity and far above the level of the turbulence.

- The current helicity x the large scale magnetic field was strongly correlated with the electromotive force after several eddy turn over times.

Legend:

B_x^2 large-scale — B_x^2 small-scale

B_y^2 large-scale — B_y^2 small-scale

B_z^2 large-scale — B_z^2 small-scale

$B_0^2 = B_x^2 = 2 \cdot 10^{-6}$ — V_x^2 small-scale

Do we really need a large scale field?

In an inhomogeneous environment, the properties of the turbulence will vary with location. We will get a nonzero divergence even for

$$j_h \sim D_T \overline{b^2} \Omega \tau$$

Is there such a term? Go back to analytic expression for the magnetic helicity flux, assume no large scale field, except for the velocity field, and take the time derivative and multiply by the turbulent correlation time. We recover this term with a positive sign (generally) and a coefficient ~0.3,

How will a realistic inhomogeneous system evolve?

- The magnetic helicity h will accumulate in separate large scale regions, growing linearly with time. (That implies a linear growth in the current helicity.)

- The large scale magnetic field will evolve via the incoherent dynamo, i.e. the random addition of eddy scale electric fields will give a root N push to B, with a sign that varies every eddy turn over time. Large scale shear will produce a net growth proportional to $t^{3/2}$

- The current helicity will dominate the kinetic helicity **induced by the environment** after roughly one eddy turn over time.

- In less than one diffusion time, the growth of the large scale field reaches the point where it can couple to the accumulated magnetic helicity. The latter cascades to large scales, driving a dynamo with a growth rate that increases as root(t), i.e. super-exponentially.

- This continues until the dynamo actually soaks up all the accumulating magnetic helicity, at which point the growth becomes roughly linear (the electromotive force is inversely proportional to the large scale field)

- As before the field lines start to inhibit helicity transport as their energy density passes the turbulent energy density

- Leading to a saturation much like the earlier case.

- The total time for growth to saturation varies, but is generally comparable to a few e-folding times in the exponential model of the dynamo.

Applications?

- In accretion disks with MRI driven turbulence, the large and small scale fields are comparable, and similar to the rms turbulent velocity. There is no distinction between using small and large scale fields to drive the magnetic helicity flux. We can recover a reasonable model for the dynamo if we consider the tendency of the eddies to flatten in the absence of vertical structure.

- Galaxies will grow their magnetic

Conclusions:

- The dynamo process can be driven by the magnetic helicity flux, which is determined by the local properties of the turbulence and the shear/rotation of the fluid.

- In general, the growth of the large scale field is not exponential. It is much faster.

- The saturation of the field is not pegged to the local rms turbulent speed, but depends on the height and shear of the system.

- Generically we expect a high