When scale-separation helps: three examples in MHD

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Pohang, November 2011

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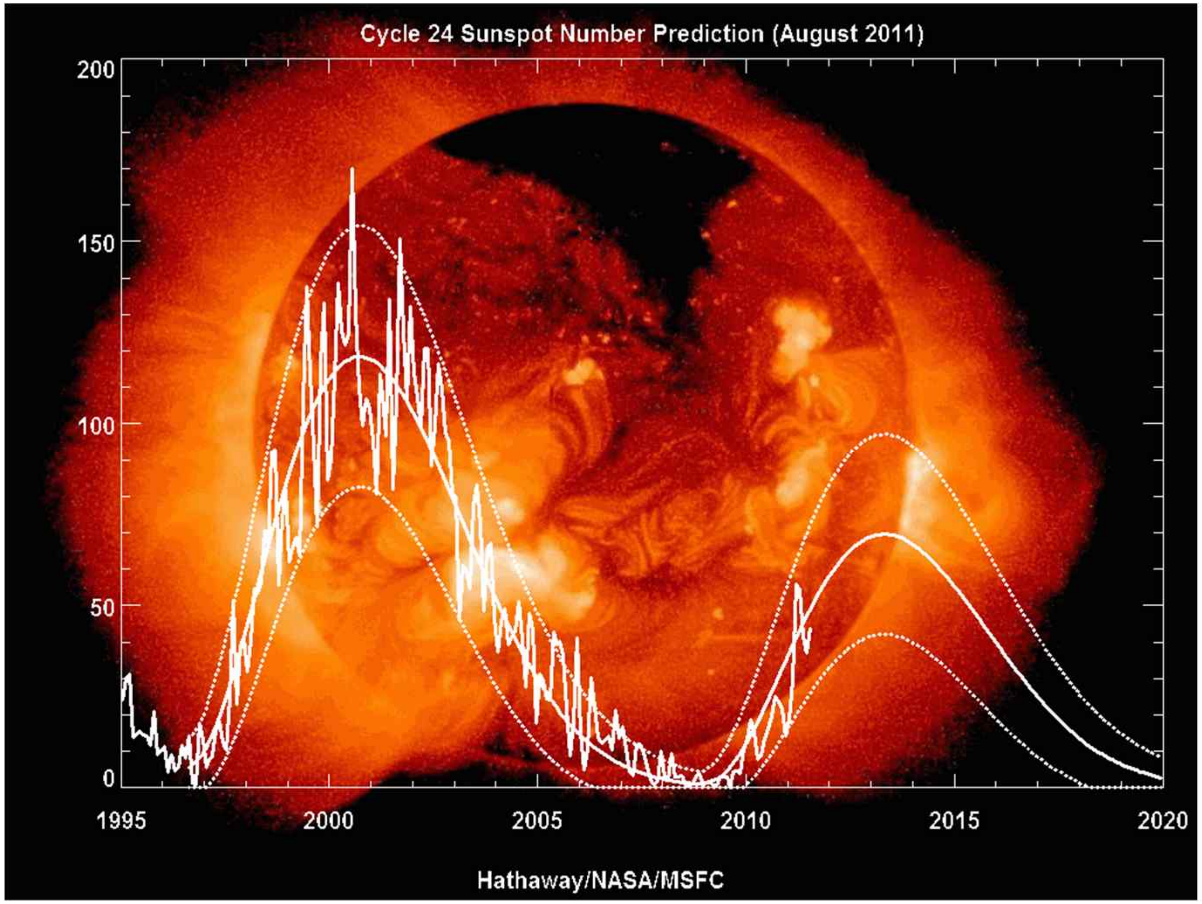
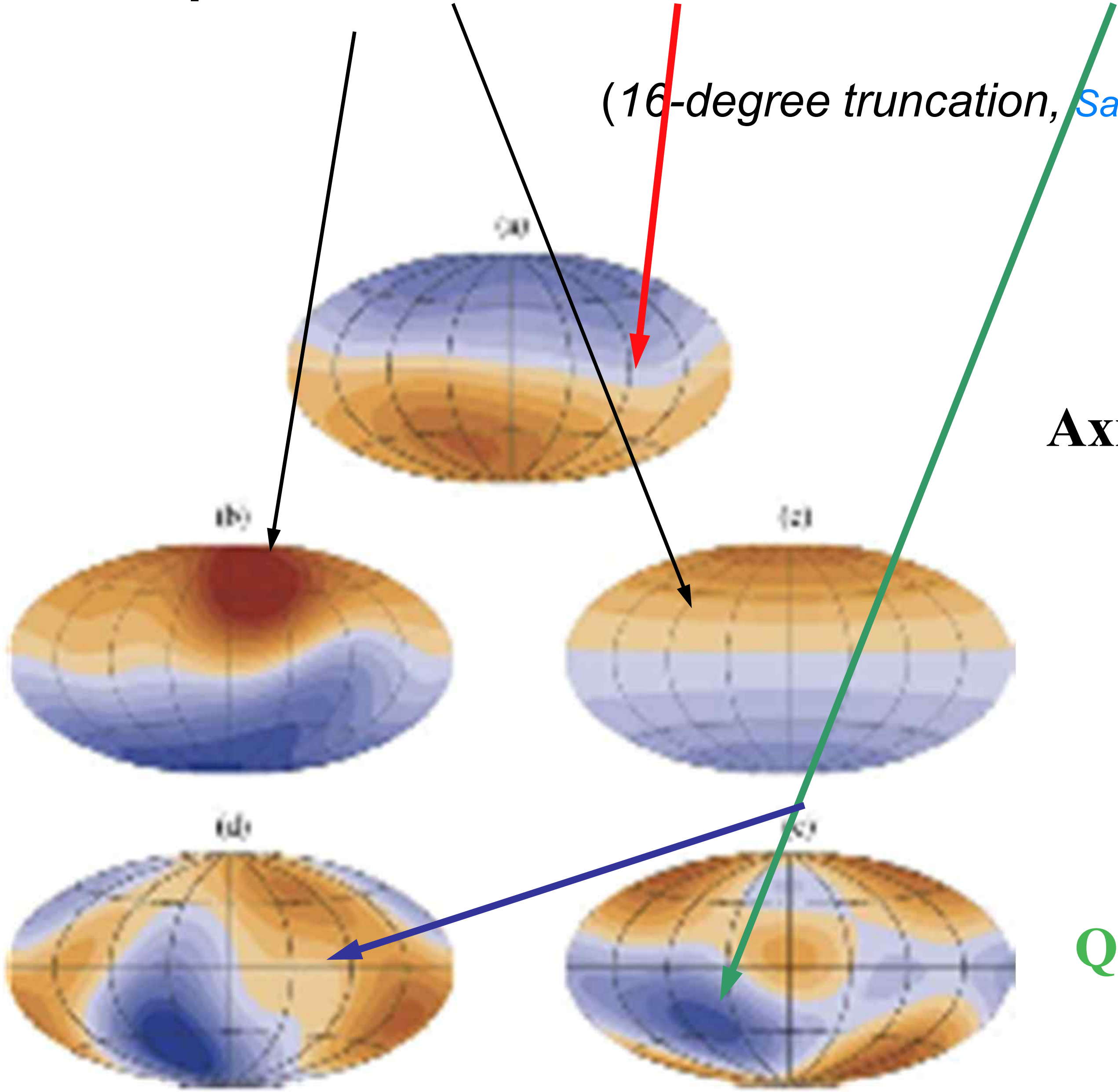


Cycle 24 Sunspot Number Prediction (August 2011)
200
150
100
50
0
1995
2000
2005
2010
2015
2020
Hathaway/NASA/MSFC

Surface (1 bar) radial magnetic fields for

Jupiter, **Saturne** & **Earth** *versus* **Uranus** & **Neptune**

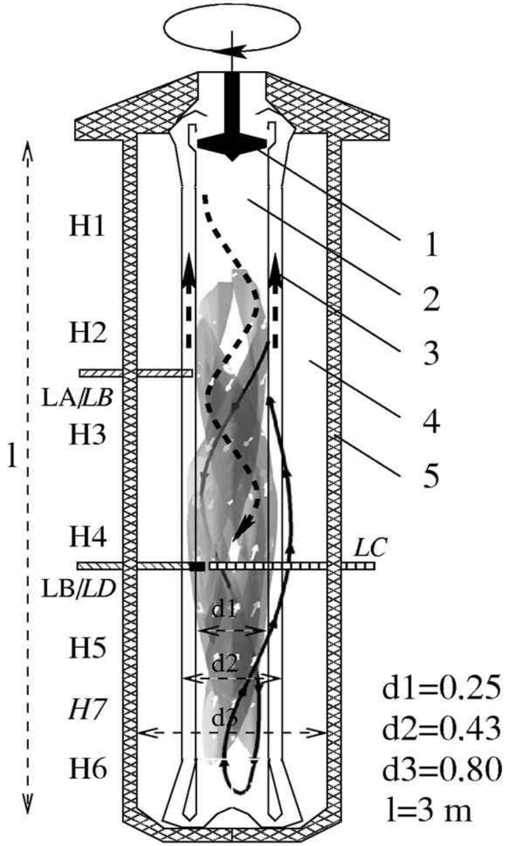
(*16-degree truncation,* *Sabine Stanley, 2006)*



Axially dipolar

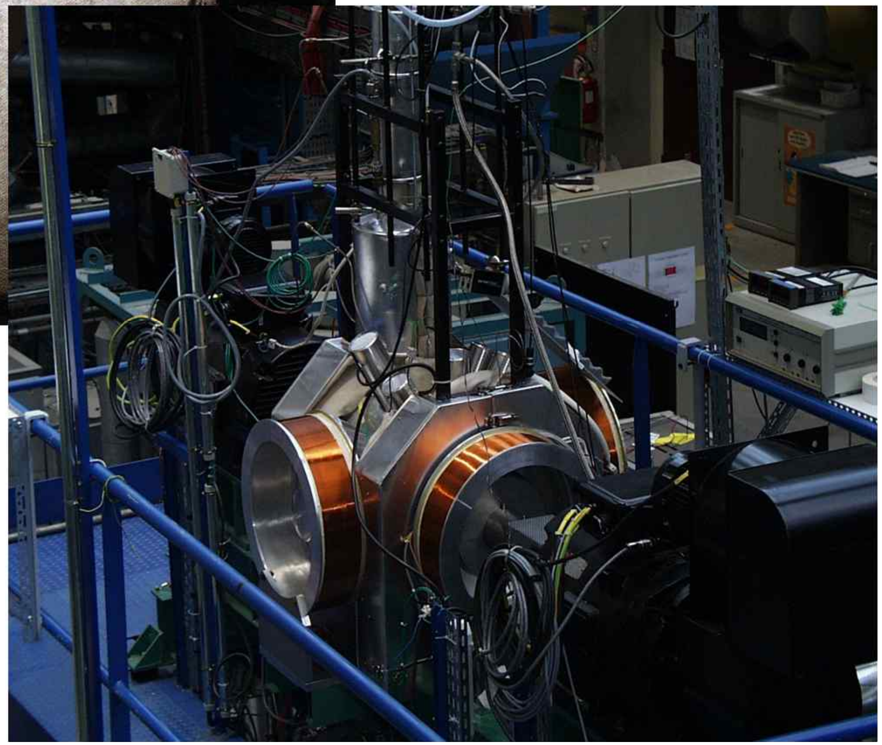
Quadrupole ~ dipole

Dynamo Experiments



Riga

Karlsruhe



Cadarache

The MHD equations

Multi-scale interactions (high Reynolds), to the detriment of all other concerns

- P is the pressure, **j** = $\nabla \times$ **B** is the current, **F** is an external force, v is the viscosity, η the resistivity, **v** the velocity and **B** the induction (in Alfvén velocity units); **incompressibility** is assumed, and div.B = 0.

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla \mathcal{P} + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{v} + \mathbf{F}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B} \ ,$$

The MHD equations

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$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{v} + \eta\nabla^2\mathbf{B}\ ,$$

Ideal case: ν=0 and η=0 → 3 quadratic invariants

Parameters in MHD

- $R_V = U_{rms} L_0 / \nu >> 1$
- Magnetic Reynolds number **$R_M = U_{rms} L_0 / \eta$**

  Magnetic Prandtl number: $P_M = R_M / R_V = \nu / \eta$

  P_M is high in the interstellar medium.

  P_M is low in the solar convection zone, in the liquid core of the Earth, in liquid metals and in laboratory experiments

  And $P_M \sim 1$ in most numerical experiments until recently …

- Energy ratio E_M/E_V *or time-scale ratio T_{NL}/T_A*

  with $T_{NL}= l/u_l$ and $T_A=l/b$
- (Quasi-) Uniform magnetic field **B_0**
- Magnetic & cross helicity $H^M = \langle A.B \rangle$ & $H_c = \langle v.B \rangle$ *(invariants, as E_M+E_V)*

- *Boundaries, geometry, rotation, stratification, cosmic rays, radiation, …*

Three examples for which scale separation helps

- Dynamics of two- & three-dimensional structures

- Dissipative turbulent behavior of a flow in the ideal non-dissipative case in two and three dimensions

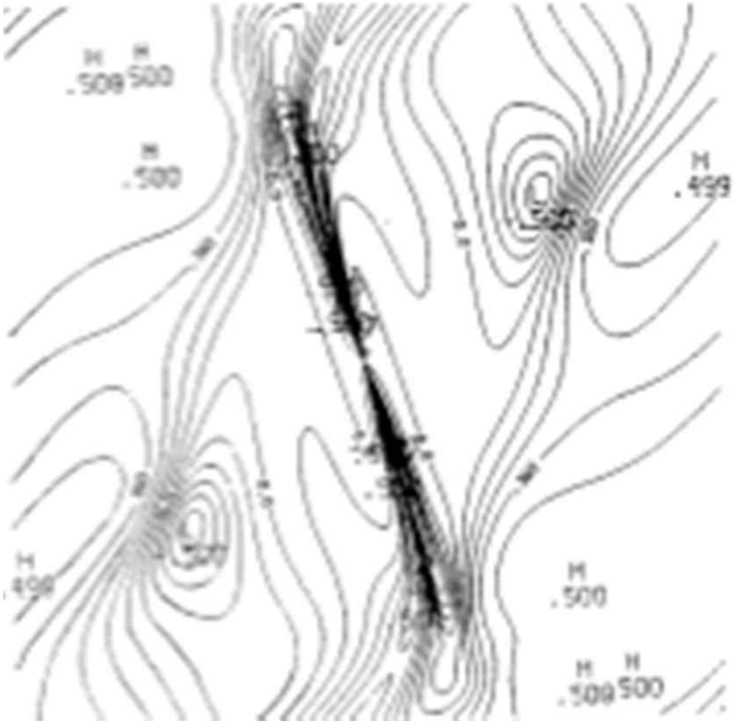
- Does scale-separation for scales larger than the forcing scale help in the large-scale helical dynamo problem, at fixed Reynolds number?

Numerical set-up for Case 1

- Pseudo-spectral codes, 2D or 3D, MHD or RMHD, up to resolutions of 1536^3 grid points, some runs with imposed B_0, initial conditions centered at large scale, mostly periodic b.c.

- 2D: Orszag-Tang (OT) vortex of a central X-point at a stagnation point

- 3D: Extension of the OT vortex, or random initial conditions



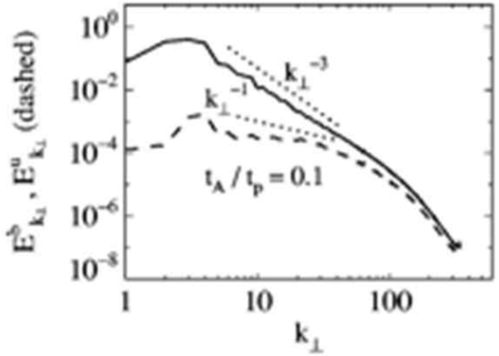
2D-MHD- Contours of $r_2(\mathbf{x})=\mathbf{v.B}/[v^2+b^2]$: local plages of maximal correlations (r_2=0.5) except in the central current sheet of the Orszag-Tang vortex -- for which globally, r_2=0.25
*(Meneguzzi et al., JCP **123**, 32 (1996)*

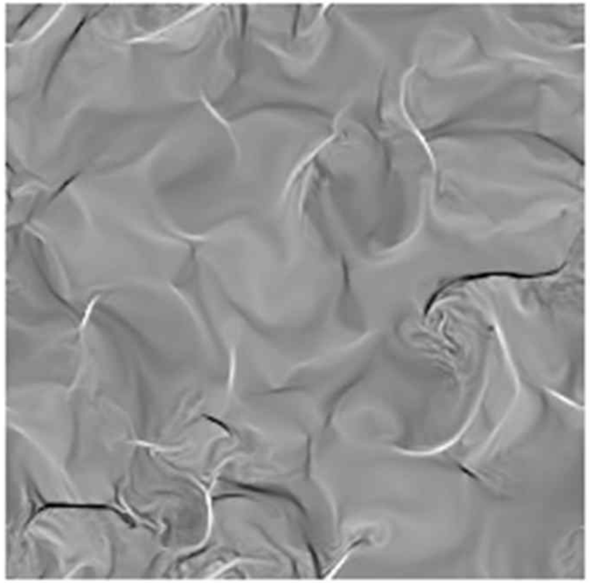
Contours of cos(**v,B**), weak global correlation of *10^{-4}*
(Matthaeus et al. , PRL 2008)

Dmitruk, Gomez & Matthaeus, PoF 2003

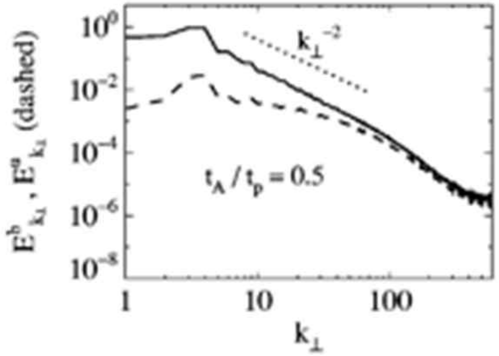
Reduced MHD Numerical data

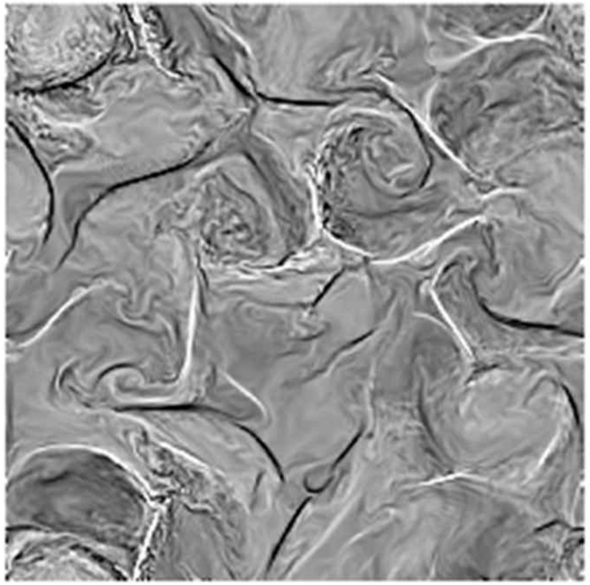
Different T_A/T_{driver}

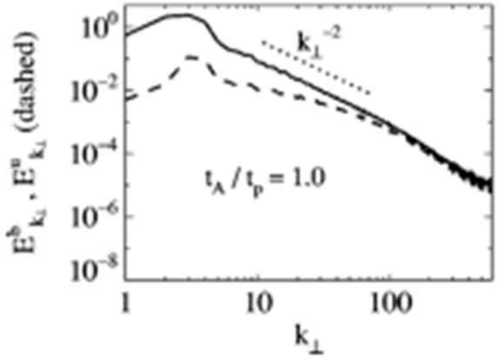




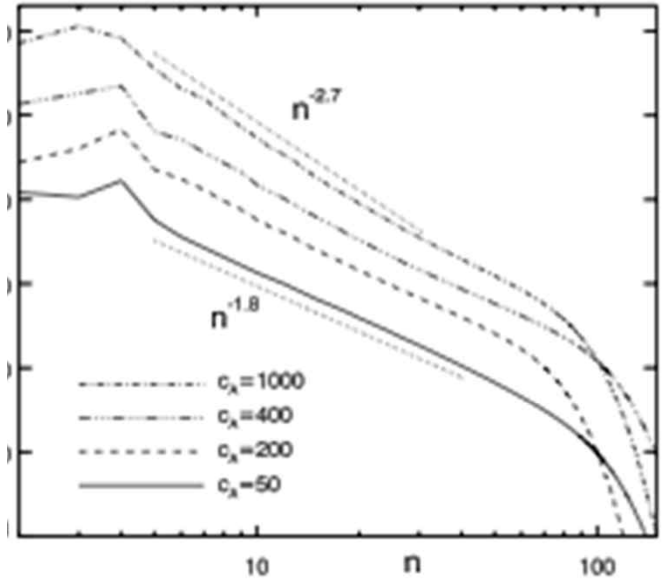
$t_A/t_p=0.5$



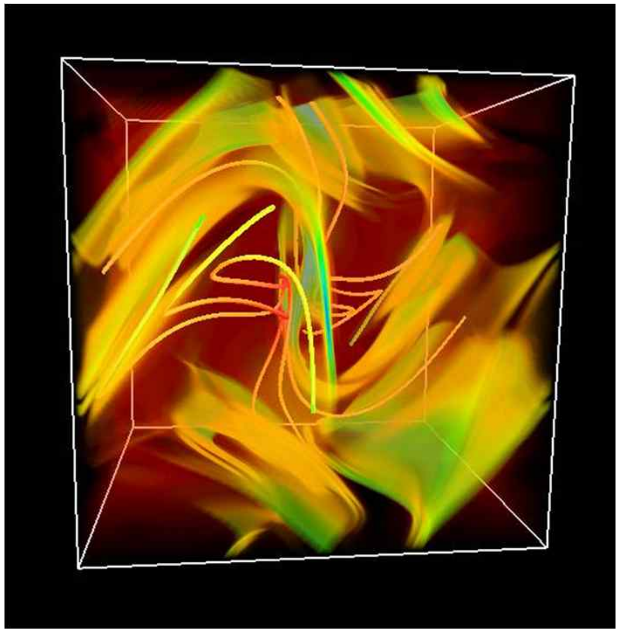




Rapazzo et al., 2008



8.—: Total energy spectra as a function of the umber n for simulations F, G, H and I. To higher of $c_A = v_A/u_{ph}$, the ratio between the Alfvén and pheric velocities, correspond steeper spectra, with l index respectively 1.8, 2, 2.3 and 2.7.

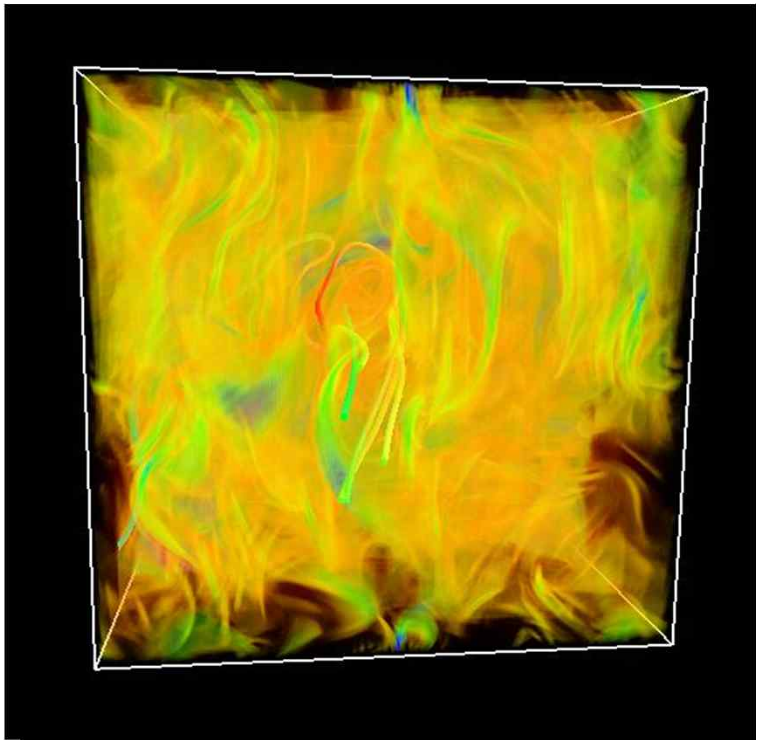


Current sheets for 3D-X point initial configuration

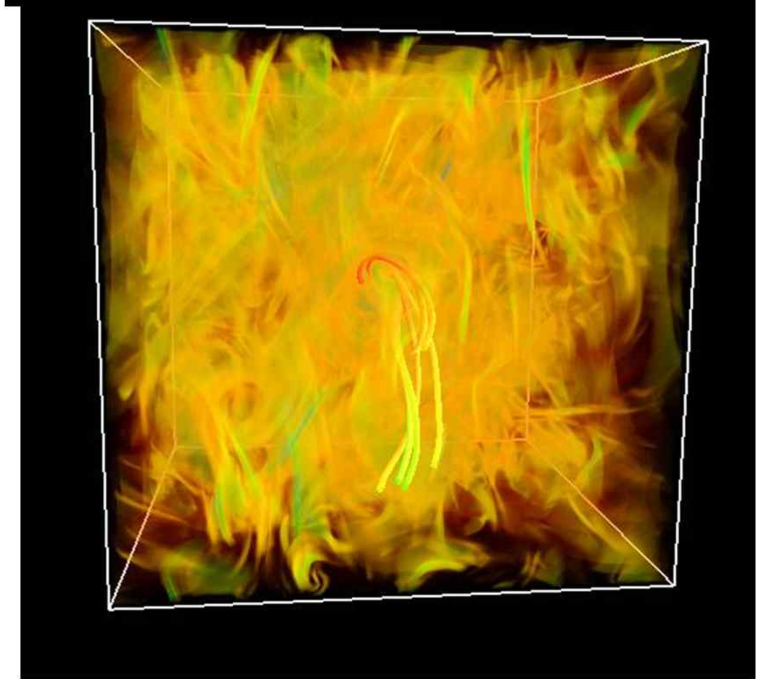
512^3 grid

<-- t=0.5

t=0.9 -->



t=1.2 -->

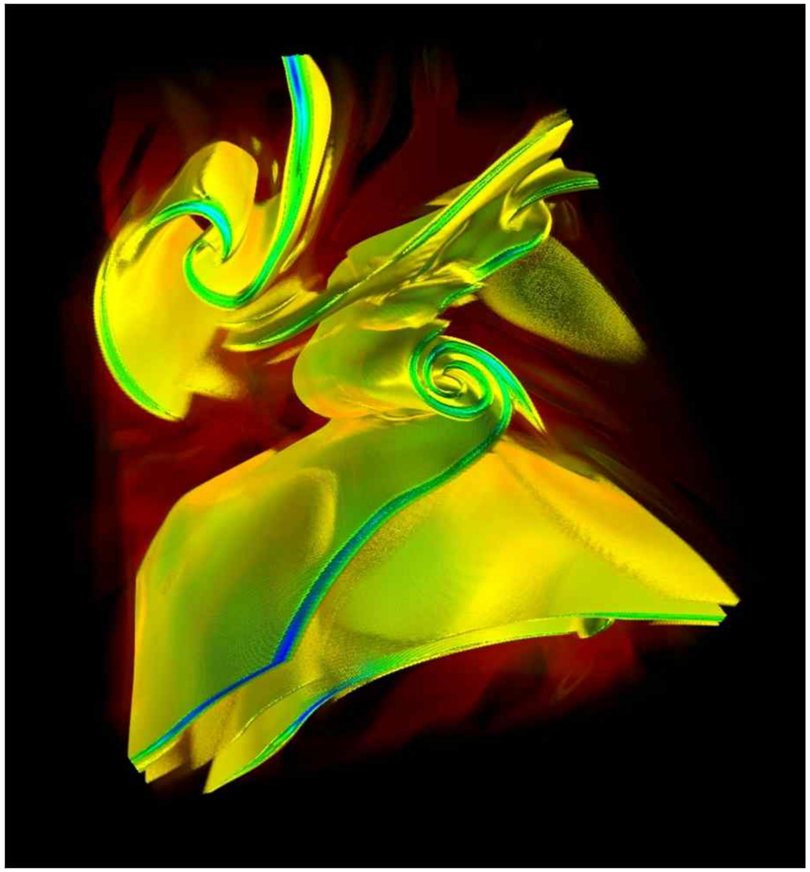


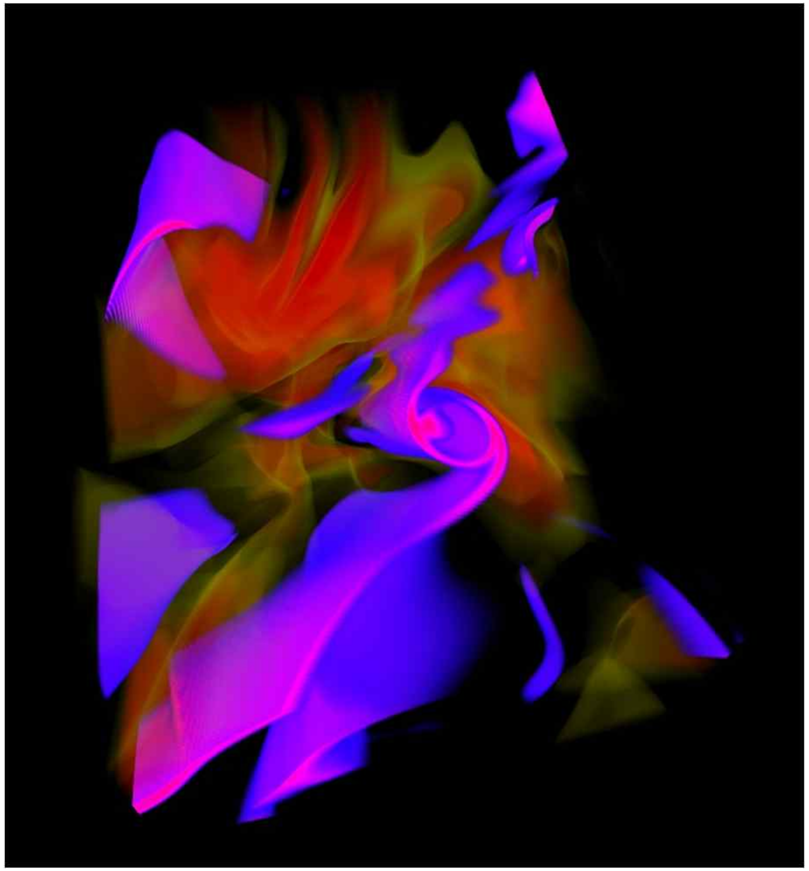
Large-scale order/memory?

V and **B** are aligned in rolled-up current sheet, *but not equal (B^2 ~$2V^2$)*

(Alexandrova et al., JGR 2006; Petviashvili & Pokhotolov, 1992)

1536^3
decay
3D MHD
run



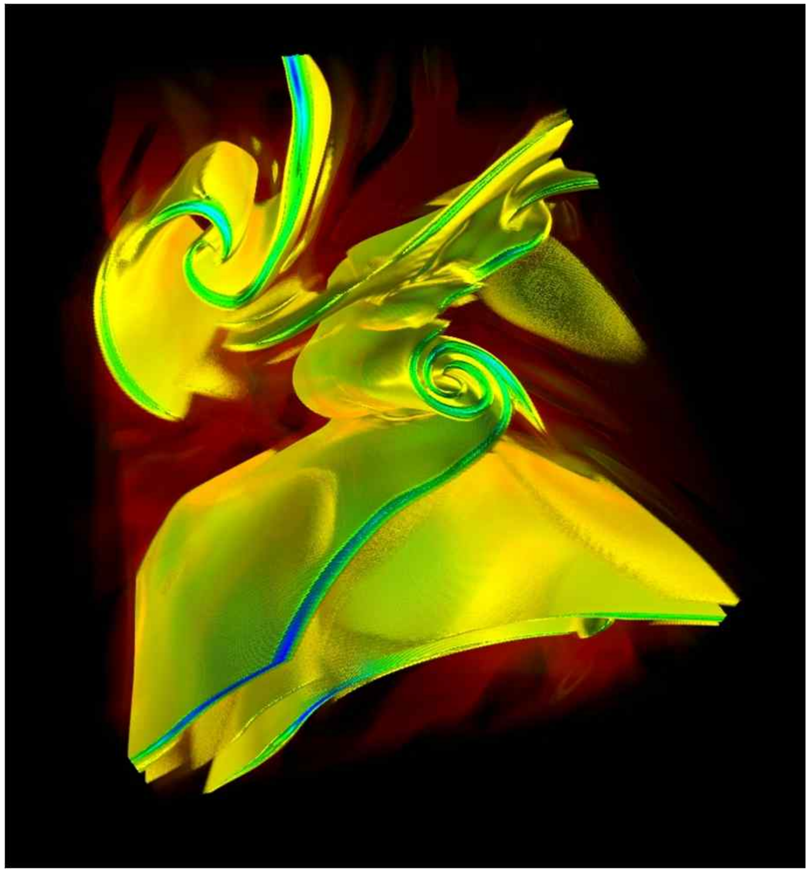


J^2 $cos(\mathbf{V}, \mathbf{B})$

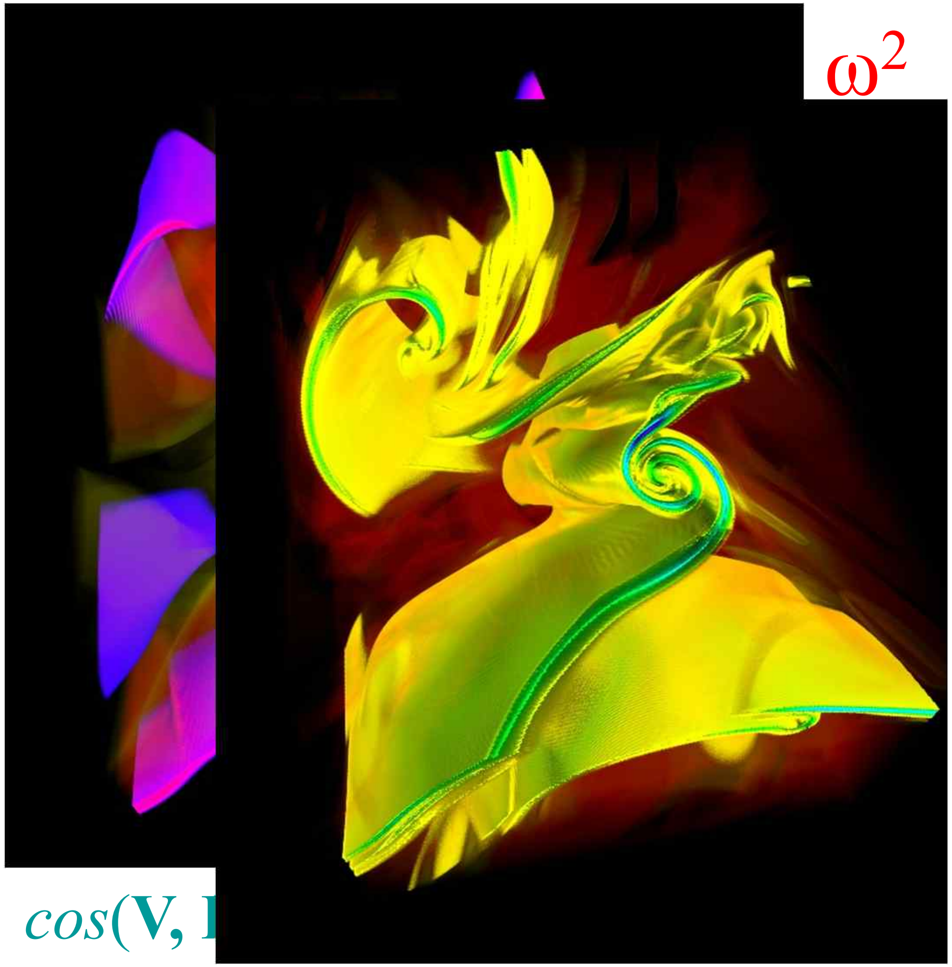
Early time (end of ideal phase) ***VAPOR*** *freeware, cisl.**ucar.**edu/hss/dasg/software/vapor*

V and **B** are aligned in rolled-up current sheet, *so are J and ω*

(Petviashvili & Pokhotolov, 1992. Solar Wind: Alexandrova et al., JGR 2006)



J^2

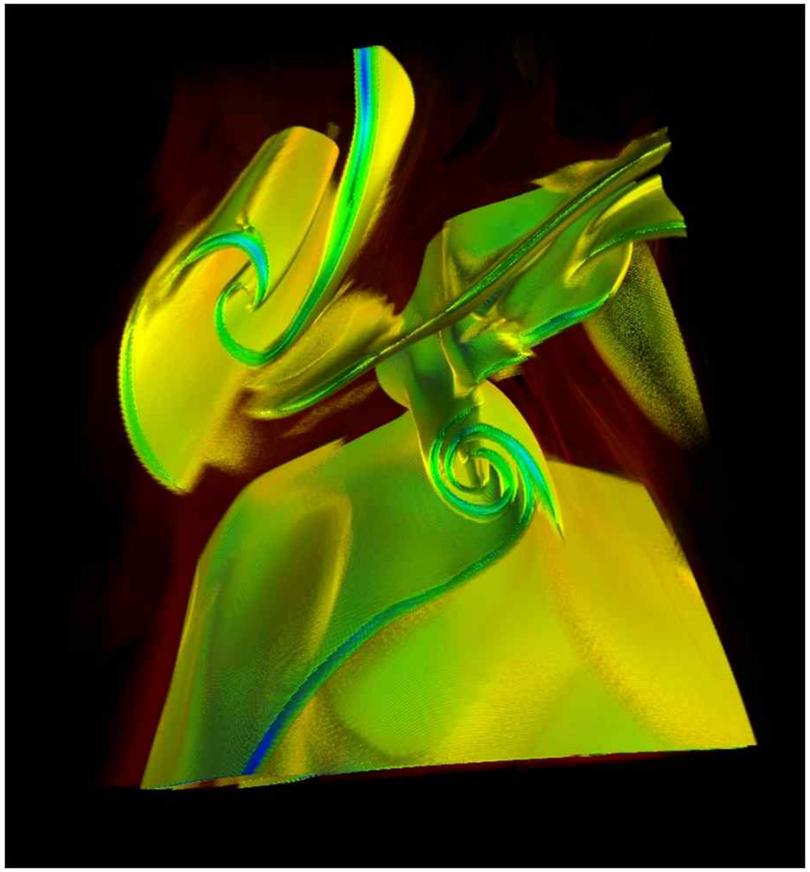


ω^2

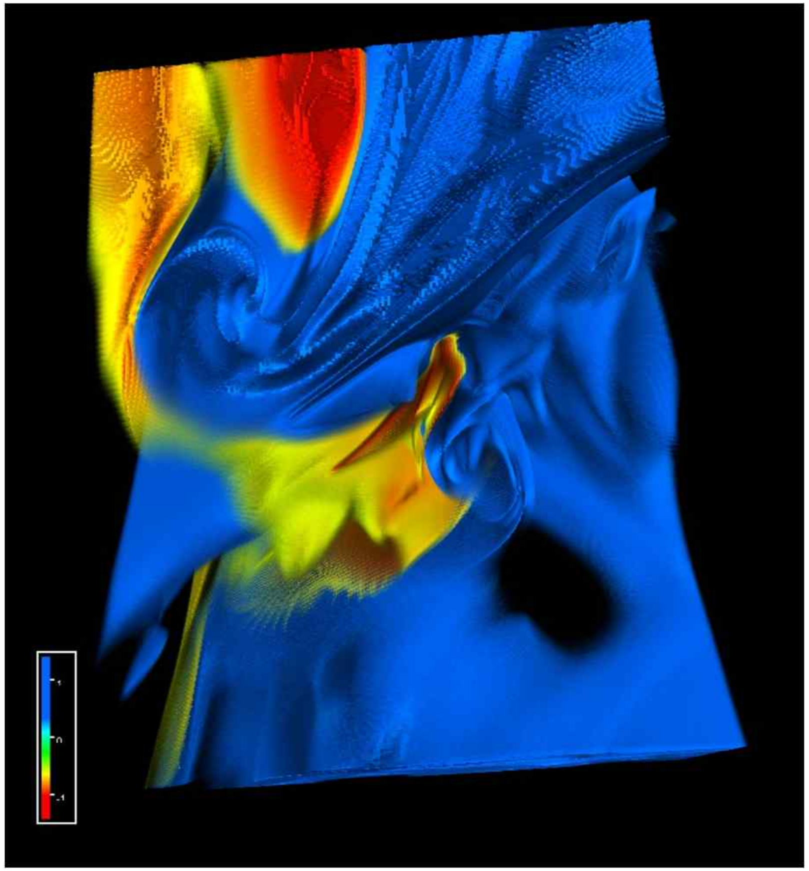
cos(**V**,

Early time (end of ideal phase)

Strong *relative* magnetic helicity (~ 1): change of topology across sheet



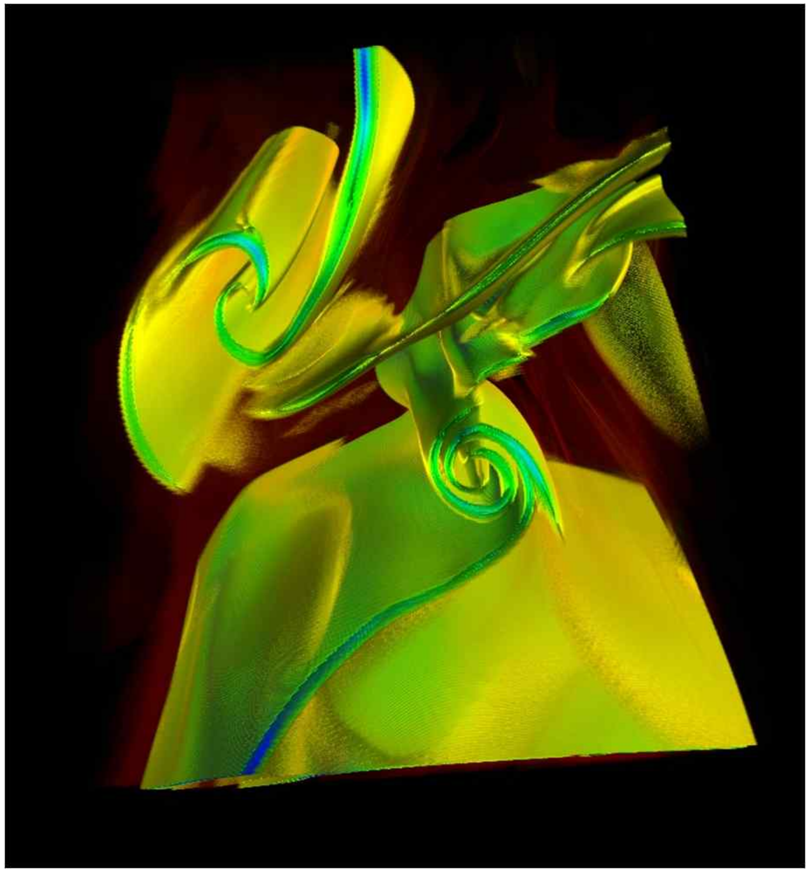
Current J^2



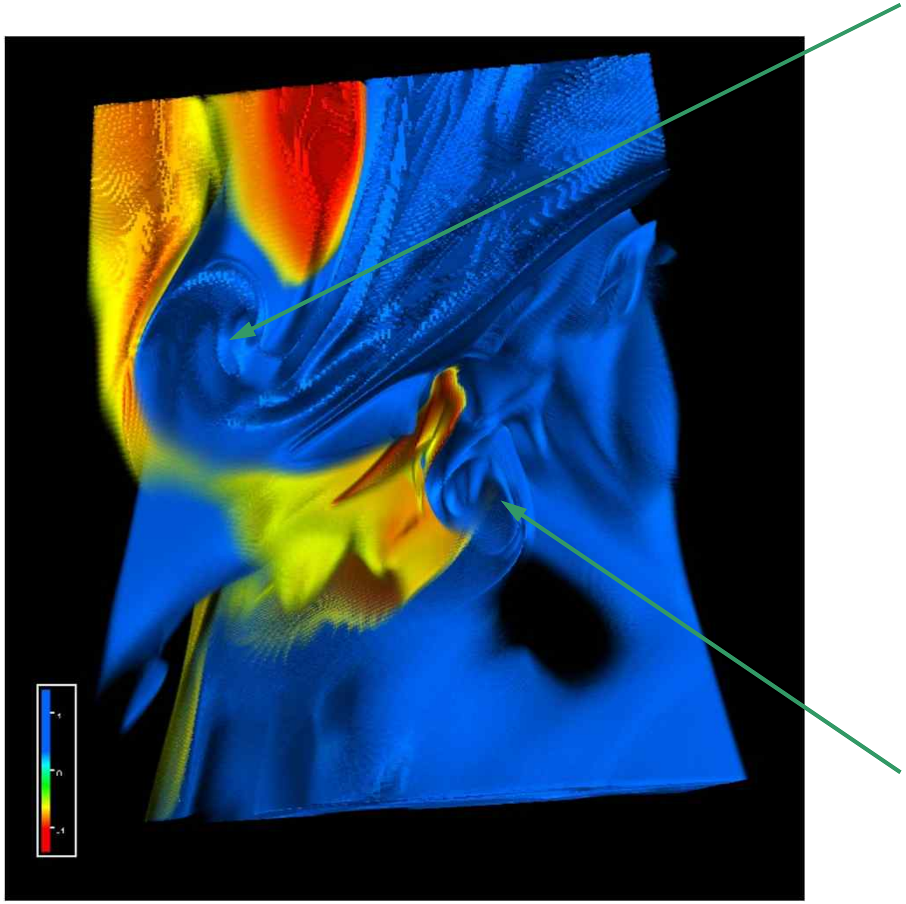
cos(**A, B**) , with **B=curl A**

1536^3 run, early time

Strong *relative* magnetic helicity (~ 1): change of topology across sheet



Current J^2

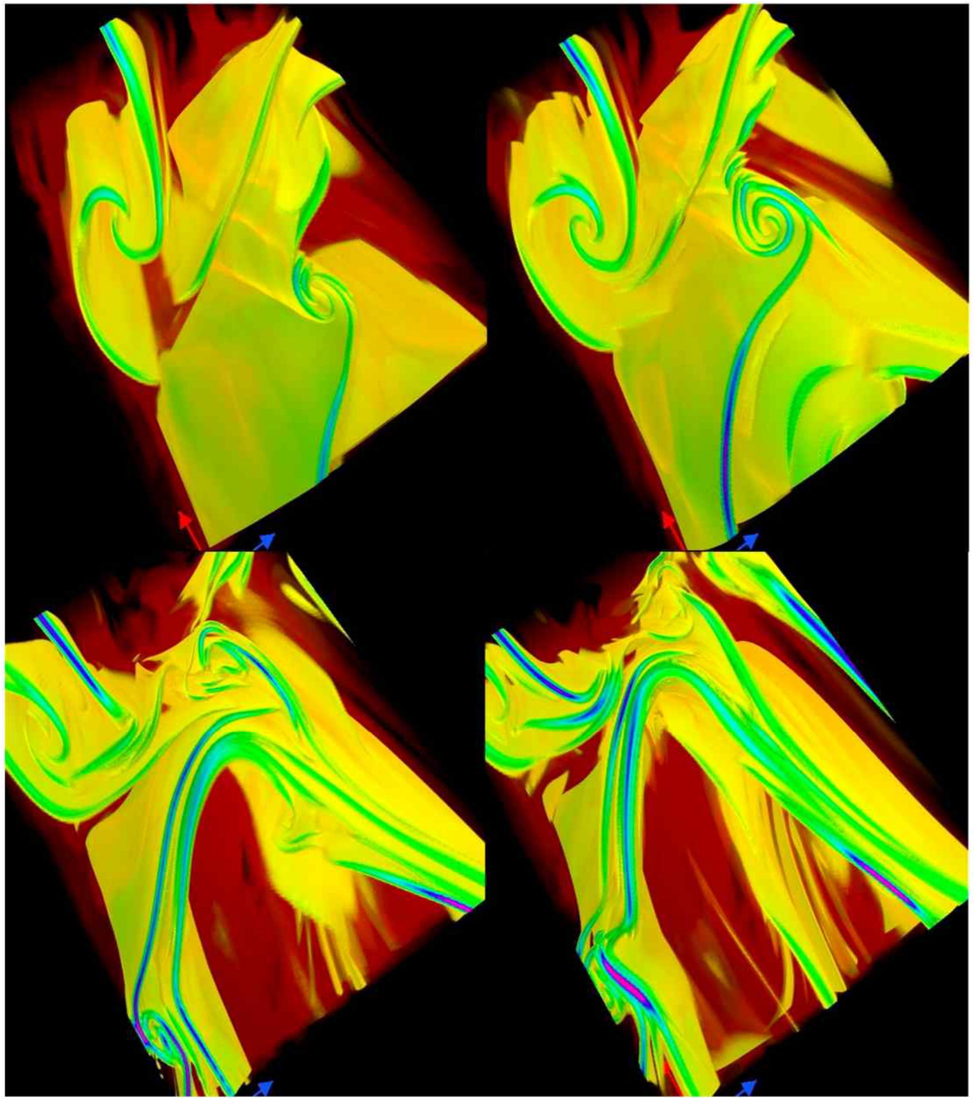


cos(**A, B**) , with **B=curl A**

1536^3 run, early time

Zoom on a current roll-up/sheet evolution

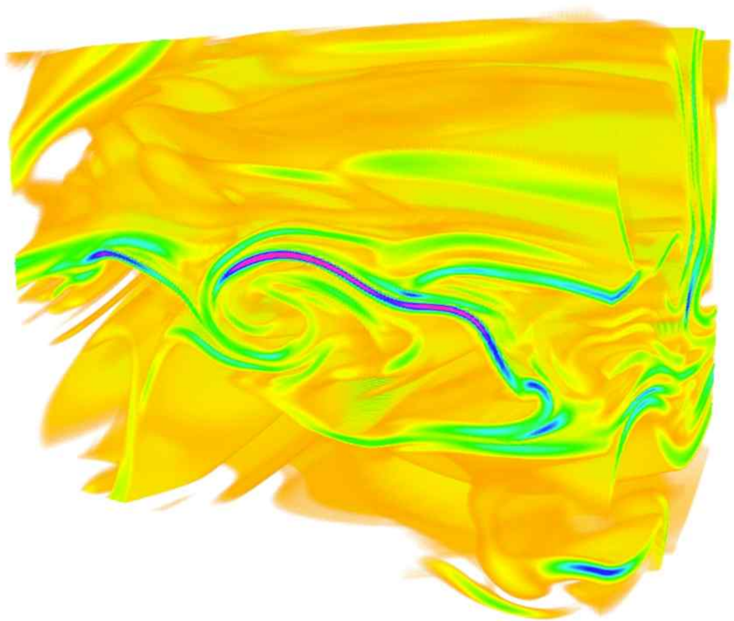
1536^3 run



Current at peak of dissipation:
Both piling-up of sheets and folding

Global view
1536^3 run

Zoom



Extreme events in direct numerical simulations of incompressible MHD

- Scaling exponents, 512^3 DNS with varying B_0:

  as B_0 increases, so does the intermittency

Müller & Biskamp, PRE 67 (2003)

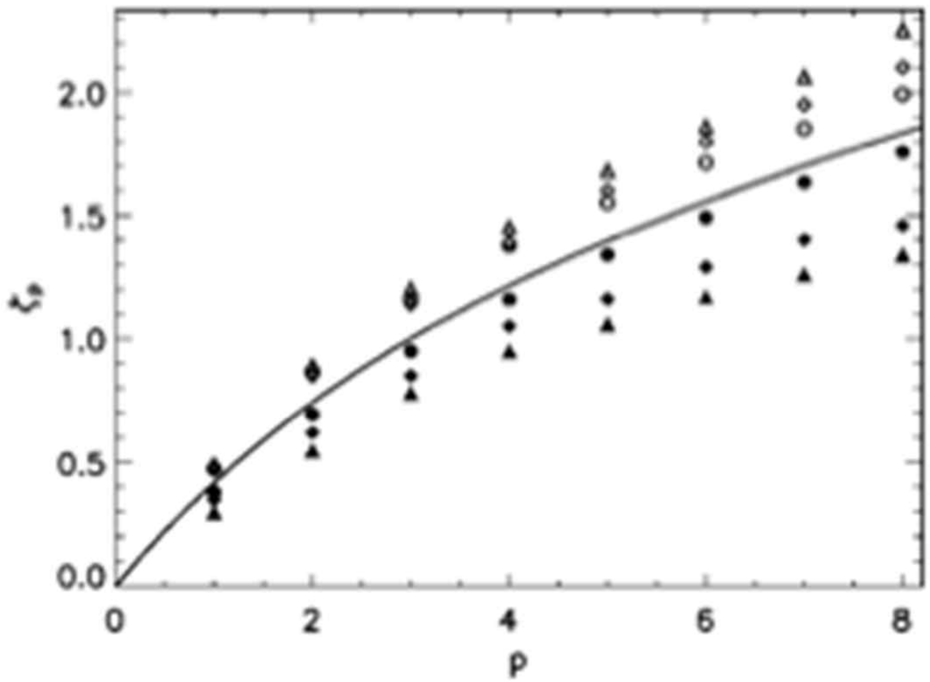


FIG. 1. Scaling exponents ζ_p of perpendicular (filled symbols) and parallel (open symbols) structure functions $S_p(\ell) = \langle |\delta z_\ell|^p \rangle$ for $B_0 = 0,5,10$ (circles, diamonds, triangles) together with isotropic

Extreme events in solar active regions

- Scaling exponents of structure functions of magnetic field (magnetograms): more intermittency (more curvature) for more energetic flares

Abramenko, review (2007)

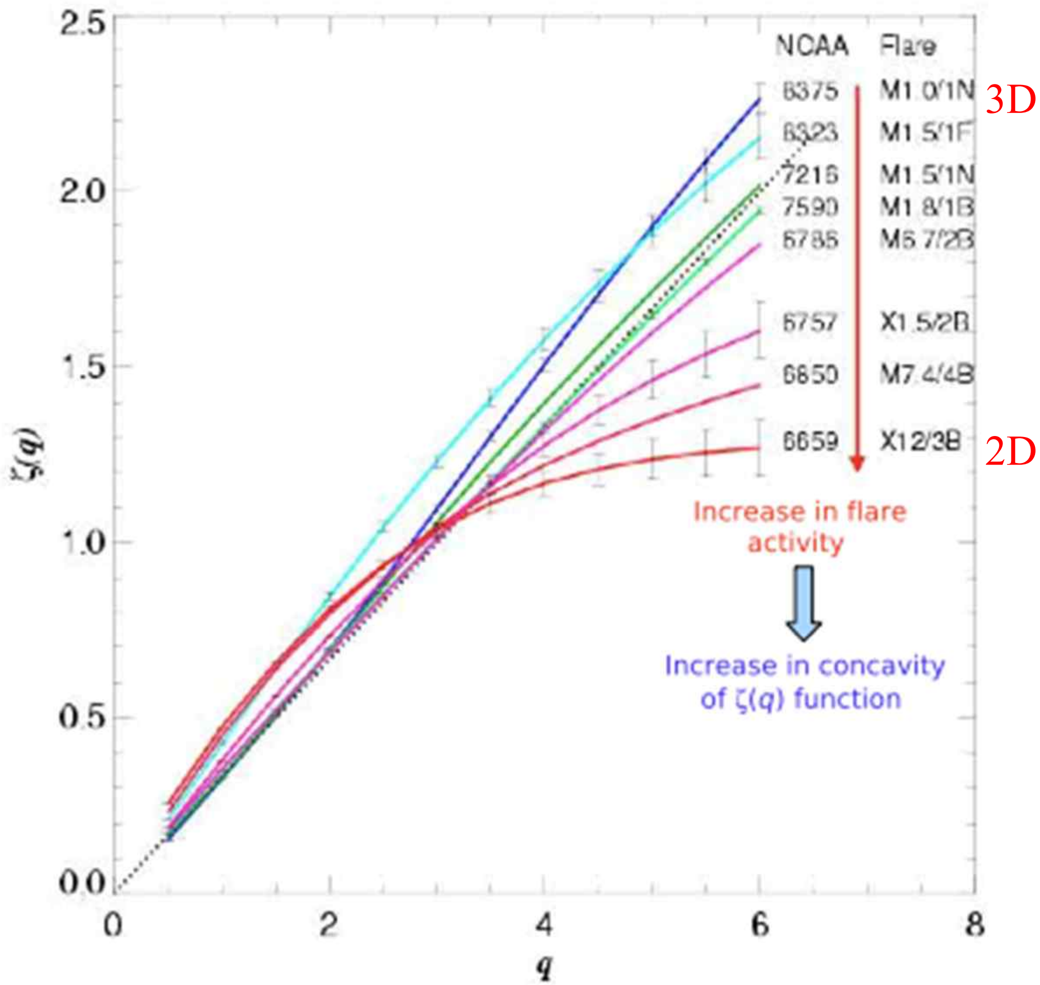
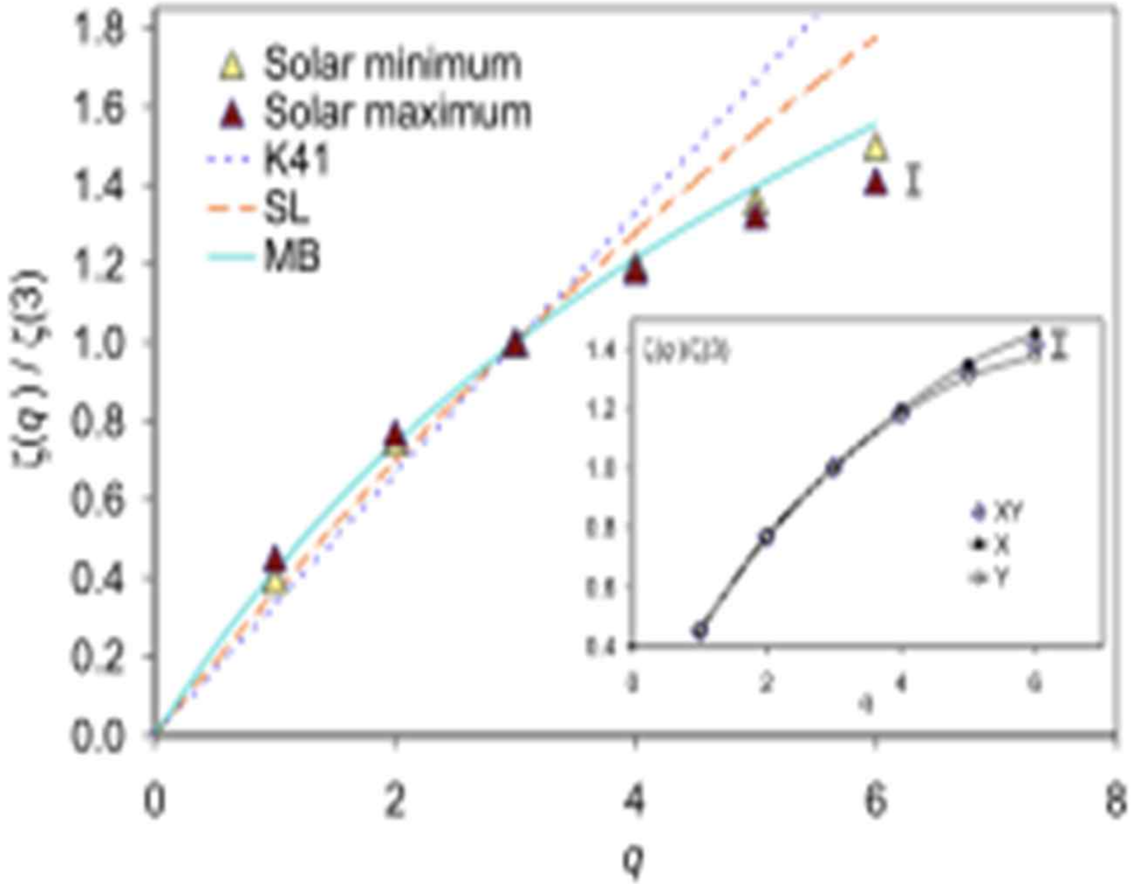


Figure 16: Scaling exponents $\zeta(q)$ of structure functions of order q calculated for eight active regions by Abramenko et al. (2002). The straight dotted line has a slope of 1/3 and refers to the state of Kolmogorov turbulence. The NOAA number and the strongest flare (X-ray class/optical class) of each active region is shown. Increase of the flaring activity of active regions (from the top down to the bottom) is accompanied by general increase in concavity of $\zeta(q)$ functions.

- Solar corona extreme events (SOHO EIT 195A) 7000+ images (central part of full-disk)



Uritsky et al., 2007

Second case study:

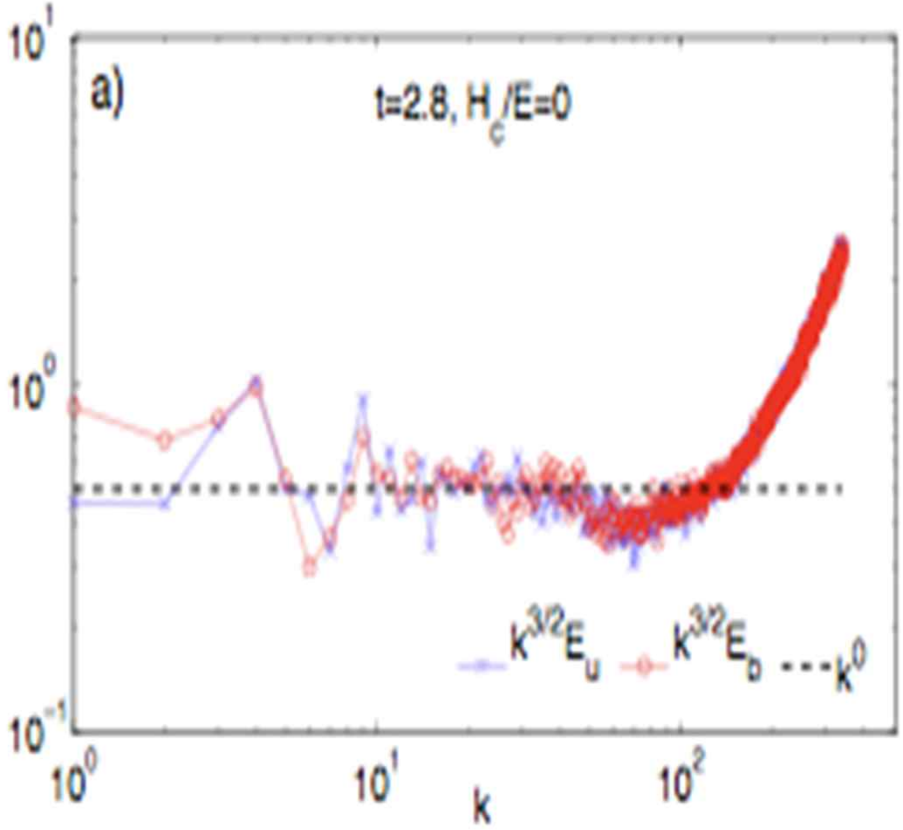
Ideal (non-dissipative) dynamics of 2D and 3D MHD flows

Numerical set-up for Case 2

- Ideal dynamics, pseudo-spectral code, de-aliasing using the 2/3 rule & periodic boundary conditions, with imposed 4-fold symmetries in 3D
- No imposed B_0, no forcing, no dissipation
- Up to 4096^2 grid points in 2D, and up to an *equivalent* resolution of 6144^3 in 3D
- 2D: Orszag-Tang vortex (OT)
- 3D: the velocity is the Taylor-Green (TG) flow, and the magnetic field has the same symmetries as TG; both are at the largest resolved scale initially

What to expect

- Long-time properties of truncated system of Fourier modes obey statistical mechanics compatible with all quadratic invariants → possibility of inverse cascades, lack of equipartition due to non-zero magnetic helicity, …
- Small-scales thermalize faster than large-scales: the small-scale spectra provide a turbulent ``dissipativity'' in a 2-fluid model (large-scale vs. small-scale)
- What is the result?

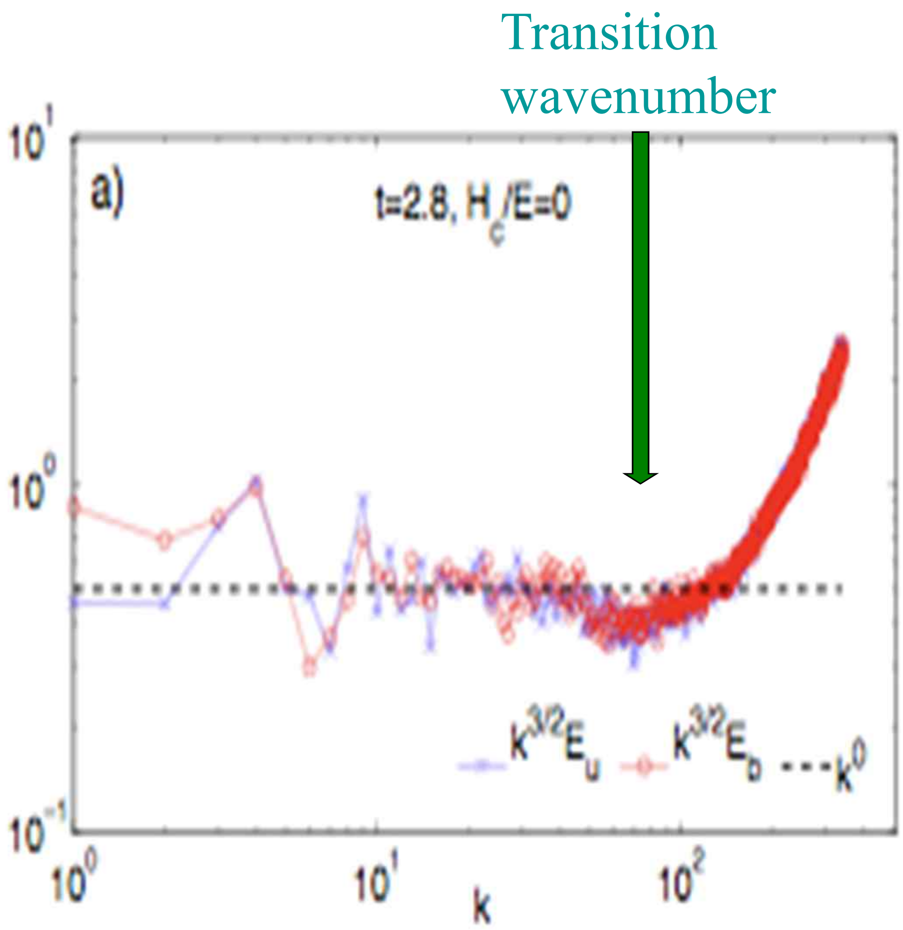


Ideal MHD in two dimensions (ν=0 and η=0):

Kinetic & magnetic energy spectra, compensated by $k^{3/2}$

Intermediate temporal phase:
the small-scale thermalized k^{D-1} spectra act as eddy diffusivities for the ``turbulent'' dynamics at intermediate scales

3D Euler: Cichowlas et al., PRL 2005
2D MHD: Krstulovic et al., PRE 2011



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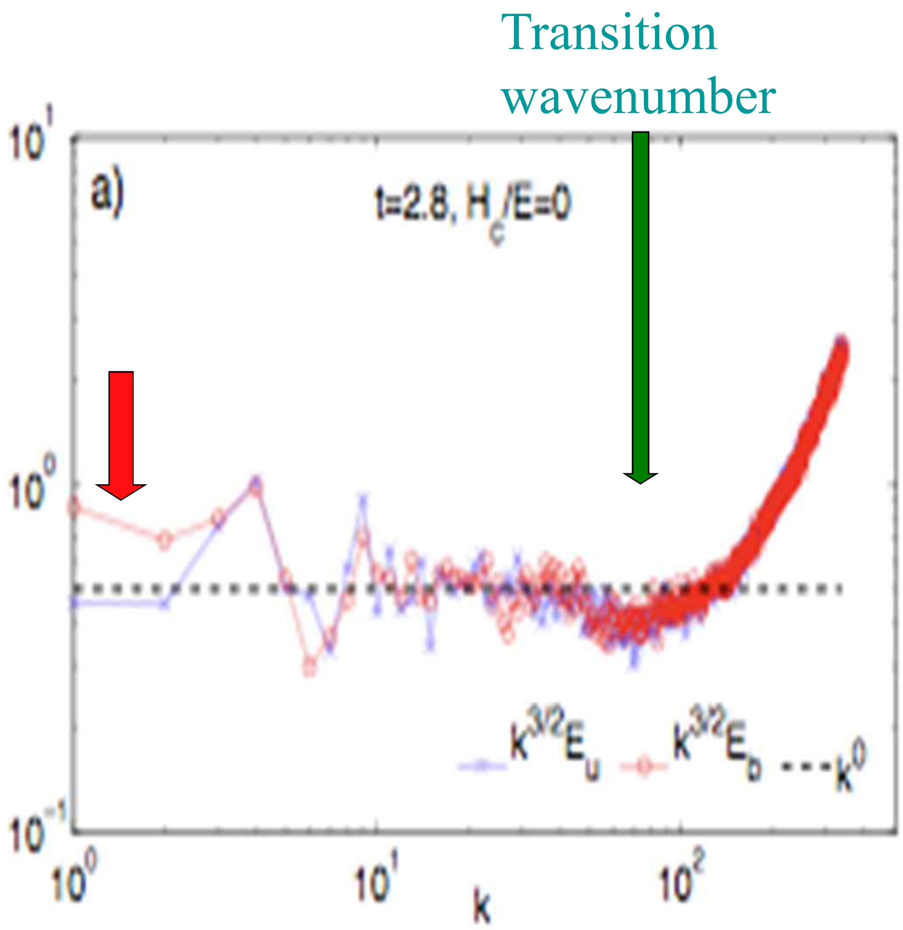

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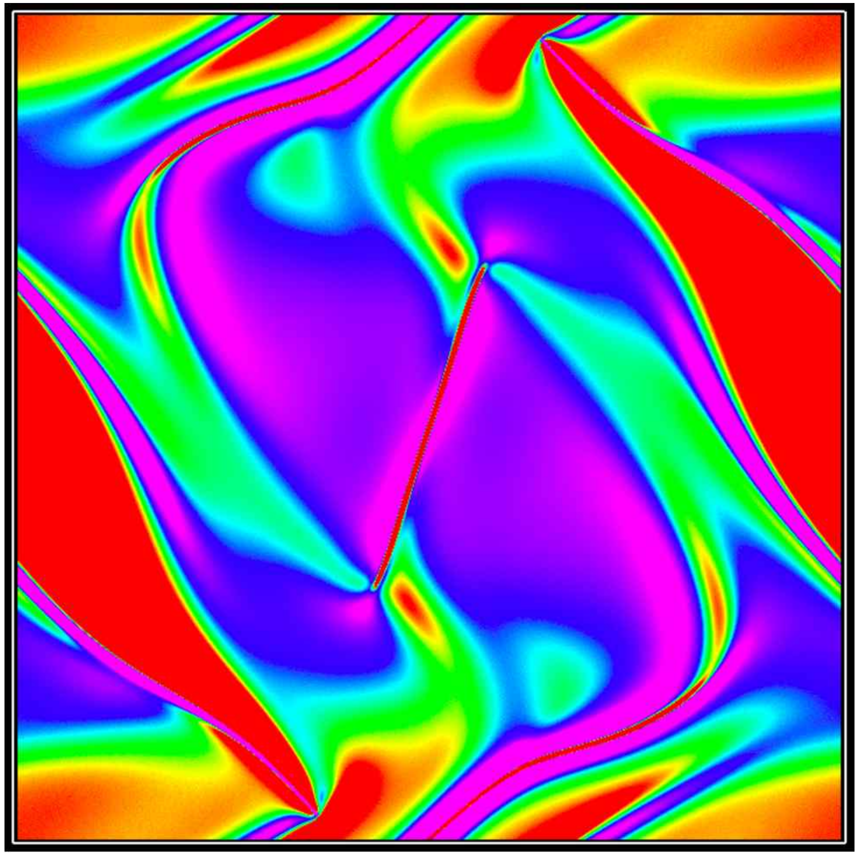
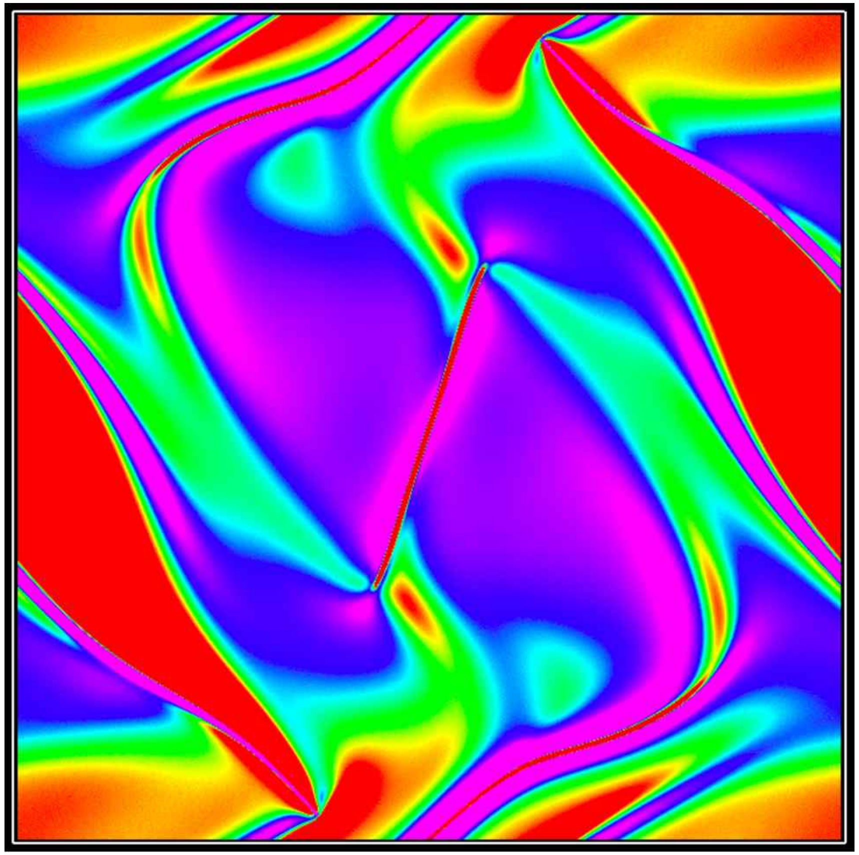
Current sheets

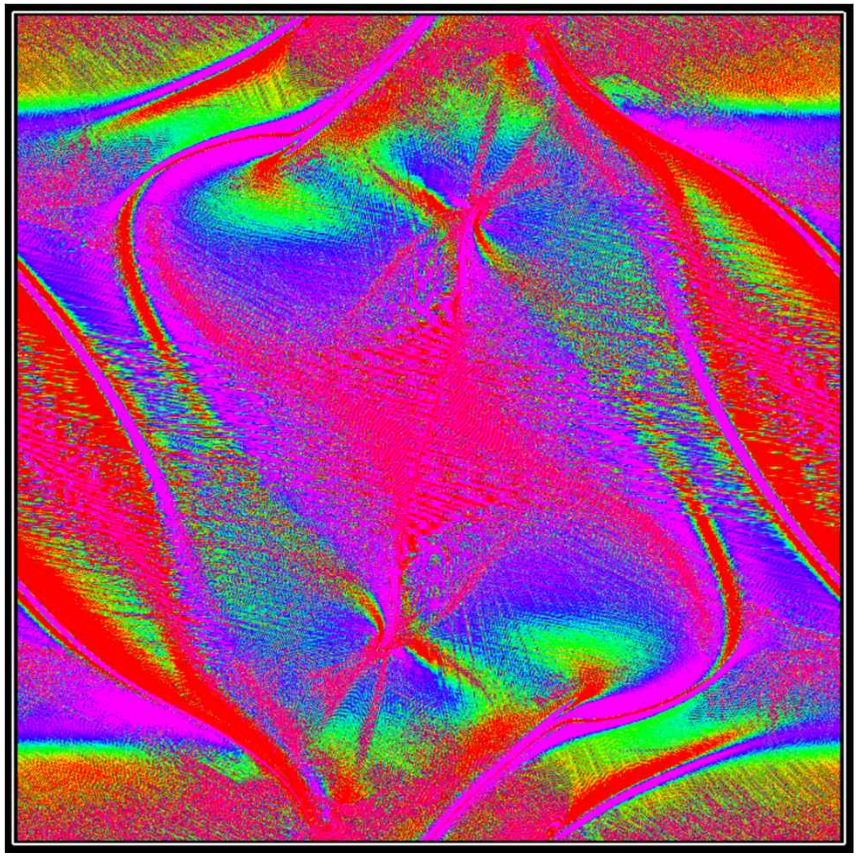


End of resolved phase

Current sheets

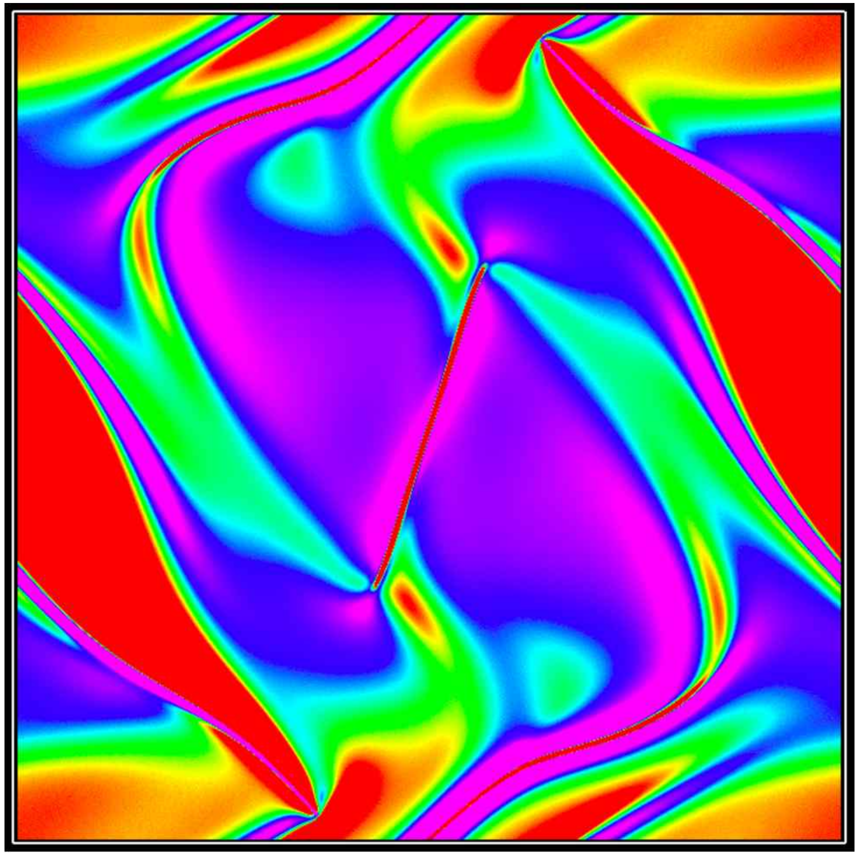


End of resolved phase

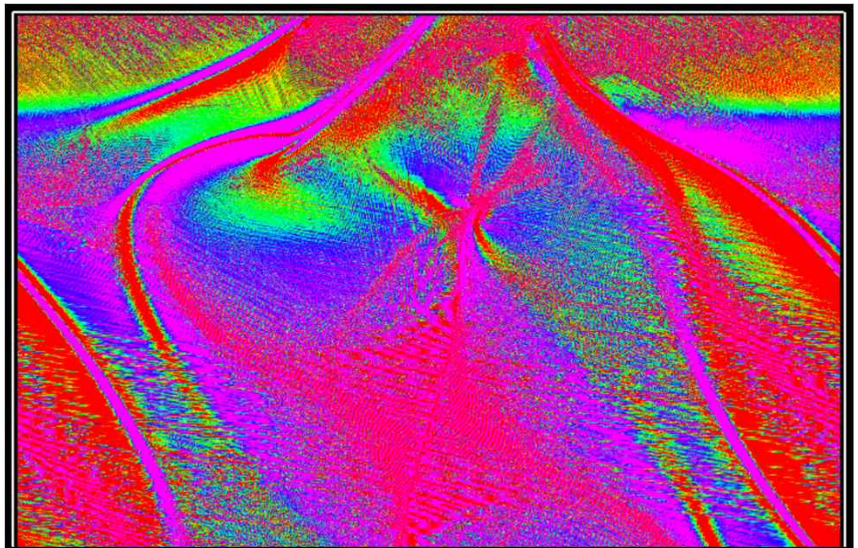


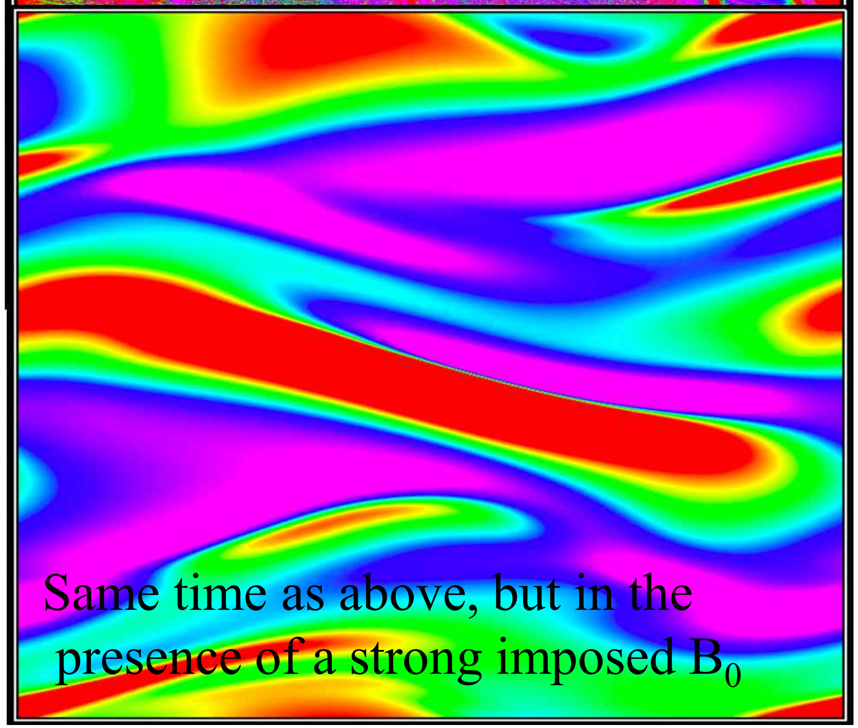
Later on, noise super-imposed
to current structures

Current sheets



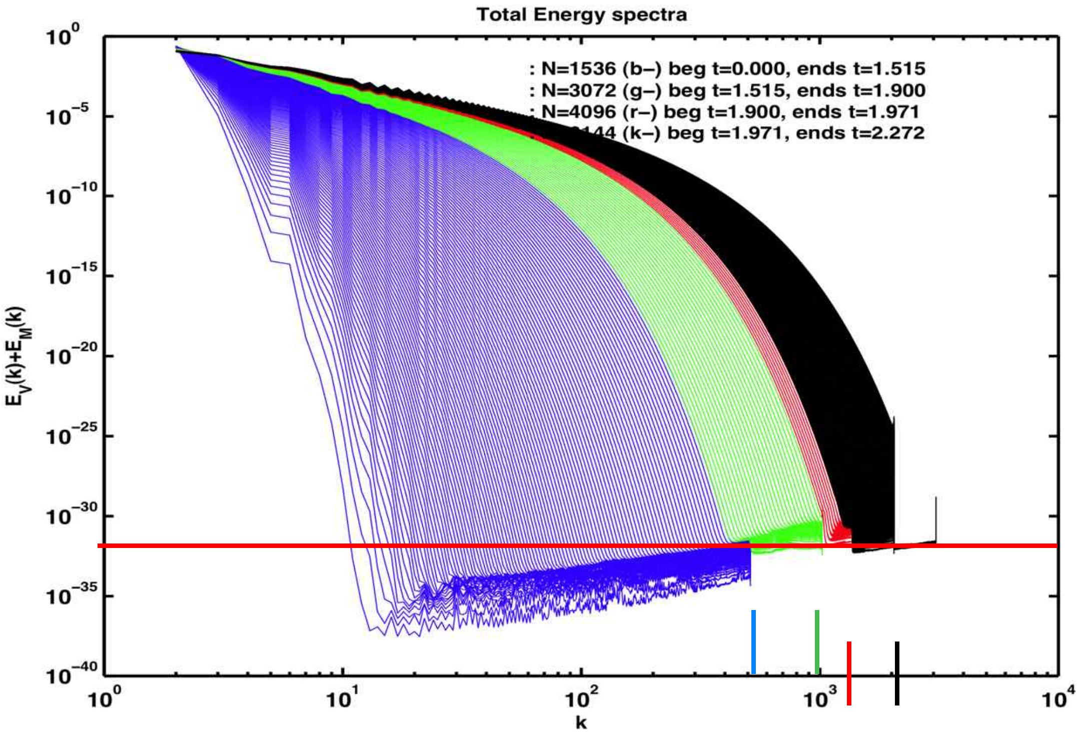
End of resolved phase





Same time as above, but in the presence of a strong imposed B_0

3D ideal dynamics in MHD, $B_0 = 0$

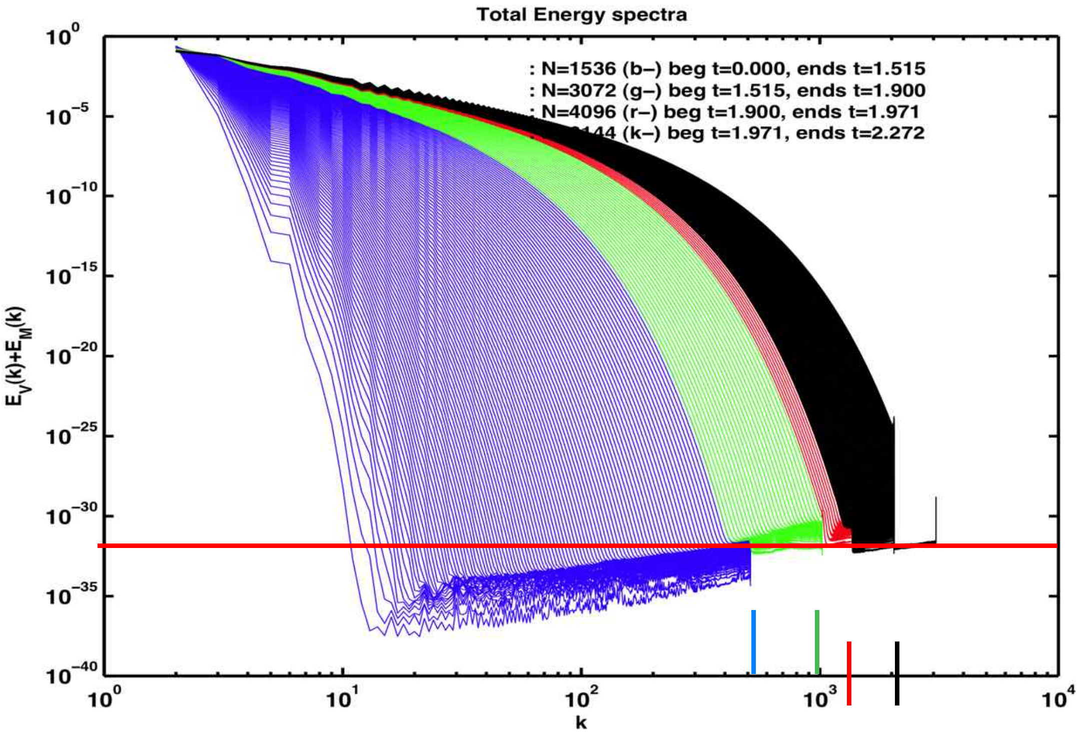


Total energy spectra, at different times and computed in a sequence of runs at different (*equivalent)*resolutions:

1536^3
3072
4096
6144^3

But is it reliable?

INCITE (DOE) award
Rosenberg et al., in preparation



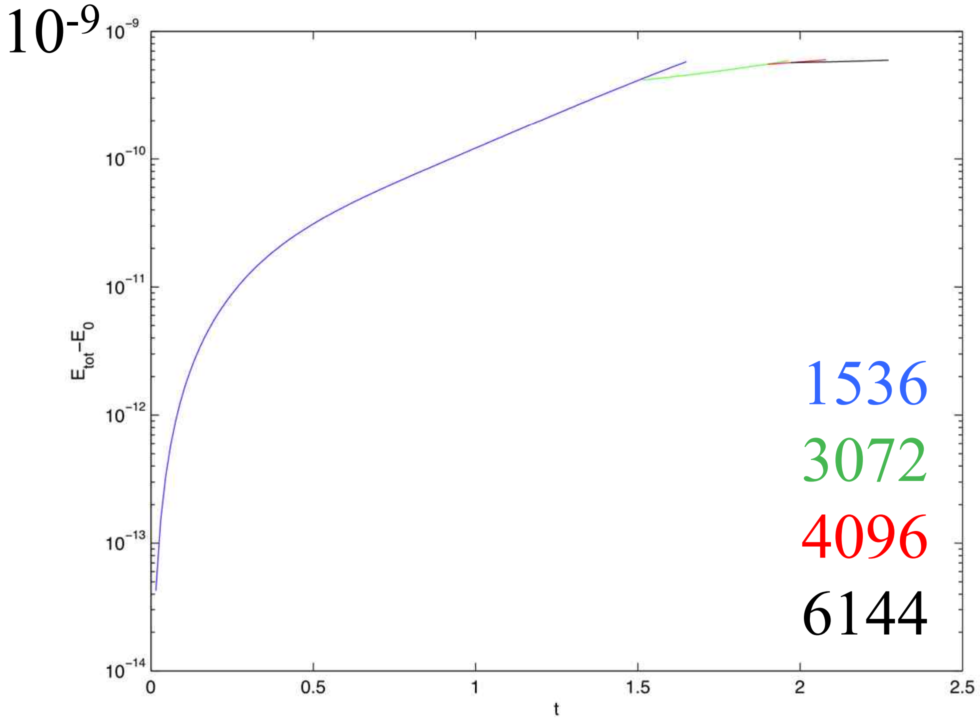
3D ideal dynamics in MHD, $B_0 = 0$

Total energy spectra, at different times and computed in a sequence of runs at different (*equivalent*)resolutions:

1536^3
3072
4096
6144^3
But is it reliable?

INCITE (DOE) award

Anisotropy spectra with moderate B_0: *Grappin Mueller PRE 2010*



Ideal MHD in 3D

Temporal evolution of the total energy error ($E_T(0)\sim 1$):

$$\mathbf{E_T(t) - E_T(0)}$$

on grids with different resolutions
(with some temporal overlap)

INCITE (DOE) award

In the future:

- Examine the ideal dynamics at high resolution (6144^3) for its singularity properties, up until the energy reaches the grid (log. decrement $\delta \sim \Delta x$)

- Continue the 4096^3 runs to long times to see the intermediate time & intermediate scale turbulent ideal dynamics with different initial conditions

Link with (fast) reconnection for the large-scale flow?

Third case study:

Small amounts of relative kinetic helicity can drive large-scale dynamos, given sufficient scale separation between the forcing scale & the largest resolved scale

Numerical set-up for Case 3

- Periodic boundary conditions, pseudo-spectral code, de-aliased with the 2/3 rule, *no imposed symmetries*
- Direct numerical simulations, from 192^3 to 512^3 points

- No imposed uniform magnetic field (B_0=0), P_M= 4
- Velocity forcing at $1< k_F / k_{min} < 6$, $T_{corr} \sim 0.1$, $T_{NL} \sim 4.2$

Relative helicity f_h of the forcing between 1% and 90%

$$\boxed{f_h = <u. \omega> /[<u\text{^}2> <\omega\text{^}2>]^{1/2}}$$

GHOST

- **G**eophysical **H**igh **O**rder **S**uite for **T**urbulence *(Gomez & Mininni)*

- Community code
- Pseudo spectral, incompressible Navier-Stokes (including rotation and passive scalar), and magnetic fields (MHD, with or w/o Hall term); it also includes some LES (the alpha model; a helical spectral model)

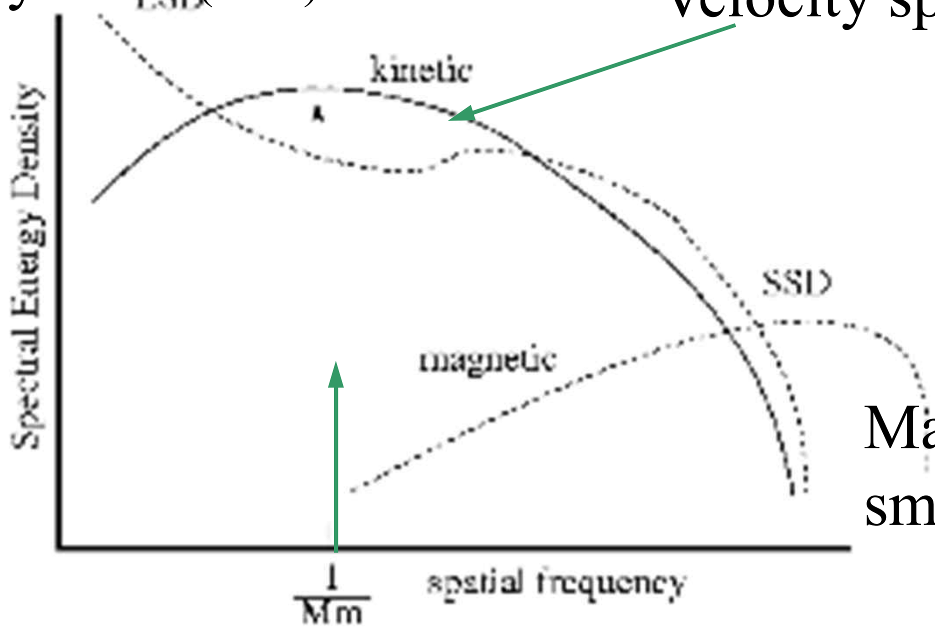
- The code parallelizes linearly up to 40,000 processors using hybrid Open-MP/MPI *(Mininni et al. 2011, Parallel Computing* ***37****)*

- Community Data *(2048^3 forced Navier-Stokes turbulence with and without helicity; 1536^3 and 3072^3 helically forced rotating turbulence; 1536^3 decaying turbulence with a magnetic field, 2048^3 MHD with symmetries). [3D visualization with VAPOR freeware]*

Small-scale (SSD) *vs.* Large-Scale (LSD) Dynamos

Magnetic spectrum:
large-scale dynamo (LSD)

Velocity spectrum



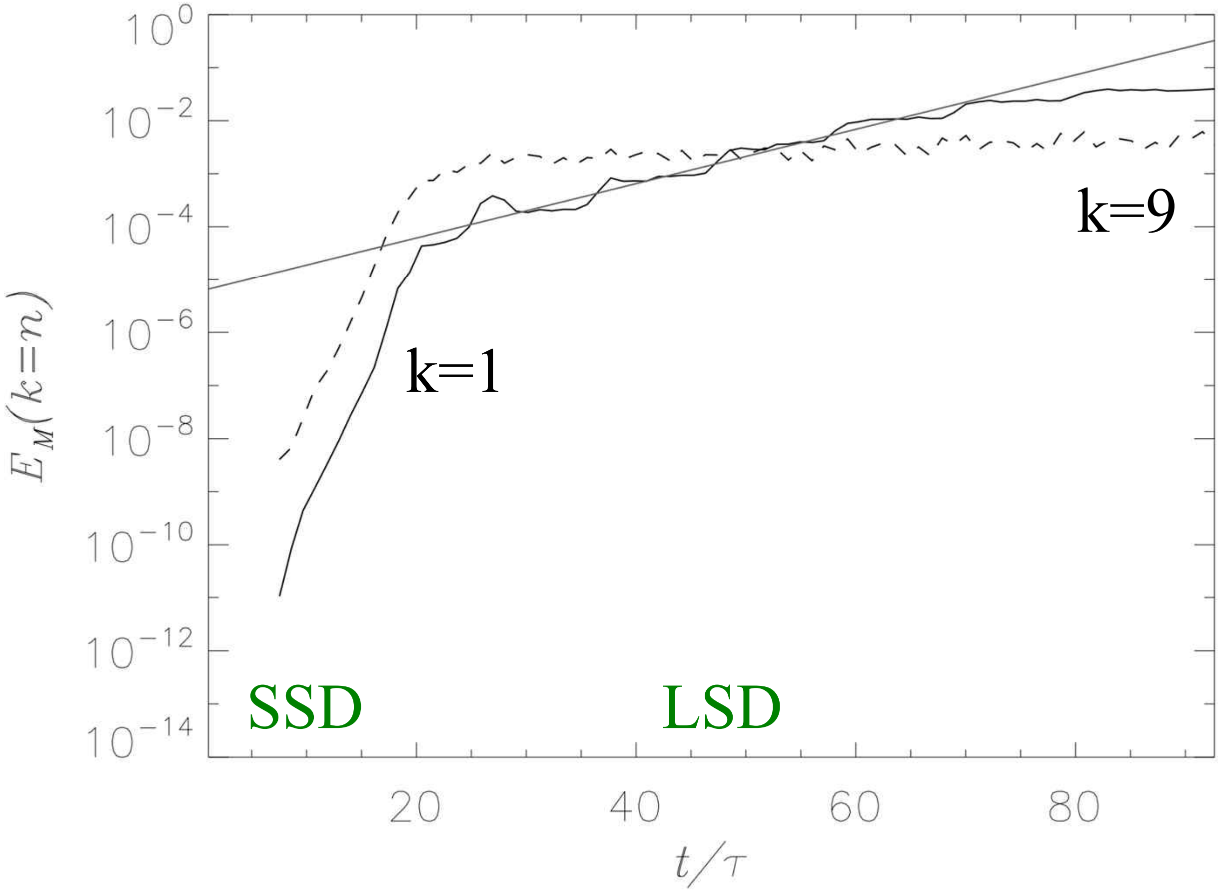
Magnetic spectrum:
small-scale dynamo (SSD)

forcing

Slide after Jonathan Pietarila-Graham

Modal magnetic energy as a function of time

Run 384-60
k_f=4



Saturation at small scale, continued slower growth at large scale

Conceptual framework

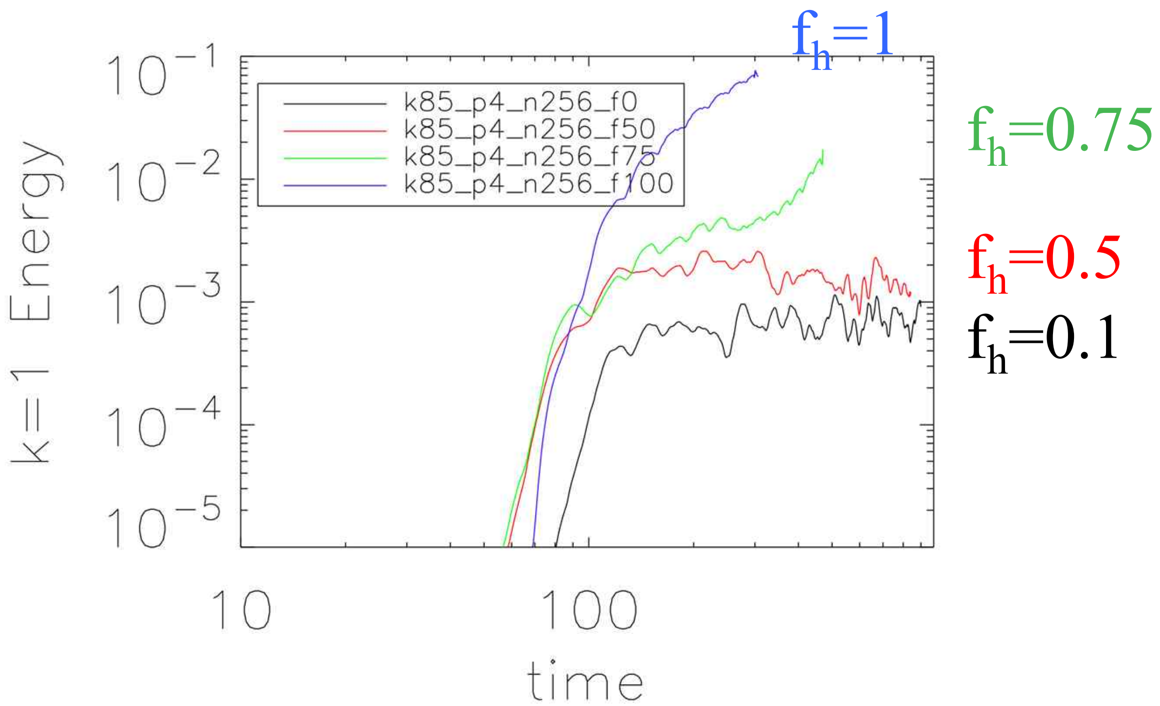
- Periodic dynamo, large-scale (LS) and small-scale (ss) fields; relative helicity $H_R(k) = H_V(k) - k^2 H_M(k) = H_V(k) - H_J(k)$ *(PFL '76)*

- Small-scale field grows through stretching of field lines, like vorticity

- Large-scale field grows through relative helicity
- Early times: kinetic helicity responsible for growth of LS helical field
- Magnetic helicity conservation implies that large-scale M-helicity leads to small-scale M-helicity of the opposite sign

- The growth of B at large scale is responsible for the decrease of small-scale H_R *(through Alfvén waves: the faster the smaller the scale), thereby stabilizing the LS field at some given scale*

Growth of large-scale field (k=1) for different relative helicity



N=256, k_f=3

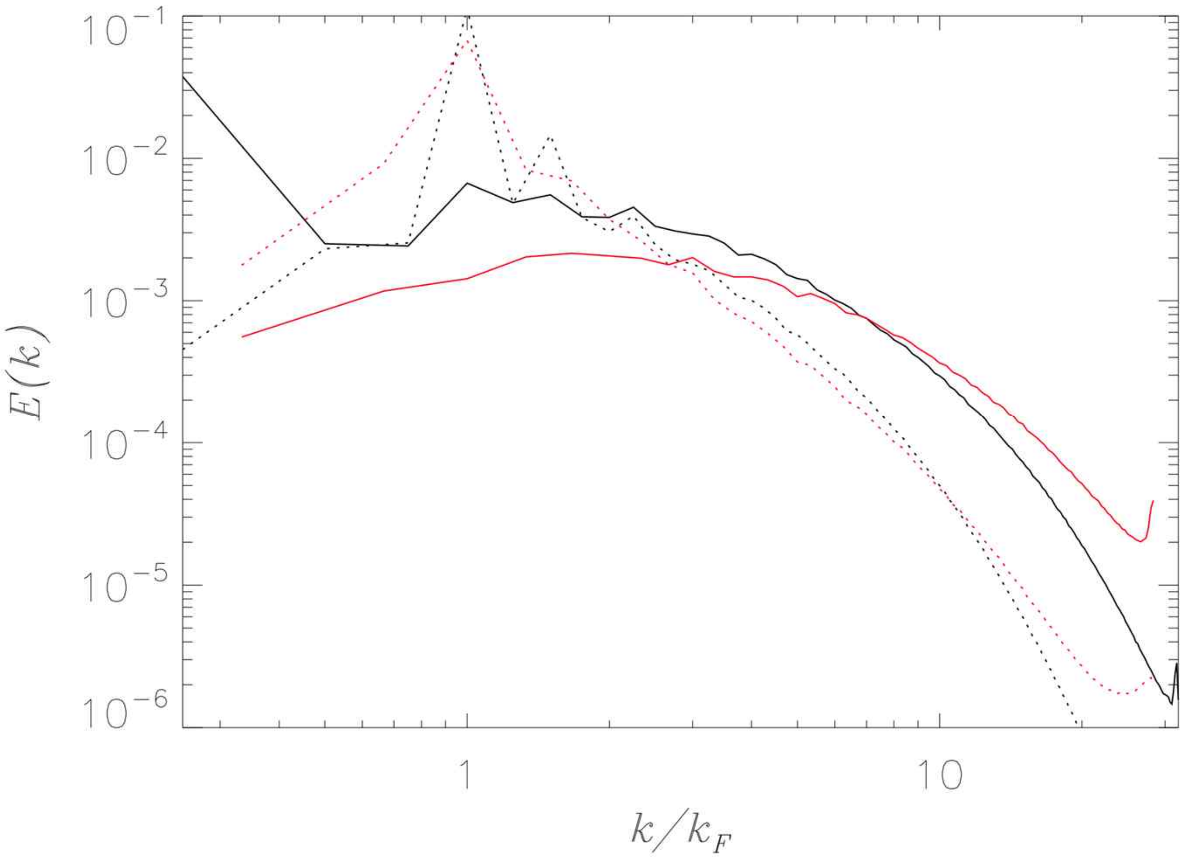
Kinetic and magnetic energy spectra as a function of k/k_f for **fixed** 60% **relative helicity**, after 90 T_{NL} and with two different scale separations

384^3, $k_f=4$

versus

256^3, $k_f=3$

Solid: E_M
Dashed: E_V



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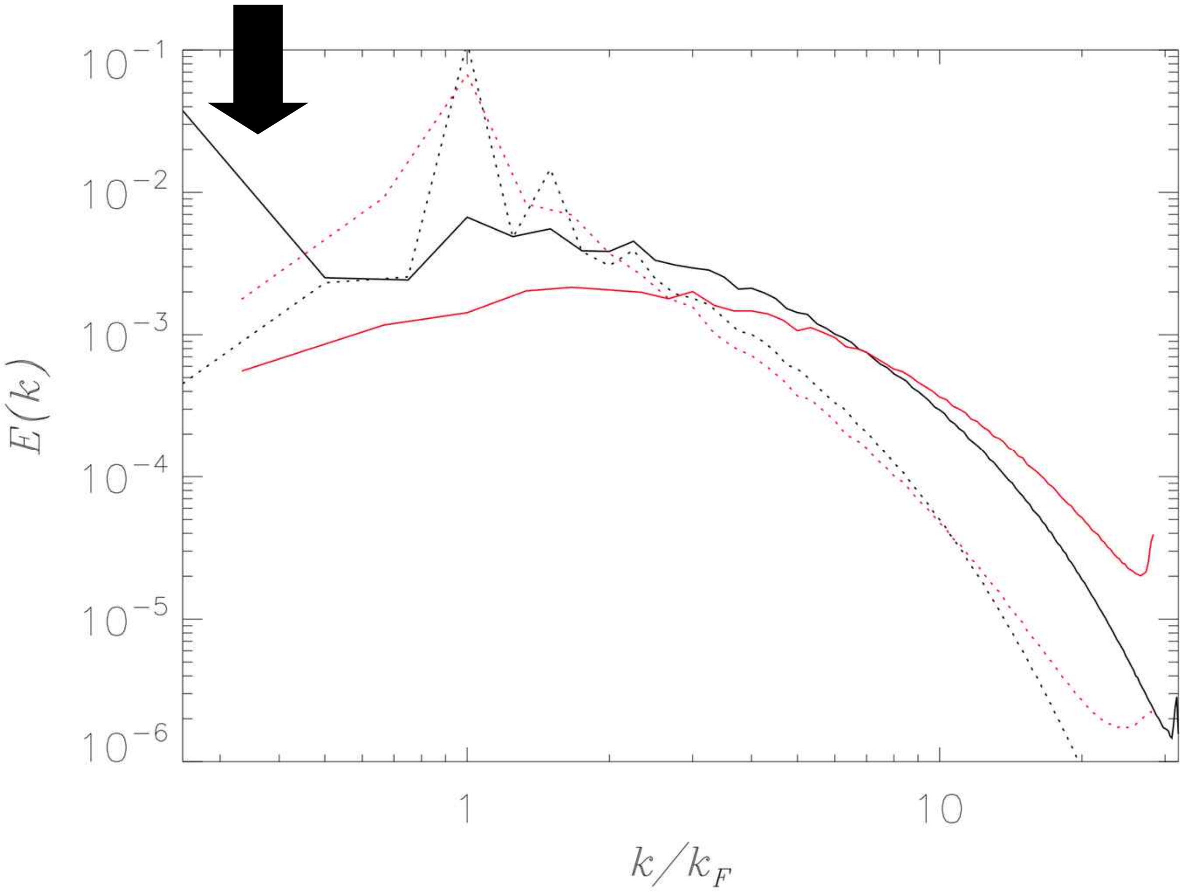
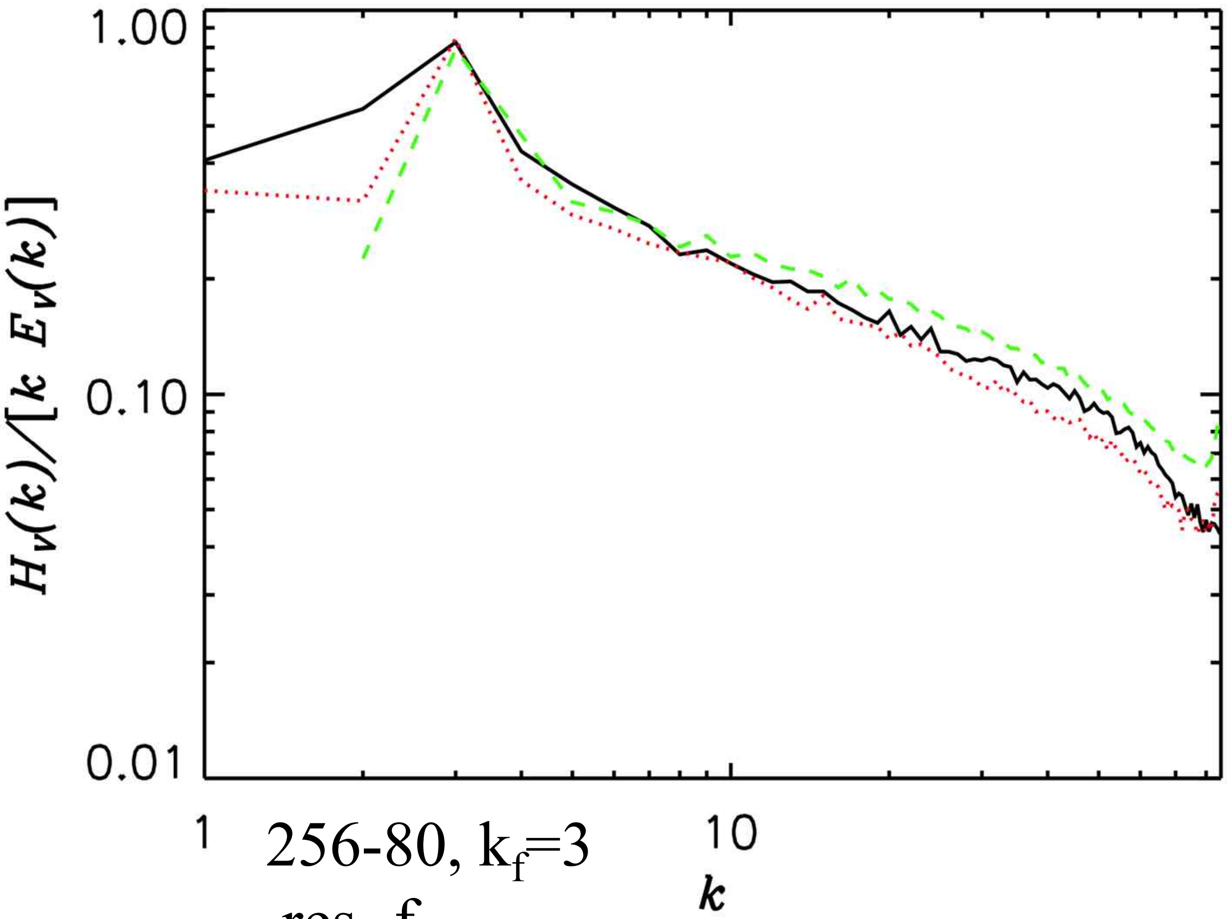


Spectra of the relative degree of alignment between the velocity field and the vorticity

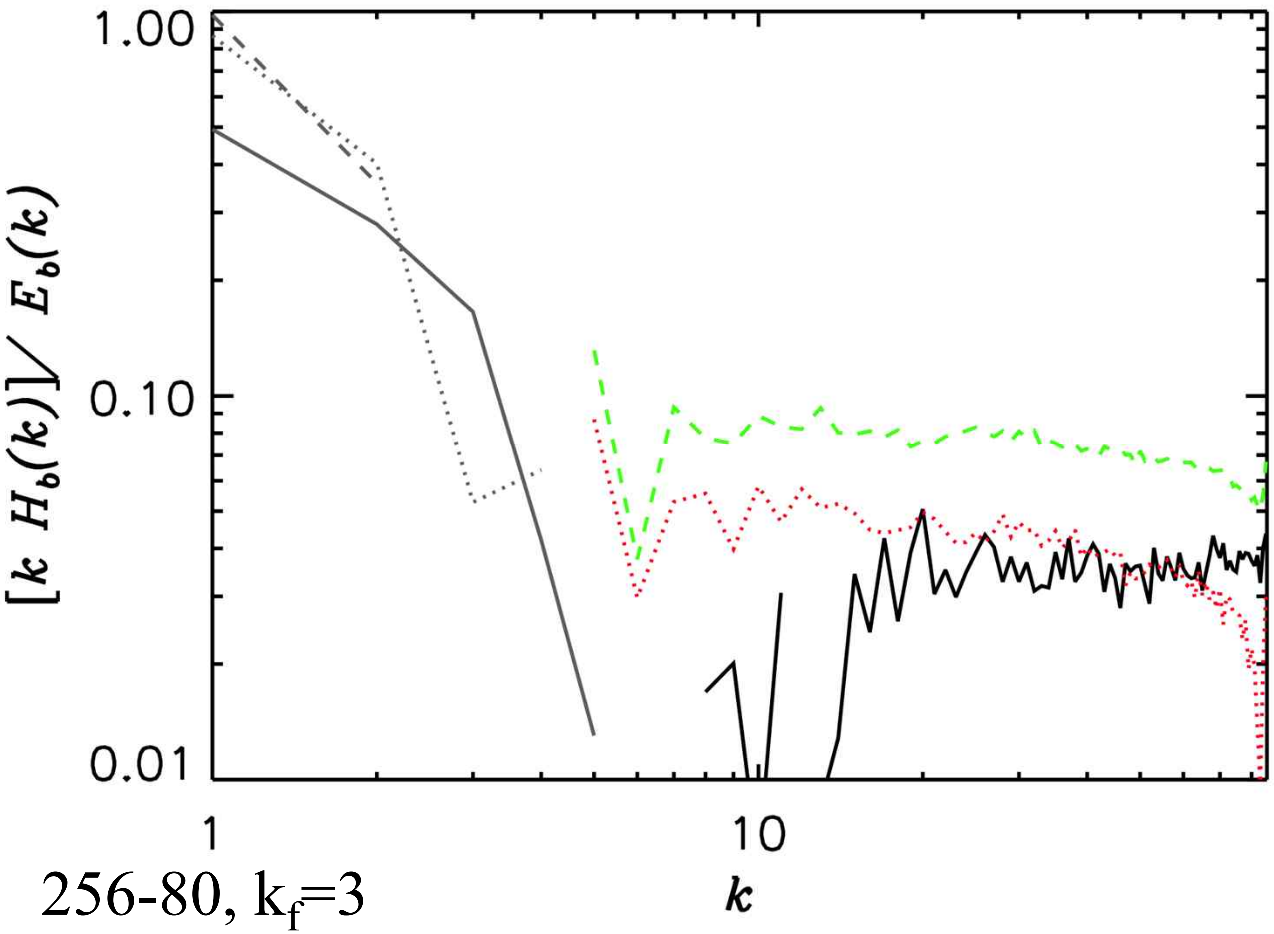


256-80, k_f=3
res.-f_h

3 different time averag.:

Black/solid: 30 - 70
Red/dots: 150 - 250
Green/dash: 900 -1000

Spectra of the relative degree of alignment of the magnetic field and magnetic potential



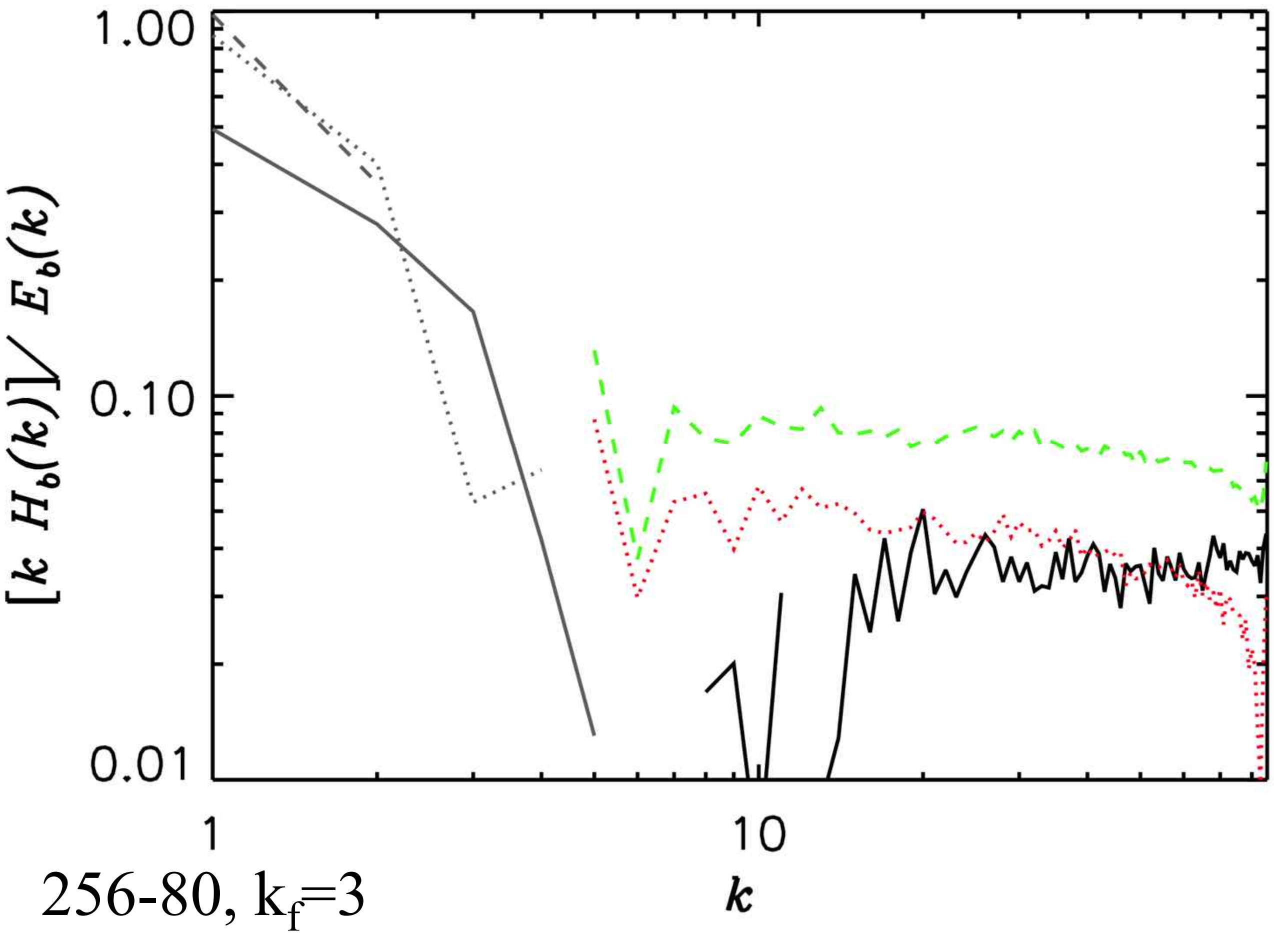
256-80, k_f=3

3 different times

$t_1 < t_2 < t_3$

solid dot dash

Spectra of the relative degree of alignment of the magnetic field and magnetic potential



256-80, k_f=3

Large-scale force-free field

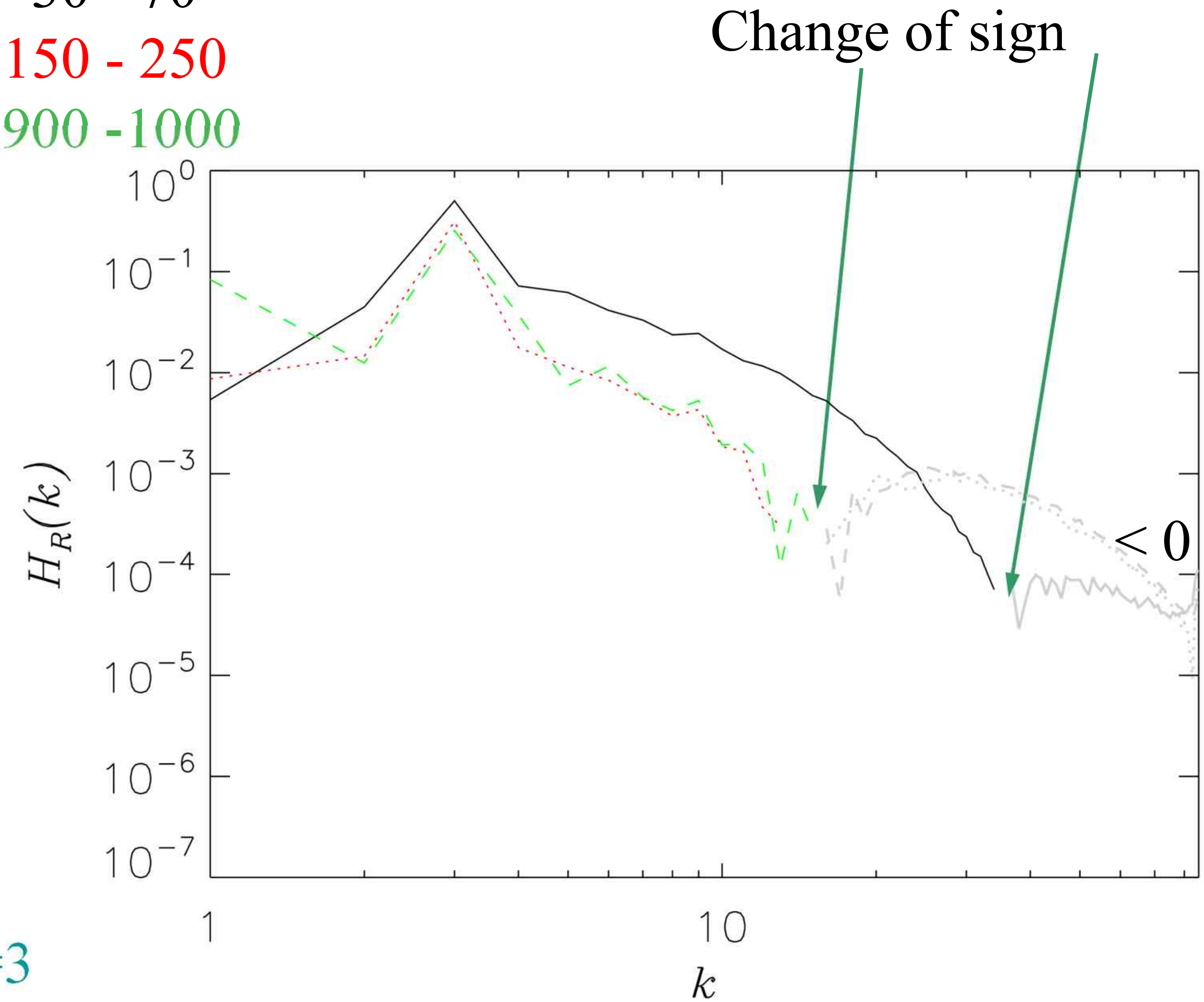
3 different times

$t_1 < t_2 < t_3$

solid dot dash

Residual helicity $H_R(K)=H_V-k^2H_M$
Temporal average in [., .]

Black/solid: 30 - 70
Red/dots: 150 - 250
Green/dash: 900 -1000



256-80, k_f=3

Previous numerical study of that issue:

Maron & Blackman 2002

- $f_h \equiv \langle \mathbf{v} \cdot \omega \rangle / \sqrt{\langle v^2 \rangle \langle \omega^2 \rangle}$
- 64^3 simulation
- Forcing wavenumber, $k_F = 4.5$
- $Re_M \sim 150$
- Critical threshold
  - $f_{h,C} \sim 0.5$ for $Pr_M = 3$
  - $f_{h,C} \sim 0.7$ for $Pr_M = 9$

→ One needs a substantial amount of relative helicity for such parameters

Slide after Jonathan Pietarila-Graham

Runs with resolutions from 192^3 to 512^3, with forcing at k_F=1 to 6, with relative helicity of the forcing between 1% and 90%, $T_{NL}\sim 4$, *and with magnetic Reynolds number Re_M of the order of 2000*

k_F	Run	Re_M	k_{seed}	γ_{SSD}	$\gamma_{k=1}$	E_b^s	$-100H_b$
2	192-70	1500	[6.7,10.7]	0.26	$(-5.6\pm2.8)10^{-3}$	0.2	$0.85f$
	192-80	1600	–	0.25	$(-8.2\pm2.8)10^{-3}$	0.25	$0.85f$
	192-90	1600	–	0.27	$(4.8\pm12)10^{-3}$	0.25	$1.5g$
3	256-40	1900	[10,16]	0.28	$(-0.7\pm7.8)10^{-4}$	0.1	$0.1f$
	256-60	1900	–	0.29	$(1.3\pm7.5)10^{-4}$	0.1	$0.2f$
	256-69	1700	–	0.26	$(5.9\pm0.7)10^{-3}$	0.1	$1.0g$
	256-80	1900	–	0.31	$(8.3\pm1.5)10^{-3}$	0.15*	$3g^*$
4	384-10	1600	[13.3,21.3]	0.24	$(-1.6\pm2.0)10^{-3}$	0.04	$0.008f$
	384-20	1600	–	0.29	$(5.9\pm0.6)10^{-3}$	0.04	$0.06g$

Res-f_h

Small-scale growth rate

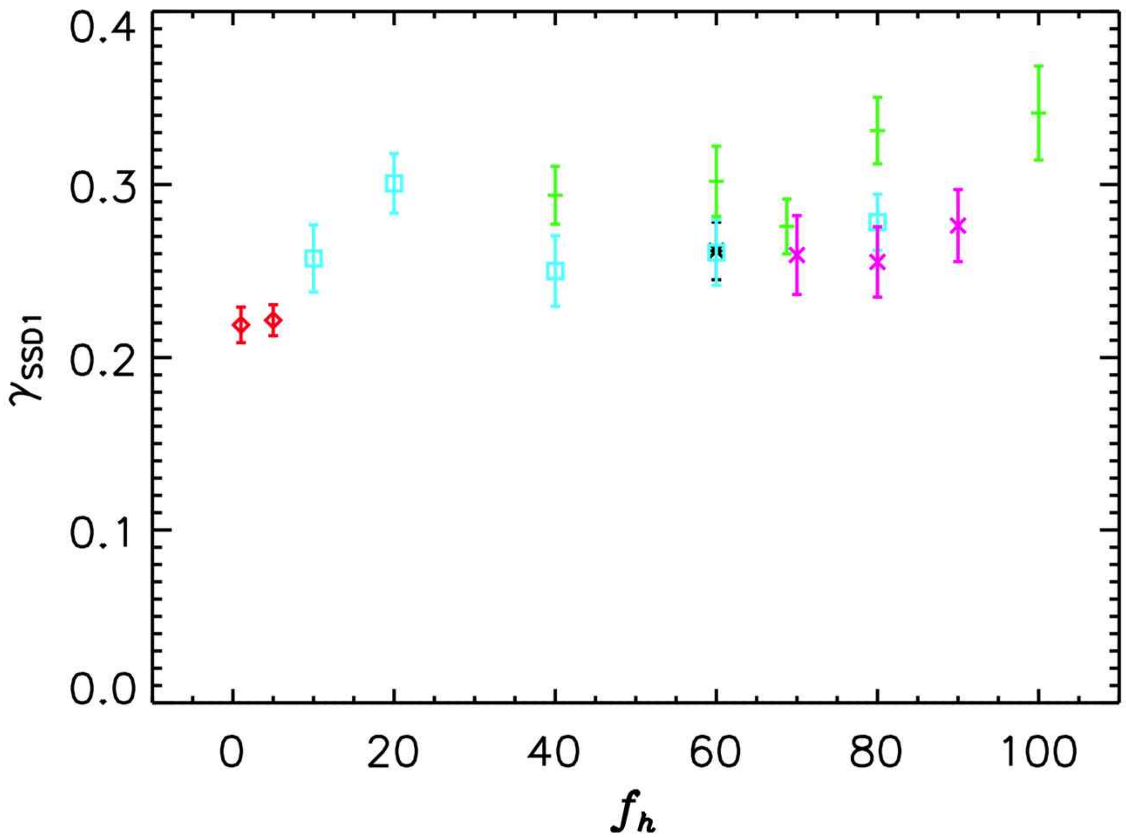
Large-scale growth rate after saturation of the SSD

Run	Re_M	k_{seed}	γ_{SSD}	$\gamma_{k=1}$	E_b^s	$-100H_b$	k_F
384-40	1500	[13.3,21.3]	0.24	$(1.5 \pm 0.1)10^{-2}$	N/A	N/A	
384-60	1500	–	0.25	$(2.8 \pm 0.2)10^{-2}$	0.1	$1.0g$	4
384-80	1500	–	0.27	$(2.8 \pm 0.3)10^{-2}$	N/A	N/A	
432-05	2100	[16.7,26.7]	0.24	$(-1.2 \pm 2.9)10^{-3}$	0.04	$0.008f$	
432-07	2000	–	0.23	$(0.6 \pm 3.9)10^{-3}$	0.03	$0.004f$	5
432-09	2000	–	0.24	$(1.1 \pm 0.4)10^{-2}$	0.03	$0.004g$	
512-01	1500	[20,32]	0.21	$(6.1 \pm 6.1)10^{-3}$	0.01	$\pm 5 \cdot 10^{-4} f$	
512-05	1500	–	0.21	$(3.3 \pm 0.7)10^{-2}$	0.01	$1.2 \cdot 10^{-3} g$	6

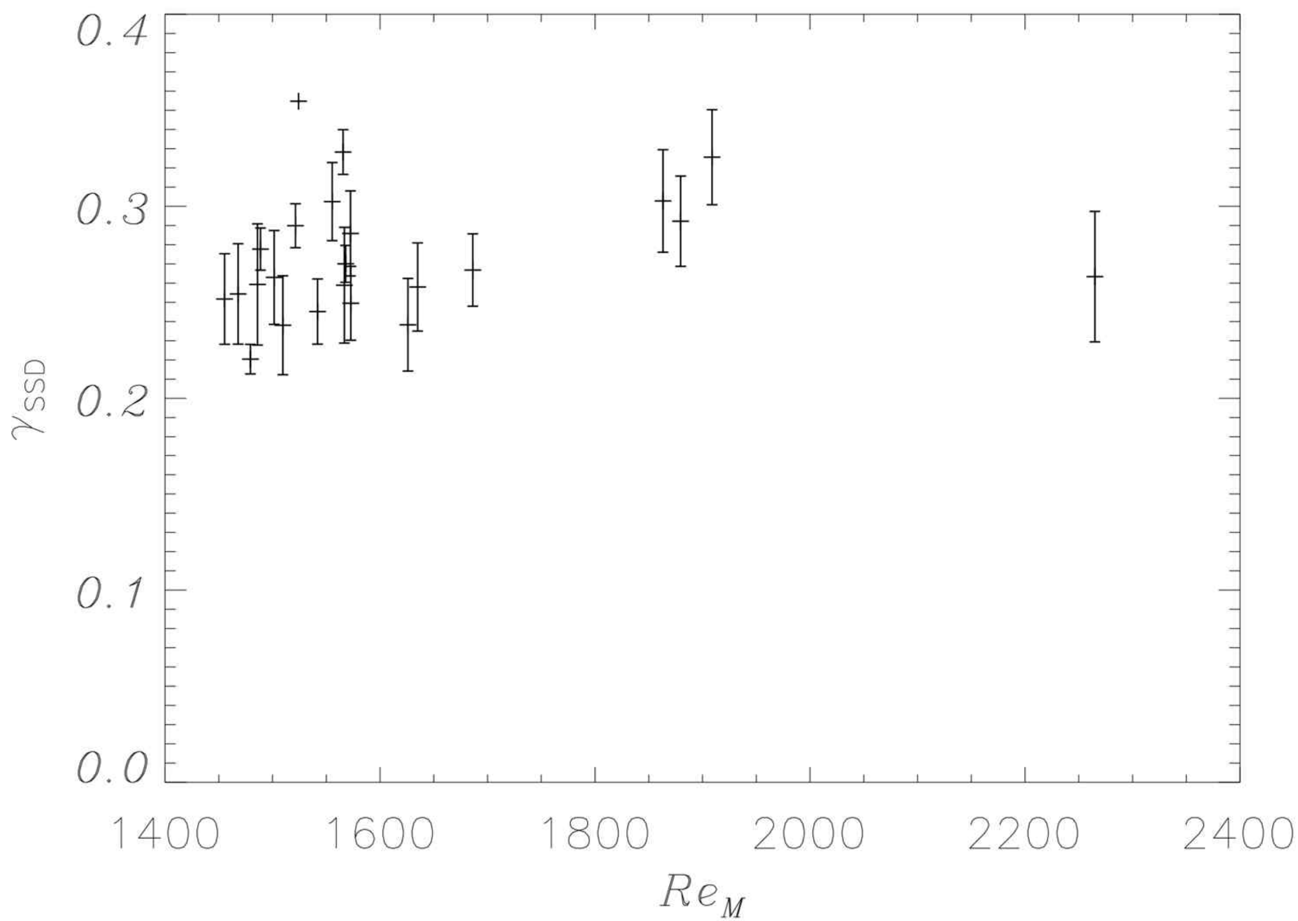
Res-f_h

Small-scale *vs.* Large-scale growth rates

Growth rate of small-scale field as a function of relative helicity

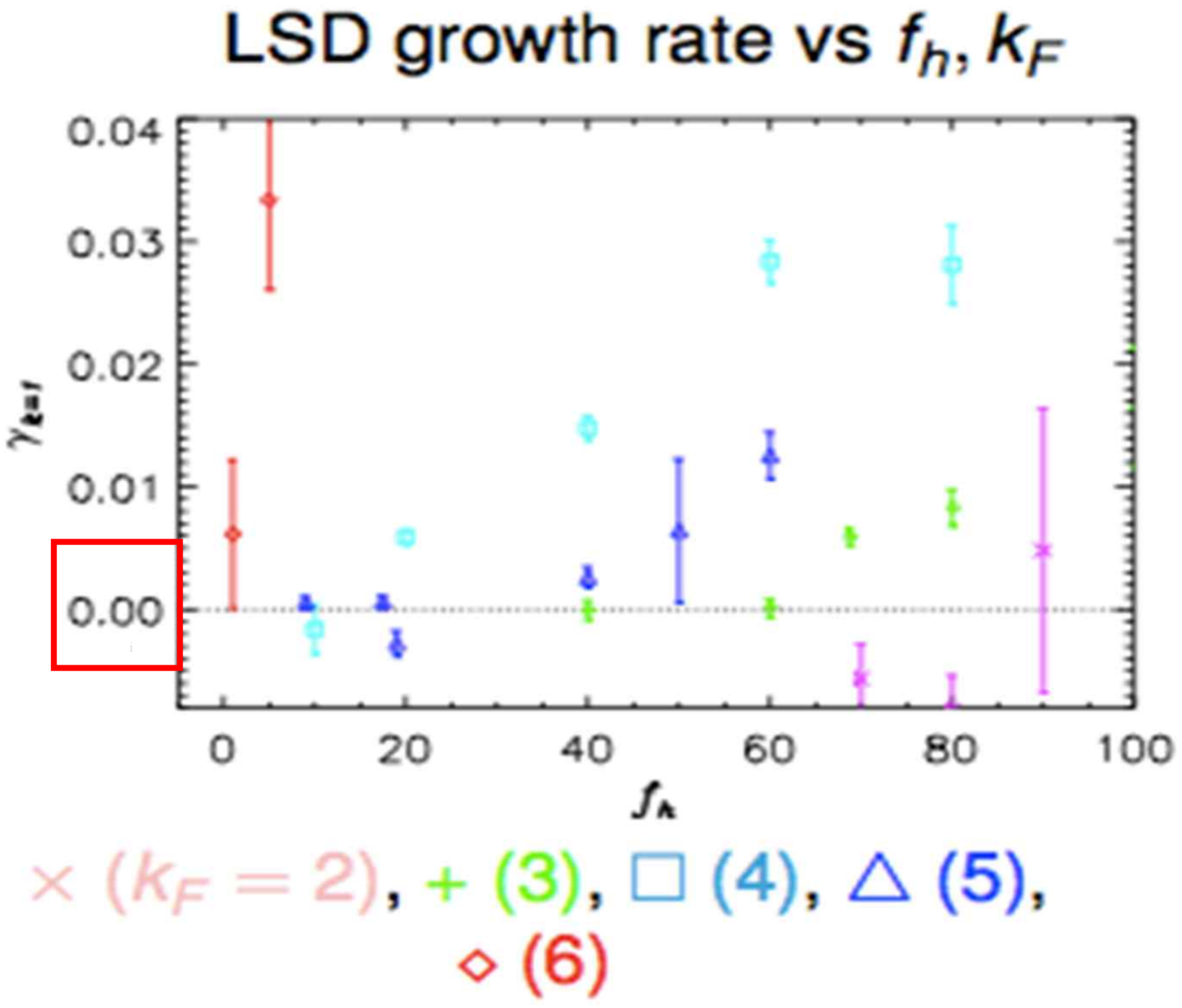


Growth rate of small-scale field as a function of magnetic Reynolds number

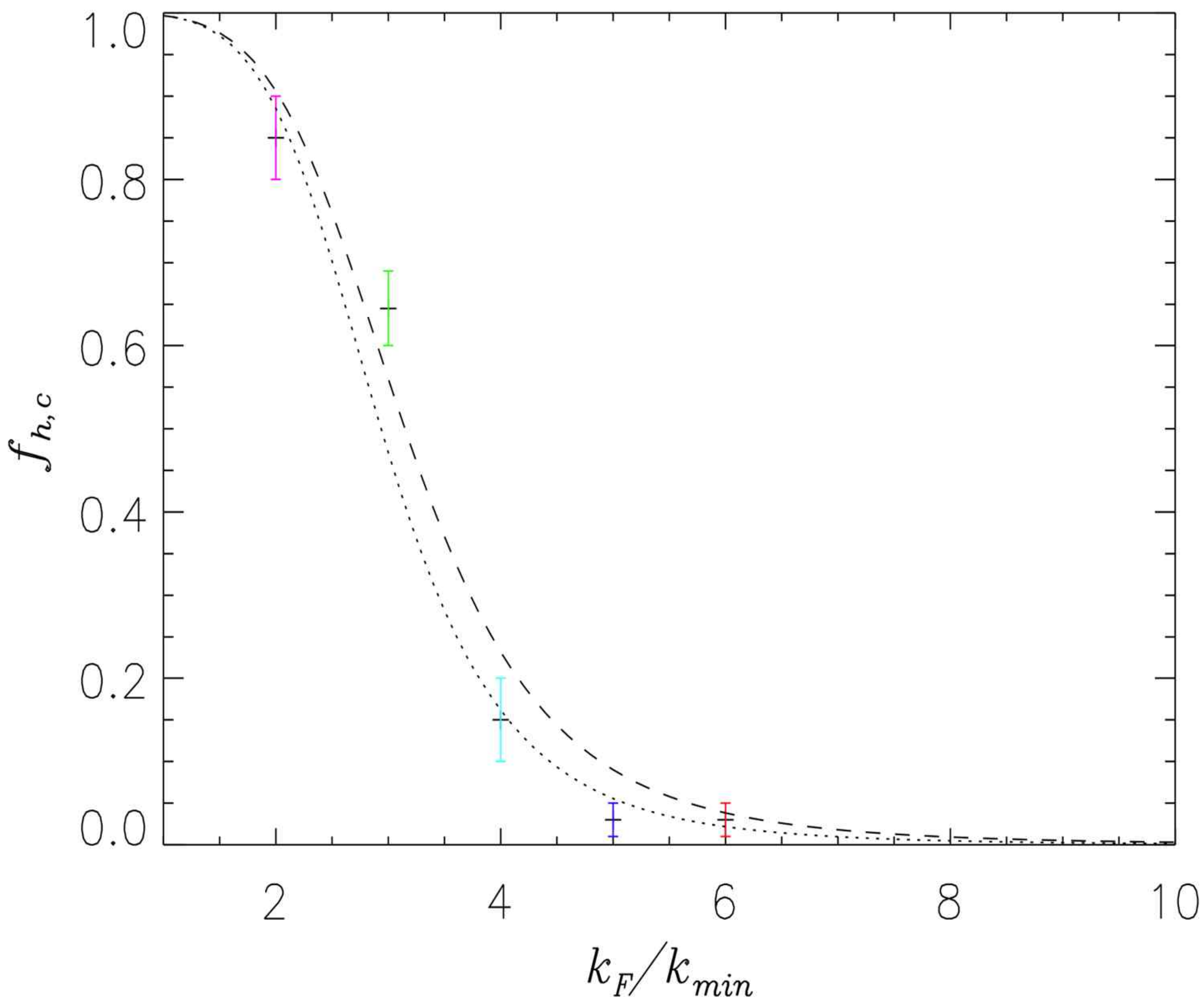


Large-scale growth-rate as a function of relative helicity f_h

for various scale separation (forcing wavenumber k_f)



Critical rate of helicity for large-scale dynamo as a function of forcing scale separation



Simple theory

- Near end of kinematic,linear SSD phase $\Rightarrow \sqrt{1-f_h}\, B^2$ non-helically-produced, $\sqrt{f_h}\, B^2$ helically-produced
- Non-helically-produced $\mathbf{j} \times \mathbf{B}$ opposes LSD
- $\boldsymbol{\omega} \times \mathbf{v}$ generates LSD
- Kazantsev $B(k)^2 \sim k^{3/2}$ spectrum $\Rightarrow$ $[\mathbf{j} \times \mathbf{B}](k_{min}) \sim (k_{min}/k_F)^{5/2}$
- $\sqrt{1-f_h}(k_{min}/k_F)^{5/2} \propto \sqrt{f_h}$
- $f_{h,C} = (1 + C^2(k_F/k_{min})^{-5})$ where $C = (k_{min}v_{rms}^2)/(k_{SS}B_F^2)$
- $f_{h,C} \sim (k_F/k_{min})^{-5}$ as $k_F/k_{min} \to \infty$

Slides after Jonathan Pietarila-Graham

Conclusion and questions

- With sufficient scale separation, at a given magnetic Reynolds number, the large-scale field grows, with $f_h^c < 0.05$ for $k_F=6$

** Does the result persist when one increases the Reynolds number?*

Park & Blackman, 2011

** What is the effect of the magnetic Prandtl number?*

** Is there an even/odd variation in growth rates & f_h^c?*

Thank you for your attention!