PV Dynamics and Phase Space Structure Evolution in Vlasov and Drift WaveTurbulence

OR:

Whatever Happened to Dynamical Friction?

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Sequence of Two Related Presentations:

- P.D.
  - ✓ Basic theory of phase space structure dynamics in Vlasov and Drift Wave Turbulence
  - ✓ Dupree-Balescu-Lenard theory for turbulent relaxation in collisionless drift wave turbulence
- Maxime Lesur
  - ✓ Simulation of *subcritical* phase structure growth and dynamics in the Berk-Breizman model

Outline

- Motivating Issues
  - Whatever happened to dynamical friction (anti-friction) in models of turbulent transport?
  - how accommodate dynamical friction in models of turbulent relaxation?
- OV of relevant paradigms
  - commonalities of Vlasov and drift wave (QG) systems
  - single structure dynamics
    - G.I. Taylor's take on Rayleigh criterion and implications for zonal flows
    - B+B model : dynamical anti-friction by *collisionless dissipative coupling*

Outline (cont'd)

- CDIA : dynamical friction by momentum exchange
- collisionless DW-ZF interaction
  - dynamical friction by ambipolarity + polarization charge flux
  - kinetic pseudo-momentum and relation to Charney-Drazin non-acceleration theorem
- multi-structure calculation
  - generic structure of mean field relaxation
  - D-B-L equation for drift wave turbulence
  - Relation to momentum theorems

- Outlook for Future Work

I.) Motivation

→ Whatever Happened to Dynamical Friction?

- In M.F.E. studies, transport usually proclaimed to be ``in agreement with quasi-linear prediction'' (?)
- Mean Field Electrodynamics (MFE?!) is effectively exercise in quasi-linear theory as applied to MHD EMF
- paradox: → Q.L.T. $\tau_{a,c} < \tau_{str,circ}$ $\tau_{ac} \sim (\Delta\bar{\omega}|_{res})^{-1}$

  requires Kubo # $K = \tau_{ac}\dfrac{\tilde{v}}{\Delta} < 1$

  → saturation levels ``$\sim$ mixing length prediction''

  M.L.T. → L.T. $\sim$ N.L.T. → $K \sim 1$

  "Critical Balance" ↔ K ~ 1 for Alfvenic turbulence

How reconcile? ⟷ transport modeling studies are often rather simplified

- Generally:
  - Q.L.T. views transport as ensemble of $\sim$ linear waves and/or wave responses
  - yet real turbulence is ``soup'' of structures, eddys, blobs, holes, etc. (c.f. Lesur, this meeting)

→ structures in the ensemble (`soup') should exert drag on one another!

- akin to collisions, but collectively mediated, via screening
  → wave radiation transmits energy/momentum from structure → fluid, as in a ship wake

- manifested in $\partial_t \langle f \rangle$ via drag

$\therefore$ enter dynamical friction! <u>i.e.</u>

$$\partial_t \langle f \rangle = \partial_v D \partial_v \langle f \rangle$$

$$\Rightarrow \partial_t \langle f \rangle = -\partial_v \left(-D \partial_v \langle f \rangle + F \langle f \rangle \right)$$

↑ drag effect $\sim \mathrm{Im}\,\varepsilon$

QLT

$$D = D_{QL} = \sum_{k,\omega} \langle \tilde{E}^2_{k,\omega} \rangle \pi \delta(\omega - kv)$$

- analogous to Cerenkov wake effect in Balescu-Lenard screened Landau collision operator

- can allow novel relaxation mechanism via momentum exchange not based on linear waves, etc

- DISCLAIMER: calculational approaches (D-B-L) *not* rigorous, but at least *try* to confront relevant effects in K ~ 1 regime

N.B.: Mean field electrodynamics validity also dubious

→ How was it missed?

- it wasn't! - pioneering theoretical work by Lynden-Bell, Dupree, Kadomtsev; '60s - '70s
  → initiated theory of phase space structure turbulence

- applications to non-trivial DWT, CTEM systems: P.D.,Terry, Hahm; '80s

- discovery of subcritical CDIA instability in simulation, Berman,et. al. '80s

- coherent structures (phase space holes), Dupree '80s-'90s

then studiously forgotten...

- Exception: B+B → reduced model of E.P. resonance dynamics akin B-O-T

  c.f. Breizman, N.F. '11 → emergent BGK mode solution
  also this conference

- SLOWLY re-remerging in glorious epoch of massively-parallelized HPC...

II.) OV of Paradigms

Comparison of QG, Vlasov/GK dynamics ↔ Role of Conserved PV

QG, GK systems structurally similar, i.e. $q = \beta y + \omega \quad f = \langle f \rangle + \delta f$

	QG system	Vlasov, GK system
Dynamical variable	PV, $q(x,t)$	distribution function, $f(x,v,t)$
Time evolution	$dq/dt = \partial_t q + \{q, \phi\} = 0$	$df/dt = \partial_t f + \{f, H\} = 0$
Circulation	$\Gamma = \oint (V + 2\Omega a \sin\theta) dl$	$\Gamma = \oint \mathbf{v} \cdot d\mathbf{x}$
Kelvin's Thm.	Yes	Yes (Lynden-Bell, '67)
Feedback	PV, $q = \nabla^2 \phi + F(\phi, n)$	GK Poisson, Pol Charge $\int d^3 v f + \rho_s^2 \nabla^2 \phi = g(\phi, n_e, \ldots)$
ZF Generation	Vorticity Flux	Pol. Charge Flux
C-D Theorem	Yes	???

Some general observations:

- GK Poisson equation links fluid vorticity to kinetic dynamics
- Spatial flux of polarization charge is underpinning of Z.F. generation mechanism in GK systems
- C-D Theorem for GK systems!? - has Kelvin's Theorem!

Yes, as has Kelvin's Theorem!

- Phase space density correlation similar to potential enstrophy

$\langle \delta f^2 \rangle \leftrightarrow \langle \delta q^2 \rangle$ → $\langle \delta f^2 \rangle \sim$ potential enstrophy in phase space

→ phasetrophy D+I+I, '10

- Phasetrophy: comparison to other physical quantity

Quasi-geostrophic system		Gyrokinetic system	
Potential enstrophy $\langle \delta q^2 \rangle$	$\Leftrightarrow$	Phase space density correlation $\langle \delta f(1) \delta f(2) \rangle$	↘ Fluctuation entropy $\int \sqrt{E} dE \langle \delta f^2 \rangle / \langle f \rangle$ $\Rightarrow$ entropy budget
↓		↓	
Wave activity density $\langle \delta q^2 \rangle / \langle q \rangle'$ → pseudomomentum	$\Leftrightarrow$	Kinetic wave activity density $\int \sqrt{E} dE \langle \delta f^2 \rangle / \langle f \rangle' \vert_0$ → pseudomomentum $\Rightarrow$ momentum budget	

II) Examples: Single Structure Dynamics → What Can We Learn?

– G.I. Taylor's take on Rayleigh criterion

- consider effect on (zonal) flow by displacement of PV: δy

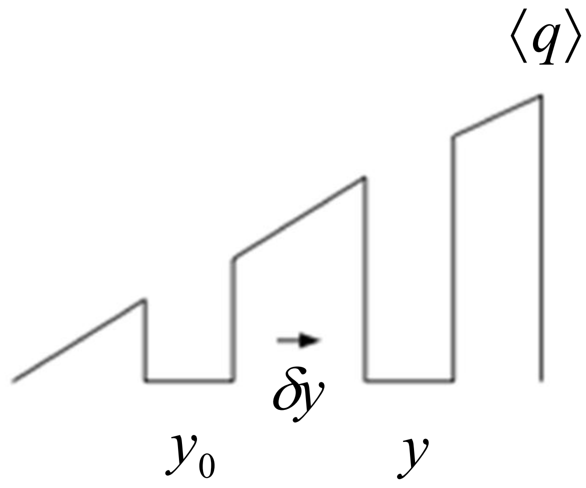
$$\frac{\partial}{\partial t}\langle v_x \rangle = \langle \tilde{v}_y \tilde{q} \rangle$$

$$\tilde{q} = (\text{PV of vorticity blob at y}) - (\text{mean PV at y})$$

$$\langle q(y) \rangle = \langle q(y_0) \rangle + (y - y_0)\frac{d\langle q \rangle}{dy}\bigg|_{y_0}$$

Small displacement

$$\therefore \frac{\partial}{\partial t}\langle v_x \rangle = -\langle \tilde{v}_y \delta y \rangle \frac{d\langle q \rangle}{dy} = -\left(\partial_t \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \frac{d\langle q \rangle}{dy}\right)$$



So, for instability

$$\begin{cases} \partial_t \langle \tilde{\varepsilon}^2 \rangle > 0 & \text{; growing displacement} \\ \dfrac{\partial}{\partial t} \displaystyle\int_{-a}^{a} dy \langle v_x \rangle = 0 & \text{; momentum conservation} \end{cases}$$

$$-\int_{-a}^{a} dy \left(\partial_t \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \right) \frac{d\langle q \rangle}{dy} = 0$$

$\dfrac{d\langle q \rangle}{dy}$ must change sign within flow interval

$\Rightarrow$ inflection point

also,

$$\frac{\partial}{\partial t} \left\{ \langle v_x \rangle + \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \frac{d\langle q \rangle}{dy} \right\} = 0 \qquad \tilde{q} = -\tilde{\varepsilon} \frac{d\langle q \rangle}{dy}$$

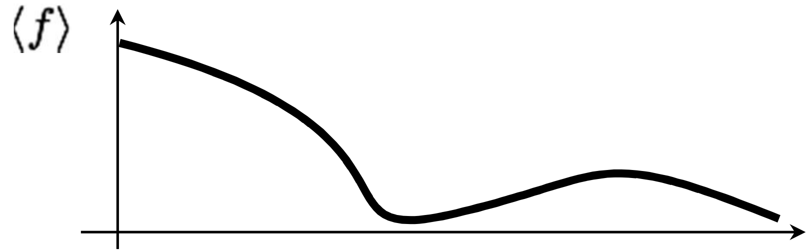
$$\Rightarrow \boxed{\frac{\partial}{\partial t} \left\{ \langle v_x \rangle - \left(- \frac{\langle \tilde{q}^2 \rangle}{2\partial \langle q \rangle / \partial y} \right) \right\} = 0}$$

$-\langle \tilde{q}^2 \rangle / 2\partial \langle q \rangle / \partial y \equiv$
Pseudomomentum for QG system

→ no slip condition of flow + quasi-particle gas

→ (significant) step toward momentum theorem

→ Examples: Single Structure, cont'd



Vlasov Plasma

b.) B+B = B.O.T. + background dissipation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial v} = C(f - f_0) \quad ; \quad C(f-f_0) = \frac{\nu_f^2}{k}\frac{\partial}{\partial v}(f-f_0) + \frac{\nu_d^3}{k^2}\frac{\partial^2}{\partial v^2}(f-f_0)$$

$$\frac{\partial E}{\partial t} = -\int dvv(f-f_0) - 2\gamma_d E$$

→ generic background dissipation
(i.e. electron L.D of A.W.)

→ Consider localized δf → hole (collisionless limit)

→ displacement in velocity δV

$$f = \langle f \rangle + \delta f$$

$$\langle f \rangle = f_0 + (v-u)\left.\frac{\partial f_0}{\partial v}\right|_u = f_0 + (v-u)\frac{\partial f_0}{\partial u}$$

$$\frac{\partial}{\partial t}\int dv\delta f^2 = -2\frac{d}{dt}\int dv\langle f \rangle \delta f$$

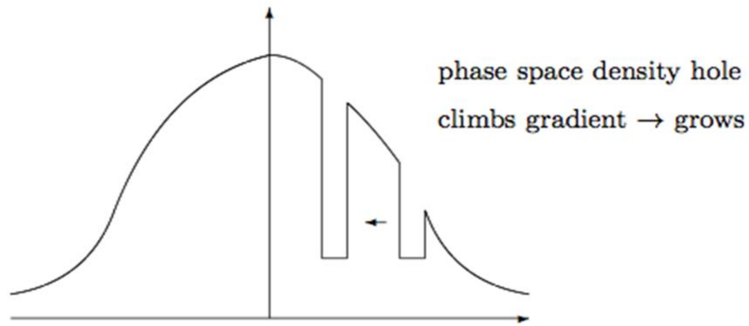


FIG. 4. hole in phase space

$$\frac{\partial}{\partial t}\int dv\delta f^2 = -2\left(\frac{\partial f_0}{\partial u}\right)\frac{d}{dt}\int dv(v-u)\delta f \equiv -2\left(\frac{\partial f_0}{\partial u}\right)\frac{d}{dt}\frac{\langle p \rangle}{m}$$

structure momentum balance is critical!

Aside: Holes and Blobs – Why? – What holds a hole together?

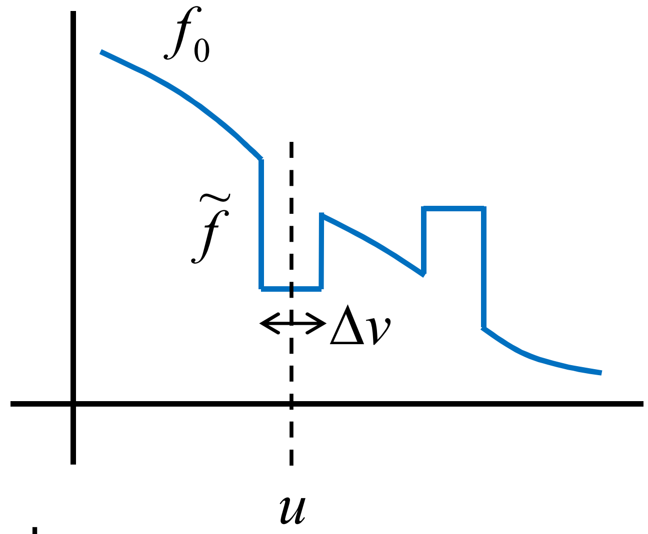
⇒ Jeans Instability

$$\omega^2 = k^2 c_s^2 - 4\pi G\rho_0$$

⇒ Plasma Electrostatic self-interaction

$$(\omega - ku)^2 = -\frac{4\pi\omega_p^2 \tilde{f}\Delta v}{\varepsilon(k,ku)} + k^2(\Delta v)^2$$

↖ screening by background



Points:

⇒ marginality ⇒ $\tilde{f} \sim \Delta v$ → size vs depth relation

⇒ ε > 0 ⇒ $\tilde{f} > 0$ is marginal ⇒ blob

ε < 0 ⇒ $\tilde{f} < 0$ is marginal ⇒ hole

$$\frac{\partial}{\partial t}\int dv\,\delta f^{2} = -2\left(\frac{\partial f_0}{\partial u}\right)\frac{d}{dt}\left(\frac{\langle p\rangle}{m}\right)$$ → structure grows by collective momentum exchange

→ For B+B

$$\partial_t \int dv \langle \delta f^2 \rangle = -\left.\frac{\partial f_0}{\partial v}\right|_u \int dv \langle \tilde{E}\delta f\rangle$$ → growth by acceleration

$$(-i\omega + 2\gamma_d)E_{k,\omega} = -\int dv v \delta f$$ → feedback

$$\partial_t \int dv \langle \delta f^2 \rangle = 2\gamma_d \left.\frac{\partial f_0}{\partial v}\right|_u \int dv' \int dv \frac{v' \langle \delta f(v')\delta f(v)\rangle}{(ku)^2 + (2\gamma_d)^2}$$

$$\boxed{\gamma \cong (\Delta v)\left.\frac{\partial f_0}{\partial v}\right|_u \frac{4\pi\gamma_d u}{(ku)^2 + (2\gamma_d)^2}}$$ basic result for structure growth rate

where $\Delta v \sim \text{hole extent} \sim (q\phi/m)^{1/2}$

- Result for B+B:

$$\gamma \cong (\Delta v) \frac{\partial f_0}{\partial v}\bigg|_u \frac{4\pi \gamma_d u}{(ku)^2 + (2\gamma_d)^2}$$

See Lesur, this meeing

observe:

- growth is nonlinear, i.e. $\gamma \sim \Delta v \sim \sqrt{\phi}$

- linear instability not required

  i.e. if δf sufficient, can have $\gamma_{L,0} < \gamma_d$ where $\gamma_{L,0} = \frac{\omega_p}{2} \frac{\pi \omega_p^2}{|k|k} \frac{\partial f_0}{\partial v}\bigg|_{\omega/k}$

  contrast $\gamma_L \sim \gamma_{L,0} - \gamma_d$

- of course, need free energy: $\partial f_0 / \partial v|_u > 0$ required

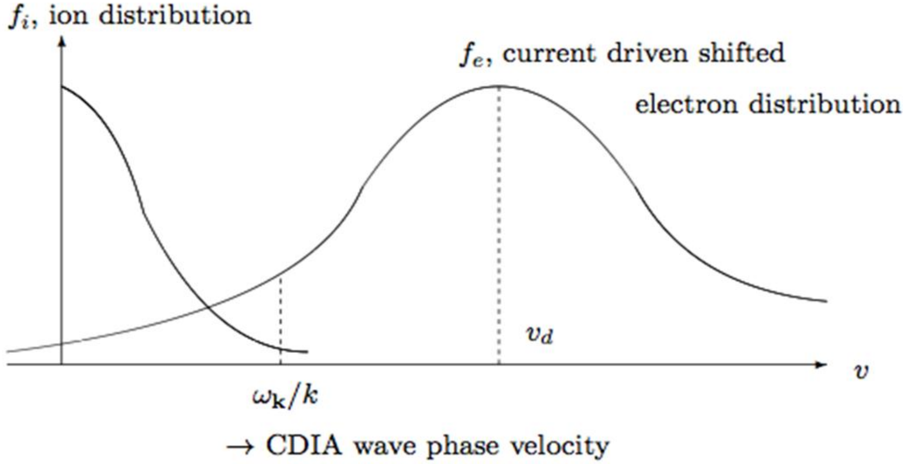
- critical δf ? $\longleftrightarrow$ likely Jeans marginality condition, but still under study

- easily extended to include collisions → c.f. Lesur c.) CDIA = Electron with current + Ions

- classic ancient paradigm for anomalous resistivity (Sagdeev, et.al., ’60s), still frequently invoked
- physics: overlap of distribution fucntions

$$\gamma = -(\epsilon^e_{IM} + \epsilon^i_{IM})/\partial\epsilon/\partial\omega|_{\omega_k}$$

$v_d > c_s$ required for free energy



- recall: $$\frac{\partial}{\partial t}\int dv\delta f_i^2 = -2\left(\frac{\partial f_{i,0}}{\partial u}\right)\frac{d}{dt}\int dv(v-u)\delta f \equiv -2\left(\frac{\partial f_{i,0}}{\partial u}\right)\frac{d}{dt}\frac{\langle p_i\rangle}{m_i}$$

(n.b.: assumed ION hole) – growth in both species possible

- now: $$\frac{d}{dt}(\langle p_i\rangle + p_e + p_w) = 0$$

→ momentum exchange with electrons and/or waves triggers relaxation!

→ For ions (with waves stationary);

$$\frac{d}{dt}\langle p_i\rangle = -\frac{1}{2}\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv \delta f_i^2 \quad ;$$

For electrons similarly;

$$\frac{d}{dt}p_e = -\frac{1}{2}\frac{m_e}{(\partial f_{0,e}/\partial v)|_u}\partial_t \int dv \delta f_e^2$$

$$\rightarrow \quad \frac{m_e}{(\partial f_{0,e}/\partial v)|_u}\partial_t \int dv \delta f_e^2 = -\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv \delta f_i^2$$

- growth possible if $\left.\frac{\partial f_{i,0}}{\partial v}\right|_u \left.\frac{\partial f_{e,0}}{\partial v}\right|_u < 0$

n.b. kinetic pseudomomentum:

$$= -\frac{1}{2}\int dv \frac{\delta f^2}{\partial f_0 / \partial v}$$

→ generalizes wave momentum density idea

- overlap of distributions allows momentum exchange to scatter δf_i producing growth

- require overlap for maximal nonlinear growth → complementary to ancient linear story...

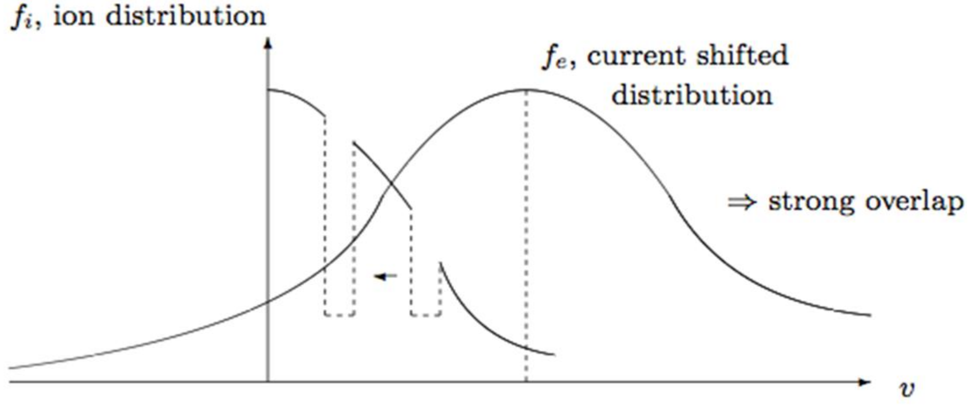


FIG. 8. Ion hole growth in current carrying Vlasov plasma

- Calculation

$$\frac{1}{2}\frac{m_i}{(\partial f_{0,i}/\partial v)|_u}\partial_t \int dv \delta f_i^2 = -\frac{d\langle p_i\rangle}{dt} = \frac{dp_e}{dt}$$

- using linear electron response for simplicity (can be expanded)

$$\frac{dp_e}{dt} = -m_e \int dv D(v)\frac{\partial}{\partial v}\langle f_e\rangle$$
$$\simeq -m_e D(u) \left.\frac{\partial f_{0,e}}{\partial v}\right|_u \Delta u$$

$$D(v) = \frac{q^2}{m_e^2}\sum_{k\omega}\langle E^2\rangle_{k\omega}\pi\delta(\omega - kv)$$

spread of structure-generated phase velocities

Results:

$$\partial_t \int dv \delta f_i^2 \cong -\frac{m_e}{m_i}\left.\frac{\partial f_{0,i}}{\partial v}\right|_u \int dv D(v)\frac{\partial}{\partial v}\langle f_e\rangle$$
$$\cong -\frac{m_e}{m_i}\Delta u D(u)\left.\frac{\partial f_{0,i}}{\partial v}\right|_u \left.\frac{\partial f_{0,e}}{\partial v}\right|_u$$

- $\langle \tilde{E}^2\rangle \sim \int dv \int dv' \langle \delta f^2\rangle/|\epsilon|^2 \sim \Delta v^2 \langle \delta f^2\rangle/|\epsilon|^2$

which brings us to:

- The result:

$$\gamma \simeq k\,\Delta v\,\left[-\frac{\partial f_{0,i}}{\partial v}\frac{\partial f_{0,e}}{\partial v}\right]_u F(\mathrm{mess})$$

- F(mess) is complicated function of BGK mode/hole structure

  NOT INSTRUCTIVE

Elements of Growth:

- rate → set by trapping time $\gamma \sim k\,\Delta v$ → nonlinear $(\sim \sqrt{\phi})$

- free energy: $-\frac{\partial f_{0,i}}{\partial v}\frac{\partial f_{0,e}}{\partial v} > 0$ → electron current required

- mechanism: electron scattering off ion structure, not related to wave scattering

- no apparent critical current if δf sufficient

- observed in simulations (R.H Berman, et.al.) 1986 d.) Collisionless Drift Wave Turbulence

Example: Darmet Model, A Simplified Interesting Prototype

- Darmet '06: Trapped Ion Induced ITG
- Bounce Averaged DKE for Trapped Ions + GK Poisson Equations

  $$\partial_t f + v_d \partial_y f + \{\phi, f\} = C(f)$$

  $$\alpha_e(\phi - \langle\phi\rangle_\theta) - \rho^2\nabla^2\phi = \frac{2}{n_{eq}\sqrt{\pi}}\int_0^\infty dE\sqrt{E}f - 1$$

- Drive: $Q = -\chi_{col}\langle T\rangle' + \int dE\sqrt{E}E\langle v_r \delta f\rangle$

  to match applied heat flux

- Irreversibility
  - trapped ion drift resonance
  - $\sim$ 1D resonance dynamics $(v_{ph\phi} \leftrightarrow v_d)$

$\rightarrow$ possibility of long wave-ion coherence time, $Kubo\# > 1$

$\therefore$ phase space structure formation, failure of QLT are *both* likely

- Drift resonance relatively coherent → $K > 1$ easily satisfied (P.D. et. al. '82)

↔ ~ 1D structure

$$K = \frac{\tilde{v}}{|d\omega/dk_\theta - \omega/k_\theta||\Delta k_\theta|\Delta_r}$$

→ strongly resonant structure formation likely

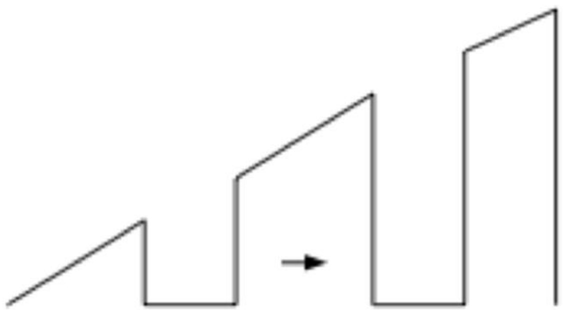
→ Dynamics → consider displacement δr:

$$\frac{df}{dt} = \frac{d}{dt}(f_0 + \delta f) = 0$$

$$\partial_t \int dE\sqrt{E}\langle \delta f^2 \rangle = -2\frac{d}{dt}\int dE\sqrt{E} f_0 \delta f \quad ; \quad f_0 = f_0(x_0) + (x - x_0)\left.\frac{\partial f_0}{\partial x}\right|_{x_0} + \ldots$$

$$\partial_t \int dE\sqrt{E}\langle \delta f_i^2 \rangle = -2\langle \tilde{v}_r \delta n_i \rangle \left.\frac{\partial f_0}{\partial x}\right|_{x_0}$$

and QN → $-\rho^2 \nabla^2 \tilde{\phi} = \delta n_i - \delta n_e$

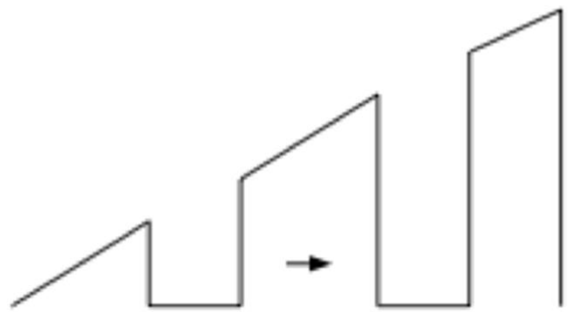


$$\delta f_i\left(\frac{x - x_0}{\Delta x}, \frac{E - E_0}{\Delta E}\right)$$

→ Physics: Ambipolarity / PV conservation

- total dipole moment conserved, including polarization charge

$$\int dx \sum_{\alpha} q_{\alpha} n_{\alpha}(x) x = const$$



$$\delta f_i\left(\frac{x-x_0}{\Delta x}, \frac{E-E_0}{\Delta E}\right)$$

- Polarization Flux → Reynolds Force

$$\partial_t \left\{ \int dE \sqrt{E} \frac{\delta f_i^2}{2\langle f \rangle' |_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \langle \tilde{v}_r \delta n_e \rangle$$

Observe:

- even for localized phase space structure dynamics → ZF coupling appears

- For TIM regime, non-adiabatic electrons dissipative (i.e. collisional response)

$$\partial_t \left\{ \int dE \sqrt{E} \frac{\delta f_i^2}{2\langle f \rangle' |_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle + D_{DT} \frac{\partial \langle n \rangle}{\partial x}$$

Observe:

- structure + Z.F. evolution

$$\partial_t \left\{ \int dE\sqrt{E}\frac{\delta f_i^2}{2\langle f\rangle'|_{x_0}} + \langle V_\theta\rangle \right\} = -\nu\langle V_\theta\rangle + D_{DT}\frac{\partial\langle n\rangle}{\partial x}$$

- Charney - Drazin Non-Acceleration Theorem for H-W model

$$\frac{\partial}{\partial t}\{\langle v_\theta\rangle - WAD\} = -\nu\langle V_\theta\rangle - \langle\tilde{V}_r\tilde{n}\rangle - \frac{1}{\langle q\rangle'}\left\{\mu\langle(\nabla\tilde{q})^2\rangle + \partial_r\langle\tilde{V}_r\tilde{q}^2\rangle\right\}$$

$$WAD = -\frac{\langle\delta q^2\rangle}{\langle q\rangle'}$$

$$q = n - \nabla^2\phi$$

→ ~ very close correspondence!

- $\int dE\sqrt{E}\delta f_i^2/2\langle f\rangle|_{x_0}$ → equivalent to structure zonal pseudomomentum

- momentum conservation of structure + ZF is fundamental ⟷ follows from flux of polarization charge

- electron flux drives NET system momentum → ambipolarity

- C-D Thm. for Darmet Model (KPD $\equiv \int \sqrt{E} dE \langle \delta f^2 \rangle / \langle f \rangle'$)

$$\partial_t \left\{ \langle v_\theta \rangle - KPD \right\} = -\nu \langle V_\theta \rangle - \int dE \sqrt{E} \left[\frac{1}{\langle f \rangle'} \{ \partial_r \langle \tilde{v}_r \delta f^2 \rangle + \langle \delta f C(\delta f) \rangle \} \right]$$

- KPD $\equiv -\int dE \sqrt{E} \langle \delta f^2 \rangle / \langle f \rangle'$, Kinetic 'Phasetrophy' Density

  In non-resonant limit:

  $\delta f_k = -\tilde{v}_{r\,k} \langle f \rangle' / (-i\omega_k)$, $KPD \sim \int \sqrt{E} dE \langle \tilde{v}_r^2 \rangle_k \langle f \rangle' / \omega_k^2 \sim -k_\theta \mathcal{E} / \omega_k$

  $\rightarrow$ corresponds to kinetic pseudomomentum

  $\rightarrow$ reduces to wave momentum in small amplitude limit, $P_k = kN_k$,

  $N_k = (\partial \epsilon / \partial \omega)|_{\omega_k} (|E_k|^2 / 8\pi)$

- Non-Acceleration: Absent KPD/spreading or collisonal dissipation,

  cannot accelerate or maintain Z.F. with stationary KPD

  $\rightarrow$ Momentum Freezing-in Law for ZF and QP gas!!

Aside: Kinetic 'Phasetrophy' Density - What Does it Mean?

- c.f. Antonov Energy Principle for collisionless Self-Gravitating Matter (Stellar Dynamics, $F_0' = \partial F_0/\partial E$)

$$\delta W = \boxed{\int d^3x d^3v \frac{\delta f^2}{|F_0'|}} - G \int d^3x d^3x' d^3v d^3v' \frac{\delta f(\mathbf{x}, \mathbf{v}) \delta f(\mathbf{x}', \mathbf{v}')}{|\mathbf{x} - \mathbf{x}'|}$$

$\rightarrow$ KPD corresponds to fluctuation dynamic pressure

$\rightarrow$ opposes self-gravity in usual Jean's balance

- Formulate as response to external force
- Appears in Kruskal-Oberman Kinetic Energy Principle

Is There a Non-Acceleration Theorem for 1D Vlasov Plasma ?

$$\partial_t f + v\partial_x f + E\partial_v f = C(f)$$

$$-\nabla^2 \phi = 4\pi \int f dv$$

$\Rightarrow$ Phasetrophy Evolution:

drives $\partial_t \langle v \rangle$

$$\partial_t \int \frac{\langle \delta f^2 \rangle}{2\langle f \rangle'} dv - \int \frac{\langle f \rangle''}{\langle f \rangle'^2} \langle \widetilde{E} \delta f^2 \rangle dv = -\langle \widetilde{E} \delta n \rangle + \int \langle \widetilde{C}(f) \delta f \rangle dv$$

Pseudomomentum: $-\int \frac{\langle \delta f^2 \rangle}{2\langle f \rangle'} dv \;\Rightarrow$ WMD, for linear perturbations

$\Rightarrow$ Non-Acceleration Theorem

Collisional phasetrophy dissipation

$$\partial_t \left\{ \langle v \rangle - \left(-\int \frac{\langle \delta f^2 \rangle}{2\langle f \rangle'} dv \right) \right\} = \int \frac{\langle f \rangle'}{\langle f \rangle'^2} \langle \widetilde{E} \delta f^2 \rangle dv + \int \langle \delta f \widetilde{C}(f) \rangle dv + \int \langle C(f) \rangle v dv$$

Collisional mean flow damping

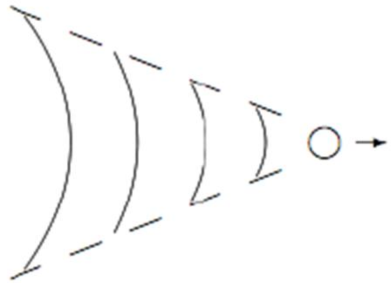
Implications for 'Anomalous Resistivity' ?

III) Relaxation + Transport in phase space turbulence for $K \sim 1$

- An approach: Dupree+Balescu+Lenard (DBL)

- Evolution of Mean by turbulent granulation

$$\partial_t \langle f \rangle = -\frac{\partial}{\partial r}\langle \tilde{v}_r \delta f \rangle = \frac{\partial}{\partial r}\left(D \frac{\partial}{\partial r}\langle f \rangle - F\langle f \rangle \right)$$



$$\delta f = \delta f^c + \widetilde{\delta f}$$

Coherent response

→ similar to quasilinear theory

Incoherent granulation → Dynamical Friction!

→ Structure = 'Macro' particle (Dupree '70)

→ Cerenkov emission, leading to wake

$$\tilde{\phi}_{k\omega} = \frac{1}{\epsilon(k,\omega)} \int dv \widetilde{\delta f}$$

- Evolution of turbulence

$$\partial_t \langle \delta f(1) \delta f(2) \rangle + \tau^{-1} \langle \delta f(1) \delta f(2) \rangle = P(1,2)$$

Life time of correlation due:

→ relative streaming, turbulent scattering, collisions

Production due to free energy relaxation

$$-\langle \tilde{v}_r \delta f \rangle \langle f \rangle'$$

mixing of phase space density

→ mixing of pol. charge → zonal flow coupling

Production term

$\delta f = -(q\tilde{\phi}/T_i)\langle f_i \rangle + \delta h$

- Phase space density fluctuation: $\delta h = \delta h^c + \widetilde{\delta h}$

  (δh^c → coherent response; $\widetilde{\delta h}$ → incoherent granulation)

$$P(1,2) = \mathrm{Re}\sum_{k\omega}(-i\omega + i\omega_*^i(1))\left\langle \frac{q\tilde{\phi}(1)}{T_i}\underline{\delta h^{c*}(2)}\right\rangle_{k\omega} e^{i\mathbf{k}\cdot\mathbf{x}_-}\langle f_i(1)\rangle + (1 \leftrightarrow 2)$$

$$+ \mathrm{Re}\sum_{k\omega}(-i\omega + i\omega_*^i(1))\left\langle \frac{q\tilde{\phi}(1)}{T_i}\underline{\widetilde{\delta h}^*(2)}\right\rangle_{k\omega} e^{i\mathbf{k}\cdot\mathbf{x}_-}\langle f_i(1)\rangle + (1 \leftrightarrow 2)$$

→ Coherent and Incoherent Production

$P(1,2)$ → $P^c(1,2)$ ← by diffusive part

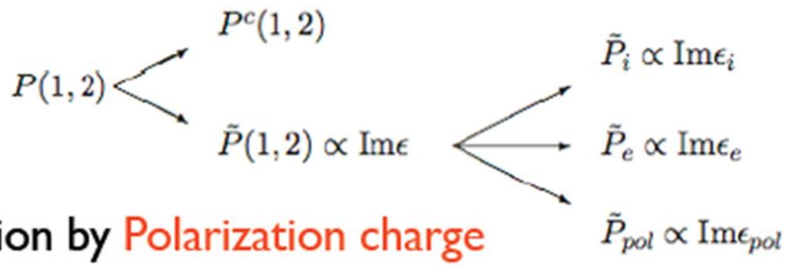
$P(1,2)$ → $\tilde{P}(1,2) \propto \mathrm{Im}\epsilon$ ↑ by dynamical friction

$\tilde{P}(1,2) \propto \mathrm{Im}\epsilon$ →

- $\tilde{P}_i \propto \mathrm{Im}\epsilon_i$
- $\tilde{P}_e \propto \mathrm{Im}\epsilon_e$
- $\tilde{P}_{pol} \propto \mathrm{Im}\epsilon_{pol}$ → Novel piece: envelope coupling, zonal flow drag on granulations

$\epsilon(\mathbf{k},\omega)$: plasma dielectric for the model

Incoherent Production (pol. charge)



- Novel piece:Incoherent Production due Dynamical Friction by Polarization charge

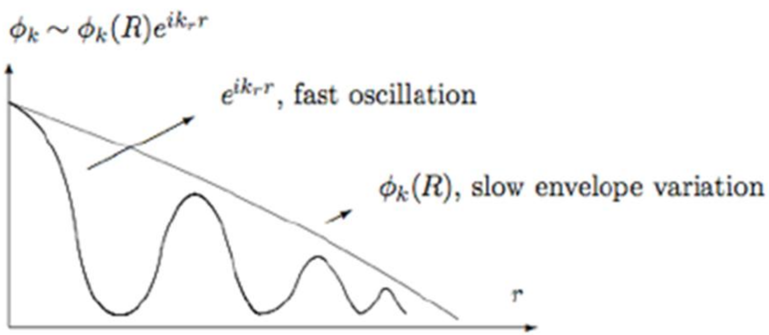
$$\tilde{P}_{pol} \simeq -2\sum_{k\omega}(\omega - \omega_*^i(E))\langle f_i(E)\rangle(-2\rho_i^2 k_r)\frac{1}{|\epsilon(k,\omega)|^2}\left\langle \frac{\widetilde{\delta n}}{n_0}\partial_r \widetilde{\delta h}^* \right\rangle_{k\omega}$$

- $\mathrm{Im}\epsilon_{pol} \propto k_r \partial_r$:envelope coupling → mesoscales

$$\phi_k \sim \phi_k(R) exp(ik_r r)$$

↙ slow envelope variation

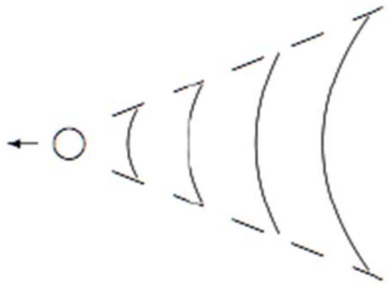
↘ fast oscillation



Radial derivative: $\partial_r \to ik_r + \partial_R$ i.e. $k_r \to k_r - i\partial_R$

→ $\mathrm{Im}k_r$

so $\mathrm{Im}\epsilon_{pol} \propto \mathrm{Im}k_\perp^2 \propto k_r \mathrm{Im}k_r$



Incoherent Production (pol. charge)

$$P(1,2) \to P^c(1,2), \quad \tilde{P}(1,2) \propto \mathrm{Im}\epsilon$$

$$\tilde{P}(1,2) \propto \mathrm{Im}\epsilon \to \tilde{P}_i \propto \mathrm{Im}\epsilon_i, \quad \tilde{P}_e \propto \mathrm{Im}\epsilon_e, \quad \tilde{P}_{pol} \propto \mathrm{Im}\epsilon_{pol}$$

- Incoherent production by pol. charge? → Zonal Flow coupling!

$$\int d^3v \frac{\tilde{P}_{pol}}{2\langle f_i\rangle} \simeq \frac{v_*^i}{v_{thi}^2}\partial_r\langle\tilde{v}_r\tilde{v}_\theta\rangle$$

→ Dynamical Friction on ion phase space density due to Zonal flow coupling
→ n.b. closely analogous to zonal flow coupling via pol. charge in single structure case
→ akin to wake of granulation in zonal flow

- Relative magnitude of dynamical friction by pol. charge

$$\frac{\tilde{P}_{pol}}{\tilde{P}_e} \sim \frac{\overline{k_r k_\theta}}{\overline{k_\theta^2}} \frac{\eta_e}{1+3\eta_e/2} \frac{\nu_e/\epsilon_0}{v_{thi}/L_{T_e}} \frac{1}{\epsilon_0^{1/2}} \sqrt{\frac{\rho_i}{L_{T_e}}}$$

where $\overline{(\ldots)} \equiv \sum_{k\omega}(\ldots)\langle\tilde{\phi}^2\rangle_{k\omega} / \sum_{k\omega}\langle\tilde{\phi}^2\rangle_{k\omega}$

- $\dfrac{\overline{k_r k_\theta}}{\overline{k_\theta^2}}$ and $\dfrac{\eta_e}{1+3\eta_e/2}$ → $O(1)$
- $\dfrac{\nu_e/\epsilon_0}{v_{thi}/L_{T_e}}$ and $\dfrac{1}{\epsilon_0^{1/2}}$ → large
- $\sqrt{\dfrac{\rho_i}{L_{T_e}}}$ → modest

→ Can be order unity!

Implication: Momentum Theorem for phase space turbulence

- Full Evolution → Extension of momentum theorem by Charney-Drazin to granulation

$$\frac{\partial}{\partial t}\int d^3v \frac{\langle \delta h^2\rangle}{2\langle f\rangle} = \int d^3v \frac{P_{i,i}+P_{i,e}}{2\langle f\rangle} + \frac{v_*^i}{v_{thi}^2}\partial_r\langle \tilde{v}_r\tilde{v}_\theta\rangle - \tau_L^{-1}\int d^3v \frac{\langle \delta h^2\rangle}{2\langle f\rangle}$$

: for GK turbulence

Forcing in turbulence due relaxation | ZF coupling | dissipation by turbulent mixing

$$\frac{\partial}{\partial t}\frac{\langle \delta q^2\rangle}{2\langle q\rangle'} = -\langle \tilde{v}_r\tilde{n}_e\rangle + \partial_r\langle \tilde{v}_r\tilde{v}_\theta\rangle - \frac{1}{\langle q\rangle'}\left(\frac{\partial}{\partial r}\left\langle \tilde{v}_r\frac{\delta q^2}{2}\right\rangle + D_0\langle(\nabla\delta q)^2\rangle\right)$$

: for Hasegawa + Wakatani system

$$dq/dt = D_0\nabla^2 q$$

$$q = n - \nabla^2\phi$$

- An implication → Non-Acceleration theorem:

In the absence of production/dissipation of phase space turbulence (granulation), stationary granulation cannot accelerate flow against collisional drag

→ With production/dissipation, granulation can drive zonal flow

Transport

- Flux of phase space density

$$J(r) \equiv \langle \tilde{v}_r \delta f \rangle = J_{i,i} + J_{i,e} + J_{i,pol}$$

→ total radial flux

$$J_{i,i} = \sum_{k\omega} (\omega - \omega_*^i(E)) \langle f_i(E) \rangle k_\theta \rho_i v_{thi} \mathrm{Re} g_{k\omega} S_{k\omega}$$

→ $\sim$ QL diffusion; $S_{k,\omega}$ includes i,i drag

$$J_{i,e} = -\sum_{k\omega} k_\theta \rho_i v_{thi} \frac{\mathrm{Im}\epsilon_e}{|\epsilon(k,\omega)|^2} \left\langle \frac{\widetilde{\delta n}}{n_0} \widetilde{\delta h}^* \right\rangle_{k\omega}$$

→ dynamical friction from electrons

$$J_{i,pol} = -\sum_{k\omega} k_\theta \rho_i v_{thi} \frac{(-2\rho_i^2 k_r)}{|\epsilon(k,\omega)|^2} \left\langle \frac{\widetilde{\delta n}}{n_0} \partial_r \widetilde{\delta h}^* \right\rangle_{k\omega}$$

→ dynamical friction from zonal flow

→ Dynamical Friction by zonal flow competes against **G.C.** fluxes

→ Unlike conventional shear decorrelation/suppression of turbulence (enter propagator)

IV.) Summary and Outlook

- Dynamical friction is alive and well in Vlasov turbulence
- Dynamical anti-friction possible ⇒ subcritical structure growth predicted
- Physics of Dynamical friction efficiently revealed by consideration of δf blob scattering in phase space
- Localized structures carry a pseudo-momentum and thus excite zonal flows
- Correspondence between single structure and statistical D-B-L calculations demonstrated
- Time to re-investigate the 'declaration of victory' for quasilinear models…

Outlook and Open Questions

- Feedback of structure generated zonal flow on violent relaxation process of structure formation? BGK structure in shear flow – Okubo-Weiss balance?
- Effect of zonal flow shear on hole growth and saturation dynamics $\Rightarrow$ yet another variation on the predator-prey theme?
- Applications to more interesting systems i.e. EPM?