Self-Similar Evolution of Cosmic-Ray Modified Shocks

base on DSA simulations

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Basic Equations for Kinetic DSA Simulations

$$\frac{\partial \rho}{\partial t} + \frac{\partial (u\rho)}{\partial x} = 0$$

(1D plane qausi-parallel shock)

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + P_g + P_c) = 0$$

ordinary gasdynamics EQs

+ P_c terms

$$\frac{\partial (\rho e_g)}{\partial t} + \frac{\partial}{\partial x}(\rho e_g u + P_g u) = -u \frac{\partial P_c}{\partial x} \cdot \qquad - L$$

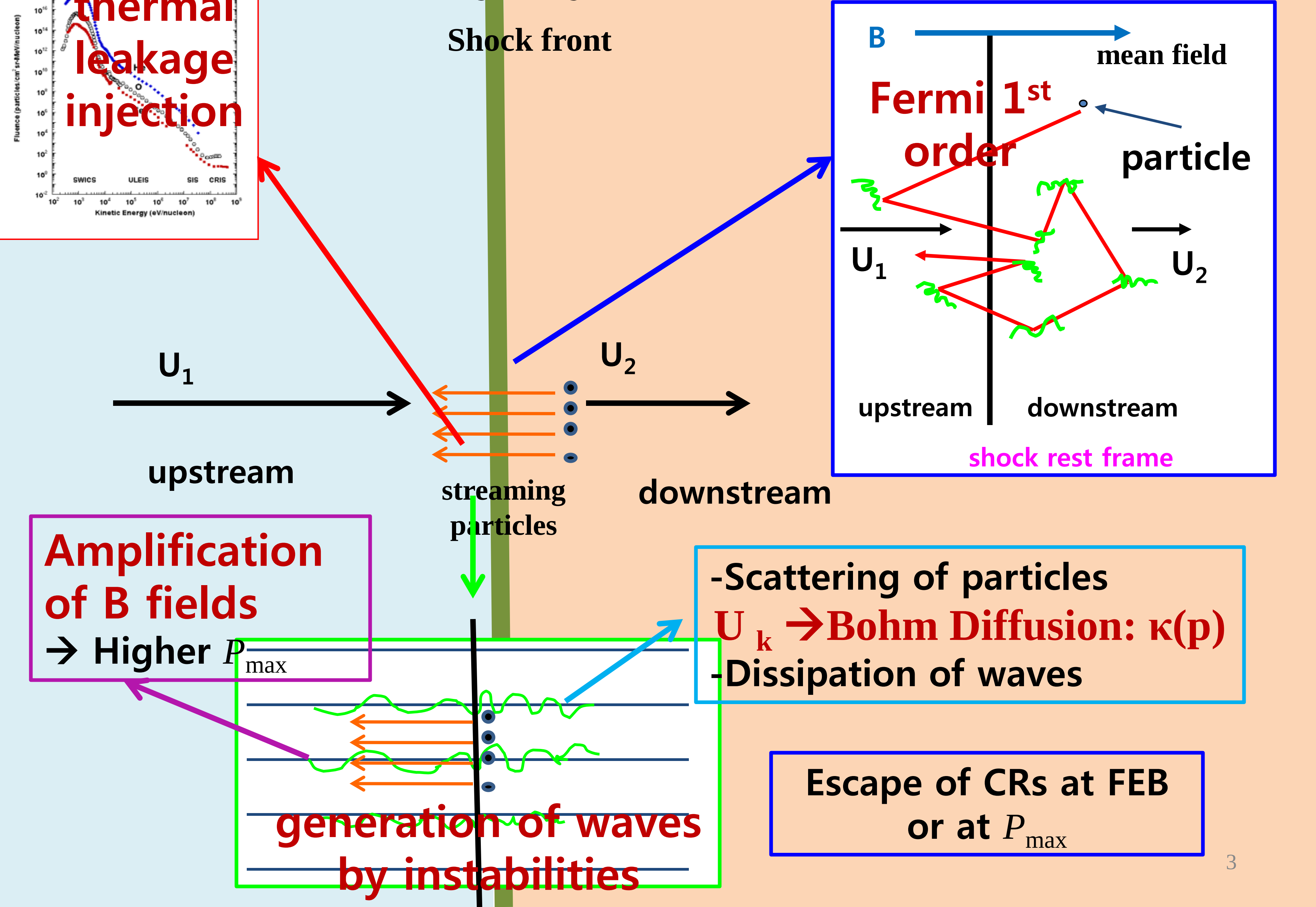
Diffusion Convection Eq. for isotropic part of *f(x,p,t)*

$$\frac{\partial f}{\partial t} + (u \qquad) \frac{\partial f}{\partial r} = \frac{1}{3} \frac{\partial}{\partial x}(u \qquad) \cdot p \frac{\partial f}{\partial p} + \frac{\partial}{\partial x}[\kappa(x,p)\frac{\partial f}{\partial x}] + Q(x,p)$$

$$P_c = \frac{4}{3}\pi m_p c^2 \int_0^\infty f(p) \frac{p^4 dp}{\sqrt{p^2+1}}$$

***L*= thermal energy loss due to injection, Q= CR injection**

Key Physics of DSA



Simple models for wave-particle interactions

$$\frac{\partial f}{\partial t} + (u+u_w)\frac{\partial f}{\partial x} = \frac{1}{3}\frac{\partial}{\partial x}[(u+u_w)]\, p\frac{\partial f}{\partial p} + \frac{\partial}{\partial x}[\kappa(x,p)\frac{\partial f}{\partial x}] + Q(x,p)$$

1) $u_w \approx -\upsilon_A = -\mathrm{B}_0/\sqrt{4\pi\rho(x)}$ in upstream, $u_w \approx 0$ in downstream,

velocity jump : $\Delta \mathrm{u} = u_1 - u_2 \rightarrow \Delta \mathrm{u} = u_1 - \upsilon_A - u_2$ smaller

steepens CR spectrum & reduces acceleration efficiency

2) $\kappa(x,p) = \kappa^* p(\rho/\rho_0)^{-1}$: Bohm - like power - law

$\delta B \sim B_0$ & compression of field

3) $Q(x,p) =$ thermal leakage injection

$$\frac{\partial(\rho e_g)}{\partial t} + \frac{\partial}{\partial x}(\rho e_g u + P_g u) = -u\frac{\partial P_c}{\partial x} + W - L$$

$W(x,t) \approx -\upsilon_A \frac{\partial P_c}{\partial x}$: wave disspation & gas heating in precursor

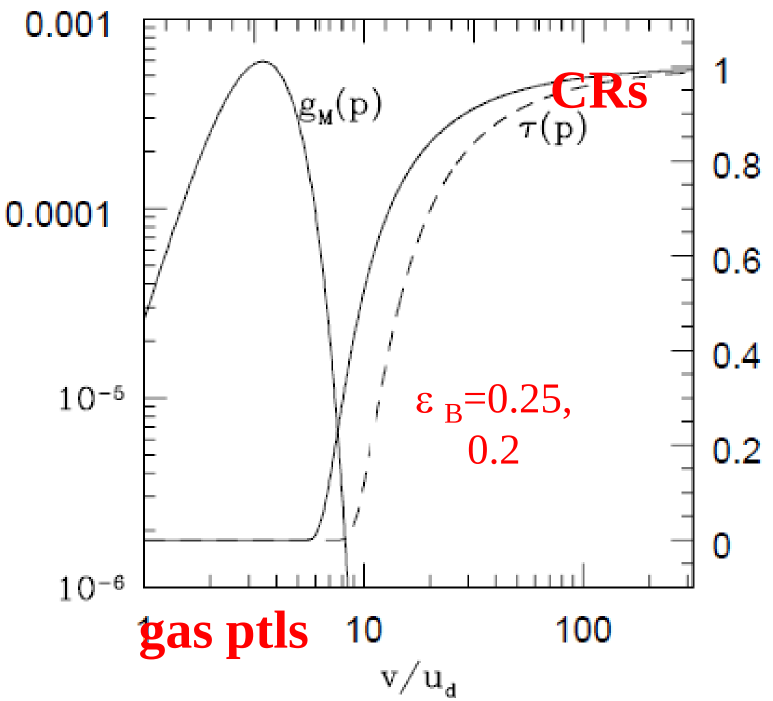
Weakens the shock & reduces acceleration efficiency

Numerical Model for Thermal Leakage Injection in CRASH

$\tau_{esc}(\varepsilon_B, M)$ **: filter function**

"Transparency function": probability that particles at a given velocity can leak upstream. e.g. $\tau_{esc} = 1$ for CRs

$\tau_{esc} = 0$ for thermal ptls



$u_d(M) =$ downstream flow speed

$$\varepsilon_B = \frac{B_0}{B_\perp} = \frac{\text{mean field}}{\text{turbulent field}}$$

$$p_{inj} \approx \left(1 + \frac{1.07}{\varepsilon_B}\right) m_p u_d$$

more turbulent $B_\perp$

$\rightarrow$ smaller ε_B

$\rightarrow$ larger p_{inj}

$\rightarrow$ smaller injection rate

cf. Malkov & Völk (1998)

Numerical Tool: CRASH Code (Kang et al. 2001)

Bohm type diffusion: $\kappa(p) \propto p$

- **wide range of diffusion length scales to be resolved!** $l_{diff} = \kappa(p)/u_s$

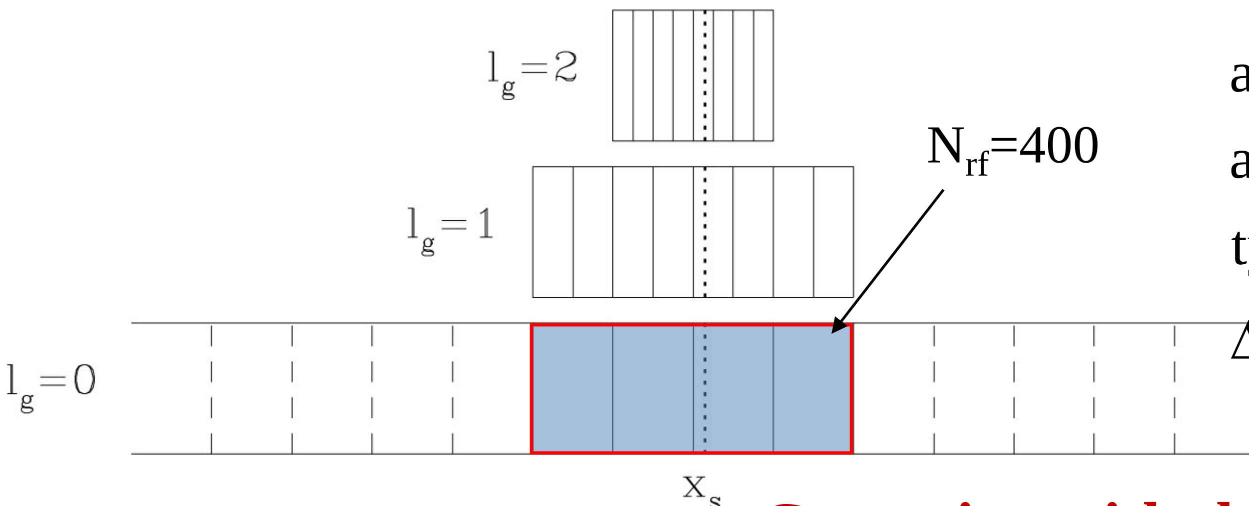
from p_{inj}/mc(~10^{-2}) to outer scales for the highest p_{max}/mc (~10^6)

1) Shock Tracking Method (Le Veque & Shyue 1995)

- **tracks the subshock as an exact discontinuity**

2) Adaptive Mesh Refinement (Berger & Le Veque 1997)

- **refines region around the subshock with multi-level grids**



a factor of two refinement at each grid level,

typically $l_{g,\max} = 8-10$

$\Delta x_{10} = \Delta x_0 / 1024$

Comoving with the shock

acceleration time scale, precursor length scale, …

for $\kappa(p) = \kappa^* p(\rho_0 / \rho) \propto p$, where p in units of $m_p c$

mean accel. time: $t_{acc}(p) \approx \frac{3}{u_1 - u_2}\left(\frac{\kappa_1(p)}{u_1} + \frac{\kappa_2(p)}{u_2}\right) \approx 8\frac{\kappa^* p}{u_s^2}$ for $M >> 1$

$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t \qquad \text{at a given shock age}$$

in an evolving CR shock with ever increasing $p_{\max}$(t) (no escaping condition),

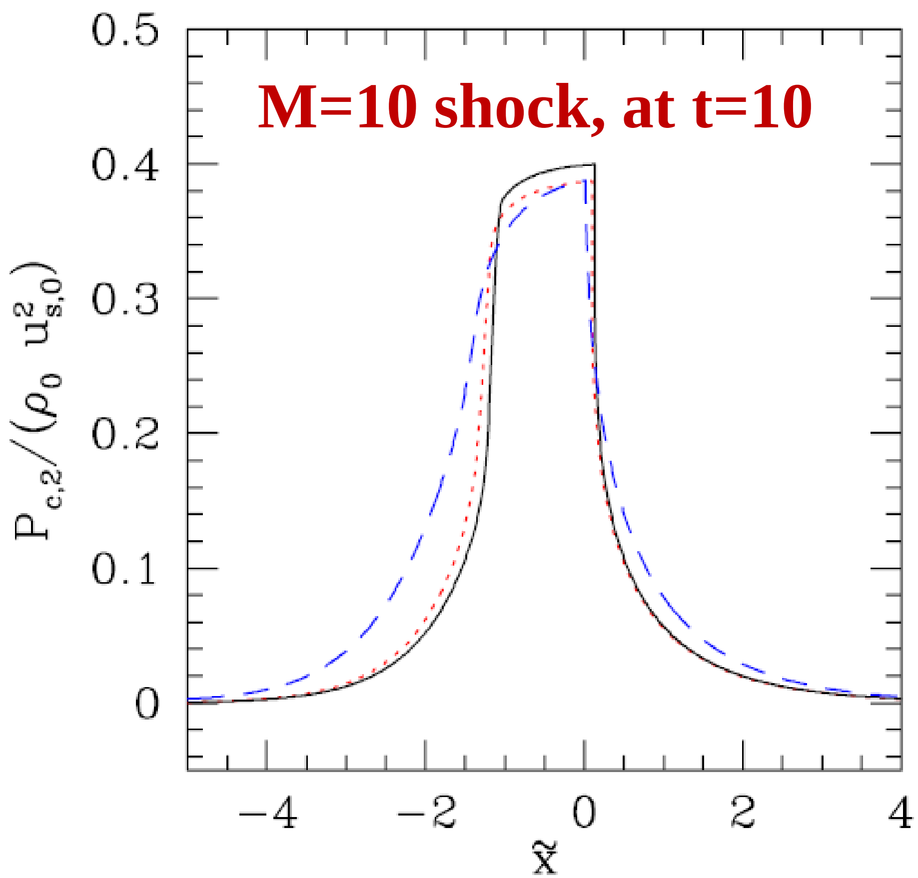
precursor scale: $\sim l_{\max} = \frac{\kappa(p_{\max})}{u_s} \approx \frac{1}{8} u_s t$ diffusion length of p_{max}

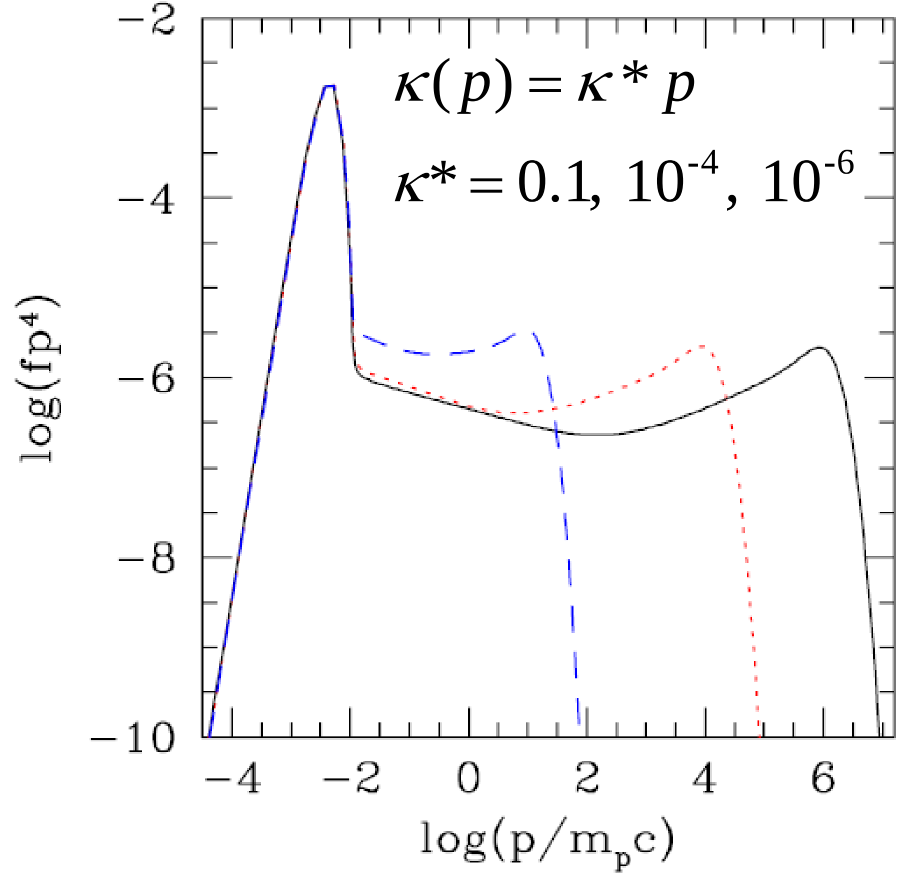
increases with $t \Rightarrow \xi \equiv \frac{x}{u_{s,i} t}$ similarity variable

So the shock structure broadens linearly with time independent of κ^*.

smaller $\kappa^* \rightarrow$ higher $p_{\max}$ at a given shock age.

But hydrodynamic structure is independent of κ^*.





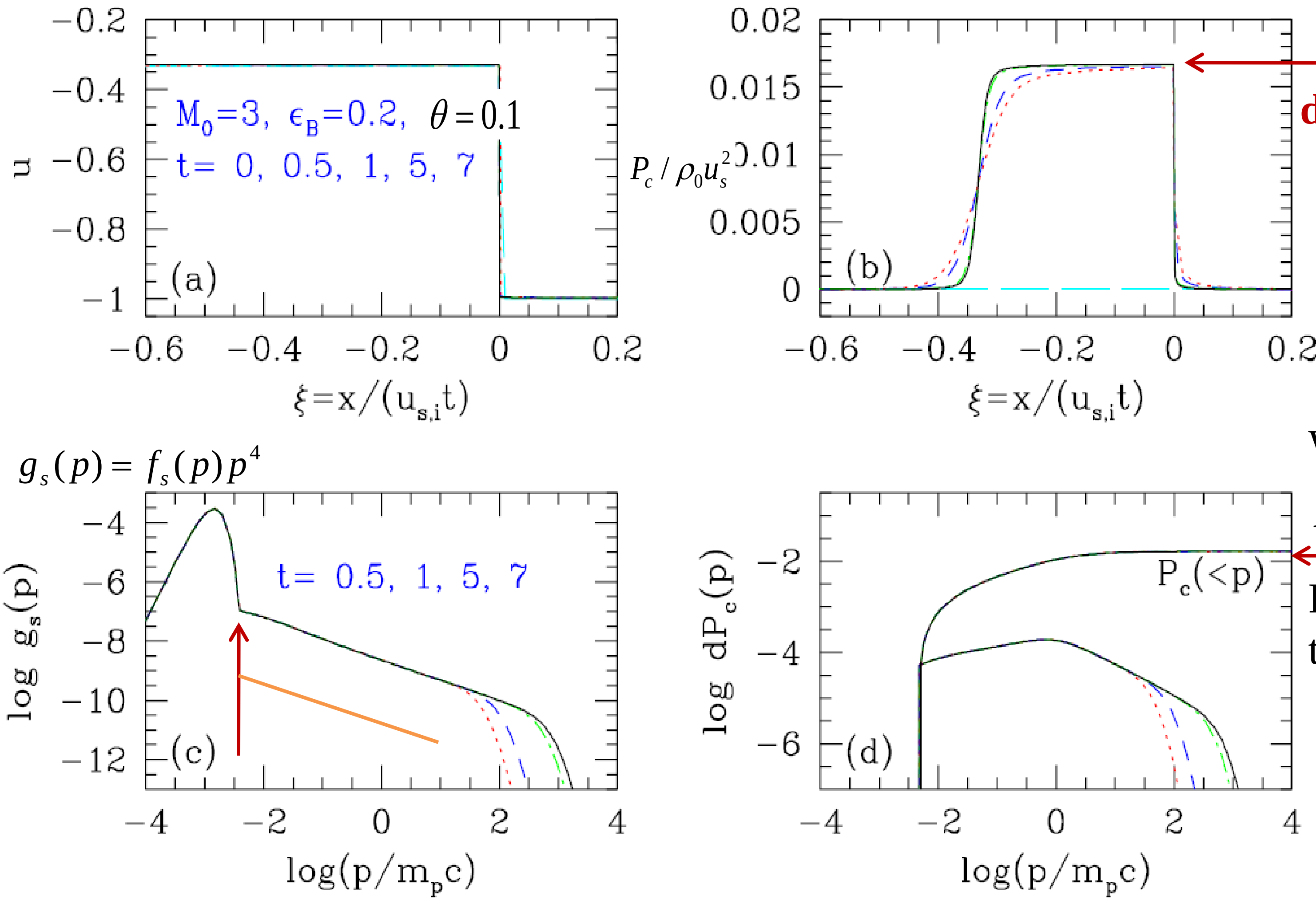
dynamic structure is independent of κ^*. **in an evolving CR shock with ever increasing** p_{max}

smaller $\kappa^* \rightarrow$ higher $p_{\max}$ at a given shock age.

$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t$$

*Minor differences are due to numerical problems.

DSA Kinetic Simulation for M=3 shock



What determines Pc ?

With increasing $p_{max}(t)$

Pc constant in time

Evolution of weak shocks is almost test-particle.
No modification to the shock flow.

$$q(p) \equiv \frac{d \ln f(p)}{d \ln p} \approx \frac{3[u(x_p) - \upsilon_A]}{[u(x_p) - \upsilon_A - u_2]}$$

where $x_p = \dfrac{\kappa(p)}{u_s}$,

$u(x_p)$: velocity profile in the precursor

Test-particle slope for weak shocks

$$q_{test} = \frac{3(u_1 - \upsilon_A)}{(u_1 - \upsilon_A - u_2)} = \frac{3(1 - M_A^{-1})}{(1 - M_A^{-1} - \sigma_s^{-1})}$$

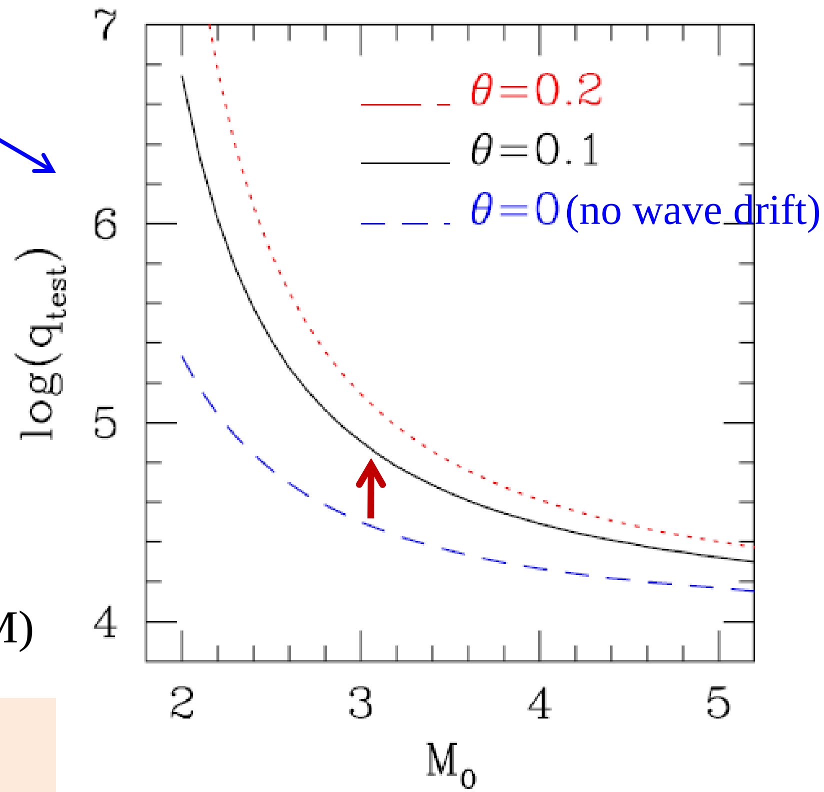
where $M_A = \dfrac{u_1}{\upsilon_A}$, $\sigma_s = \dfrac{u_1}{u_2}$

$$\theta = \frac{E_B}{E_{th}} = \frac{8\pi / B^2}{1.5(P/\rho)}$$

for $\theta = 0.1$, $M_A = \sqrt{\dfrac{5}{9\theta}} M_0 = 2.36 M_0$

important for weak shocks (e.g. shocks in the ICM)

Wave drift steepens CR spectrum & reduces acceleration efficiency



Slope of test-particle spectrum

Test-particle spectrum for weak shocks

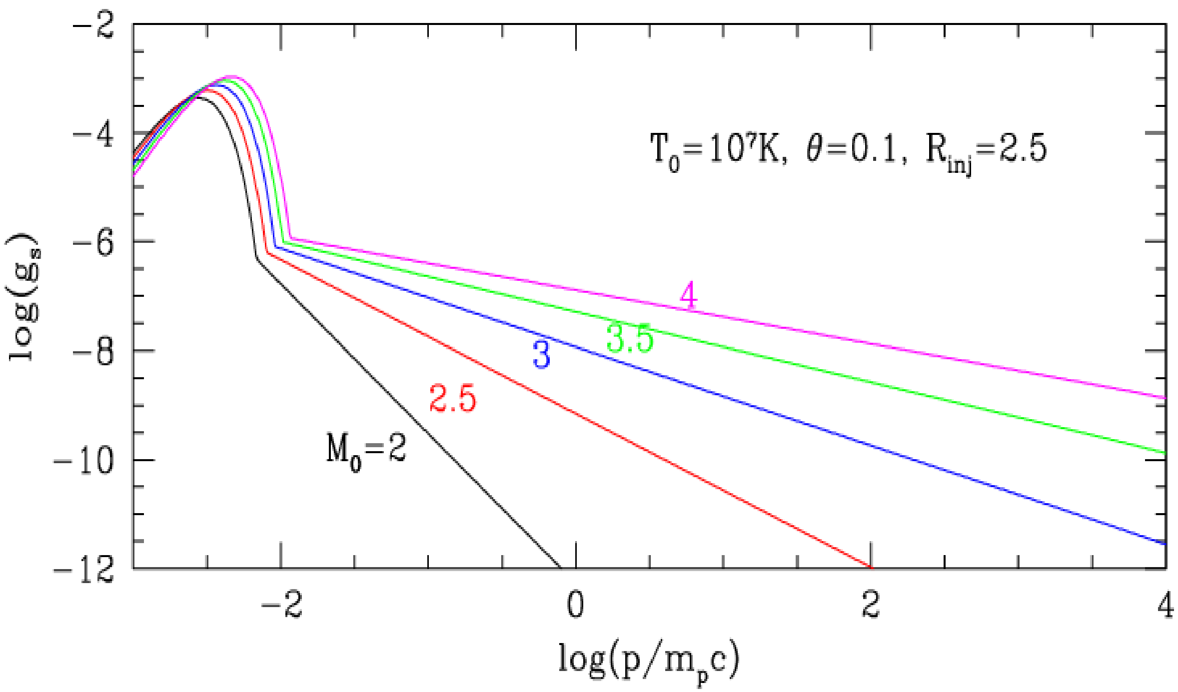
$$f_{test}(p,t) = f_{th}(p_{inj})\left(p/p_{inj}\right)^{-q_{test}} \exp\left[-(p/1.5p_{\max})^2\right]$$

$$p_{th} = m_p \upsilon_{th} = 2\sqrt{k m_p T_2}$$

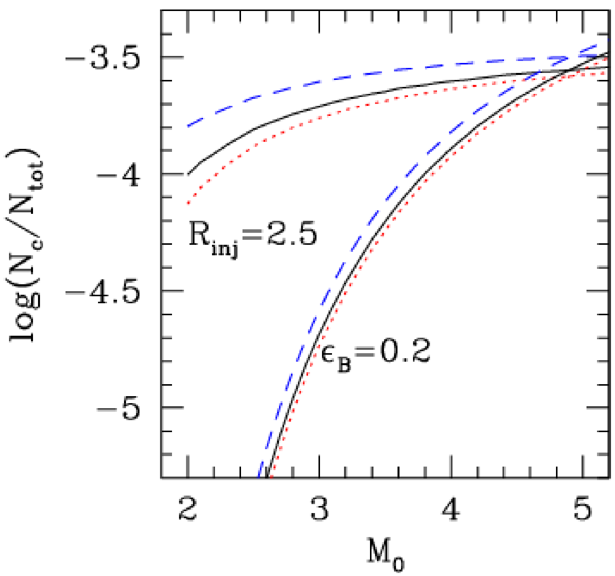
$$p_{inj}/p_{th} = R_{inj} \approx 2.5-2.7$$

$$q_{test} = \frac{3(u_1 - \upsilon_A)}{(u_1 - \upsilon_A - u_2)}$$

$$f_{th}(p_{inj}) = \frac{n_2}{(2\pi T_2)^{1.5}} \exp(-2R_{inj}^2) \text{ Maxwell distribution}$$



CR injection fraction



$$p_{inj}/m_p u_d \approx (1 + \frac{1.07}{\varepsilon_B})$$

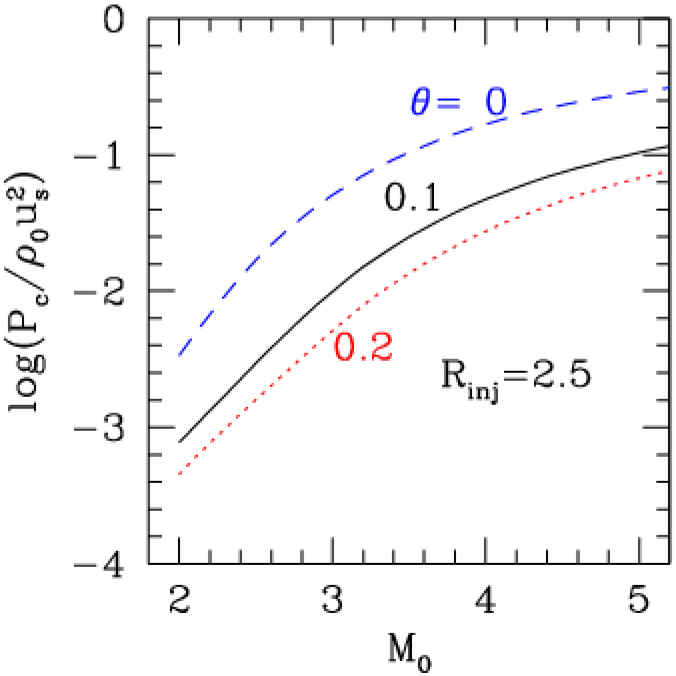
Test-particle spectrum for weak shocks

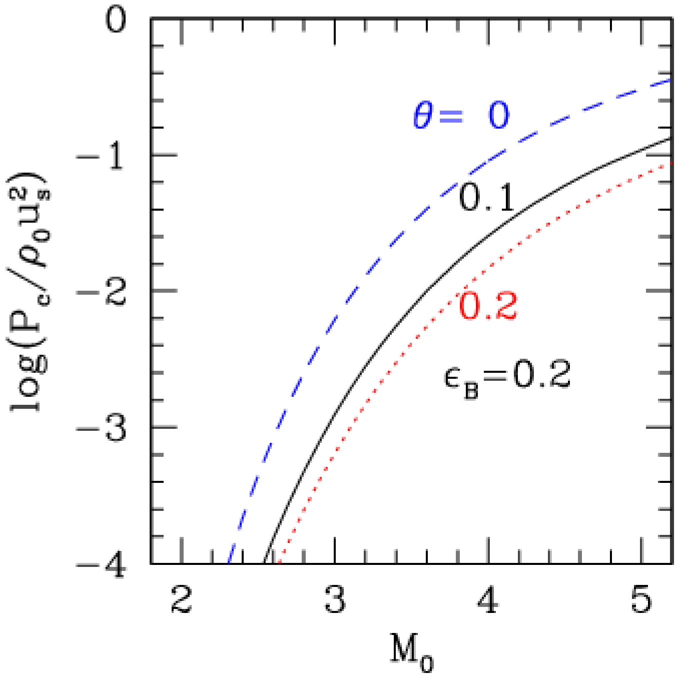
$$f_{test}(p,t) = f_{th}(p_{inj})\left(p / p_{inj}\right)^{-q_{test}} \exp\left[-(p/1.5p_{\max})^2\right]$$

injection model 1) $p_{inj} / p_{th} = R_{inj} \approx 2.5-2.7$ (e.g. Blasi)

2) $p_{inj} / m_p u_d \approx (1+\frac{1.07}{\varepsilon_B})$ (e.g. KJ)

Postshock CR pressure

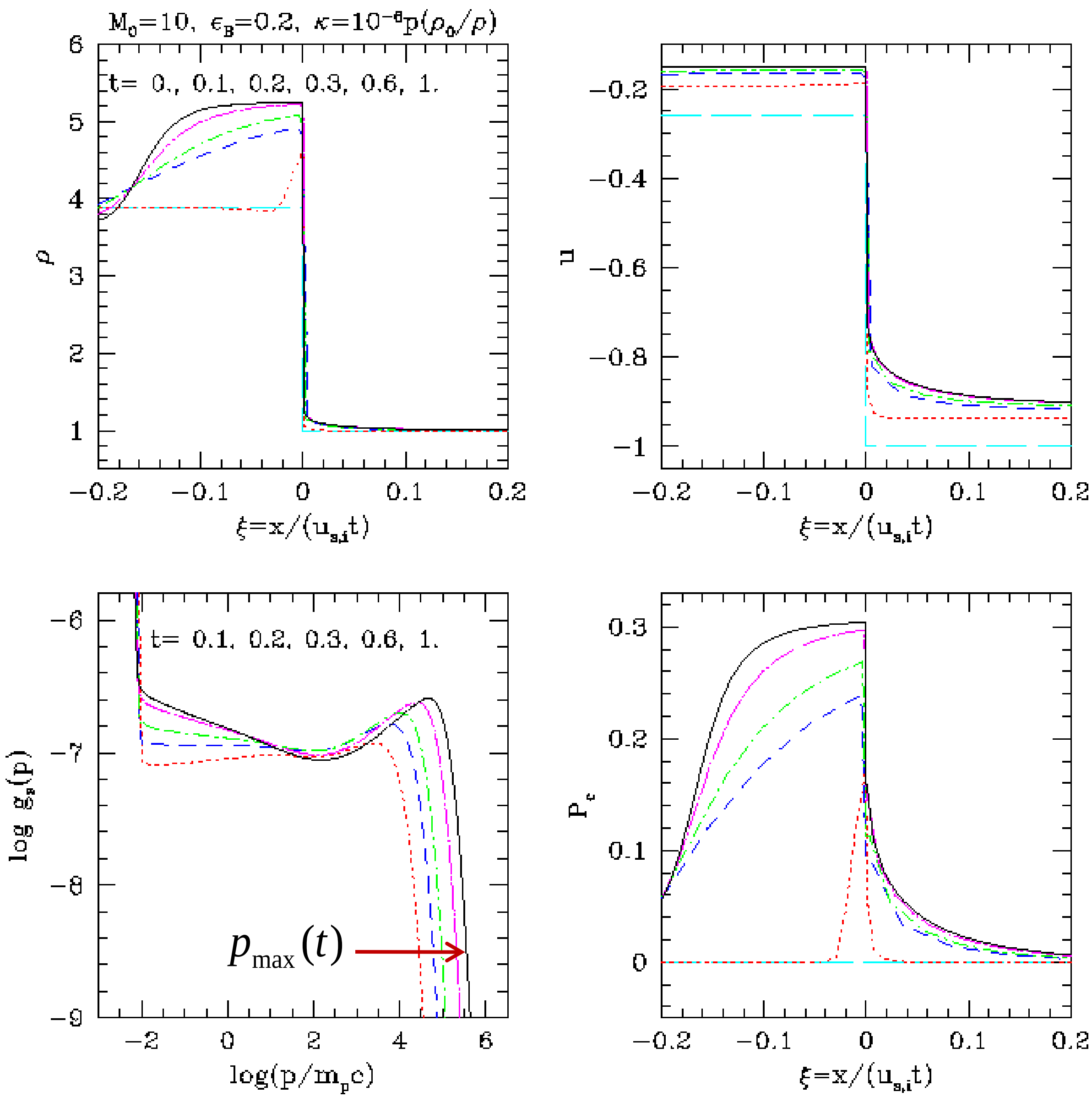




$$\theta = \frac{E_B}{E_{th}} = \frac{8\pi / B^2}{1.5(P/\rho)}$$

For weak shocks M<4, test-particle approximations is valid.
Wave drift reduces the CR acceleration efficiency

How does a CR shock evolve ?



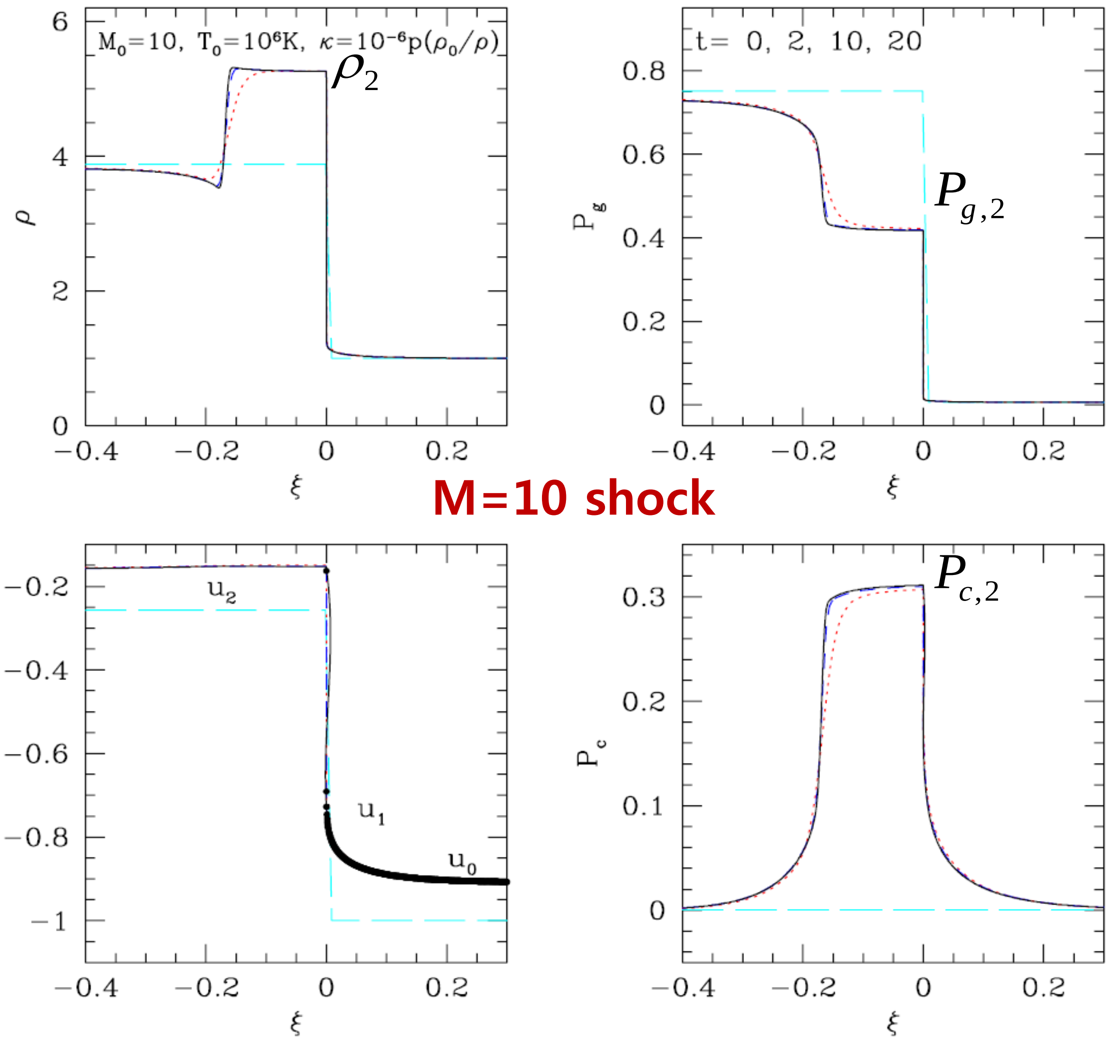
A gasdynamic shock in plane-parallel geometry is a self-similarly evolving structure.

initial state: M=10 gasdynamic shock with Pc=0

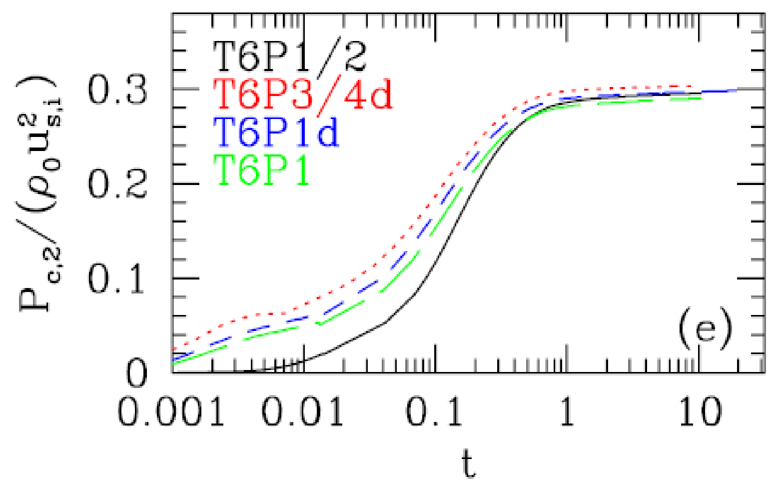
CRs are injected at shocks
→Accelerated to higher E.
→ Pc increases,
Pg decreases,
→ precursor develops

Q: Does the shock reach an equilibrium state with ever increasing p_{max} ?

DSA Kinetic Simulation for M=10 shock

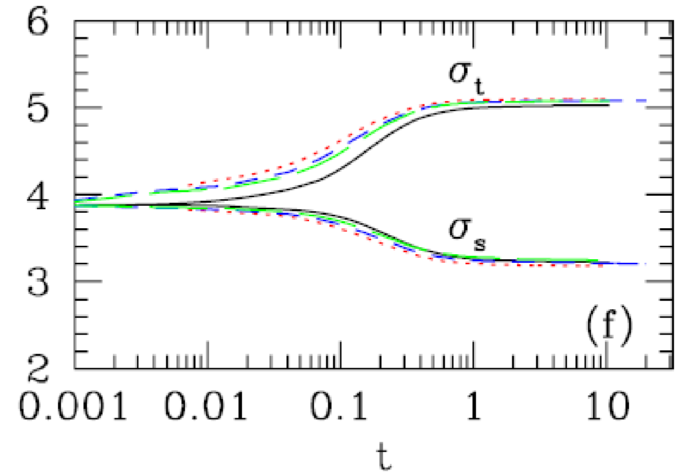


$P_{g,2}$, $P_{c,2}$ $\rightarrow$ constant in time



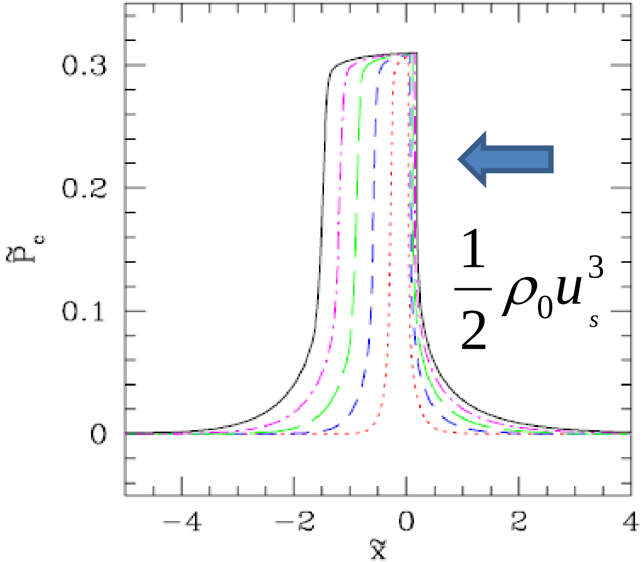
$\sigma_t = \rho_2 / \rho_0$ $\rightarrow$ constant in time

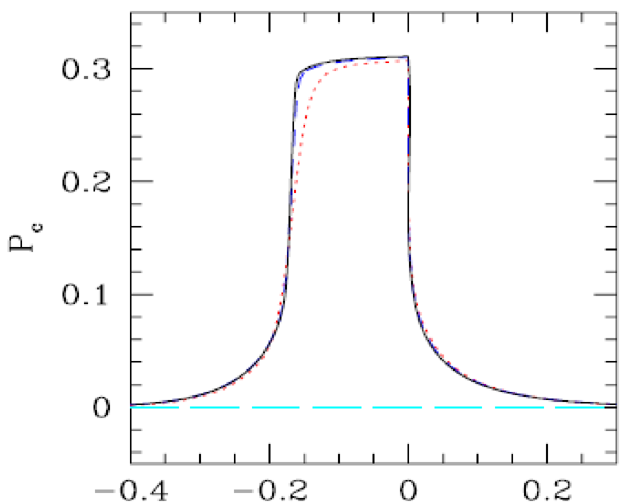
$\sigma_s = \rho_2 / \rho_1$



$$\xi = \frac{x}{l_{max}(t)} \propto \frac{x}{u_s \cdot t}$$ = similarity variable

DSA Kinetic Simulation for M=10 shock





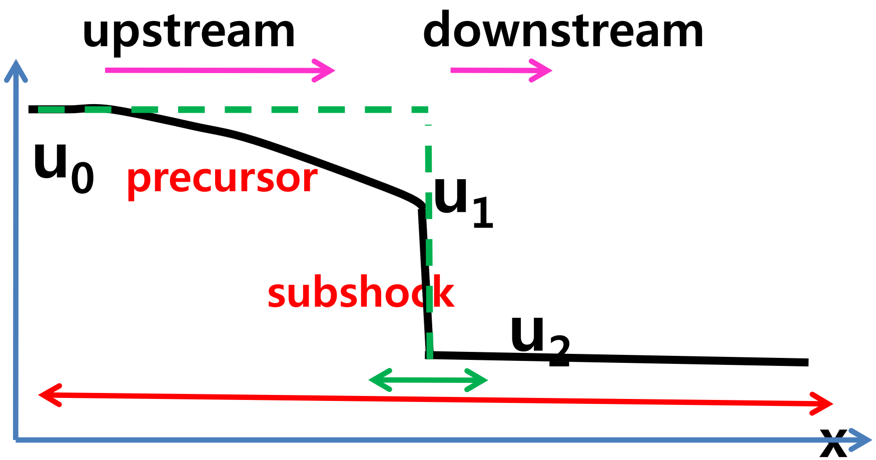
$$\xi = \frac{x}{u_s t}$$

-Precursor compression, subshock transition, and postshock Pc evolve to *self-consistent* dynamical equilibrium states.

-The shock structure broadens linearly with time.

- CRs are injected at subshock and accelerated to higher pmax, but they are also advected downstream and diffuse further upstream.

$$\int E_c dx \propto (\rho_0 u_s^3) \cdot t$$



$$\sigma_s = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \text{subshock comp.}$$

$$\sigma_t = \frac{u_0}{u_2} = \frac{\rho_2}{\rho_0} = \text{total shock comp.}$$

for $\kappa(p) = \kappa^* p^{\alpha}$

momentum dependent

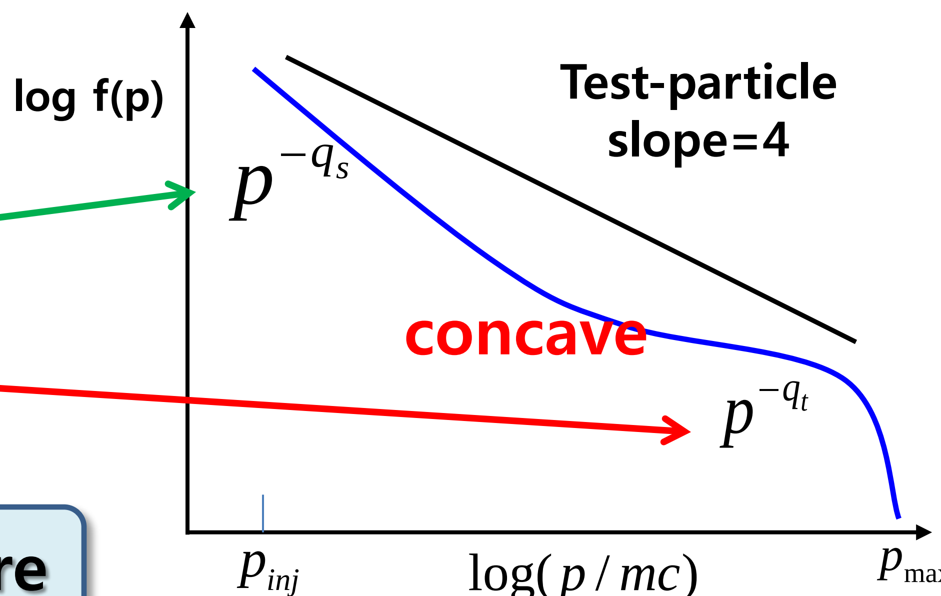
$$l_{diff}(p_{\min}) = \frac{\kappa(p_{\min})}{u_s}$$ → **feel only σ_s**

$$l_{diff}(p_{\max}) = \frac{\kappa(p_{\max})}{u_s}$$ → **feel σ_t**

Particles with different *p* experience different Δu.

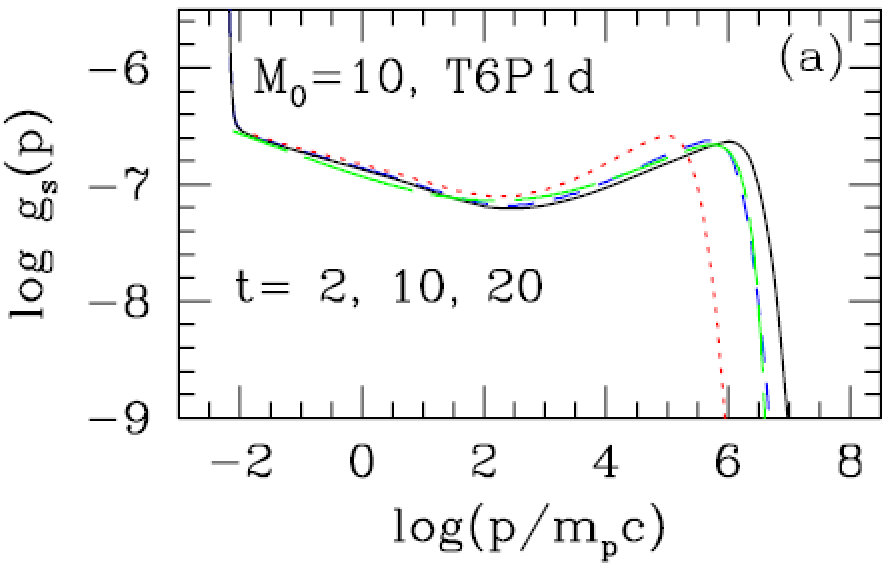
$$q_s = \frac{3(u_1 - \upsilon_A)}{(u_1 - \upsilon_A - u_2)} > 4 : \text{subshock}$$

$$q_t = \frac{3(u_0 - \upsilon_A)}{(u_0 - \upsilon_A - u_2)} < 4 : \text{total shock}$$



CR modified shock structure

CR spectrum during the Self-similar stage for M=10 shock

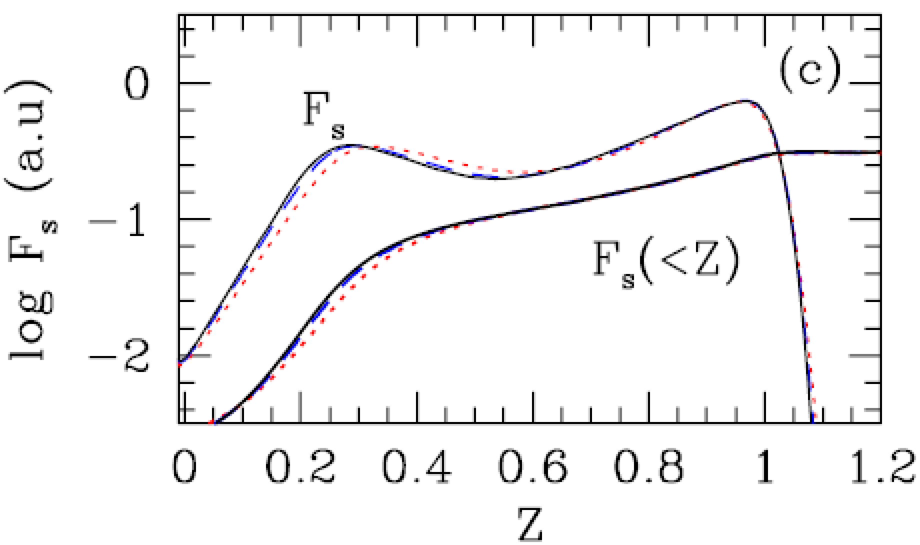


$$q_s = \frac{3(u_1 - \upsilon_A)}{u_1 - \upsilon_A - u_2}$$ subshock jump at low p

$$q_t = \frac{3(u_0 - \upsilon_A)}{u_0 - \upsilon_A - u_2}$$ total shock jump at high p

$$p_{\min} = p_{\text{inj}} \approx 2.5 p_{th}$$

$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t\,, \quad \text{where } \kappa(\text{p}) = \kappa^* p\left(\frac{\rho}{\rho_o}\right)^{-1}$$



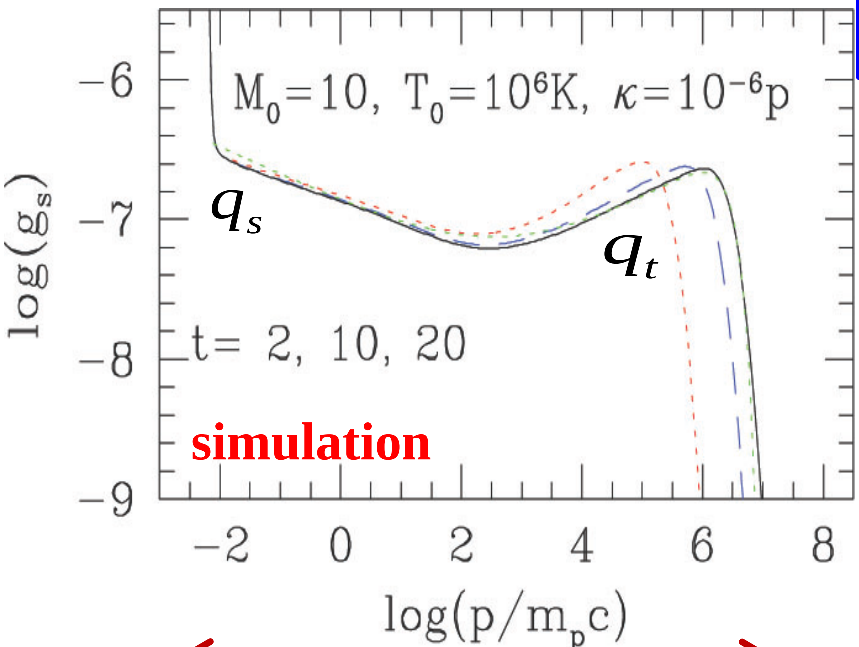
Partial pressure

$$F(Z) \equiv g(Z)\frac{p}{\sqrt{p^2+1}} \ln\left(\frac{p_{\max}}{p_{\text{inj}}}\right)$$

$$Z \equiv \ln(p/p_{\text{inj}})/\ln[p_{\max}(t)/p_{\text{inj}}]$$

constant in time

CR distribution function at shock



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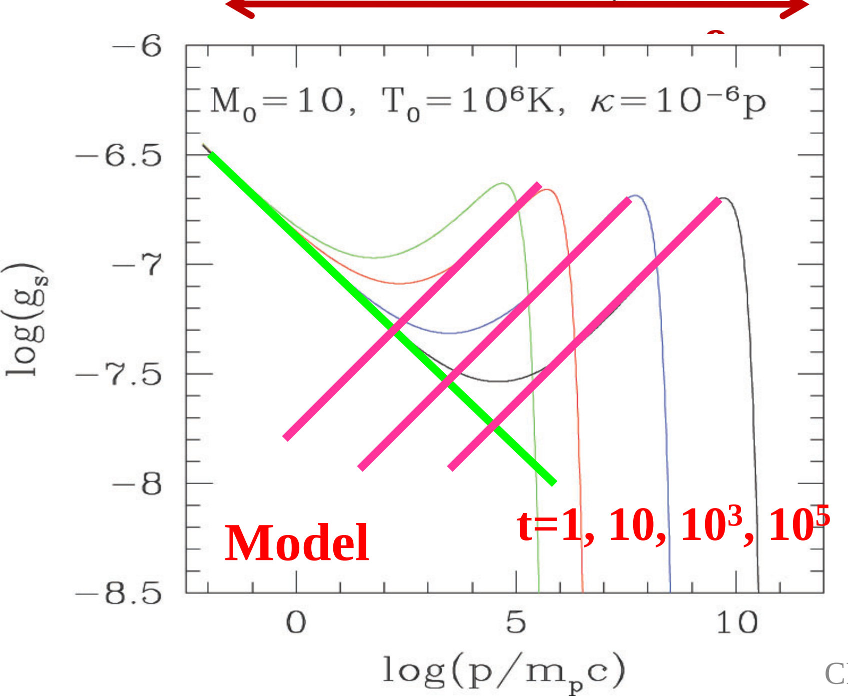


possible analytic form : **two power-laws**

$$f(x_s, p) = [\, f_1 \cdot (p / p_{\min})^{-q_s} + f_2 \cdot (p / p_{\max})^{-q_t} \,] \cdot \exp[-(p / 1.5\, p_{\max})^2]$$

where $f_1 = f_{th}(p_{\text{inj}})$ at thermal tail

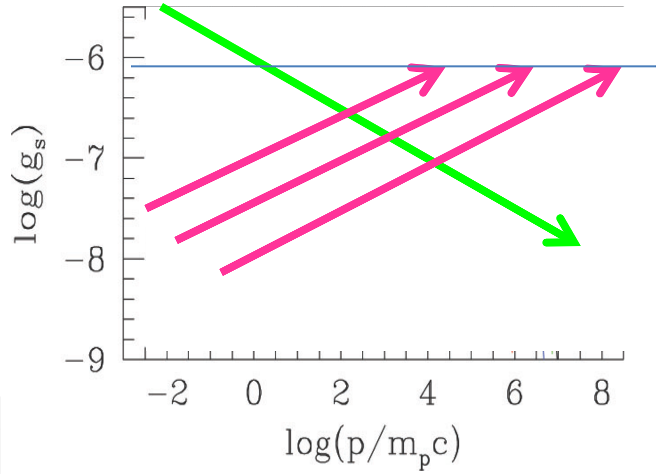
$f_2 \approx \text{const}$

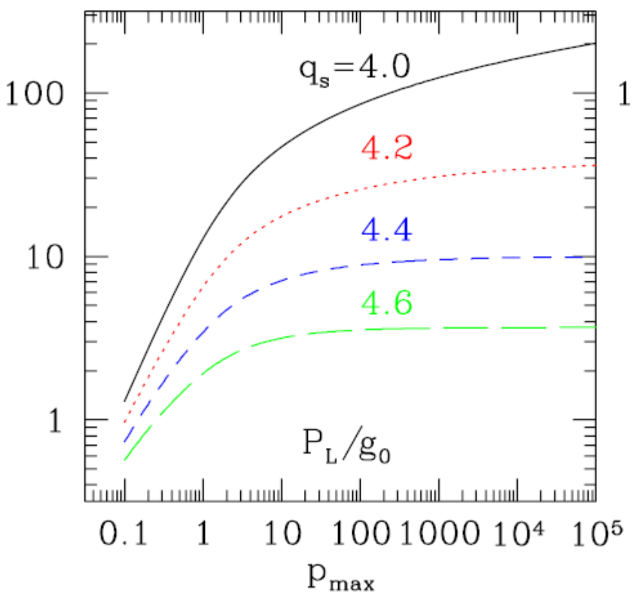
Kang et al 2009

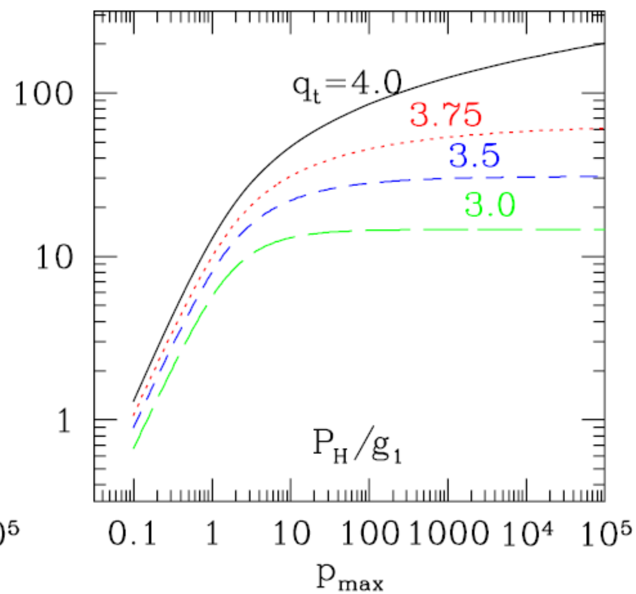
Why CR modified shocks become self-similar ?

$$P_c = \frac{4\pi}{3} m_p c^2 \int_{p_{inj}}^{\infty} g(p) \frac{p}{\sqrt{p^2+1}} \frac{dp}{p}. \quad = P_L \quad + \quad P_H$$

$$g_s(p) = \left[g_0 \cdot \left(\frac{p}{p_{inj}}\right)^{-q_s+4} + g_1 \cdot \left(\frac{p}{p_{max}}\right)^{-q_t+4} \right] \exp\left[-\left(\frac{p}{1.5 p_{max}}\right)^{2\alpha} \right]$$







if $q_s > 4.0$ (i.e. $\sigma_s < 4.0$),

$P_L \rightarrow$ constant at large p_{max}

if $q_t < 4.0$ ($i.e.\ \sigma_t > 4.0$),

$P_H \rightarrow$ constant at large p_{max}

in the limit of $t \rightarrow \infty$,

$p_{max} \rightarrow \infty$

postshock $P_{c,2} \rightarrow$ constant

shock structure $\rightarrow$ steady,

so $P_{g,2}$, ρ_1, $\rho_2 \rightarrow$ constant

P_c = constant requires

g_1 = constant

How to combine the two formulae for weak and strong shocks ?

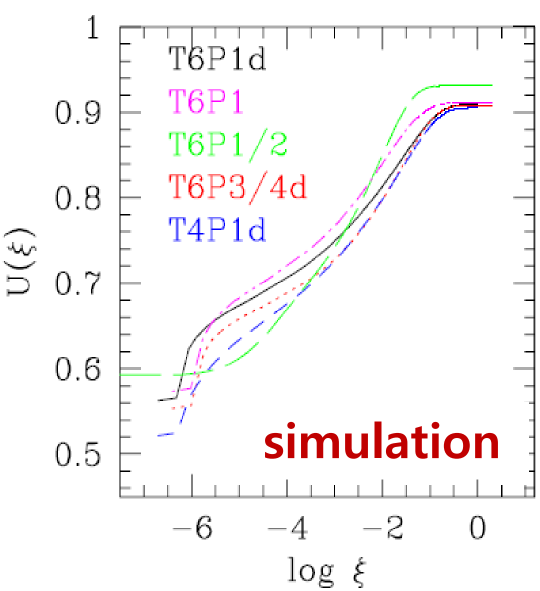
linear approximation for velocity profile in the precursor

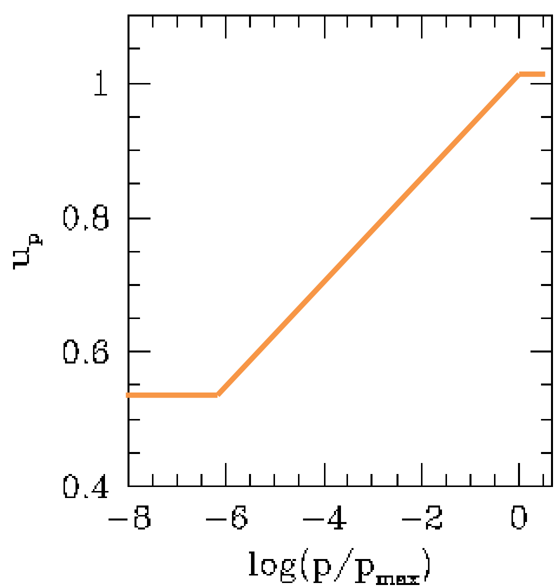
$$u_p \approx u_0 + (u_1 - u_0)\frac{\ln(p/p_{\max})}{\ln(p_{\min}/p_{\max})} \Rightarrow q(p) \approx \frac{3(u_p - \upsilon_A)}{(u_p - \upsilon_A - u_2)}$$

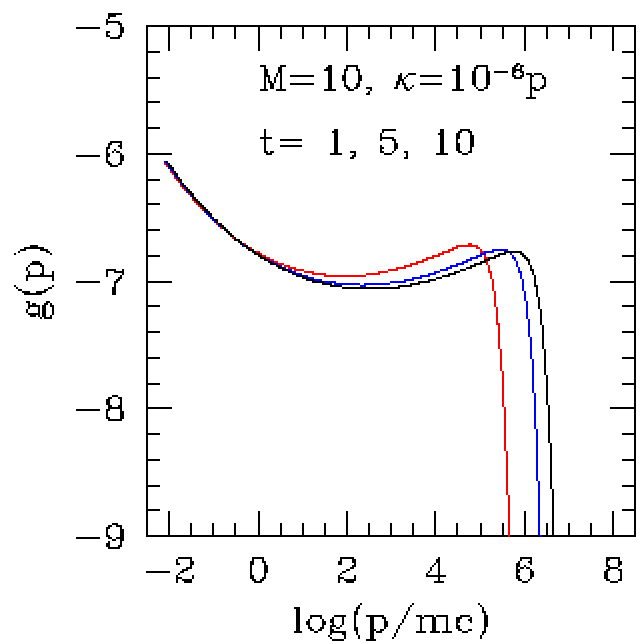
$$\Rightarrow f(p) \approx f_{th}(p_{inj})\exp[-\int_{\ln p_{\min}} q(p)d\ln p]\cdot\exp[-(p/1.5p_{\max})^2]$$

$$q(p) = \frac{d\ln f(p)}{d\ln p} : q(p_{\min}) = q_s, \quad q(p_{\max}) = q_t$$

This analytic approximation can be used for both weak and strong shocks

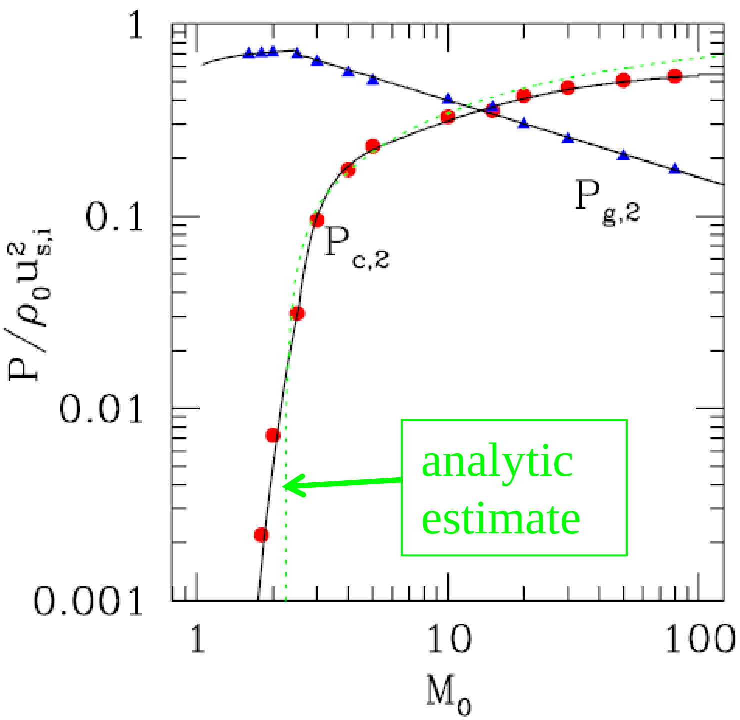


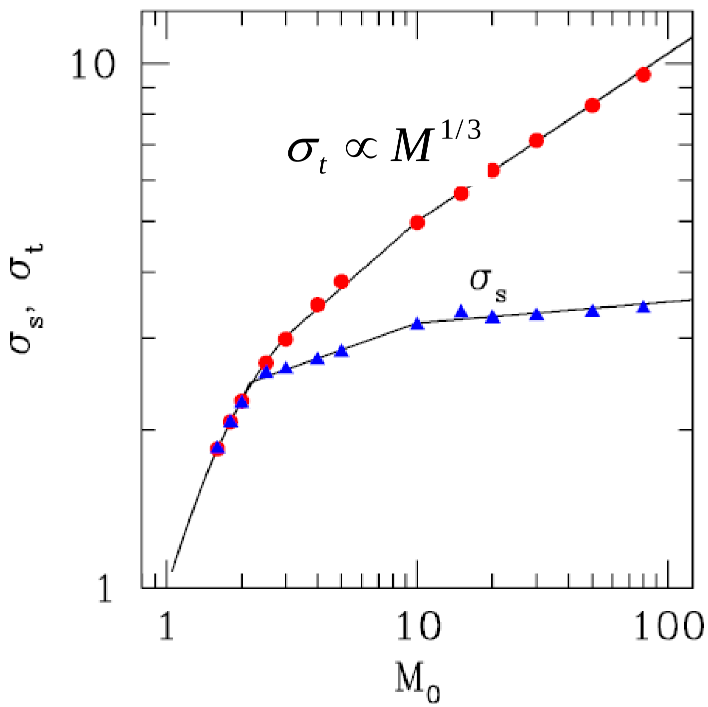




The approximate form of *f(p)* requires time asymptotic solutions from DSA simulations

$P_{g,2}$, $P_{c,2}$, ρ_2, u_1, u_2





total compression

$$\sigma_t = u_0 / u_2$$

subshock compression

$$\sigma_s = u_1 / u_2 \sim 3$$

$$\frac{P_{c,2}}{\rho_0 u_s^2} \approx 0.5 \text{ for } M_0 \geq 20$$

These solutions depend on details of the models for injection, wave generation, drift, and dissipation models, especially for weak shocks.

SUMMARY

- In evolving CR modified plane shocks even with ever increasing p_{max}, the precursor & subshock transition approach time-asymptotic states.

$$P_{g,2}, P_{c,2}, \sigma_t = \rho_2/\rho_0, \ \sigma_s = \rho_2/\rho_1 \rightarrow \text{constant}$$

- Then precursor/shock structure evolves in a self-similar fashion, depending only on similarity variable, $\xi = x/(u_s t)$.

- Wave drift increases the power-law slope from canonical test-particle values and reduces the CR acceleration efficiency.

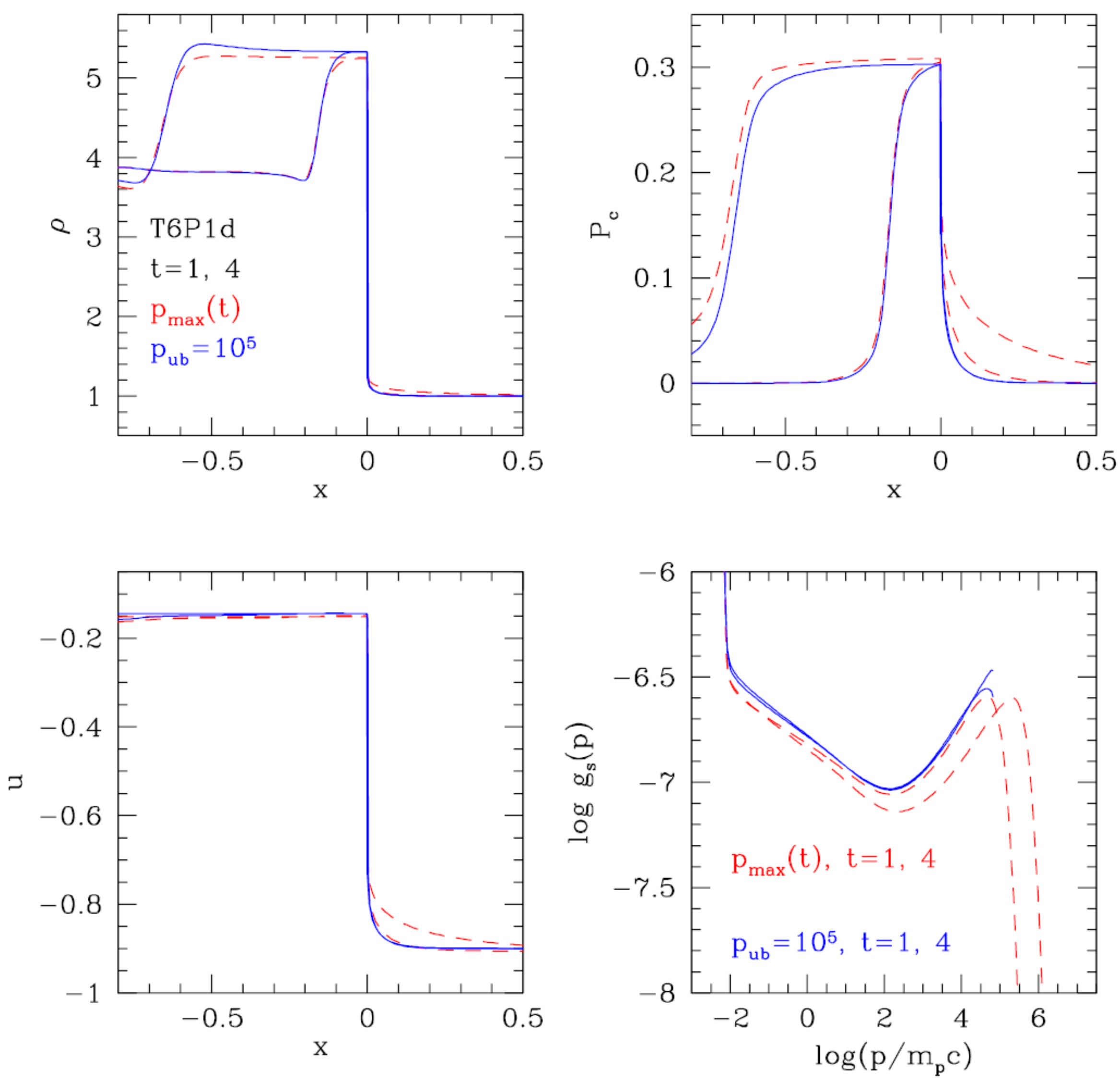
Wave dissipation (heating of gas) in the precursor also reduce the CR acceleration efficiency.

→ total compression is $\sigma_{tot} = R_{tot} \approx 5(M/10)^{1/3} < 10$

(even with escaping particles, or with fixed p_{max})

- CR acceleration may not be “too efficient”.

time-dependent solution and steady-state solution with the same pmax are the same.



Steady state solution is achieved by setting a upper momentum boundary at $p_{ub} = 10^5$ (blue solid lines)

Precursor becomes steady.
Highest energy particles escape through the upper momentum boundary.

Time-dependent solution at t=1, 4 → $p_{max}(t) \approx 10^5$ (red dashed lines)

Precursor broadens as pmax increases with time.

Falle & Giddings, 1987, numerical DSA simulations

final steady state

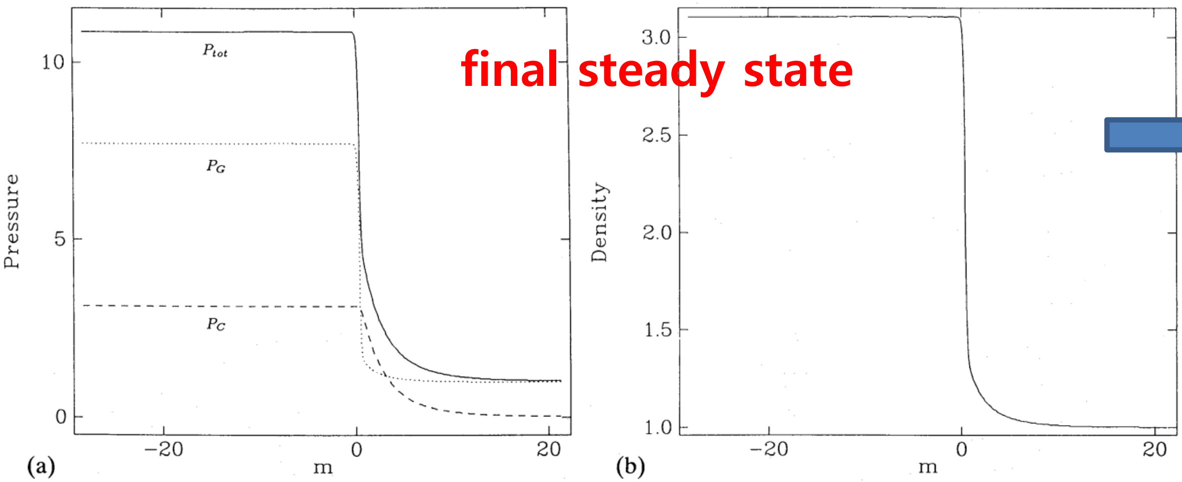
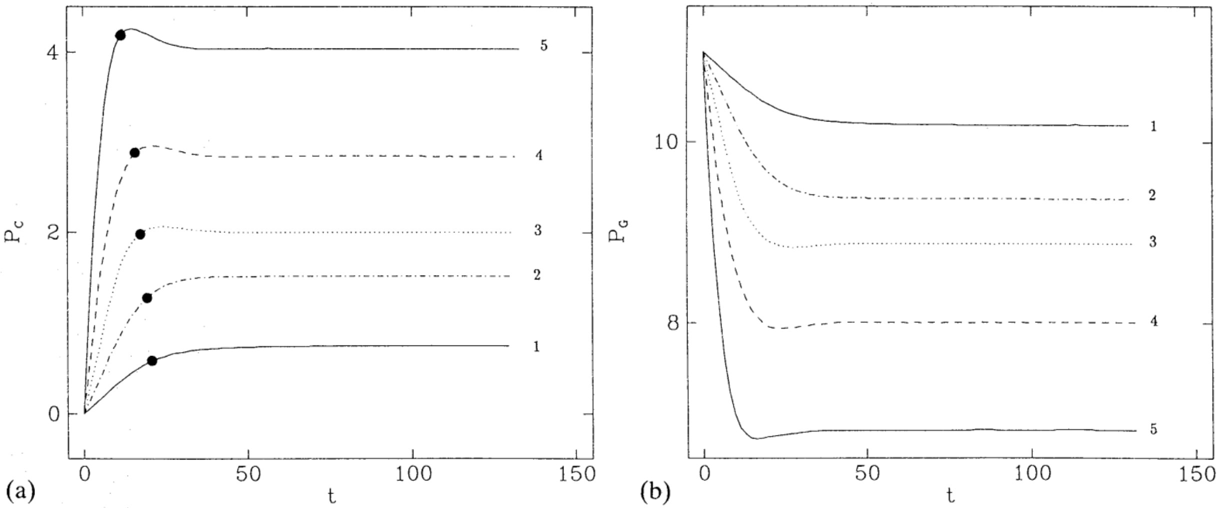


Figure 5. Shock structure in the steady state for a piston Mach number $M_p=2.0$ and upstream state $P_G=1.0$, $P_C=0$, $\rho=1.0$, $f=0$. The injection rate $\varepsilon=0.009$ and the injection momentum $p_1=0.01m_pc$. $\varkappa \propto p^{1/4}/\rho$. m is the Lagrangian coordinate defined by $dm=\rho\,dx$.

a piston (M_p=2.0) driven shock.

with $\kappa \propto p^{1/4}\rho^{-1}$

fixed injection rate



Pc increases,
Pg decreases,
Pc/Pg → constant in time.
The shock reaches a steady state.

During this self-similar stage, the CR distribution at the subshock maintains a characteristic form.

analytic approximation for CR spectrum at the shock

$$f(p,t) \approx f_{th}(p_{inj}) \exp[-\int_{\ln p_{\min}} q(p) d\ln p] \cdot \exp[-(p/1.5 p_{\max}(t))^2]$$

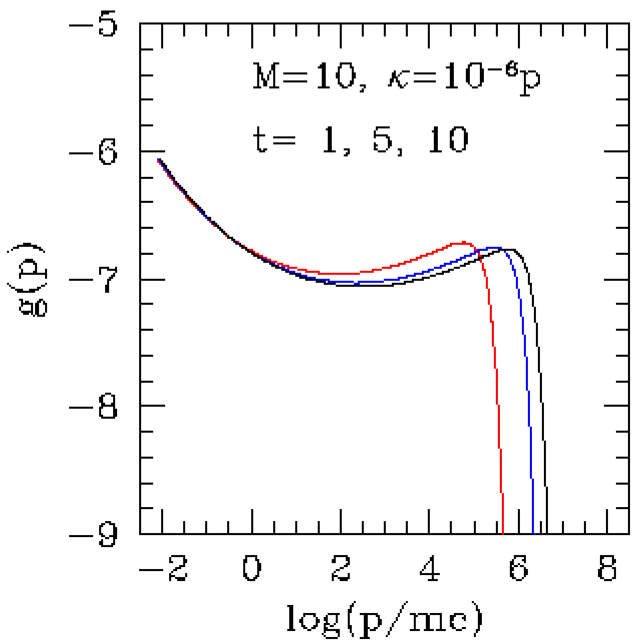
where $q(p) \approx \frac{3(u_p - \upsilon_A)}{(u_p - \upsilon_A - u_2)}, u_p \approx u_0 + (u_1 - u_0)\frac{\ln(p/p_{\max})}{\ln(p_{\min}/p_{\max})}, \; p_{\max}(t) \approx \frac{u_s^2}{8\kappa_n} \cdot t$

For weak shocks in test-particle limit this recovers

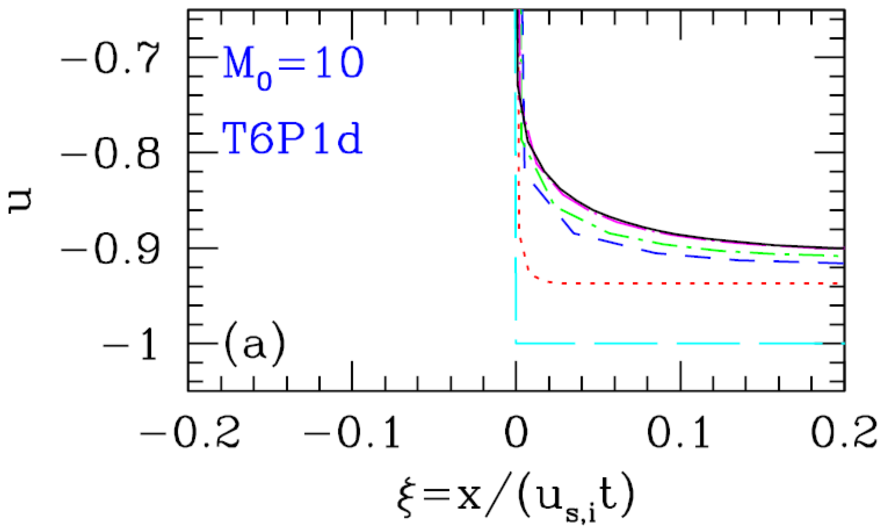
$$f_{test}(p,t) = f_{th}(p_{inj})\left(p/p_{inj}\right)^{-q_{test}} \exp\left[-(p/1.5 p_{\max})^2\right]$$

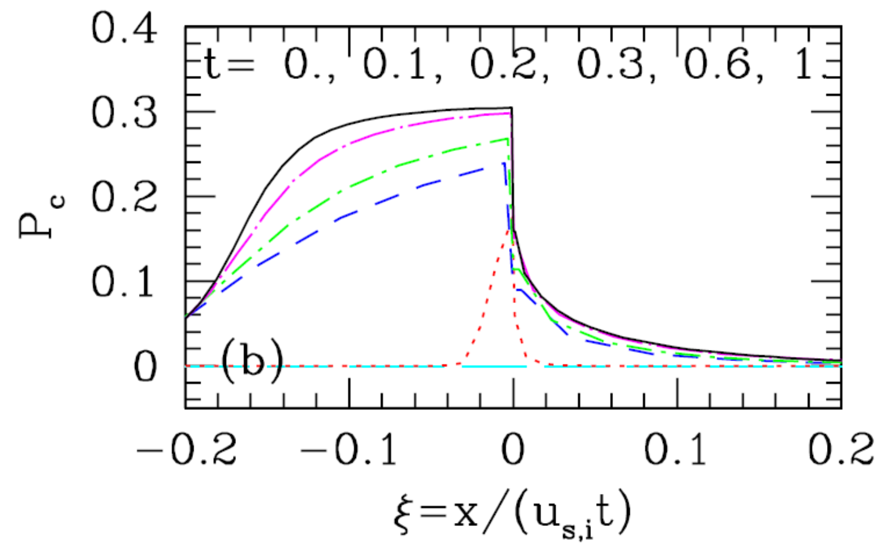
providing that we know the time asymptotic states:

$$P_{g,2}, \; P_{c,2}, \; \rho_2, \; u_1, \; u_2$$

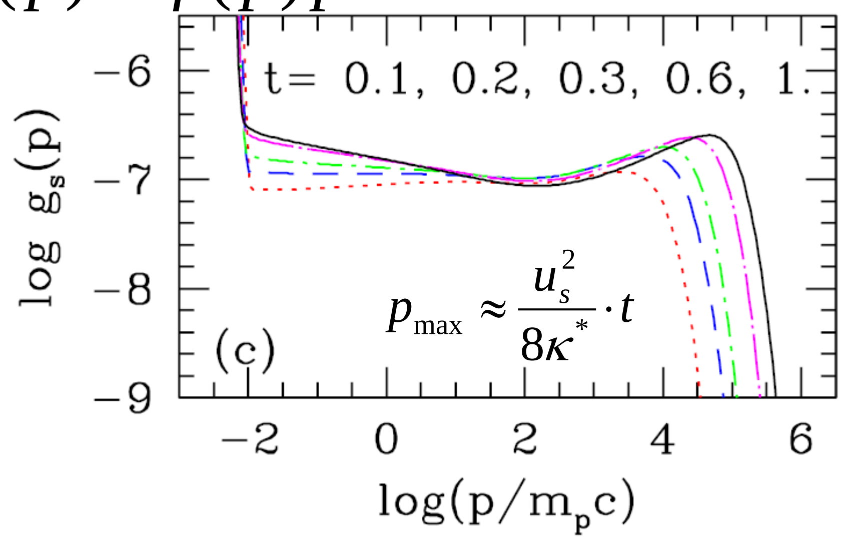


DSA kinetic simulation for M=10 shock: Early Evolution





$$g(p) = f(p)p^4$$



$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t$$

Initial conditions at t=0

M_0=10 gasdynamic shock

No pre-existing CRs

ε_B=0.2, θ=0.1

$\kappa(p)=10^{-6}p(\rho/\rho_0)^{-1}$

$$\theta = \frac{E_B}{E_{th}}, \quad \frac{\upsilon_A}{c_s} = \frac{M_0}{M_A} = \sqrt{\frac{9\theta}{5}}$$