Spectra of MHD Turbulence and Particle Transport in Nonlinear Shocks

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Nonlinear Processes in Astrophysical Plasmas
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Goals

- Construct a model of **magnetic field amplification** (MFA) in nonlinear shocks that describes in detail the **spectra of magnetic fluctuations**;
- Focus on **self-consistent modeling of the non-linear connections** between the system components (thermal plasma, turbulence, superthermal particles);
- Here, I will demonstrate a nonlinear shock model with B-field amplified simultaneously by **three different mechanisms (resonant and non-resonant)**;
- **Two models of particle diffusion** in strong magnetic turbulence will be presented and compared.

Method

The Nonlinear Model

- Particle transport modeled with a **Monte Carlo simulation**;
- **Analytic, semi-phenomenological description** for magnetic field amplification, self-consistently coupled to CR distribution and MHD flow;
- Fundamental **conservation laws** used to iteratively derive a nonlinear shock modification that conserves mass, momentum and energy;

Reasoning

- We describe a **large dynamic range** in turbulence scales and particle energies;
- Elements of the model **tested** against spacecraft observations of heliospheric shocks;
- Works for highly **anisotropic** particle distributions (particle escape and injection; large gradients of u and B).
- Ability to incorporate **non-diffusive** particle transport (future work).

Plasma Instabilities Induced by CR Streaming

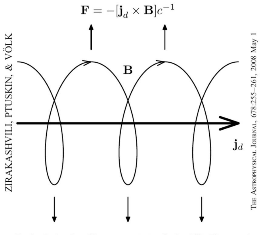
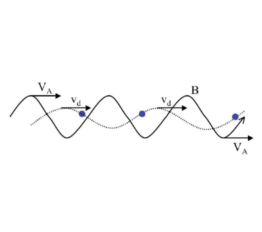
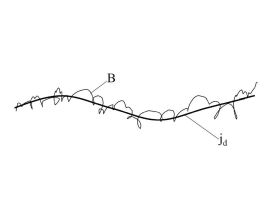


FIG. 1.—Explanation of the nonresonant streaming instability. The magnetic spiral (*thin solid line*) is stretched by the Lorentz force $F = -(j_d \times B)c^{-1}$ that appears due to the diffusive cosmic-ray electric current j_d.





Bell's nonresonant current-driven instability works on small scales by stretching magnetic field lines (Bell, 2004).

Resonant CR streaming instability. Alfvén waves gain energy from resonant particles streaming faster than the waves. (Skilling, 1975).

Nonresonant long-wavelength instability arises from the back-reaction of the amplified waves on the current of the streaming CRs (Bykov et al., 2009).

Evolution of Waves in the Precursor

Definitions

We describe turbulence by $W(x,k)$ – spectral energy density of turbulent fluctuations, and separate it into

$$W = W_M + W_K = \sum_{i \in \mathrm{modes}} W_M^{(i)} + \sum_{i \in \mathrm{modes}} W_K^{(i)}.$$

W_M – magnetic fluctuations, W_K – associated plasma velocity fluctuations, and (i) runs over the three types of waves (A – Alfvén waves, B – Bell's modes, C – Bykov's modes).

Equations

Evolution for each mode is given by the equation for $W^{(i)} = W_M^{(i)} + W_K^{(i)}$:

$$u\frac{\partial W^{(i)}}{\partial x} = \gamma^{(i)} W^{(i)} - L^{(i)} + \left[-\alpha^{(i)} W^{(i)} + \frac{\partial}{\partial k}\left(kW^{(i)}\right)\right]\frac{du}{dx} - \frac{\partial \Pi^{(i)}}{\partial k} \qquad (1)$$

Resonant Streaming Instability

Quasi-Linear Theory

- Growth rate at a wavenumber k is

$$\gamma^{(\mathrm{A})} W^{(\mathrm{A})} = v_A \left[\frac{\partial P_{\mathrm{cr}}(x,p)}{\partial x} \left| \frac{dk}{dp} \right| \right]_{p=\frac{eB_{\mathrm{ls}}}{ck}} ;$$

- Wavelengths resonant with CR particles: $r_g^{-1}(p_{\mathrm{max}}) < k < r_g^{-1}(p_{\mathrm{min}})$;
- Equal energy in magnetic and kinetic fluctuations.

Strong Fluctuations (Nonlinear Theory)

- We include transit time damping (Lee & Völk 1973, Achterberg & Blandford, 1986):

$$L^{(\mathrm{A})} = \sqrt{\frac{\pi}{2}} k r_{g,\,\mathrm{th}} \frac{\left[W^{(\mathrm{A})}\right]^2}{B_0^2/(8\pi)} \omega_B .$$

- Saturation at $\Delta B \approx$ a few B_0 may occur (Lucek & Bell 2000). We do not include this effect.

Nonresonant Short Scale Current Driven Instability (Bell)

Quasi-Linear Theory

- Growth rate at k is

$$\gamma^{(\mathrm{B})} = v_A k \sqrt{\frac{4\pi j_d}{cB_0 k} - 1},$$

- Amplifies short-scale fluctuations: $k > r_g^{-1}(p_{\min})$;
- Velocity fluctuations contain a few times more energy than magnetic fluctuations.

Strong Fluctuations (Nonlinear Theory)

Simulations[a] show for $\Delta B \gtrsim B_0$:

- Growth slows down (saturation at $r_{\mathrm{g\,min}} \approx k$?):
- Dominant k decreases (dissipation at large k, inverse cascade, or both)

We assume, following Riquelme & Spitkovsky 2009, nonlinear dissipation

$$L^{(\mathrm{B})} = C\left[W^{(\mathrm{B})}\right]^{\frac{3}{2}} \rho^{-\frac{1}{2}} k^{\frac{3}{2}}.$$

[a] Bell, Reville+, Zirakashvili+, Niemec+, Riquelme+

Nonresonant Large Scale Current Driven Instablity (Bykov)

Quasi-Linear Theory

- Fastest growing mode has

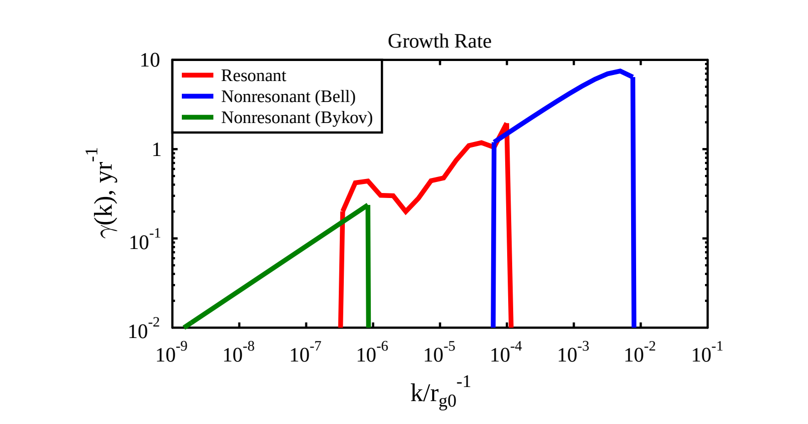
$$\gamma^{(\mathrm{C})} = v_A \sqrt{\frac{a}{\eta} \frac{4\pi j_d}{c B_0} k}$$

- Operates on large scales: $k < r_g^{-1}(p_{\max})$;
- Predominantly velocity fluctuations in the eigenmodes.

Strong Fluctuations (Nonlinear Theory)

- Slow growth: does not enter the strongly nonlinear regime $\Delta B \gg B_0$;
- Saturation via velocity fluctuations may be important.

At any point in the precursor, the 3 instabilities operate in non-overlapping regions of k-space[1].



- The short-wavelength instability (Bell) is the fastest, but short-scale turbulence does not efficiently scatter particles;
- The resonant instability is slower, but produces stronger stcattering;
- The long-wavelength instability (Bykov) is slow, but amplifies waves that may be important for the highest energy (escaping?) particles.

[1] The small overlap visible in the plot is for continuity in the numerical scheme

Calculating the Particle Diffusion Coefficient

Prescription

- Our model for diffusion coefficient uses **magnetic field re-normalization**;
- Reproduces known **asympthotic behavior** for some cases with $\Delta B \ll B_0$ and $\Delta B \gg B_0$, smoothly interpolates between them;
- Details in AV's dissertation.

Features

This prescription reproduces:

- The $\lambda \propto p^2$ scattering in **small-scale** fluctuations for $B_{\rm ss} \gg B_0$ (highest energy particles)
- The **resonant scattering** regime $\lambda \propto p^{2-s}$ for $W \propto k^{-s}$ (intermediate energies)

Two Models

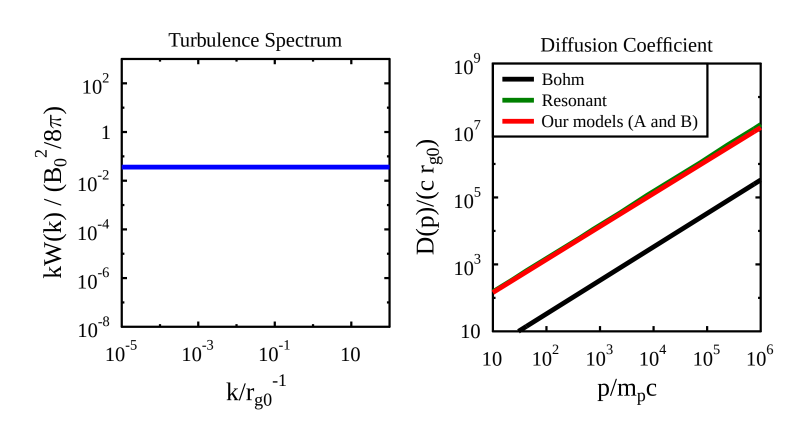
Two possibilities for diffusion of magnetized particles ($r_g < l_{\rm cor}$):

- **Model A**: $\lambda = \text{const}$ for the **smallest** particle energies;
- **Model B**: $\lambda \propto p$ for the **smallest** particle energies.

Examples of Difusion Coefficient Calculation

Example 1.

Weak white noise $W \propto k^{-1}$ + uniform B_0:

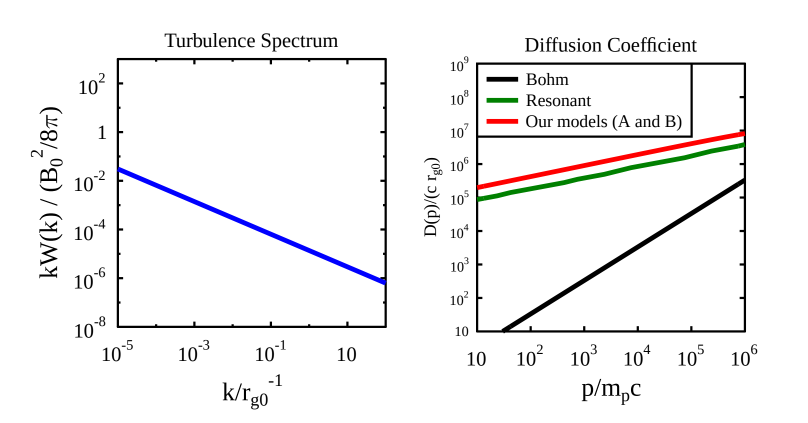


Our model reproduces the well known resonant scattering rate: if $W(k) \propto k^{-\alpha}$, then $D(p) \propto p^{2-\alpha}$.

Examples of Difusion Coefficient Calculation

Example 2.

Kolmogorov-like power law spectrum of turbulence $W \propto k^{-5/3}$:



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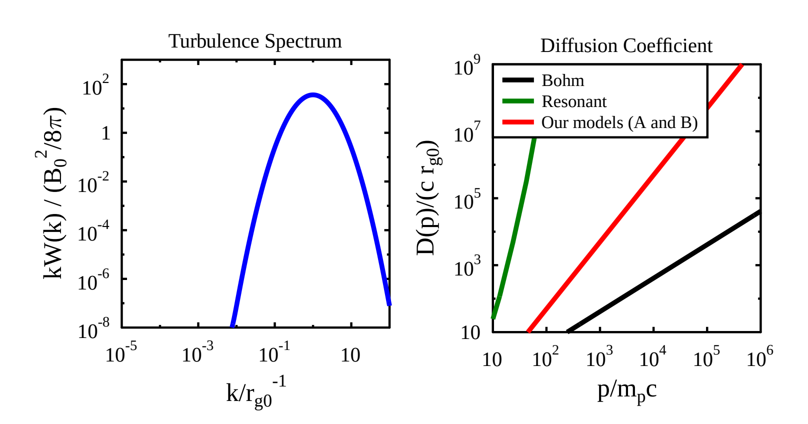
Examples of Difusion Coefficient Calculation

Example 3.

Strong, short scale turbulence:

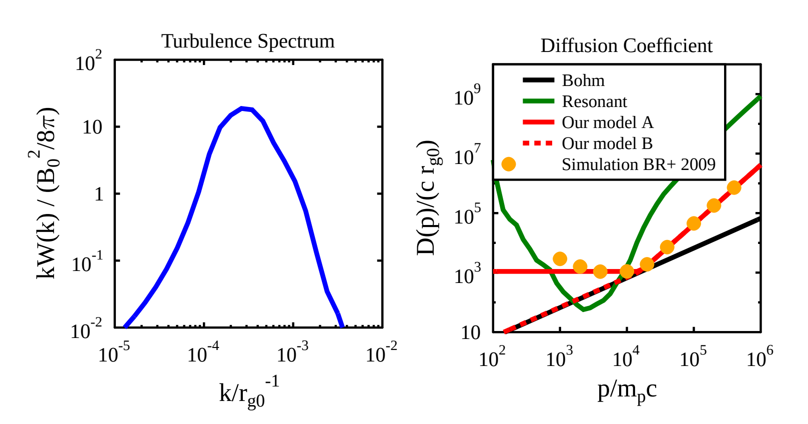


Our model correctly describes the $D(p) \propto p^2$ behavior (notice how the resonant scattering and Bohm models fail).

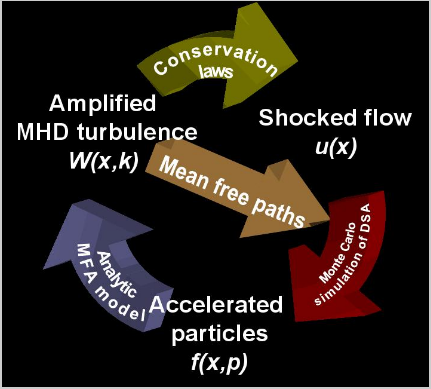
Examples of Difusion Coefficient Calculation

Example 4.

Synthetic spectrum:



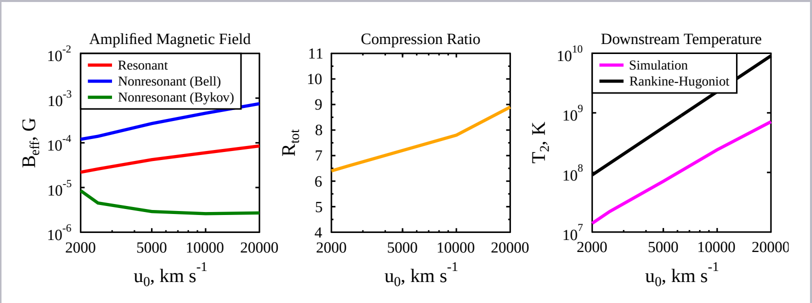
The spectrum and the orange data points are from the simulation by Reville et al. 2008. Our calculations agree in the range where $D(p)$ increases with p.



- Our model simulates **particle acceleration**, **turbulence generation** and **shocked flow** all consistently with each other;
- A **Monte Carlo (MC) code** describes particle transport and acceleration;
- **Diffusion coefficient** used in the MC code coupled to turbulence spectrum;
- **Turbulence generation** driven according to particle transport simulated in MC.

Summary of a Set of Simulations

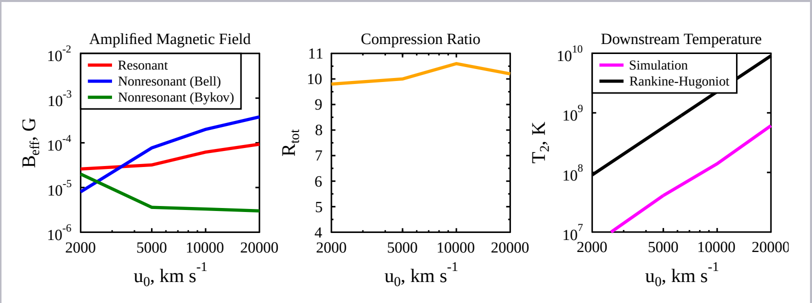
Model A. Magnetized particles have $\lambda(p) = \mathrm{const}$



Physical parameters in these simulations represent a SNR shock expanding into a cold interstellar medium ($n_0 = 0.3\,\mathrm{cm}^{-3}$, $T_0 = 10^4\,\mathrm{K}$, $B_0 = 3\,\mu\mathrm{G}$). Acceleration is size limited, $D_{\mathrm{FEB}} = 0.03\,\mathrm{pc}$, parallel geometry, steady state.

Summary of a Set of Simulations

Model B. Magnetized particles have $\lambda(p) \propto p$



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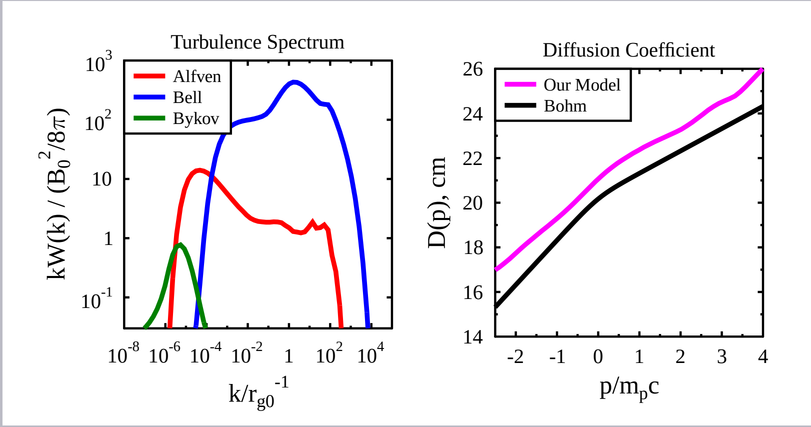
Energy spectra of magnetic fluctuations downstream of the $u_0 = 2500\,\mathrm{km\,s^{-1}}$ shock shown below.

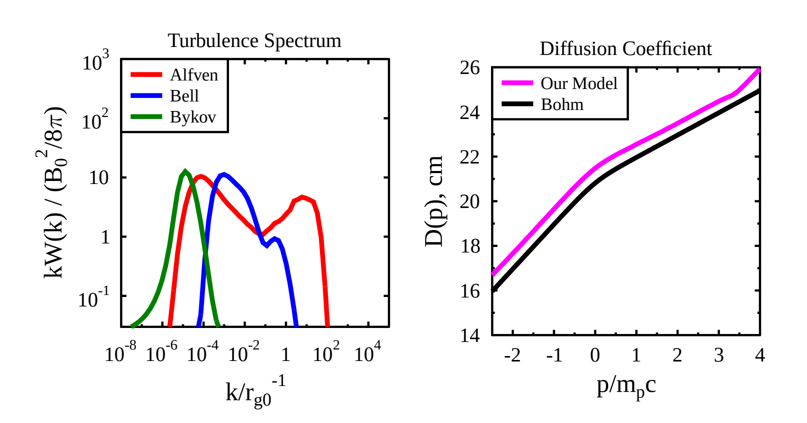
Model A. Magnetized particles have $\lambda(p) = \mathrm{const}$



Growth rates of instabilities are finite in non-overlapping k-space regions, but an overlap in the spectra occurs due to the integration through the precursor.

Energy spectra of magnetic fluctuations downstream of the $u_0 = 2500\,\mathrm{km\,s^{-1}}$ shock shown below.

Model B. Magnetized particles have $\lambda(p) \propto p$



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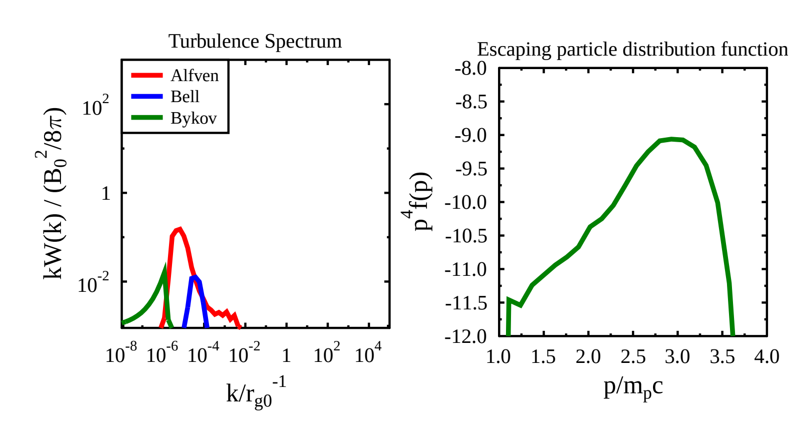
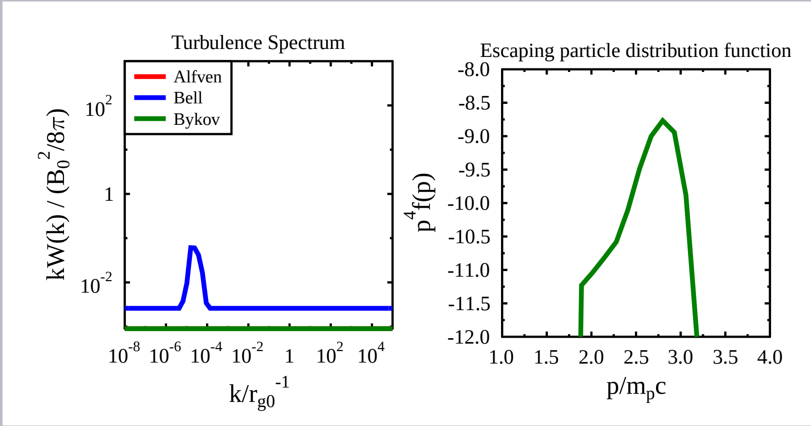
Model A. Particle escape in the presence of all three instabilities



Here, turbulence spectrum is shown at the upstream free escape boundary for the shock with $u_0 = 2000\,\mathrm{km\,s^{-1}}$.
B_{eff} downstream in this case is dominated by Bell's modes, but particle escape upstream is shaped by the resonant and the long-wavelength modes.

Model A. Particle escape in the presence of only Bell's instability



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Spectrum of Magnetic Fluctuations and Synchrotron Radiation

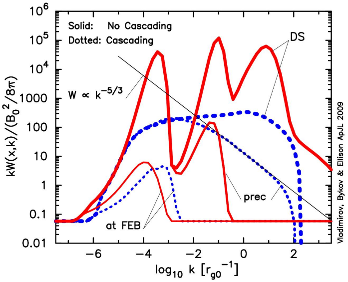

Figure: AV+, 2009, ApJL

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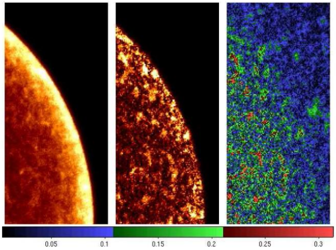

Figure: Bykov+, 2009, MNRAS

In strong turbulence, fluctuating **B** changes direction **as well as magnitude**. This modifies emitted synchrotron spectrum and may lead to time variable X-ray dots and clumps (Bykov, Uvarov, & Ellison. 2008, MNRAS), depending on the turbulence spectrum.

Summary of our Findings

Model of Nonlinear DSA with Magnetic Field Amplification

- **Self-consistently** couples particle acceleration, MFA and nonlinear flow modification;
- Combines **3 plasma instabilities** (resonant and 2 non-resonant) induced by accelerated particles;
- Simulates **particle transport based on the turbulence** energy spectra.

Results

- Macroscopic shock parameters (T_2, $R_{\rm tot}$, $B_{\rm eff}$, etc.) depend on the connection between turbulence spectra and particle transport;
- For high u_0, Bell's nonresonant instability dominates $B_{\rm eff}$ and, therefore, determines the synchrotron emission;
- Highest energy particles (esp. escaping upstream) are sensitive to the turbulence produced by the resonant and long-wavelength nonresonant mechanism;
- Synchrotron emission modified by time-variability of B may be a diagnostic of turbulence spectra produced by SNR shocks.

Open Questions

How do the instabilities evolve in the nonlinear regime($\Delta B \gg B_0$)?

- Nonlinear growth rate;
- Wave dissipation;
- Spectral energy transfer;
- Other MFA mechanisms [e.g., dynamo (Beresnyak, Cho, Ryu...) or magnetosonic instability (Malkov & Diamond), and other].

What happens at the subshock?

- Transmission of turbulence;
- Effect on particle injection.

Which model best describes low energy particle transport?

- Is either of our models ($\lambda \propto p$ or $\lambda = \mathrm{const}$) valid for $r_g < l_{\mathrm{cor}}$?
- Are time variability of **B** and velocity fluctuations important here?
- May the particle transport be non-diffusive?

Non-stationary solutions?

- E.g., work by Jones, Kang & Ryu and others.

Some References Mentioned in This Talk

- Nonresonant short-wavelength instability: Bell, MNRAS 2004;
- Nonresonant long-wavelength instability: Bykov, Osipov, & Toptygin, AstL 2009;
- Dissipation of Bell's instability: Riquelme & Spitkovsky, ApJ 2009;
- Model of diffusion discussed here and the Monte Carlo method: Vladimirov, arXiv:0904.3760.v1 2009;
- Particle transport simulation: Reville et al. MNRAS 2008;
- Dissipation of Alfvén waves: Lee & Völk, Ap&SS 1973; Achterberg & Blandford, MNRAS 1986
- Nonlinear shocks with Bell's instability: Vladimirov, Bykov, & Ellison, ApJL 2009;
- Synchrotron emission in turbulent B-field: Bykov, Uvarov, & Ellison, ApJ 2008; Bykov et al., MNRAS 2009, v. 399.