Hybrid Simulations of Particle Acceleration at Shocks (or, the injection problem)

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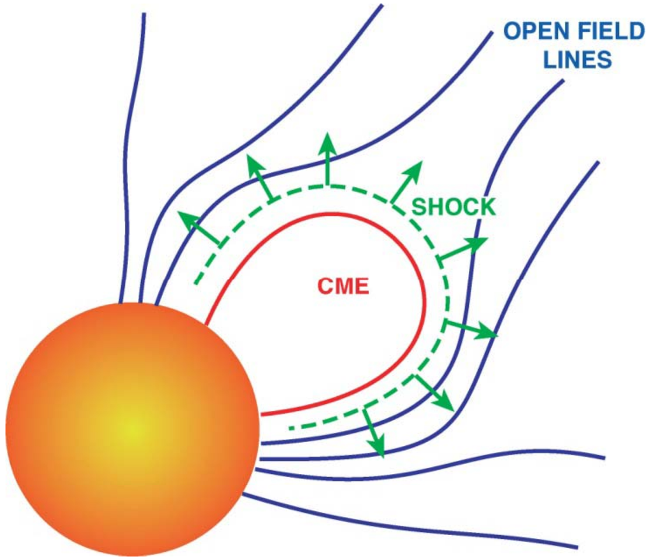
- In this talk, we will focus on the problem of accelerating low-energy and/or thermal particles by shock, which is not generally described by the diffusive transport equation

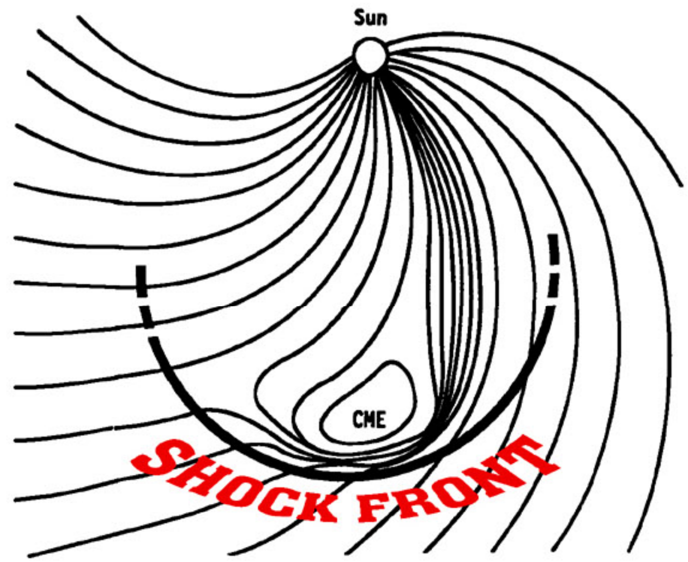
- Self-consistent plasma simulations can be used to study particle acceleration from thermal energies to energies that are generally considered to be acceptably treated by the diffusive transport equation

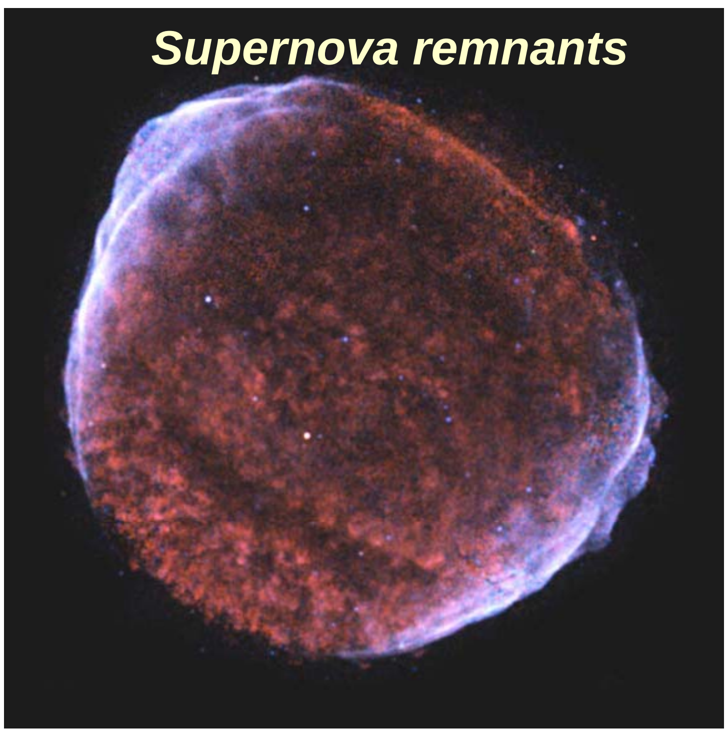
- This work is relevant to our understanding of the role of the magnetic-field angle in shock acceleration

CME – Solar Corona

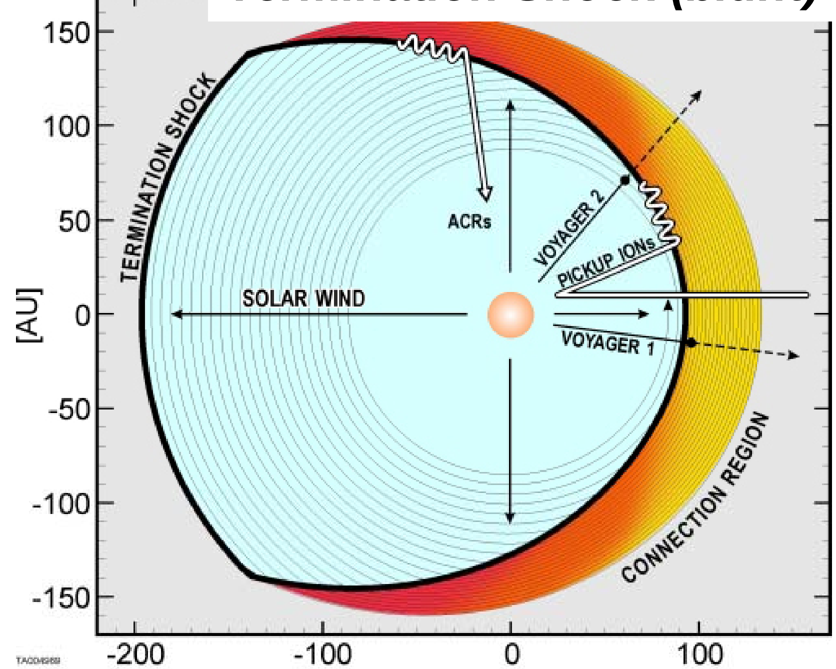


CME – Interplanetary Space





Termination Shock (blunt)



Quantitative predictions of Diffusive Shock Acceleration are obtained by solving the cosmic-ray transport equation

$$\frac{\partial f}{\partial t} = -V_{w,i}\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial x_i}\kappa_{ij}\frac{\partial f}{\partial x_j} - V_{D,i}\frac{\partial f}{\partial x_i} + \frac{1}{3}\frac{\partial V_{w,i}}{\partial x_i}\frac{\partial f}{\partial \ln p} + Q$$

advection ***diffusion*** ***drift*** ***energy change***

This equation is valid if scattering is sufficient to drive the angular distribution to near-isotropy. Hence, it can apply to energetic partices at shocks.

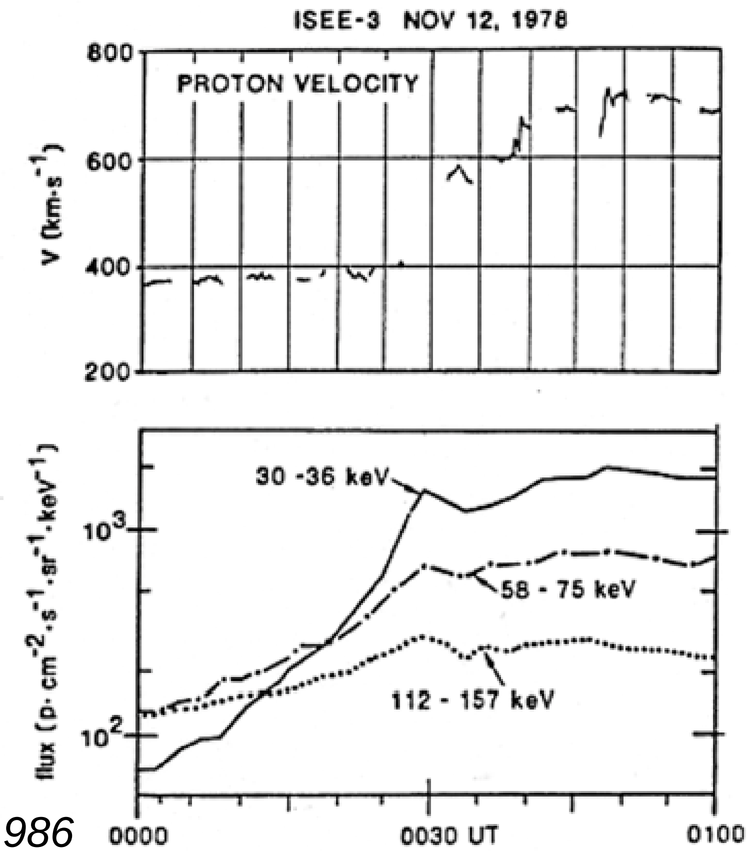
- The steady-state solution to Parker's equation at a planar shock at x = 0, for an infinite system, is given by

$$f(x,p) = \begin{cases} f_0\left(\frac{p}{p_0}\right)^{-\gamma} \exp\left(-\frac{U_1|x|}{\kappa_{xx,1}(p)}\right) & x < 0 \\ f_0\left(\frac{p}{p_0}\right)^{-\gamma} & x \geq 0 \end{cases}$$

where $\gamma = 3U_1/(U_1 - U_2)$

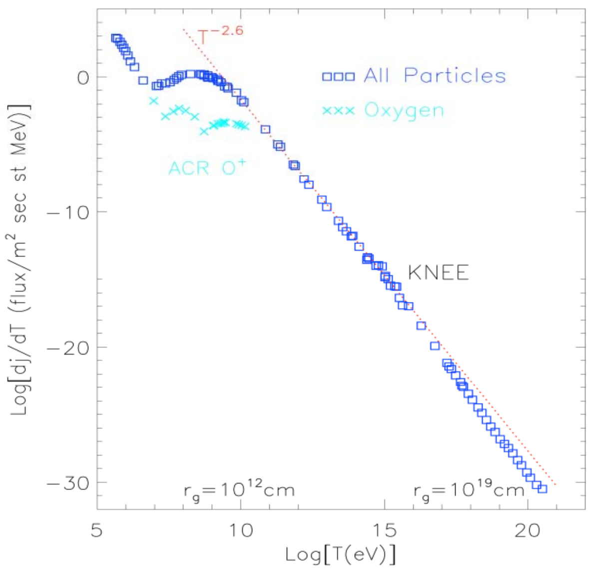
The downstream distribution is power law with a spectral index that depends only on the shock compression ratio (nearly universal spectral index).

Krymsky, 1977
Axford et al., 1977
Bell, 1978
Blandford and Ostriker, 1978

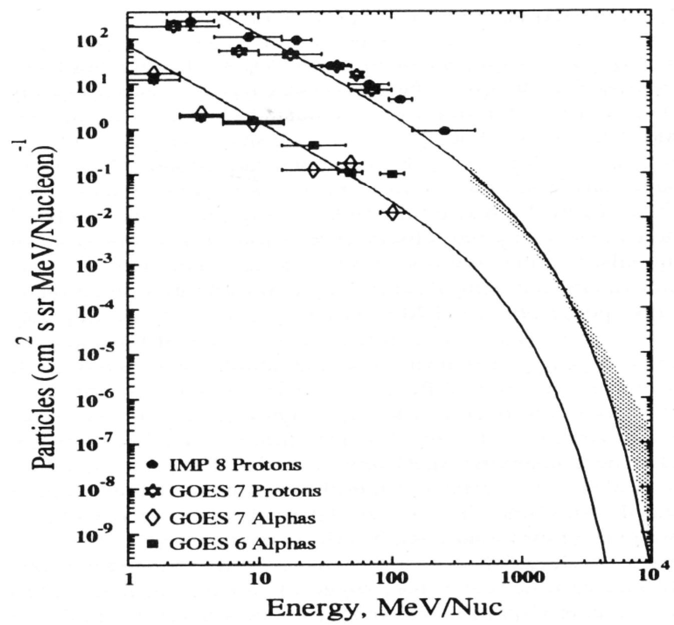


Kennel et al, 1986

The observed energy spectra of cosmic rays are remarkably similar everywhere they are observed.



Galaxy



Sun: CMEs

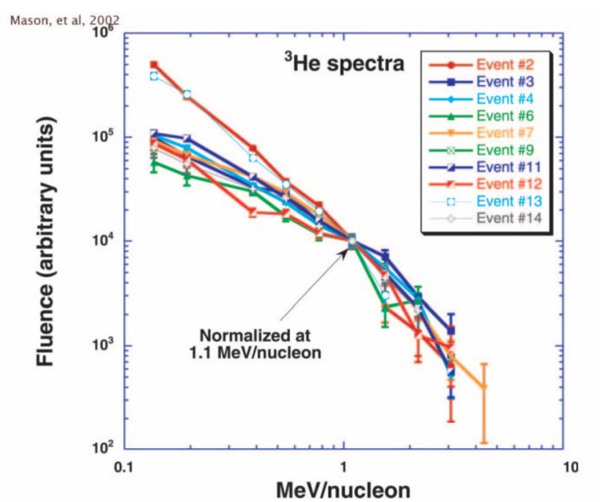
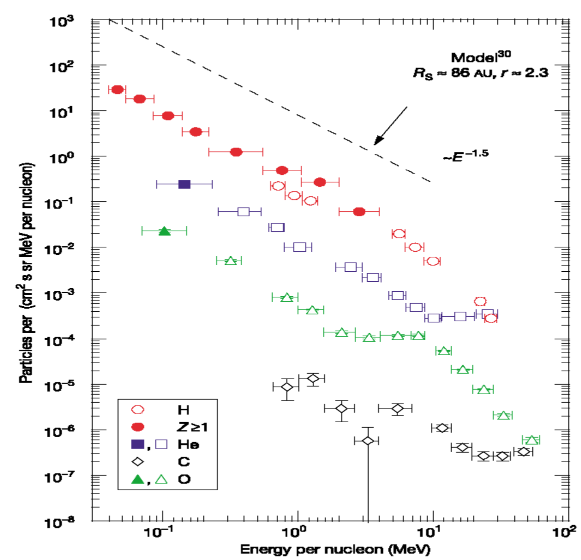


Fig. 10.—Spectra of ^{3}He of events 2–9 and 11–14 in Table 1, normalized to a fluence of 10^4 at 1.1 MeV nucleon^{-1}

Sun: Impulsive Solar Flares



ACR

The maximum energy

- The energy is limited by both the size and age of the system

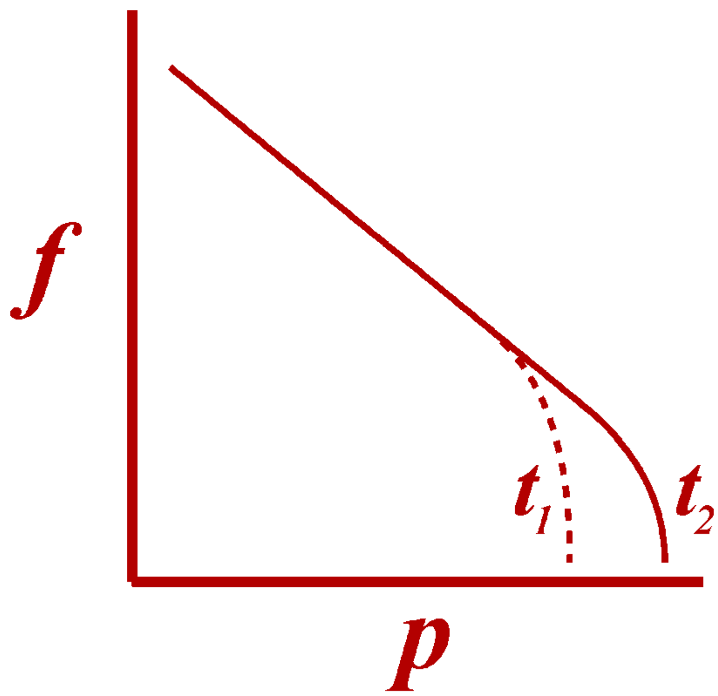
- Acceleration takes time. The ideal power-law energy spectrum is not created instantly.

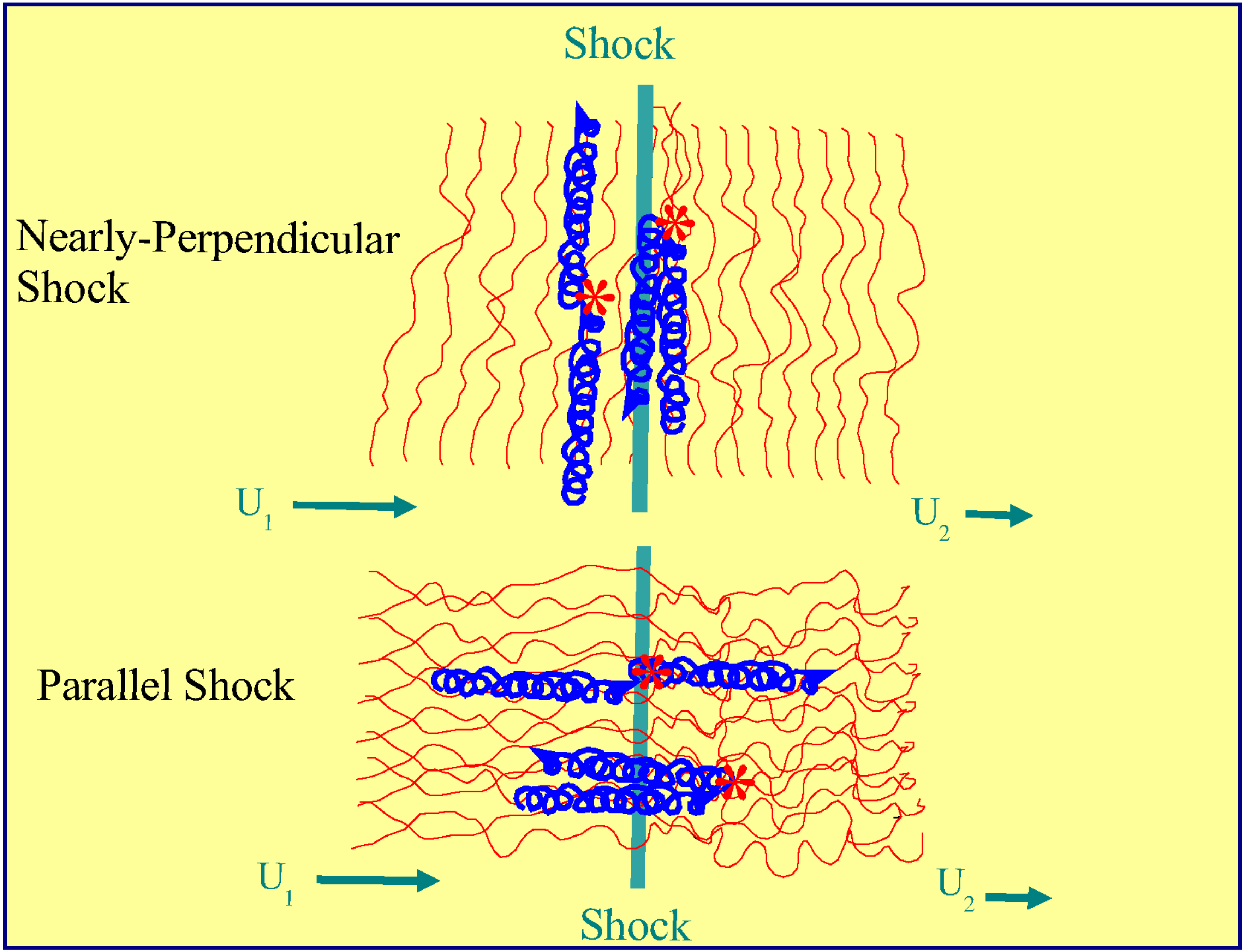
  Parallel shocks → slow

  Perpendicular shocks → fast

- The maximum energy over a given time interval strongly depends on the shock-normal angle

  for any given situation, a perpendicular shock will yield a larger maximum energy than a parallel shock.



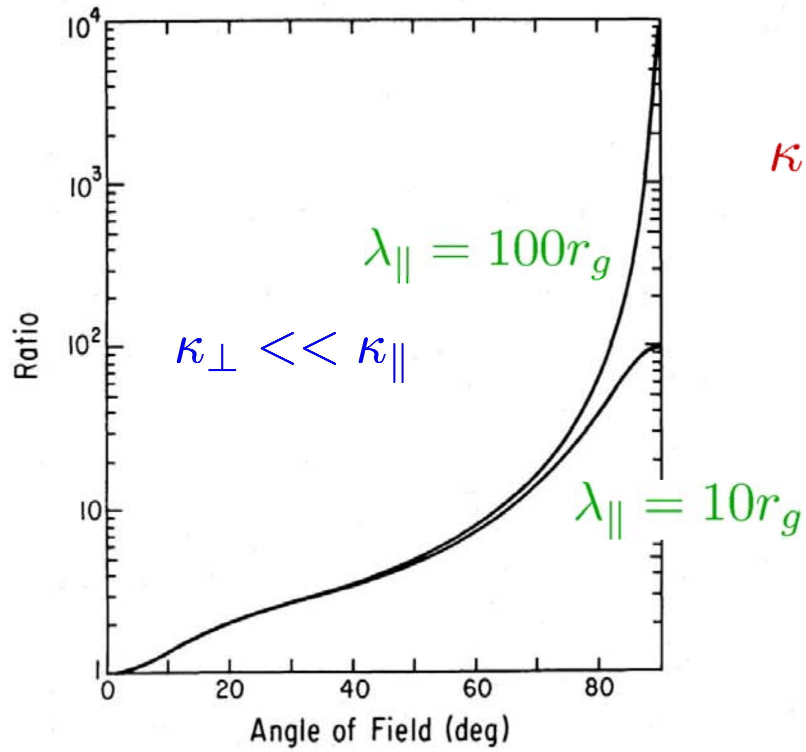


Shock
Nearly-Perpendicular Shock
U1
U2
Parallel Shock
U1
U2
Shock

Acceleration Rate as a Function of Shock-Normal Angle:
(assumes the billiard-ball approximation)

The acceleration rate depends inversely on the diffusion coefficient

$$\frac{1}{\tau_{acc}} \propto \frac{U_{sh}^2}{\kappa_{xx}}$$



$$\kappa_{\perp} = \left(\frac{r_g}{\lambda_{\|}}\right)^2 \kappa_{\|}$$

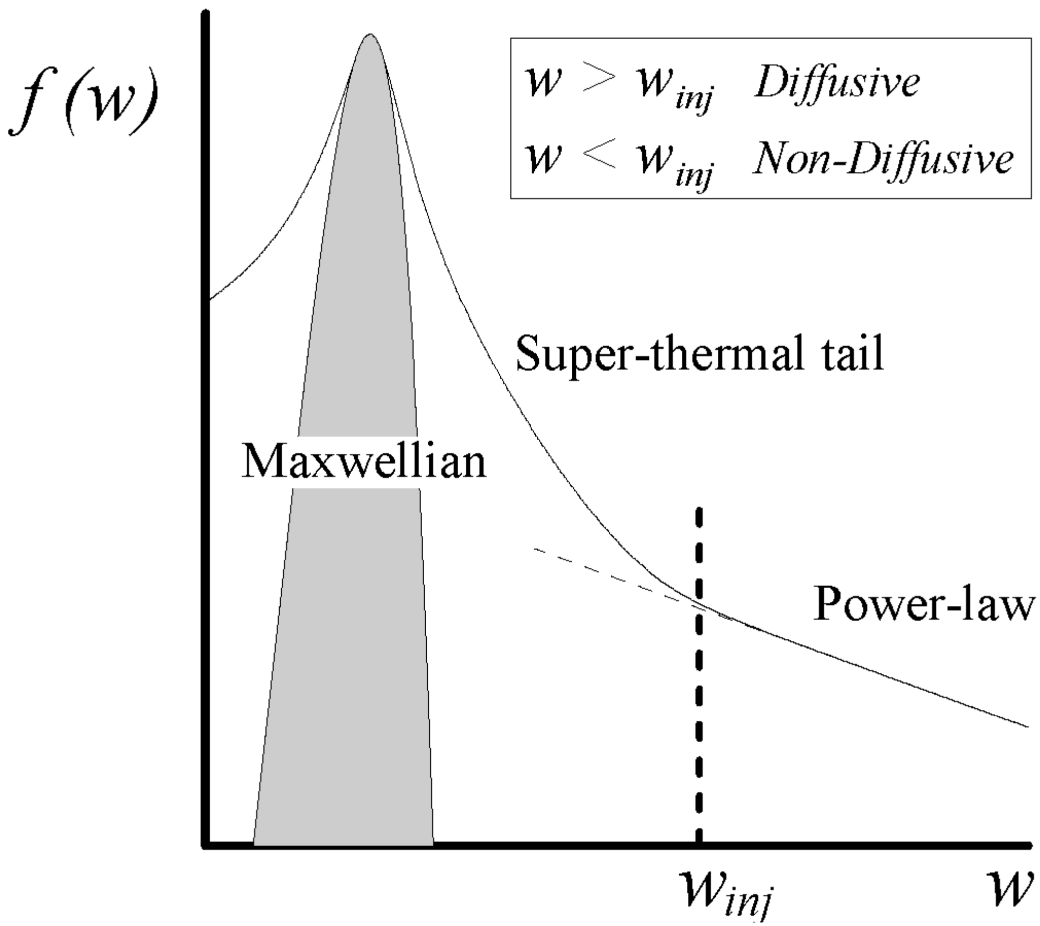
FIG. 1.—Plot of the ratio of energy gain rate with a transverse magnetic field to that neglecting the magnetic field given in eq. (8), as a function of angle between the upstream magnetic field and shock normal, θ_1. The upper curve is for a scattering mean free path $\lambda_{\|}$ equal to 100 times the gyroradius r_g, and the lower is for $\lambda_{\|} = 10\, r_g$.

Jokipii, 1987

The perpendicular diffusion may be different from that used above, but, in general, $\kappa_{\perp} << \kappa_{\|}$

Acceleration at low energies:

The injection problem: Particles must have speeds greater than the shock speed to be accurately described by Parker's equation



Diffusive shock acceleration at highly oblique shocks

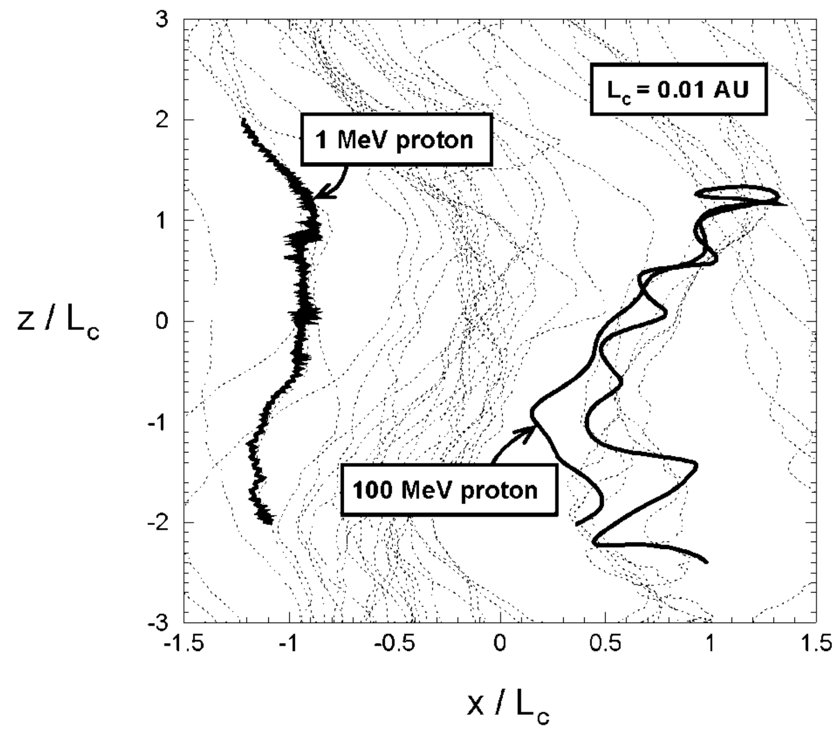
- An often-invoked injection criterion is

$$v_{inj} > U_{sh} \sec \theta_{Bn}$$

- This is incorrect since it assumes that there is NO motion normal the average magnetic field.

- It has led to a widely held misconception that perpendicular shocks are inefficient accelerators of particles.

- In general, particles move normal to magnetic fields.
  - Field-line random walk leads to a much larger diffusion coefficient that expected from hard-sphere scattering.
  - Numerical simulations show that $\kappa_\perp / \kappa_{||}$ is large and nearly independent of energy.
- The injection criterion *must* include perpendicular diffusion.



Because the distribution should be nearly isotropic, we require that the diffusive streaming anisotropy δ_i be small.

This is the criterion for the validity of the Parker equation

The general expression for the pitch angle anisotropy upstream of a shock, using the solution derived above, is:

$$|\delta_i| = \frac{3U_1}{v}\left\{1 + \frac{\left(\frac{\kappa_A}{\kappa_\parallel}\right)^2 \sin^2\theta_{Bn} + \left(1 - \frac{\kappa_\perp}{\kappa_\parallel}\right)^2 \sin^2\theta_{Bn}\cos^2\theta_{Bn}}{\left[\left(\frac{\kappa_\perp}{\kappa_\parallel}\right)\sin^2\theta_{Bn} + \cos^2\theta_{Bn}\right]^2}\right\}^{\frac{1}{2}}$$

Special Cases of the general limit:

Case 1. Weak Scattering ($\kappa_{||} \gg \kappa_A, \kappa_\perp$)

$$\frac{3U_1 \sec\theta_{Bn}}{v} \ll 1$$

Case 2. Parallel Shock ($\theta_{Bn} \to 0$)

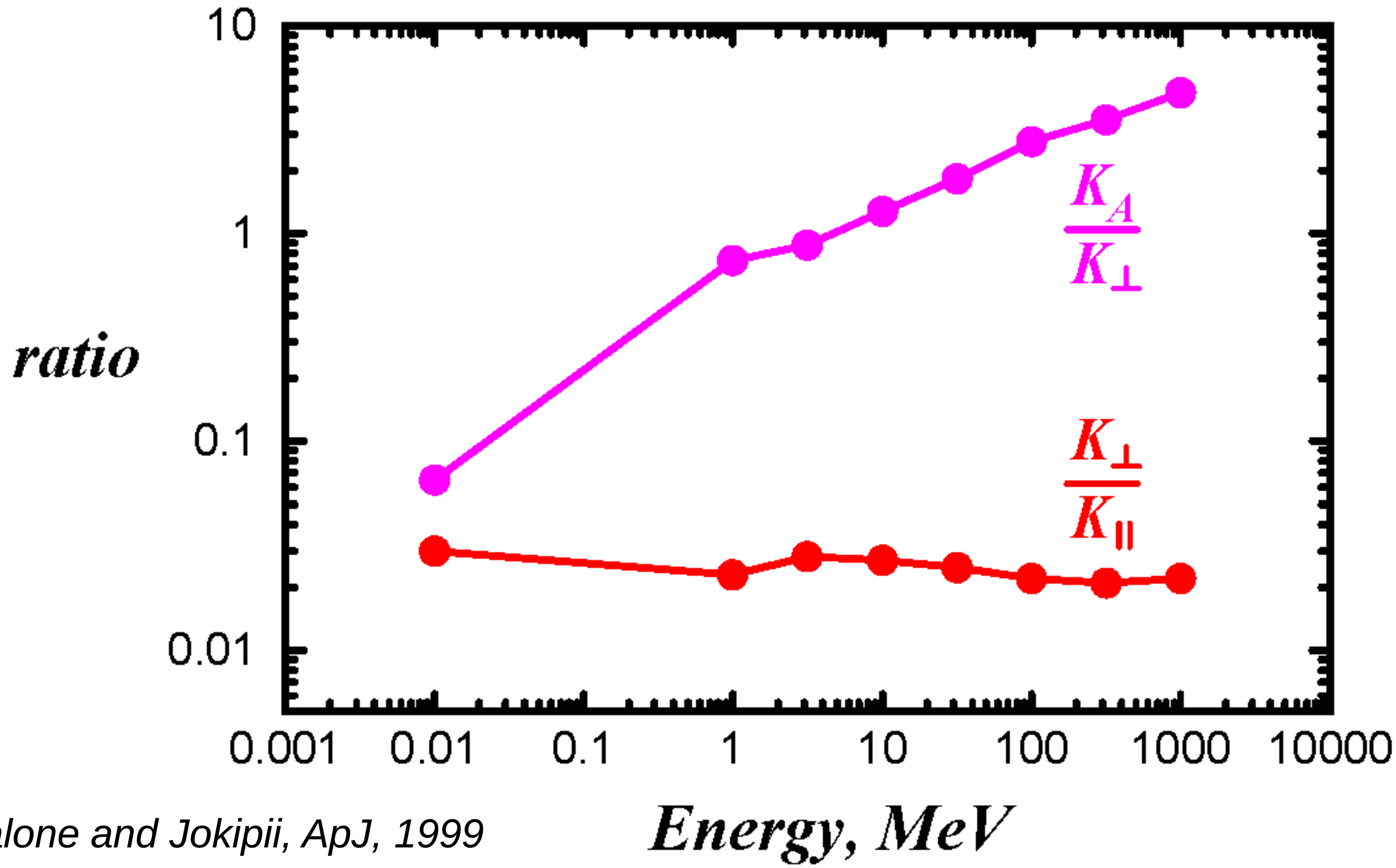
$$\frac{3U_1}{v} \ll 1$$

Case 3. Perpendicular Shock ($\theta_{Bn} \to 90°$)

$$\frac{3U_1}{v}\left[1 + \left(\frac{\kappa_A}{\kappa_\perp}\right)^2\right]^{1/2} \ll 1$$

Test-particle simulations of particle diffusion coefficients using synthesized magnetic turbulence.

At low energies, perpendicular transport is dominated by field-line random walk.



Giacalone and Jokipii, ApJ, 1999

The case of field-line random walk

Thus, for a perpendicular shock, we find

$$v_{inj} = 3U_1 \left[1 + \left(\frac{\kappa_A}{\kappa_\perp} \right)^2 \right]^{1/2}$$

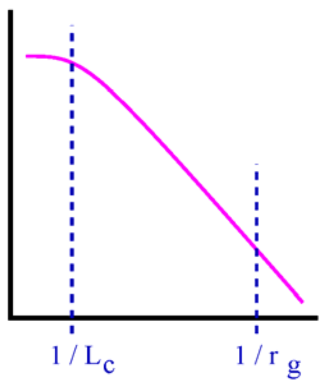
$$\simeq \; 3U_1$$

$\longrightarrow$ **The same as for a parallel shock.**

Injection is NOT more difficult for perpendicular shocks!

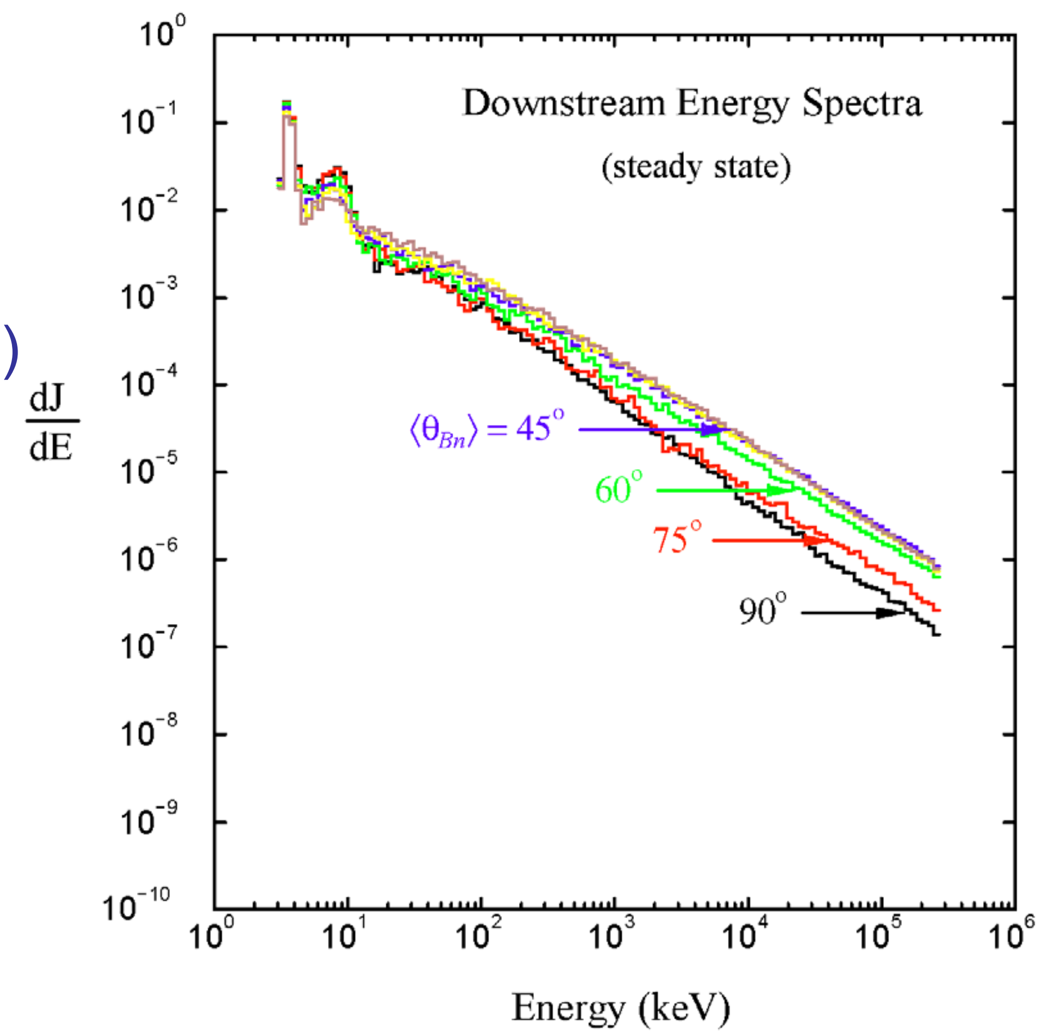
Test-particle simulations show a fairly weak dependence of the injection threshold on magnetic-field angle

- The shock moves through a plasma with a magnetic field composed of a mean plus a random component derived from an assumed power spectrum (ΔB/B ~ 1)



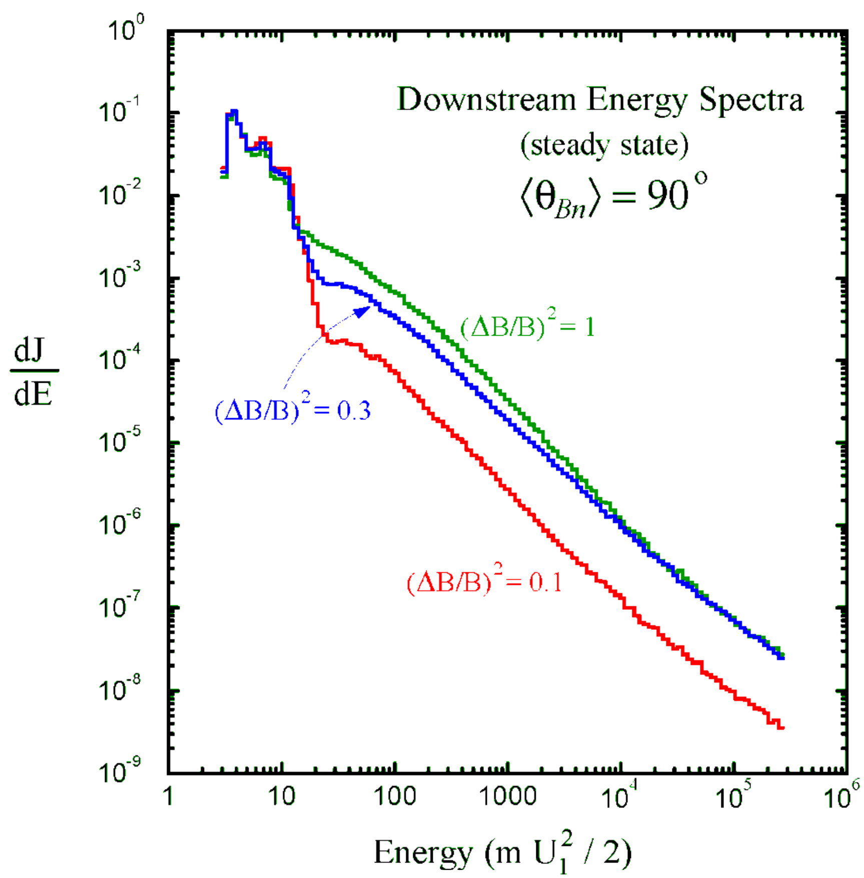
Assumed power spectrum

Steady-State Calculation

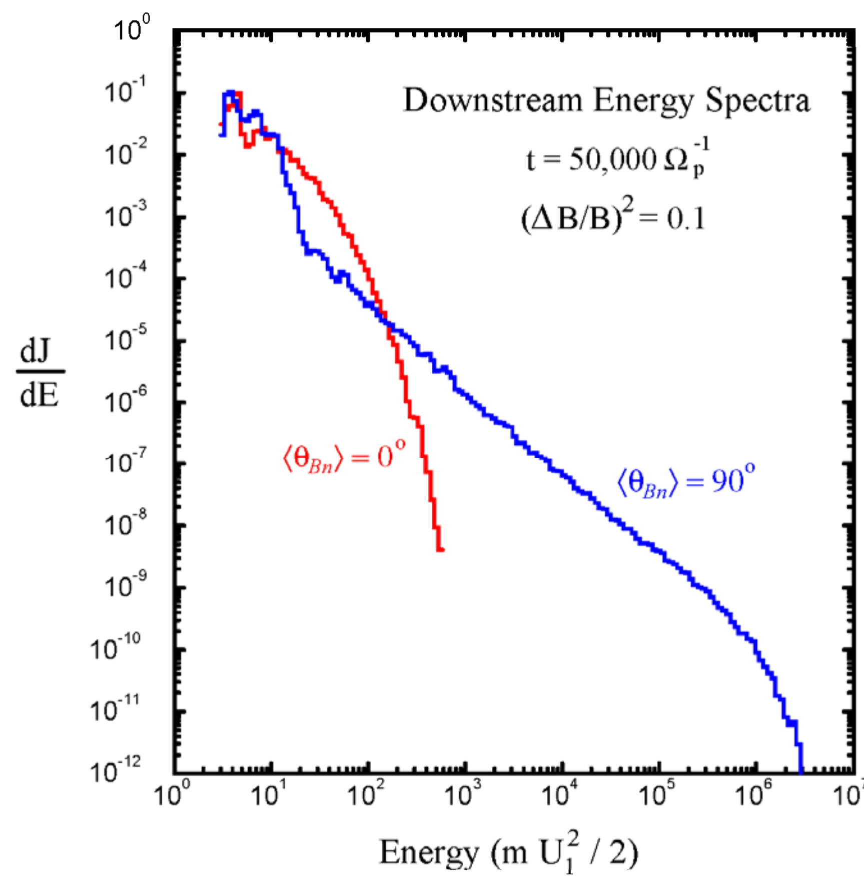


More results from test-particle simulations

Effect of Turbulence Amplitude



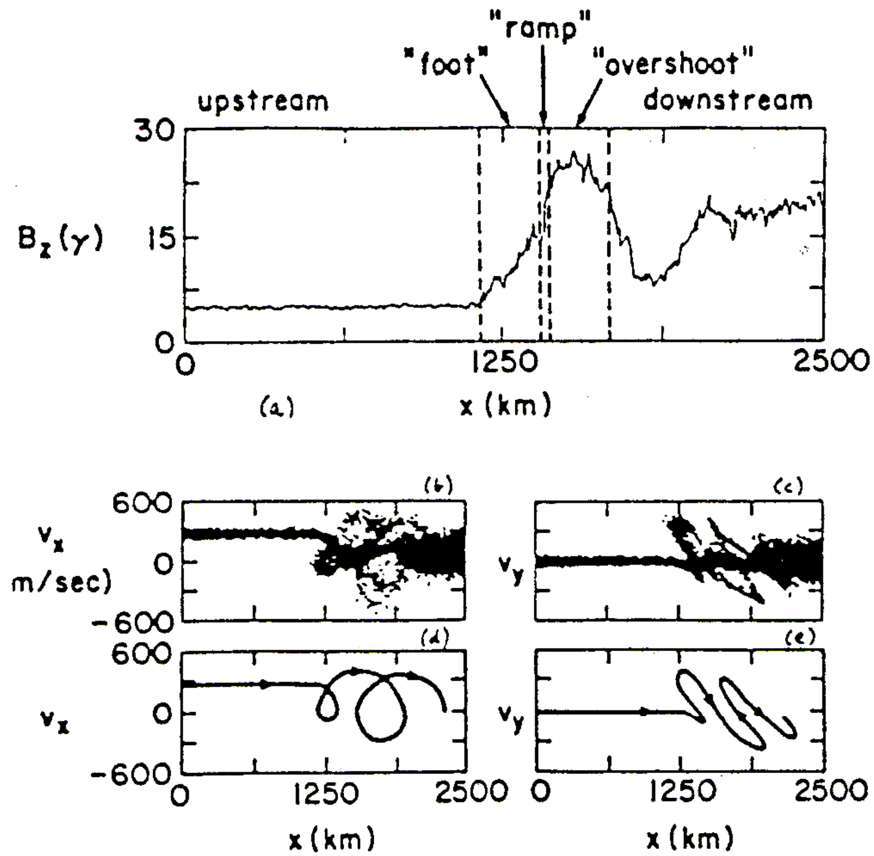
Time dependent case



Self-consistent hybrid simulations

- To better handle the physics of acceleration from near-thermal energies, we need a self-consistent treatment

- The hybrid simulation treats the ions kinetically and the electrons as a massless fluid
  - Used to study the structure of collisionless shocks, as well as the acceleration of thermal ions to high energies.



Numerical considerations for high-energy particles

- Must improve statistics at high energies, by incorporating *particle splitting*

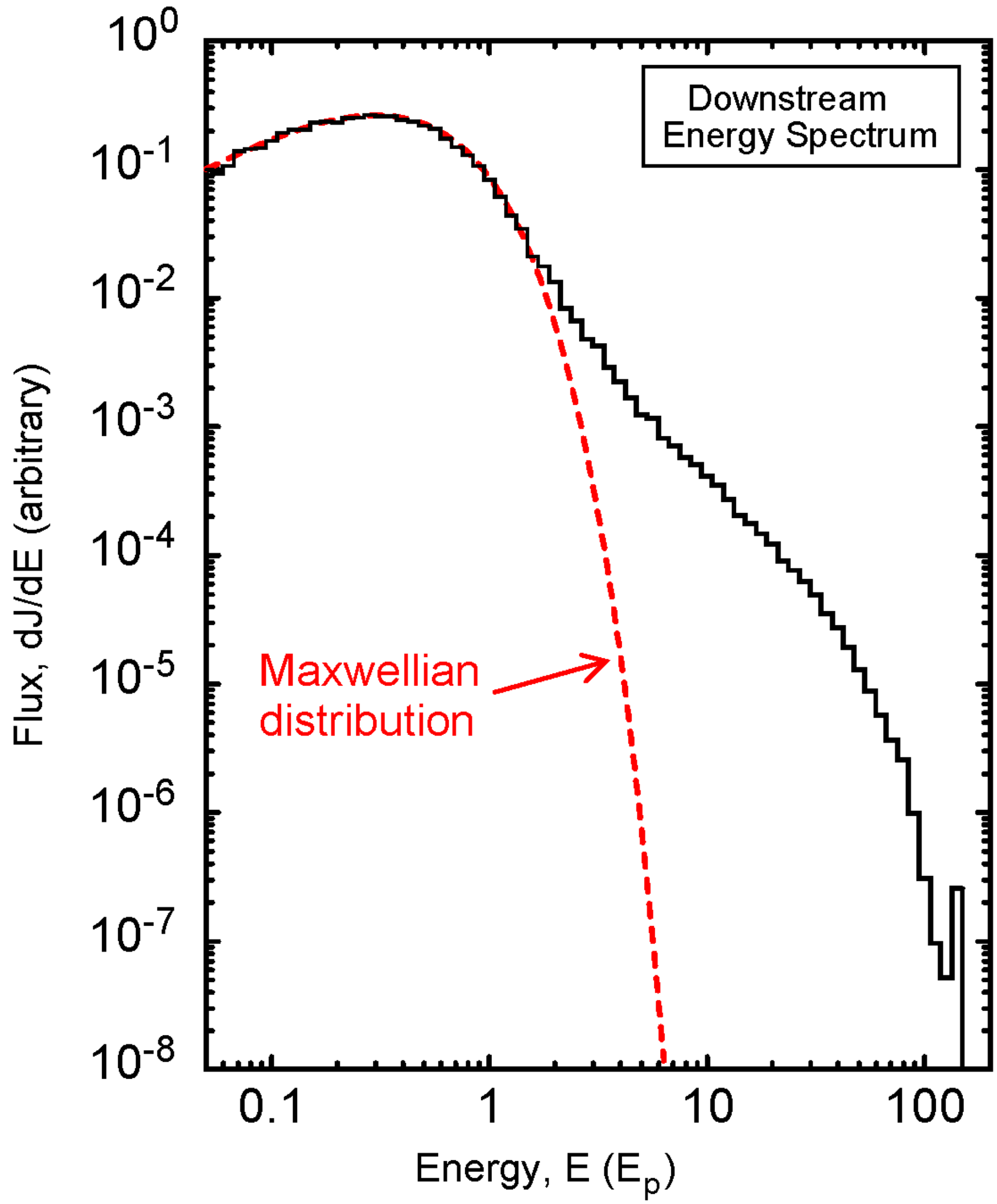
- Must use large simulation domains because:

$$\lambda_{||} \gg c/\omega_p$$

- It takes time to generate the fluctuations that scatter the high-energy particles. It is often necessary to put them in at the start of the simulation

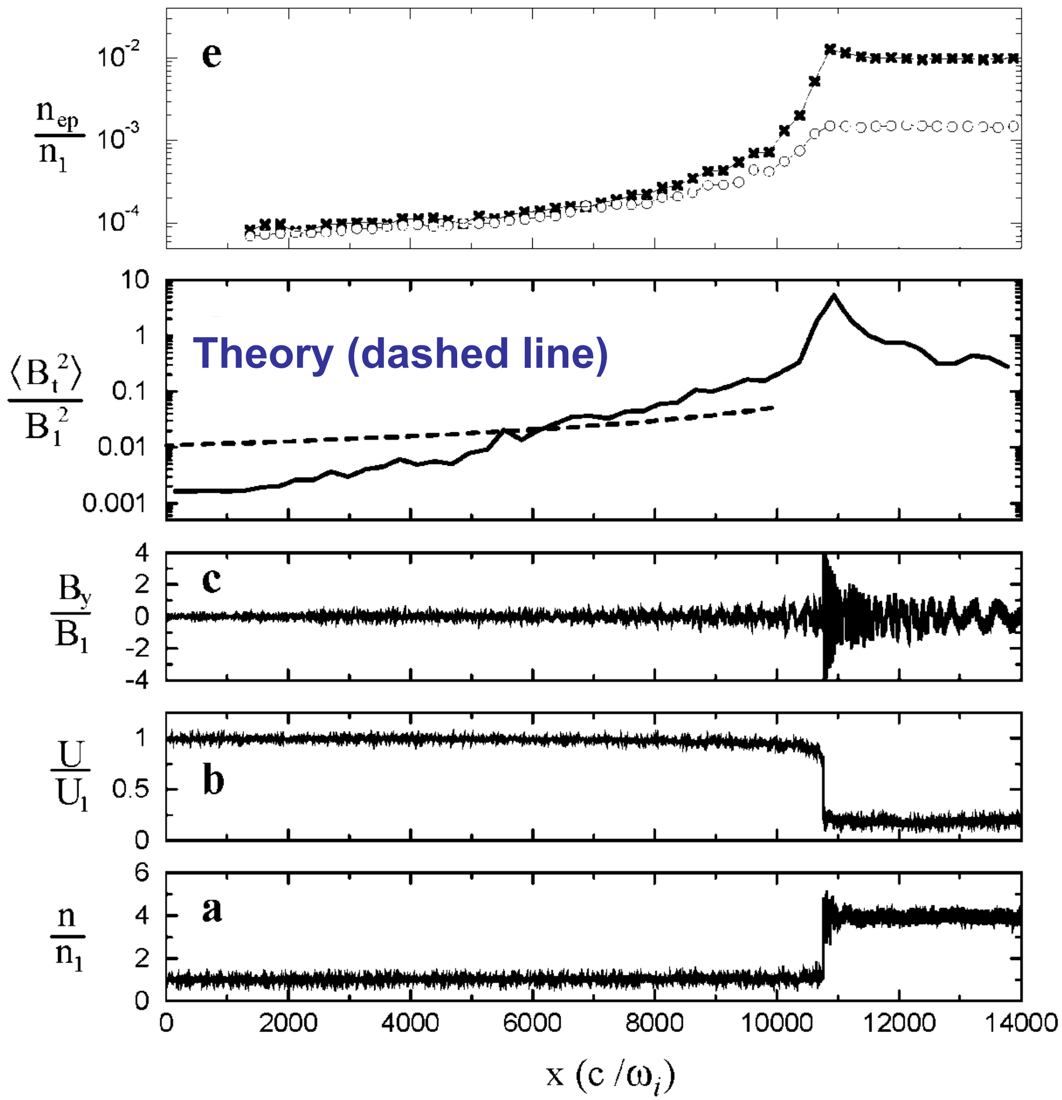
- Ideally we would like to do 3D to overcome a restriction on particle motion normal to the field (tied to field lines)
  – hard to do!

Hybrid simulation of the energy spectrum downstream of a parallel shock



Quest, 1988
Scholer, 1991
Giacalone et al., 1992,93

Parallel Shock



High-energy particles accelerated directly from thermal population.

The self-generated waves are generally weaker than expected from theory

Giacalone, ApJ, 2004

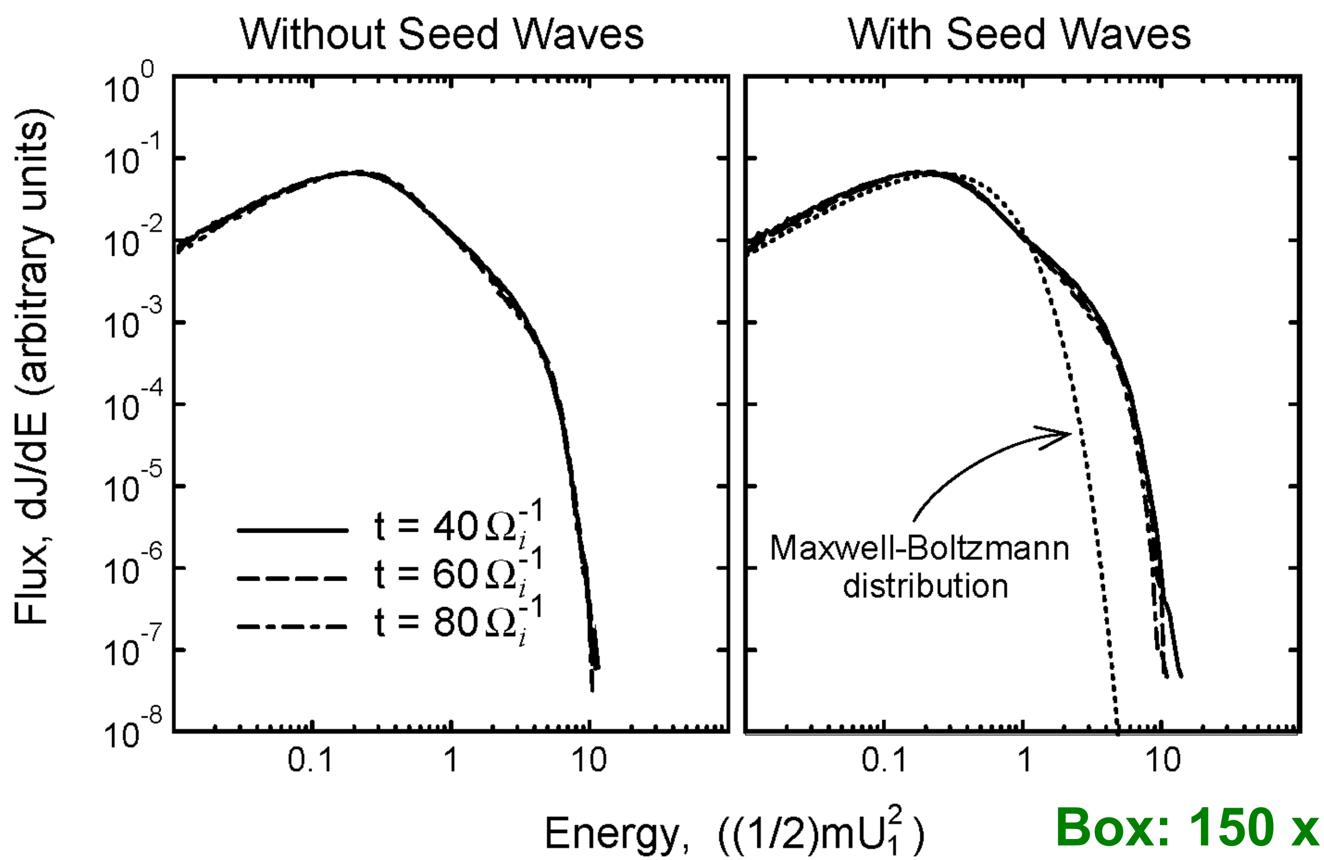
The Magnetic-Field Angle and Hybrid Simulations

- The previous simulations showing high-energy particles accelerated directly from thermal energies were for quasi-parallel shocks

- Until recently, it has been thought that quasi-perpendicular shocks were not efficient accelerators.

- Recent hybrid simulations have also shown efficient acceleration for perpendicular shocks, but it is found that the size of the simulation domain is very important to be able to demostrate this.

First 3D hybrid simulations of perpendicular shocks to study injection/acceleration of thermal particles.



Box: 150 x 10 x 10 c/ω_p

No significant acceleration

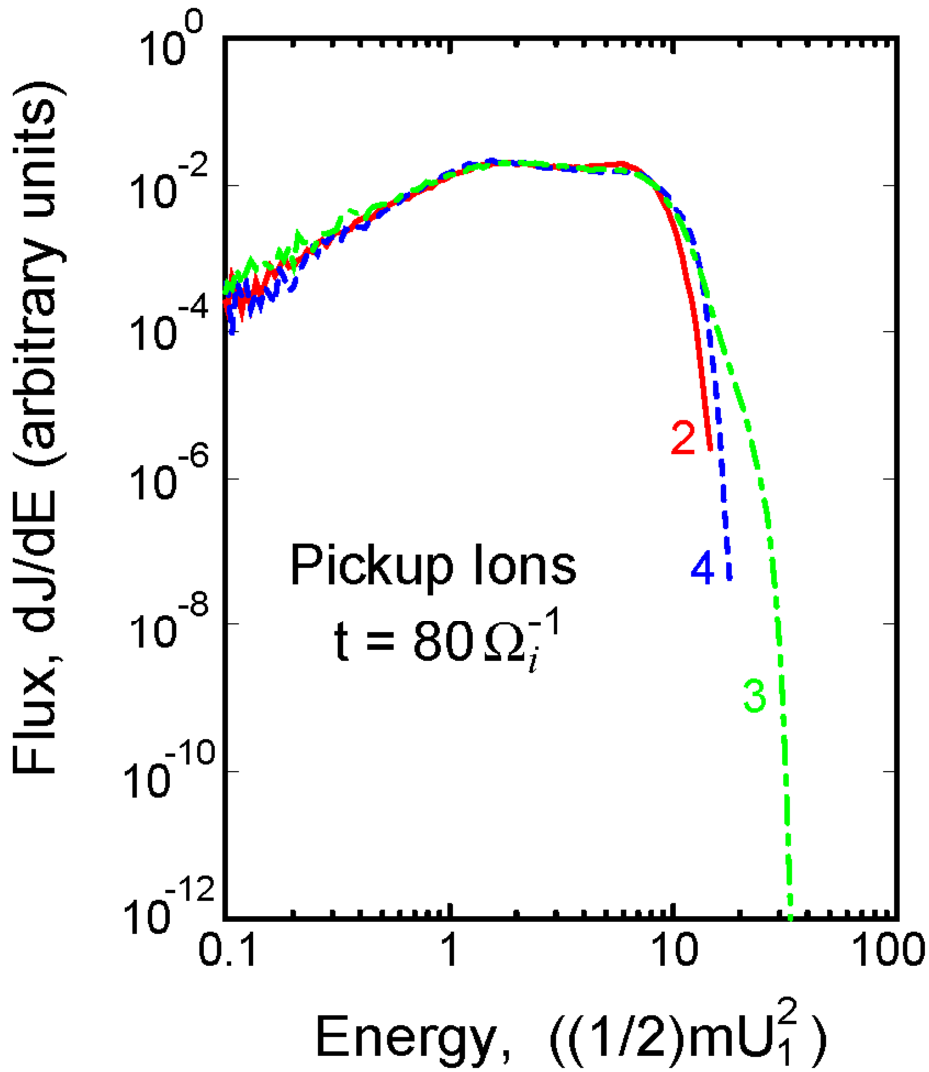
Giacalone and Ellison, 2000

Effect of Simulation Dimensions

2 – 150 x 10 x 10 c/ω_i

4 – 150 x 50 x 2 c/ω_i

3 – 150 x 2 x 50 c/ω_i

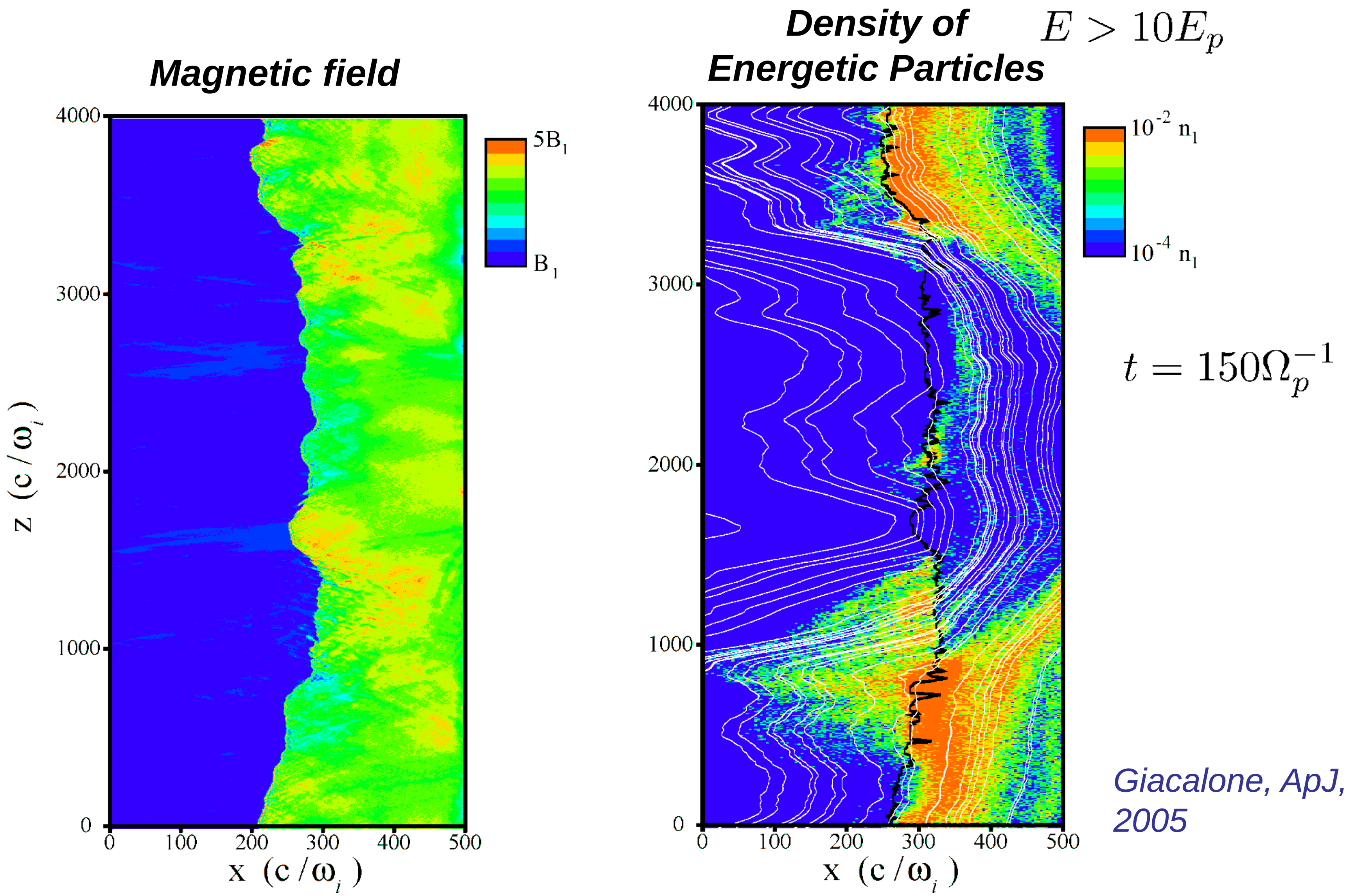


Giacalone and Ellison, 2000

New 2D Hybrid Simulations

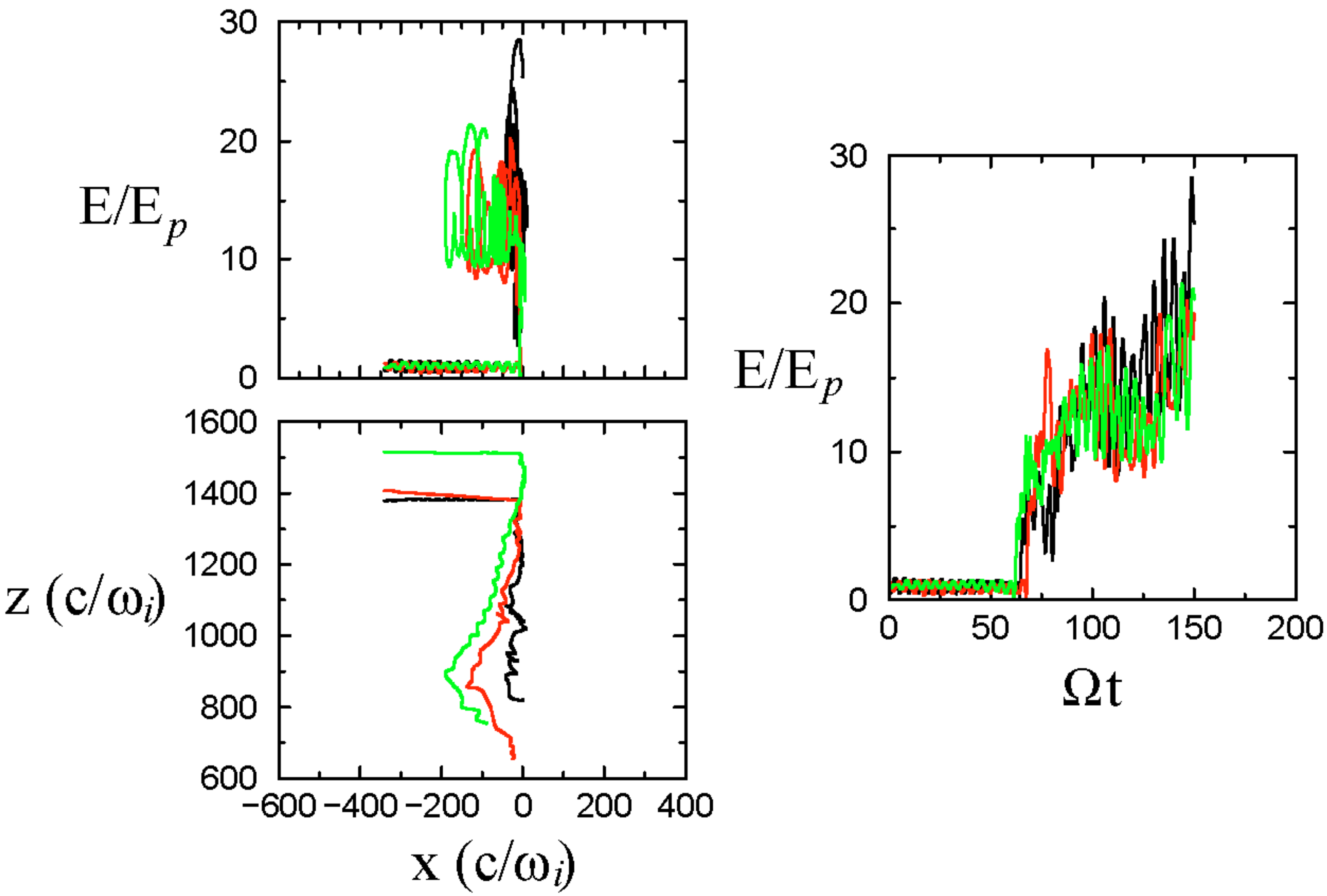
- We have performed large 2D simulations (500 × 4000 c/ω_i) to investigate the effect of long-wavelength magnetic fluctuations on the acceleration of thermal ions at a *perpendicular* shock.
- "Seed" or pre-existing upstream magnetic fluctuations are imposed on the system.
- Particles are tied to field lines, but move normal to the mean field by following meandering lines of force "partial" perpendicular diffusion

Perpendicular shock



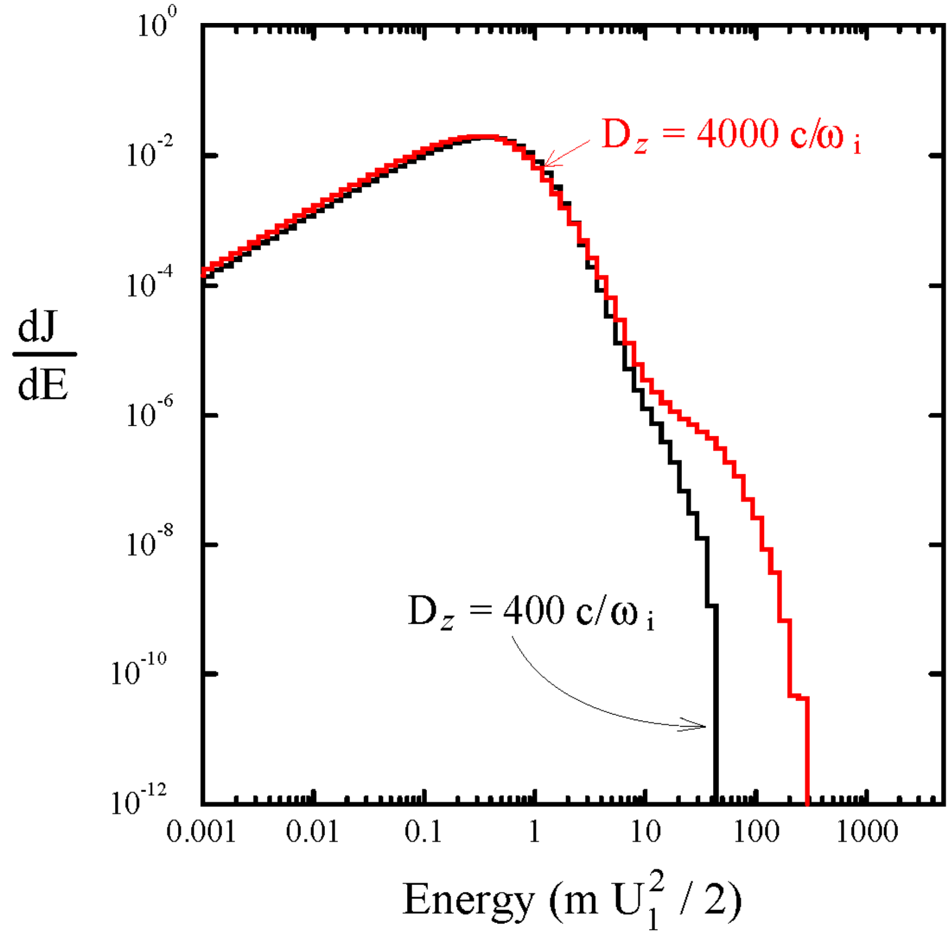
Giacalone, ApJ, 2005

Individual Particle Trajectories

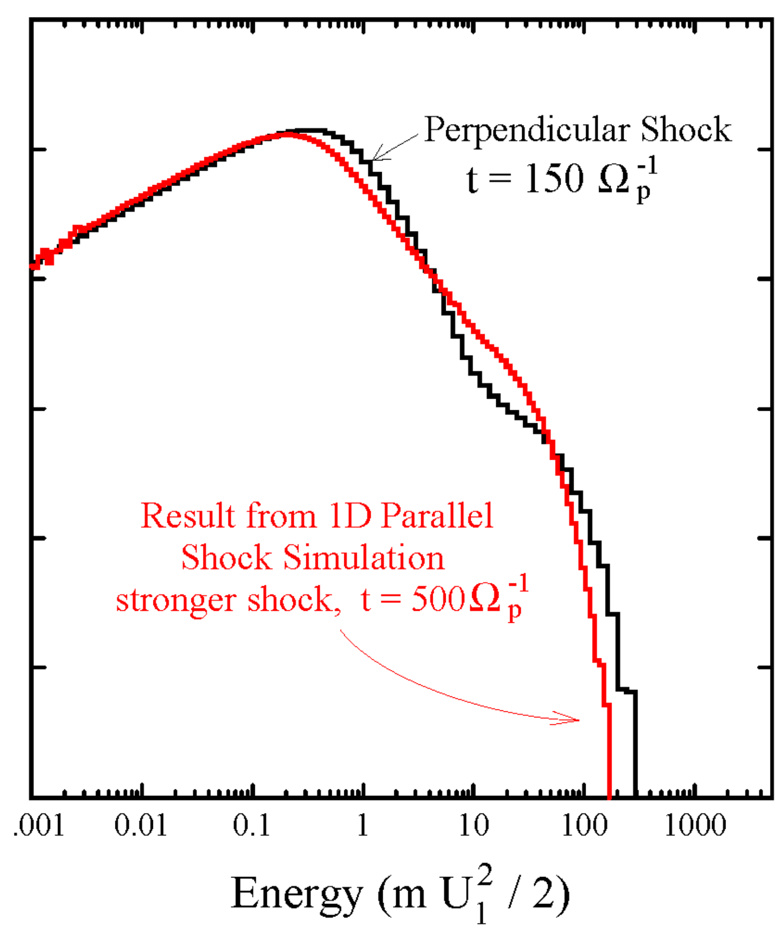


Effect of:

Domain Size

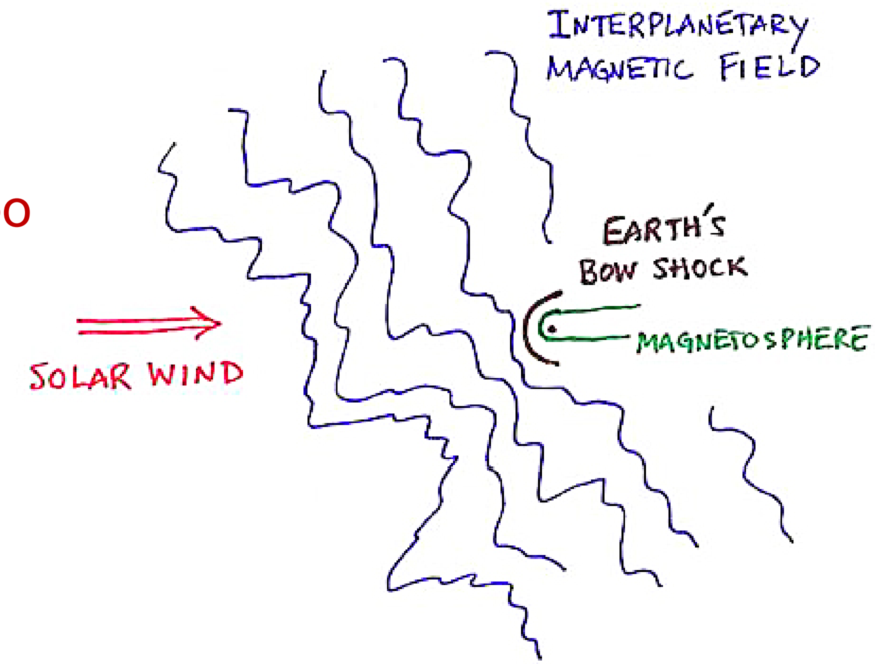


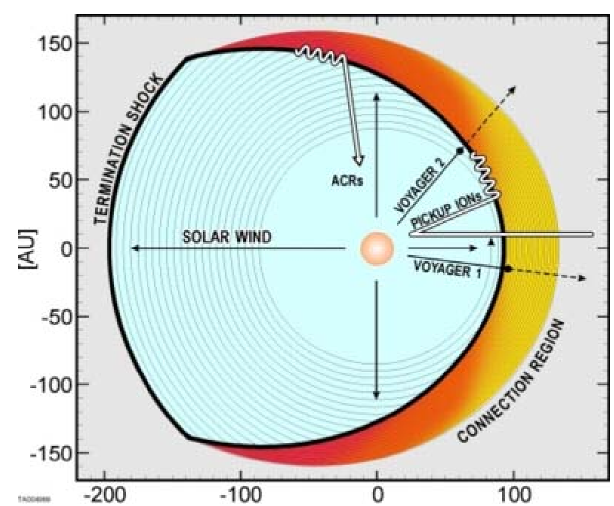
Magnetic-field angle

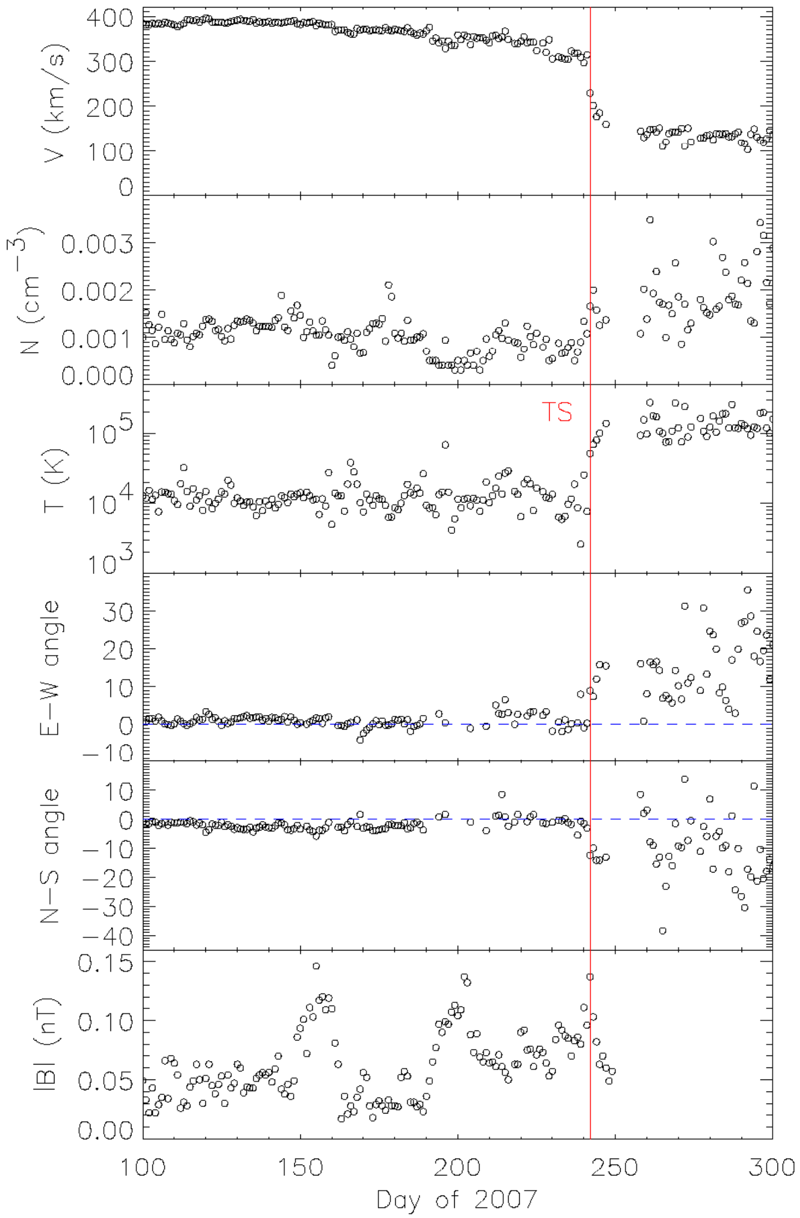


Direct observational tests ?

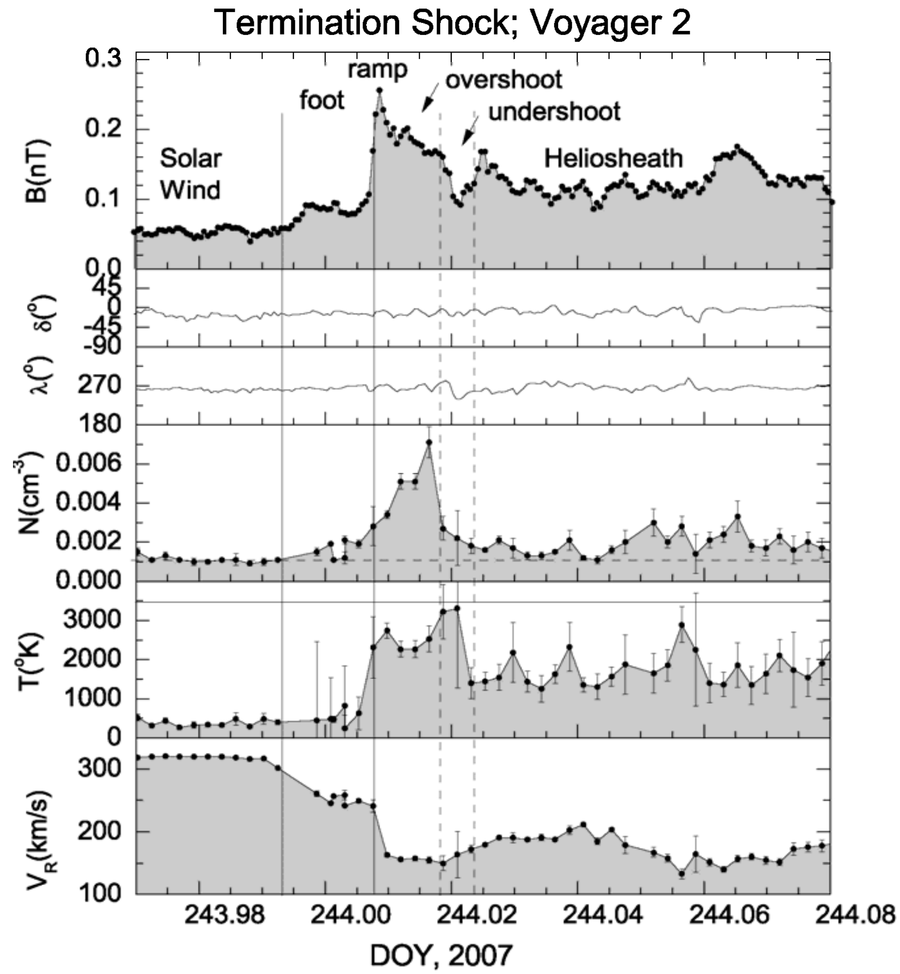
- Earth's Bow shock
  - Not a good test because it is too small compared to the I.M.F. coherence scale
- Interplanetary shocks
  - difficult to unravel time dependence in source population, shock evolution
- Bow shocks of outer planets
  - Possibly, but only a few encounters
- Solar-wind termination shock
  - Yes, but only 2 crossings





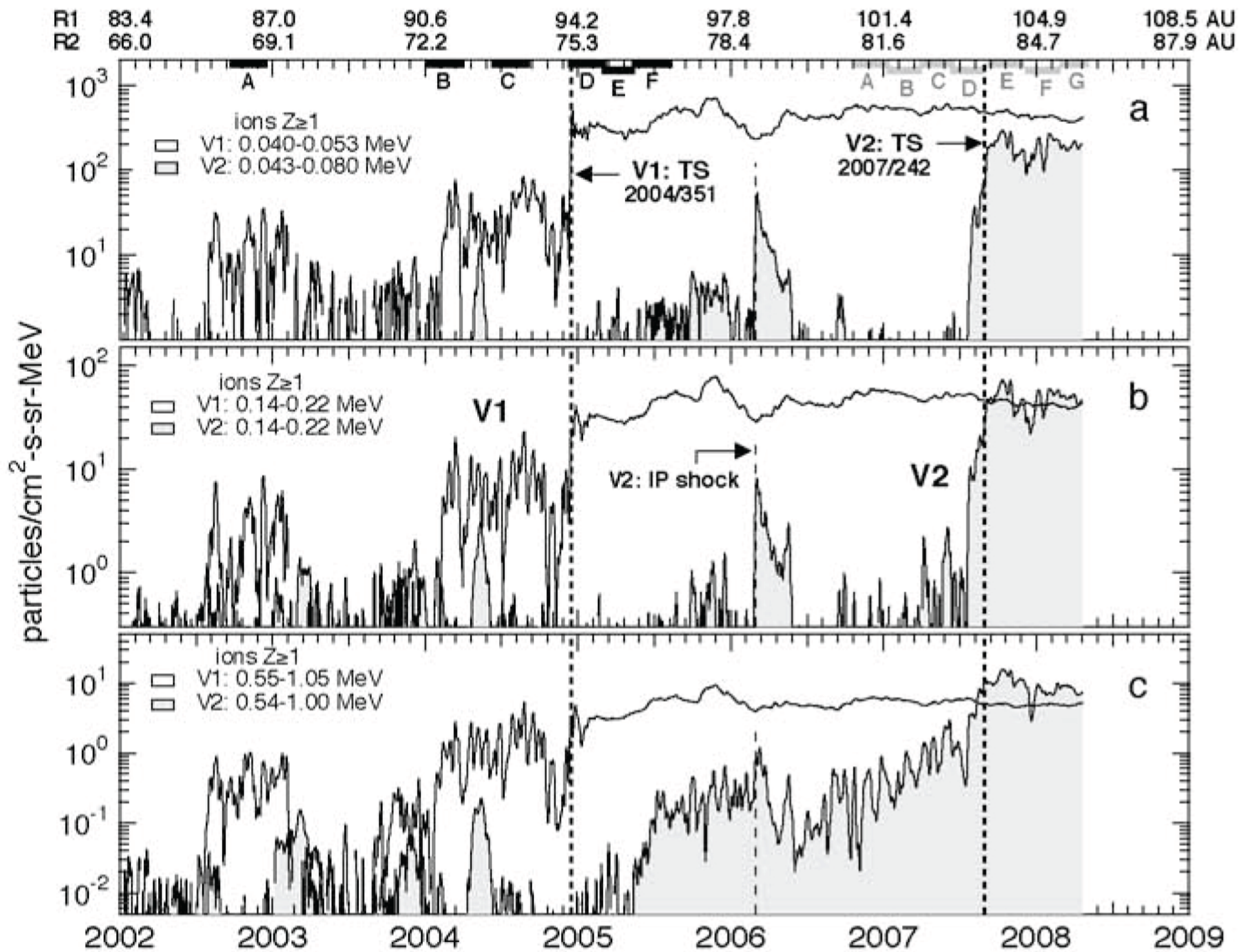


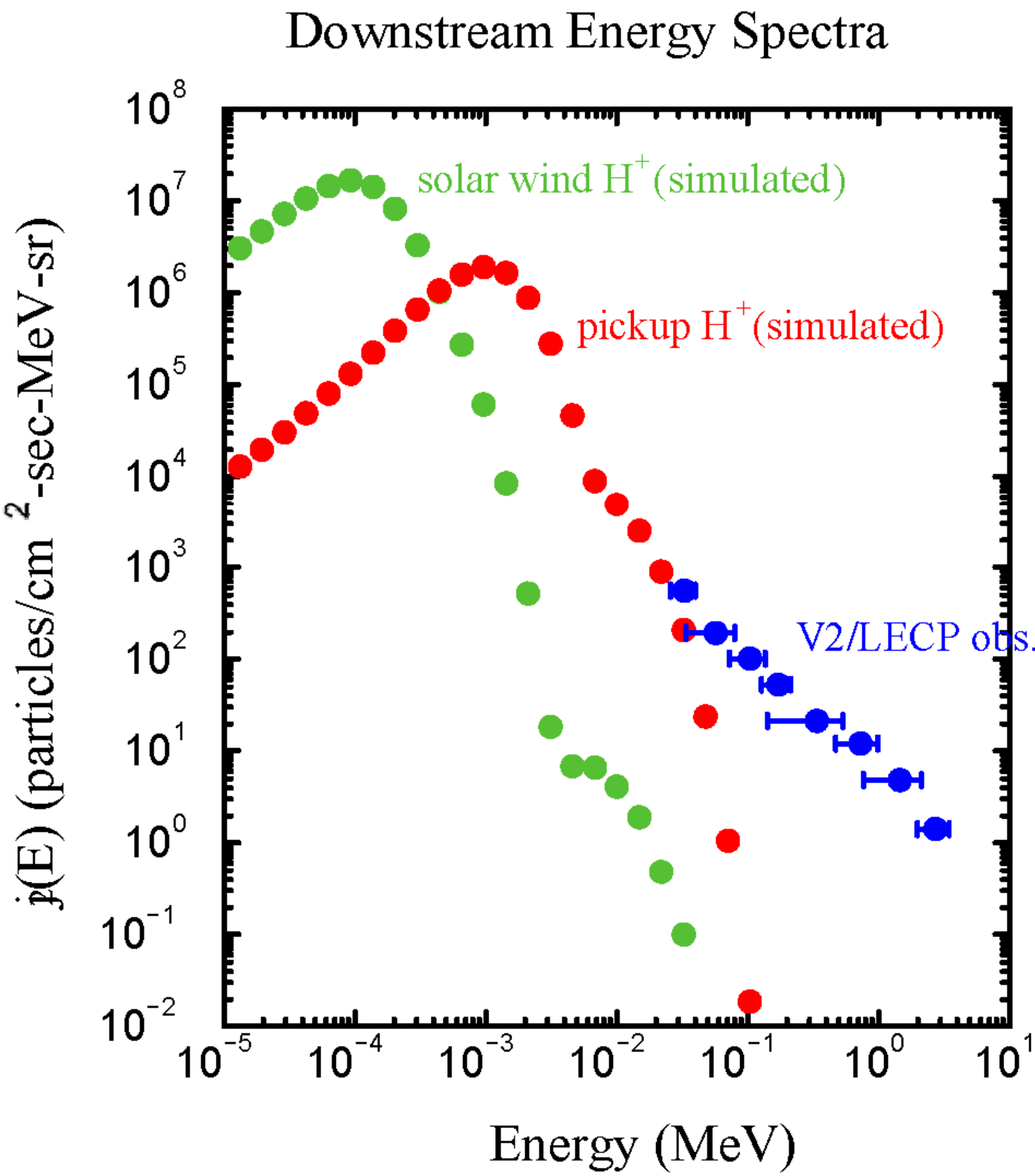
Richardson et al., 2008



Burlaga et al., 2008

Voyager Observations of Energetic Ions





Downstream Energy Spectra
solar wind H+(simulated)
pickup H+(simulated)
V2/LECP obs.
j(E) (particles/cm 2-sec-MeV-sr)
Energy (MeV)

Conclusions

- Shocks moving into a plasma with large-scale magnetic turbulence accelerate low-energy particles with high efficiency. There is not a significant injection problem.
- Perpendicular shocks readily accelerate low-energy particles, perhaps even as efficiently as parallel shocks.
- Perpendicular shocks have a higher rate of acceleration.
  - for a given time to accelerate particles, the highest energy ones originate from regions on the shock that are nearly perpendicular to the average mag. field