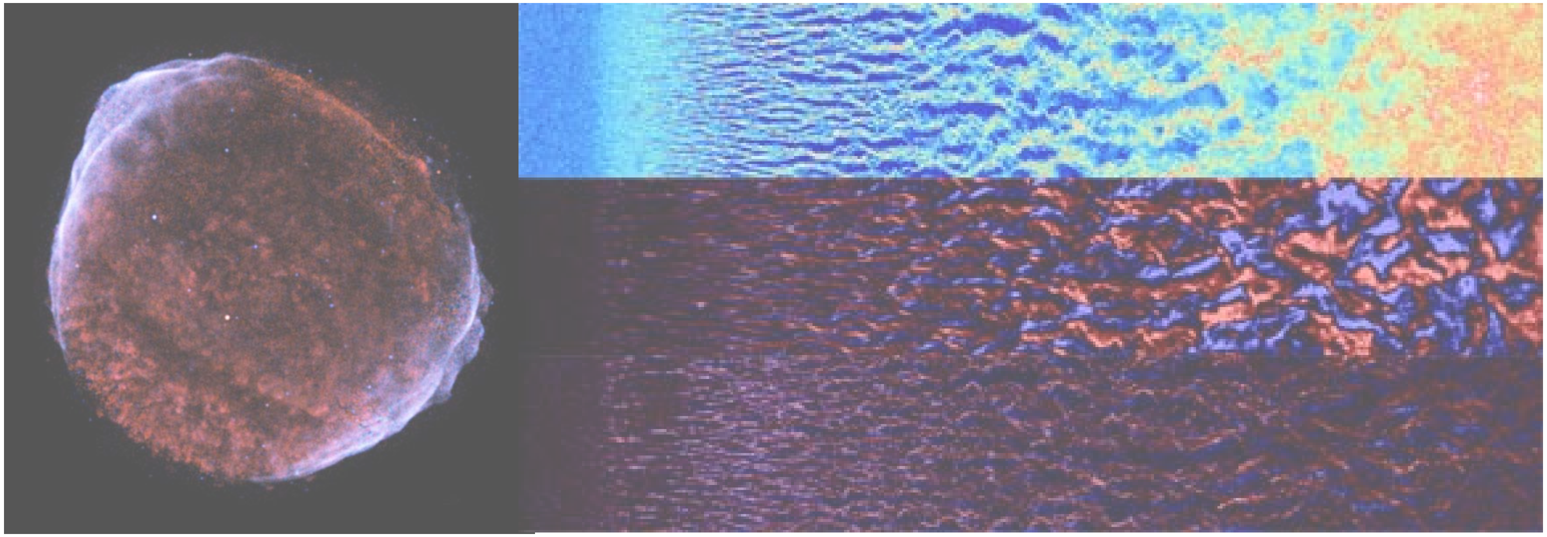
Non-Relativistic Collisionless Shocks in Unmagnetized and Weakly Magnetized Electron-Ion Plasmas



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Nonrelativistic collisionless shock in electron-ion plasmas

1. **Unmagnetized** shock (Weibel shock)
2. **Weakly magnetized** shock (perpendicular)

Method: 2D PIC simulations of electron-ion plasmas

Simulation of Collisionless Shocks

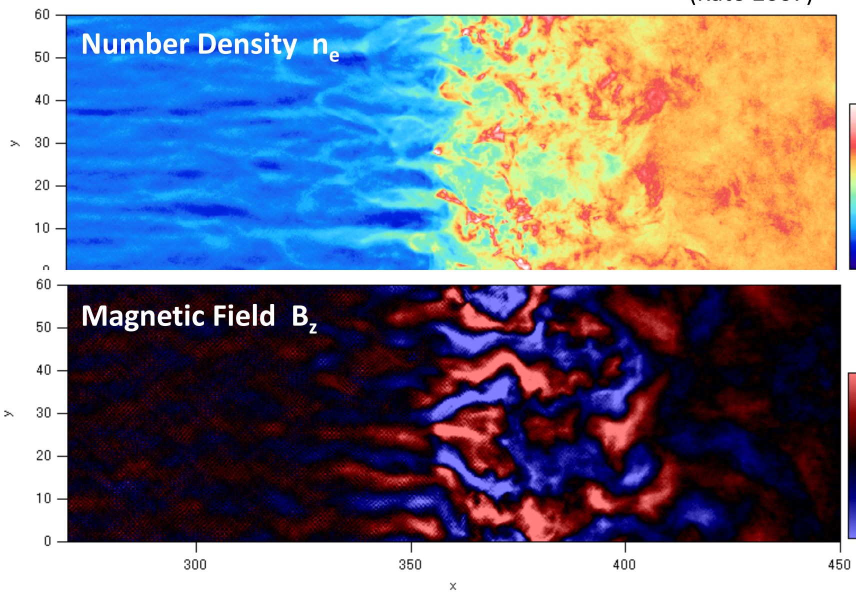
In Electron-Ion Plasma
<u>without</u> Background Magnetic Field

Relativistic Collisionless Shocks in Unmagnetized Plasmas

"Weibel-mediated" Shocks

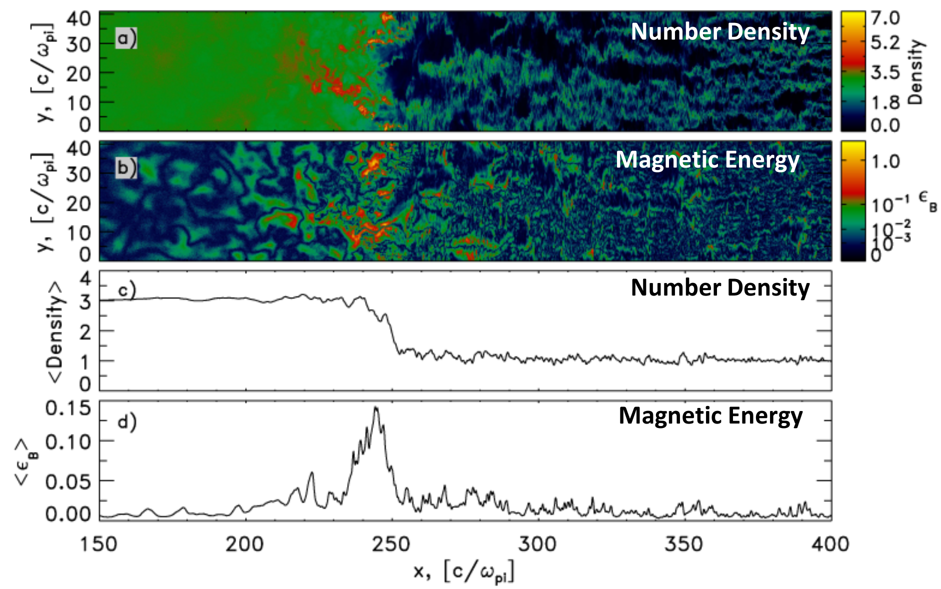
in Electron-Positron Plasma (Γ=2.2)

(Kato 2007)



in Electron-Ion Plasma (Γ=15)

(Spitkovsky 2008)



The **Weibel instability** generates **strong magnetic fields** and provides an effective **dissipation mechanism** for the shock formation

Motivation

The shocks in supernova remnants are **non-relativistic**

Typically, **V ~ 3000 km/s ~ 0.01c**

Do the "Weibel-mediated" collisionless shocks exist in **non-relativistic** regime?

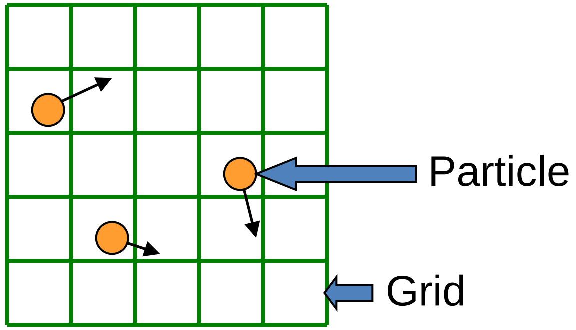
Particle-in-Cell (PIC) Simulation

Particle in Cell Method

- **Particles:** calculating individual trajectory
- **Electromagnetic field**: solving Maxwell's equations on grid

Steps of calculation

$$(\mathbf{E},\mathbf{B}) \rightarrow (\mathbf{x},\mathbf{p}) \rightarrow (\mathbf{J},\rho) \rightarrow (\mathbf{E},\mathbf{B}) \rightarrow \ldots$$



$$\mathbf{E},\mathbf{B},\mathbf{J},\rho$$

Basic Equations

Field Maxwell's equations

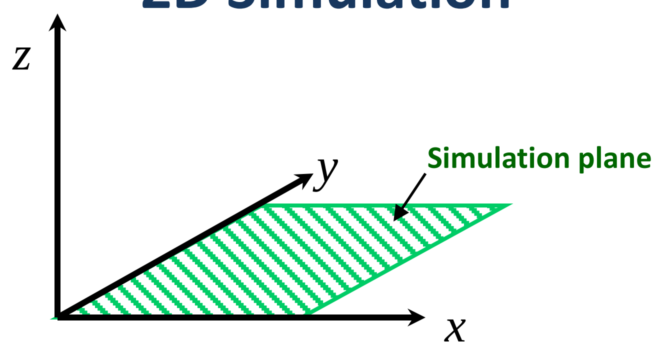
- $$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{J}$$
- $$\nabla \cdot \mathbf{E} = 4\pi\rho$$
- $$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
- $$\nabla \cdot \mathbf{B} = 0$$

Particle Equation of motion

- $$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{\mathbf{p}\times\mathbf{B}}{\gamma mc}\right)$$

Simulation Settings

2D Simulation



- The simulation plane = the x-y plane
- The z axis is perpendicular to it

Units

Time	$\tau_0 = 1/\omega_{pe}$
Length	$\lambda_e = c/\omega_{pe}$ (skin depth)
EM Fields	$E_* = B_* = c\sqrt{4\pi n_{e0} m_e}$

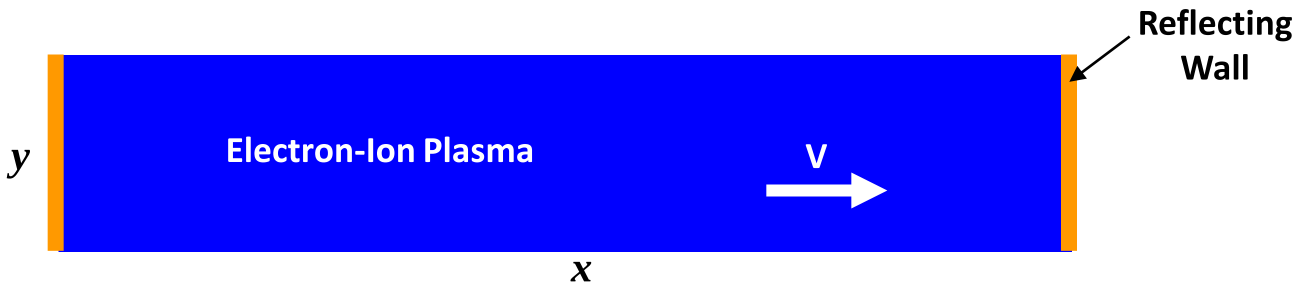
Settings

Composition	Electron, Ion
Physical Size	2240×280
Grid Size	4096×256
Particle Number	5×10^7 particles / species (**27** particles / cell)
B.C.	Periodic
Magnetic Field	None

Mass Ratio	20
Bulk Velocity	0.45c

How to Drive a Shock Wave

“Injection Method”



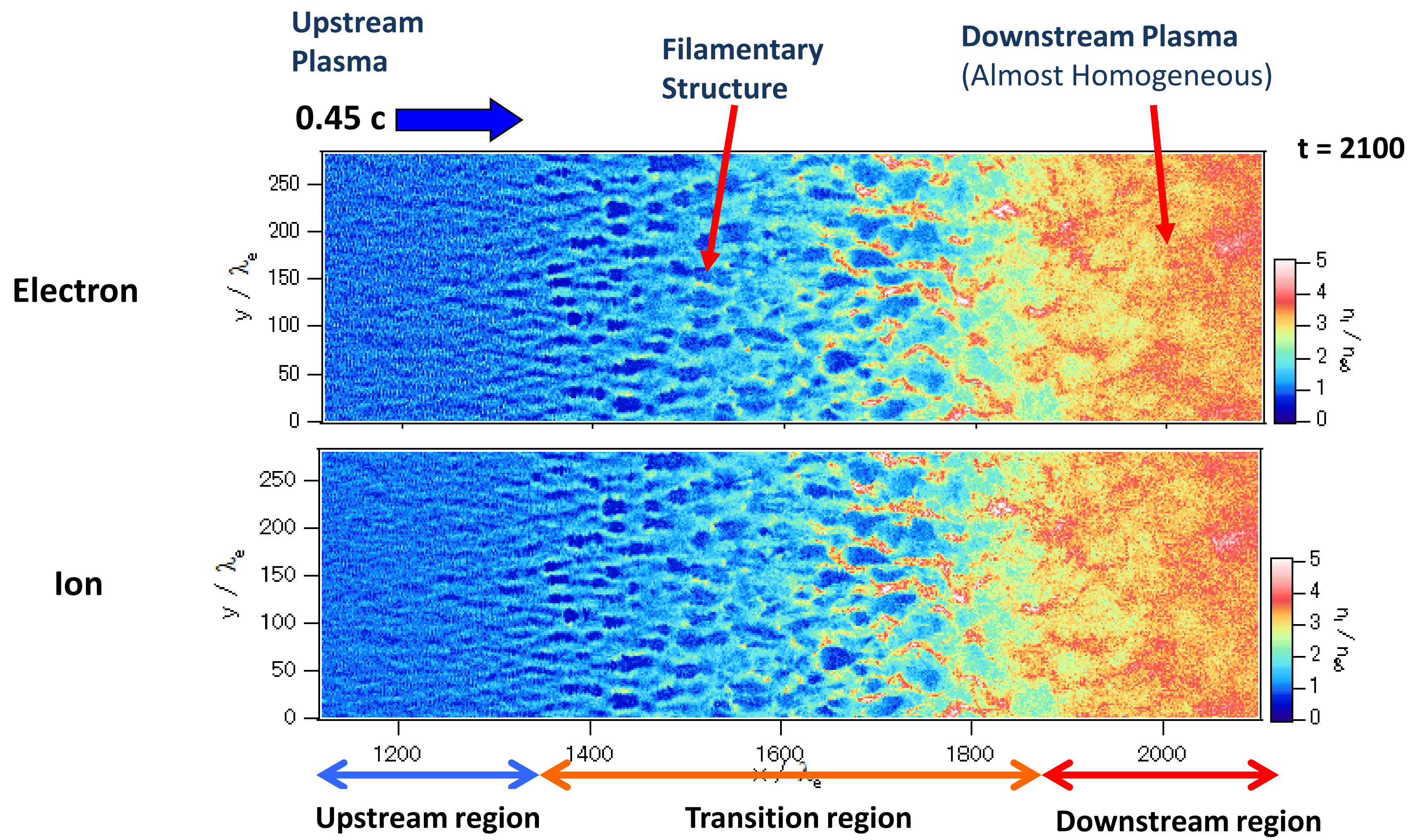
1. Initially, particles are loaded in the area between two reflecting walls with bulk velocity of V to the right.
2. Particles that strike **the right wall** are **reflected**.
3. The reflected particles interact with the incoming particles to **cause some plasma instabilities**.
4. Then, a **shock wave** is formed and propagates to the left.

Note: the simulation frame corresponds to <u>the downstream rest frame</u>.

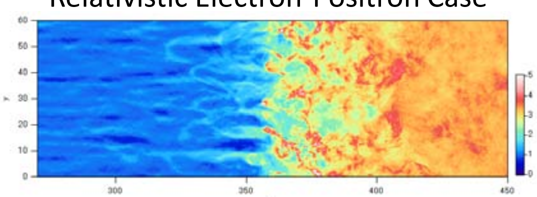
Number Density

V=0.45c

Kato & Takabe, ApJ, 2008, 681, L93



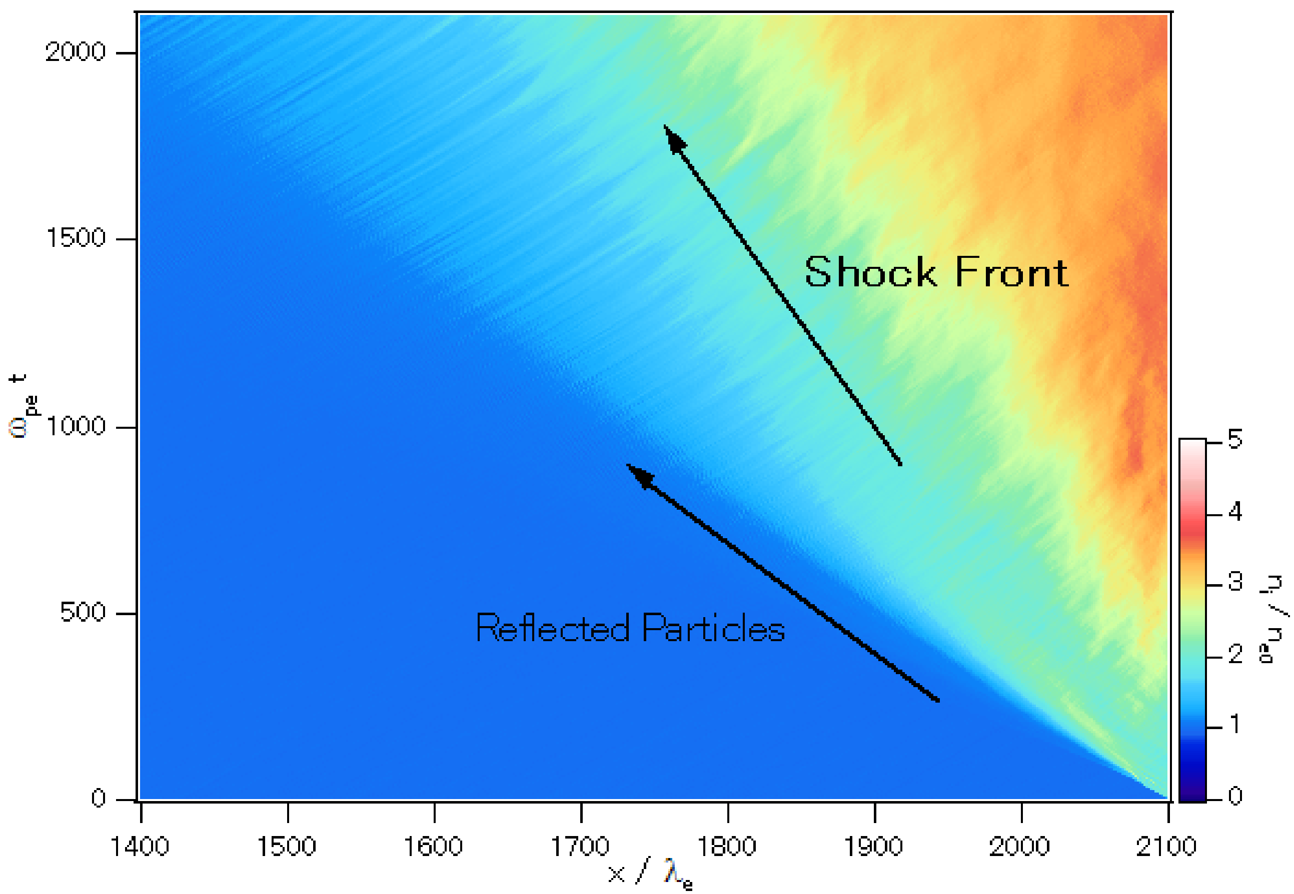
Relativistic Electron-Positron Case



- A collisionless shock is formed
- Structure is **similar** to that in the **relativistic cases**
- There are a lot of **filaments** in the transition region

Propagation of the Shock

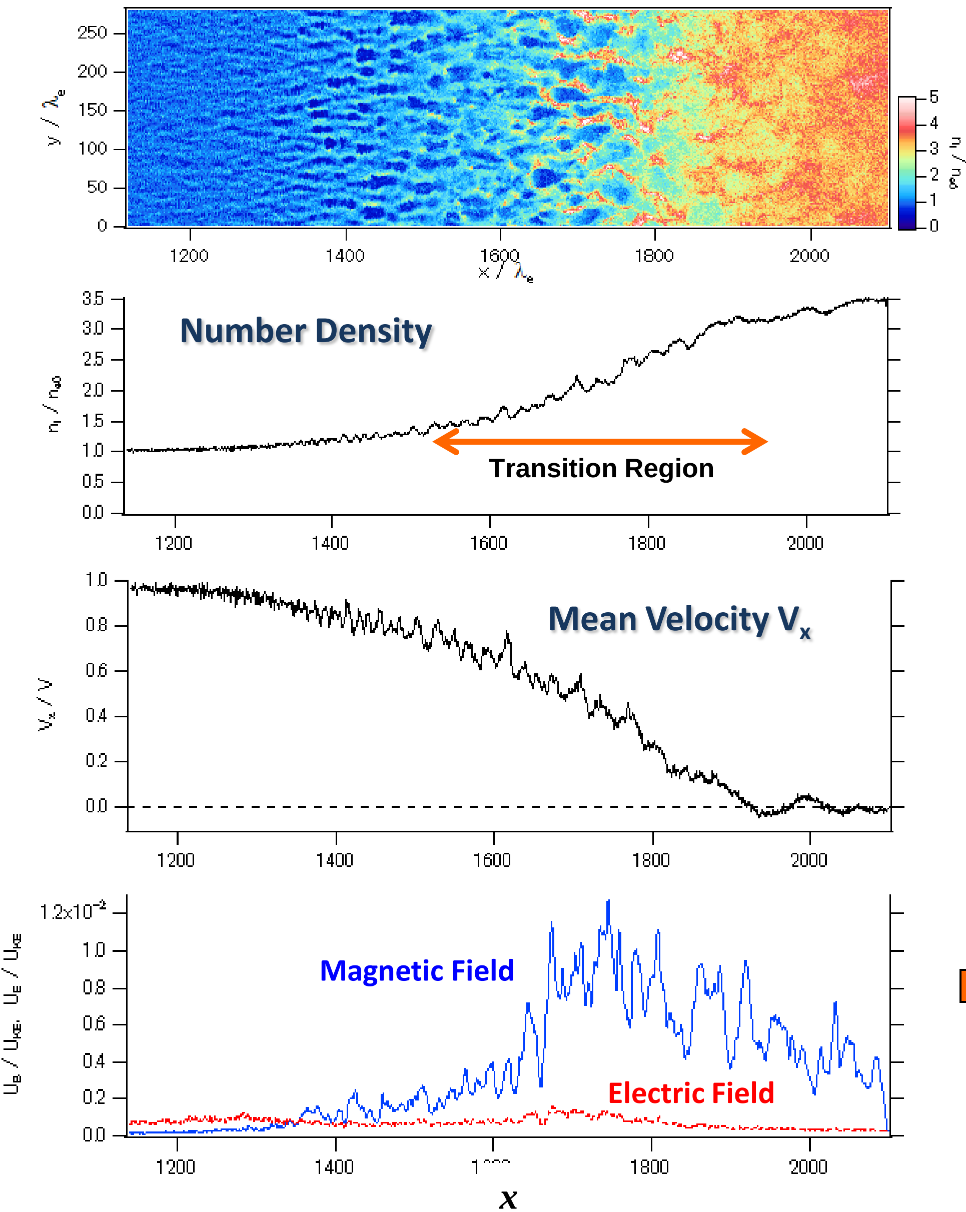
Ion Number Density



The shock propagates into the upstream region at **an almost constant speed**.

Shock speed $V_{\mathrm{sh,d}} = -0.18c, \quad V_{\mathrm{sh,u}} = -0.63c$

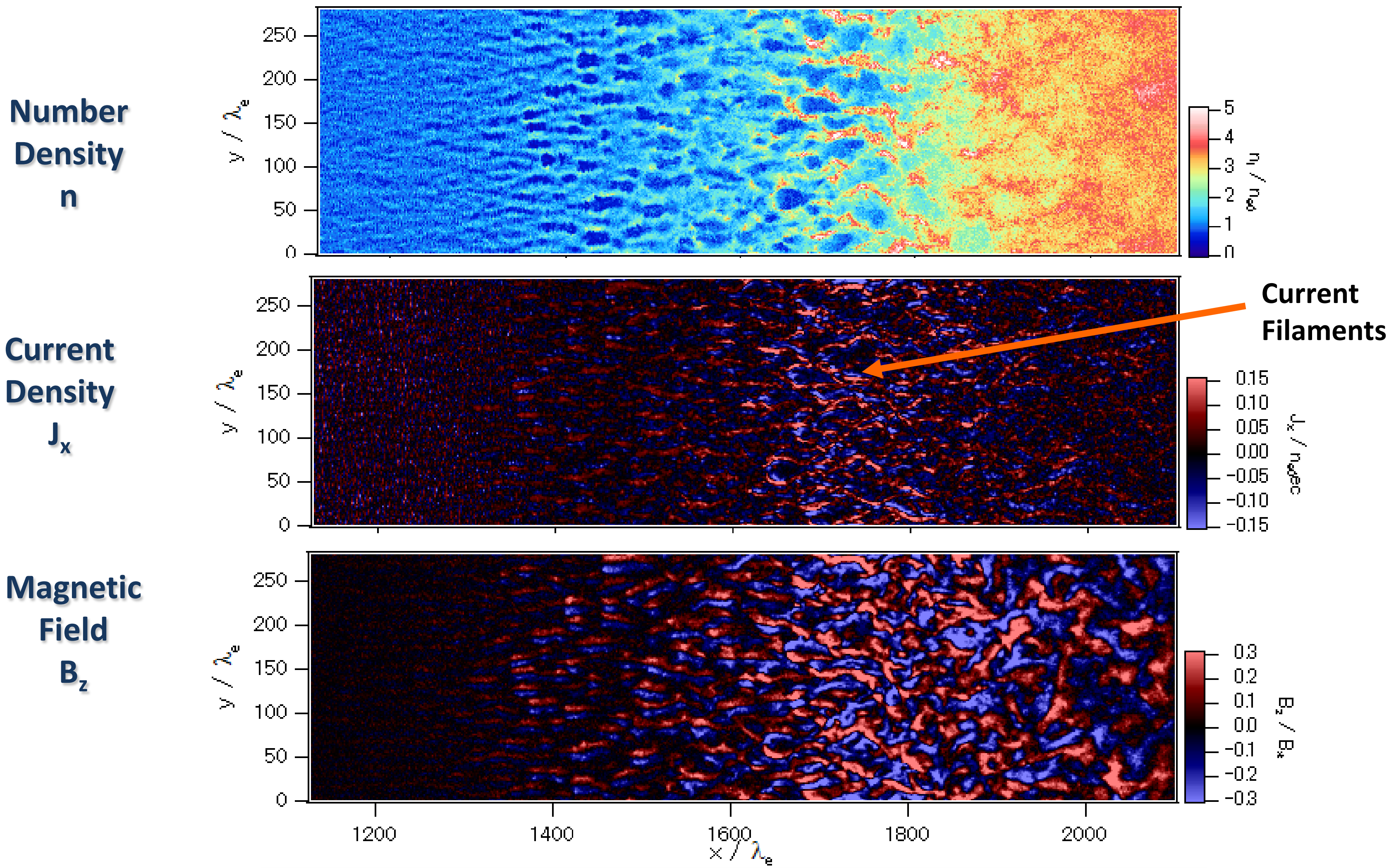
Profiles



- The transition region extends *x* = 1400 – 1900.
- The compression ratio is about 3.5.
- The mean velocity is rapidly decelerated within the transition region.
- There is **strong magnetic field** in the transition region. (**~1%** of the upstream bulk kinetic energy density)

Strong magnetic field provides an effective dissipation mechanism for the upstream plasma

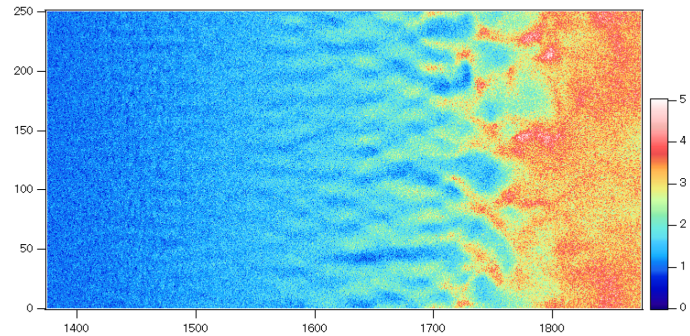
Generation of Magnetic Field



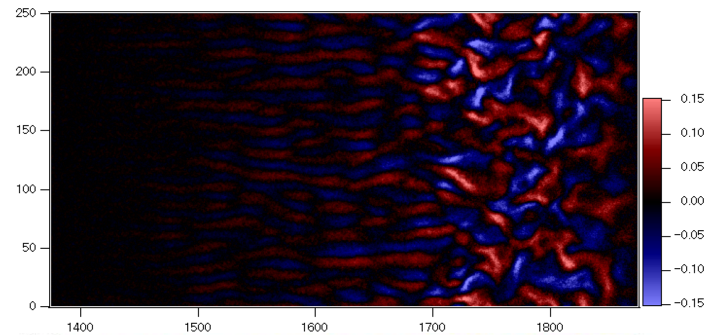
Current filaments generate **strong magnetic fields** within the transition region → **Weibel instability**

Case of V=0.1c

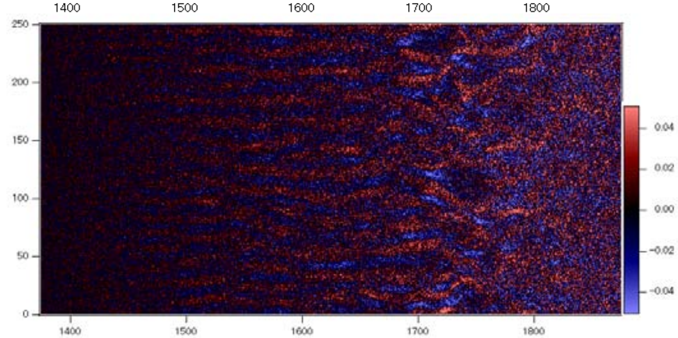
Number Density
n



Current Density
J_x



Magnetic Field
B_z



The Weibel-mediated shock forms even for **V = 0.1c**

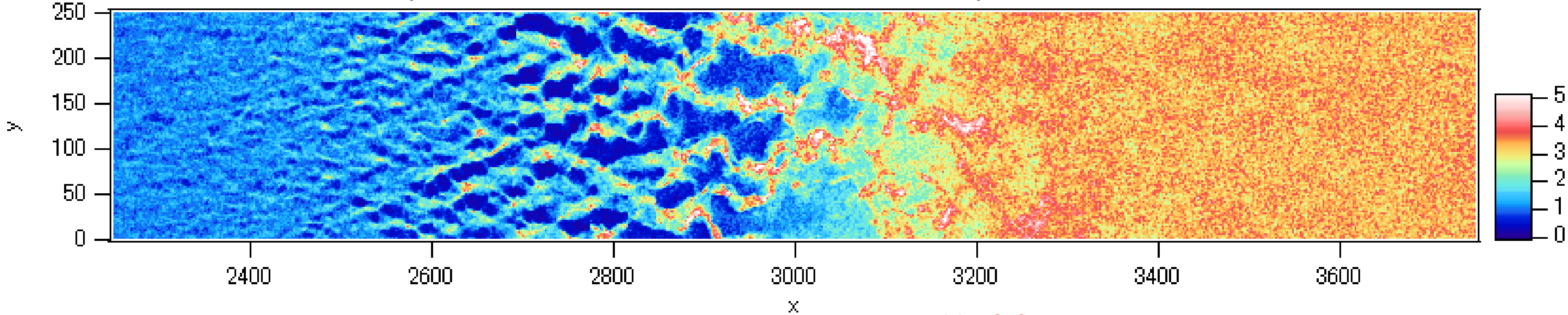
Shock Structure

$m_p/m_e = 20$

Ion Number Density

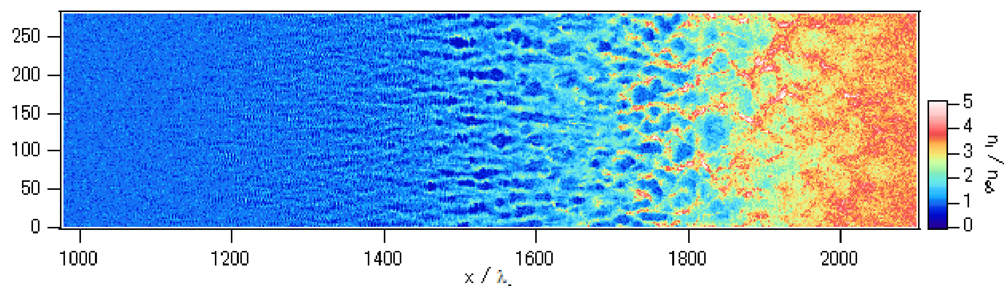
V = **0.9c**
Γ=2.2

100 λ_i



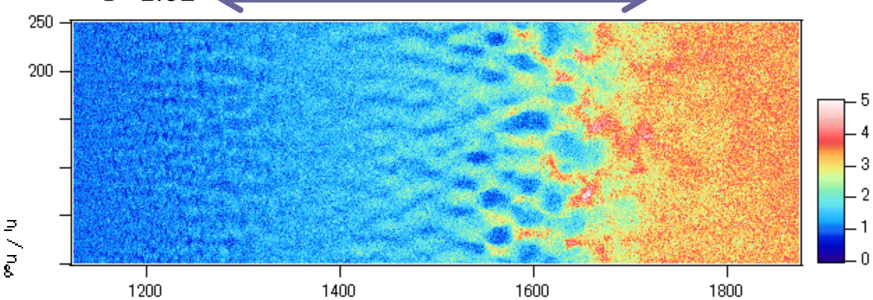
V = **0.45c**
Γ=1.1

100 λ_i



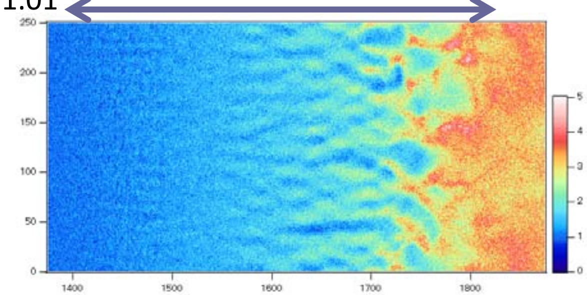
V = **0.2c**
Γ=1.02

100 λ_i



V = **0.1c**
Γ=1.01

100 λ_i

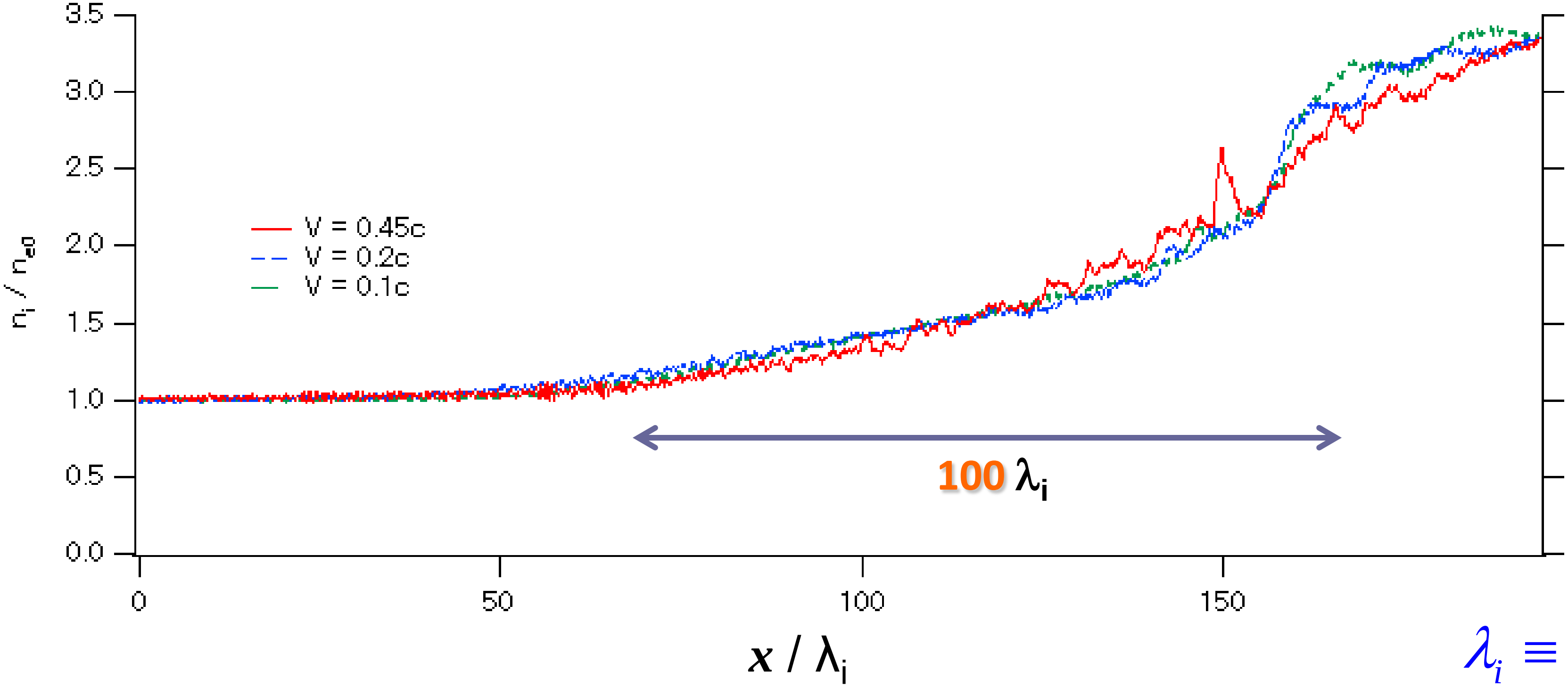


The width of the transition region is approximately **100 ion inertia length λ_i**

$$\lambda_i \equiv c / \omega_{pi}$$

Number Density Profiles

n_i / n_{e0} for **V = 0.45c, 0.2c, 0.1c**



$\lambda_i \equiv c / \omega_{pi}$

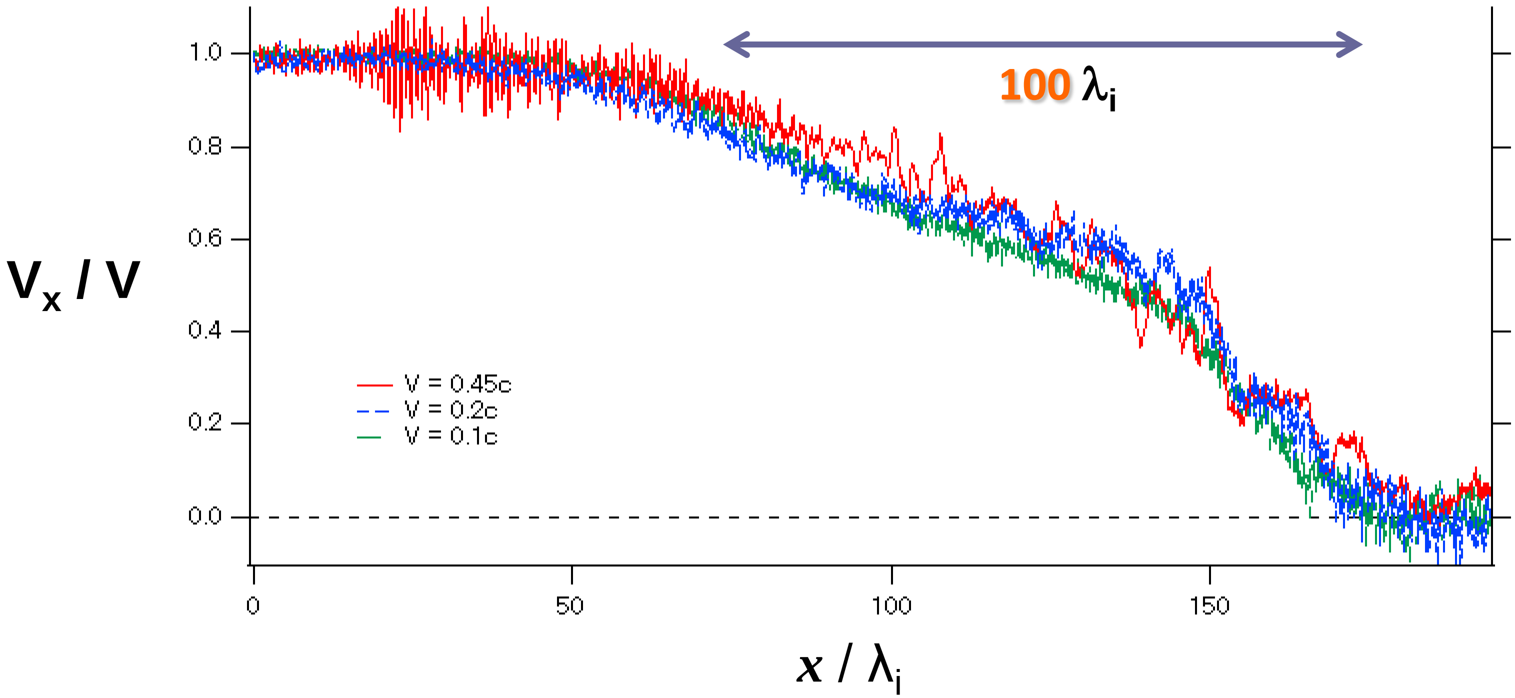
- **Normalized by the upstream number density**
- The spatial scale is normalized by the ion inertia length

The profiles are **almost identical independent of velocity**

In particular, W ~ **100** λ_i , n_2 / n_1 ~ **3.4** in all cases

Normalized Velocity Profiles

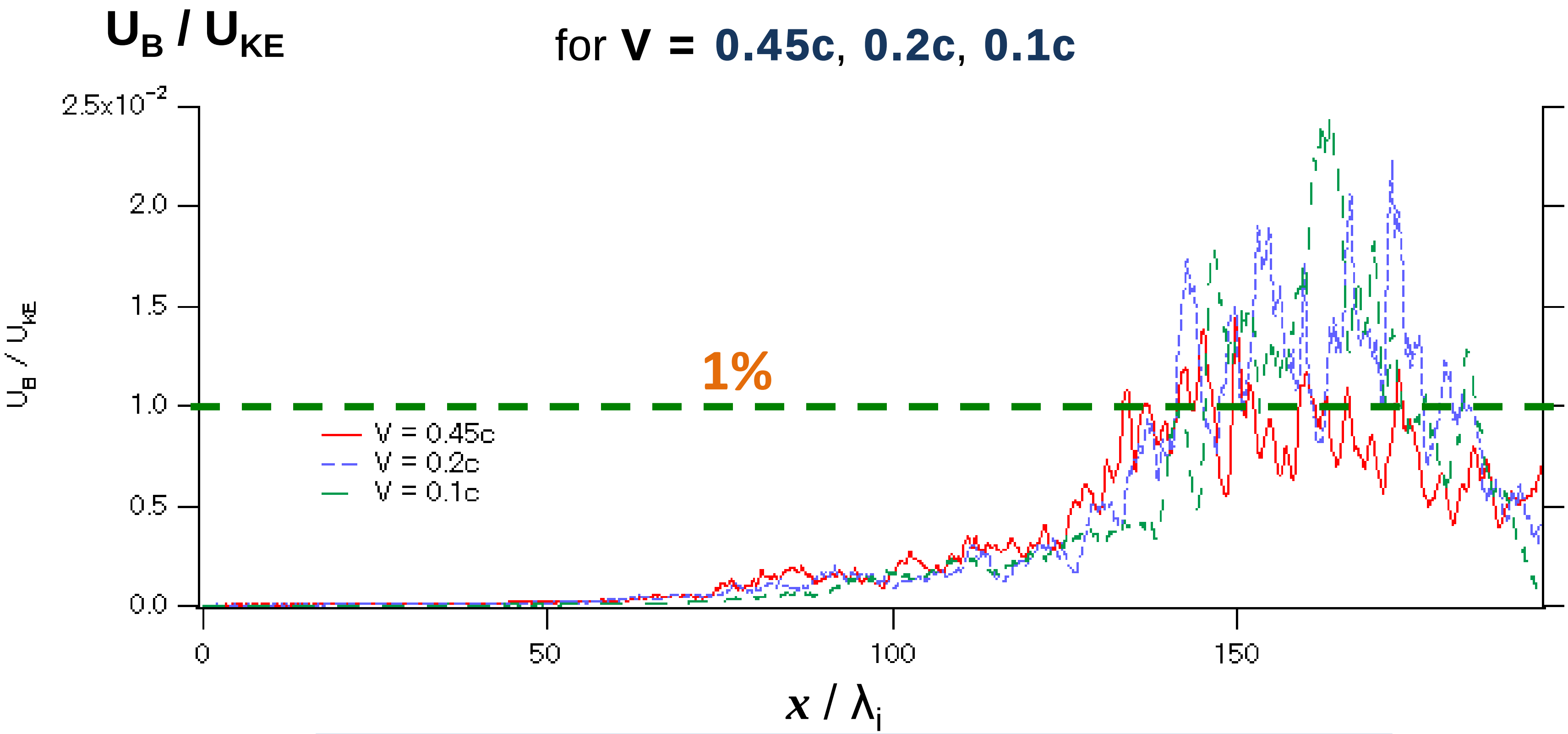
for **V = 0.45c, 0.2c, 0.1c**



- **Normalized by the upstream velocity**
- The spatial scale is normalized by the ion inertia length

Almost identical profiles

Profiles of Magnetic Energy Density



- **Normalized by the upstream bulk kinetic energy density**
- The spatial scale is normalized by the ion inertia length

The energy density of generated magnetic fields reaches **~1%** of the upstream bulk kinetic energy in all cases.

Simulation of Collisionless Shocks

In Electron-Ion Plasma
<u>with</u> Background Magnetic Field

Field Strength around SNRs

There exist **weak magnetic fields** in ISM

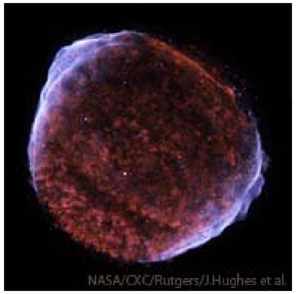
- Magnetic Field: $B_0 \sim 3\mu G$
- Number Density: $n \sim 0.1\ cm^{-3}$

The magnetization parameter:

$$\sigma \equiv \frac{U_B}{U_{KE}} = \frac{B_0^2/8\pi}{\frac{n}{2}\left(m_p + m_e\right)V^2}$$

$$\sigma \approx 4.3\times10^{-4}\left(\frac{B}{3\mu G}\right)^2\left(\frac{n}{0.1cm^{-3}}\right)^{-1}\left(\frac{V}{1000km/s}\right)^{-2}$$

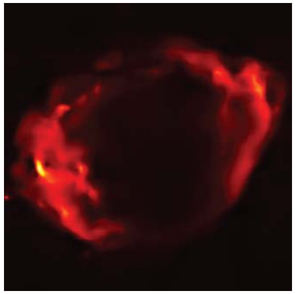
SN1006 (1003 years old)



$V_s \sim 3000$ km/s

⇒ $\sigma \sim 5 \times 10^{-5}$

G1.9+0.3 (~140 years old?)

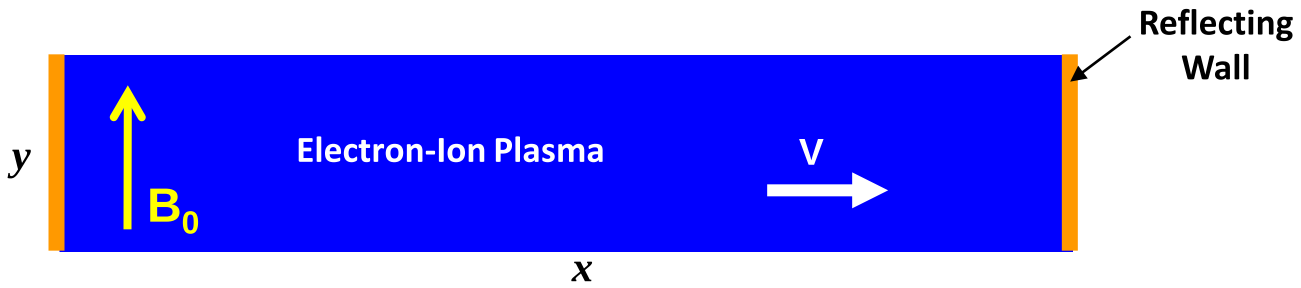


$V_s \sim 14{,}000$ km/s

⇒ $\sigma \sim 2 \times 10^{-6}$

Typically, $10^{-6} < \sigma < 10^{-3}$, in young SNRs

Initial Condition for Shocks in Magnetized Plasmas



A background magnetic field is set in the **y-direction**

"Perpendicular Shock"

Simulation Parameters

- **Perpendicular Shock** (θ=90˚)
- σ = **10^{-4}** **(Low-sigma)**
- M_A^* = **100** **(High Mach Number)**

$M_A^* = V / v_A$

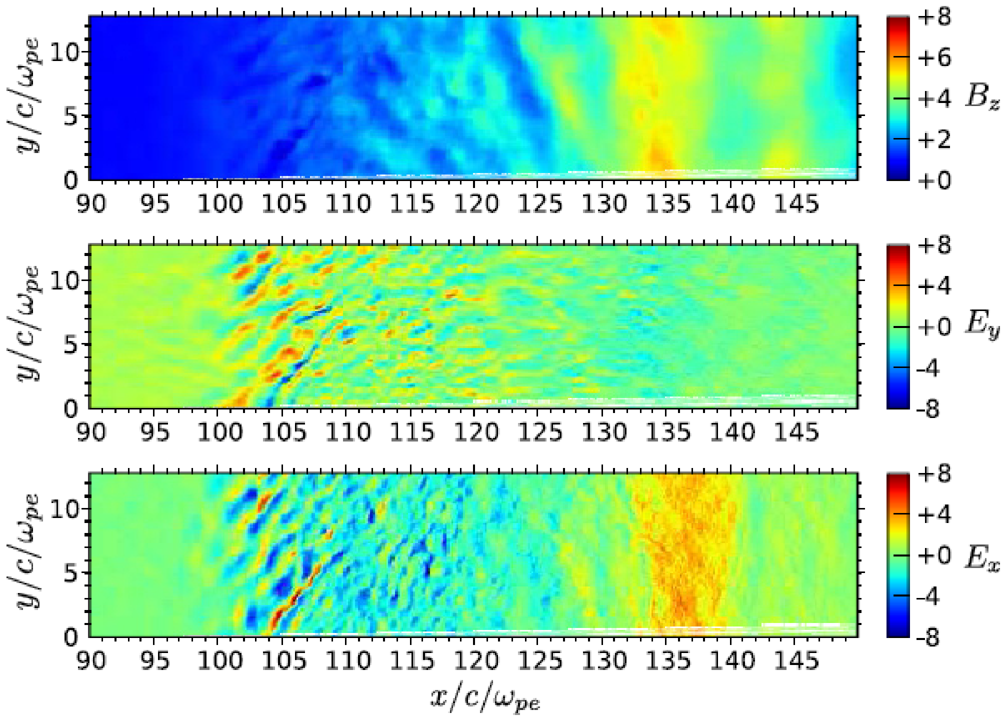
- 2D PIC simulation
- V = **0.25c**
- m_i/m_e = **30**
- (L_x, L_y) = **($3200\lambda_e$, $200\lambda_e$)**

λ_e: electron skin depth

2D Perpendicular Shock Simulation

Amano & Hoshino, ApJ, 2009, 690, 244

Nonrelativistic perpendicular 2D PIC simulation



- **Perpendicular Shock** (θ=90˚; **out of plane**)
- **σ ~ 10^{-2}**
- **M_A = 14**

- 2D PIC simulation
- V = **0.2c**
- m_i/m_e = **25**
- (L_x, L_y) = (**204**λ_e, **12.8**λ_e)

λ_e: electron skin depth

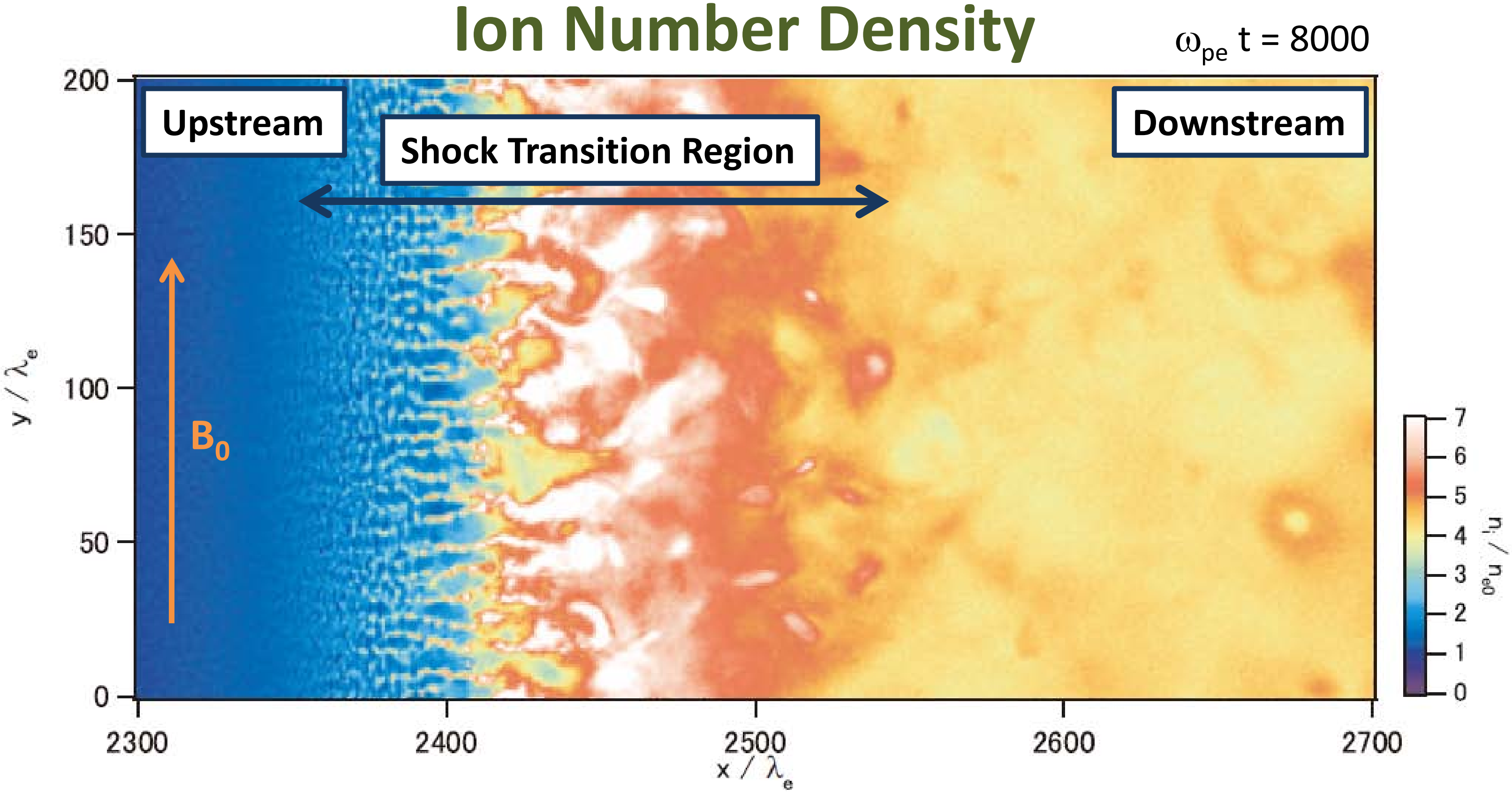
Electron acceleration is observed

⟺ Our simulation: **on-plane** magnetic field, **low-sigma** (**σ=10^{-4}**), and high Mach number (**M_A^*=100**)

Results

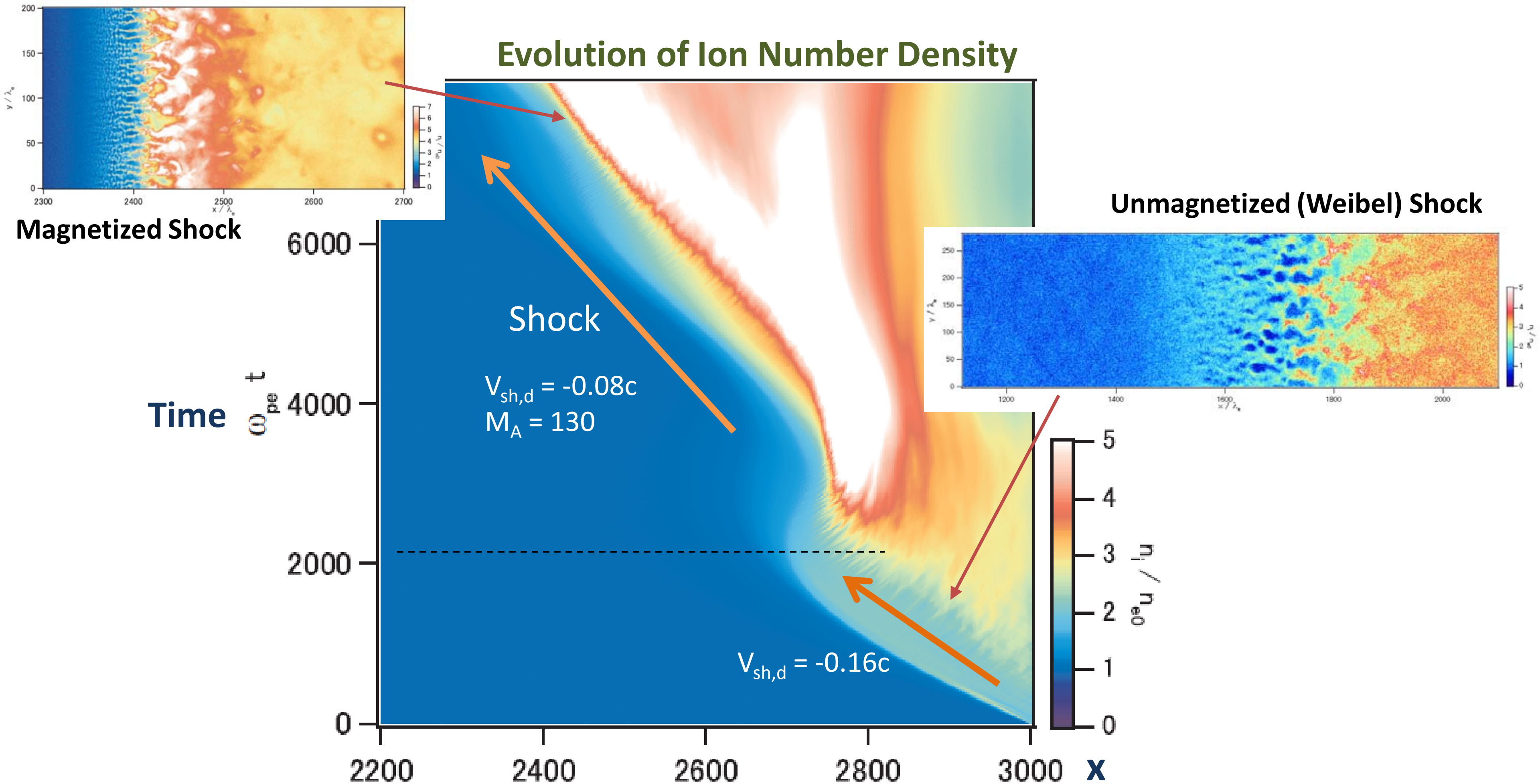
V=0.25c, σ=10^{-4}, θ=90°

Ion Number Density



- Collisionless shock is formed
- **Filamentary structures** in the leading edge of the transition region
- **Highly inhomogeneous** structure in the transition region
- Downstream region is almost **homogeneous**

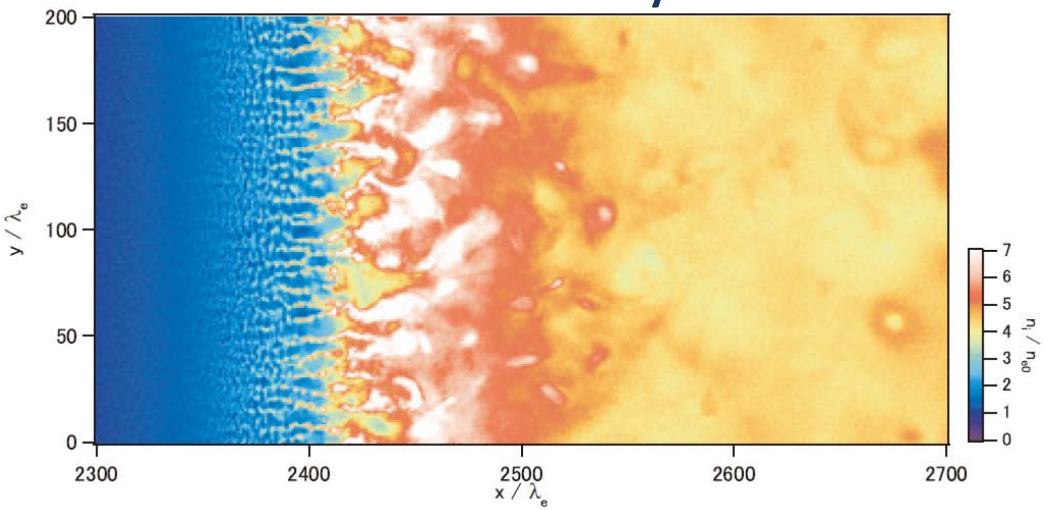
Propagation of Shock

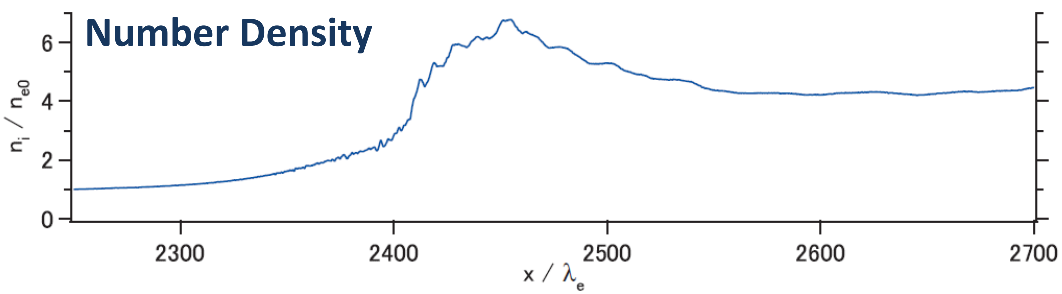


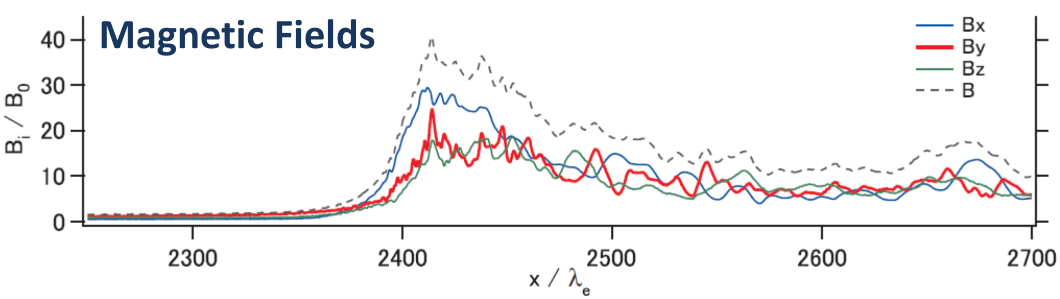
- First, an **unmagnetized (Weibel-mediated) shock** is formed. Then, a **magnetized shock** is formed
- Both shocks propagate at almost **constant speeds**
- **No clear shock reformation** is observed

Profiles

Number Density



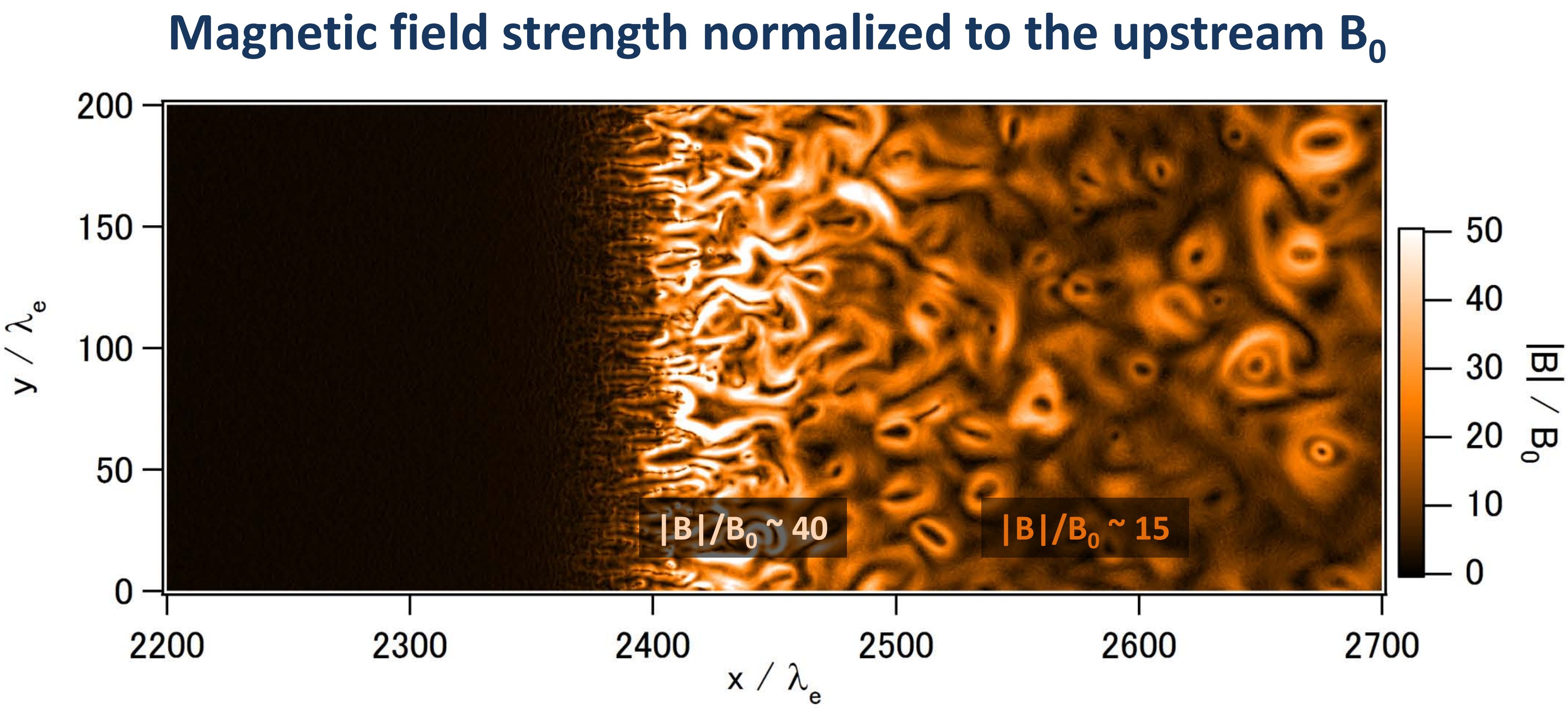




- Density jump is **~4**
- **B_x,B_z** components are **generated** as well as the **compression** of the upstream field **B_y**
- In the transition region, **B_x is dominant**
- In the downstream region, mean magnetic field strength is **~15 times** the upstream strength

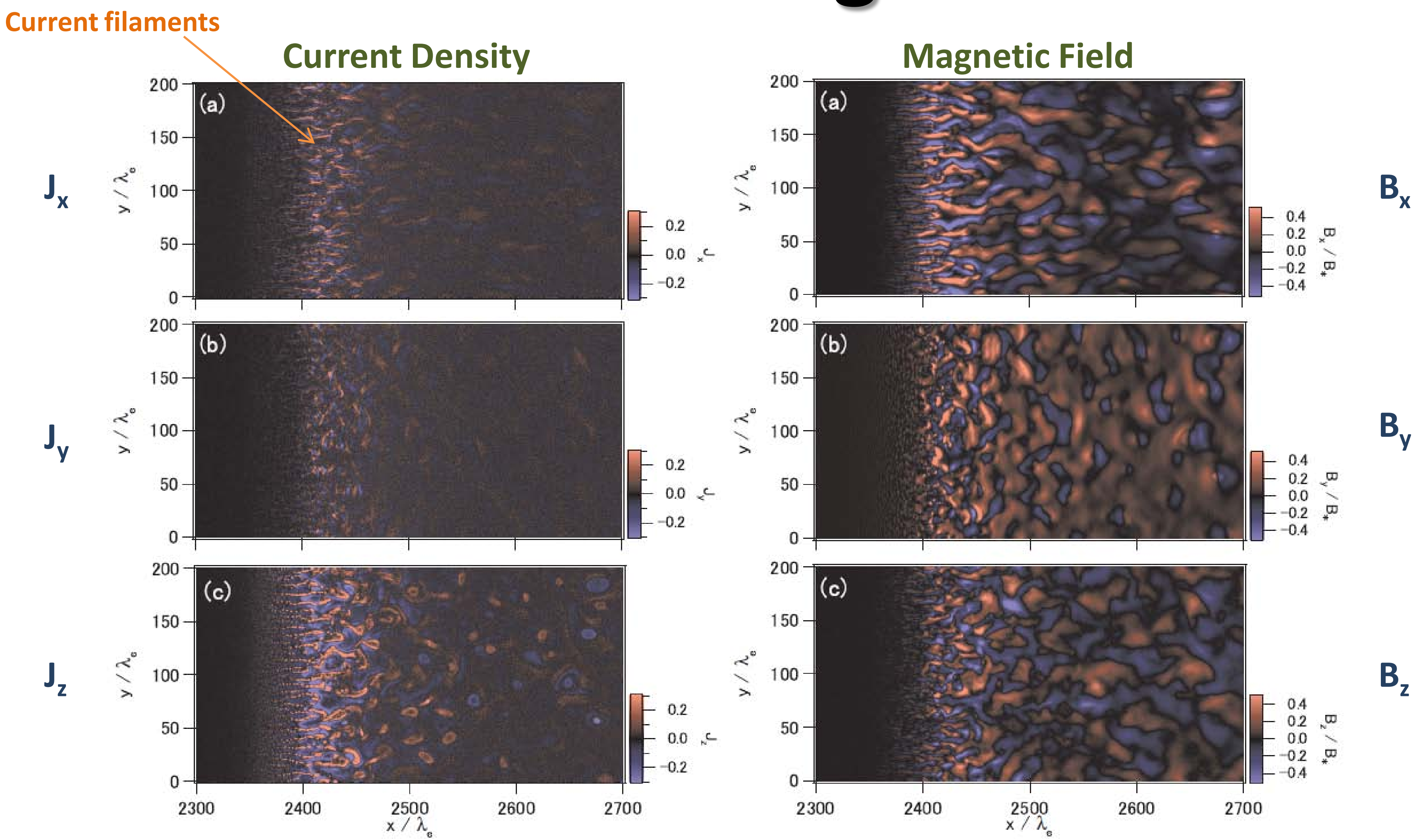
Magnetic field generation in the transition region

Magnetic Field Generation



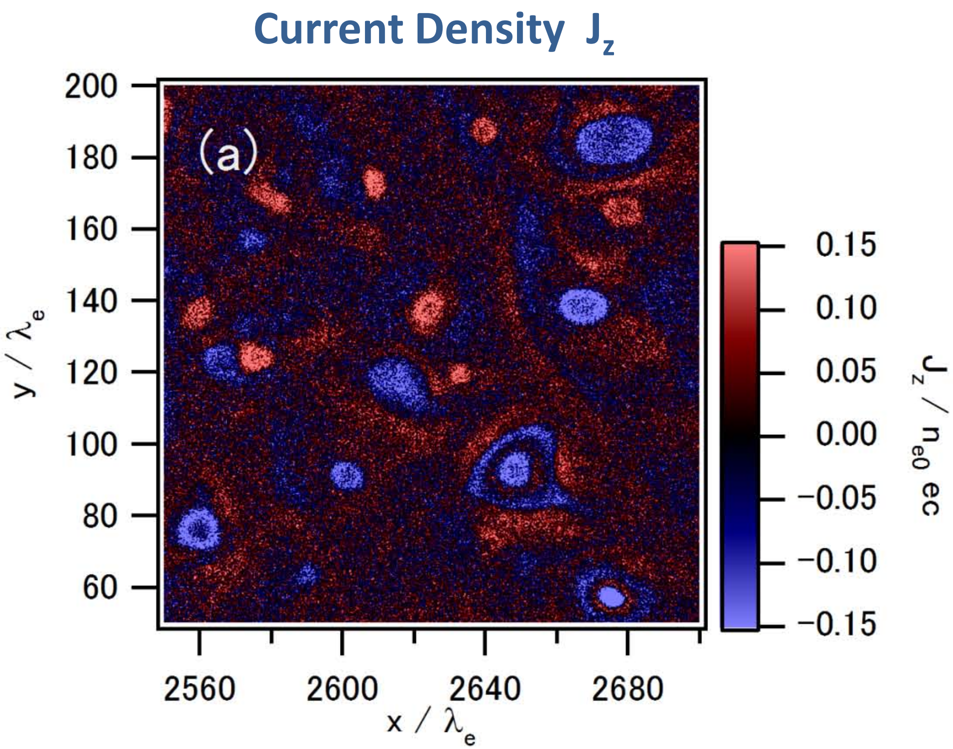
Highly tangled strong magnetic field is generated, in the **shock transition region** (**$|B|/B_0$~40**) and in the **downstream region** (**$|B|/B_0$~15**)

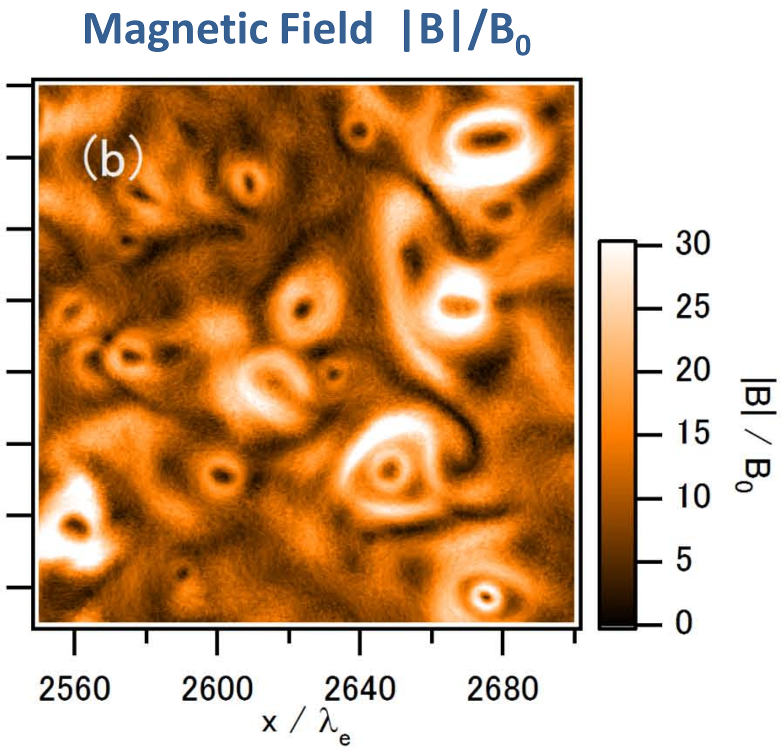
Currents and Magnetic Field



- In the transition region, there are **current filaments** generated by **Weibel instability**
- The current filaments **generate magnetic field**, while the **background field** is also **compressed**

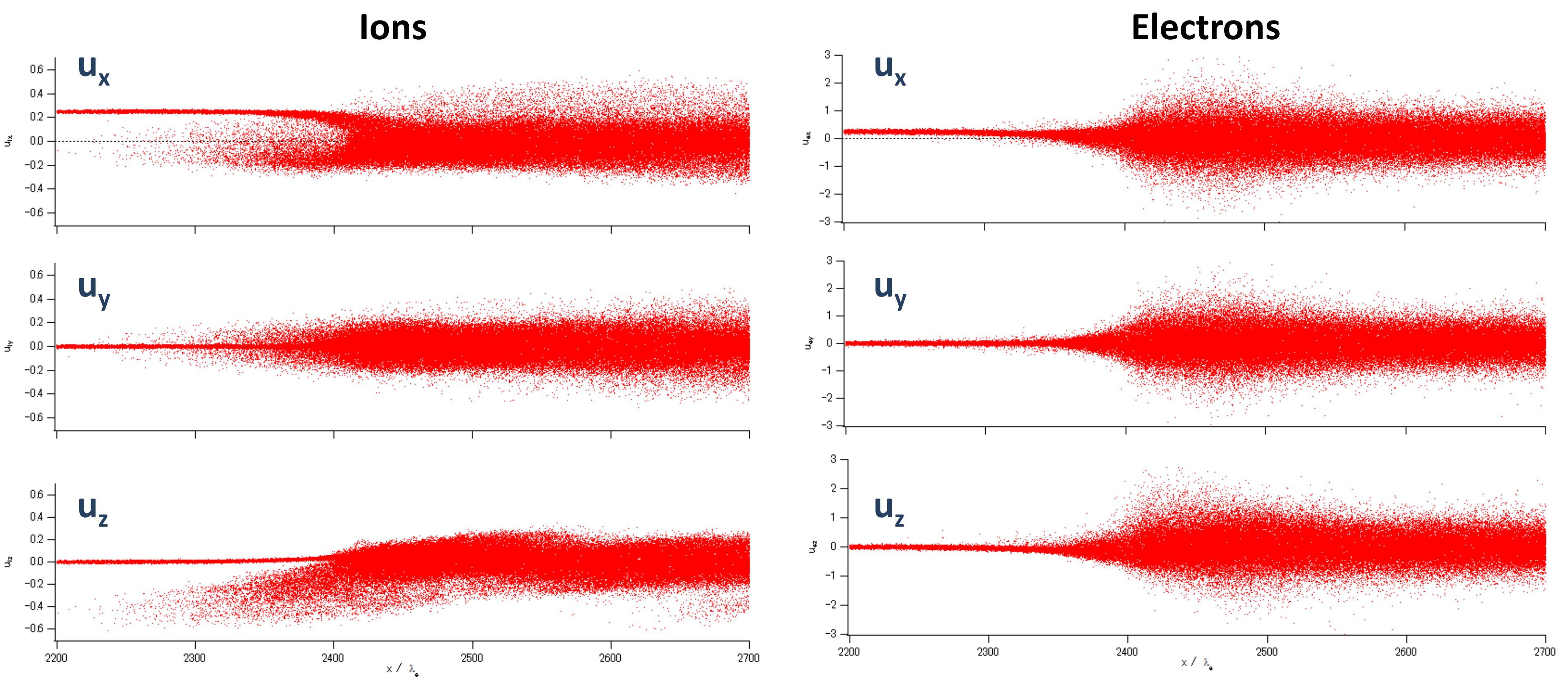
Downstream Magnetic Fields





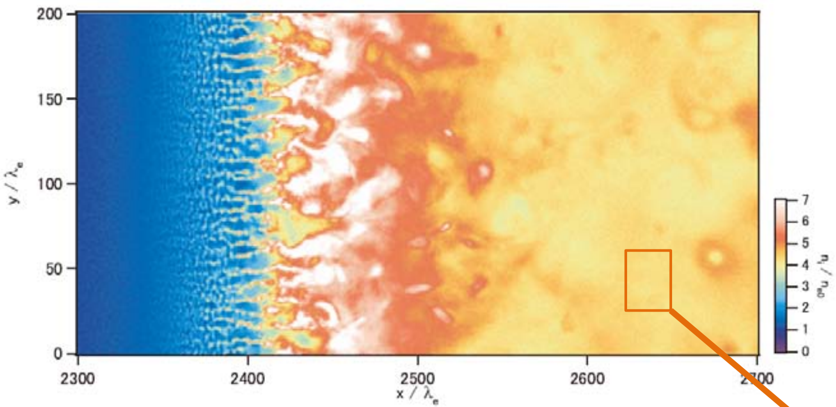
- Downstream magnetic fields are generated by a lot of **current filaments in z-direction, J_z**
- Some of the filaments have **coaxial** structure

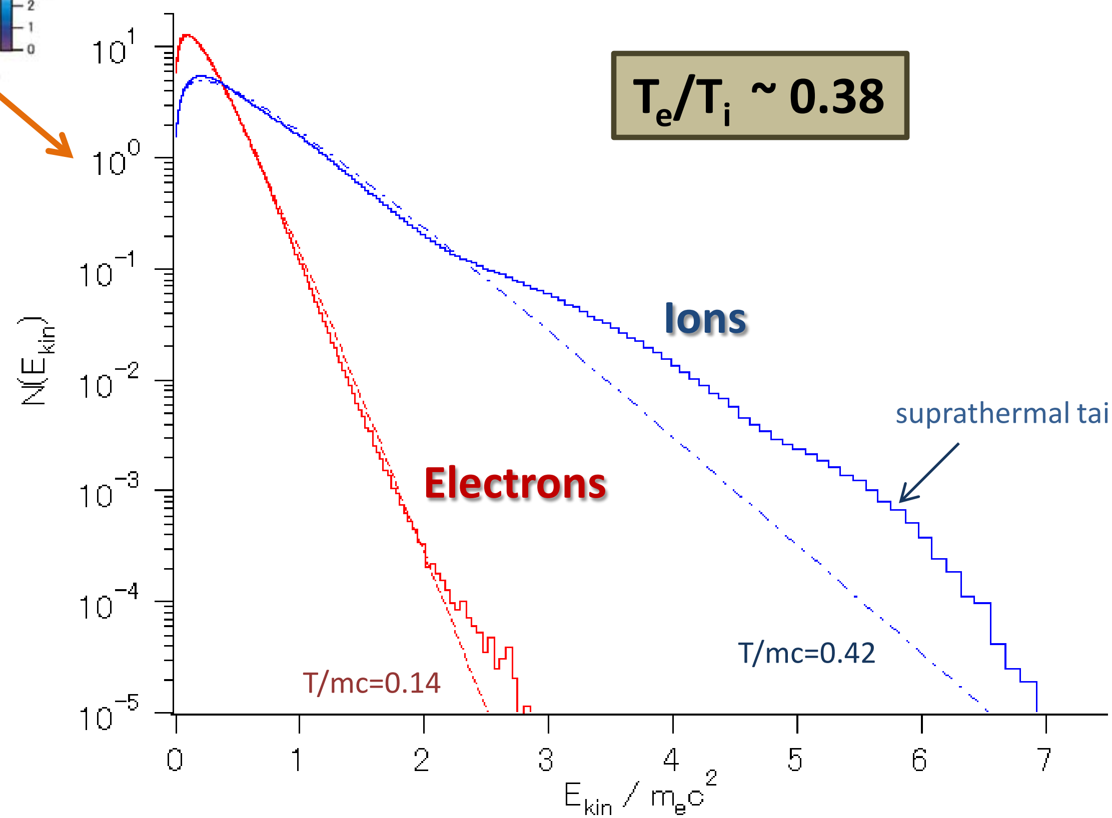
Phase Space Plots



- Both electrons and ions are **isotropically thermalized** in the downstream region
- Thermalization would be due to mainly **magnetic field** and partly **Buneman instability** (for electrons)

Downstream Energy Distribution





- Both electrons and ions are almost **Maxwell distribution**
- Electron temp. is **lower** than ion temp.
- There are some **suprathermal ions**
- **No** prominent **acceleration** for electrons

Jump Condition

From the simulation, we obtain

$$n_2/n_1 \sim 4.1,\ \ V_1/V_2 \sim 3.9,\ \ ((T_e+T_i)\ /\ m_i)^{1/2} \sim 0.14c$$

On the other hand, the (MHD) Rankine-Hugoniot relation gives

$$n_2/n_1 \sim 4,\ \ V_1/V_2 \sim 4,\ \ ((T_e+T_i)\ /\ m_i)^{1/2} \sim 0.15c$$

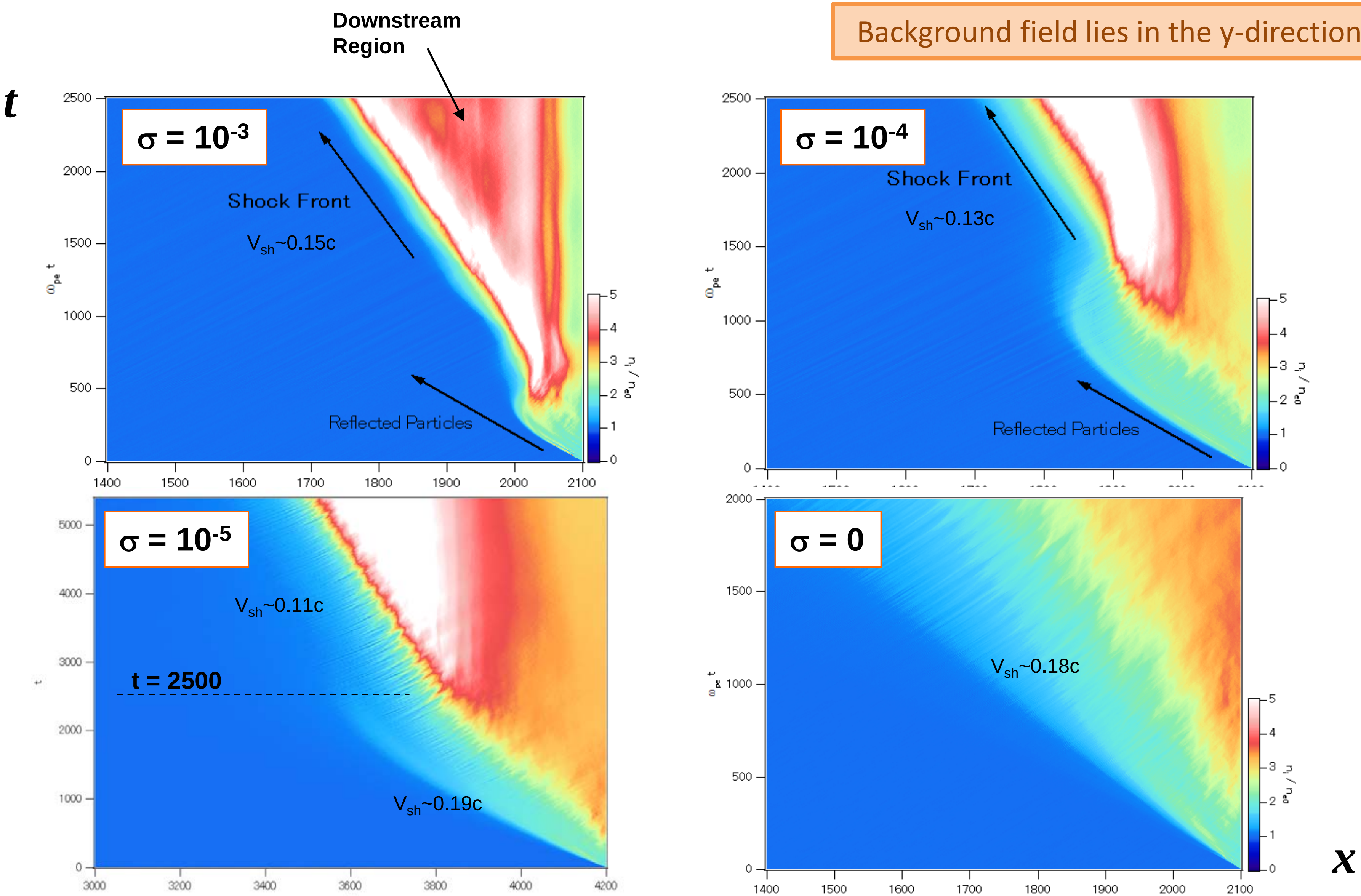
(e.g., Tidman & Krall 1971)

R-H relation holds very well

Although **$|B|/B_0$ ~ 15** in the downstream, the **plasma beta** is **~25** and **the magnetic field is negligible** for the jump condition

σ –dependence

Background field lies in the y-direction



Shock structure changes at about ¼ **the ion gyro-motion**

Summary

2D PIC simulation of **nonrelativistic** collisionless shocks in **unmagnetized** and **weakly magnetized** electron-ion plasmas

Unmagnetized (Weibel) shock

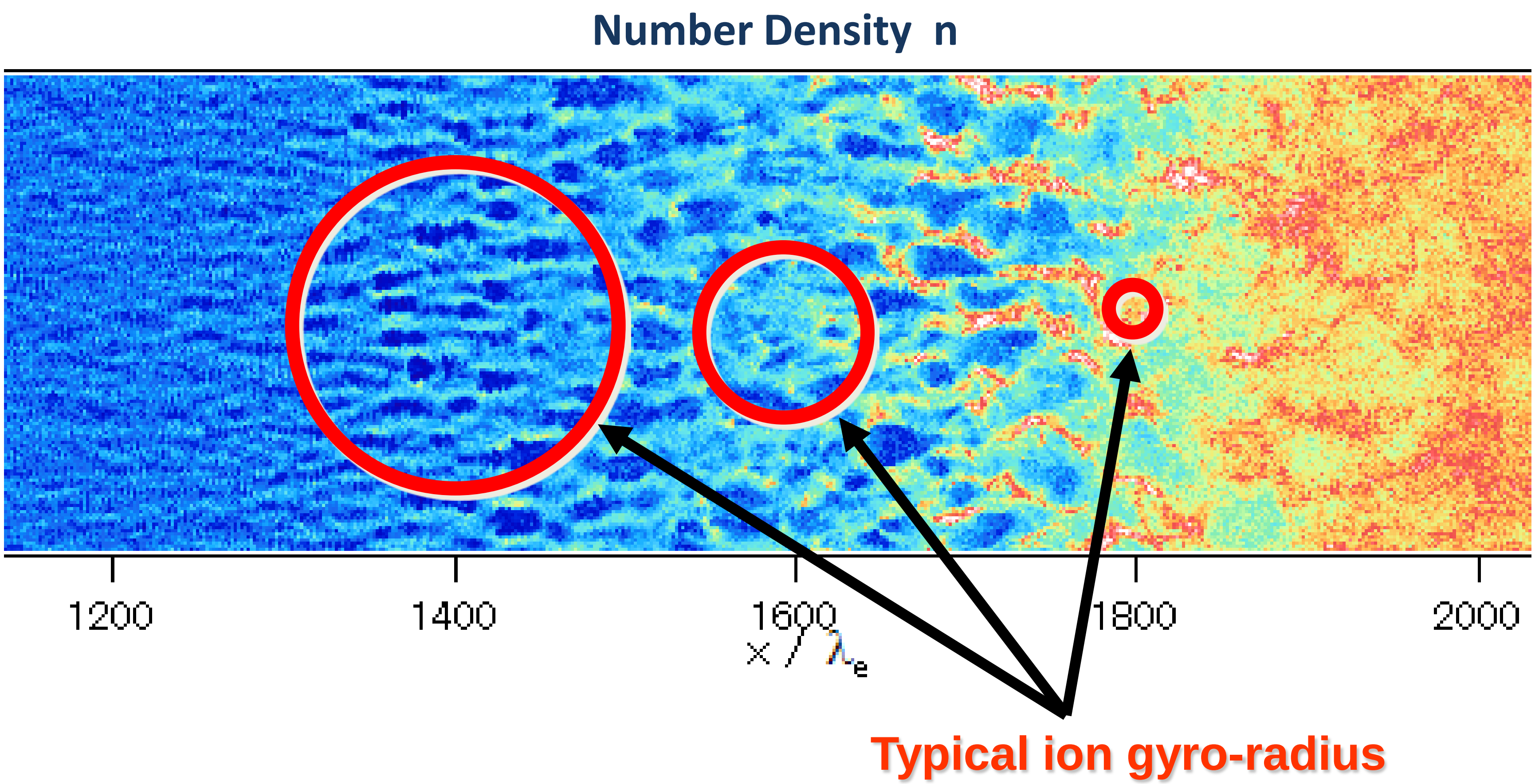
- The **Weibel-mediated shocks exist** in the **non-relativistic** regime
- The structure is **similar** to those in the **relativistic** cases
- The shock exist at least **V > 0.1c**
- **Profiles of number density, normalized velocity, normalized magnetic field** are almost **independent of V**

Weakly magnetized shock ($\sigma=10^{-4}$)

- First, **Weibel-mediated shock**, then **magnetized shock** are formed
- In the **transition region**, **current filaments** and **strong magnetic fields ($|B|/B_0$ ~ 40)** are generated by the **Weibel instability**
- In the downstream region, **highly tangled magnetic fields ($|B|/B_0$ ~ 15) remain**
- **No clear shock reformation**
- Electrons and ions are **thermalized** and well fitted by **Maxwellian** distributions in the downstream. Temp ratio is **T_e/T_i~0.4**
- **No** prominent **particle acceleration** is observed

Thank you for your time and your attention

Saturation of Magnetic Field



Magnetic field saturates when **filament radius=ion gyro-radius**

➡ **Isotropization of particles** ➡ **Dissipation**

Filament Radius at Saturation

R_f : radius of current filament

Magnetic field generated by the current filament

$$B \approx 2\pi R_f qnV / c$$

Gyro-radius

$$R_g \approx \frac{\gamma m_i cV}{eB} \approx \frac{\gamma m_i c^2}{2\pi nq^2 R_f}$$

Condition for saturation: $R_f \sim R_g$ **(corresponding to the ion Alfven current)**

$$\Rightarrow \quad R_f \approx \sqrt{2\gamma}\,\frac{c}{\omega_{pi}}$$

~ ion inertial length (including relativistic effect)

Independent of **V**

Energy density of magnetic field

$$U_B = \frac{B^2}{8\pi} \approx \frac{1}{4}\gamma m_i V^2 \quad \Rightarrow \quad \frac{U_B}{U_{KE}} \approx \frac{1}{4}\frac{\gamma V^2}{\gamma - 1}$$

sub-equipartition

Model for Current Filament

Coalescence of two current filaments

$$J \sim \eta e n V, \quad B \sim 2\pi \eta e n R \frac{V}{c}$$

V: Flow velocity
R: Filament radius
l: Distance between two filaments

$$\frac{d^2 l}{dt^2} = \frac{\eta}{2} \omega_{pi}^2 \left(\frac{V}{c}\right)^2 R$$

E.O.M for two filaments

Order estimate

Time scale of coalescence: $\tau \sim (\alpha/\eta)^{1/2} \omega_{pi}^{-1} \left(\frac{V}{c}\right)^{-1}$ $\quad (l = \alpha R)$

Independent of **R, l**

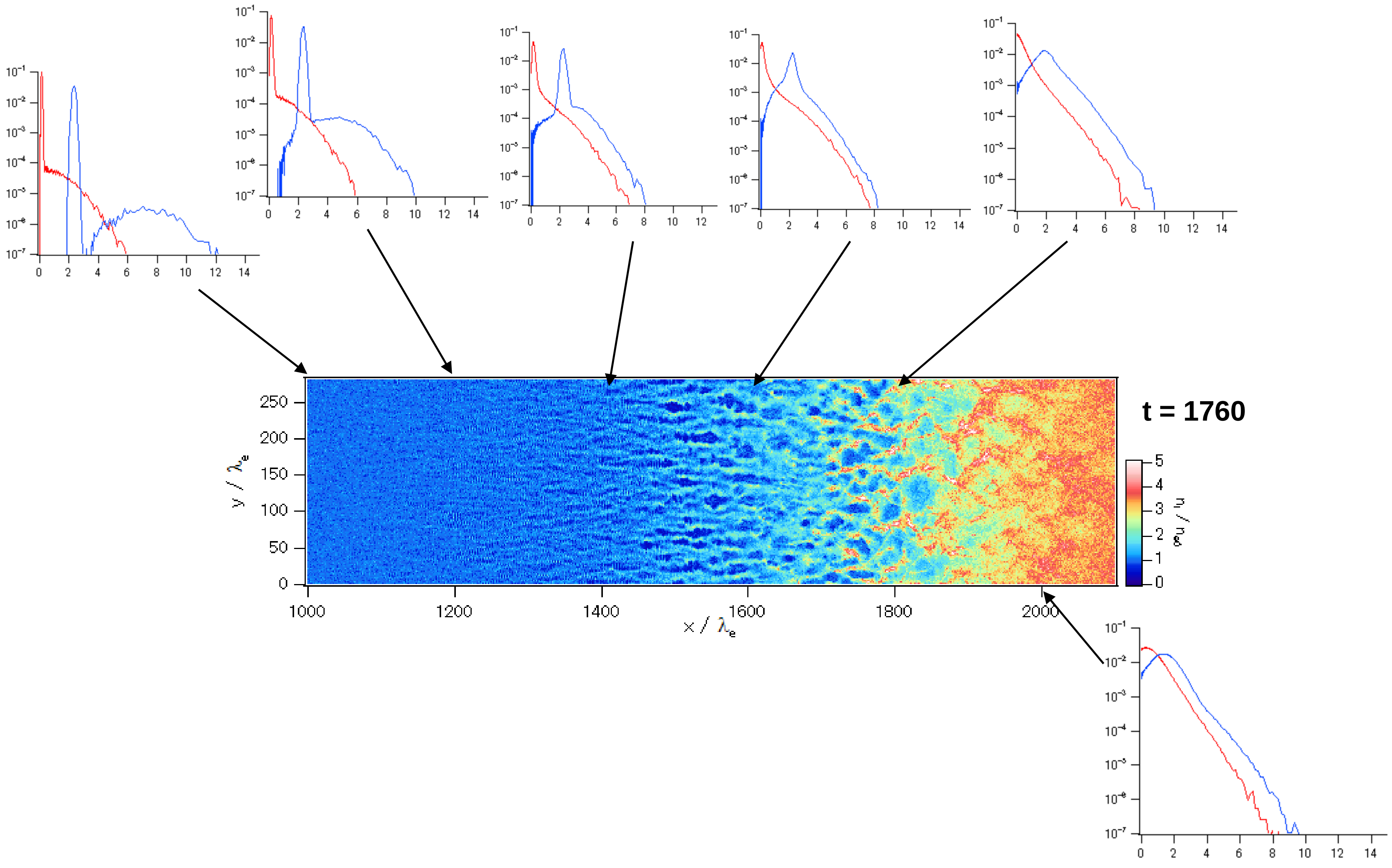
Time scale for which filament radius becomes twice:

$$\Delta W \sim V\tau \sim (\alpha/\eta)^{1/2} \lambda_i$$

Independent of **V**

If $R_0 \sim c/\omega_{pe}$, $R_1 \sim c/\omega_{pi}$, then **W** is **independent of V**

Energy Distribution



Electrostatic Mode

Buneman instability

- **Condition**

$$V_D > V_{\mathrm{th,e}}$$

- **Wave length**

$$\lambda \approx 2\pi \frac{2V_{\mathrm{sh}}}{\omega_{\mathrm{pe}}} \approx 4\lambda_e$$

- **Growth rate**

$$\gamma \approx 0.7 \times \left(\frac{m_e}{m_i} \right)^{1/3} \approx 0.2\omega_{\mathrm{pe}}$$

Ion acoustic instability

- **Condition**

$$T_e / T_i \gg 1$$

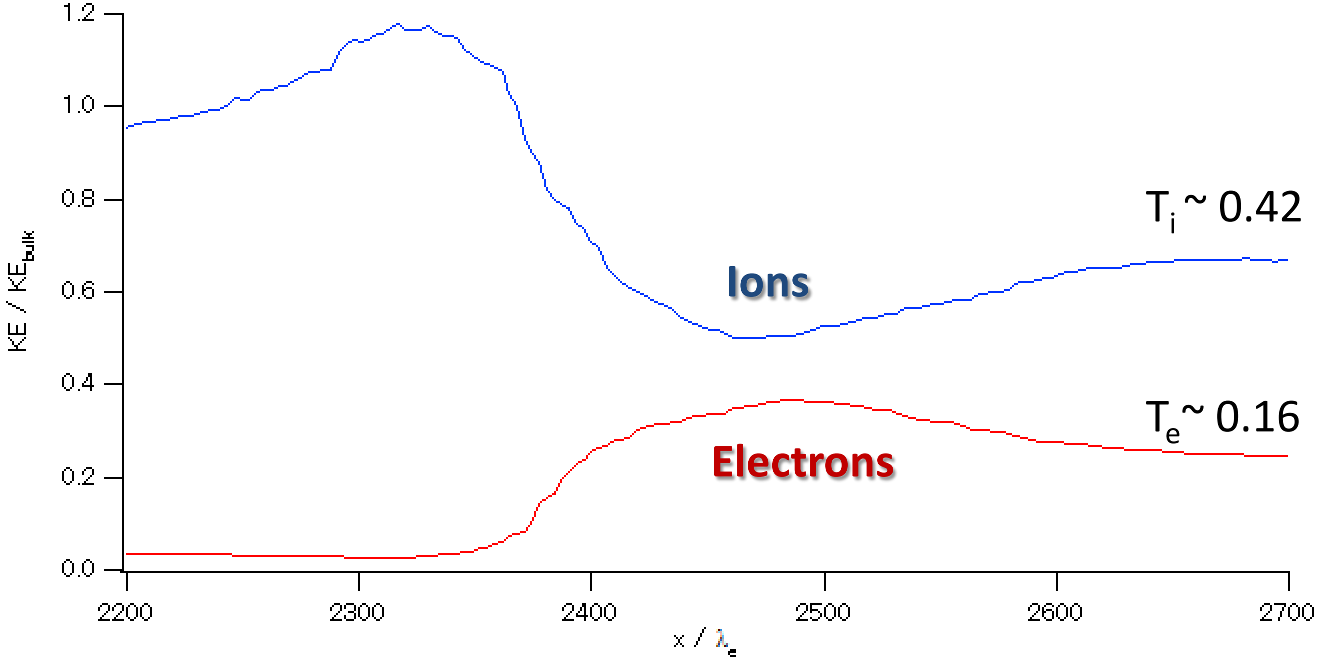
- **Wave length**

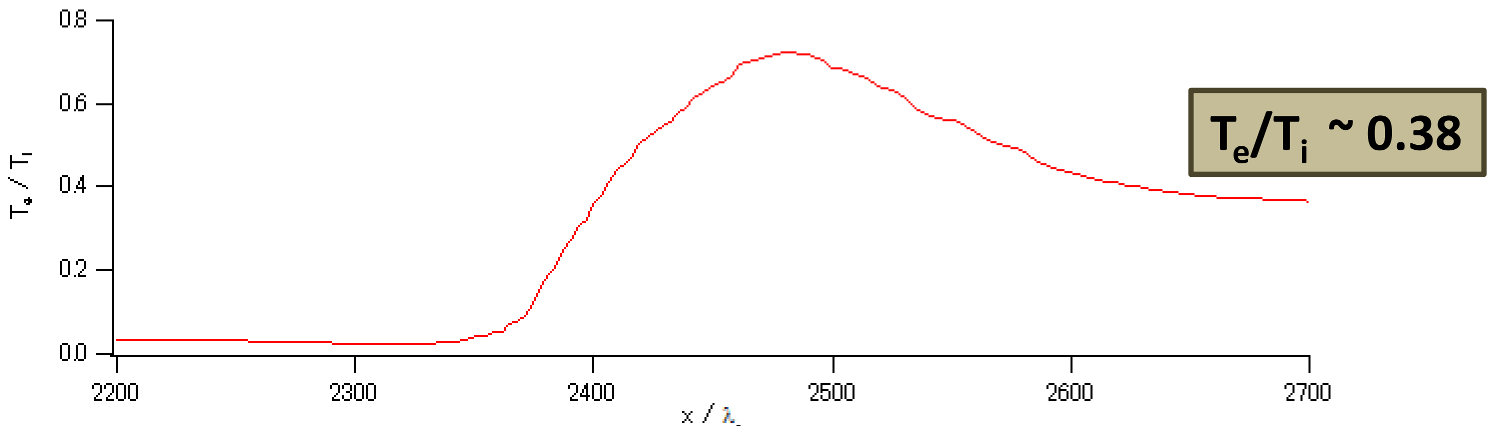
$$\lambda \approx 2\pi \frac{2V_{\mathrm{sh}}}{\omega_{\mathrm{pe}}} \approx 4\lambda_e$$

- **Growth rate**

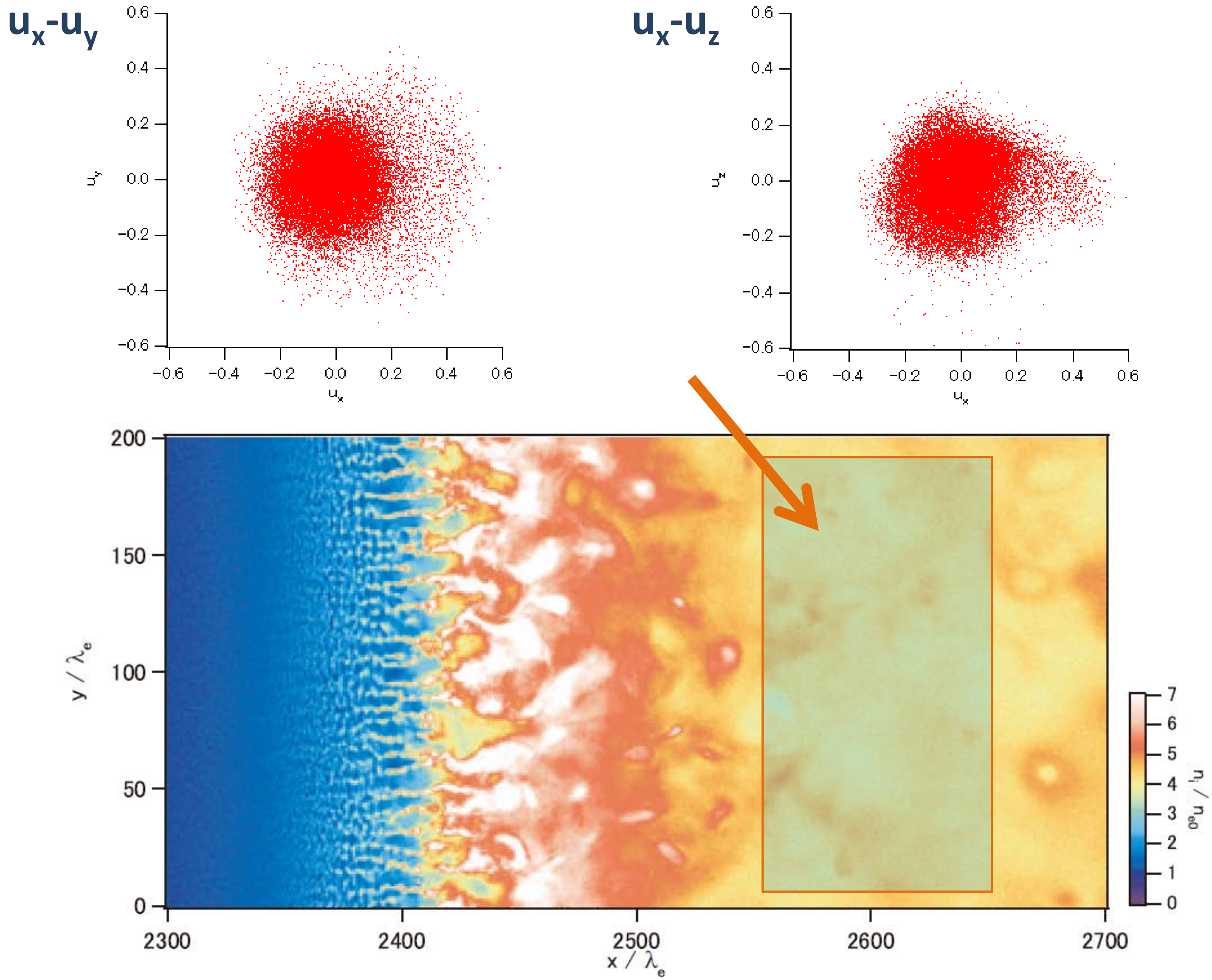
$$\gamma \approx 0.01\omega_{\mathrm{pe}}$$

Averaged Particle Kinetic Energy



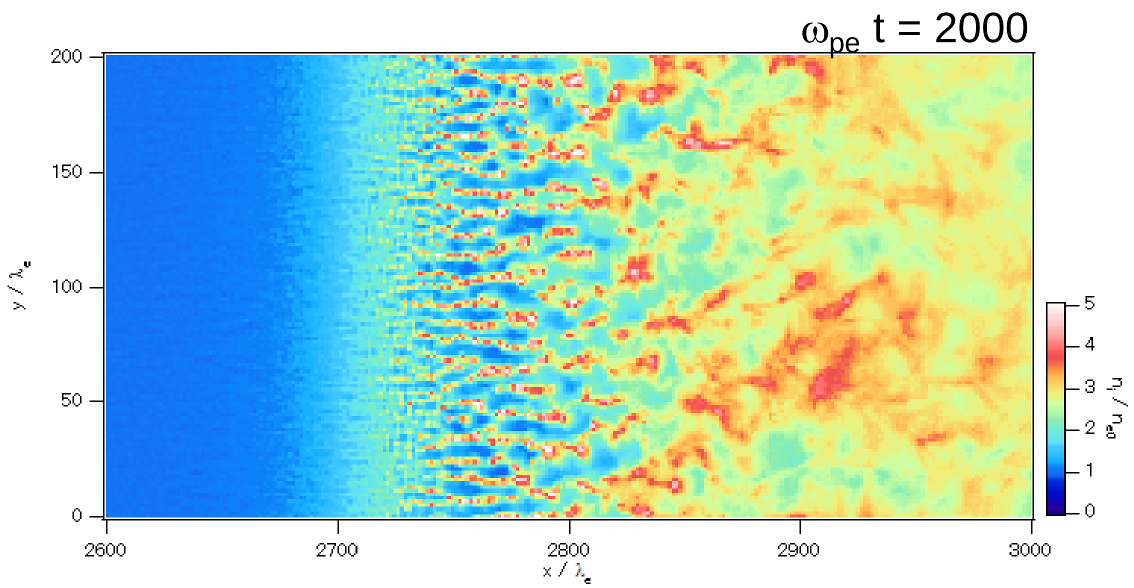


Ion Velocity Distribution

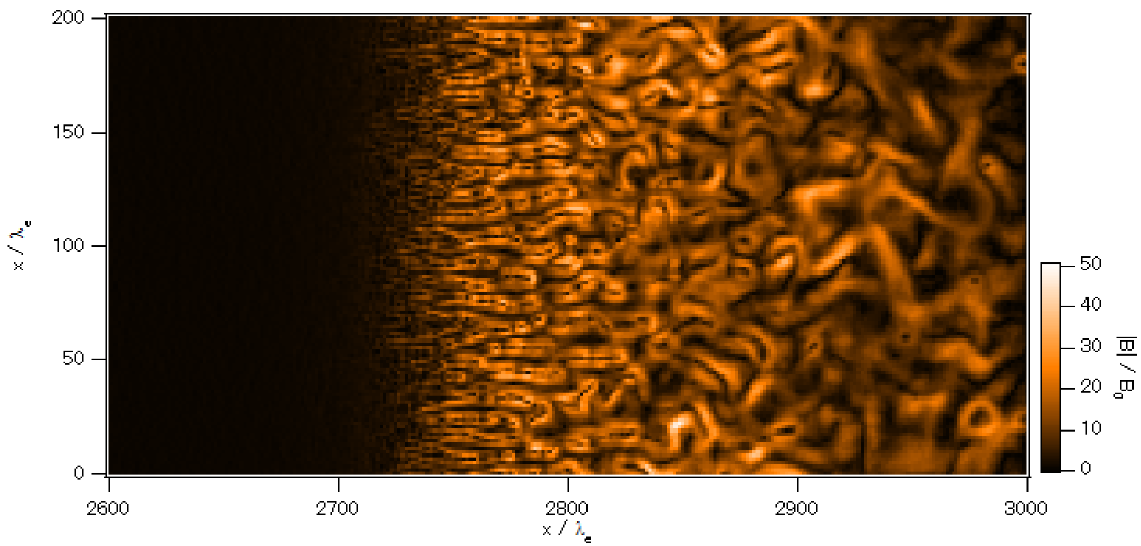


Early Evolution

Ion number density

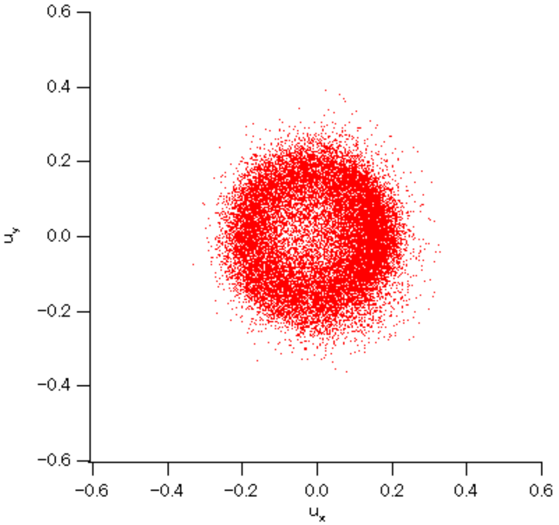


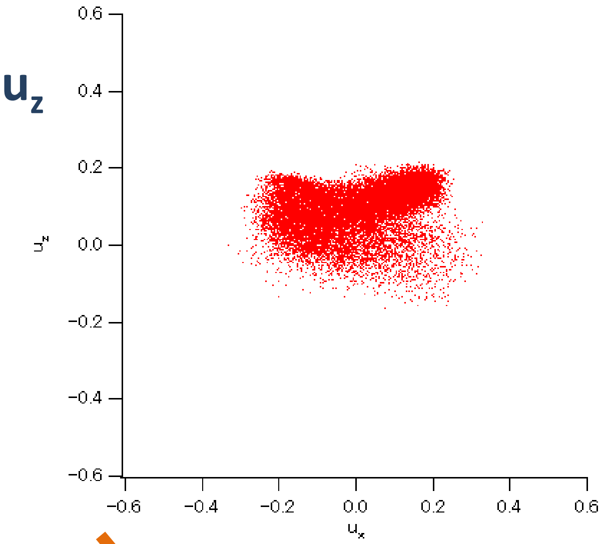
Magnetic field strength
$|B|/B_0$ ~ 30

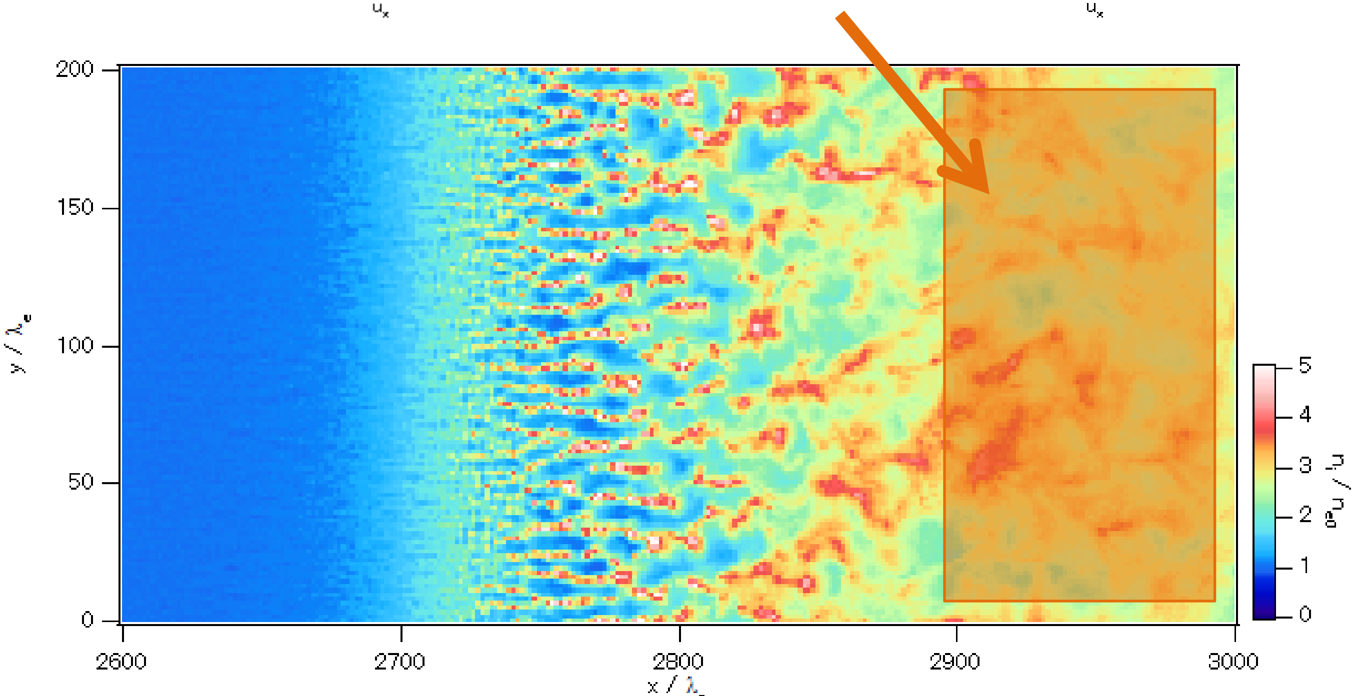


Early Evolution

almost isotropic in x-y plane, but not thermalized

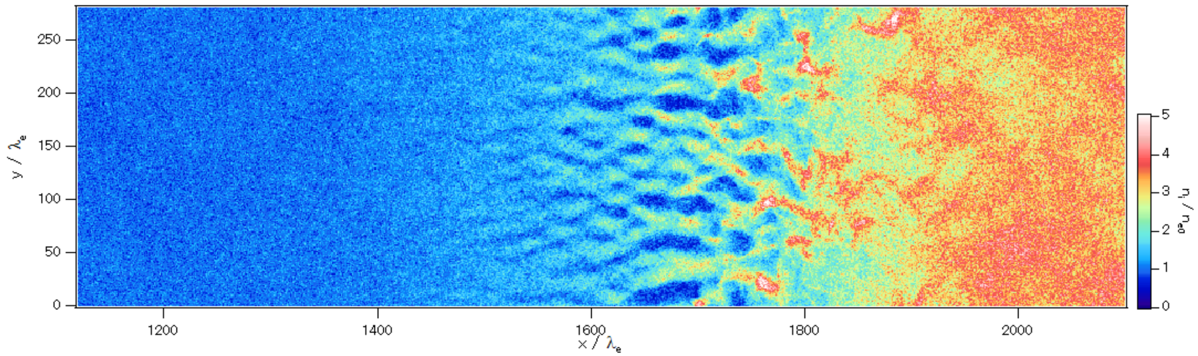




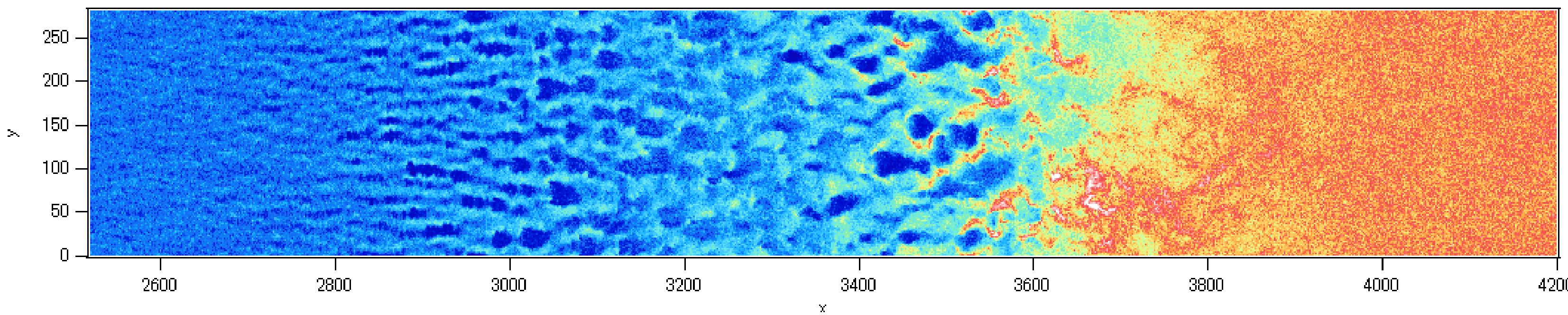


Parallel Background Field

$\sigma = 10^{-4}$

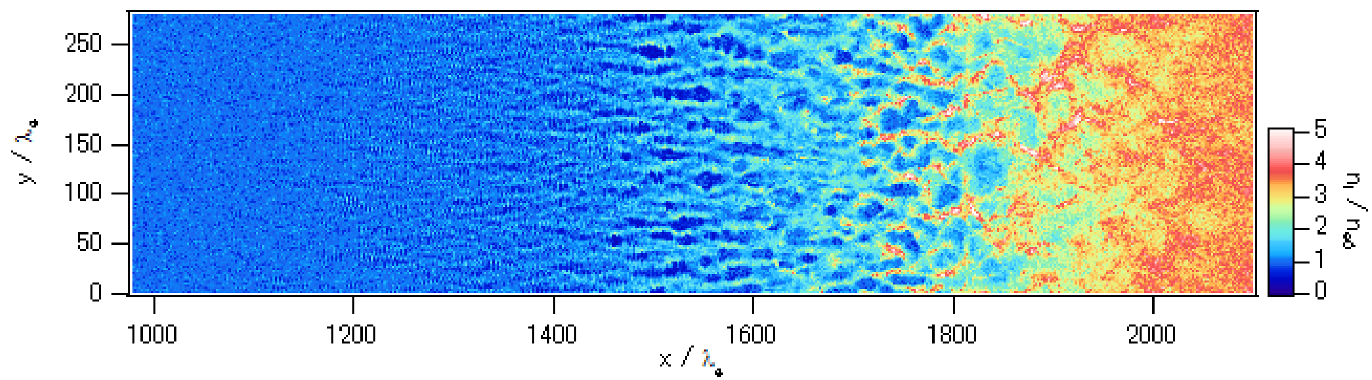


$\sigma = 10^{-5}$



$\sigma = 0$

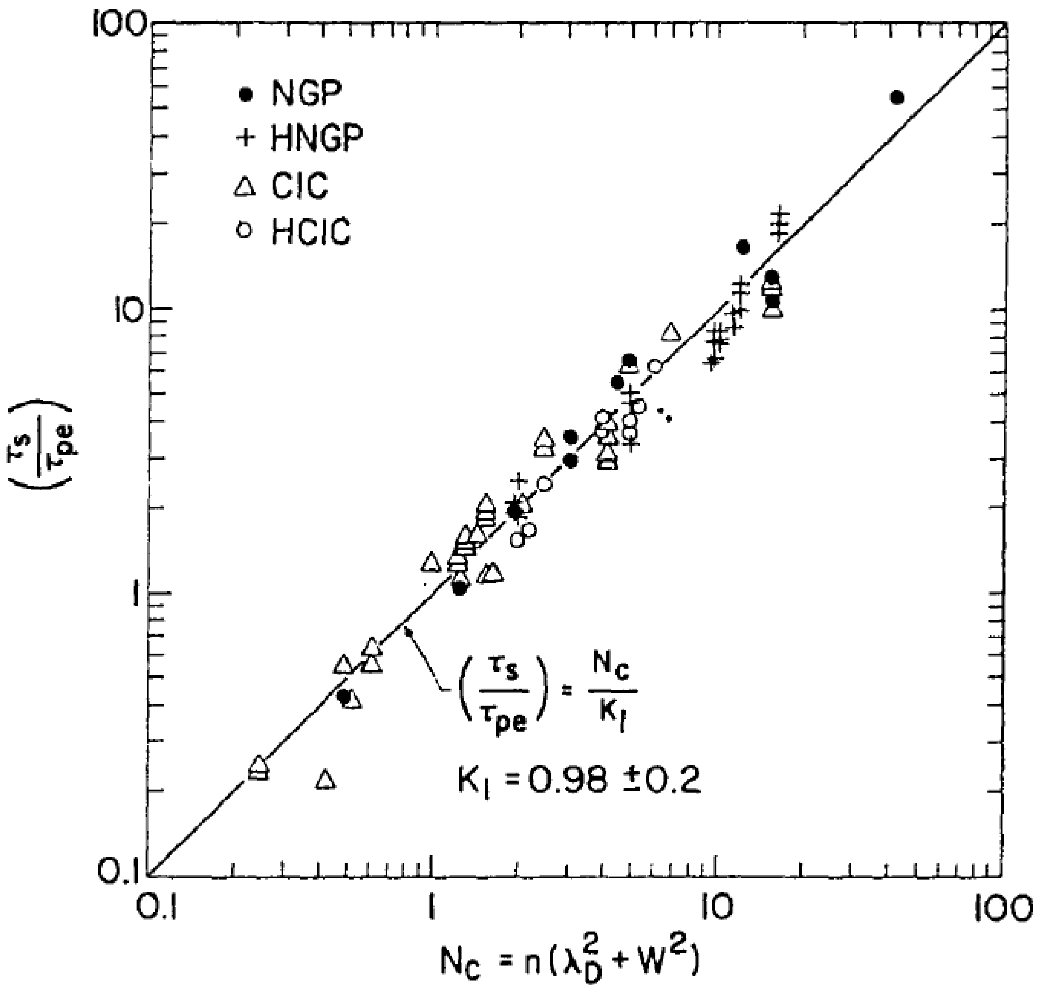
(Weibel Shock)



Almost the **same structure** as that of **unmagnetized Weibel shocks**

Numerical Collision Effect (2D)

Hockney, 1971, J. Comput. Phys., **8**, 19



Experimental Formula

$$\frac{\tau_{\text{coll}}}{\tau_{\text{pe}}} \approx N_C$$

$$N_C = n\left[\lambda_D^2 + R^2\right]$$

Effective size of particle

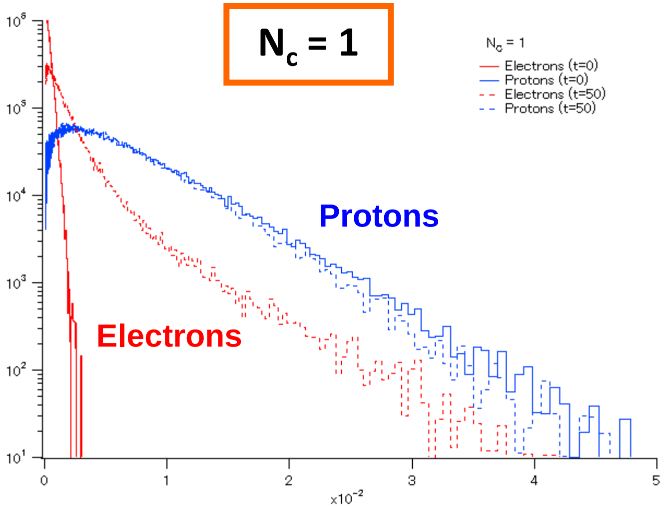
N_c must be large enough for long-term simulations

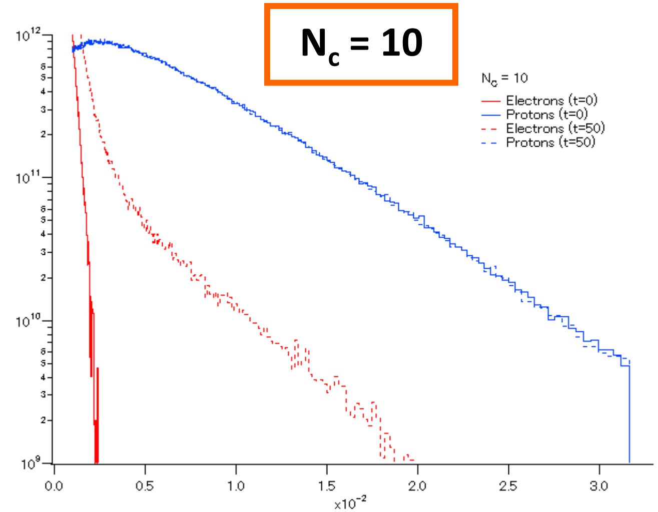
PIC Simulation of Two-Temp e-p Plasma

$m_p / m_e = 20$

$(R = \Delta x = 15\,\lambda_D)$

Solid: **t = 0**, Dotted: **t = 50**

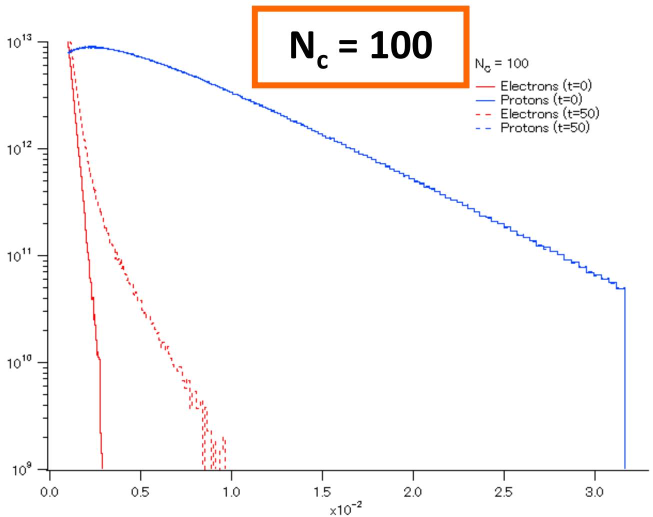




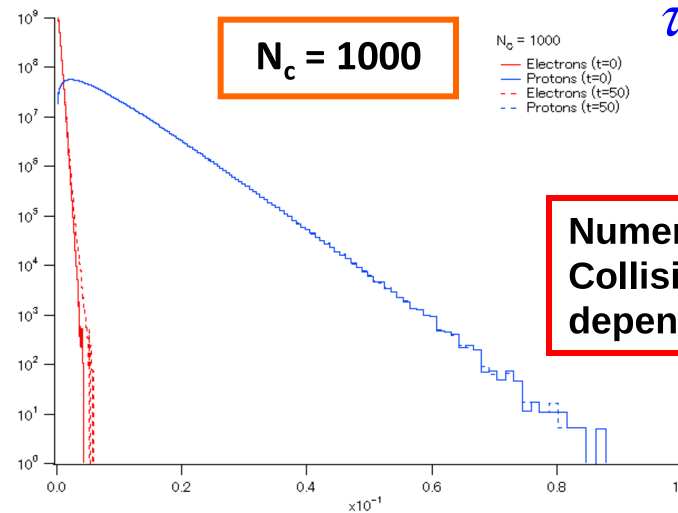
N(E)

Numerical Collision Timescale

$$\tau_{ep} \approx 120 N_c$$



$E / m_e c^2$

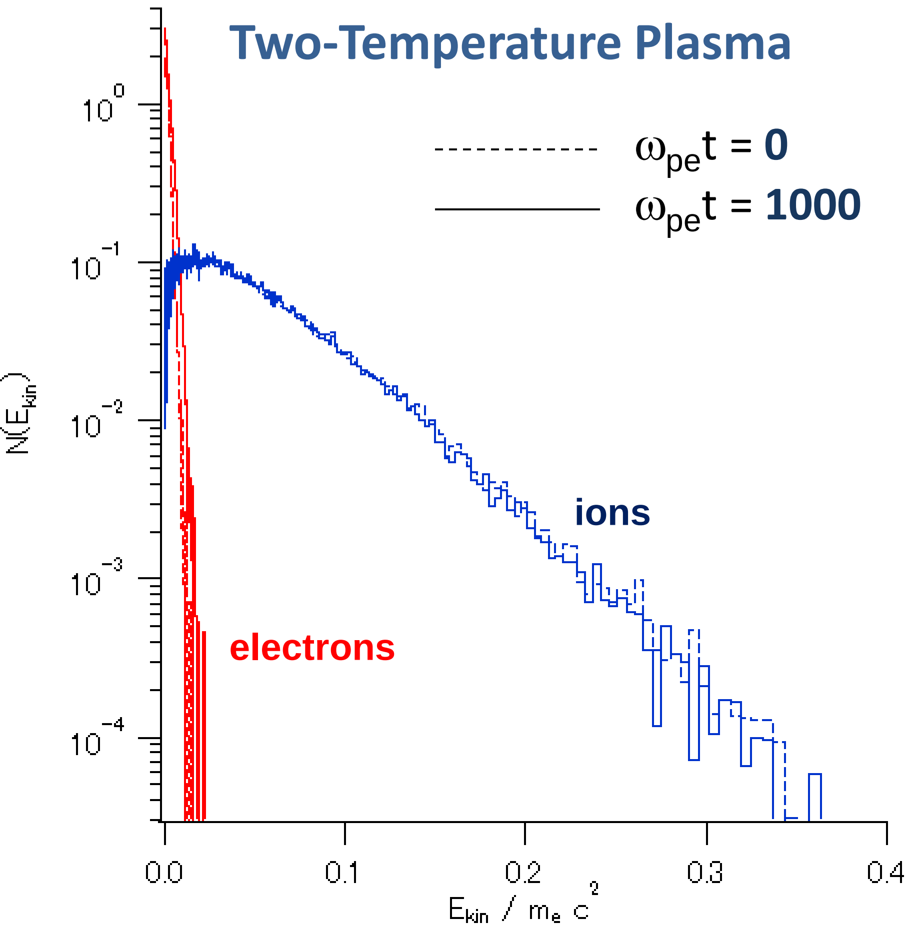


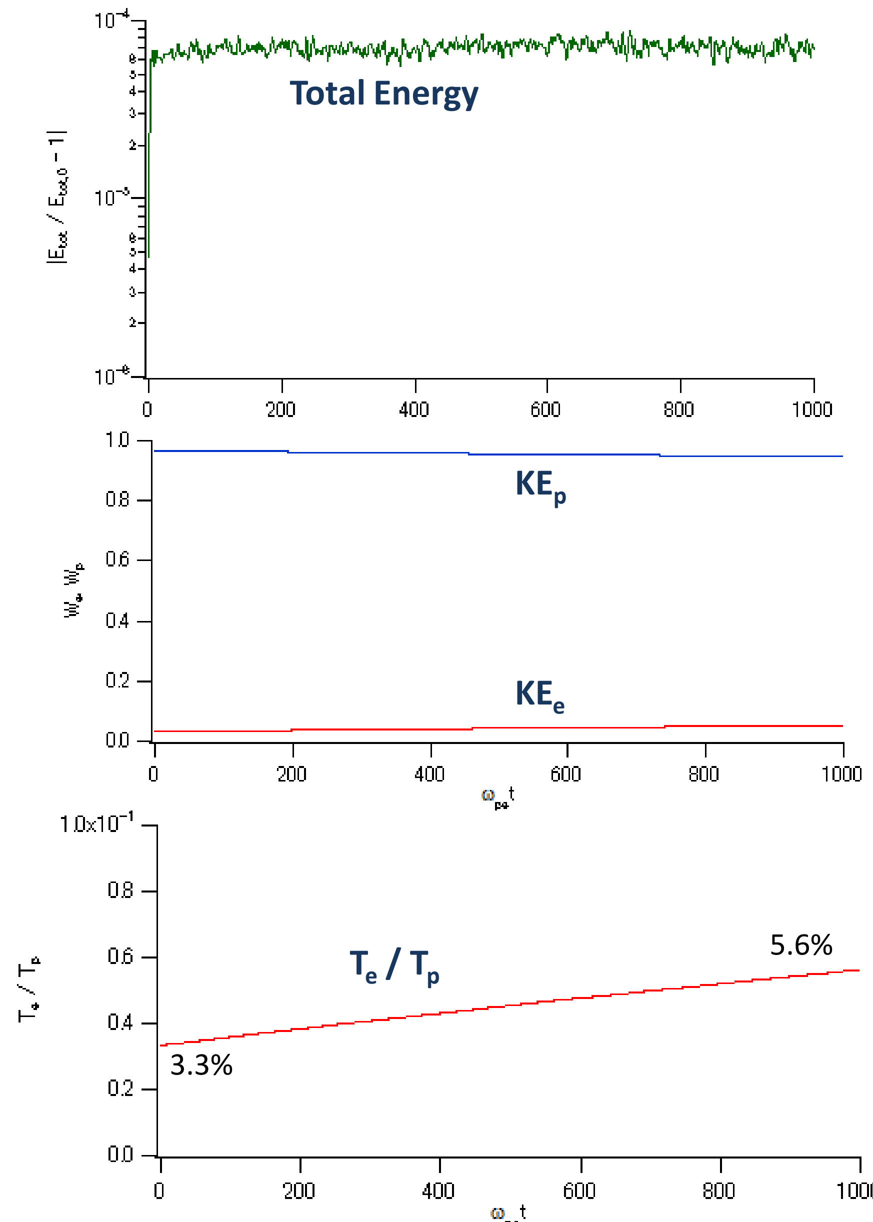
Numerical Collision occurs depending on N_c

Numerical Collision Effect

N_{PPC} = **40**, PQS

m_i/m_e=**30**, v_{the}=v_{thi}=**0.2c**





Simulation Parameters

In the simulations

$m_i / m_e = 20$

$V = 135{,}000$ km/s

However,
in real situations

$m_i / m_e = 1836$

$V \sim 3{,}000$ km/s

Scaling Law

What happens when m_i ,V, and B are changed **with fixed σ** ?

$$\sigma \approx \frac{B^2/8\pi}{\frac{1}{2} n_i m_i V^2} = \text{const.} \quad \Rightarrow \quad n_i \propto \frac{1}{m_i}\frac{B^2}{V^2}$$

Time scales

Ion gyro time

$$T_c \propto \frac{m_i}{B}$$

Same dependence

Growing timescale of the Weibel instability

$$T_W \propto \frac{1}{\omega_{\rm pi} V} \propto \frac{m_i^{1/2}}{n_i^{1/2} V} \propto \frac{m_i}{B}$$

Ratio of the two time scales does not change
→ Structure would not change significantly , too.

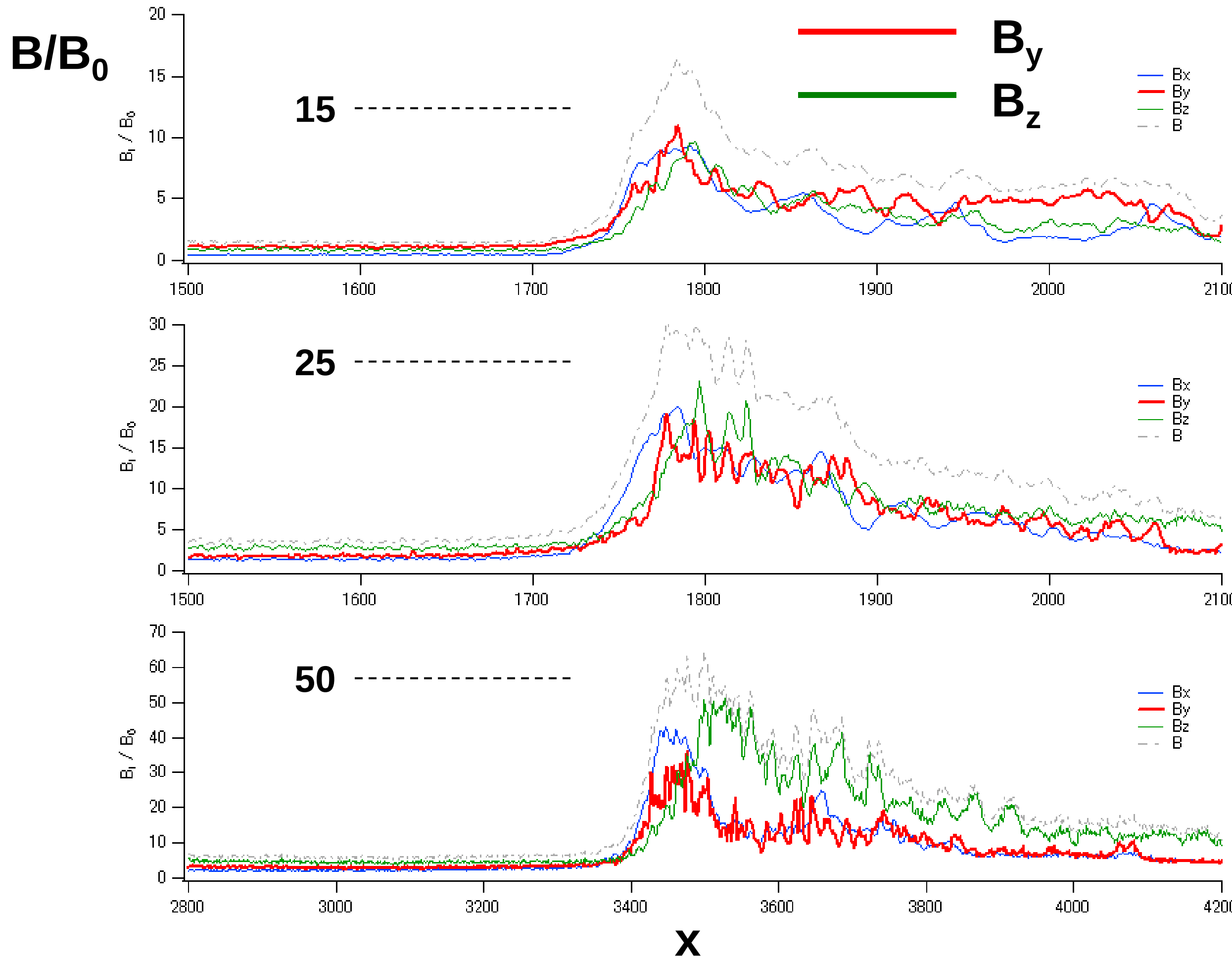
σ is the fundamental parameter to determine the shock structure?

Magnetic Field Profiles

$\sigma = 10^{-3}$

$\sigma = 10^{-4}$

$\sigma = 10^{-5}$

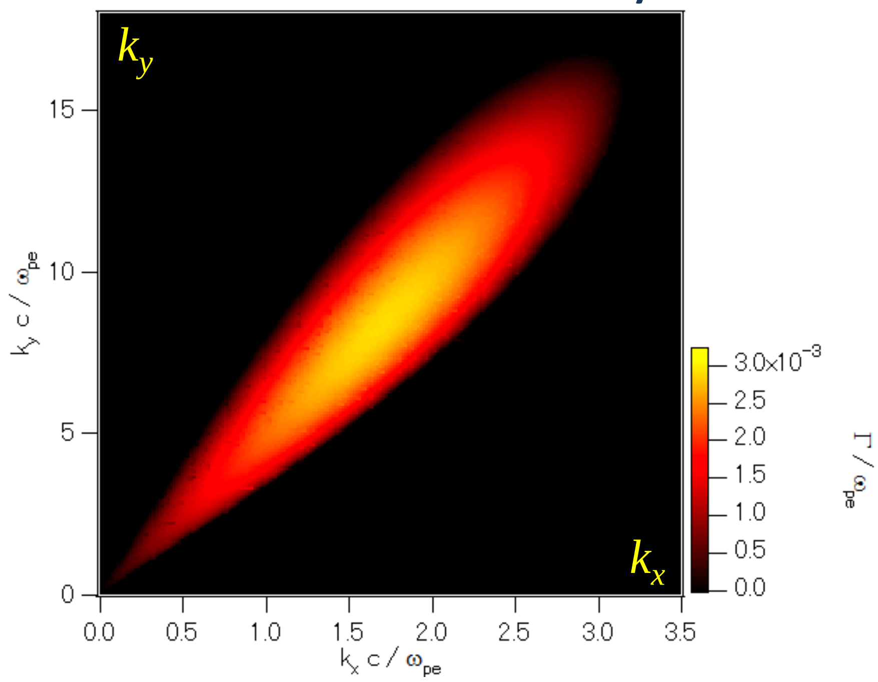


The **smaller σ**, the **larger** the contribution from **B_z**, which is generated by the current filaments

Linear Growth Rate of Instabilities

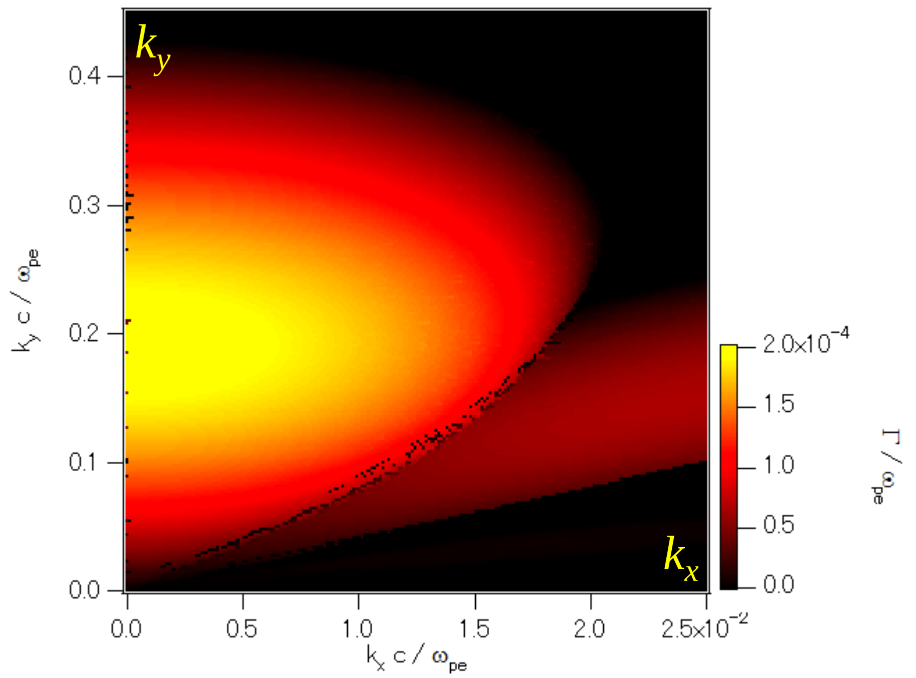
Counter-streaming plasma (T_e / T_i = 9, V = 3000km/s, m_i / m_e=1836)

Electristatic Ion-Ion Instability



Growth rate: **~ $3x10^{-3}$** ω_{pe}
Wave length: ~ 0.7 $c\omega_{pe}^{-1}$

Weibel Instability



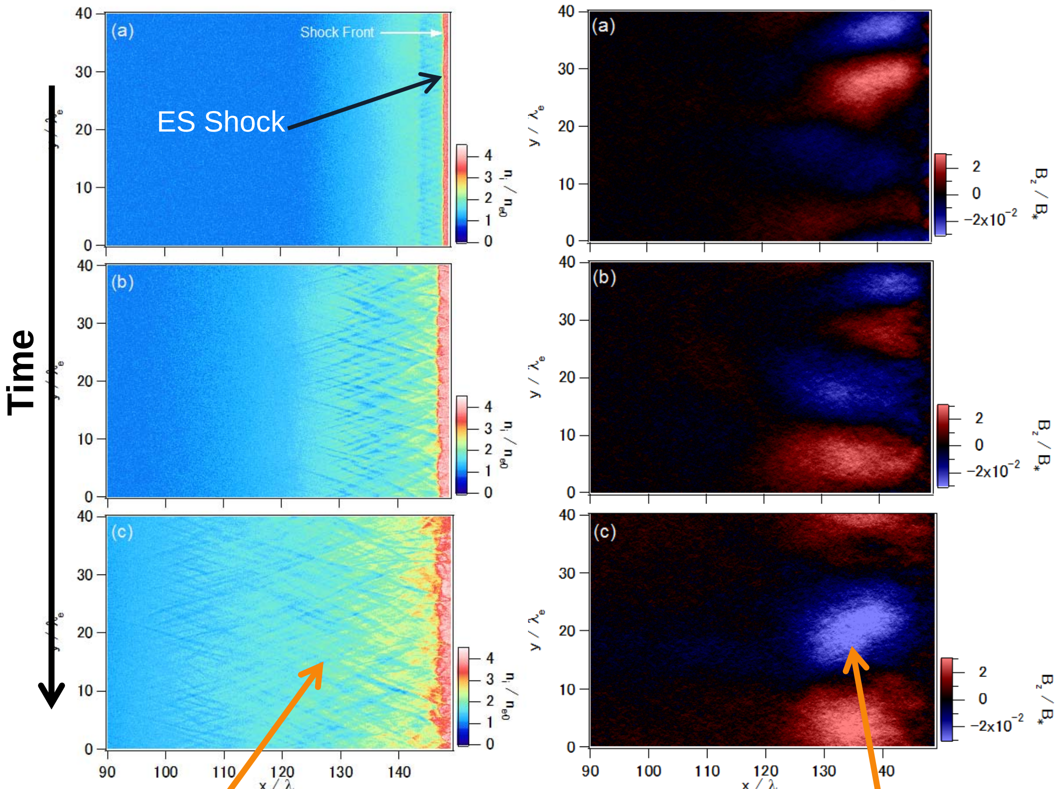
Growth rate: **~ $2x10^{-4}$** ω_{pe}
Wave length: ~ 33 $c\omega_{pe}^{-1}$

Electrostatic ion-ion instability grows much faster than **Weibel instability**

2D PIC Simulation of Electrostatic Shock

$T_e / T_i = 9$, $V = 3000km/s$, $m_i / m_e=1836$

Ion Number Density | **Magnetic Field B_z**



Filaments generated by the **electrostaic ion-ion instability**

Magnetic field generated by **Weibel instability**

Electrostatic ion-ion instability grows, then **Weibel instability** grows slowly

The **Weibel instability** can grow for **V~3000km/s** and **m_i/m_e~1800**

Plasma Beta

The **plasma beta** can be another **important parameter** for the **shock structure**

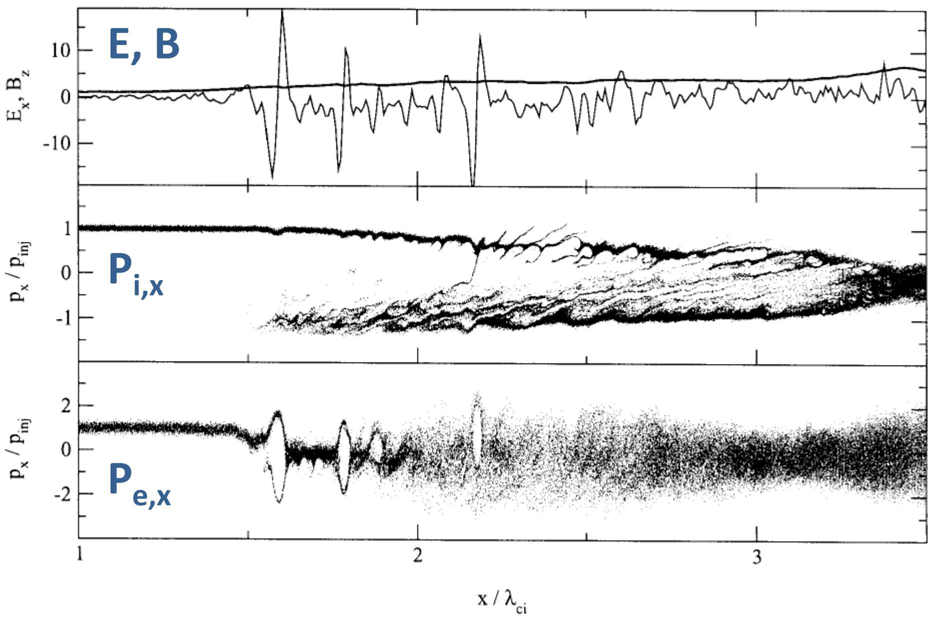
Plasma beta

$$\beta \equiv \frac{P_{particle}}{P_{\mathrm{magnetic}}} = \frac{\sum_i n_i k_B T_i}{B_0^2 / 8\pi}$$

Plasma Beta

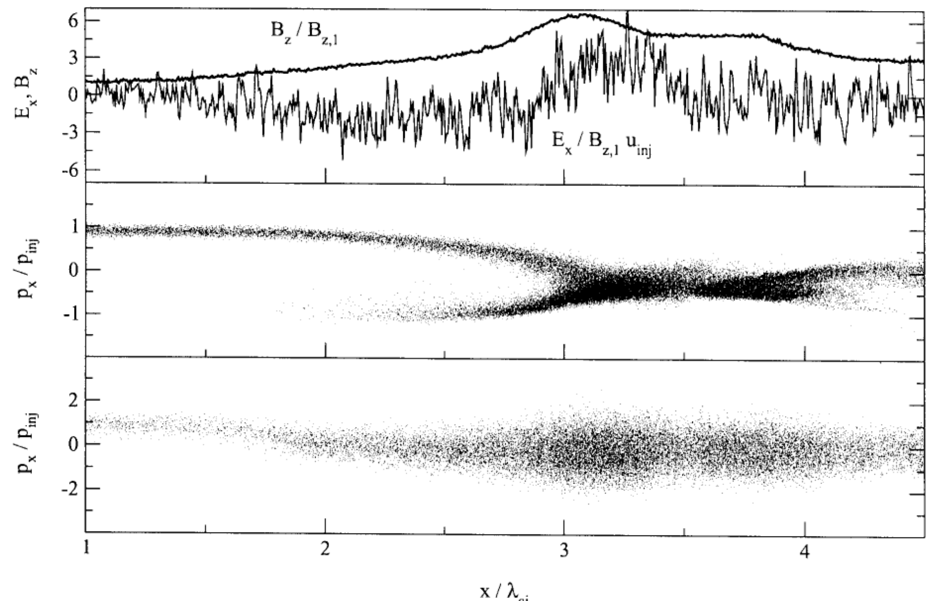
1D PIC Simulation (Schmitz et al., 2002)

β=0.15



- Shock reformation
- Buneman instability
- Electron holes
- Surfing acceleration

β=1



- **No** Shock reformation
- **No** Buneman instability
- **No** electron holes
- **No** Surfing acceleration

The shock structures are **completely different**