Mach Number Dependence of Electron Heating in $Q_{\perp}$ shocks

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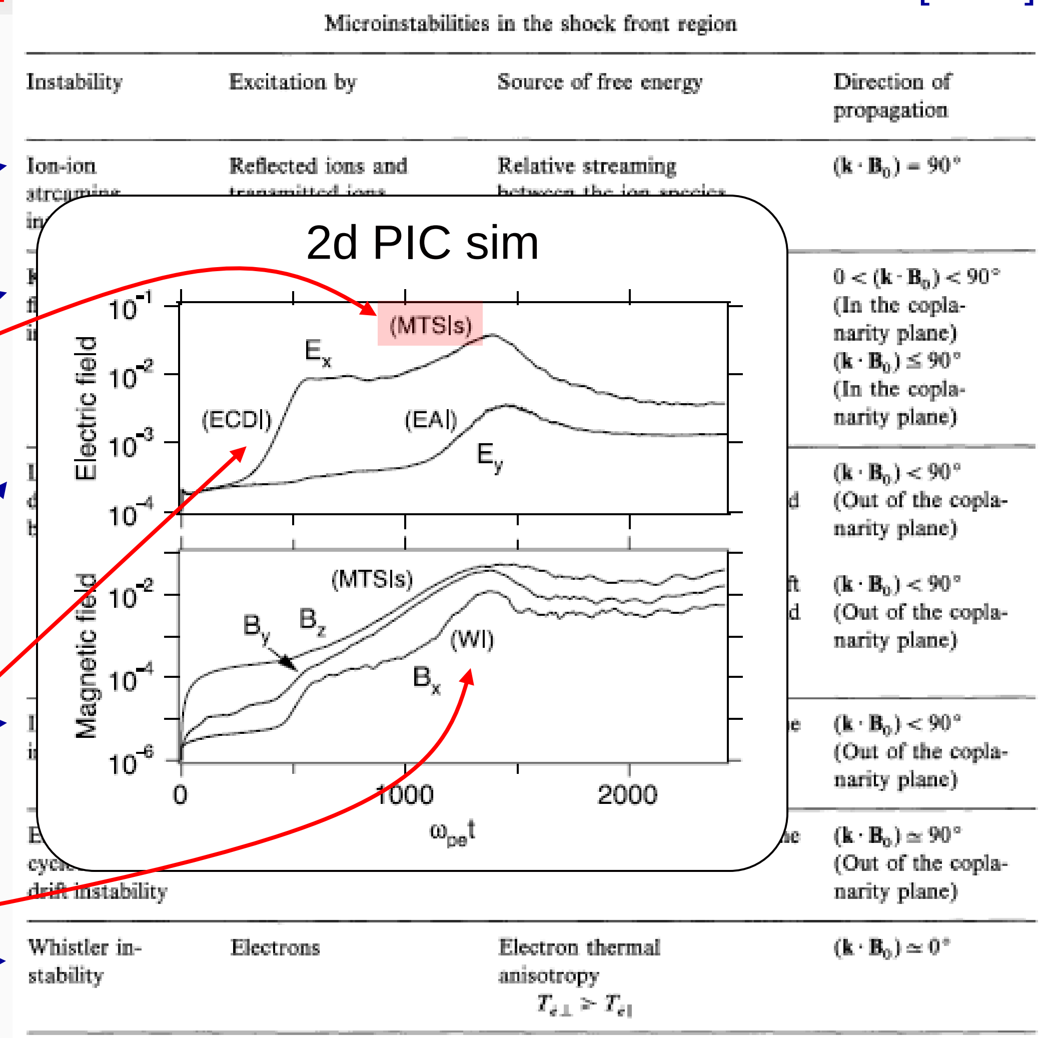
Electron heating in collisionless shocks

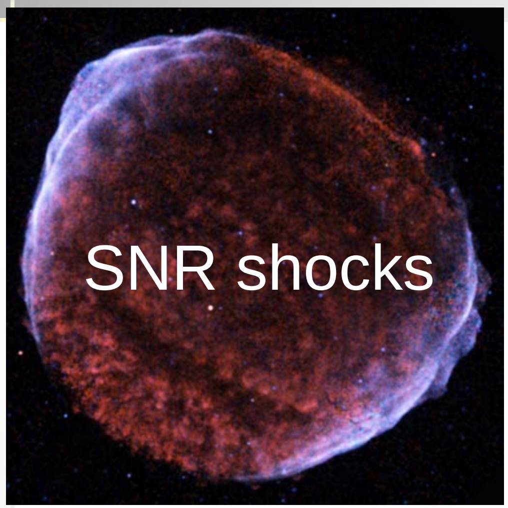
Review of instabilities in the earth's bow shock

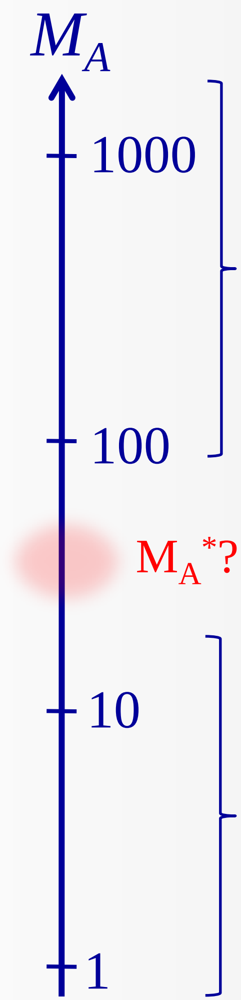
Wu et al. [1984]

- ion-ion streaming Inst.
- kinetic cross-field streaming inst. (modified two-stream inst.)
- lower-hybrid-drift inst.
- ion acoustic inst.
- electron cyclotron drift inst.
- whistler inst.



Unknown M_A dependence of e^- heating/acc.





From observations:

➔ quite efficient

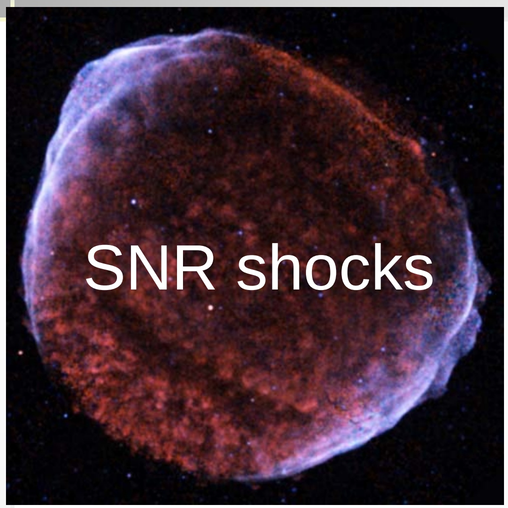
Different dissipation processes

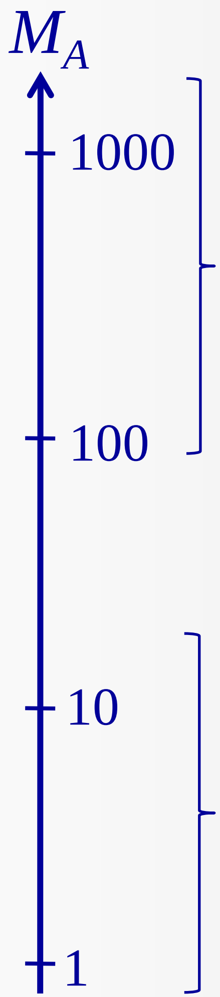
⬇

Differences in
basic structure,
wave activity,
plasma heating/acc.,

➔ inefficient

Unknown M_A dependence of e^- heating/acc.

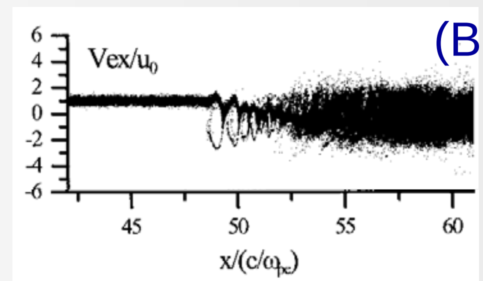




From simulations:

dominant inst. ➔ BI

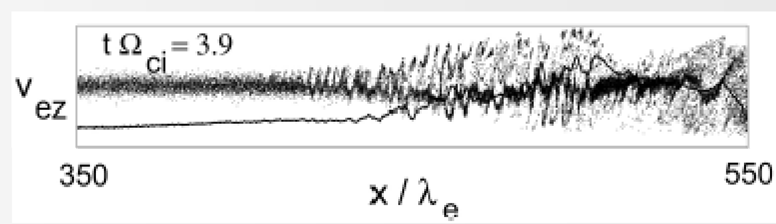
(Buneman Inst.)



Shimada & Hoshino [2000]

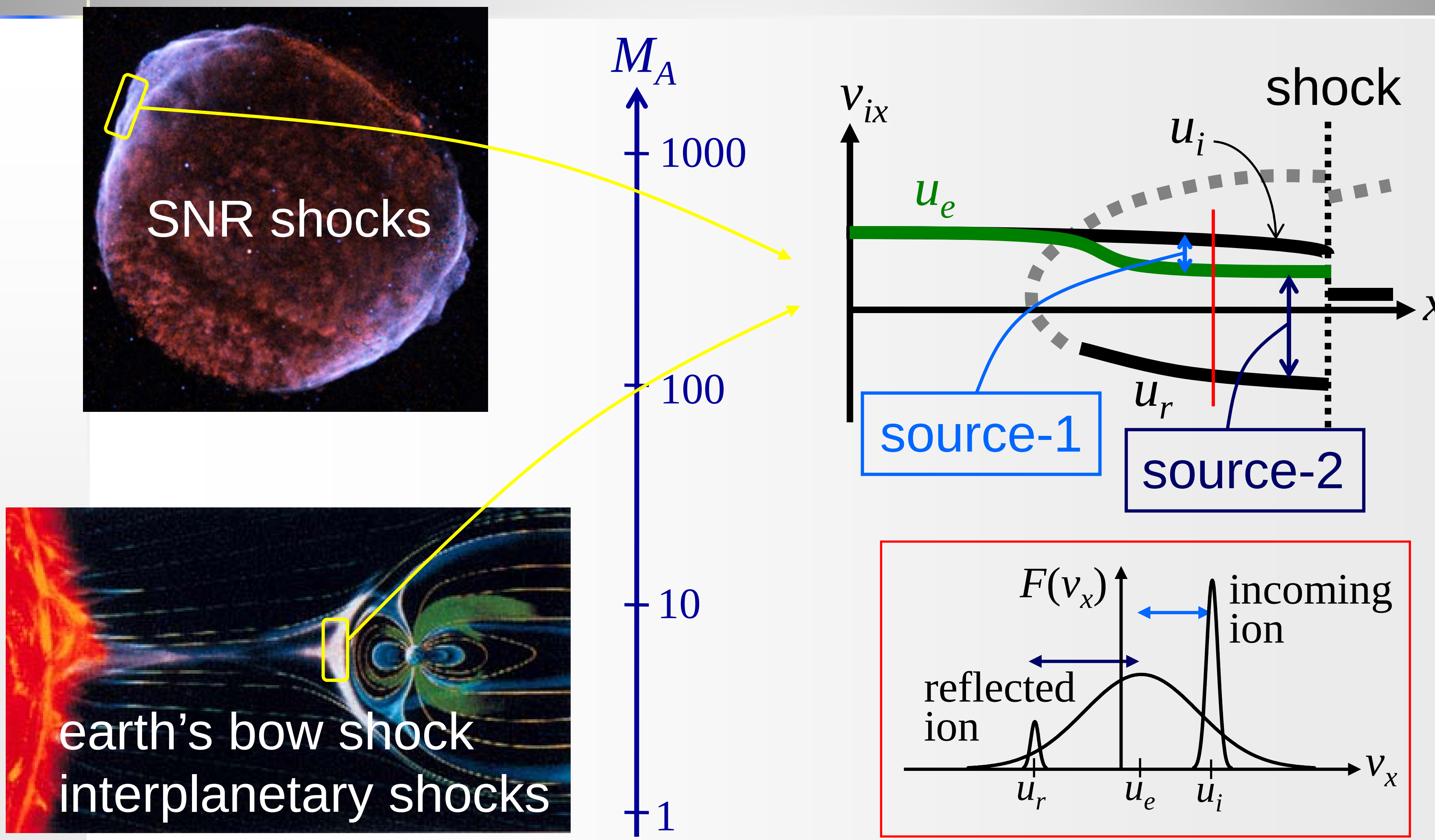
dominant inst. ➔ MTSI

(modified two-stream inst.)



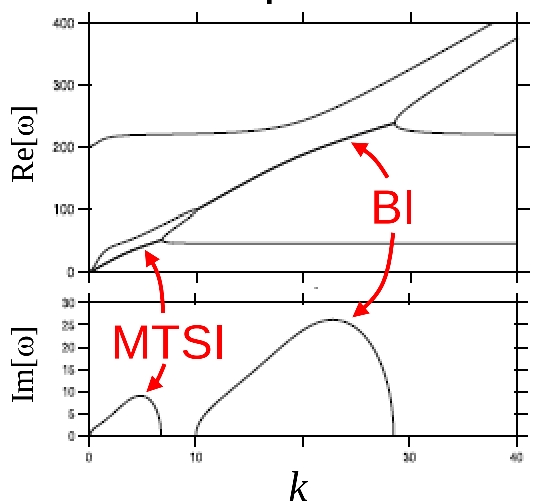
Scholer & M. [2004]

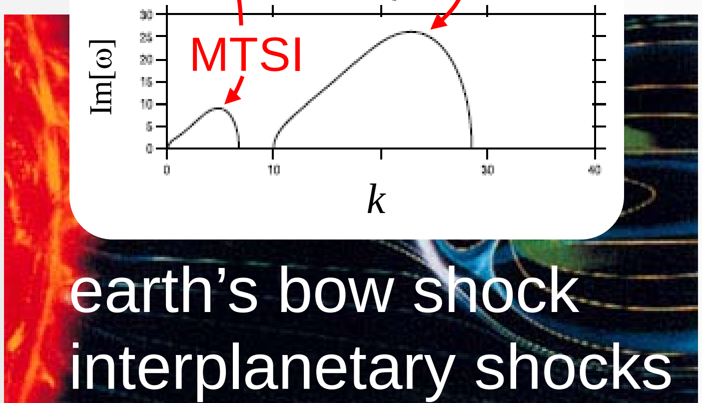
Two-stream instabilities in the foot of a $Q_\perp$ shock

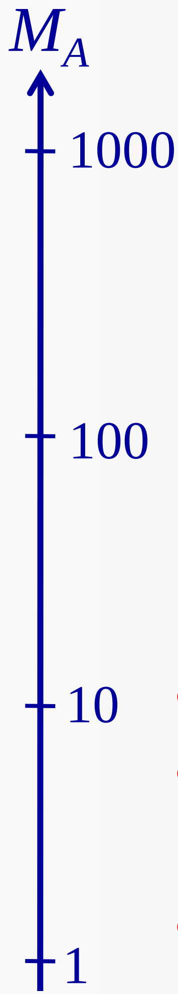


BI vs MTSI

Dispersion relation in a cold plasma







Electron heating/acc. via BI

Shimada & Hoshino [2000, 2004]
Dieckmann et al. [2000]
Hoshino & Shimada [2002]
McClements et al. [2005]
Ohira & Takahara [2008]
Umeda et al. [2008]
Amano & Hoshino [2009]
Dieckmann & Bret [2009]

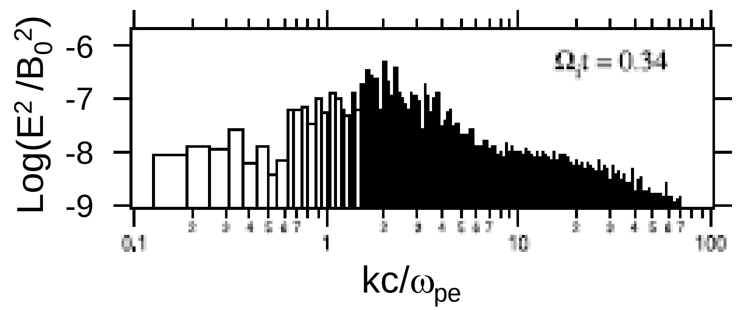
← Highly nonlinear process

- Electron heating via MTSI ?
- Transition of dominant heating process at a certain M_A ?
- Other kinetic inst.?

e^- heating through MTSI : model analysis

- broad wave spectrum
- small amplitudes

Wave energy spectrum of MTSI



M & Scholer [2003]

➔ quasi-linear approach

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$f = F + f_1$$
$$\mathbf{E} = \mathbf{E}_1$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$$

Taking ensemble average with remaining 2nd order terms of products of the 1st order quantities, ➔

Quasi-linear (QL) eqs.

QL evolution of a distribution function (eg. Stix [1992]):

$$\frac{\partial F(v_\perp, v_\parallel, t)}{\partial t} = -\frac{q}{m}\left\langle \int_0^{2\pi} \frac{d\phi}{2\pi} \nabla_{\mathbf{v}} \cdot \left[\left(\mathbf{E_1} + \frac{\mathbf{v} \times \mathbf{B_1}}{c} \right) f_1 \right] \right\rangle$$

$$\Rightarrow \lim_{V \to \infty} \pi \frac{q^2}{m^2} \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{k}}{V} L v_\perp \delta(\omega_r - k_\parallel v_\parallel - n\Omega) |\psi_{n,k}|^2 v_\perp L F(v_\perp, v_\parallel, t)$$

$$L = \left(1 - \frac{k_\parallel v_\parallel}{\omega_r}\right) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega_r} \frac{1}{v_\perp} \frac{\partial}{\partial v_\parallel}$$

$$|\psi_{n,k}|^2 = \frac{E_1^2}{1 + A_\perp^2 + A_\parallel^2} \left[\frac{(1 + A_\perp)^2}{4} J_{n-1}^2(z) + \frac{(1 - A_\perp)^2}{4} J_{n+a}^2(z) + \frac{v_\parallel^2}{v_\perp^2} A_\parallel^2 J_n^2(z) \right]$$

$$\left(z = k_\perp v_\perp / \Omega,\ A_\parallel \equiv E_{1z} / E_{1x},\ A_\perp \equiv iE_{1y} / E_{1x} \right)$$

Extended QL approach

Time evolution of kinetic energies are obtained by taking the second order (v^2) moments.

$$\frac{\partial K_{\parallel}^{e}}{\partial t} = \frac{m_e}{2}\int d^3\mathbf{v}\, v_{\parallel}^2 \frac{\partial F_e}{\partial t}$$

$$\frac{\partial K_{\perp}^{e}}{\partial t} = \frac{m_e}{2}\int d^3\mathbf{v}\, v_{\perp}^2 \frac{\partial F_e}{\partial t}$$

$$\frac{\partial K_{x}^{i}}{\partial t} = \frac{m_i}{2}\int d^3\mathbf{v}\left(v_x - u_0\right)^2 \frac{\partial F_i}{\partial t}$$

F_j : (shifted-) Maxwellian with the thermal energy K^j.

γ_k : linear growth rate calculated in each time step by using Emdisp (dispersion solver).

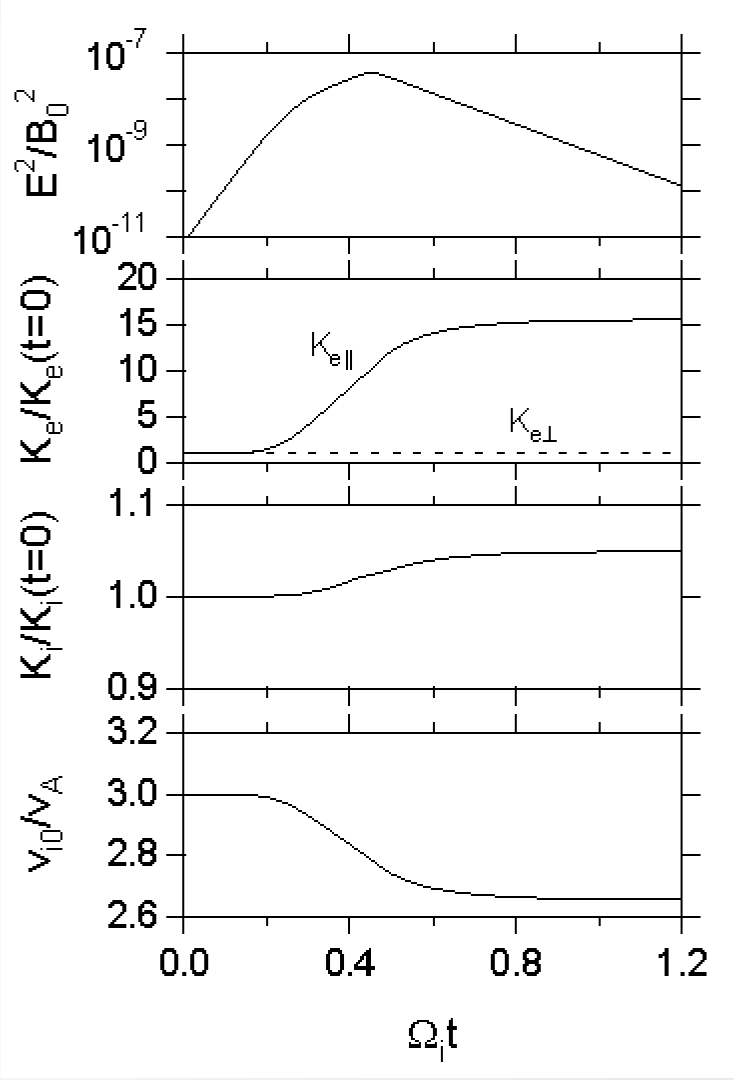
ref. ion

$$\frac{\partial E_{1,k}^2}{\partial t} = 2\gamma_k E_{1,k}^2$$ --- Evolution of field energy

$$\left(\frac{n_i m_i}{2} u_0^2 + K_x^i\right) + \left(K_{\parallel}^e + K_{\perp}^e\right) + \frac{E_1^2}{8\pi} = const.$$ --- Energy conservation

Ext. QL (typical example)

Time history



Initial conditions:

M_{A_foot} = 6

m_i/m_e = 1836

$(\omega_{pe}/\Omega_e)^2 = 10^4$

β = 0.4 ($\beta_e = \beta_i$)

θ_{Bk} = 85.5

ion reflection ratio:

$\alpha = n_r / n_i = 1/3$

(kc/ω_{pe} = 1 (fixed))

Saturation occurs when the field is damped out.

$E^2 \sim 0$

Ext. QL (typical example)

Kinetic effects

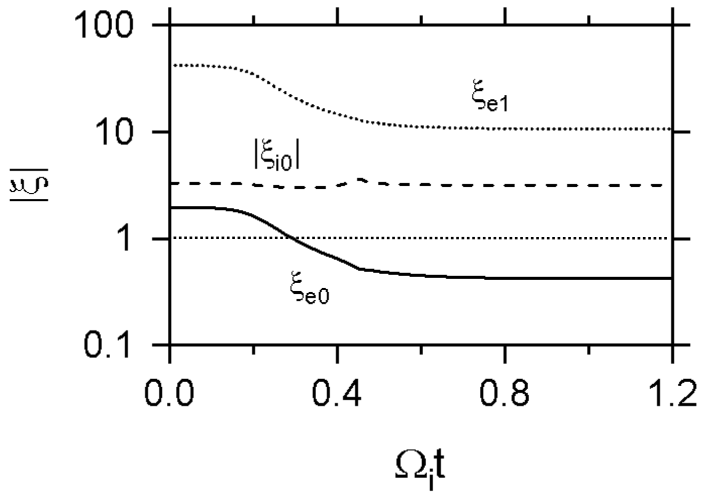
$$\gamma \propto -\xi_{jn} \exp\left(-\xi_{jn}^2\right)$$

$$\xi_{en} \equiv \frac{\omega - n\Omega_e}{k_{\|} v_{th,e}}, \quad \xi_{i0} \equiv \frac{\omega - k u_0}{k v_{th,i}}$$

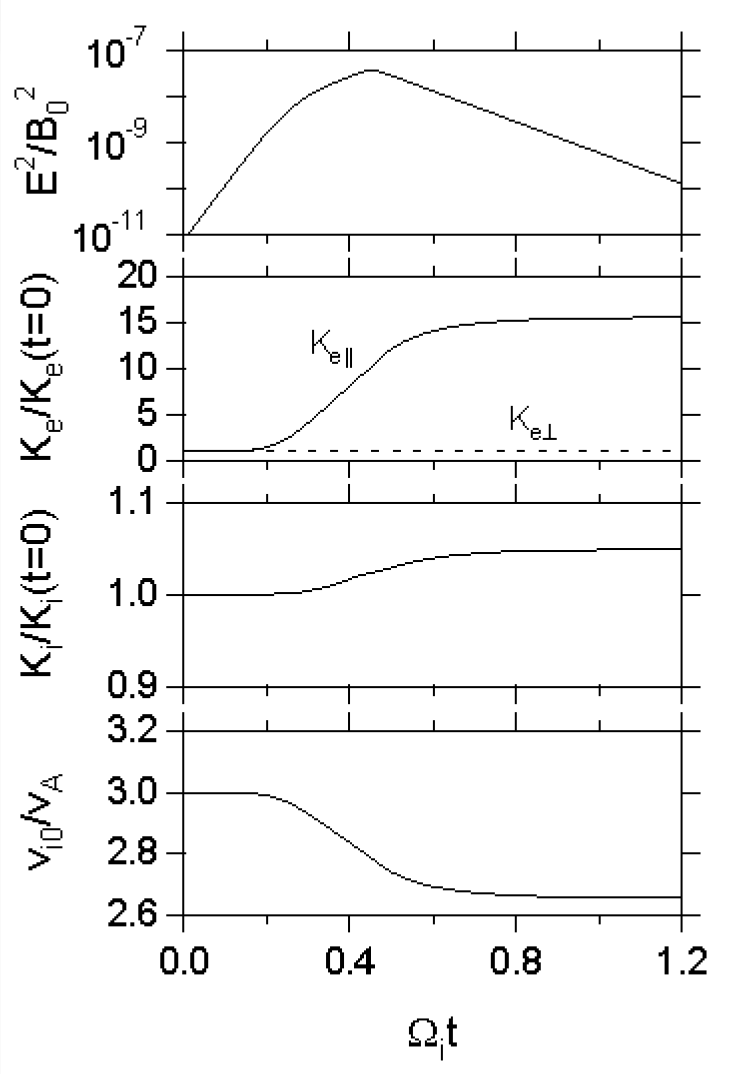
$|\xi_{e0}|$~1 : elec. Landau damp.

$|\xi_{e1}|$~1 : elec. cyclotron damp.

$|\xi_{i0}|$~1 : ion Landau res.



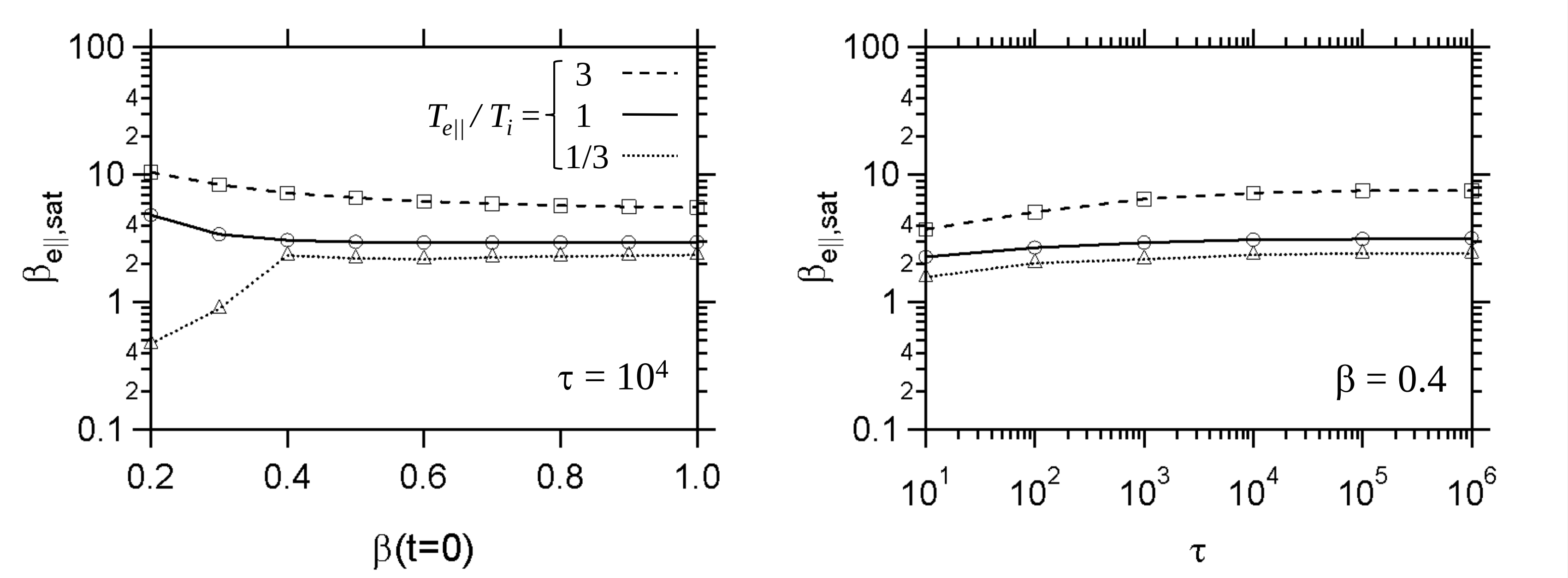
Time history



Ext. QL (β & τ dependence)

($M_A = 6$, $\theta_{Bk} = 85.5$, $0 < kc/\omega_{pe} < 3$)

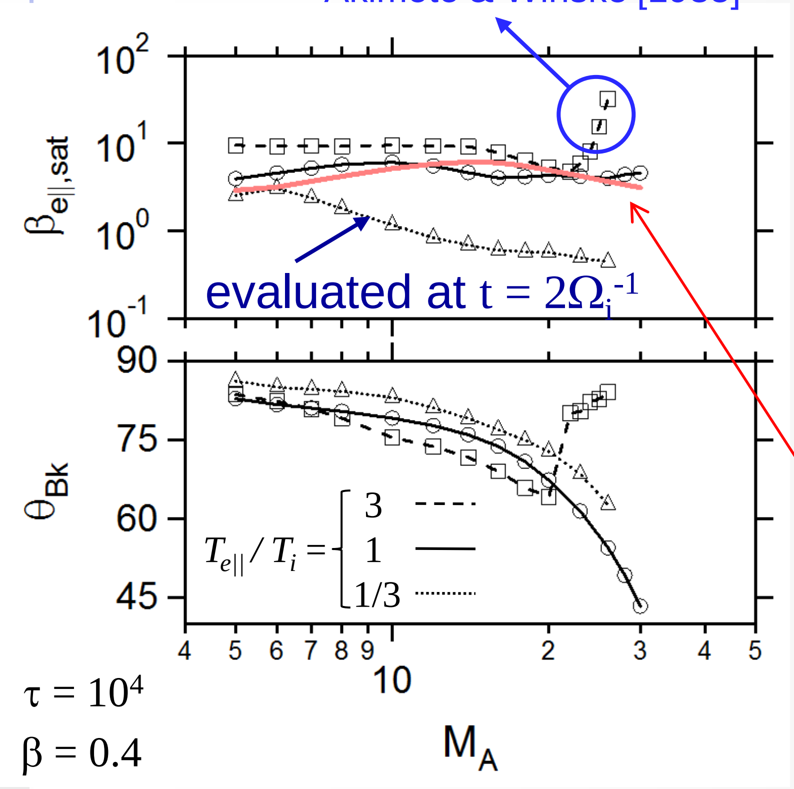
weak dependence on β for $\beta > 0.4$ | weak dependence on τ



$\tau = (\omega_{pe} / \Omega_e)^2$

Ext. QL (M_A dependence)

oblique ion-acoustic-like inst.
Akimoto & Winske [1985]



- $\beta_{e\|}(t=t_{sat})$
  --- weak dependence on M_A

- Saturation occurs when the field is damped out (ξ_{e0}~1).

$$\Rightarrow v_{te\|} \approx \omega / k_{\|}$$

$$\approx u_0 / \cos\theta_{Bk}$$

⬇

$$\beta_{e\|,sat} \approx \frac{2M_A^2}{(m_i/m_e)\cos^2\theta_{Bk}} \left(\frac{2\alpha}{1+\alpha} \right)^2$$

$(\alpha = 1/3 : ion\ reflection\ ratio)$

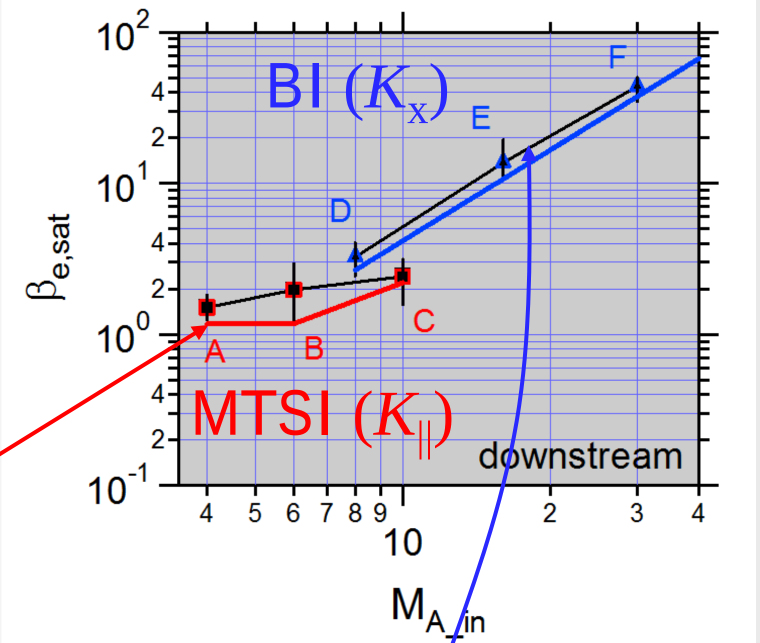
Ext. QL vs 1D PIC sim.

Shock parameters for PIC simulations

Run	M_{Ain}	ω_{pe}/Ω_e	m_i/m_e	β_e	β_i	Θ_{Bn}(deg.)	
A	4	4	625	0.3	0.1	84	MTSI
B	6	4	625	0.3	0.1	81	MTSI
C	10	4	625	0.3	0.1	79	MTSI
D	8	10	64	0.3	0.1	90	BI
E	16	10	64	0.3	0.1	90	BI
F	30	10	64	0.3	0.1	90	BI

$$\beta_{e\|,sat} \approx \frac{2M_A^2}{(m_i/m_e)\cos^2\theta_{Bk}}\left(\frac{2\alpha}{1+\alpha}\right)^2$$

PIC

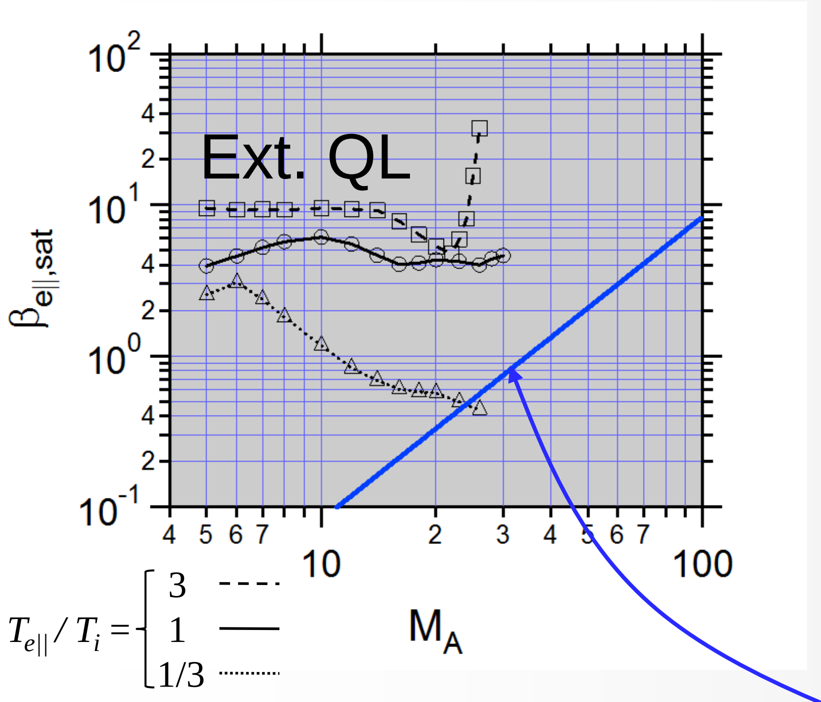


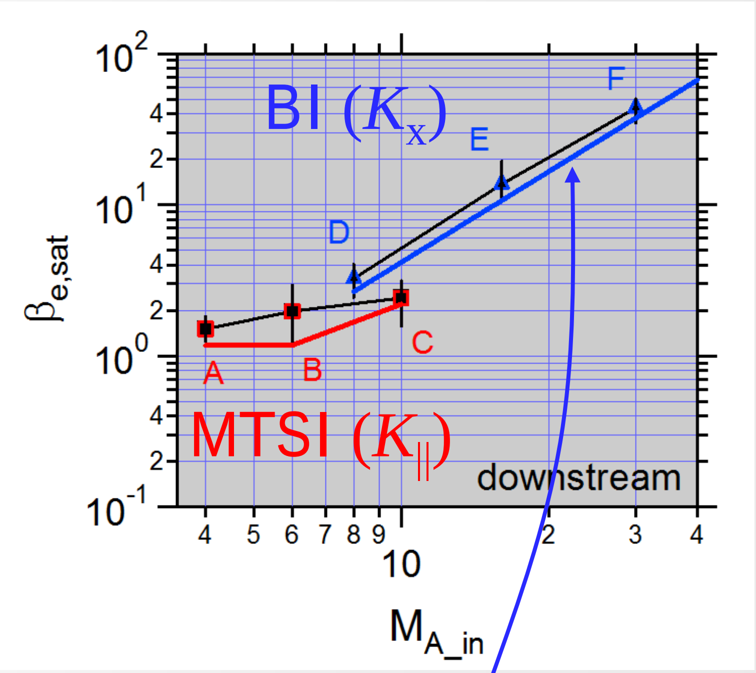
Trapping theory:

$$\beta_{e,sat} \sim M_A^2\left(\frac{m_e}{m_i}\right)^{7/6}$$

MTSI vs BI

PIC



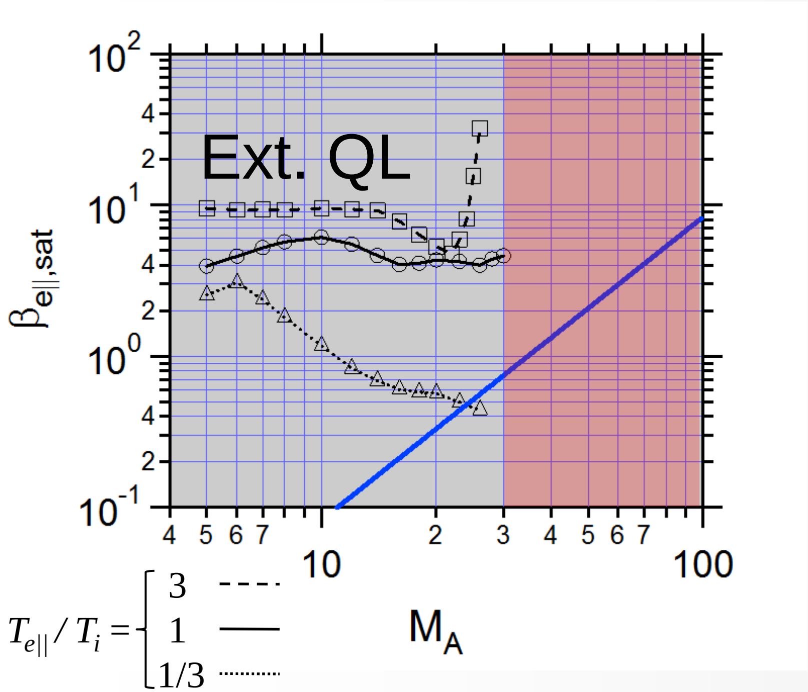


- Dominant heating process may switch at $M_A > 20$.

Trapping theory:

$$\beta_{e,sat} \sim M_A^{2}\left(\frac{m_e}{m_i}\right)^{7/6}$$

Other possible inst.



MTSI is linearly stable for $M_A > 30\text{-}40$.
--- oblique ion acoustic inst.
--- electron cyclotron drift inst.
--- lower hybrid drift inst.
--- whistler inst.
--- ion-ion two stream inst.
⋮

- Dominant heating process may switch at $M_A > 20$.

Summary

Electron heating through MTSI & BI in high Mach number quasi-perpendicular shocks was discussed.

- **Ext. QL analysis for MTSI**
  --- Heating mechanism is Landau damping.
  --- Saturation temperature does not depend much on M_{A_foot}.
  --- Consistent with PIC simulations

- **Simple trapping theory for BI**
  --- Saturation temperature increases with M_A^2.
  --- Lower limit of PIC simulations

➔ M_A dependence of saturation electron temperature is qualitatively different between the MTSI dominant & the BI dominant cases

➔ Dominant heating process may switch at M_A ~ a few tens.