Saturation of CR-streaming instabilities at supernova shocks

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Overview

- CR-streaming instabilities: kinetic theory
- Saturation mechanisms
  –feedback effects constrain magnetic field amplification
- Implications for SNR forward shocks

Generation of magnetic fields

- Strong magnetic fields at SNR shocks
- Electromagnetic instabilities due to CR streaming
- Resonant

  -Affvén wave instability (Skilling 1970, ...)
- Nonresonant instabilities

  -CR-streaming instability $\omega \rightarrow 0$ (Bell 2004)

  -Weibel instability $\omega \rightarrow 0$ (Weibel 1959)

  -Whistler instability (e.g. Gary & Cairns 1999)

  -Filamentation instability (FI) $\omega \rightarrow 0$ (Bell 2005)
- These instabilities can generate magnetic turbulence

CR-streaming instability

- Kinetic model: background plasma+CR current $J_{CR} = qn_{CR}v_{CR}$
- Neutralisation conditions

$$e(n_e - n_p) = qn_{\mathrm{CR}}, \quad e(n_e v_e - n_p v_p) = qn_{\mathrm{CR}} v_{\mathrm{CR}}$$

$\rightarrow$ compensating (or return) current $-J_{CR}$

- Purely growing mode due to the compensating current with condition

$$J_{CR} > \frac{k_{\|} B_0}{\mu_0}$$

- Growth rate at $k_{\|} r_g \gg 1$

$$\Gamma \approx k_{\|} v_A \left(\frac{2k_m}{k_{\|}} - 1 \right)^{1/2}, \quad k_{\|} < 2k_m \equiv \frac{\mu_0 J_{\mathrm{CR}}}{B_0}$$

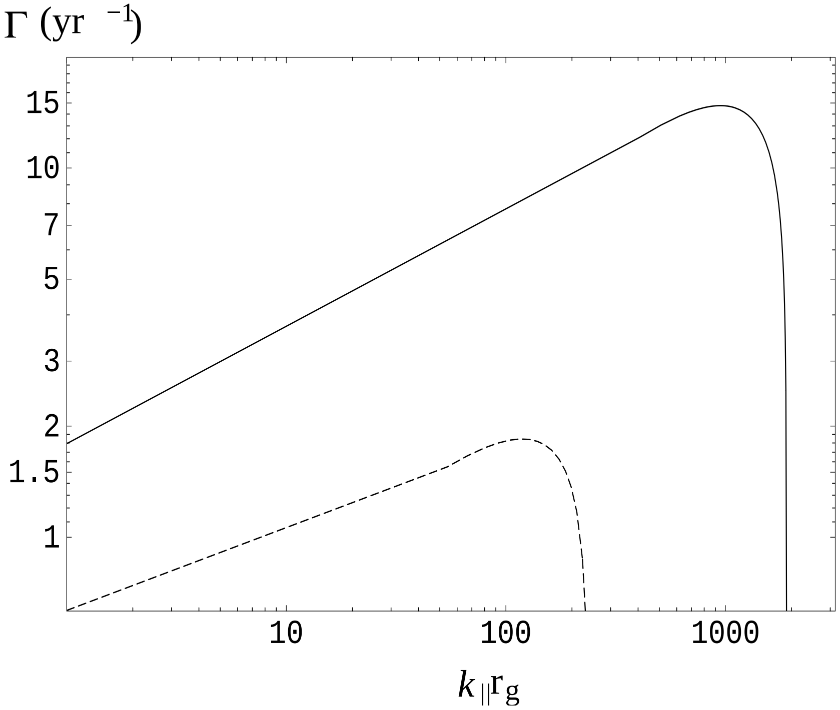
(Bell 2004; Reville et al. 2006; Amato & Blasi 2009; LM09)

- Applied to SNR forward shocks

$$v_{\mathrm{CR}} \sim v_s$$

$$\eta_s = \frac{U_{CR}}{\rho_0 v_s^2/2} = 0.1$$

$(B_0 = 10^{-10}\,\mathrm{T},\ \gamma_{\mathrm{CR}} = 10^2$
$v_s = 3 \times 10^6,\ 5 \times 10^6\,\mathrm{m\,s^{-1}})$



Saturation

- Numerical simulations clearly show saturation but predict different B

  -MHD simulation (Bell 2004, 2005)

  -PIC simulation $B/B_0 \sim 1-30$
  (Niemiec et al 2008; Riquelme & Spitkovsky 2009; Stroman et al. 2009)

- Limitation of numerical kinetic models by $n_{\mathrm{CR}}/n_p \ll 1$

- Determining of saturation important for the relevance of the instability

- Saturation mechanisms

  -Stabilizing due to feedback effects (LM09)

  -Magnetisation of CR beam (Medvedev & Loeb 1999)

  -Magnetic trapping (Davidson 1972; Achterberg et al 2007)

Saturation due to diffusion

- Feedback on CR current treated as quasilinear diffusion (QLD)

  -General formalism (e.g. Shapiro & Shevchenko 1964)

  -App to Weibel instability (Davidson et al 1972)
- Nonresonant/resonant diffusion reduces anisotropy–changes $v_{\rm CR}(t)$
- Nonresonant diffusion, similar to the firehose instability
- Resonant diffusion, pitch angle scattering

  The case $kr_g \sim 1$ well discussed (e.g. Melrose 1980)
- For $k_\parallel r_{g0} \gg 1$, only small fraction of CRs with $\alpha \sim \pi/2$ strongly affected (LM09)
- Pitch angle scattering dominant in reducing $v_{\rm CR}(t)$

A streaming model

- CR distribution

$$f(p,\alpha) = n_{CR}\left(1 + 3\frac{v_{CR}}{v}\cos\alpha\right)\frac{g(p)}{4\pi p^2}$$

$$g(p) = \begin{cases} \dfrac{b-1}{p_1}\left[1 - \left(\dfrac{p_1}{p_2}\right)^{b-1}\right]^{-1}\left(\dfrac{p}{p_1}\right)^{-b}, & p_1 \le p \le p_2 \\ 0 & \text{otherwise} \end{cases}$$

- Streaming velocity $\langle v_{\|}\rangle = v_{CR}$

- Evolution of CR treaming motion

$$\frac{dv_{CR}}{dt} = \frac{\int p_\perp dp_\perp dp_\parallel v_\parallel (dF/dt)}{\int p_\perp dp_\perp dp_\parallel F}$$

- Scattering time

$$t_s \sim \frac{\gamma_0}{\Omega}\left(\frac{B_0}{B}\right)^2 k_0 r_{g0}$$

compared to growth time $1/\Gamma$

- Saturated magnetic field

$$\frac{B}{B_0} \sim \xi (k_0 r_{g0})^{3/4} (k_m r_{g0})^{1/4} \beta_A^{1/2} \leq 2^{3/4} \xi k_m r_{g0} \beta_A^{1/2}$$

- For SNR shocks, $r_{g0} \leq R_s$, $k_0 < 2k_m$, $v_A = 10\,\mathrm{km\,s^{-1}}$

$$\frac{B}{B_0} \approx 16, \quad R_s = 3\,\mathrm{pc}, \quad k_m \approx 300\,\mathrm{pc}^{-1}, \quad \eta_s = 0.1$$

- Amplification modest, similar to recent PIC simulations (e.g. Riquelme & Spitkovsky 2009; Stroman et al. 2009)
- Prediction of scaling $B \propto v_s^{\alpha}$, α depends on how k_0 evolves

Magnetisation of CR beam

- Deflection due to self-generated B
  (Medvedev & Loeb 1999; Riquelme & Spitkovsky 2009)
- Larmor radius $\sim 1/k_m$
- Saturation

$$\frac{B}{B_0} \approx k_m r_{g0}$$

- Higher than the limit by QLD mechanism

$$\frac{B_m}{B_d} \sim \left(\frac{k_0}{k_m}\right)^{1/4} \left(\frac{c}{v_A}\right)^{1/2} \gg 1$$

Summary

- The nonresonant instability can grow efficiently to nonlinear regime, due to the return current
- Saturation due to diffusion occurs well before magnetisation of CRs
- Application to SNR forward shocks, predicting modest amplification, consistent with the result from recent numerical kinetic models (Riquelme & Spitkovsky 2009; Stroman et al. 2009)
- Analytical result not limited by $n_{\mathrm{CR}}/n_p \ll 1$
- Limitation of QLD, but as the feedback mechanism generic it may still provide reasonable estimate