Properties of small-scale MHD turbulence
(EMHD Turbulence)

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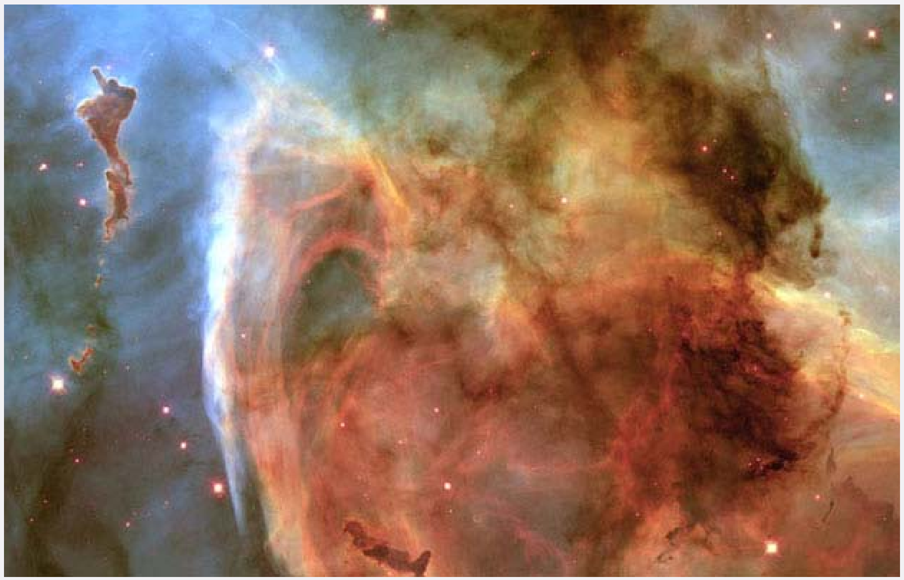
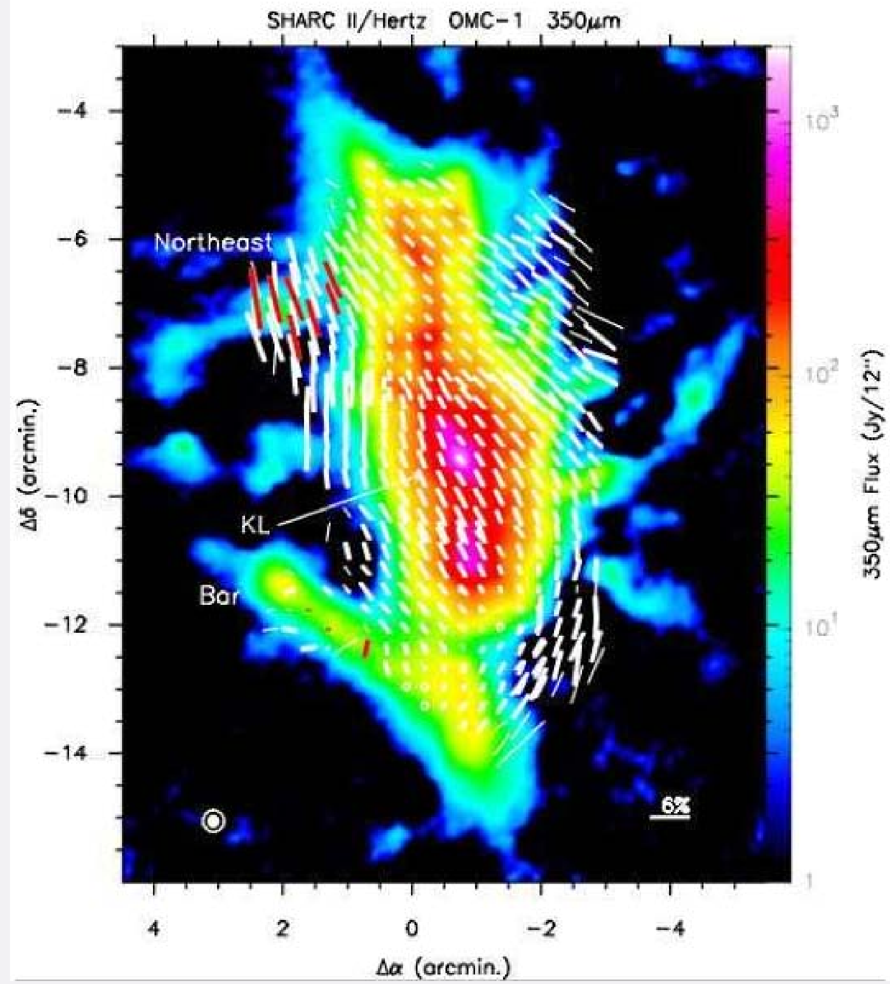
Cho & Lazarian, ApJ, 615, L41, (2004)
Cho & Lazarian, ApJ, 701, 236 (2009)

Astrophysical fluids

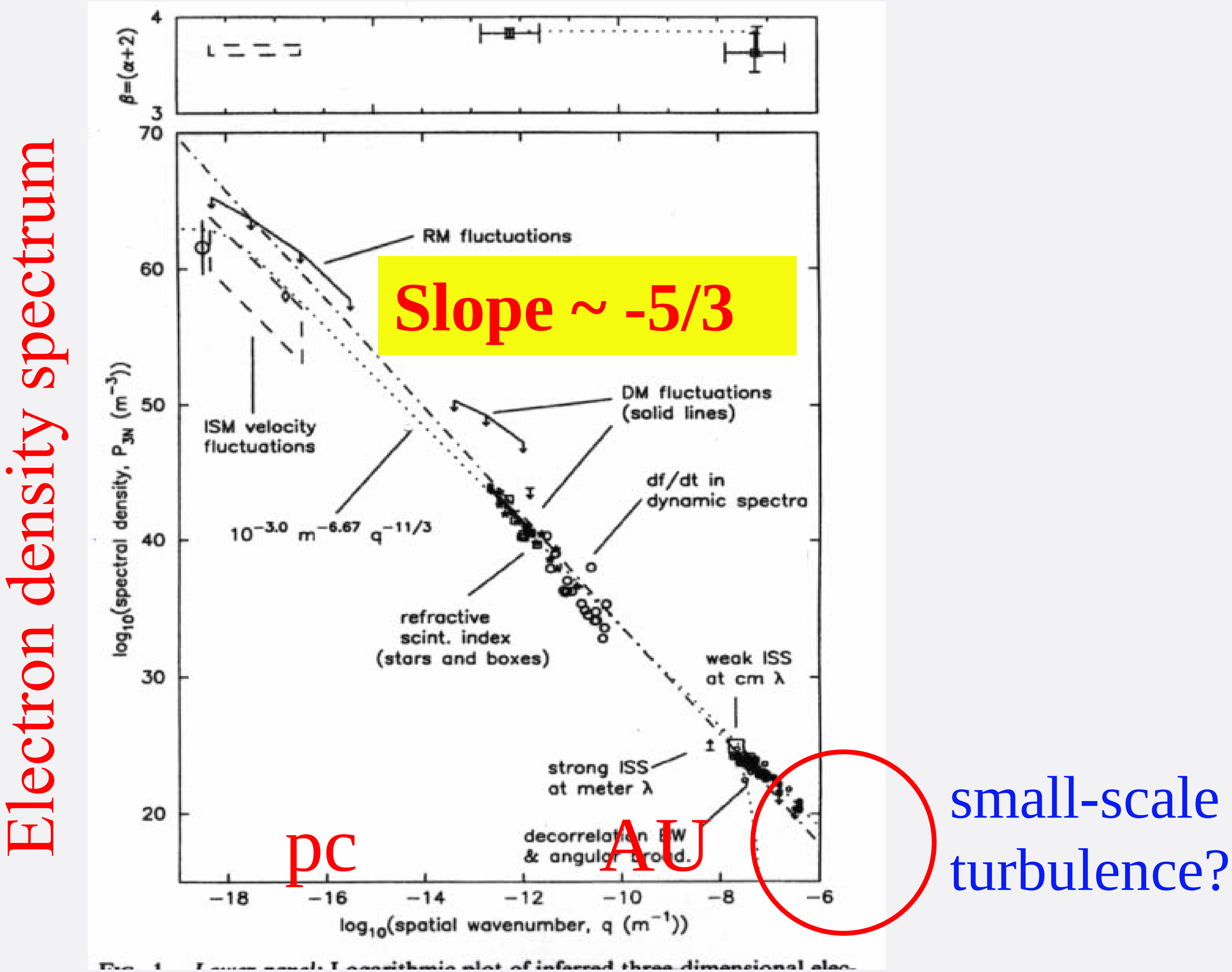
Orion nebula





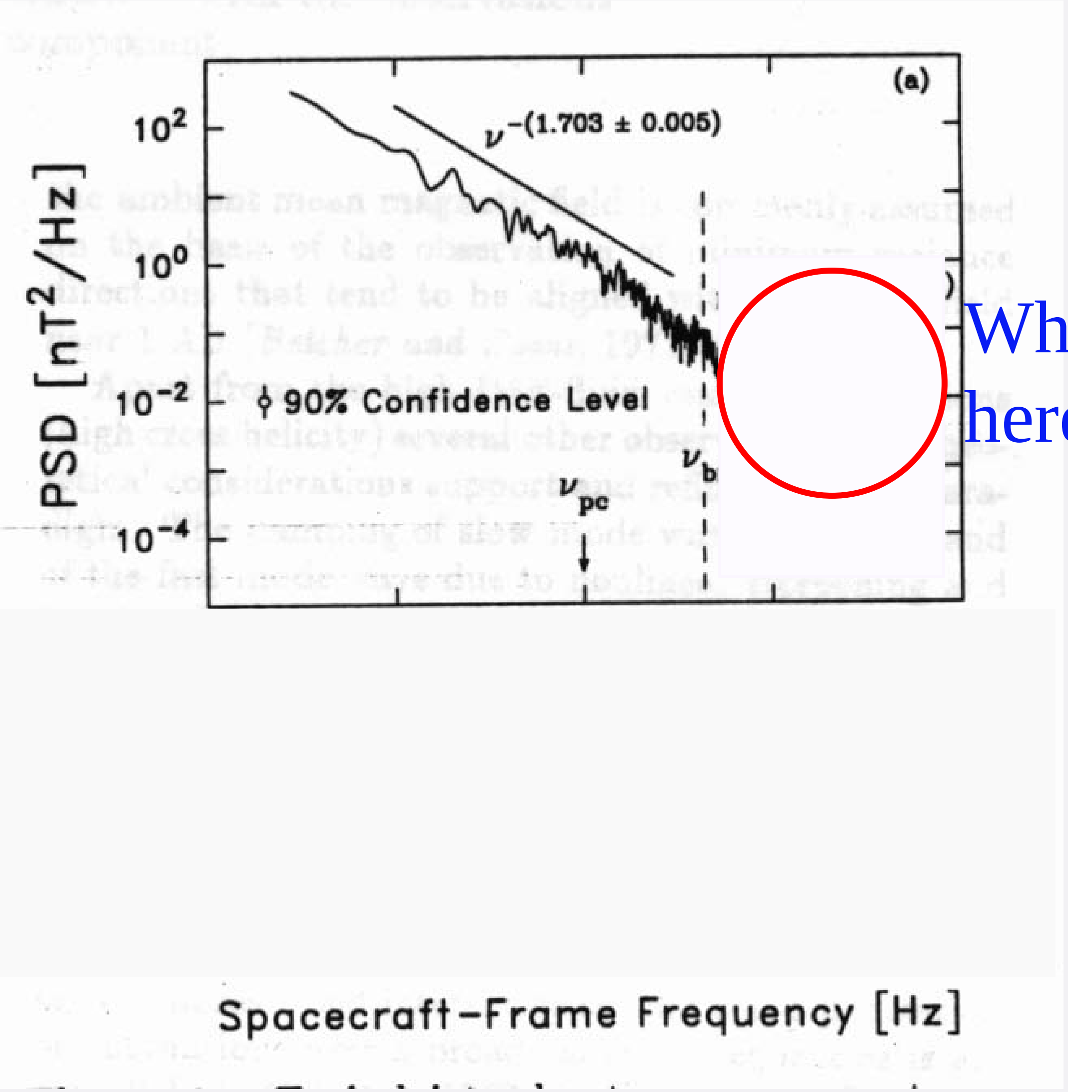
turbulence + B

ISM : Armstrong & Spangler (1995)

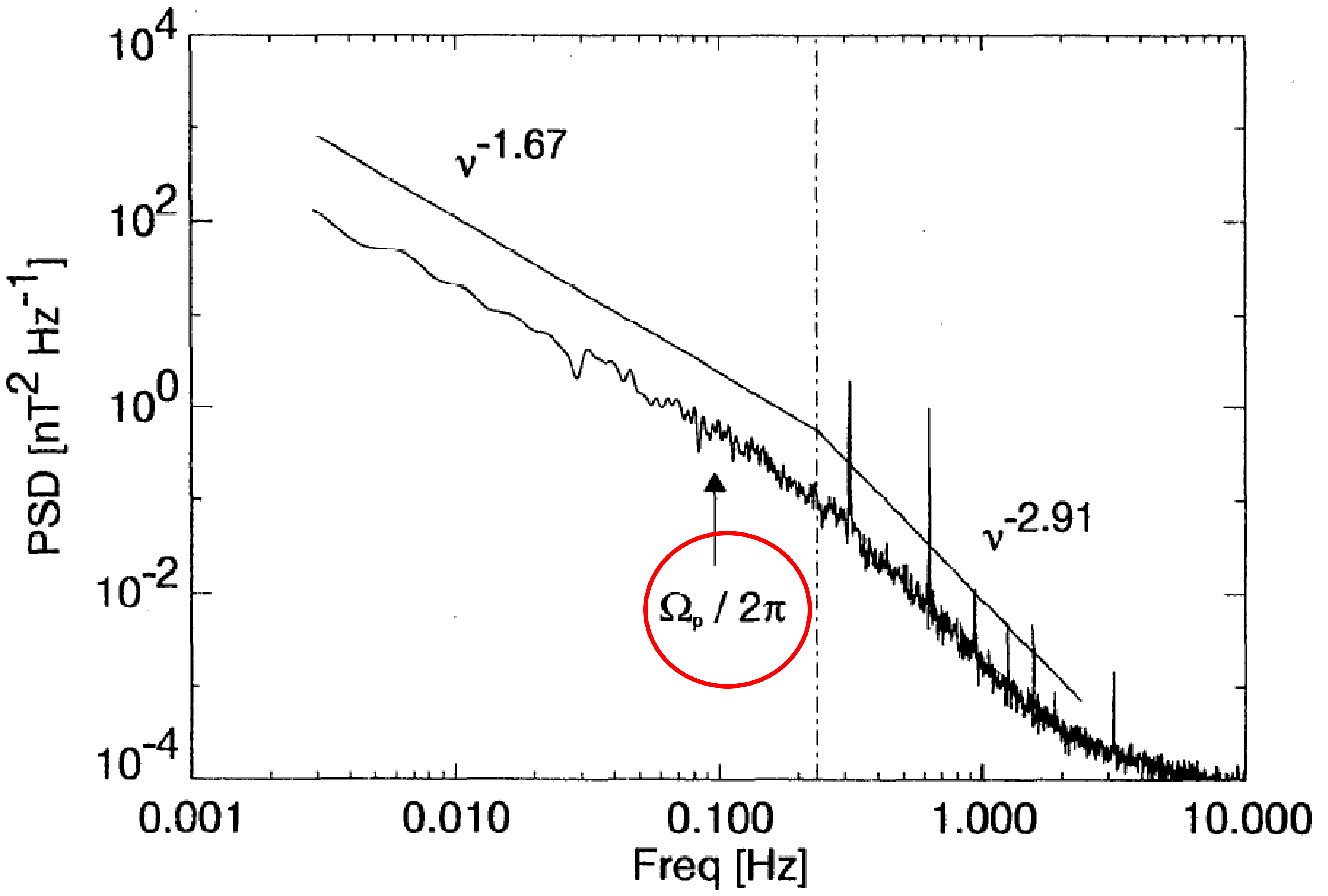


Magnetic fluctuations in the solar wind:

Leamon et al (1998)

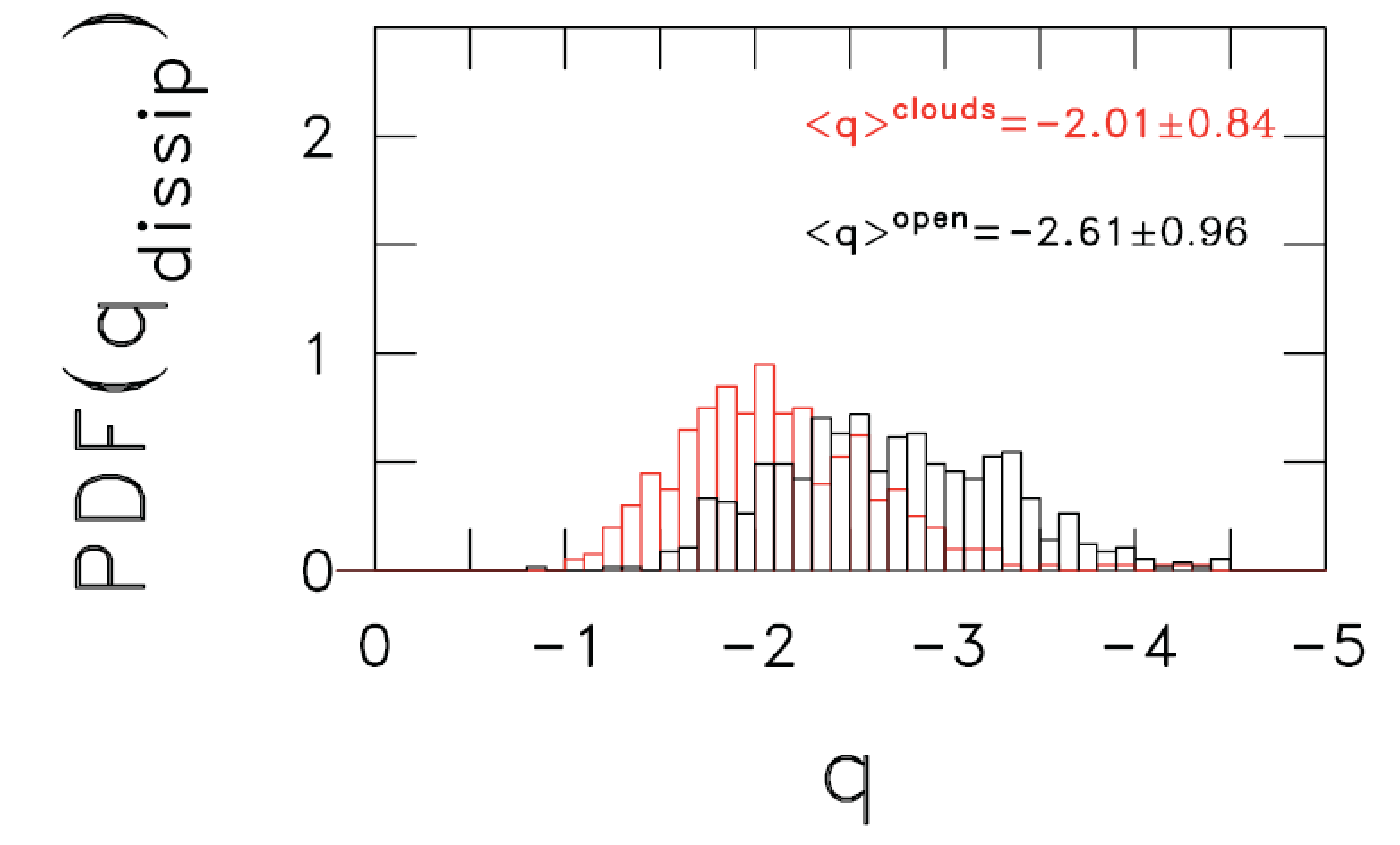


What's going on here?



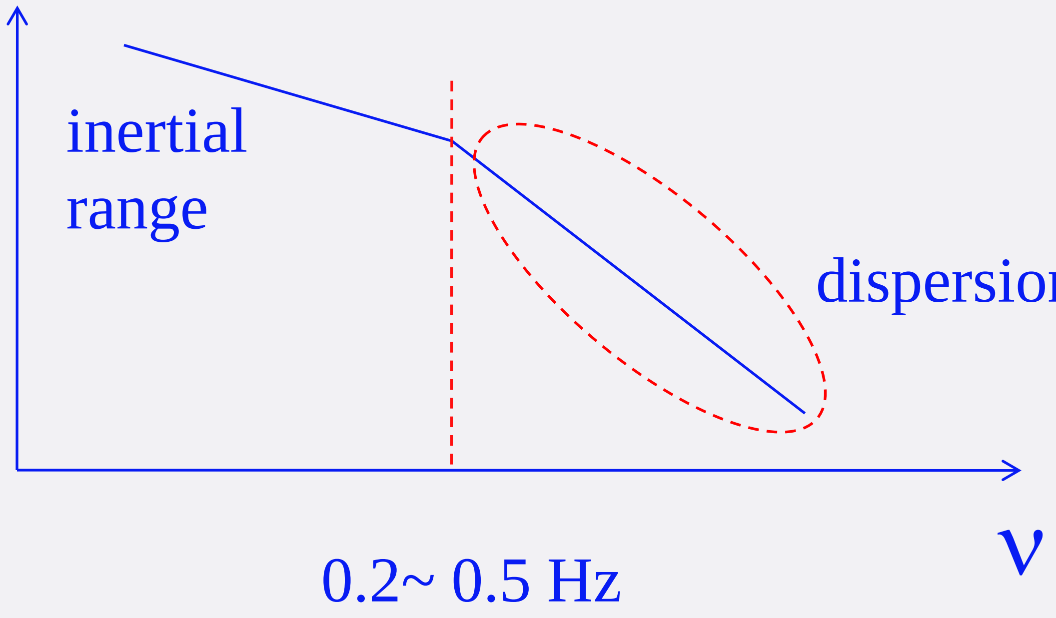
Leamon et al (1999)

slopes



Smith et al (2006)

- In general, the spectrum breaks at 0.2Hz < ν < 0.5Hz.
- The power index below the break: between -2 and -4.
- This range is termed "dispersion range"
- Recent studies: Dmitruk & Matthaeus (2006); Schekochihin et al (2009), Howes et al (2008), Saito et al (2008), Gary, Saito & Li (2008), …



Electron MHD: Introduction

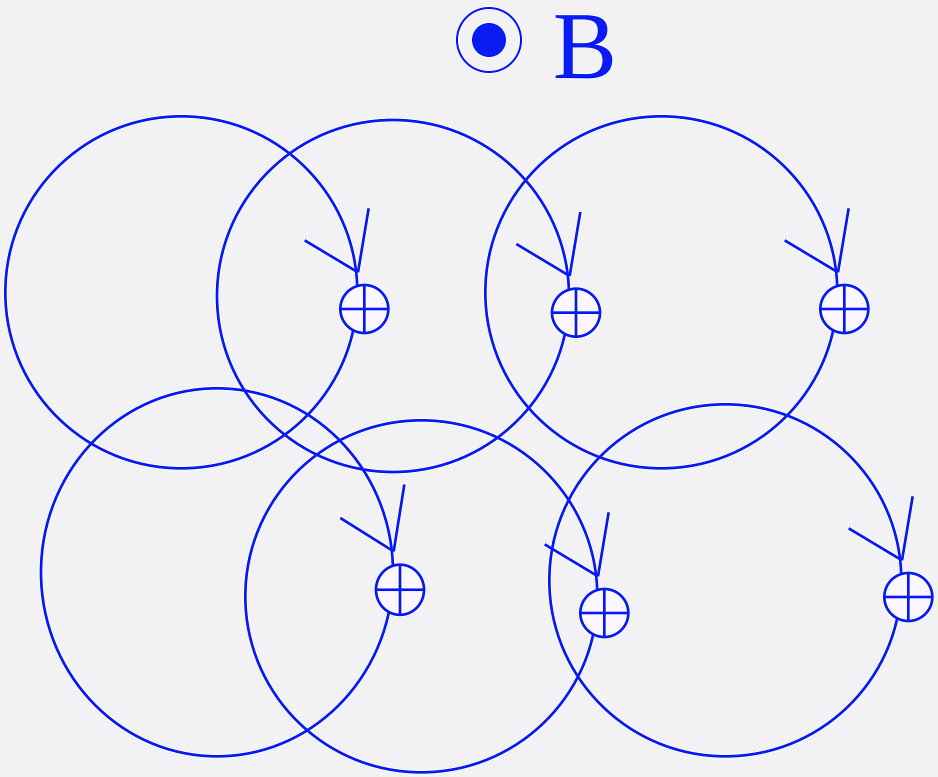
- How can we deal with small-scale physics?
- EMHD is a simple fluid-like description of small-scale physics

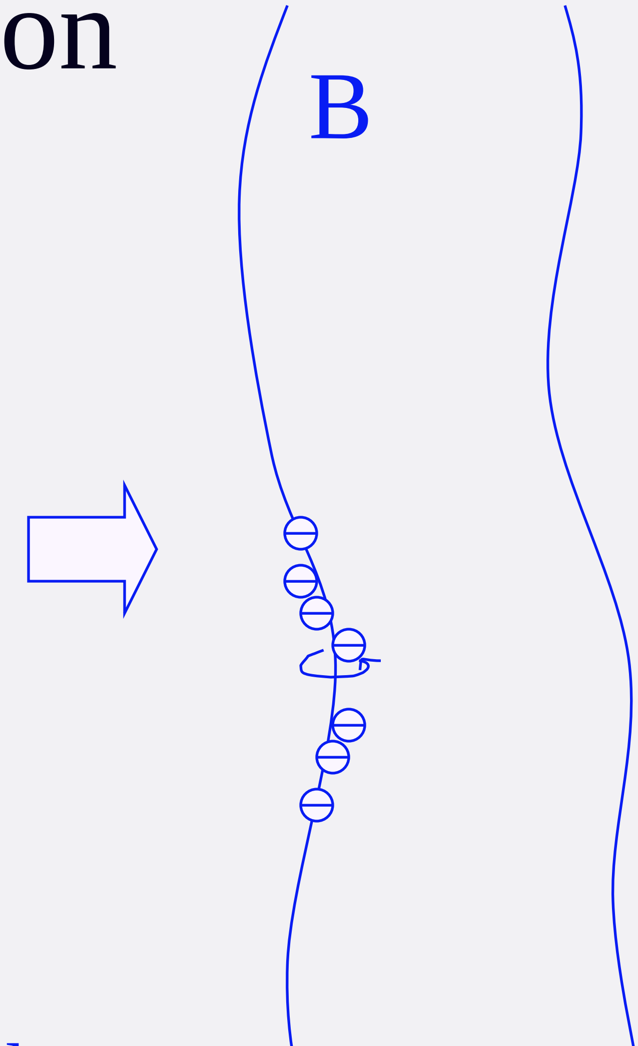
  (cf. PIC or gyro-kinetic simulations)

- The starting point is the magnetic induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

EMHD: Introduction





Protons ➔ smooth background

Electrons carry current

➔ $J \propto v$

Electron MHD eq

$\mathbf{J} \propto \mathbf{v}$

+

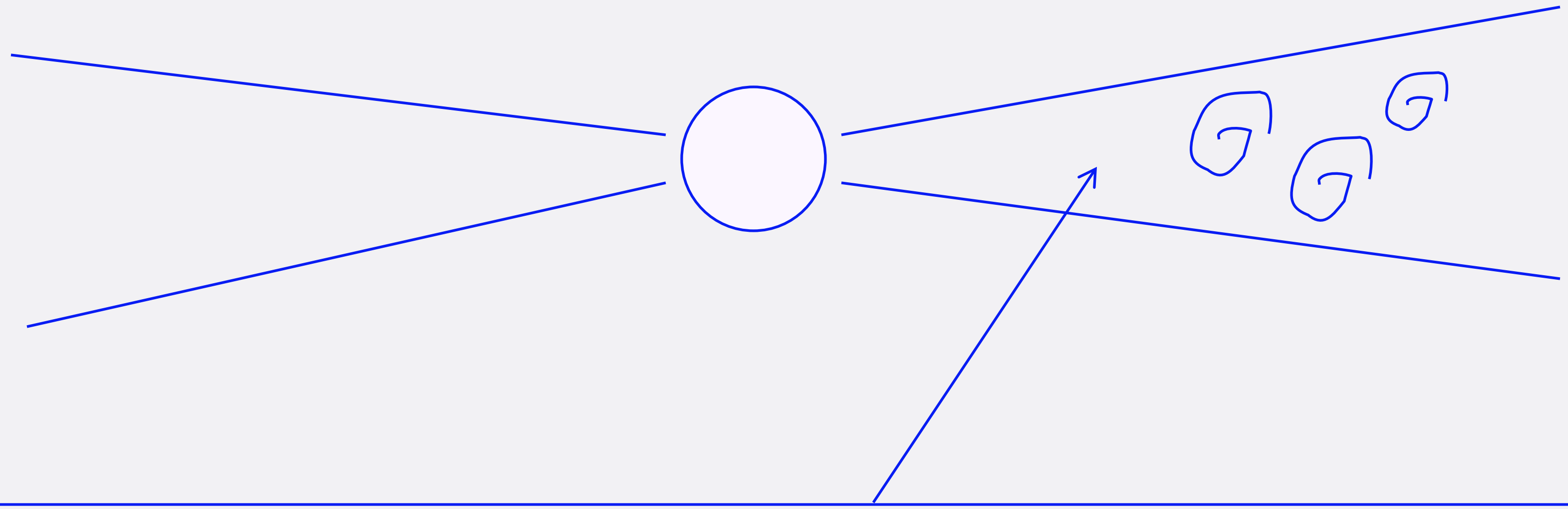
0

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},$$

$\mathbf{v} \propto \nabla\times\mathbf{B}$

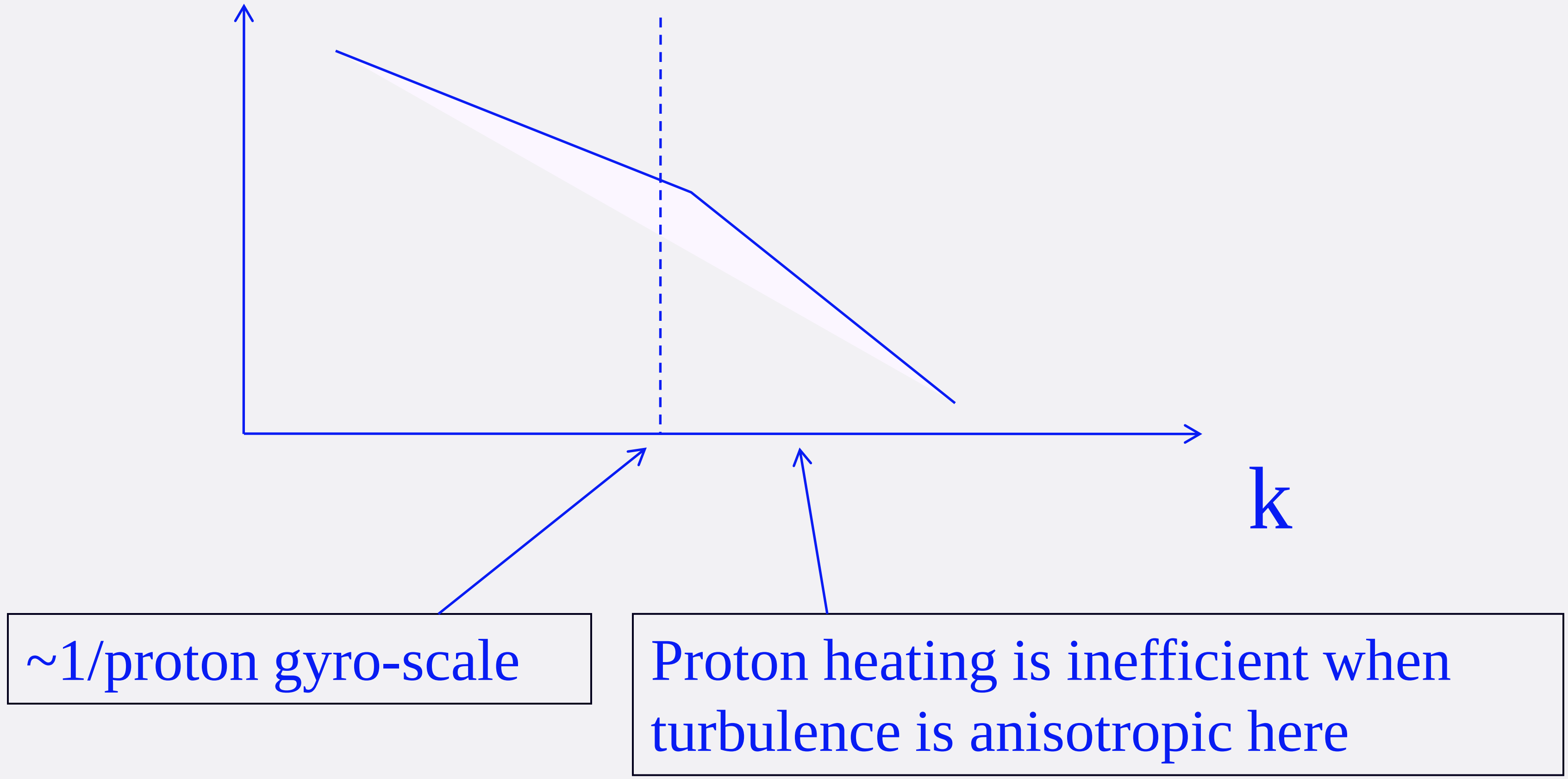
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

EMHD & collisionless plasma
(e.g. ADAF (advection dominated accretion flow))

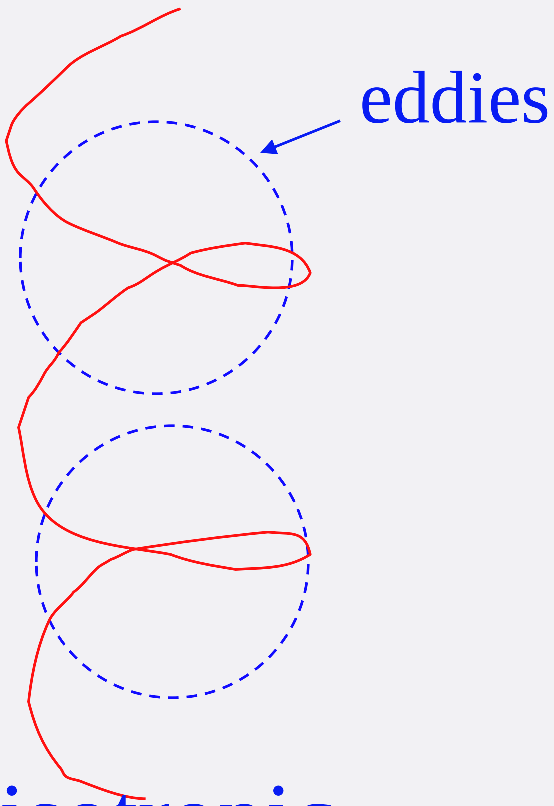


If turbulence energy goes to p, then it will have a low luminosity. (←inefficient energy transfer p→e)

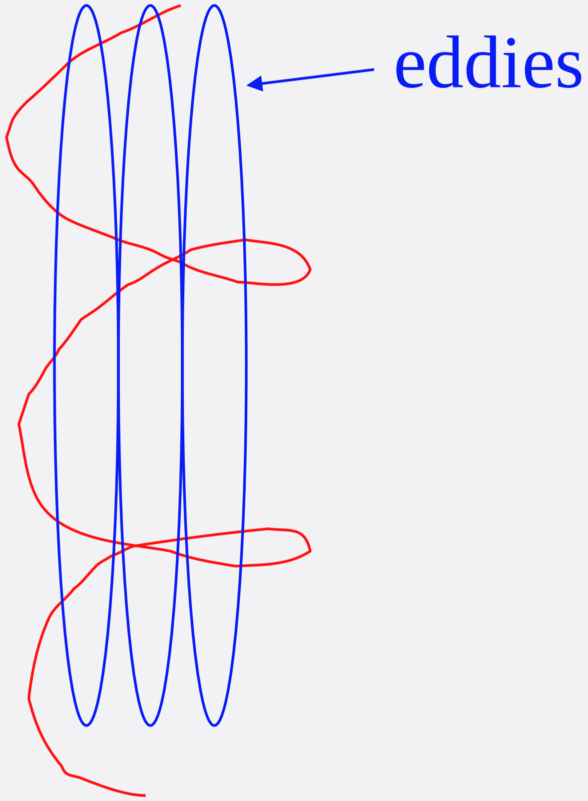
EMHD & collisionless plasma
(e.g. ADAF (advection dominated accretion flow))



Small-scale anisotropy and proton heating

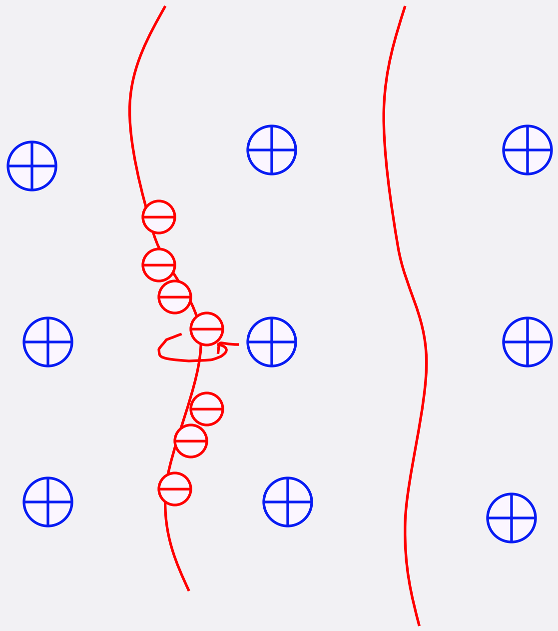


If isotropic,
resonance can
be possible!



If anisotropic,
resonance is not
possible!

EMHD & crust of neutron stars



crust of a neutron star

Protons=motionless; only electrons move

See Goldreich & Reisenegger (1992); Cumming, Arras & Zweibel (2004)

Ordinary incompressible MHD vs. EMHD turbulence

$$\frac{\partial \boldsymbol{v}}{\partial t} = -(\boldsymbol{\nabla} \times \boldsymbol{v}) \times \boldsymbol{v} + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{v} + \boldsymbol{f} + \boldsymbol{\nabla} P' ,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} ,$$

-Studied since 1960's

-Goldreich & Sridhar 1995

$E(k) \propto k^{-5/3}$

$k_{\parallel} \propto k_{\perp}^{2/3}$

-Numerical test:

Cho & Vishniac 2000

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

-Studied since 1990's

-Energy spectrum:

Biskamp-Drake group:

$E(k) \propto k^{-7/3}$

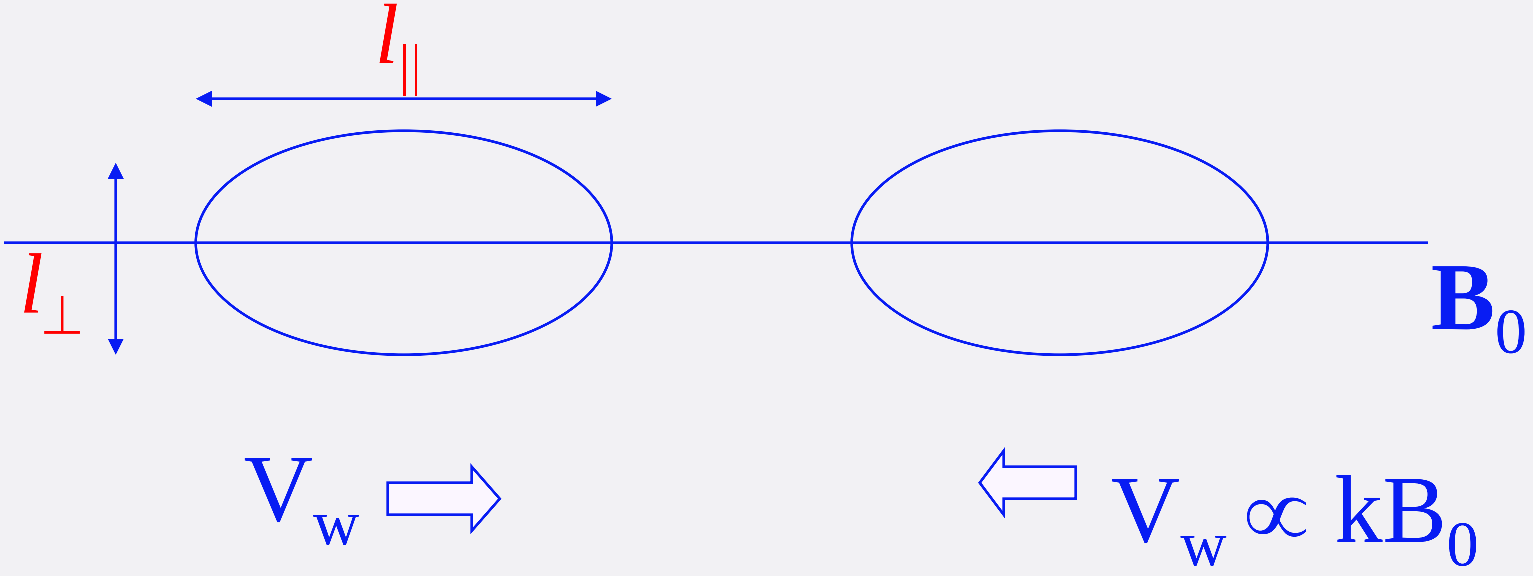
-Anisotropy:

Cho & Lazarian 2004

$k_{\parallel} \propto k_{\perp}^{1/3}$

Scaling of EMHD turbulence

Consider two EMHD wave packets:



When they collide, a packet loses energy of

$\Delta E \sim (dE/dt)\Delta t \sim (b^3/l_\perp^2)t_{coll}$

$\sim (b^3/l_\perp^2)(l_\parallel/V_w)$

$\sim (b^3/l_\perp^2)(l_\parallel/k_\perp B_0)$

Therefore $\Delta E/E \sim (b\, l_\parallel / l_\perp B_0)$

$= (l_\perp l_\parallel/B_0)/(l_\perp^2/b)$

$= t_w/t_{eddy} = \chi$

NOTE: $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta' \nabla^2 \mathbf{B}$$

➔ $db/dt \sim b^2/l_\perp^2$

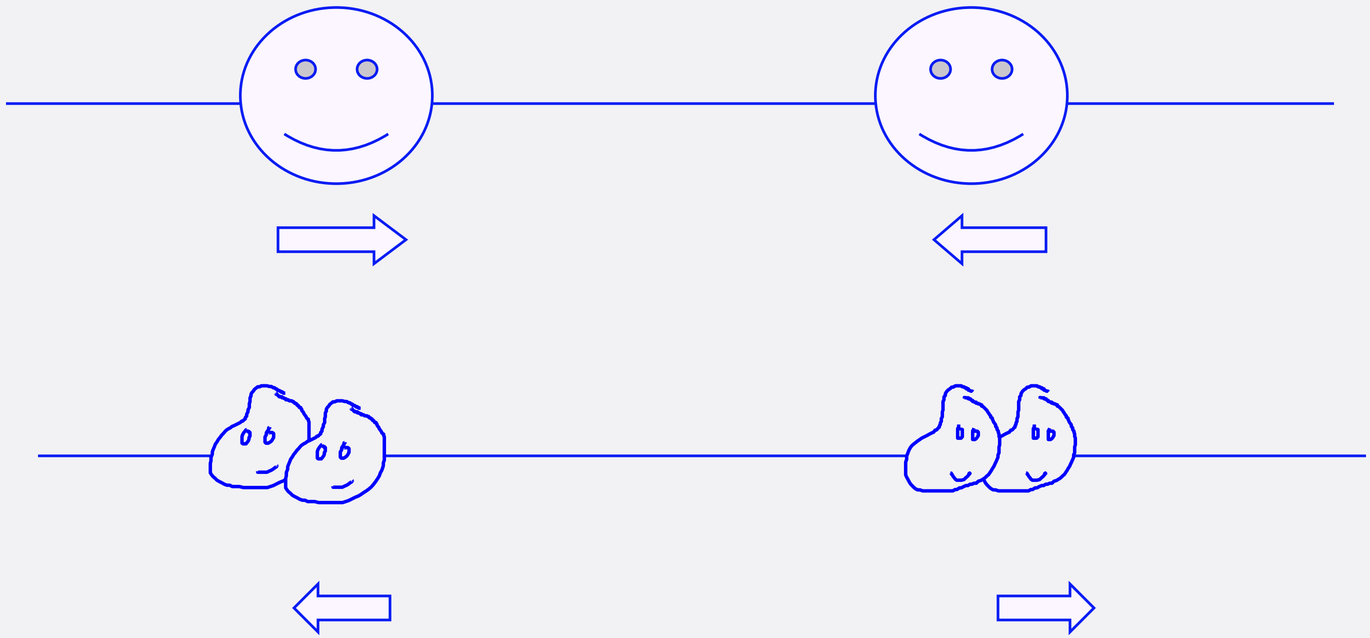
➔ $dE/dt \sim b^3/l_\perp^2$

Critical balance

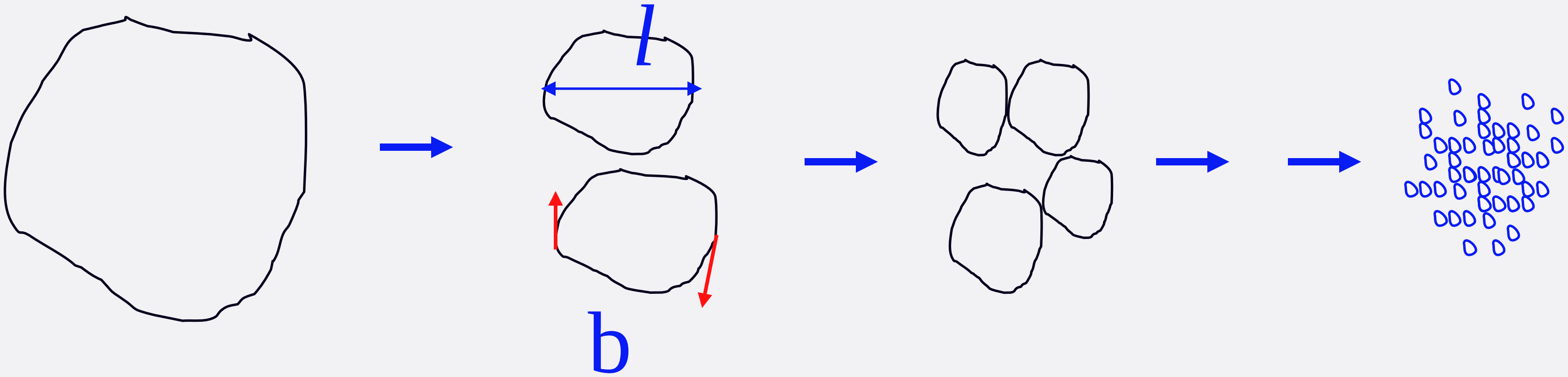
Suppose that the critical balance holds true:

$$\Delta E / E \sim t_w / t_{eddy} = \chi \sim 1$$

➔1 collision is enough to complete cascade!



Energy Cascade



$$b_l^2/t_{cas} = \text{constant}$$

↑

cascade time at scale *l*

Cho & Lazarian (2004)

- Critical balance

$$\frac{l_\perp^2}{b_{\perp l}} = \frac{l_\perp l_\parallel}{B_0}$$

- Constancy of energy cascade rate

$$\frac{b_{\perp l}^2}{t_{cas}} = \text{const}$$

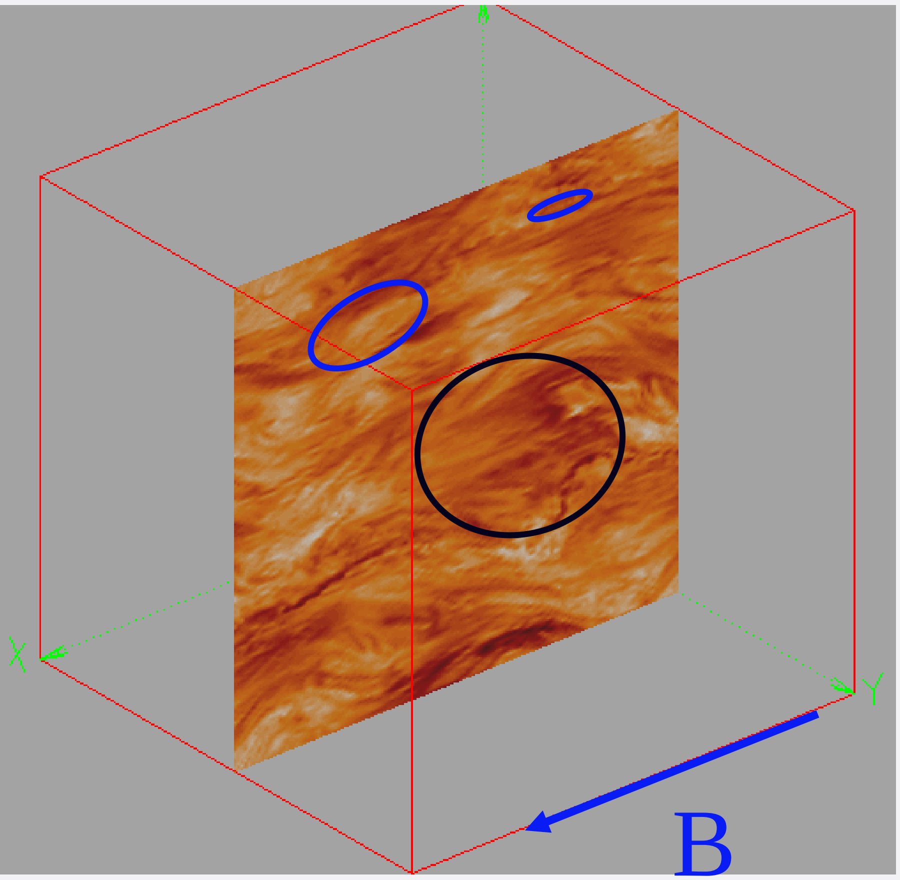
$$\frac{b_{\perp l}^2}{(l_\perp^2/b_{\perp l})} = \text{const}$$

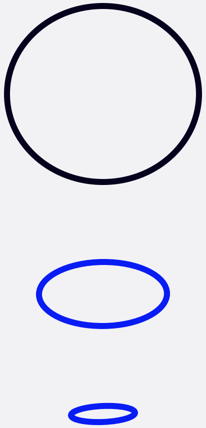
$$b_{\perp l} \sim l_\perp^{2/3}$$

Or, $E(k) \sim k^{-7/3}$

$$l_\parallel \sim l_\perp^{1/3}$$

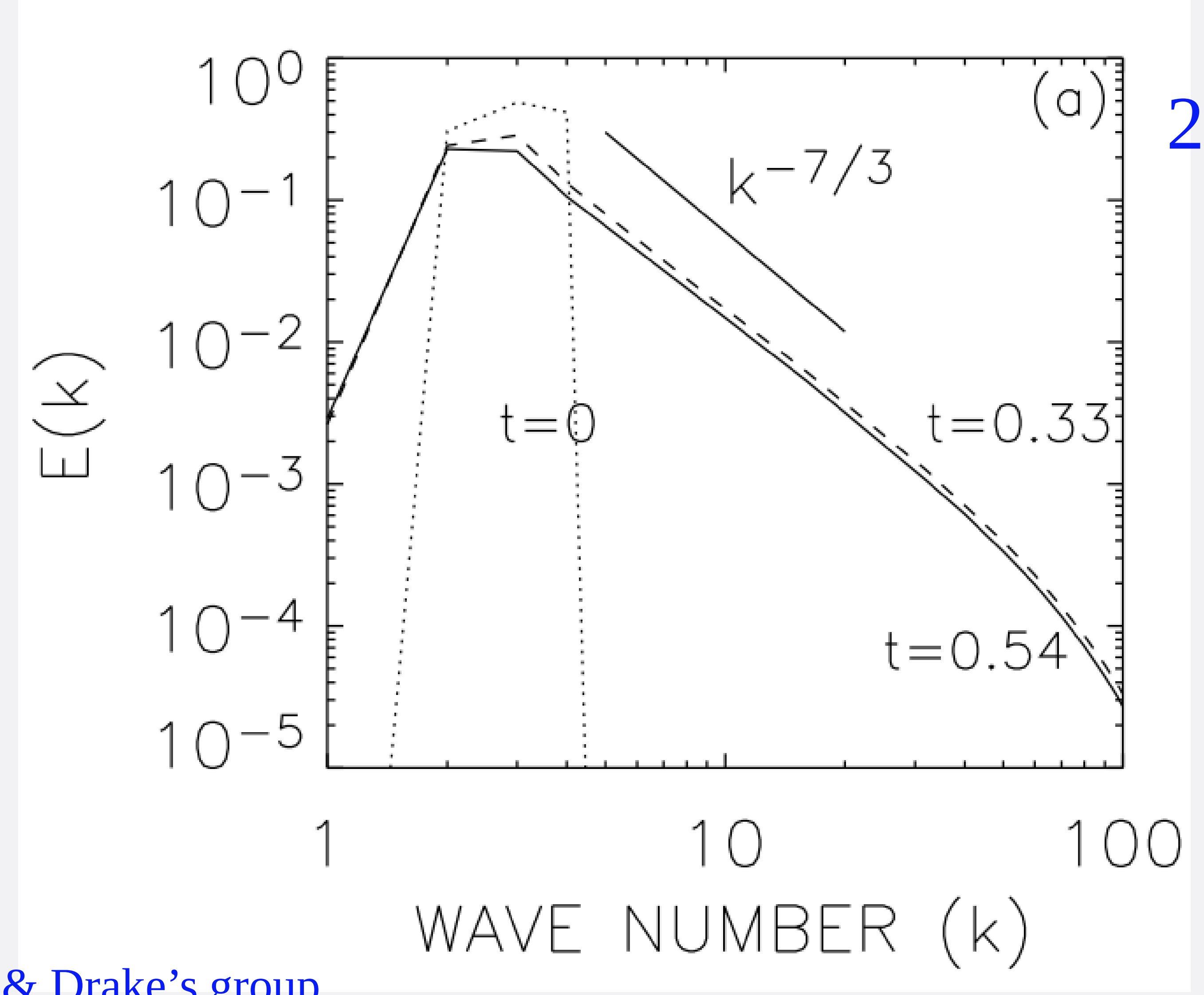
What do they mean by anisotropy?





Anisotropy = Relation between parallel size (~1/$k_{\parallel}$) and perpendicular size (~1/ $k_{\perp}$)

Numerical Results: spectrum

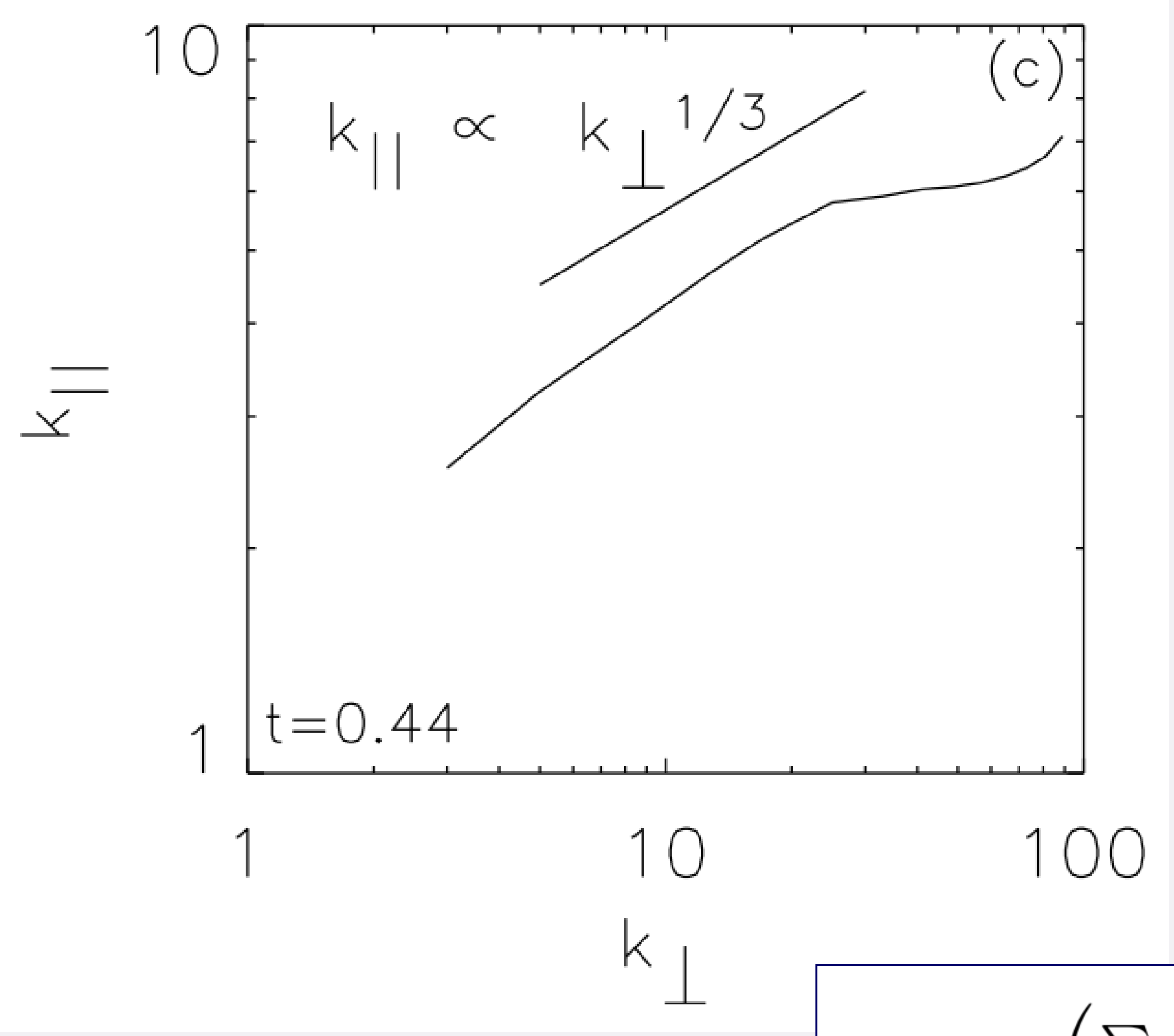


288^3

Biskamp & Drake's group obtained this in late 90's.

Cho & Lazarian (2004)

Numerical Results: anisotropy



$$k_{\|}(k_\perp) \approx \left(\frac{\sum_{k \le |\mathbf{k}'| < k+1} |\widehat{\mathbf{B}_L \cdot \nabla \mathbf{b}_l}|^2_{\mathbf{k}'}}{B_L^2 \sum_{k \le |\mathbf{k}'| < k+1} |\hat{\mathbf{b}}|^2_{\mathbf{k}'}} \right)^{1/2}$$

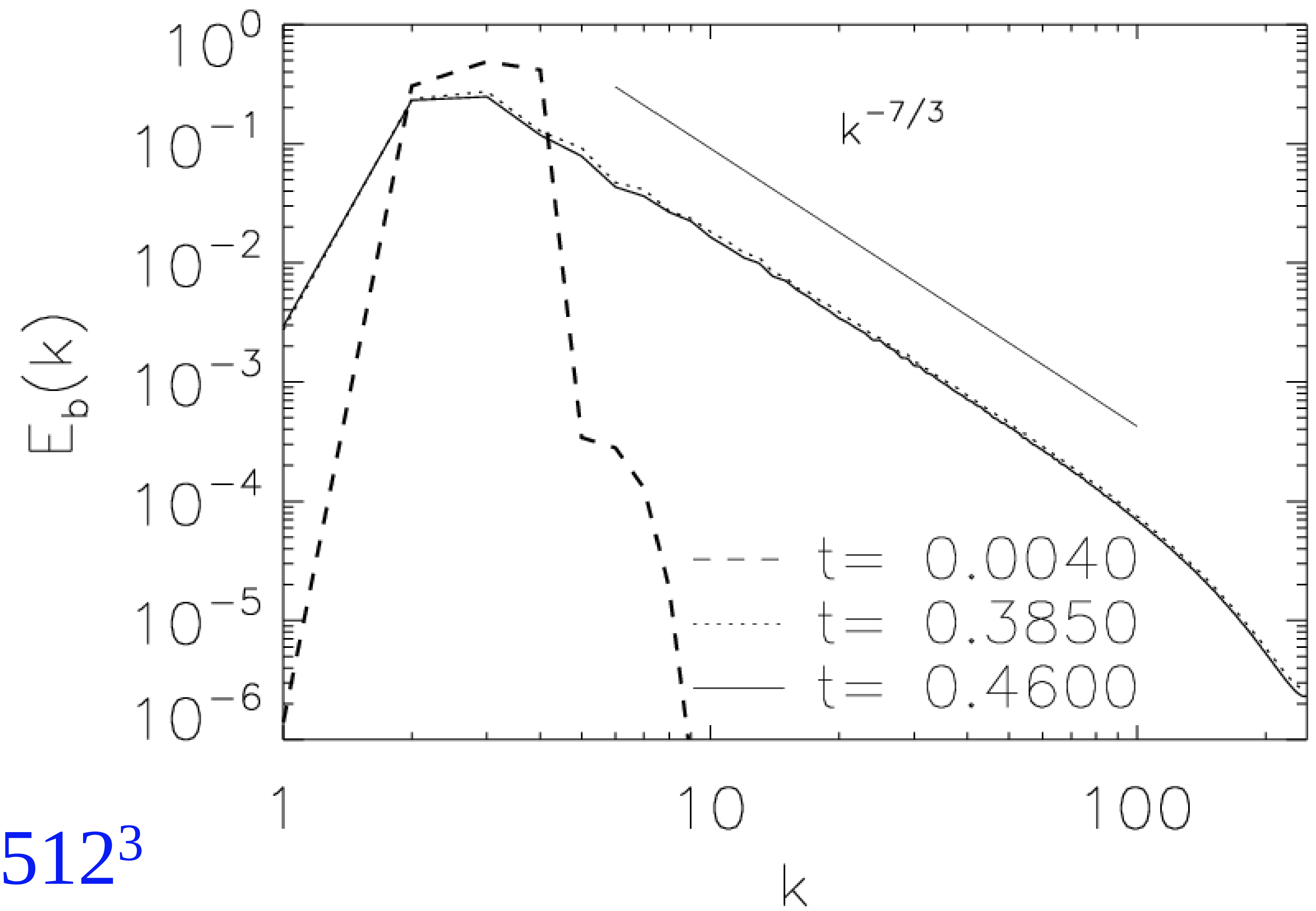
Cho & Lazarian (2004)

More results from Cho & Lazarian (2009)

- Higher resolution ($288^3 \rightarrow 512^3$)
- New techniques for anisotropy
- EMHD vs. ERMHD
- …

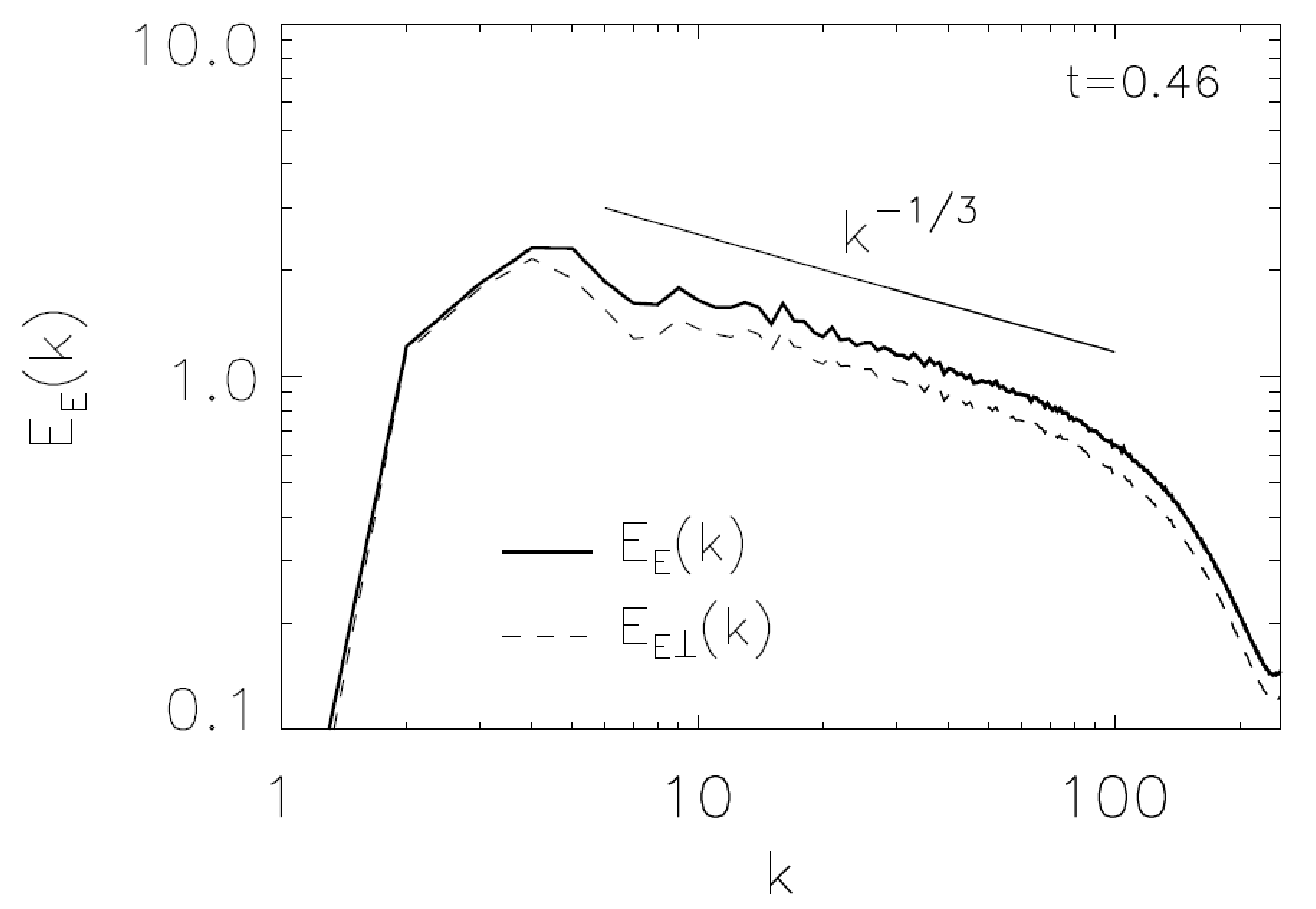
*We consider strong turbulence only.
For weak turbulence see for example
Galtier & Bhattacharjee (2003)

Spectra of decaying EMHD



512^3

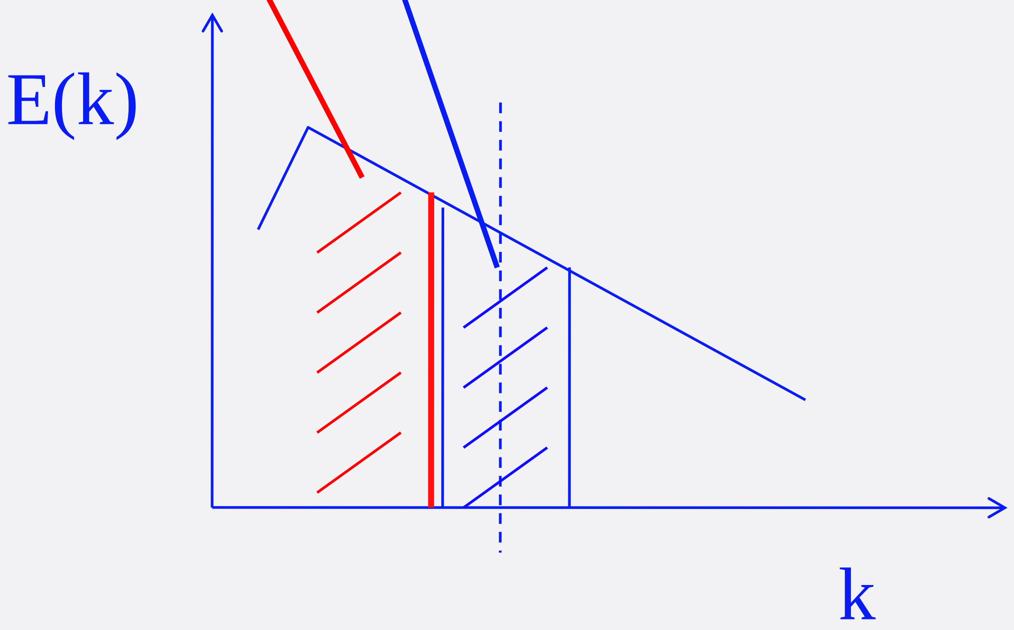
Spectrum of E



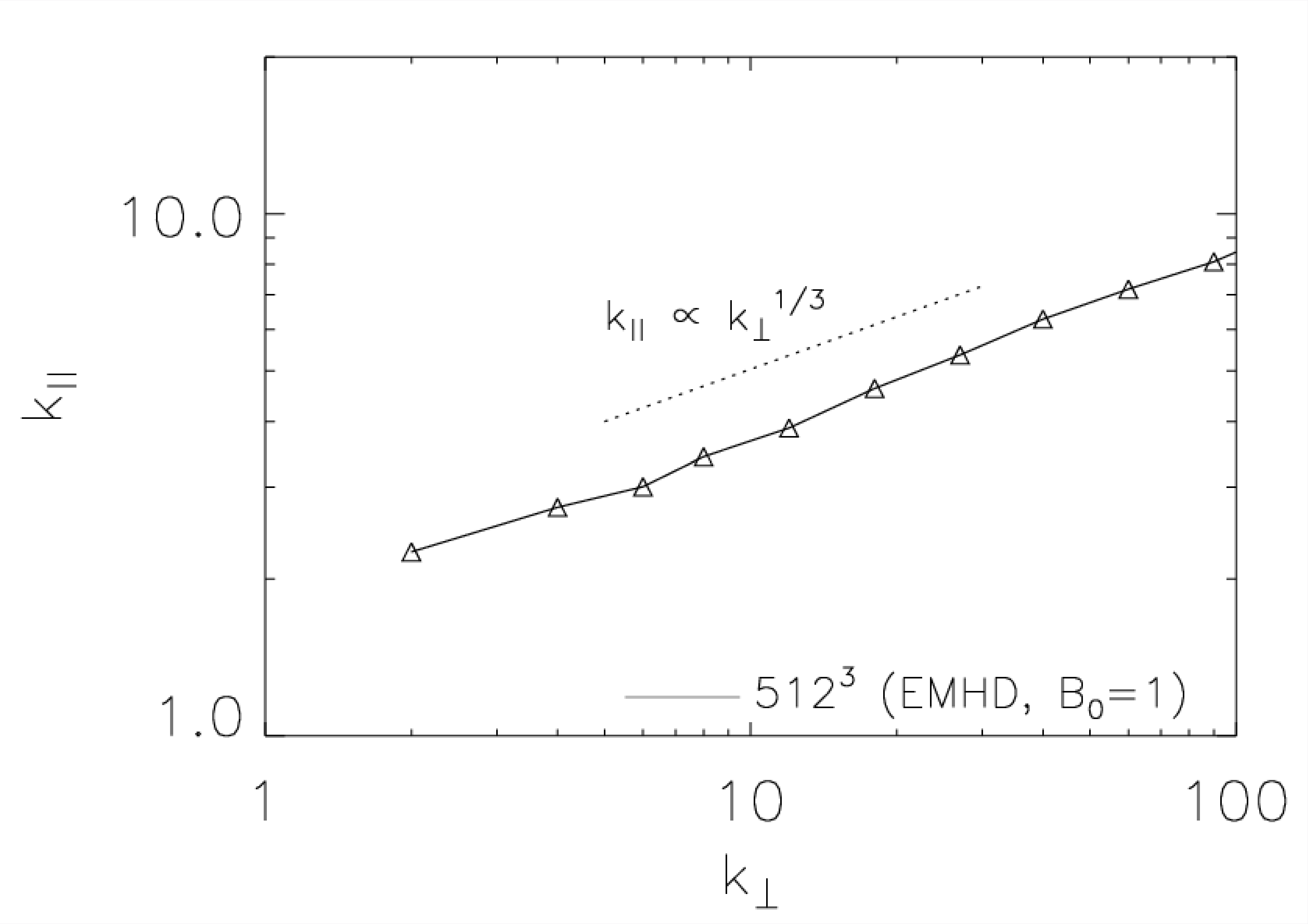
Consistent with observations (Bale at al 2005) and earlier simulations (Howes et al 2008; Dmitruk & Matthaeus 2006)

Anisotropy: method 1

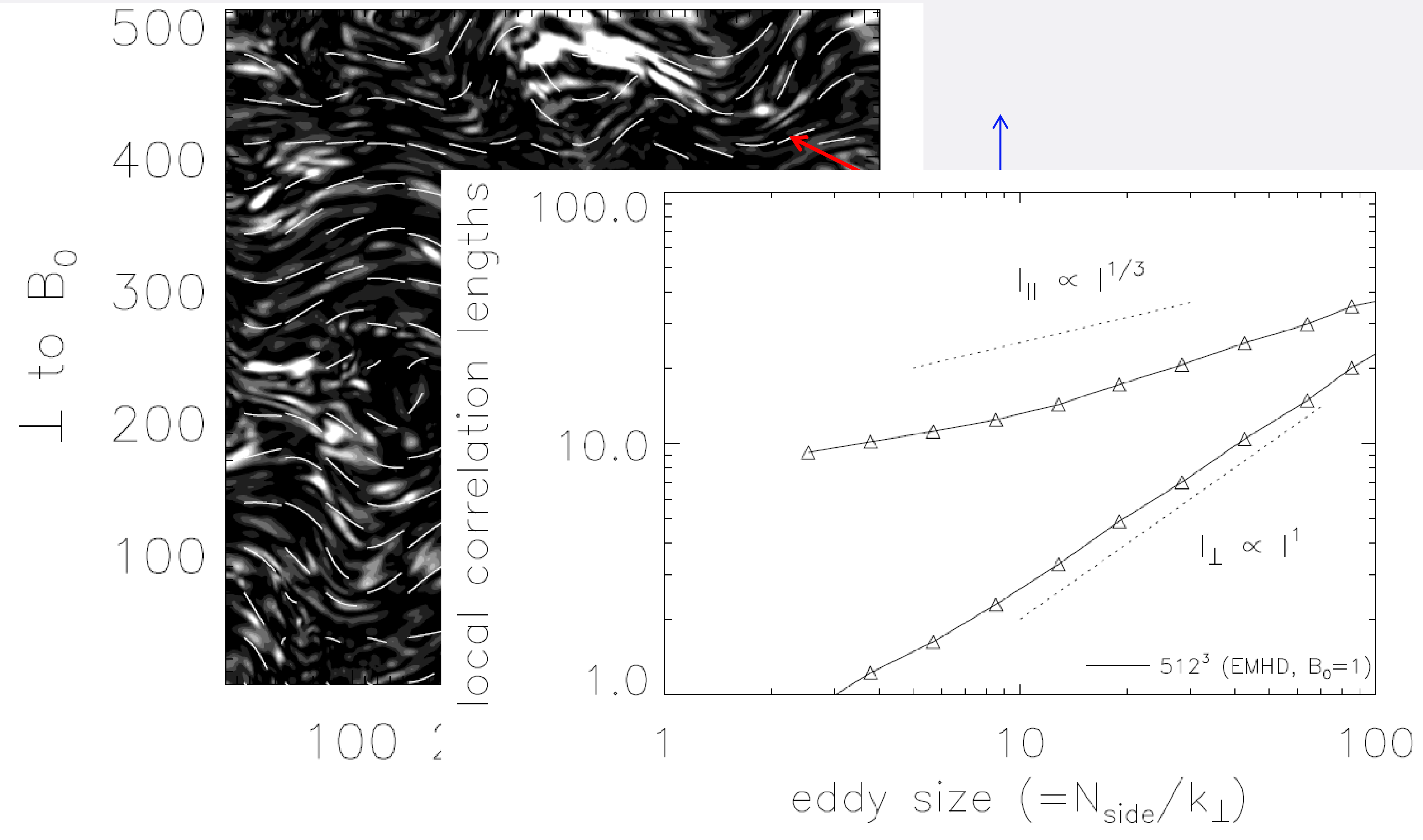
$$\mathbf{B}_L \cdot \nabla \mathbf{b}_l \approx B_L k_{||} b_l \rightarrow k_{||} \approx \frac{\left(< \left|\mathbf{B}_L \cdot \nabla \mathbf{b}_l\right|^2 >\right)^{1/2}}{B_L b_l}$$



Result



Anisotropy: method 2



Notes on EMHD vs ERMHD

$$\mathbf{E} = -\frac{\mathbf{v}_i}{c} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{n_e e c} + \frac{\mathbf{J}}{\sigma}$$

Generalized Ohm's law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

Induction equation

$$\rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B}) - \nabla \times \frac{c(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi n_e e} + \eta \nabla^2 \mathbf{B}$$

Let's assume $v_i \sim 0$.

EMHD: $$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{4\pi e n_e} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta \nabla^2 \mathbf{B}$$

ERMHD: $$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{B}\nabla \cdot \mathbf{v}_i - \frac{c}{4\pi e n_e} \nabla \times [\mathbf{B} \cdot \nabla \mathbf{B}] + \eta \nabla^2 \mathbf{B}$$

ERMHD

Schekochihin et al (2009):

$$\frac{\partial \mathbf{B}_\perp}{\partial t} = -\frac{c}{4\pi e n_e}\nabla_\perp \times [\mathbf{B}\cdot\nabla \mathbf{B}_\parallel],$$

$$\frac{\partial \mathbf{B}_\parallel}{\partial t} = -\frac{\beta_i(1+Z/\tau)}{2+\beta_i(1+Z/\tau)}\frac{c}{4\pi e n_e}\nabla_\perp \times [\mathbf{B}\cdot\nabla \mathbf{B}_\perp]$$

$=\alpha$ ← $\tau = T_i/T_e$

Let $\tilde{\mathbf{b}} = \mathbf{B}_\perp + \mathbf{B}_\parallel / \sqrt{\alpha}$

$$\frac{\partial \mathbf{B}_\perp}{\partial t} = -\frac{\sqrt{\alpha}\, c}{4\pi e n_e}\nabla_\perp \times [\mathbf{B}\cdot\nabla \mathbf{B}_\parallel]\frac{1}{\sqrt{\alpha}},$$

$$\frac{1}{\sqrt{\alpha}}\frac{\partial \mathbf{B}_\parallel}{\partial t} = -\frac{1}{\sqrt{\alpha}}\,\alpha\,\frac{c}{4\pi e n_e}\nabla_\perp \times [\mathbf{B}\cdot\nabla \mathbf{B}_\perp]$$

→ $$\frac{\partial}{\sqrt{\alpha}\,\partial t}\tilde{\mathbf{b}} = -\frac{c}{4\pi e n_e}\nabla_\perp \times (\mathbf{B}\bullet\nabla\tilde{\mathbf{b}})$$

ERMHD

$$\frac{\partial}{\sqrt{\alpha}\,\partial t}\tilde{\mathbf{b}} = -\frac{c}{4\pi e n_e}\nabla_\perp \times (\mathbf{B}\bullet\nabla\tilde{\mathbf{b}})$$

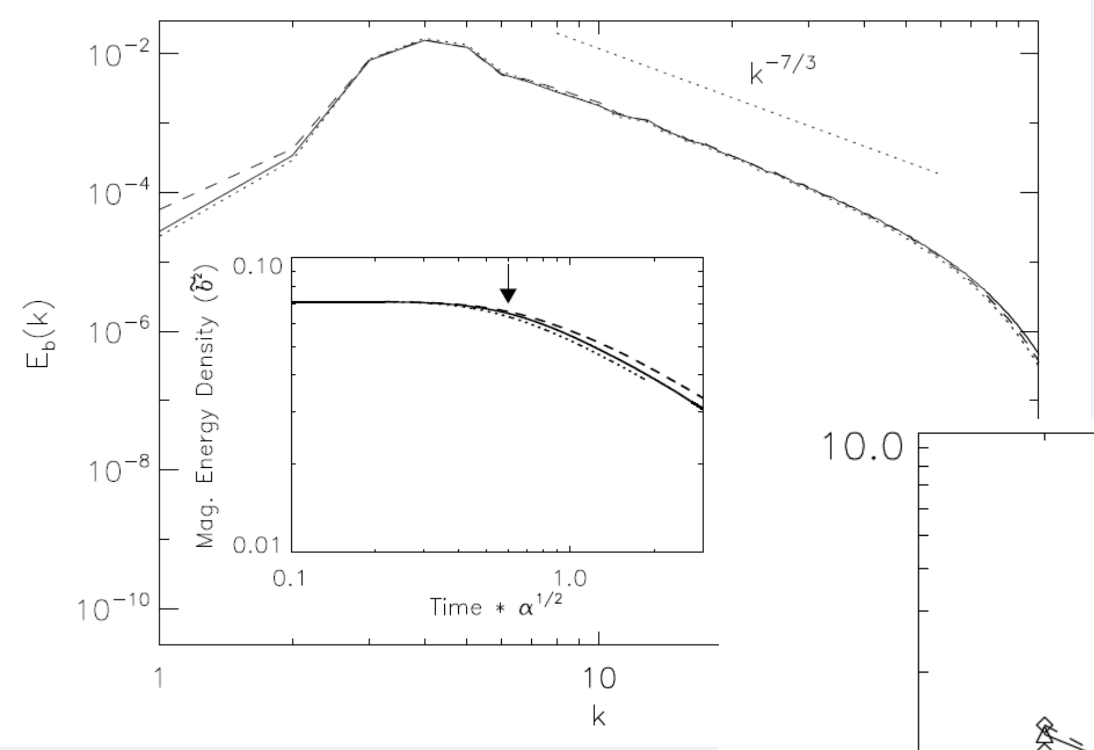
$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = -\nabla_\perp \times (\mathbf{B}\cdot\nabla\tilde{\mathbf{B}}) + \eta'\nabla^2\tilde{\mathbf{B}}$$

$$\tilde{\mathbf{B}} \equiv \mathbf{B}_0 + \tilde{\mathbf{b}},$$
$$\tilde{\mathbf{b}} = \mathbf{b}_\perp + \sqrt{\frac{1}{\alpha}}\, b_\parallel \hat{\mathbf{z}},$$
$$\tilde{t} \equiv \sqrt{\alpha}\, t,$$

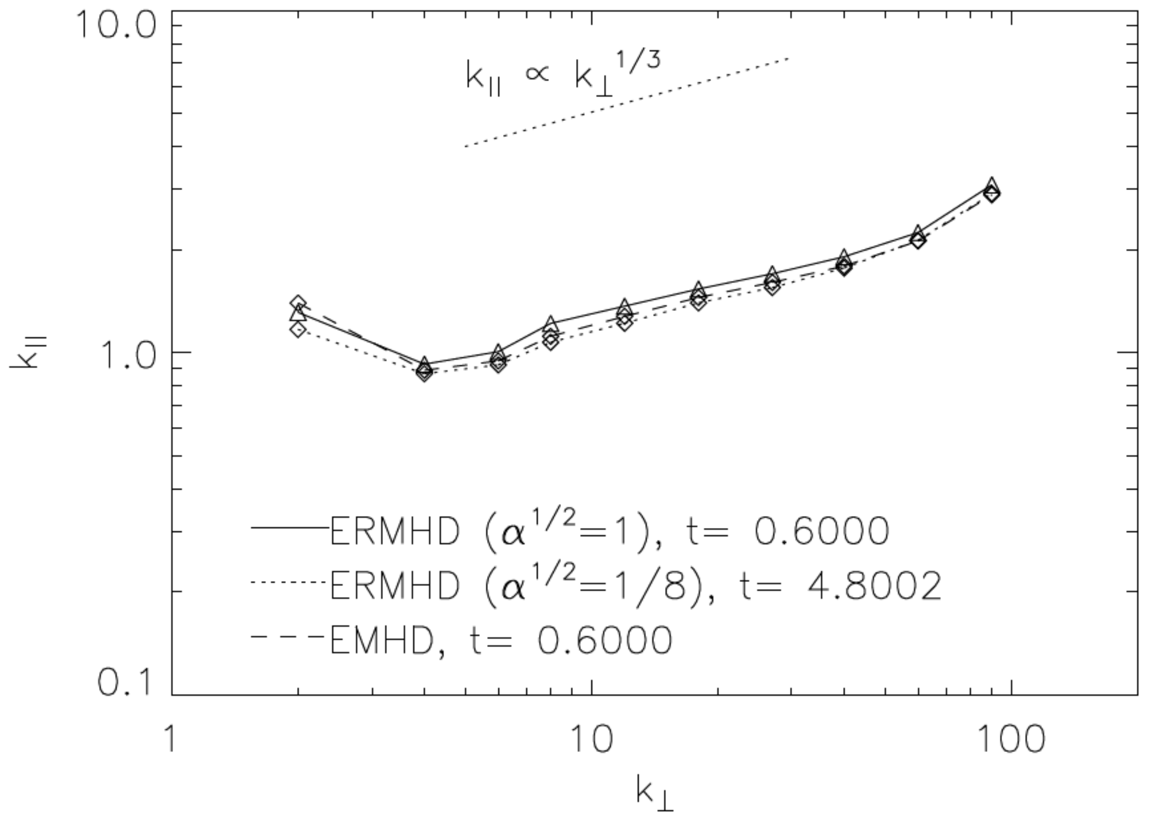
cf. EMHD: $$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{4\pi e n_e}\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta\nabla^2\mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] + \eta'\nabla^2\mathbf{B}$$

EMHD ≈ ERMHD



$E_b(k)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$k^{-7/3}$
1
10
k
Mag. Energy Density ($\tilde{b}^2$)
0.10
0.01
0.1
1.0
Time * $\alpha^{1/2}$



$k_{||}$
10.0
1.0
0.1
$k_{||} \propto k_{\perp}^{1/3}$
ERMHD ($\alpha^{1/2}$=1), t= 0.6000
ERMHD ($\alpha^{1/2}$=1/8), t= 4.8002
EMHD, t= 0.6000
1
10
100
$k_{\perp}$

Summary

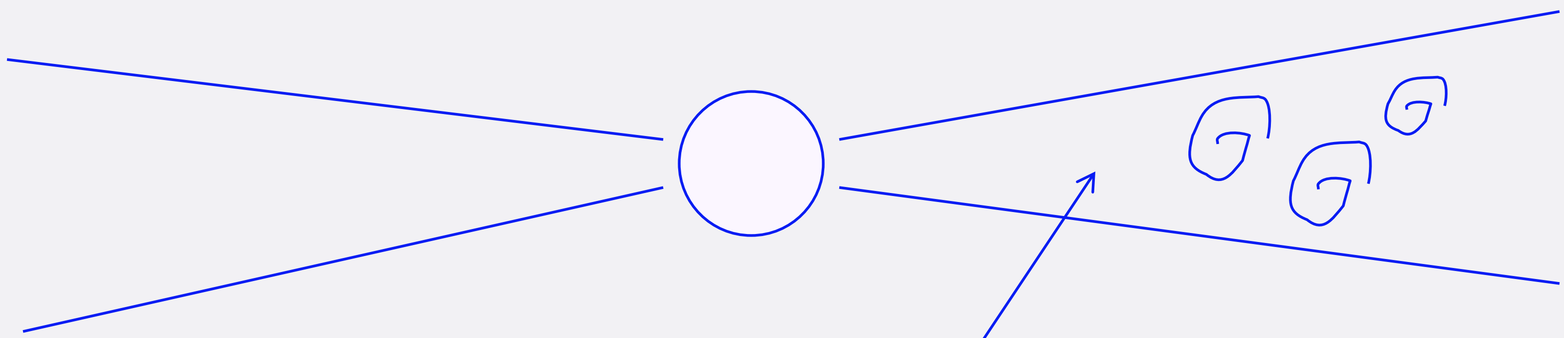
1. Scaling relations of EMHD turbulence:
   - Spectrum of B : $E(k) \sim k^{-7/3}$
   - Spectrum of E : $E(k) \sim k^{-1/3}$
   - Anisotropy: $l_{||} \sim l_{\perp}^{1/3}$

2. ERMHD ~ EMHD

Implications

- Anisotropy ➔ In ADAFs, electron heating is important when β ($=P_{gas}/P_{mag}$) < 10.
- Strong anisotropic cascade in neutron star crust ➔ dissipation of magnetic field within one whistler period (=Hall time).

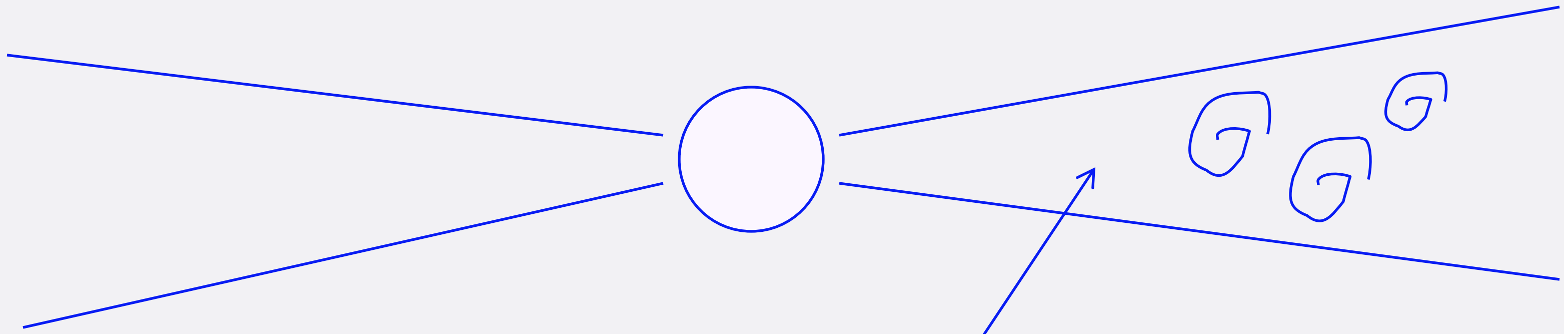
EMHD & ADAF (advection dominated acc. flow)



Collisionless
Turbulence energy goes to p
Inefficient energy transfer p→e

⇒ Low luminosity

EMHD & ADAF (advection dominated acc. flow)



Collisionless
Turbulence energy goes to p
Inefficient energy transfer p→e

⇒ Low luminosity

Is it true?

Answer: No, if small scale eddies are anisotropic and β~1.
See Quataert & Gruzinov (1999)