Turbulence and the Early Generation of Magnetic Fields in the Early Universe

Also:

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What is this all about?

- Traditional (more than 10 years ago) mean-field dynamo theory relies on the kinematic limit (no backreaction) and assumes only linear cross talk between scales - the growth of the large scale field is proportional to the large scale field amplitude.
- This fails at amplitudes close to equipartition, and also at very small amplitudes.

In terms of the galactic dynamo this means:

- We cannot follow the evolution of the field at recent times.
- We cannot follow the evolution of the field at early times - if we use the kinematic growth rate (about 0.1 Ω) then the available seed fields are too small to grow to equipartition in just a few rotations of the galactic disk.

When did galactic magnetic fields become strong?

- Faraday rotation of distant AGN can be correlated with intervening gas.
- Several studies along these lines, starting with Kronberg and Perry 1982 and continuing with efforts by Kronberg and collaborators and Wolfe and his. (Also, Bernet Nature 2009)
- Most recent work finds that galactic disks must have been near current levels of magnetization when the universe was ~ 2 billion years old (redshifts well above 3).

What are the relevant physical issues?

- The generation of a disordered magnetic field in the presence of a very weak seed field.
- The inverse cascade of the magnetic field in the absence of a large scale field.
- The generation of a coherent large scale magnetic field over annular domains.
- The generation of a coherent large scale field over an entire galactic disk. (Not going say much about this one)

An idealized problem

- We will pretend that galaxies start with their current properties rather than try to reconstruct the complicated history of real galactic disks and pregalactic gas motions
- For concrete numbers, we will take

$$v_{turn} \sim 10 \text{ km/sec}$$
$$\lambda_{turn} \sim 30 \text{ pc}$$
$$H_{disk} \sim 300 \text{ pc}$$

so that

$$\Omega\tau_{corr} \sim 0.1$$
$$\Omega\tau_{diff} \sim 10$$

How do we make seed fields?

- Use scalar perturbations in the early universe (Vishniac 1982). Nonlinear interactions plus photon shear viscosity gives vector modes. Photon drag gives magnetic fields (Mattarese et al. 2005). Galactic scale magnetic fields $\sim 10^{-21}$ Gauss.
- Galaxy rotation (or vortical motions generated during galaxy formation) plus compton drag. Zel'dovich et al. estimate $\sim 10^{-21}$ G, but they put fully developed disks at redshifts of 10.

More rotation

- Galaxy rotation - but instead of drag we appeal to vertical gravity to produce preferential settling of the ions. This can give very slightly larger numbers.
- Baroclinic forces (grad(ρ) X grad(P)) plus drag. This depends on geometry, but for a galactic disk once again gives broadly similar answers.

How about the dynamo?

- Averaging the induction equation we have where the critical piece is in the azimuthal direction. $\partial_t \overline{B} = \nabla \times \left(\overline{U \times B} \right)$
- The piece that depends linearly on the magnetic field can be written as

$$\left\langle \vec{\mathrm{v}} \times \vec{b} \right\rangle_{\varphi} \sim \left(k^2 h_B - h_k \right) \frac{\tau}{3} B_{\varphi}$$

- But this neglects the inverse cascade, and the generation of a small scale magnetic field.

What about turbulence?

- A magnetic field embedded in a turbulent medium will get stretched, and amplified, by the local shear (Bachelor) if the field is weak.
- We can divide up a turbulent cascade into the very small scales where the field is strong, and resists stretching, and where it is weak and stretches. This defines a scale of equipartition.

Turbulence

- The magnetic field gains energy at roughly the same rate that energy is fed into the energy cascade, which is

$$\rho \frac{V^3}{L}$$

- This doesn't depend on the magnetic field strength at all.
- The scale of the field increases at the equipartition turn over rate

$$\propto l^{-2/3}$$

Turbulence

- Energy flows through a turbulent cascade, from large scales to small and in stationary turbulence we have a constant flow

$$\rho u^2 \frac{u}{l}$$

- At the equipartion scale

$$B^2 \approx \rho u^2$$

- So the rate at which the magnetic energy grows is a fraction of the energy cascade rate, a constant.

Turbulence

- After a several eddy turn over rates the field scale is the large eddy scale (~30 pc) and the field strength is at equipartition.
- This is seen in numerical simulations of MHD turbulence e.g. Cho et al. (2008).
- This does not (by itself) explain the Faraday rotation results since the galactic disk is a few hundred pc thick and has a radial scale an order of magnitude larger.

An added consideration….

- The growth of the magnetic field does not stop at the eddy scale. Turbulent processes create a long wavelength tail. Regardless of how efficient, or inefficient it is, it's going to overwhelm the initial large scale seed field.
- For magnetic fields this is generated by a fluctuating electromotive force, the random sum of every eddy in a magnetic domain (Vishniac and Brandenburg 1997)

The fluctuation-dissipation theorem

- The field random walks upward in strength until turbulent dissipation through the thickness of the disk balances the field increase. This takes a dissipation time.
- This creates a large scale B_r^2 which is down from the equipartition strength by N^{-1}, the inverse of the number of eddies in domain.
- Here a domain should be an annulus of the disk, since shearing will otherwise destroy it.

The large scale field

- Choosing generic numbers for the turbulence, we have about 10^5 eddies in a minimal annulus, implying an rms $B_r \sim 10^{-8}$ G.
- The eddy turnover rate is about 10^{-14}, 10x faster than the galactic shear, and the dissipation time is about $10\Omega^{-1}$, or a couple of galactic rotations.

The randomly generated seed field

- Since the azimuthal field will be larger than B_r by $\Omega\tau_{diss}$ this gives a large scale seed field somewhere around 0.1 μG, generated in a dissipation time, a bit less than two rotation periods.
- The local field strength reaches equipartition much faster, within a small fraction of a galactic rotation period.

Magnetic helicity driven dynamos

- When the kinematic dynamo fails due to the increase in the local magnetic helicity, we can invoke the constraint equation

$$\partial_t h_b + 2\vec{B}\bullet\left\langle \vec{u}\times\vec{b}\right\rangle = -\nabla\bullet\vec{j}_h \qquad h_B \equiv \left\langle \vec{a}\bullet\vec{b}\right\rangle$$

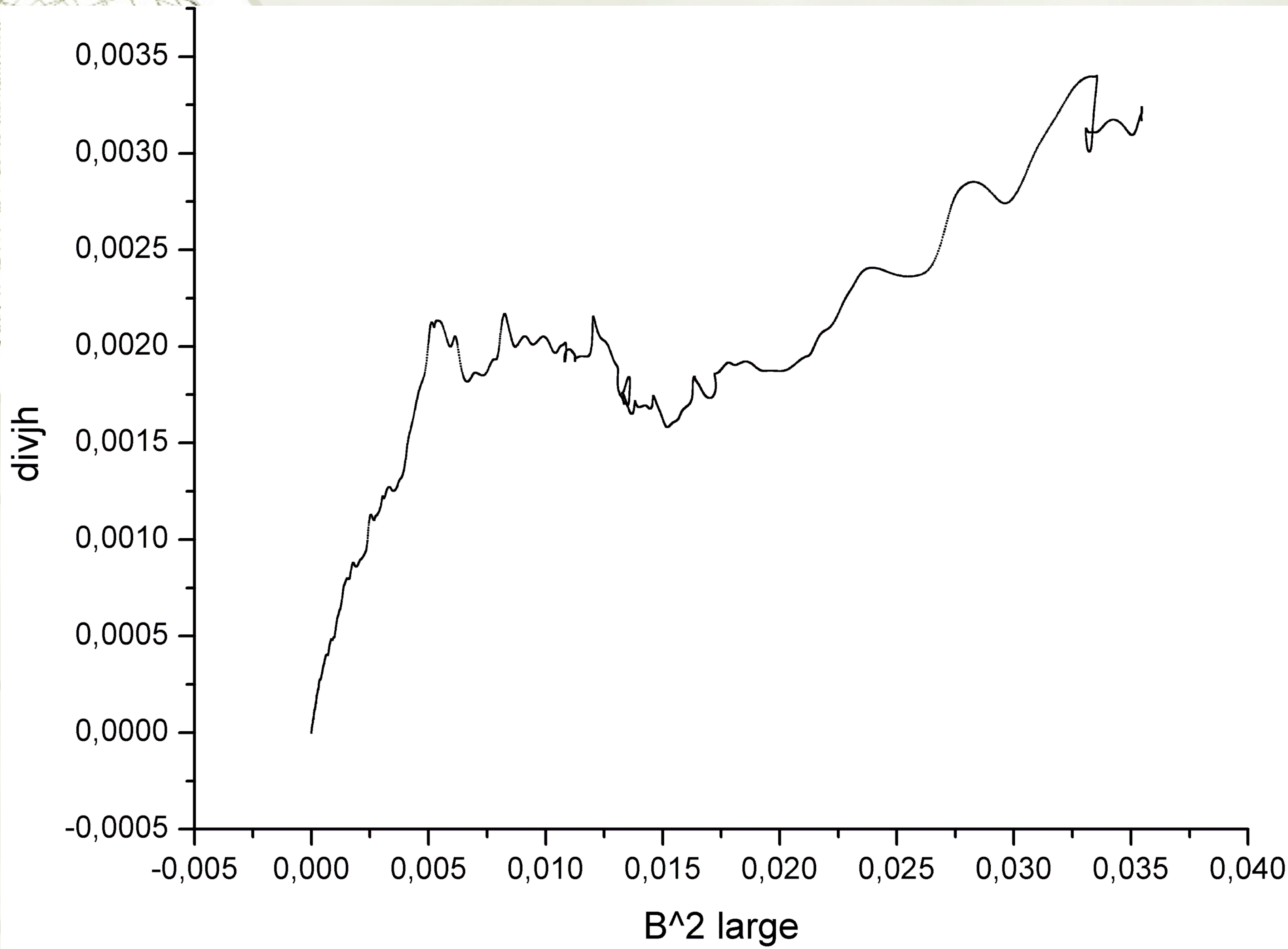
- On dimensional grounds we can estimate

$$\vec{j}_h \sim \frac{\lambda_z{}^2}{\lambda^2} u^2 \tau(\Omega\tau) B_{total\,??}{}^2 \hat{z}$$

- Assuming we're just dealing with the large scale field in this expression, we get a growth rate of $\Gamma \sim \frac{u}{H}\Omega\tau$

However….

- The magnetic helicity current does not actually depend on the existence of large scale field.
- The existence of turbulence and rotation produces a strong flux of magnetic helicity once the local field is in equipartition.
- The inverse cascade **does** depend on the existence of a large scale field, but the consequent growth of the field is super-exponential.



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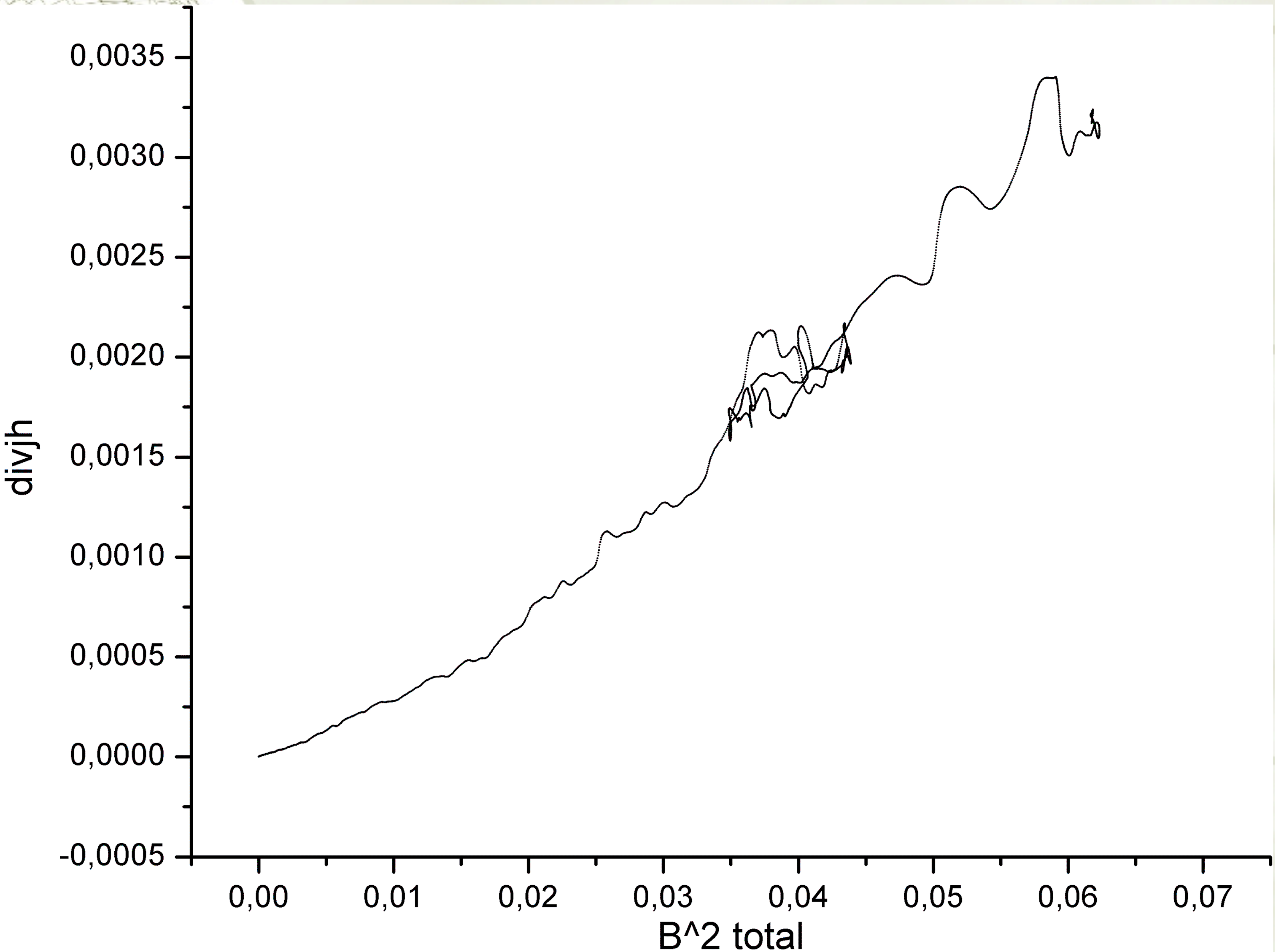
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B^2 total

- In other words, the large scale field is important for the magnetic helicity flux only in a homogeneous background.
- Consequently we expect the galactic dynamo to evolve through five stages:

1. Linear increase in small scale magnetic energy
2. Random walk increase in large scale magnetic field.
3. Coherent driving while h increases linearly.

   (Roughly exp $(t/t_g)^{3/2}$ growth.)

4. Divergence of helicity flux balanced by

   inverse cascade. (Roughly linear growth in field amplitude.)

5. Saturation when $B \sim H\Omega$.

Timescales?

- The buildup of the eddy scale field may require as much as 30-40 eddy turnover times, or $3/\Omega$, roughly half an orbital period.
- The transition to coherent growth occurs at roughly $2/\Omega$ later.
- Saturation sets in after 1 “e-folding time”, or at about $10/\Omega$, roughly two orbits.

Yet more Complications

- While a strong Faraday signal requires only the coherent magnetization of annuli in the disk, local measurements seem to show that many disks have coherent fields with few radial reversals.
- This requires either radial mixing over the life time of the disk - or that the galactic halo play a significant role in the dynamo process.

Conclusions

- The early universe is not responsible for the magnetization of galaxies, and the magnetization of galaxies tells us nothing about fundamental physics.
- Large scale magnetic fields will be near saturation within one large scale dynamo growth time. Attempts to find disk galaxies with sub-equipartition field strengths at high redshift are likely to prove disappointing for the foreseeable future.
- More generally, the exponential growth of large scale fields may never be seen once realistic dynamics is included.