Analytic Studies of DSA

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Why analytic (again) in computer era?

- **Conceptual problems of DSA are possible**

  Hillas '05 review:

  All round, the model of diffusive shock acceleration seems to become more persuasive, though the flatter spectrum predicted at high energies may yet turn out to be a severe Problem for cosmic rays

  …a more steeply falling proton spectrum in the SNR would alleviate the isotropy problem for galactic cosmic rays… → *preferred spectrum* $E^{-2.4}$

  This, though, would involve a drastic change in the pressure balance of cosmic rays in current models of diffusive shock acceleration, in which the most energetic particles play a large role

  → **Challenge to 'low injection– high acceleration efficiency' NL concept**

- **Performance issues**

  Lagage & Cesarsky '83: *maybe too slow*

Way to overcome: trade in efficiency for performance → Spectral break!
Steeper spectrum at HE, less back reaction, shorter CR precursor (beneficial in terms of observations, S. Reynolds SNR1006)
→ more rapid acceleration

Diffusive Shock Acceleration

Trilogy

- Injection
- Acceleration
- Escape

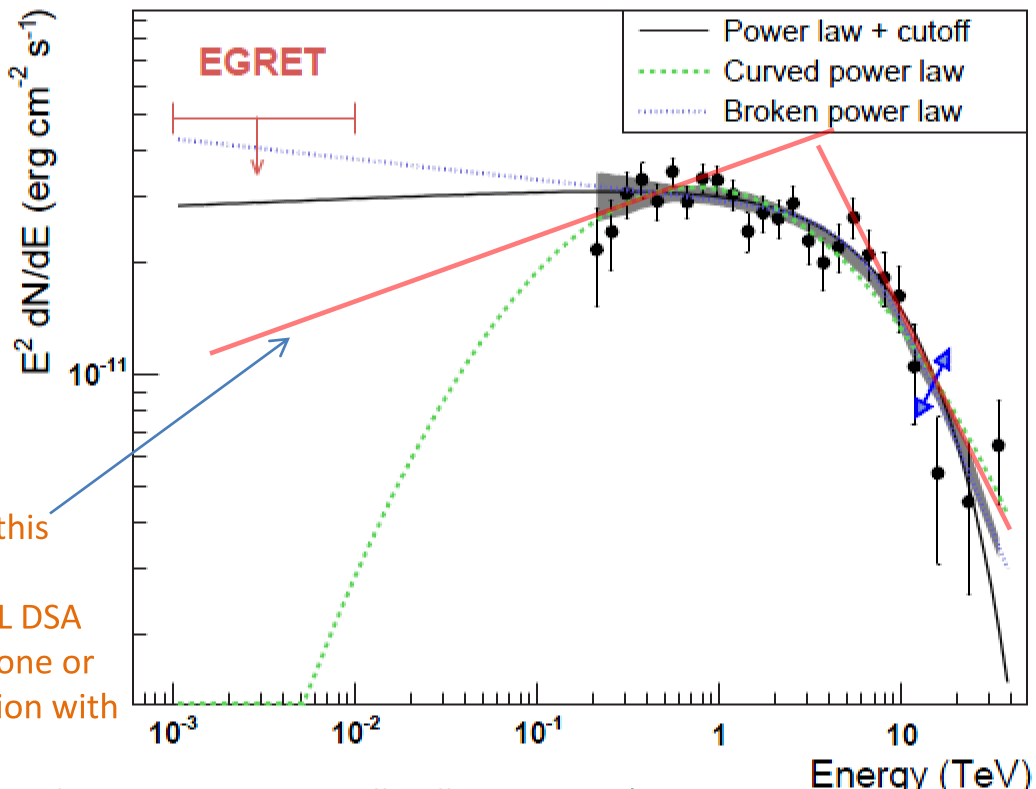
All three processes are strongly interrelated

new study:
overlap if acceleration and escape regions in momentum space
Escape as a direct result of acceleration, not of external conditions

→Phase space fragmentation :gyro-phase (normally averaged out) is as Important as pitch-angle and momentum

→ Spectral break

Tentative evidence for the break



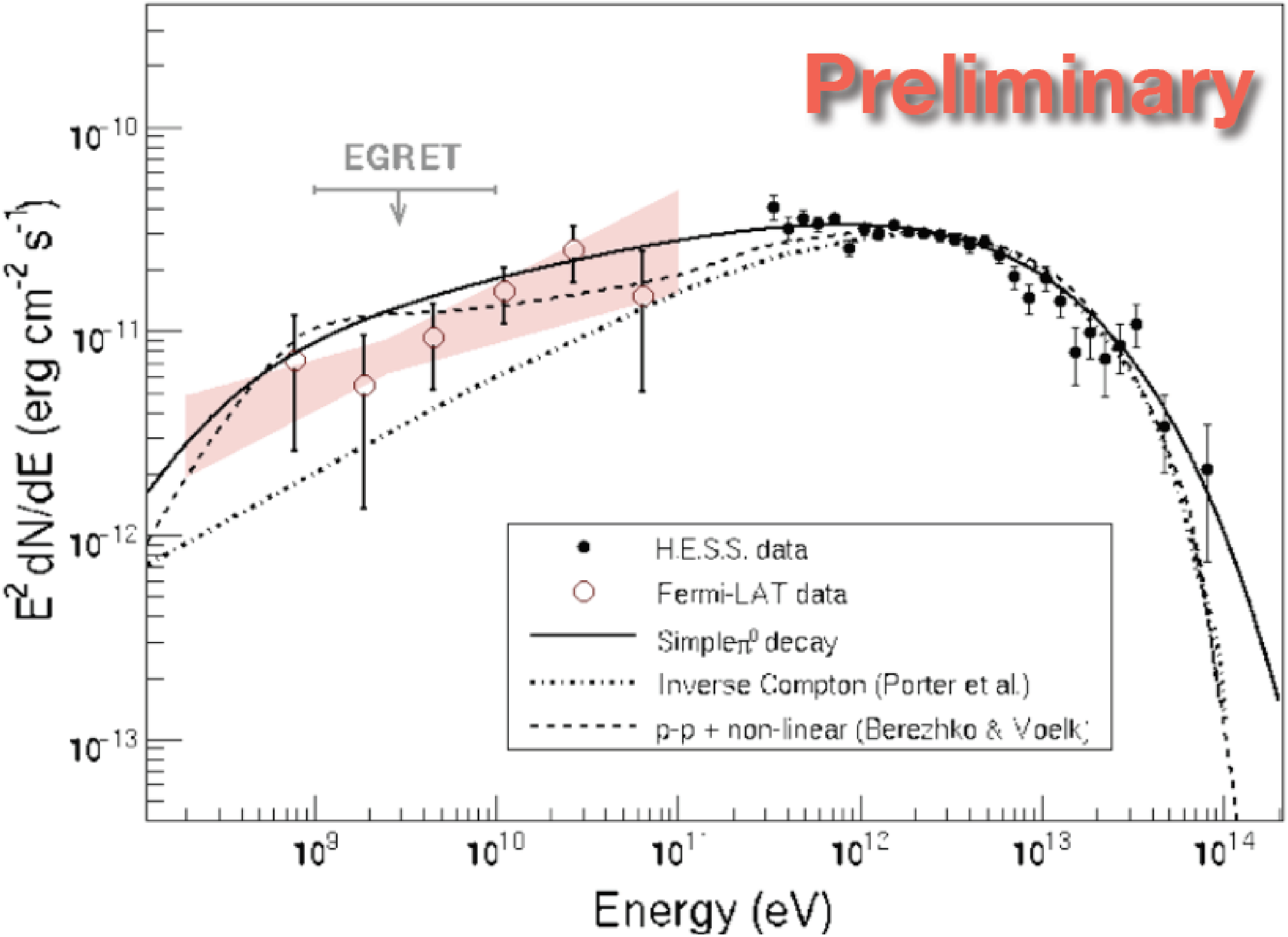
If Fermi reveals something like this

→triumph of NL DSA over the linear one or
→Just a confusion with IC scenario?

(Reynolds, Vink, Uchiyama , Acero , Voelk talks on Fri pm)

Aharonian, F. and HESS team (2006). A detailed spectral and morphological study of the gamma-ray supernova remnant RX J1713.7-3946 with HESS. *Astronomy and Astrophys.*, 449:223–242.

From S. Funk talk, Fermi Symposium Nov 2009

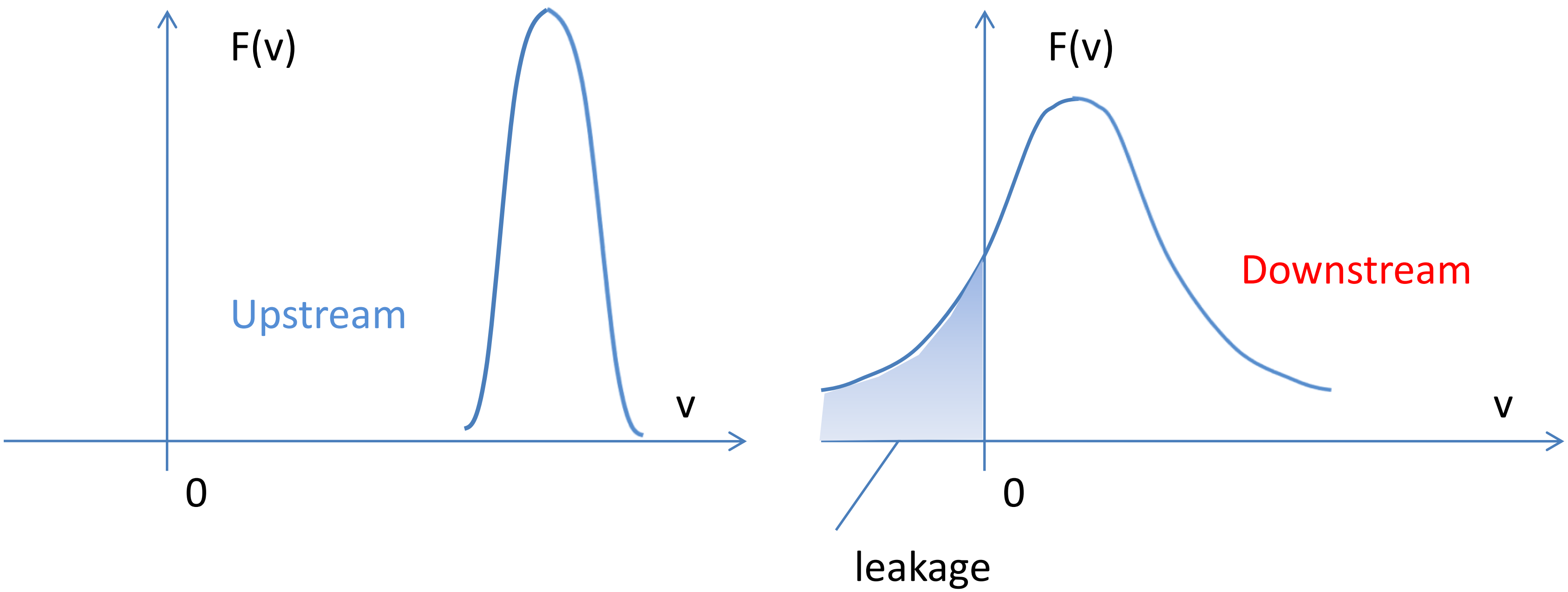


Part I: injection

Proton thermal leakage:

(e-injection—separate story: Laming, Amano, Hoshino Thu am)

Particle velocity distribution, shock frame



Leakage rate critically depends on:

- heating upon shock crossing → collisionless shock mechanism
- Shock energy left from HE particle acceleration → shock modification

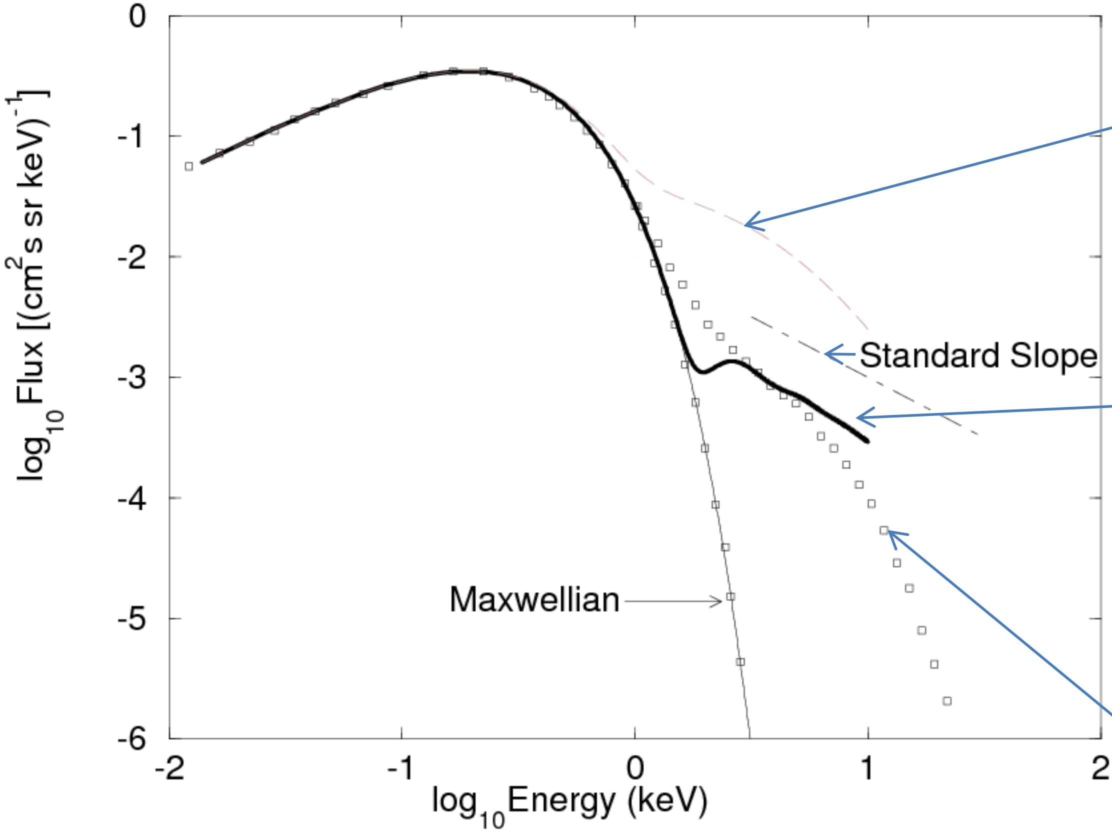
Injection: physical phenomena to include in calculations

- ❑ back-reaction of the over-injected particles on the flow; modified flow → suppression of injection (MC scenario)
- ➢ taken in isolation → overinjection

  Example: Earth's bow- and IP-shocks are not modified, yet injection is in check

- ❑ calculate scattering **self-consistently**:
- ➢ Leaking particles drive a coherent, quasi-monochromatic MHD-wave upstream that, being convected (and compressed) downstream, traps supra-thermal particles and suppresses leakage by ~90%
- ❑ thermal pool cooling due to injection: included (along with the above two items) in hybrid (numerical/analytical) advanced schemes (Kang, Jones, Ryu, Gieseler)

Injection: comparison with hybrid simulations (no significant shock modification)



Without trapping effect

Quasi-monochromatic wave Driven by leaking particles (self-Consistent calculations with trapping)

Bennett & Ellison 95 Hybrid code

Injection bottom line

- Generates correct spectral slope (consistent with the standard DSA predictions at higher energies where the distribution becomes isotropic and the diffusion-convection equation may be applied)
- Considerable overlap of injection and 'standard' DSA spectra

  → artificial 'injection momentum' is no longer required (smooth transition)
- Successfully benchmarked to Hybrid simulation with no free parameters (only downstream thermal fit)
- Clear self-regulation mechanism: too strong injection→ big wave, strong trapping → weaker injection
- Limitation: Q-parallel shock

Part II: acceleration

NL shock response to particle injection/acceleration

(long known in HD-two fluid approx: Axford, Leer and Skadron '77, particularly in Drury and Voelk '81)

Kinetetic treatment:

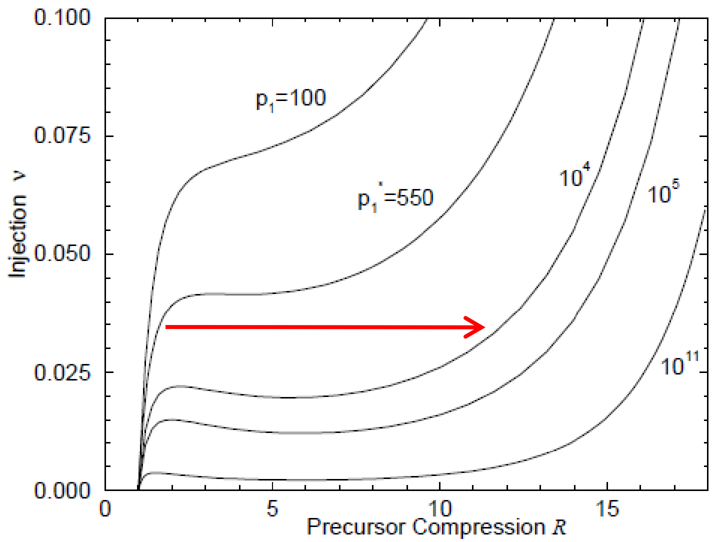


FIG. 1.— The nonlinear response of an accelerating shock (characterized by the precursor compression R) to the thermal injection ν given in the form of the function $\nu(R)$ calculated for the fixed injection momentum $p_0 = 10^{-3}$, Mach number $M = 150$ and for different $p_1 = 100; 550; 10^4; 10^5; 10^{11}$. The critical value (see text) $p_1^* = 550$. The TP regime is limited to the region $R \simeq 1$.

- consider injection as a control parameter
- flow modification (acceleration efficiency) as an order parameter
  - ➢ phase transition to high efficiency acc'n regime (velocity gradient)
  - ➢ new (acoustic) instability follows

Can the calculated injection rate stay the same if the compression strongly increases? NO (sub-shock reduction)

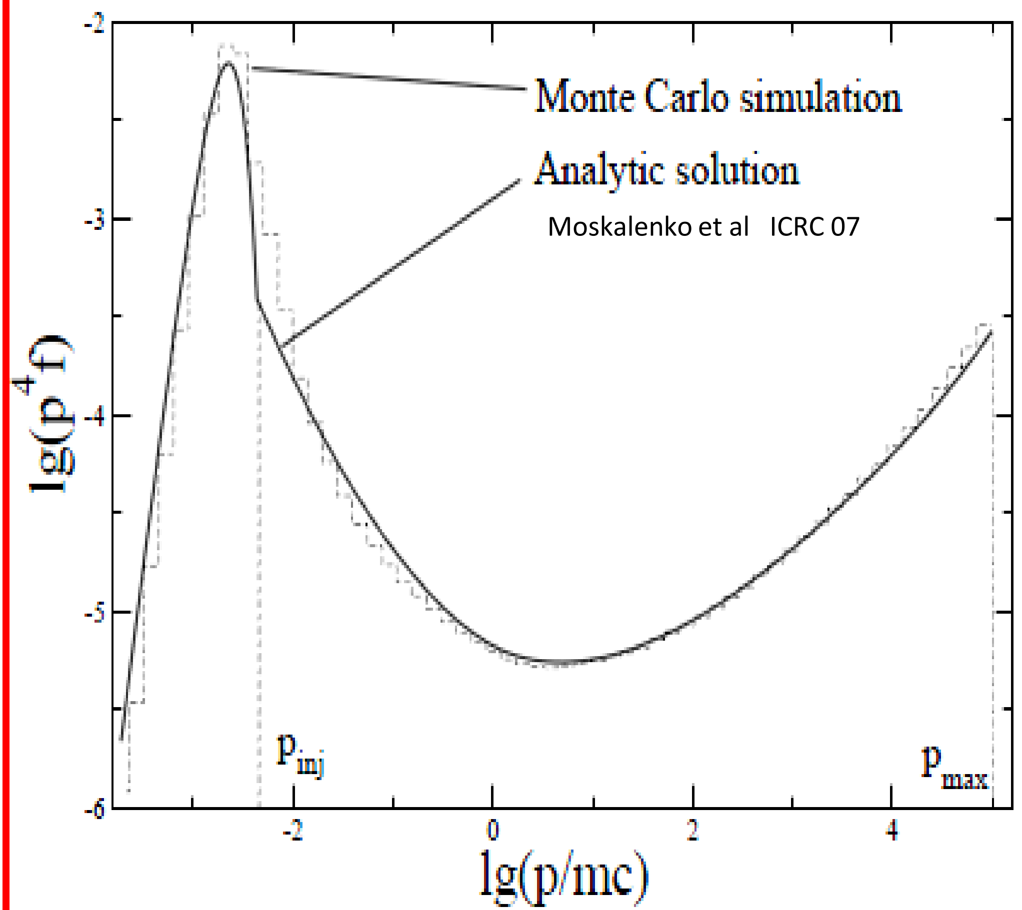
Is solution multiplicity real? YES, if the injection is fixed (Contr. Par.)

Solution multiplicity: Evidence #1

The same analytic solution that points at multiplicity and bifurcation, produces absolutely correct spectrum

MC Simulations:

Ellison, D. C., Berezhko, E. G., & Baring, M. G. 2000, *Astrophys. J.* , 540, 292



Analytic solution: NL integral equation

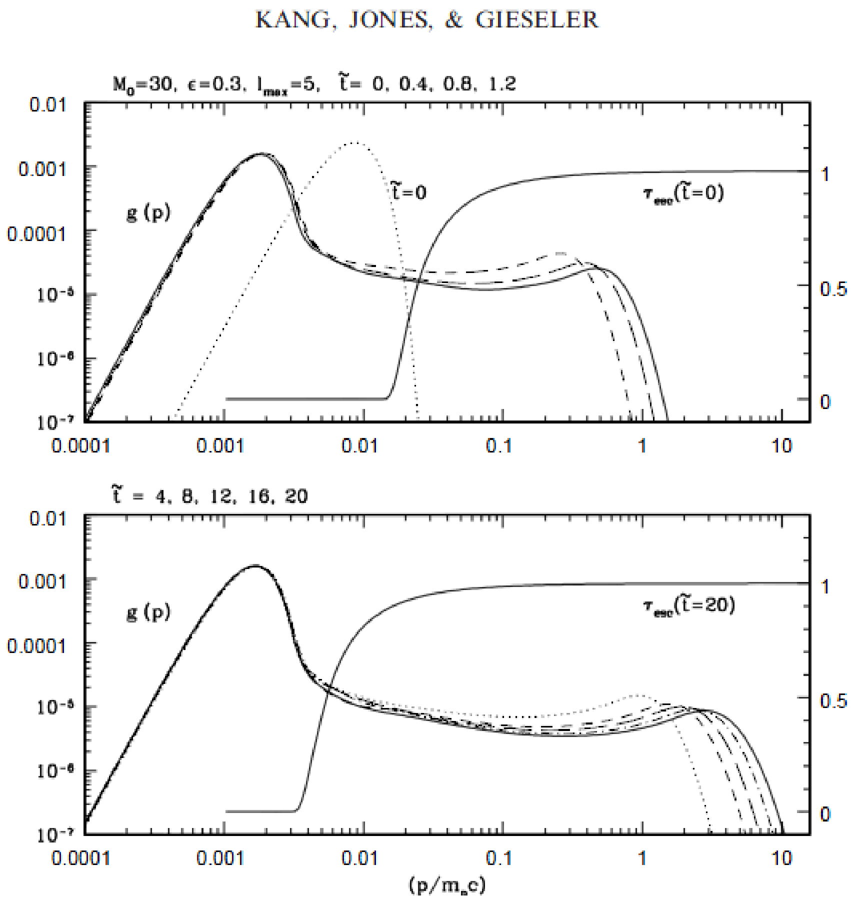
→treats particle spectrum and the shock flow structure self-consistently MM '97

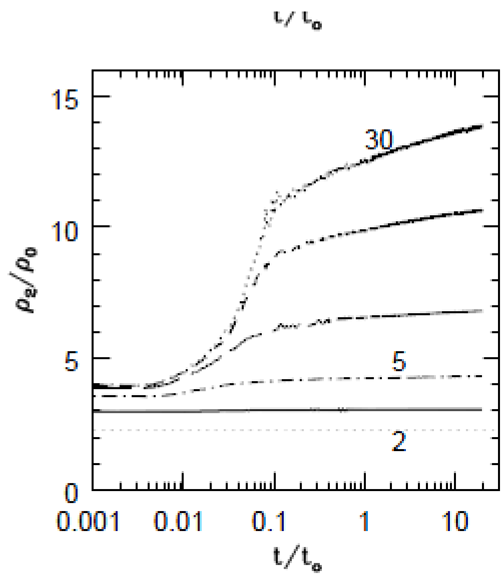
Different approximations:

→ similar results: Blasi, Gabici, Amato, Vannoni, Reville, Kirk, Duffy 2002-2009

Evidence #2

Bifurcation of the acceleration regime (phase transition) in time dependent numerical solutions

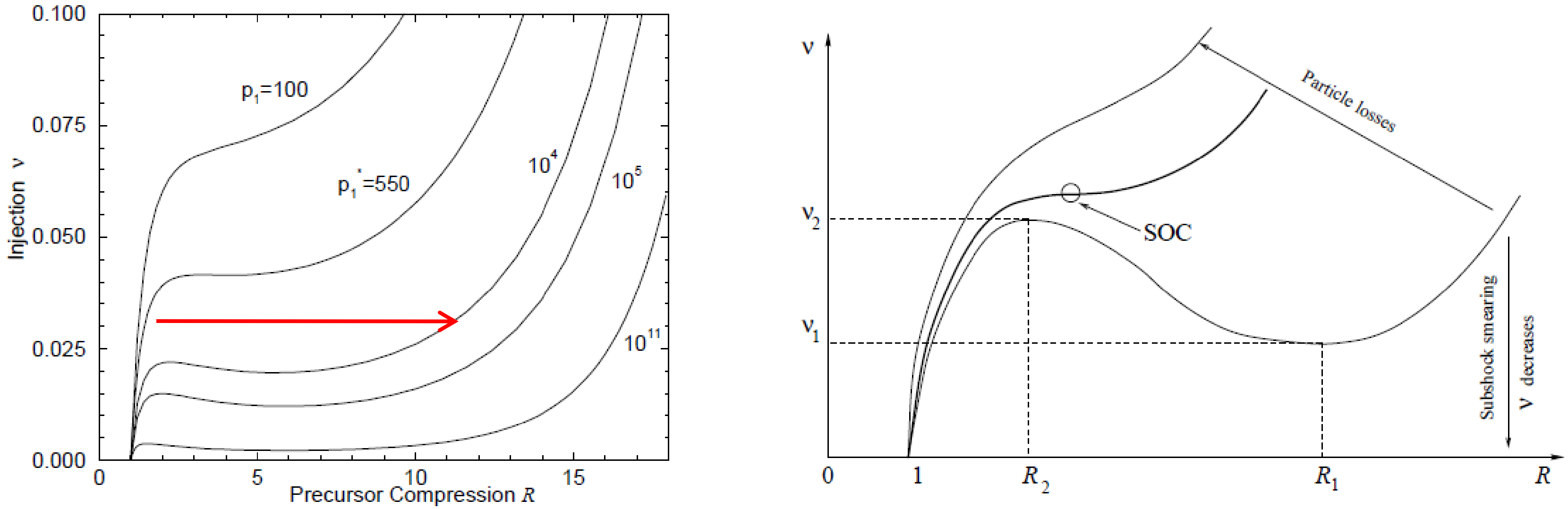




NL shock response to particle injection/acceleration

Self-organization of acceleration/shock structure

→ ~50% acceleration efficiency (CR/shock ram pressure)



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NL sub-shock (injection) reduction, enhanced particle losses at HE's

→weaker NL response of the shock structure to acc'n

→ S-curve straightening → critical self-organization (SOC)

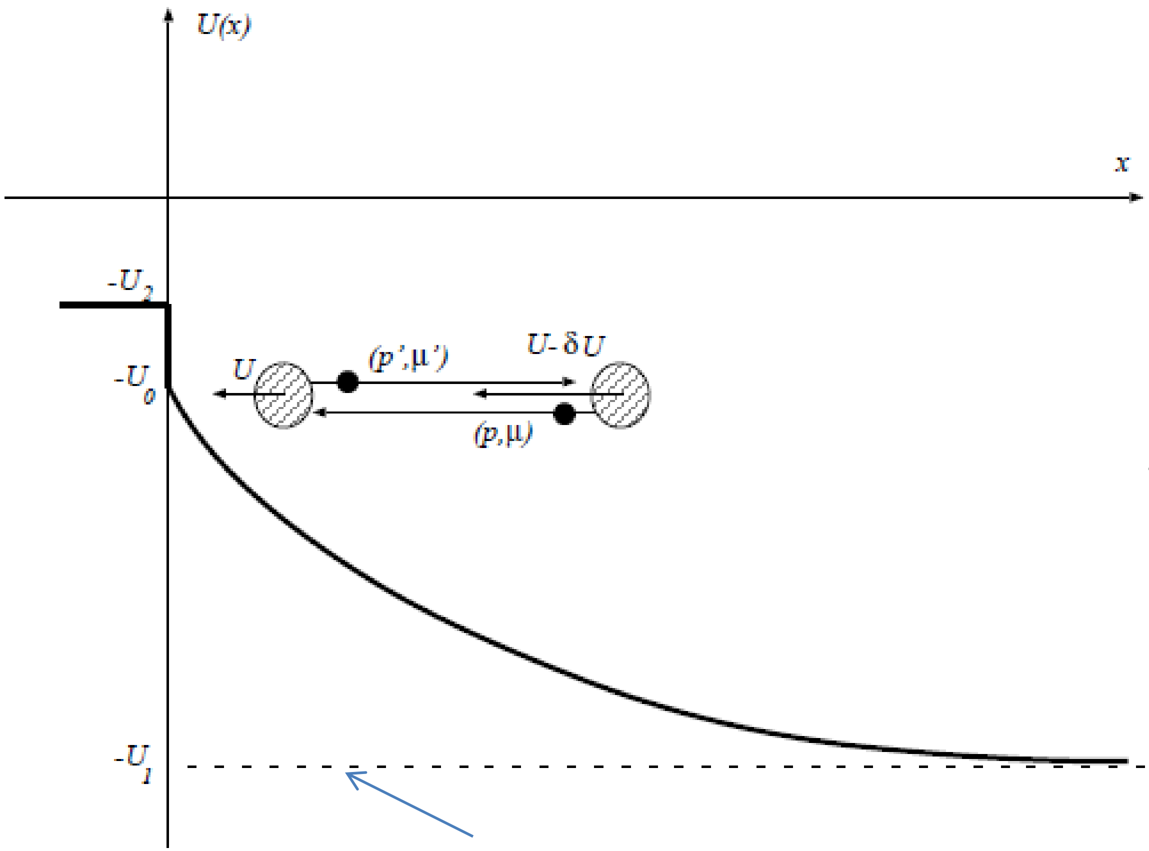
Bonus: faster acceleration process

Part III: escape and spectral break

Reconsider acceleration mechanism after the phase transition, Physically similar to the standard DSA

Krymsky '77, Axford et al '77
Bell 78
Blandford and Ostriker 78

But: acceleration in the CR precursor

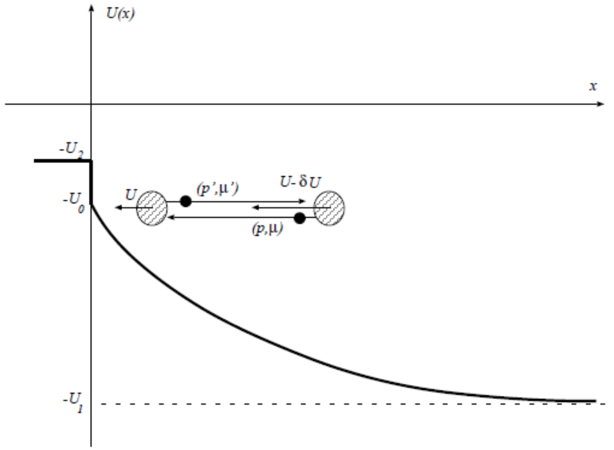


Unmodified classical shock

CRs communicate information upstream (akin to ionizing front, radiative shock)

Two basic ways of communication
- upstream plasma instabilities
- upstream flow modification

Momentum gain



$$\delta U \ll c$$ (+ isotropy in pitch-angle μ)

after one shock crossing cycle

$$\frac{\langle \delta p \rangle}{p} \simeq 2\frac{\delta U}{c}\langle \mu \rangle = \frac{4}{3}\frac{\delta U}{c}$$

Discontinuity crossing

$$\frac{\Delta p}{p} = \frac{4}{3}\frac{(u_1 - u_2)}{c}$$

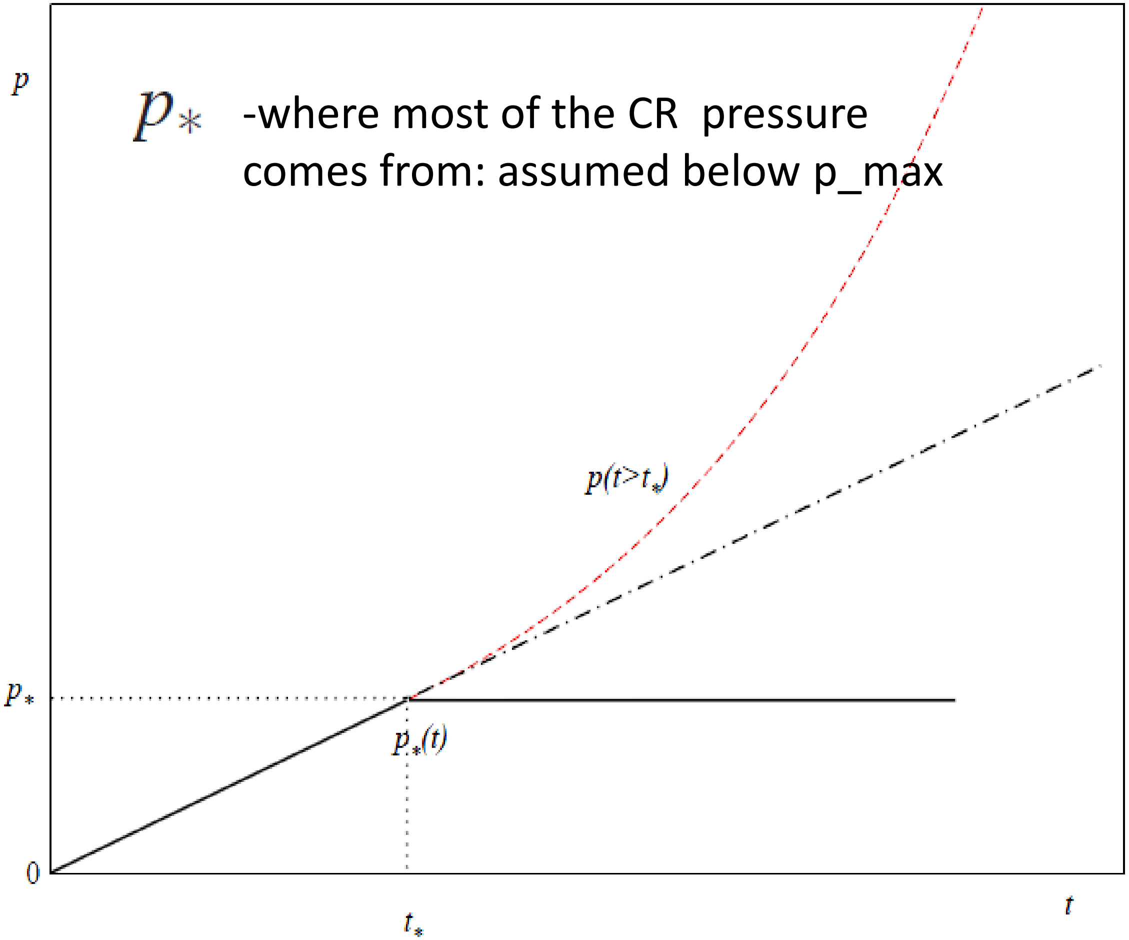
$$\frac{dp}{dt} \equiv \dot{p} \simeq \frac{\langle \delta p \rangle}{\langle \delta t \rangle} = \frac{1}{3}p\frac{\delta U}{\lambda}$$

$$\delta U \simeq -\lambda \frac{\partial U}{\partial z}$$

$$\dot{p} \simeq -\frac{1}{3}p\frac{\partial U}{\partial z}$$

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\kappa\frac{\partial f}{\partial z} = \frac{1}{3}\frac{\partial U}{\partial z}p\frac{\partial f}{\partial p}$$

New approach: acceleration within (smooth) precursor



Linear acceleration time

$$\tau_{acc} \sim L_{dif}(p)/U_1$$

slow due to idling U/D and infrequent shock crossing

NL acceleration time

$$\tau_{acc} \sim L_p/U_1$$

slow due to precursor growth, But:

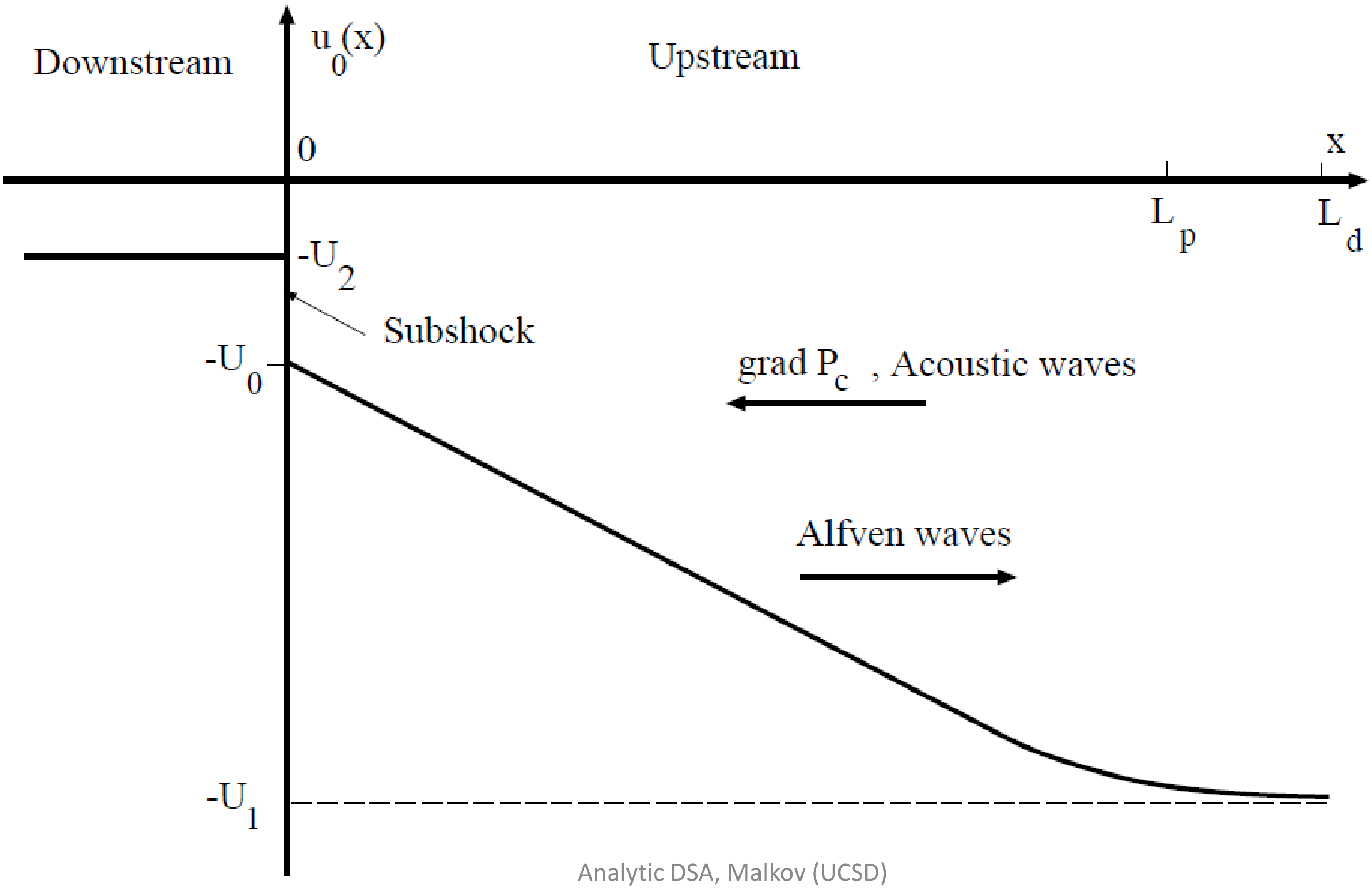
$$L_p \sim \kappa(p_*)/U_1$$

Acceleration dose not slow down (in smooth part of the shock transition) for

$$p > p_*$$

However, p_* must not grow any further!

Instabilities, important for particle transport in CRP



Instabilities

- cyclotron resonance, Alfven waves, $k_{\|} v_{\|} \approx \Omega_c(p)$, i.e., $k \sim r_g^{-1}(p)$ →Bell '78
- nonresonant (firehose): maximum growth in a very short wave range (not good for particle scattering)

→Achterberg '83, Shapiro and Quest '98, Bell and Lucek '01, 04, Reville et al 08

- →hydrodynamic, CR pressure gradient driven (Drury's) instability

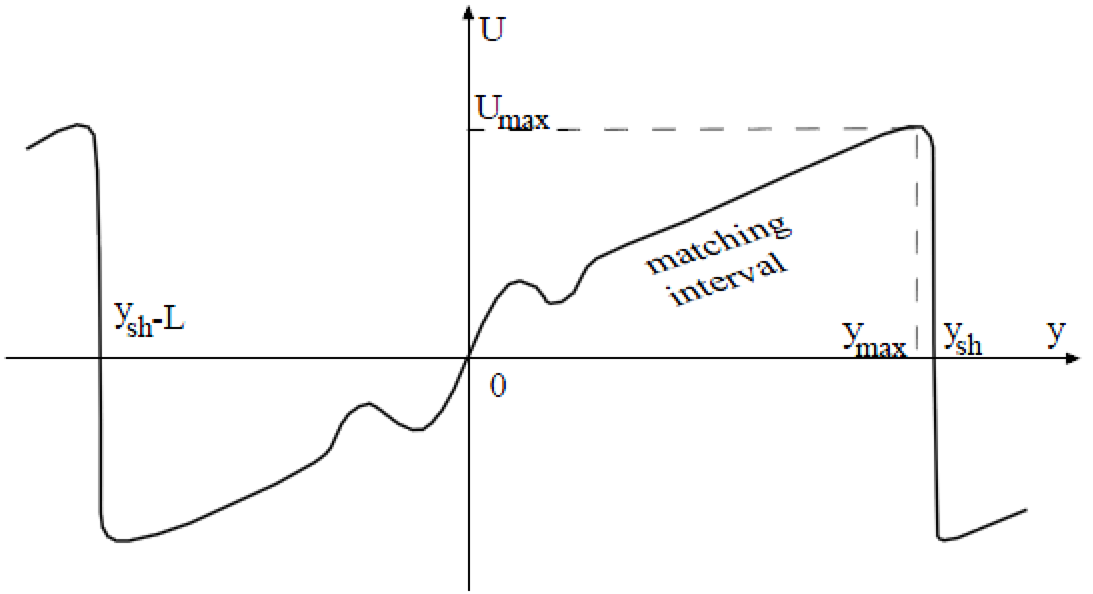
→Drury 84, Drury and Falle 86, Zank et al 90, Kang, Ryu and Jones 92…

Advantages:

- drive all wave numbers, $\gamma(k) \approx const$
- insensitive to CR distribution function
- staibilizes only nonlinearly (not quasi-linearly)
- long scales, much needed for particle confinement are naturally produced → Diamond, preceding talk

Traveling wave solution driven by acoustic and cyclotron instabilities

$$\frac{\partial \hat{\rho}}{\partial t} + \hat{\rho}\frac{\partial \hat{\rho}}{\partial \zeta} - \gamma \hat{\rho} - \mu \frac{\partial^2 \hat{\rho}}{\partial \zeta^2} = Q\left(\zeta - vt\right)$$

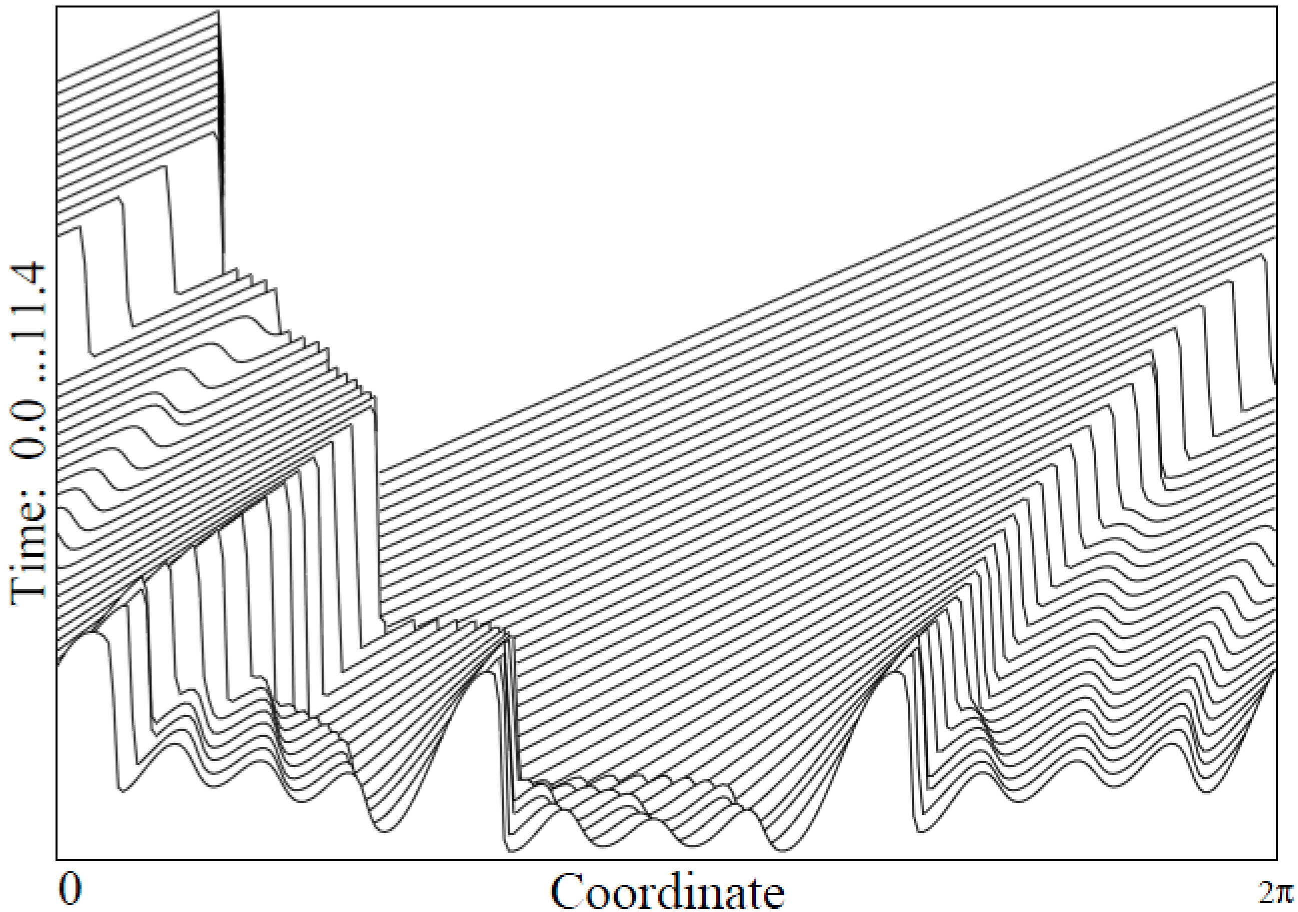


More general, 'magnetic' version of this solution but with a cyclotron-unstable driver only (no acoustic instability term)

→ Kennel et al JETP Let. '88,

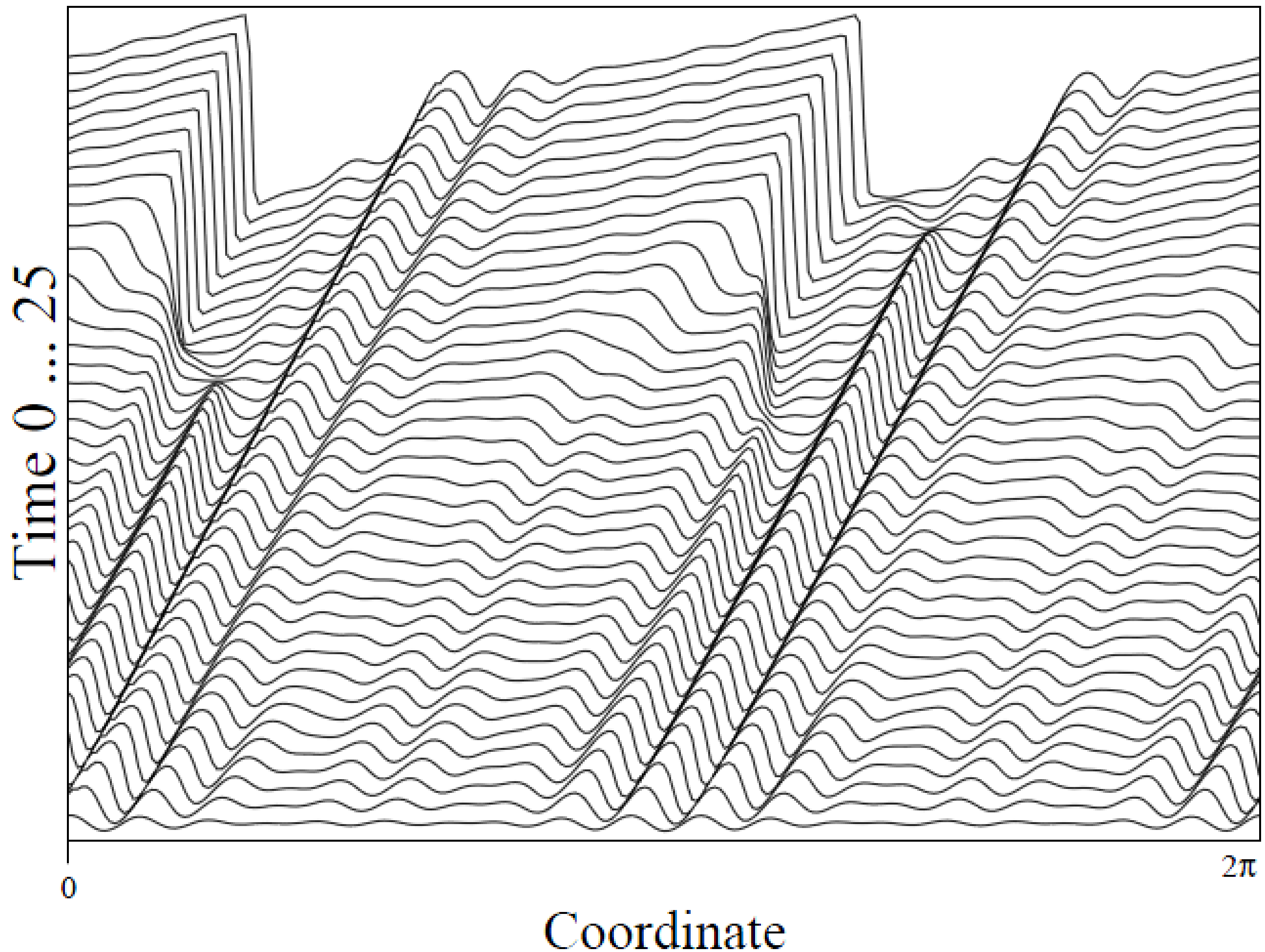
→ MM et al PFL '90

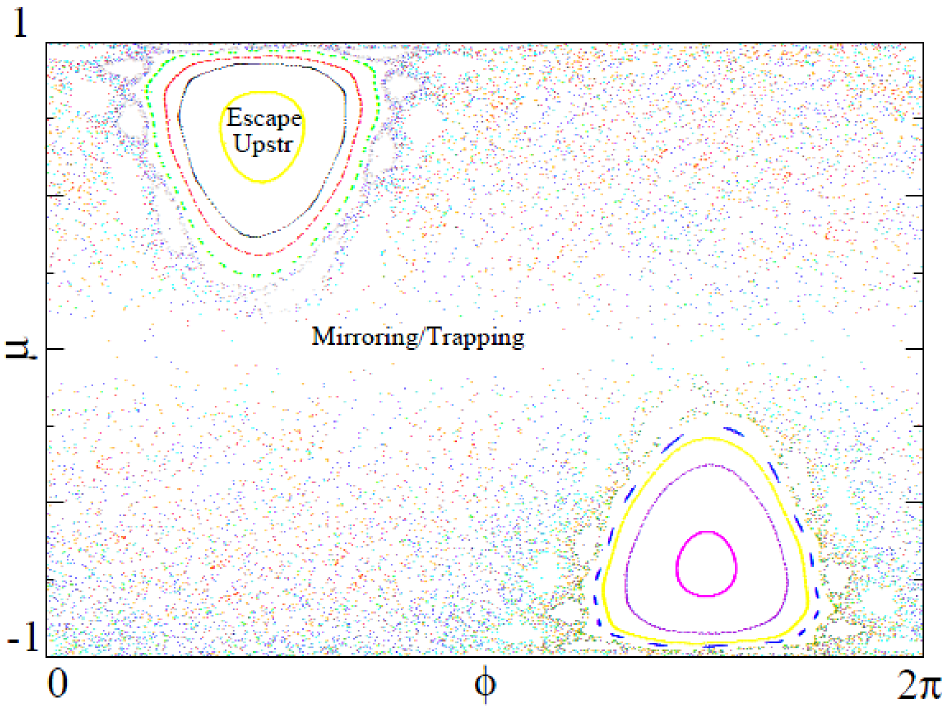
Numerical verification of the traveling wave solution (acoustic instability only)



Initial perturbation profile steepens into 3 relatively weak shocks
They merge to form one strong shock

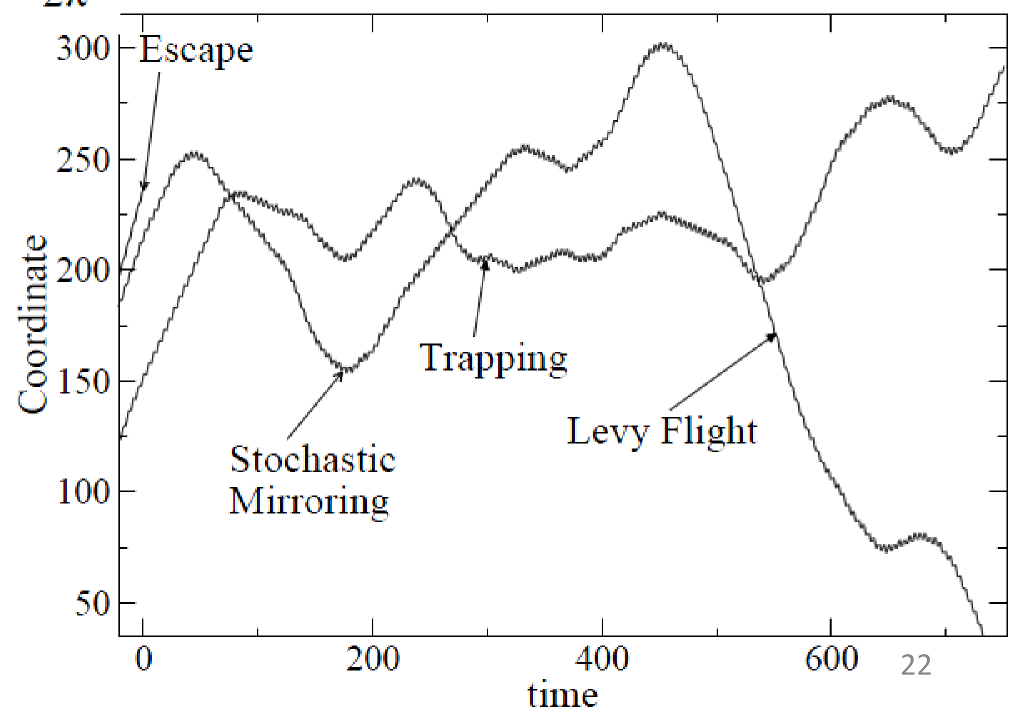
Numerical verification of the traveling wave solution (acoustic instability +IC instability)





Pitch-angle/Gyro-phase
Poincare map
(Pitch-angle wrt shock normal, 45 deg here)

$r_g(p)/L \simeq 3$



Particle trajectories

Particle spectrum

For particles with momentum below the break $p=p_*$ the spectrum should be determined From nonlinear self-consistent solution of kinetic and HD equations.

Above the break at $p=p_*$ the spectrum can be approximated by a test particle solution (no significant contribution of those particles to the CR pressure)

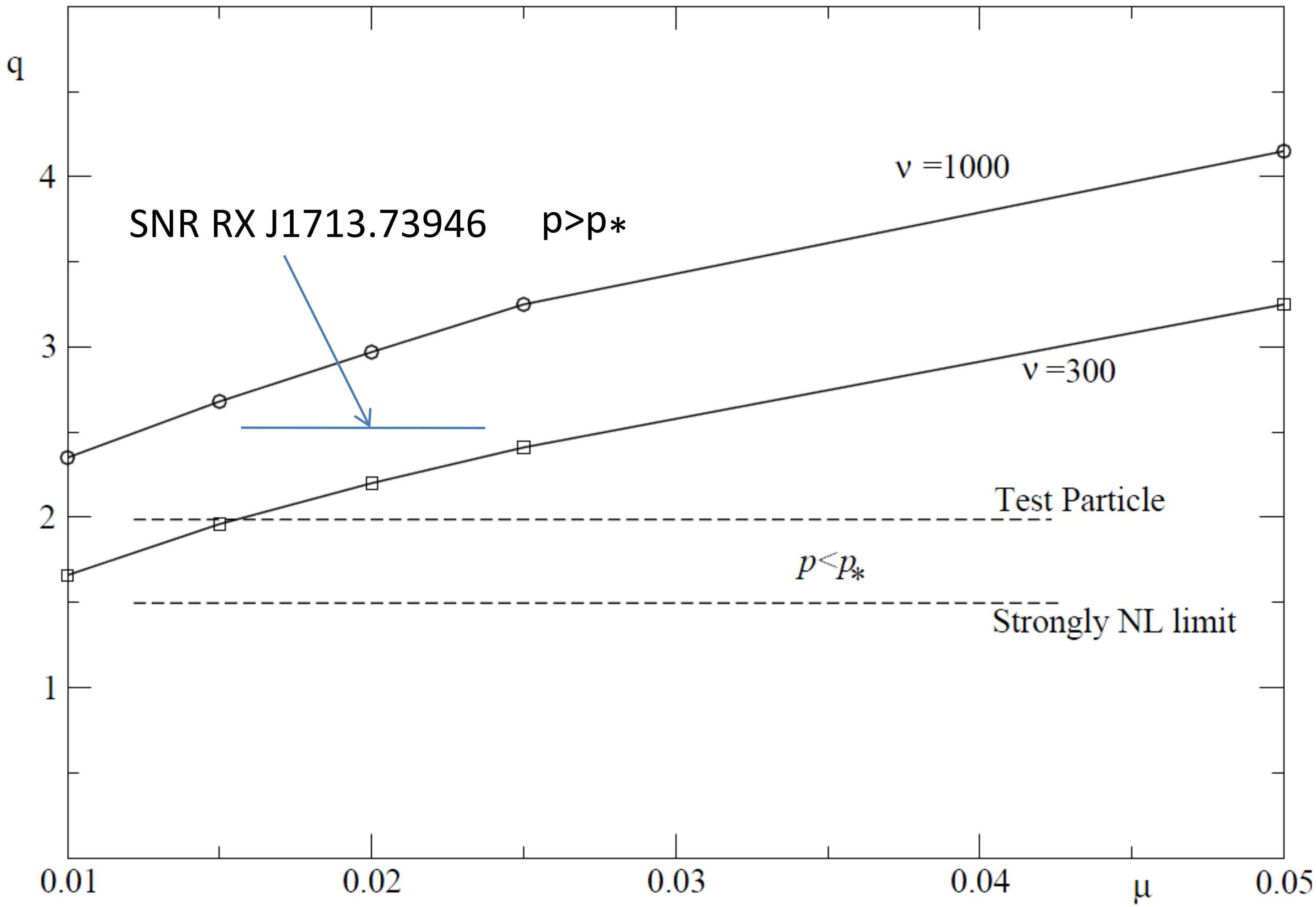
Fermi '49 general spectral index

$$q = 3 + \tau_{acc}/\tau_{conf}$$

$$q_e = 3 + \frac{\ln\left[(U_+/U_0)\left(\mathscr{P}_L^2/\mathscr{P}_{tr}\right)\right]}{K(\vartheta)\ln(1/\mathscr{P}_L)}$$

$\mathscr{P}_{tr}$ Trapping probability

$\mathscr{P}_L$ Detrapping probability (Levy flight)



$$\nu = \frac{U_+ \beta}{U_0 \alpha_+} \simeq \frac{U_+}{U_0} \frac{\tau_L \tau_L^+}{\tau_{tr}^2} \sim \frac{c}{U_0} \gg 1$$

$$\mu \equiv \frac{\beta \kappa_0}{2\pi \eta^2 p_0 p_* (U_0 - U_2)^2} .$$

Conclusions

• acoustic instability is robust (compared to cyclotron and firehose/mirror) in that it is hydrodynamic in nature and cannot be stabilized by kinetic (e.g. quasilinear) effects (isotropization, trapping) or by the modulational instability (as Alfven waves)

• magnetic shocktrains trap and mirror particles
→quick isotropization of momentum distribution →suppression of other instabilities

• shock merging generates longer scales
– crucial for confinement of highest energy particles
– prevents the magnetic energy from rapid damping

• shock merging (3D) generates vorticity → magnetic field amplification

• almost independent of the cyclotron instability, the acoustic instability creates a more efficient scattering environment which substantially improves particle confinement and enhances particle acceleration

• the spectrum of accelerated particles is softer than in a 'standard' (resonant waves, QL) theory