Multi-Scale Interaction in Diffusive Shock Acceleration

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Outline

i.) Short Commentary on DSA Theory

- Very Brief Overview
- Two Critical Issues in DSA → Multi-Scale Interaction

ii.) #1: Confinement on Mesoscales $(kr_g < 1)$

primary focus

→ Inverse "Cascade" of $\langle \tilde{B}^2 \rangle_{\mathrm{k}}$ via Modulational Instability

- Alfven Wave Packet Dynamics in Shocklet Scatterer Field
- Stochastic Refraction / "Inverse Cascade" Induced Amplification of Precursor Density Fluctuations

→ Quasi-steady and Unsteady States

P. D. and Malkov
Ap. J. '07

iii.) #2: Enhanced Acceleration Efficiency (the knee and beyond…)

relation to (ii.)

- Classical Fermi → "Pinball Process"
  i.e. scattering by shocklets in convergent precursor flow
- Confinement → Trapping and Loss Islands in Phase Space → {Fractional kinetics

→ DSA not entirely "D"

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iv.) Where Next?

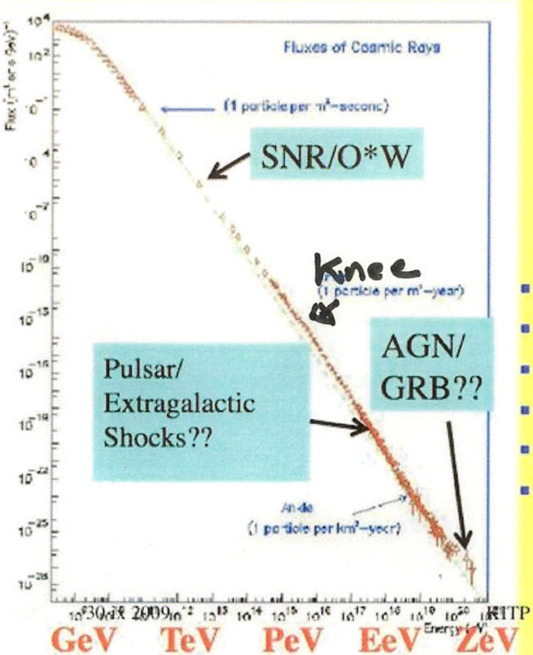
- "Radiation Hydro" Theory of Drury Instability
- Analysis of System States

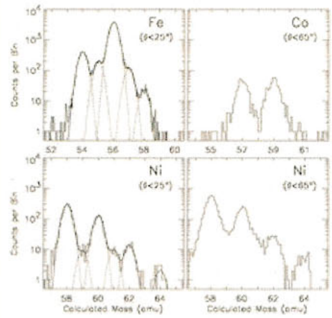
and ?

N.B.: Observations at the Outset

- 2 fluctuation populations
  - $kr_g \sim 1$ (small scale) → resonant Alfven waves
  - $kr_g \ll 1$ (mesoscales) → shocklets
- ∇P_{CR} is crucial drive
- Shocklets directly link
  - Enhanced acceleration frequency
  - "Inverse Cascade"

Cosmic Ray Spectrum





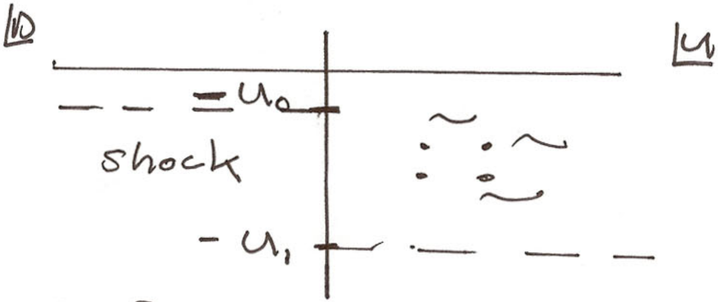
- $N \sim E^{-2.6}$ below "knee"
- $U_{CR} \sim U_{GeV} \sim U_{ISM} \sim U_{CMB} \sim U_{star}$
- 0.1 Myr $< t <$ 15 Myr
- L/M $\lambda \sim 5\ E_9^{-.3}$ g cm^{-2}
- $\Rightarrow S \sim E^{-2.3}$
- $L_{CR} \sim U_{CR} M_{gas}\ c\ \lambda^{-1}$
  $\sim 3\times10^{33}W \sim 0.03 L_{SNR} \sim 10^{-3} L_{gal}$

Can SNR accelerate up to PeV "knee"?

→ "knee"
→ spectral break $\sim 10^{15}_{eV}$
→ can DSA
– accelerate particles to knee?
– explain/encompass knee?

VG. by R.D. Blandford

Usual Story of DSA - I.)



Shock + Confinement
(circa late 70's)

Confinement:

- resonant pitch & scattering by CR-emitted AW's
- resonance: $kp/m = \Omega_g$?
  high energy $\Leftrightarrow$ large scale $\Leftrightarrow$ $\left\{ \begin{array}{l} k r_g \sim 1 \\ \text{(max growth)} \end{array} \right.$
- scattering: $D \sim c^2/\nu_{eff}$
  $\nu_{eff} \sim \Omega \, (\delta B/B_0)^2$
  typically $\delta B/B_0 \leq 1$.

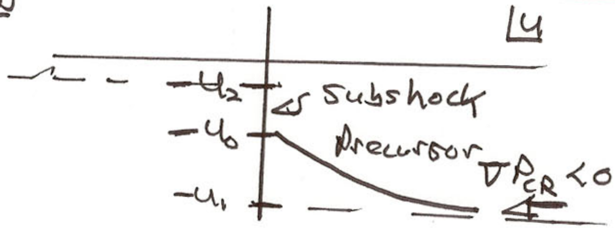
$\left\{ \begin{array}{l} \text{efficiency ?} \\ 1/\tau_{acc} \sim u_{sh}^2/D \end{array} \right.$

- convection-diffn. model

$$\partial f/\partial t + u \, \partial f/\partial x - \frac{\partial}{\partial x} D \frac{\partial f}{\partial x} = + \frac{1}{3} \frac{\partial u}{\partial z} p \frac{\partial f}{\partial p}$$

p

Usual Story II – Self-Consistent Shock
+ Precursor ('80's →)



→ discontinuity
+
CR cloud (P_{CR})
(⇒ flow)

Fluctuations:

- AW's → high frequency, (micro)
($k r_g \sim 1$)

- (magneto) acoustic modes ⇒ "shocklet trains/turbulence"
→ low frequency (meso)

$\boxed{\dfrac{r_g}{L_{pre}} < k r_g \ll 1}$

How? – linearly, $\boxed{\text{Drury Instability}}$

$$\gamma_D^{\pm} = -\underbrace{\frac{\gamma_c P_{CR}}{\rho D}}_{\text{(diffn)}} + \frac{P_{CR}}{c_s \rho}\left(1 + \partial \ln D / \partial \ln \rho\right)$$

– or ---- ?

↔ role in confinement ?

$\boxed{r_g < \ell < L_{pre}}$
mesoscale range

→ Issues in DSA ?

① – how confine high energy particles ?

⇒ require populating larger scales AW range

$k\,\rho/m = \Omega$

② – how improve the efficiency of acceleration ?

③ – is DSA "D" ? – if not, how represent physics?

① how generate AW's with $k r_g < 1$?

⇒ requires spectral energy transfer to long wavelength

– counter to lore of MHD Alfven cascade

② { increased P_{CR} "inflates" shock structure
time to cross ↑ with energy

⇒ exploit precursor ?

③ speculations ...

A Path Forward → Multi-Scale Interaction

– Two Component Turbulence (á la Langmuir Turbulence)

Acoustic modes Alfven waves

{stochastic refraction}

$k > 1/L_{pre}$ $\qquad$ $1/L_{pre} \ll k < 1/r_g$

{ linear – Drury Mech.
nonlin. – pumped by AW

stochastically refract/modulate Alfven waves

{ resonant growth
scattered/modulated by acoustics ⇒
decay instability

$k r_g \sim 1$
population inversion

– upshot:

→ pumping of wave population on mesoscales

→ excitation of large scale "Drury" Modes by decay (vitiates need for robust Drury inst.).

→ Key Physics I: Stochastic Refraction

- Consider Alfven wave packet in field of acoustic scatterers:

$\omega = k v_A$

$$\boxed{\frac{dk}{dt} = -\frac{\partial}{\partial x} k v_A \cong \frac{k v_A}{2} \frac{\partial}{\partial x}(\tilde{\rho}/\rho_0)}$$

{ scatterers: density bumps ("shocklets")

∴ For random ensemble shocklets, diffusion in k ⇒ spectral spreading

i.e.

$$\boxed{D_k \cong \frac{k^2 v_A^2}{4} \sum_q |\tilde{\rho}_q/\rho_0|^2 q^2 \tau_{cq}.}$$

{ Diffusivity for stochastic Refraction

∴ - population density flux:

$\Gamma_N = -D_k \, \partial\langle N\rangle/\partial k$ ⇒ { populates $k r_g < 1$ from inversion.

- $\frac{d\mathcal{E}}{dt} < 0$ (pop/n. imbalance)

Key Physics II : Ponderomotive Feedback
(Stochastic Analogue to NLS)

- How does stochastic refraction pump large scales?

$$\frac{\partial \tilde{v}}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}\left(c_s^2\tilde{\rho} + \tilde{P}_{rad}\right)$$

$$\frac{\partial \tilde{\rho}}{\partial t} = -\rho_0 \frac{\partial \tilde{v}}{\partial x} \qquad P_{rad} = \sum_{\underline{k}} \omega_{\underline{k}} \left(N^+ + N^-\right)$$

$$\Rightarrow \quad \omega^2 - q^2 c_s^2 = q^2 \sum_{\underline{k}} \frac{\omega_{\underline{k}}}{2\rho_0} \, \partial q k V_A \, \tilde{\tau}^{\pm}_{c,q,k} \frac{\partial \langle N^\pm \rangle}{\partial k}$$

$$\boxed{\gamma^{\pm} = \frac{q^2}{4\rho_0} \frac{V_A}{c_s} \sum_{\underline{k}} k\,\omega_{\underline{k}}\, \tilde{\tau}^{\pm}_{c,q,k} \frac{\partial \langle N \rangle}{\partial k}}$$

- $\tilde{\tau}_{c\,kq} \sim \Delta\omega_{\underline{k}} / q^2 v_{gr}^2 + \Delta\omega_{\underline{k}}^2$

- $\partial \langle N \rangle / \partial k > 0$ required for maximal $\tilde{\tau}_{c\,q,k}$

- NL pumping 'cascades' downward (i.e. $\gamma^{\pm} \sim q^2$)

Energetics :

$\rightarrow$ stoch. scattering by acoustics

$$\rightarrow \frac{\partial \mathcal{E}_{AW}}{\partial t} = \int dk\, \omega_k \frac{\partial}{\partial k} D_k \frac{\partial}{\partial k} \langle N \rangle_{AW}$$

$$\cong - \int v_{gr}\, D_k \left[\left| \frac{\tilde{\rho}}{\rho_0} \right|^2 \right] \frac{\partial \langle N \rangle_{AW}}{\partial k}$$

$\hookrightarrow$ acoustic field

$$\frac{\partial \mathcal{E}_{AC}}{\partial t} = \int dq\, \gamma_q \left[\frac{\partial \langle N \rangle_{AW}}{\partial k} \right] \left| \frac{\tilde{\rho}}{\rho_0} \right|^2$$

$\hookrightarrow$ AW wave field.

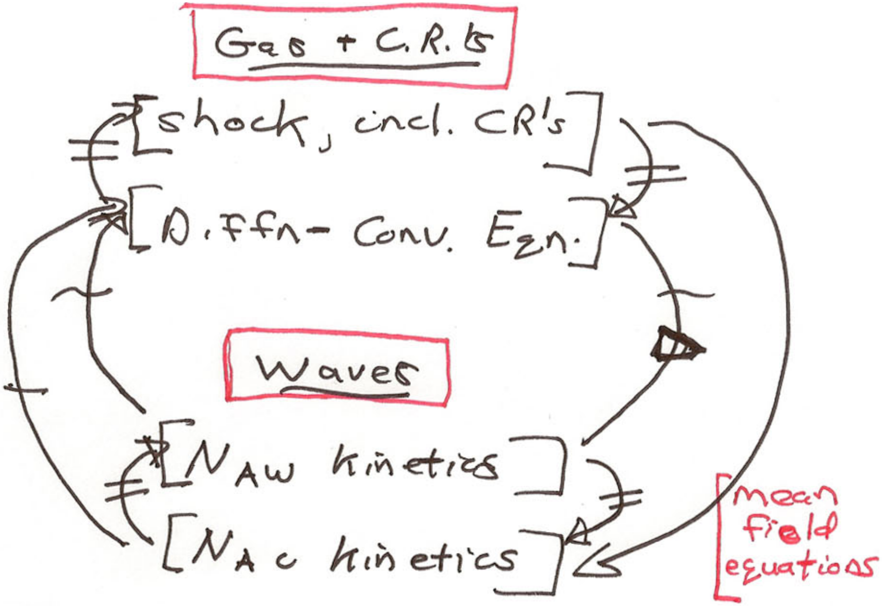
Books balance $\rightarrow$ Manifestly ...

$$\partial_t \left(\mathcal{E}_{AW} + \mathcal{E}_{AC} \right) = S_0 + S_c.$$

$\rightarrow$ Akin to $\partial_t \left(RPKED + WED \right) = 0$ in quasi-linear theory ...

→ A Bit More Depth . . . .

- Where Does this lead us? → {multi-population system with feedback loops



– modulational instability feedback

– CR-AW feedback for confinement

→ CR-shock feedback

– Drury Process

– Acoustic effect on confinement.

[Bottom line: evolve AW, AC populations on [redacted] equal footing with f_{CR}, gas dynamics

→ How?

→ **Deriving Wave Population Evolution**

Eqns? — $\left\{\begin{array}{l}\text{Mean Field Theory} \\ \quad + \\ \text{wave Kinetics}\end{array}\right.$

i.e. $\left\{\begin{array}{l}\text{Consider AW wave kinetics} \\ \text{in AC scatterer field}\end{array}\right.$

$$\partial_t \langle N^\pm \rangle + (u \pm v_A) \frac{\partial}{\partial x} \langle N^\pm \rangle - k u_x \frac{\partial}{\partial k} \langle N^\pm \rangle$$

$$+ \frac{\partial}{\partial k} \underbrace{\left\langle k v_A \frac{\tilde{\rho}_x}{\rho_0} \tilde{N}^\pm \right\rangle}_{\hookrightarrow \text{closure needed}} = \gamma_k^\pm \langle N^\pm \rangle + \langle C(N^\pm) \rangle$$

— proceed a la Chapman-Enskog:

$$\gamma_k^\pm N^\pm + C^\pm\{N^\pm\} = 0 \qquad \underline{\text{l.o.}}$$

$$L^\pm \tilde{N}^\pm = -k v_A \frac{\tilde{\rho}_x}{2\rho_0} \frac{\partial}{\partial k} \langle N^\pm \rangle \qquad \underline{1^{st}\ \text{O.}}$$

$\underline{\text{propagator}}$

$$\left[L^\pm = L_{linear} + \Delta\omega_{\underline{k}}. \right. \qquad \gamma^\pm \tilde{N}^\pm + \frac{\delta C}{\delta N} N^\pm \approx -\Delta\omega_{\underline{k}} \tilde{N}^\pm.$$

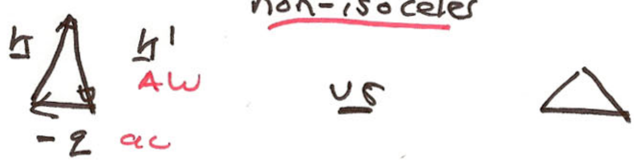
$$\frac{\partial}{\partial t}\langle N^{\pm}\rangle + u\frac{\partial}{\partial x}\langle N^{\pm}\rangle - k u_x \frac{\partial}{\partial x}\langle N^{\pm}\rangle$$

$$-\frac{\partial}{\partial k} D_k \frac{\partial \langle N^{\pm}\rangle}{\partial k} = \gamma_k^{\pm}\langle N^{\pm}\rangle + \langle C(N^{\pm})\rangle$$

$$D_k = \frac{1}{4} k^2 V_A^2 \left\langle \frac{\tilde{\rho}_x}{\rho_0} L^{-1} \frac{\tilde{\rho}_x}{\rho_0} \right\rangle$$

→ $\langle N \rangle$ evolves via anisotropic mode couplings
non-isoceles

i.e.



→ akin W.T.T. decay, with resonance broadening

→ recovers wave packet diffusion (no surprise)

→ finite amplitude can overcome mis-match

$$\Delta\omega \cong \left[\frac{k^2 V_A^2}{4} \frac{\langle (\nabla\tilde{\rho})^2 \rangle}{\rho_0^2 (\Delta k)^2} - \tilde{\Omega}^2_{MM} \right]^{1/2}$$

→ Observation

– like many related systems, N_{AW}, N_{AC} map to ecology fed by P_{CR} i.e. "Predator + Prey"

n.b.: {Contrast Mean Field Electrodynamics

– Prey : N_{AW} → $\frac{dN_{AW}}{dt} = \gamma_{AW}[P_{CR}] - \gamma_{decay}[N_{AC}]\,N_{AW}$

→ prey fed by linear resonant instability

"inverse cascade" process

N_{AC} → $\frac{dN_{AC}}{dt} = \gamma_{decay}[N_{AW}]\,N_{AC} - \alpha N_{AC}^2 + \gamma_{Drury} N_{AC}$

→ steepening

Drury drive

→ $\gamma_D \leq 0$ ⇒ usual 2 species fixed point ⇒ N_{AW} ⇔ γ_D → diffusive damping of Drury modes is regulator

but:

→ $\gamma_D > 0$ ⇒ simple story lost ----?
→ enhanced breaking
→ non-stationary state ?

→ Something Specific: Effectiveness of Diffusive Spectral Spreading in Shock driven systems ?

- concern: effectiveness on $t \simeq L_p/u$? → precursor crossing time

- $I_k \equiv |\tilde{B}_k / B_0|^2$

spectral spreading AW growth/drive

$$\partial_t I_k + u\,\partial I_k/\partial x - \frac{\partial}{\partial k} D_k \frac{\partial I_k}{\partial k} = \frac{2u}{kV_A}\frac{\partial P_{CR}}{\partial x}$$

where:

$$D_k = \frac{k^2 V_A^2}{4} \sum_q \frac{q^2 \Delta\omega}{q^2 V_{gr}^2 + \Delta\omega^2} \left|\frac{\tilde{\rho}_q}{\rho_0}\right|^2$$

- take: $|\tilde{\rho}_q/\rho_0|^2 \cong \overset{\text{str.}}{A}\,(q_0/q)^2$ (cf. Kang, '92) { shock amplitude $\sqrt{A}$, { separation $1/q_0$

⇒

∴ $D_k \cong \frac{3\pi}{4} A q_0 \frac{V_A^2}{V_g} k^2 \equiv \nu_c k^2$ → for strong scattering limit

↳ scattering rate

for $y^3 = D_k V_g' / q_0 V_g^3 \gg 1$.

- then, intensity:

$$\frac{\partial}{\partial k} k^2 \frac{\partial I}{\partial k} - \frac{u_1}{r_c}\frac{\partial I}{\partial x} = -2 M_A \frac{u_1}{r_c k}\frac{\partial p}{\partial x}\Big|_{\substack{\text{res.} \\ pk = m\Omega_c}}$$

if $r \to 0 \Rightarrow I \sim p|_{\text{reson.}}$, as usual

- a bit of algebra $\Rightarrow$ $Q = 2M_A P/k$

$$I = \int_{-\infty}^{T} dT' \frac{\exp[-(1/4)(T-T')]}{(4\pi(T-T'))^{1/2}} \int_{-\infty}^{+\infty} dv' \exp\left[-\frac{(v-v')^2}{4(T-T')} + \frac{1}{2}(v-v')\right] \frac{\partial Q(v')}{\partial T'}$$

Governing parameter: $\left\{ \begin{array}{l} \delta = r_c L_p / u_1 = r_c T_{conv.} \\ \equiv t_{\substack{prec. \\ conv.}} / t_{scattering} \end{array} \right.$

$\delta \to 0 \Rightarrow I = Q \sim P_{CR}$, as usual.

$\delta \gg 1$?

$s > 1 \Rightarrow$

$$I(x) = 2\,\frac{M_A}{s}\,\epsilon(x)\begin{cases} 1 & p > p_{max} \\ p/p_{max} & p < p_{max} \sim p_b \end{cases}$$

→ upper c.o. (critical spectrum)

→ lower c.o.

recall $kp \sim$ const, so

→ high $p \Rightarrow$ low $k \Leftrightarrow I(k) \sim$ const (~ flat spectrum)

→ low $p \Rightarrow$ high $k \Leftrightarrow k^2\, dI/dk \sim$ const (~ const. flux)

∴ diffusive refraction in k generates low k and confinement of higher p particles, with $p > p_{max}$. !

n.b. here: $\epsilon(x) = \dfrac{P_c(x)}{\rho_1 u_1^2} = \dfrac{4\pi}{3}\dfrac{mc^2}{\rho_1 u_1^2}\displaystyle\int_{p_b(x)}^{p_{max}} dp\, p^3 f_0(p)$

→ what is σ, really ?

- σ determined by structure, spacing, etc. of scatterers

i.e. acoustic shocks (i.e. standard scenario) $\Delta\rho \sim \rho_0 \Rightarrow \sigma \sim q_0 L / M_A$

$\sim N_s / M_A$

shocklets in train

$\wedge \;\; \wedge \;\; \wedge$

- for realistic parameters, $\sigma \gg 1$ confirmed.

- Generally, both short AW wave and particle N.L. dynamics depend sensitively on structure of scatterer field in nonlinear regime !

n.b. studies of impact of shock coalescence in nonlinear regime especially interesting !

Partial Summary

- modulational decay via stochastic refraction identified as important multi-scale mechanism

- modulational decay:
  - → generates larger scale AW
  - → enhances shocklets (vitiates need for linear Drury mechanism)

- spectrum of density gradients $|\tilde{\rho}_k/\rho_0|^2$ is crucial

- critical parameter $\delta = r_c \tilde{\gamma}_{conv}$.
  - → set by scatterer configuration (i.e. spatial structure)

- demonstrated $\delta > 1 \Rightarrow$ spectral broadening & enhanced acceleration + conf.

Next steps:

→ self-consistent theory of Drury instability coupled to AW spectrum

i.e. - modulate R and $\langle \delta B^2 \rangle$, ...

- retain decay feed, etc.

→ Analysis of coupled states / feedback loops in shock + AW + AC system

II.) Enhanced Efficiency ?!

- another weakness of DSA is low efficiency at high energy i.e. $1/\tau_{accel} \sim u_{sh}^2 / D(p)$

- difficulty arises from long time for multiple shock crossings ?

$\Rightarrow$

- Malkov '06 (and following talk)

→ foresake traditional Fermi process

→ explore scattering in convergent precursor flow – "Pinball Process"

→ scatterers : (magneto) acoustic shocklets in precursor flow.

- relation to this talk → <u>both</u> $\left\{ \begin{array}{l} \text{enh conf.} \\ \text{enh. eff} \end{array} \right. \Leftarrow\Rightarrow$ effects of DPCR driven shocklets.

→ Relation to Multi-Scale Interaction ?

- M-S I generates precursor scattering field !

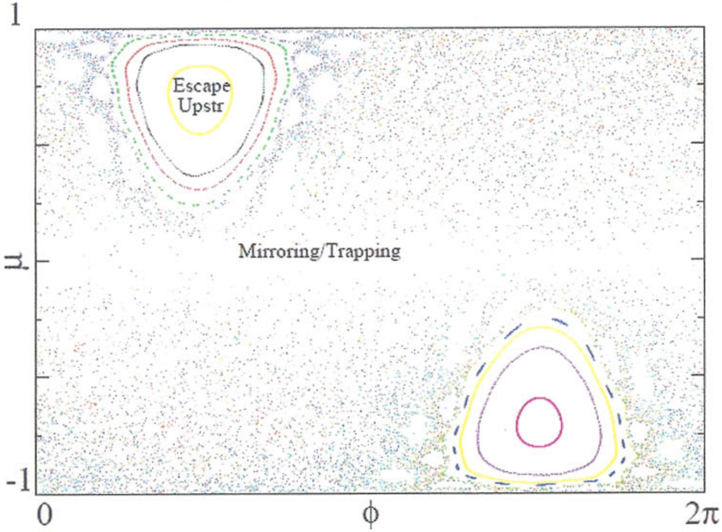
  i.e. Drury + NL decay + ? ⇒ shocklet configuration
  ⟺ acceleration

- P_* ⟺ knee → location of enhanced loss, in p
  → loss islands (a la' loss cones) in CR phase space, cf.
  ⟺ scatterer configuration !
  (i.e. loss cone ⟺ scale, mirror ratio)

- Orbits of CR's no longer simply diffusive ..
  - trapping osc.
  - mirroring; stoch + non-stoch.

  ⇒ - Levy flights → super-diffusive
  - sticking → sub-diffusive

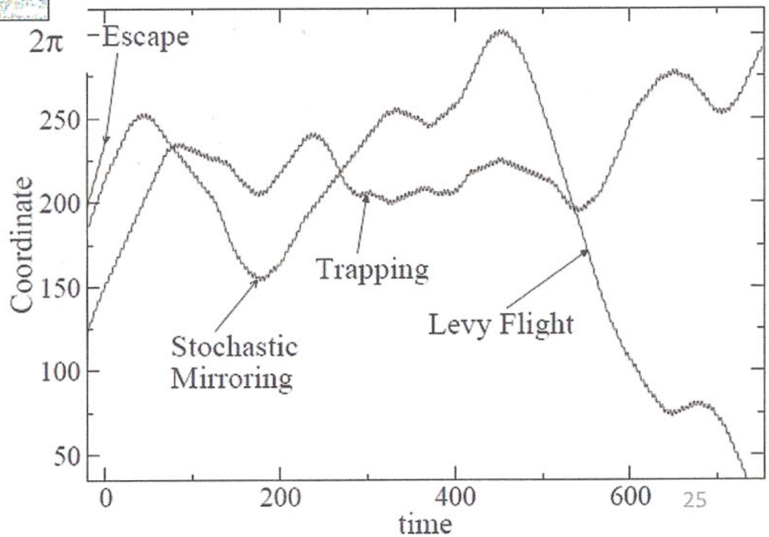
Particle dynamics and transport in a shock train inside of CR precursor



Pitch-angle/Gyro-phase
Poincare map
(Pitch-angle wrt shock normal, 45 deg here)

$r_g(p)/L \simeq 3$

Particle trajectories



→ A speculation ?! — { convergence of attention on mesoscale scatterer field ⇒

– time to junk time-honored diffusion–convection equation ....

– alternative strategy:

→ develop simple characterization of scatterer distribution of scale, separation, etc. for shocklets, 'solenoidlets' (c.f. Bell)

n.b.: linear instability won't suffice

i.e. ↔ shock coalescence ⇒ scatterer field

→ compute particle scattering using scatterer pdf model

i.e. Fractional kinetics (c.f. Zaslavsky)

expect: $\frac{\partial}{\partial x} D \frac{\partial}{\partial x} f \rightarrow \frac{\partial}{\partial x} \int dx' \, K(x, x') \frac{\partial f}{\partial x'}$

→ nonlocal interaction kernel

Memory!

Conclusions

- ∇P_{CR} shocklets (Drury or nonlinear) promising route toward:
  - → higher energy confinement ⇔ via stochastic refraction
  - → enhanced efficiency
- linear Drury mechanism not critical ⇒ AW pumping...
- Crucial to understand: ⇔ self-consistency!
  - scale
  - strength
  - spacing ⇔ packing fraction

  of mesoscale scattering field

  need for D_{k}, D, K etc. ⇒ spectral structure.
- replace/extend DSA to FKSA