The Acceleration and Transport of Cosmic Rays with Heliospheric Examples

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Presented at the 3rd Korean Astrophysics Workshop, August 17, 2004

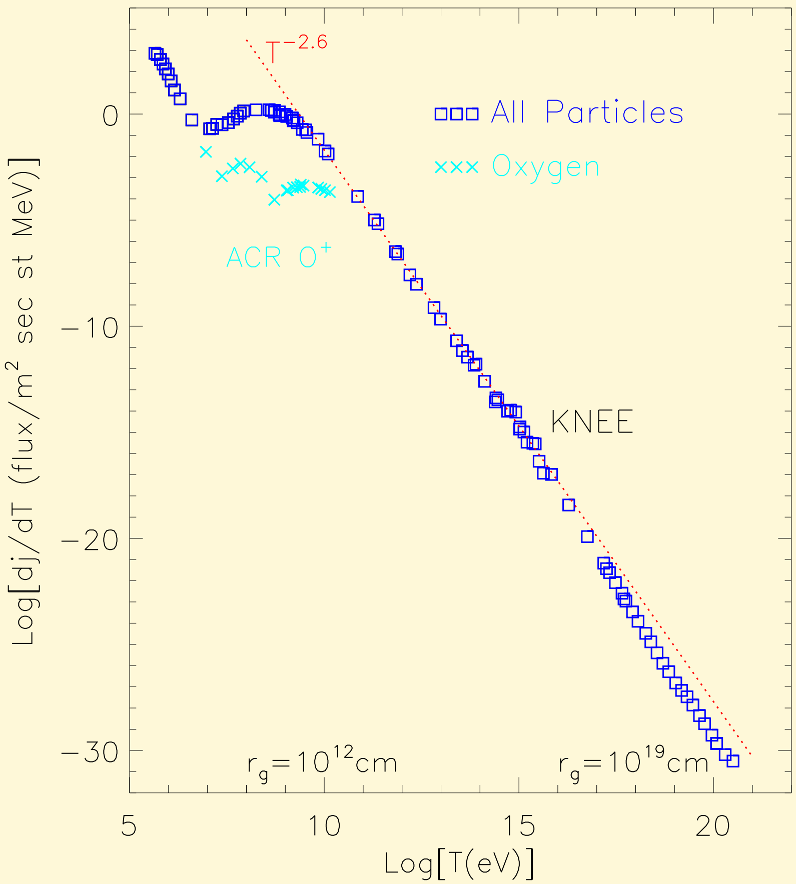
- Understanding the transport and acceleration of cosmic rays remains one of the most important problems in astrophysics.

- Although diffusive shock acceleration has proved to be very successful and explains much, there remain areas where it does not apply.

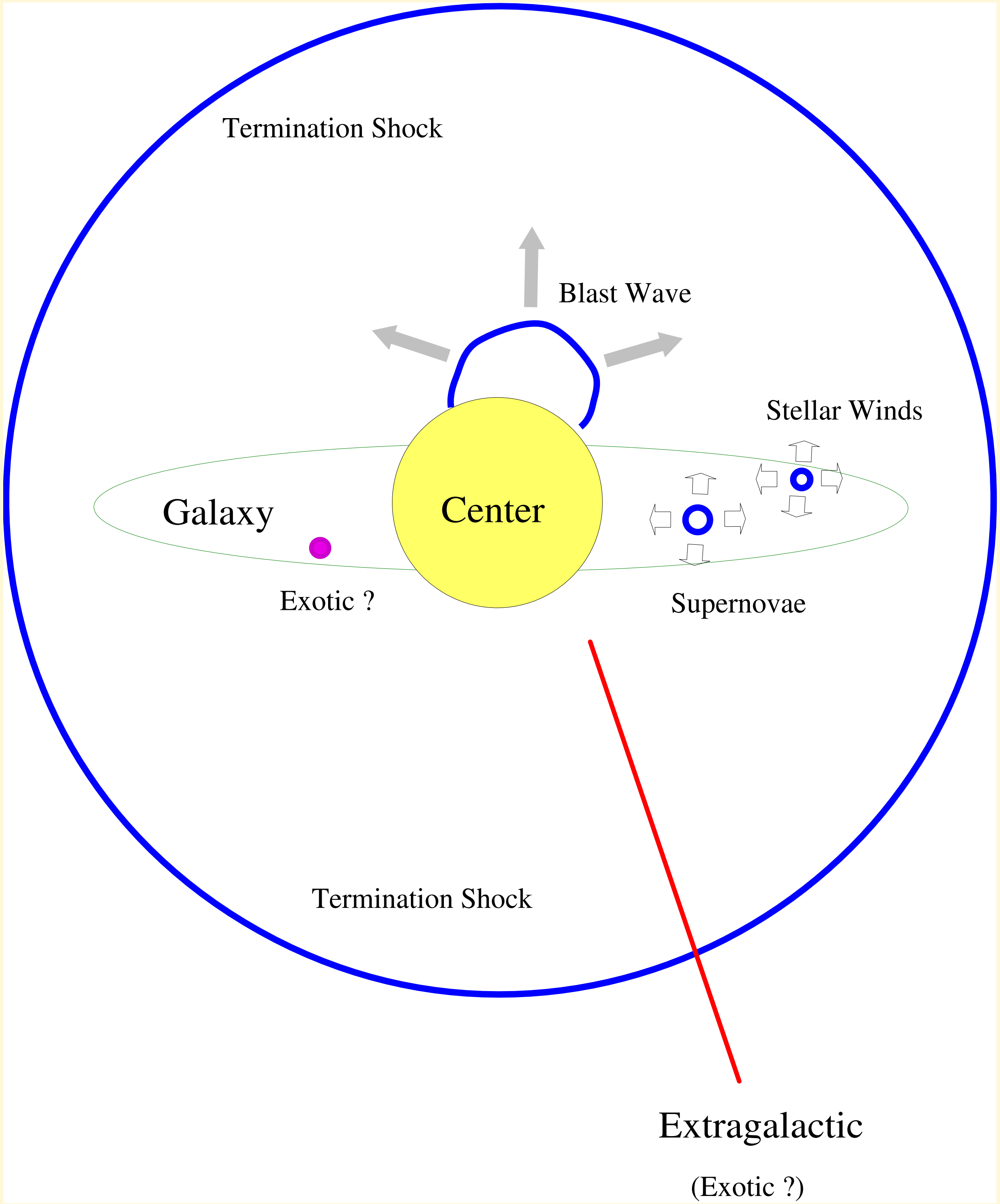
- I will review the basic physics of charged-particle acceleration and introduce a "new" acceleration mechanism which may contribute to solving the remaining problems.

- In this I will use *in situ* results from the heliosphere to inform and constrain our results.

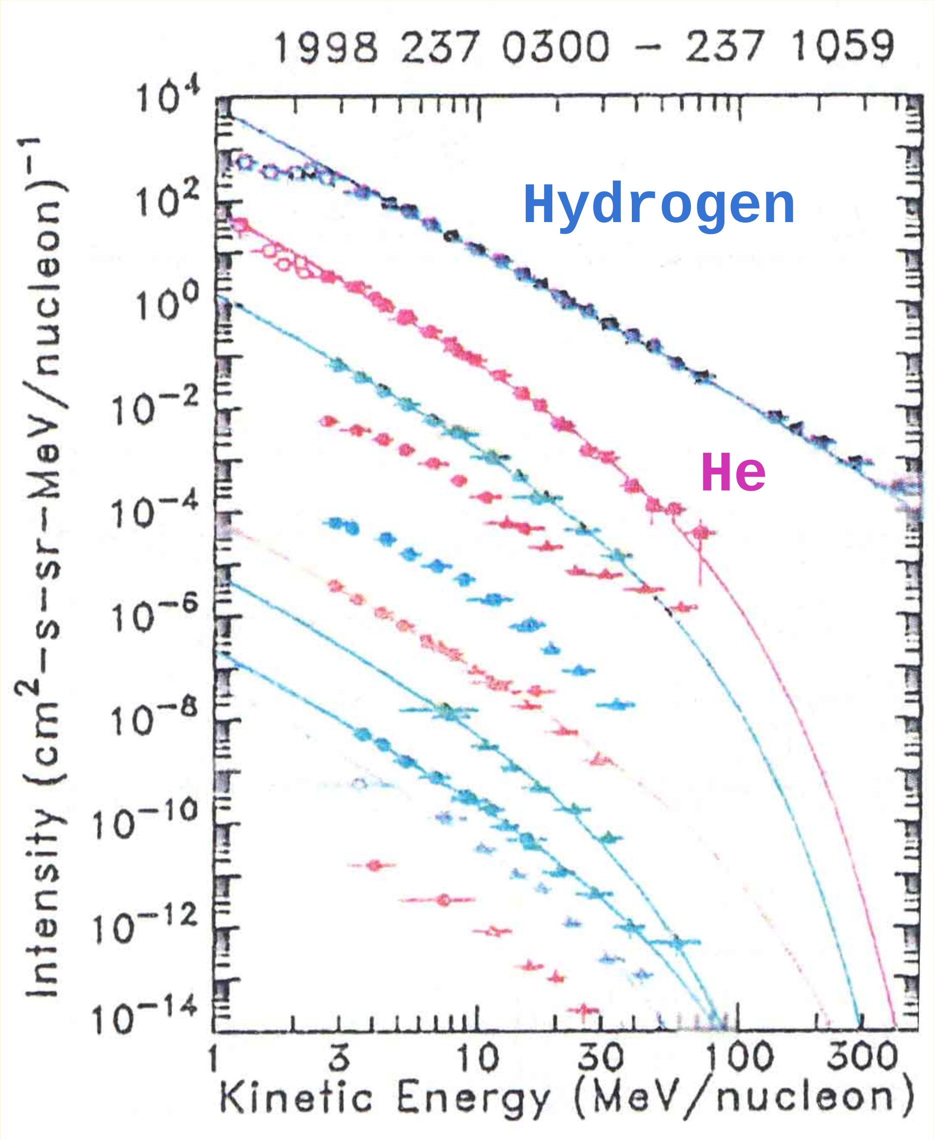
The observed quiet-time (no significant solar activity) cosmic-ray spectrum.

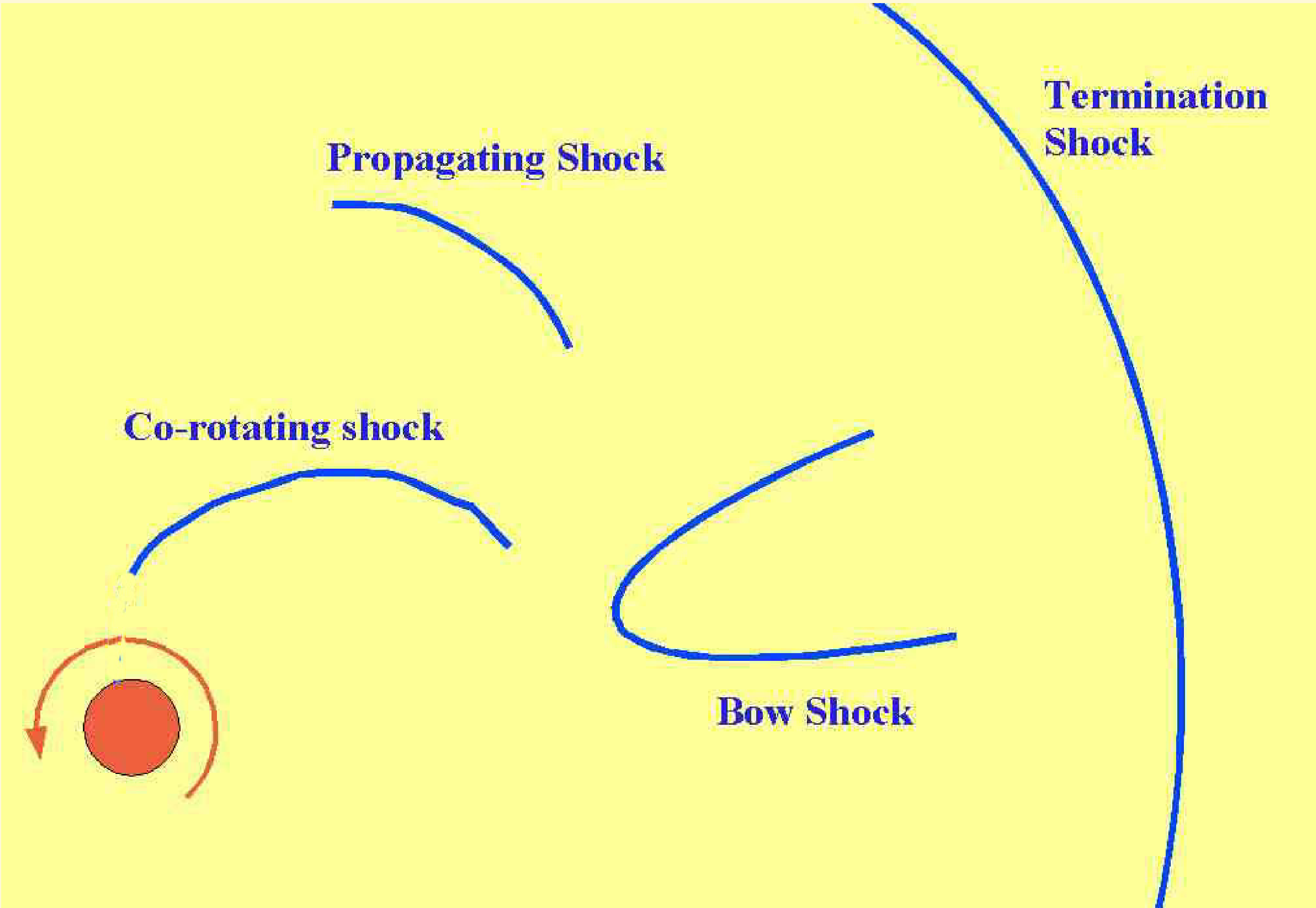


Could be called "The Great Power Law in the Sky"


Termination Shock
Blast Wave
Stellar Winds
Galaxy
Center
Exotic ?
Supernovae
Termination Shock
Extragalactic
(Exotic ?)

Typical solar flare spectrum. Note Power Laws.

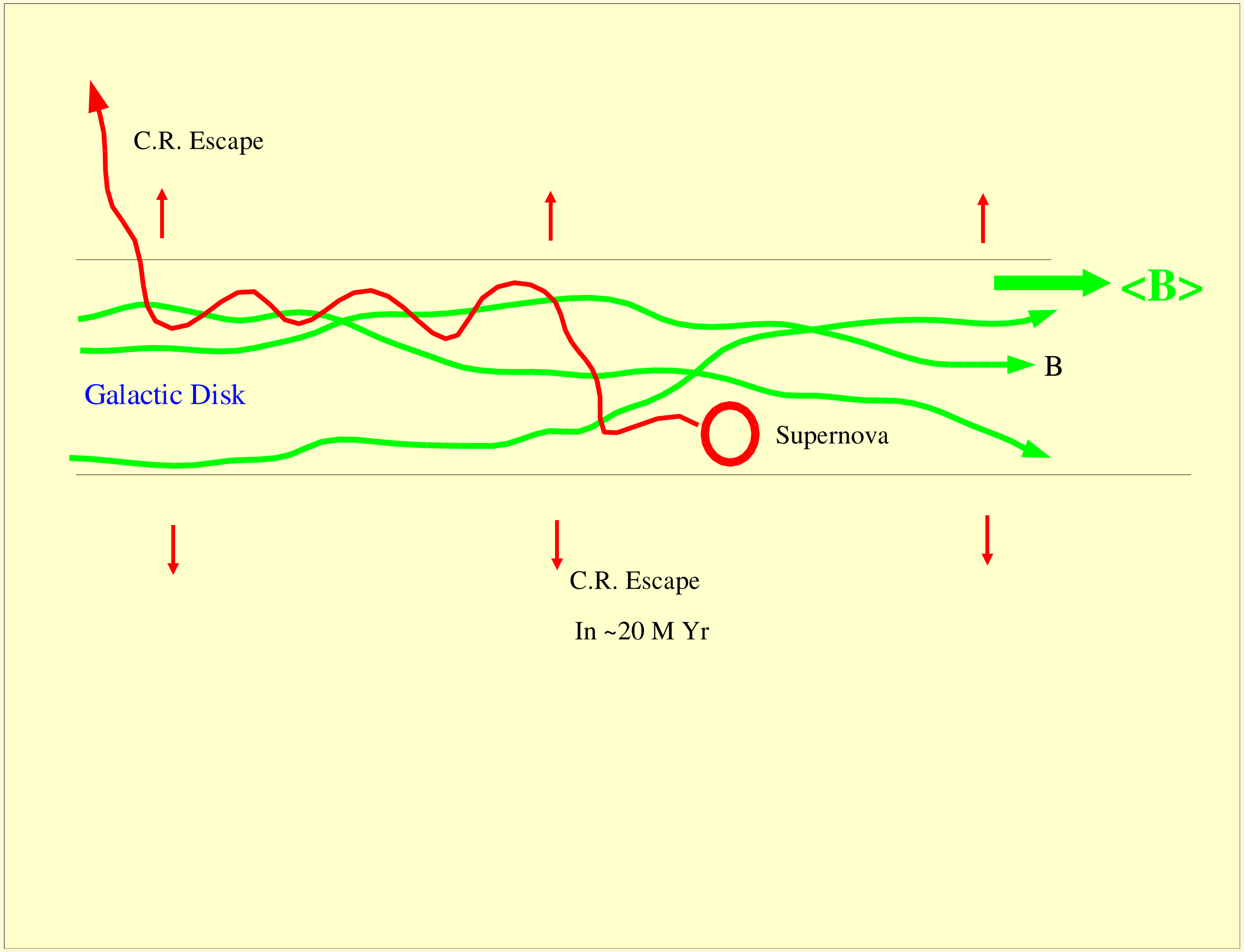




Heliospheric Sources of Energetic Particles

$\Rightarrow$ Universal power law spectrum and accelerator.

Schematic view of cosmic-ray acceleration and loss from the galaxy.



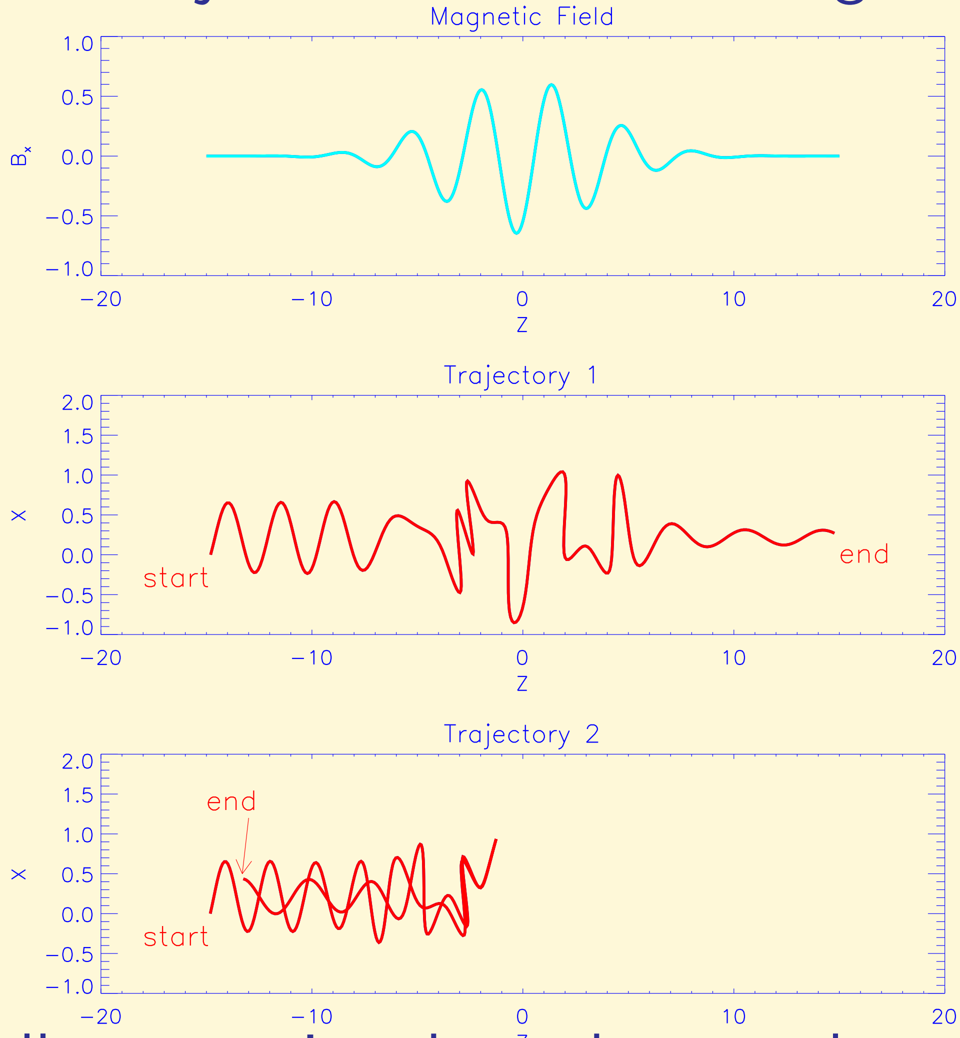
Models of cosmic-ray transport in the galaxy are still crude.

The energy spectrum, **very generally**, imposes significant constraints on cosmic-ray origin:

- The accelerator(s) must produce nearly-universal smooth power-law spectra over many decades.
- Confinement (or transport) must also vary smoothly with particle energy over at least the same range.
- These two mechanisms should both be very common.

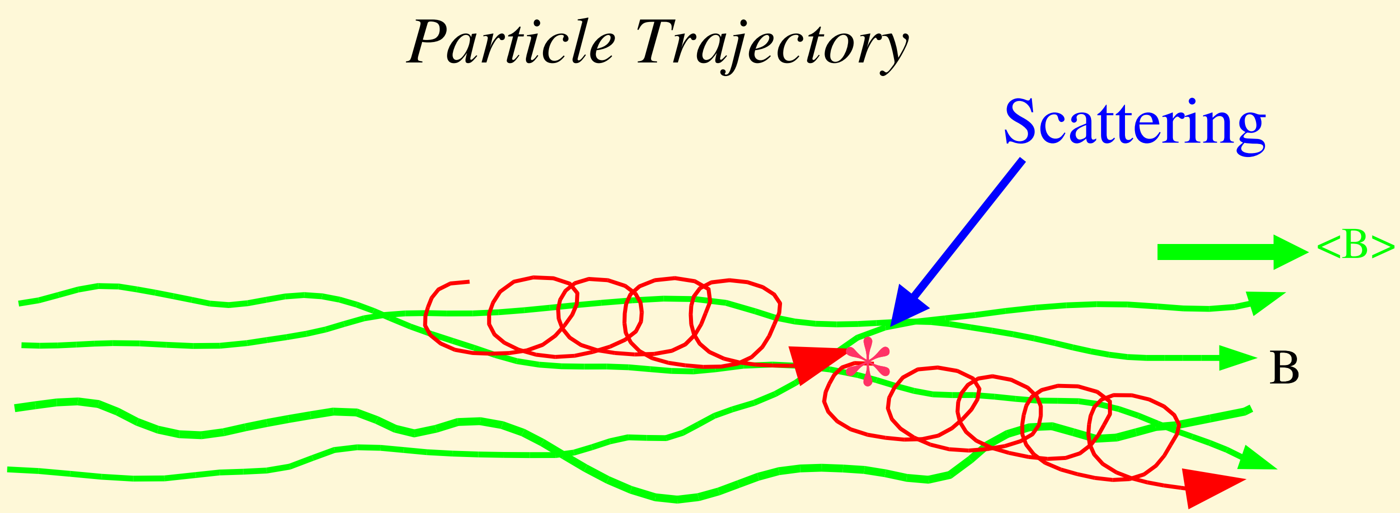
Look first at confinement and transport.

Chaotic particle trajectories in a fluctuating magnetic field.

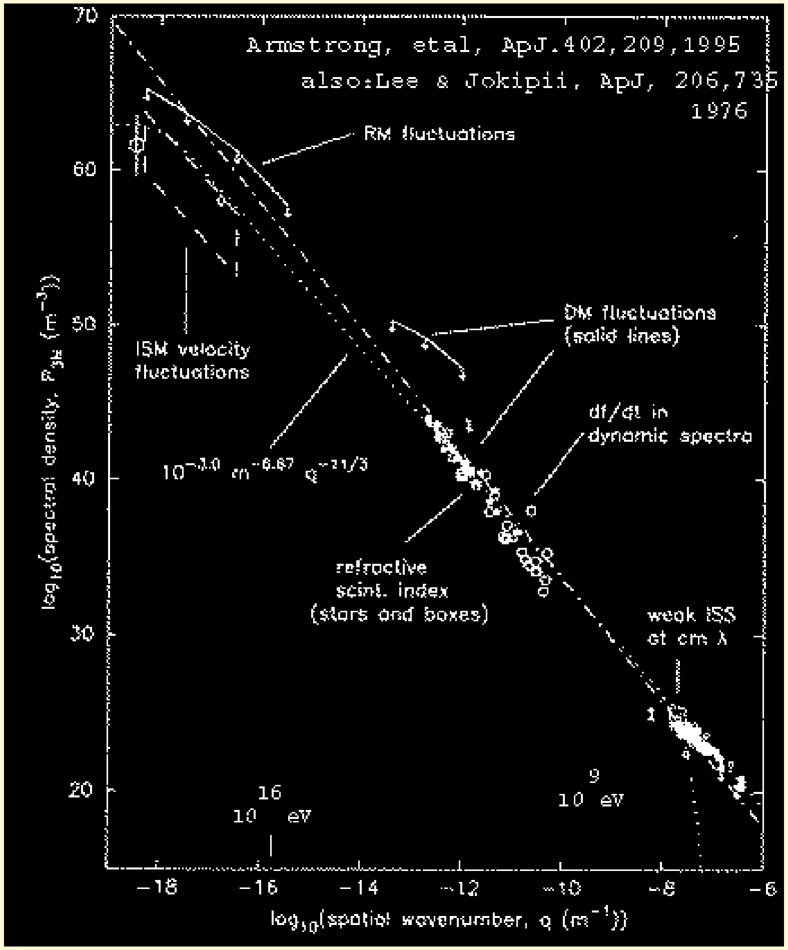


Quite generally, scattering depends on scales $\approx r_g$

Quasilinear approximation $\Rightarrow$ Scattering Rate $\nu \propto P_B(k \approx 1/r_c)$



A smooth power-law turbulence spectrum over the corresponding scales produces smooth variation of confinement. This is observed in the interstellar medium.



Interstellar turbulence has a smooth Kolmogorov spectrum over the relevant scales. This is the 2nd great power law!

The Parker equation for the phase-space density f is:

$$
\begin{aligned}
\frac{\partial f}{\partial t} \;=\; & \frac{\partial}{\partial x_i}\left[\kappa_{ij}\frac{\partial f}{\partial x_j}\right] && (diffusion) \\
& - U_i \frac{\partial f}{\partial x_i} && (convection) \\
& - V_{di} \frac{\partial f}{\partial x_i} && (guiding-center\ drift) \\
& + \frac{1}{3}\frac{\partial U_i}{\partial x_i}\left[\frac{\partial f}{\partial \ell n p}\right] && (energy\ change) \\
& + Q(x_i, t, p) && (source)
\end{aligned}
$$

This equation contains both spatial transport and acceleration.

Proposed acceleration mechanisms.

- Swann's Mechanism (The first! Uses d B / d t)
- 2nd Order Fermi (statistical) Mechanism (Historically Very Popular)
- Parallel Electric Fields (difficult to Set up)
- Shock waves (The Most Successful)
- Velocity Shear (Acceleration Generally Small)

2nd-Order Fermi Acceleration can be incorporated by simply adding momentum diffusion

$(1/p^2)\partial/\partial p[p^2 D_{pp} \partial f/\partial p]$ to Parker's equation.

But this has two significant problems.

- It is probably too slow.

- The shape of the spectrum is strongly dependent on poorly-understood parameters. For Example:

Reprinted from Soviet Phys. JETP, Vol. 13, No. 6, pp. 1257–1260, Dec. 1961

SPECTRUM OF GALACTIC AND SOLAR COSMIC RAYS

S. I. SYROVAT-SKII

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 4, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 1788-1793 (June, 1961)

$$N(E)\,dE = KE^{-\gamma}dE, \tag{1}$$

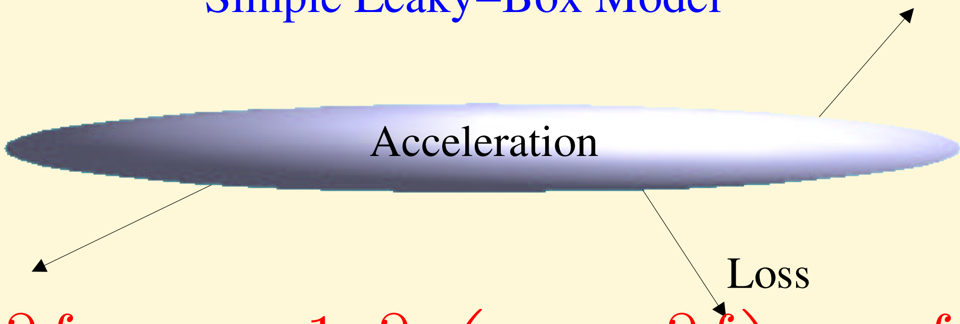
where E is the total energy of the particle, K is a constant, and γ is the spectrum exponent ($\gamma = 2.5$ for galactic cosmic rays), the scheme first suggested by Fermi[1] is usually employed. In this scheme γ is obtained under the assumptions that the energy of the particles increases exponentially with a time constant $1/\alpha$ during the acceleration process and that the absorption of the particles follows an experimental law with a lifetime T. Then

$$\gamma = 1 + 1/\alpha T. \tag{2}$$

Subsequently, the lifetime with respect to nuclear collisions, which led to a strong unobserved dependence of γ on the atomic number of the cosmic-ray nucleus, was replaced by the mean time T in which the particles leave the region of acceleration.[2] In order to obtain the spectrum (1) and (2) in this scheme, it has to be assumed, moreover, that the conditions of acceleration, i.e., the parameters, α and T, and, what is particularly important, the injection of the particles do not change over the time interval necessary for the particles to acquire the maximum observed energy. Under these assumptions and with a suitable choice of parameters α and T, one can obtain the value of γ required by experiment.

It should be noted that the rather severe assumptions on the character of the acceleration and injection processes and chiefly the strong dependence of γ on specific values of α and T make the foregoing scheme for the production of the cosmic-ray spectrum highly unconvincing.

Simple Leaky–Box Model



$$\frac{\partial f}{\partial t} = 0 = \frac{1}{p^2}\frac{\partial}{\partial t}\left(p^2 D_{pp}\frac{\partial f}{\partial t}\right) - \frac{f}{\tau_{loss}}$$

where, typically, $D_{pp} = \frac{1}{2} < \frac{(\Delta p)^2}{\Delta t} >= p^2/\tau_{acc} = (V_a^2/c^2)(p^2/\tau_{coll})$.

If τ_{coll} and τ_{loss} are constant (0nly if), we obtain a power law.

$$f(p) = Ap^{-\alpha}$$

where

$$\alpha = \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{\tau_{acc}}{\tau_{loss}}}$$

Unfortunately, the highly uncertain τ's enter into the exponent α. $\Rightarrow$ cannot explain the very similar power laws in a variety of situations.

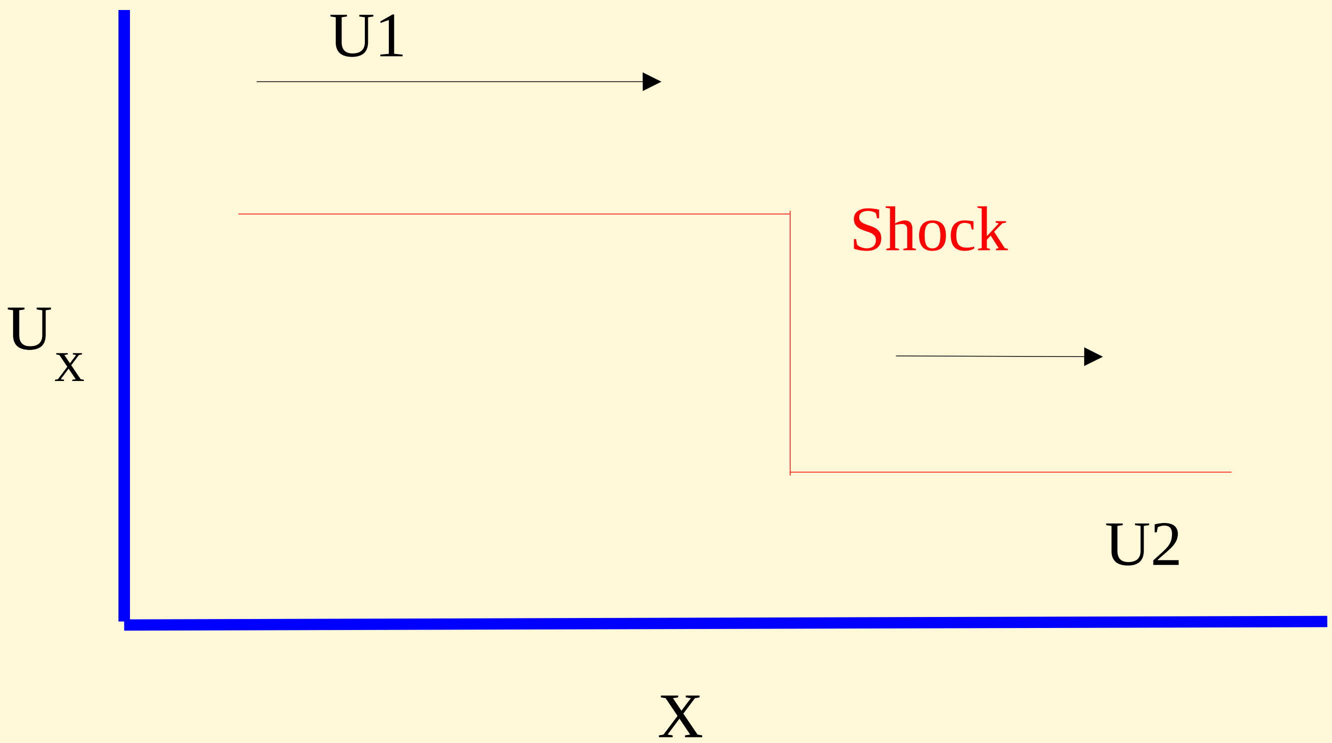
It turns out that shock waves are powerful accelerators. Consider a one-dimensional system in which case the Parker equation becomes:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[\kappa_{xx}\frac{\partial f}{\partial x}\right] - U_x\frac{\partial f}{\partial x} + \frac{1}{3}\frac{\partial U_x}{\partial x}\left[\frac{\partial f}{\partial \ln p}\right] + Q(x,t)$$

This can be applied to a shock propagating in the x direction, as illustrated.

Consider a one−dimensional flow $U_x(x)$ as shown.
The shock is at rest in this coordinate frame.

The shock ratio $r = U1/U2 \leq 4$.

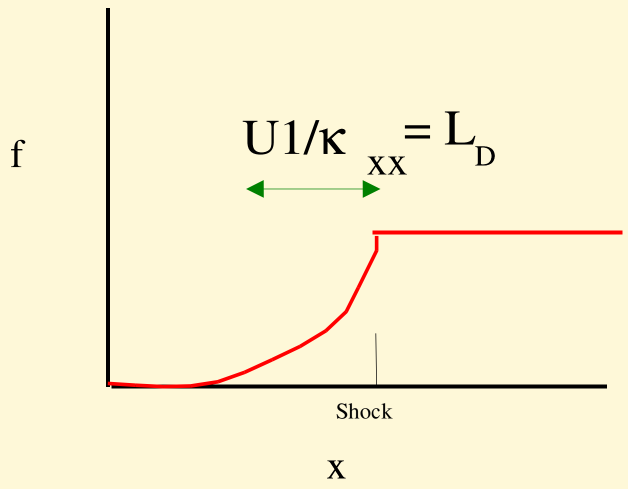


For this system, the steady solution to Parker's equation for particles injected at a low momentum p_0 has the characteristic form (above p_0; nothing below p_0):

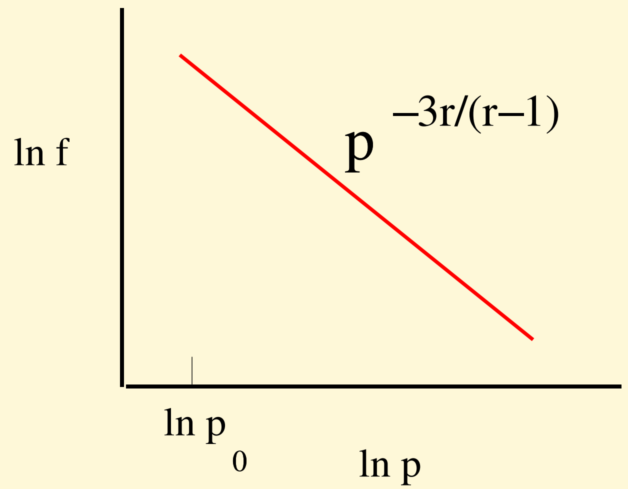
$$
\begin{aligned}
\frac{dj}{dT} &= p^2 f(x,p) \\
&= Ap^{-q+2}\ exp(\frac{U1\ x}{\kappa_{xx}}) \qquad x < x_{shock} \\
&= Ap^{-q+2} \qquad\qquad\qquad\qquad x \geq x_{shock},
\end{aligned}
$$

where $q = \frac{3r}{r-1} \approx 4$ for strong shocks $\rightarrow dj/dT \propto p^{-2} \approx T^{-2}$.

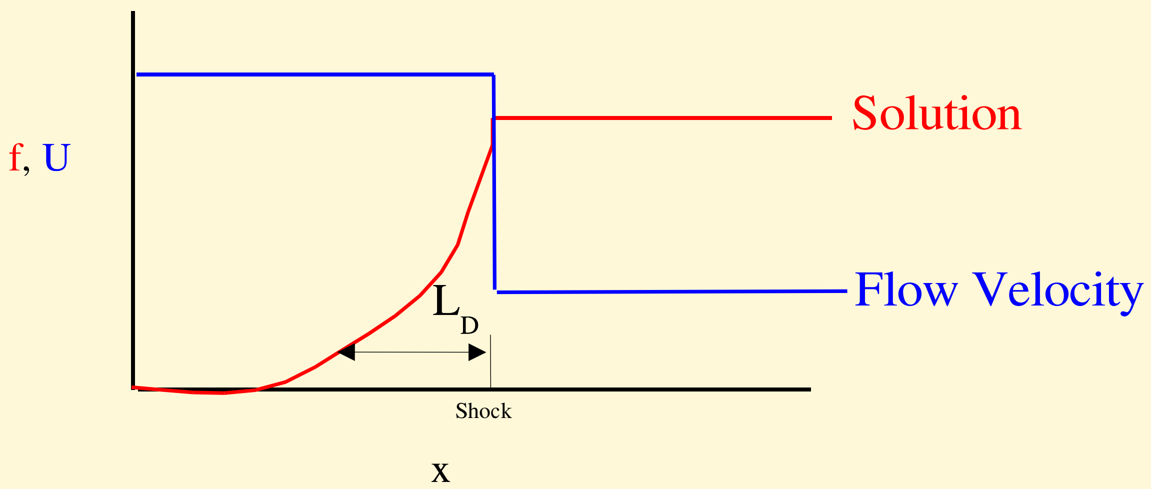
This seems to be the desired accelerator – it produces a near-universal power law energy spectrum!!



f
U1/κ xx = L D
Shock
x

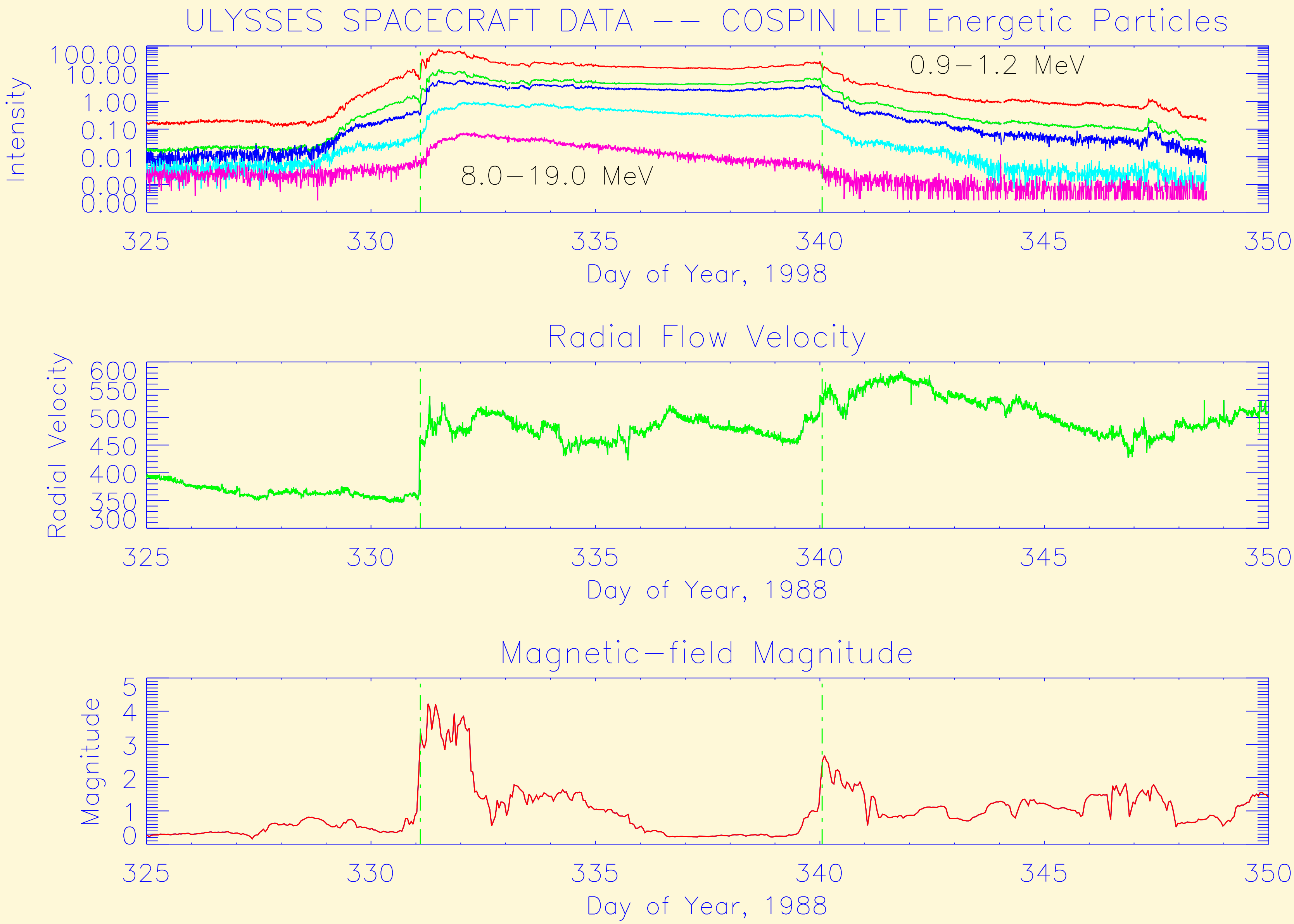


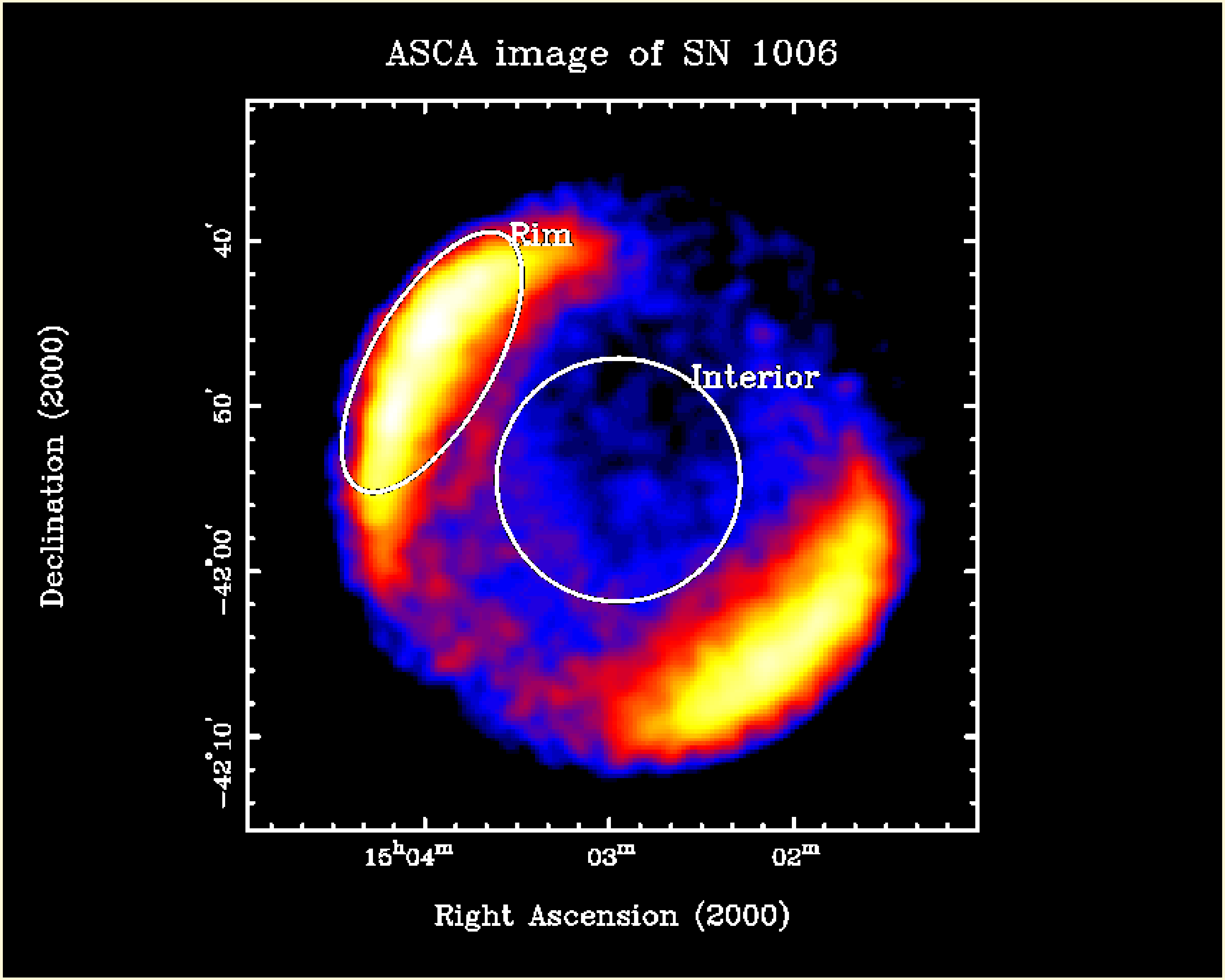
ln f
p −3r/(r−1)
ln p 0
ln p



f, U
Solution
Flow Velocity
L D
Shock
x

Shock acceleration is well-documented by observations, both *in situ* (from spacecraft) and remotely.


ULYSSES SPACECRAFT DATA -- COSPIN LET Energetic Particles
Intensity
100.00
10.00
1.00
0.10
0.01
0.00
0.00
0.9−1.2 MeV
8.0−19.0 MeV
325
330
335
340
345
350
Day of Year, 1998
Radial Flow Velocity
Radial Velocity
600
550
500
450
400
350
300
Day of Year, 1988
Magnetic−field Magnitude
Magnitude
5
4
3
2
1
0
Day of Year, 1988


ASCA image of SN 1006
Rim
Interior
Declination (2000)
40′
50′
−42°00′
−42°10′
15h04m
03m
02m
Right Ascension (2000)

We have achieved our objectives.

- The observed smooth, power-law (Kolmogorov) turbulence spectrum, over a wide range of wave numbers provides the needed smooth confinement mechanism as a function of energy (and position).
- Diffusive shock acceleration produces a near-universal spectrum having the correct shape.
- Both turbulence and shocks are ubiquitous.
- Given their many successes, maybe shocks are the *only* accelerators.

Unfortunately, — there are some important loose ends.

- At best, supernovae only work up to the knee ($\approx 3 \times 10^{15}$ eV). But the spectrum extends smoothly beyond this.

- There may be a difficulty even in going up to the knee.

- Buckley, et al. find that TeV γ-ray observations reveal serious problems with the supernova origin of cosmic rays. (A&A 329, 639, 1998).

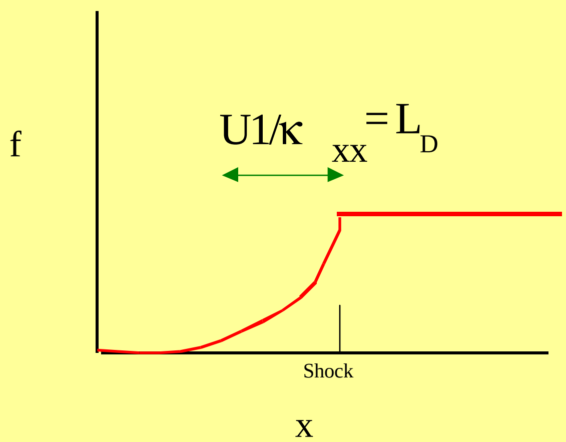
- Observations in the heliosphere show significant charged-particle acceleration where there are no shock waves present or nearby.

What about the maximum energy attainable?

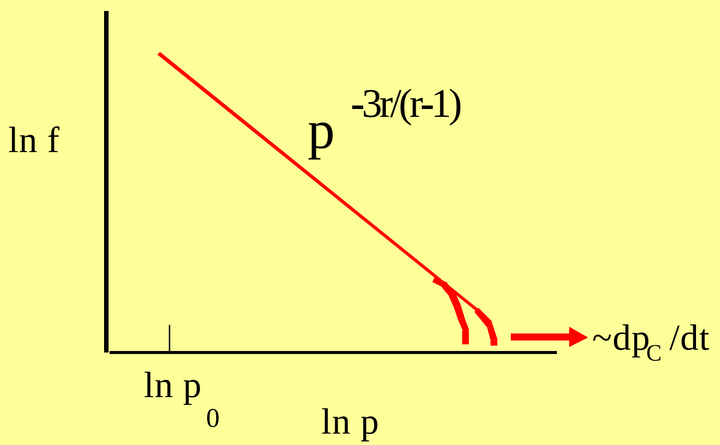
- The energy is limited either by geometry $(r_c < L)$ or by a finite time.

- Usually the geometry is not the principal limiter.

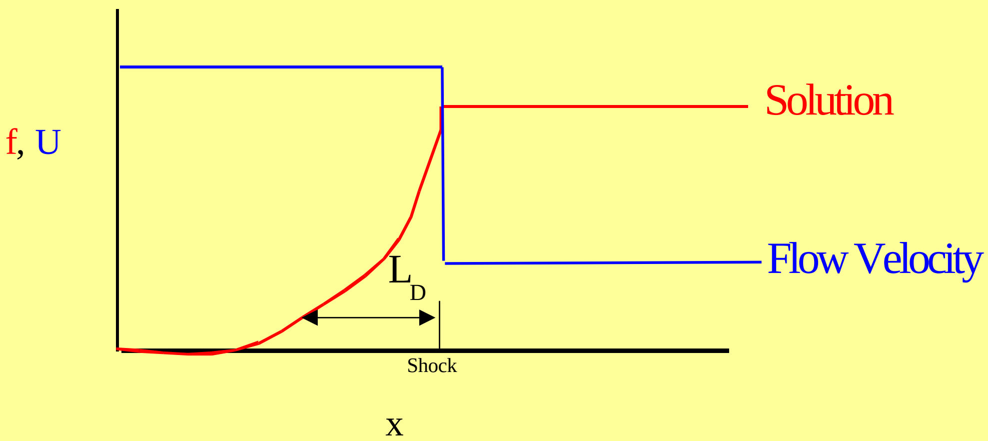
- However, acceleration takes time. The ideal power law energy spectrum is not created instantly.



f
U1/κ
xx
= L
D
Shock
x



ln f
p
-3r/(r-1)
~dp
C
/dt
ln p
0
ln p



f, U
Solution
Flow Velocity
L
D
Shock
x

The rate dp_c/dt depends on κ_{xx}, the diffusion coefficient normal to the shock front and on the speed of the shock relative to the upstream gas, U_{sh},

$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{U_{sh}^2}{4\kappa_{xx}}.$$

Here $\kappa_{xx} = \kappa_{\|} cos^2(\theta_B) + \kappa_{\perp} sin^2(\theta_B)$, where θ_B is the angle between the shock normal and the magnetic vector.

For a parallel shock $\theta_B = 0$ and $\kappa_{xx} = \kappa_{\|}$. and for a perpendicular shock $\theta_B = \pi/2$ and $\kappa_{xx} = \kappa_{\perp}$.

Also, generally $\kappa_{\perp} \ll \kappa_{\|}$.

One may conclude:

- The rate dp_c/dt depends on κ_{xx}.

- $\kappa_\| \gg \kappa_\perp$

- The rate of change of the maximum energy is much larger for perpendicular shocks.

- Hence, for any given situation, a perpendicular shock will yield a larger maximum energy than a parallel shock.

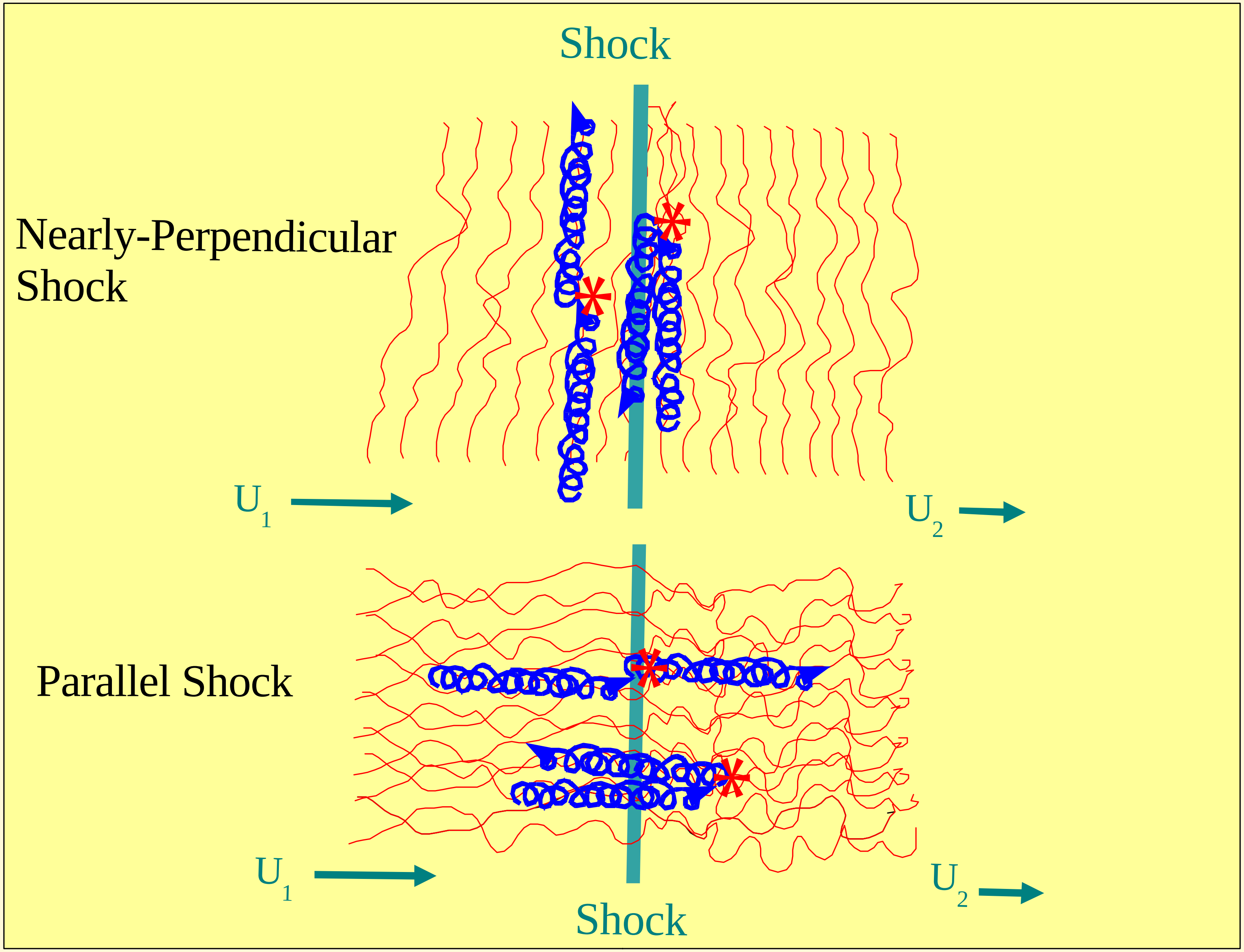
Consider the well-known Lagage and Cesarsky result. This only applies to a parallel shock. We have

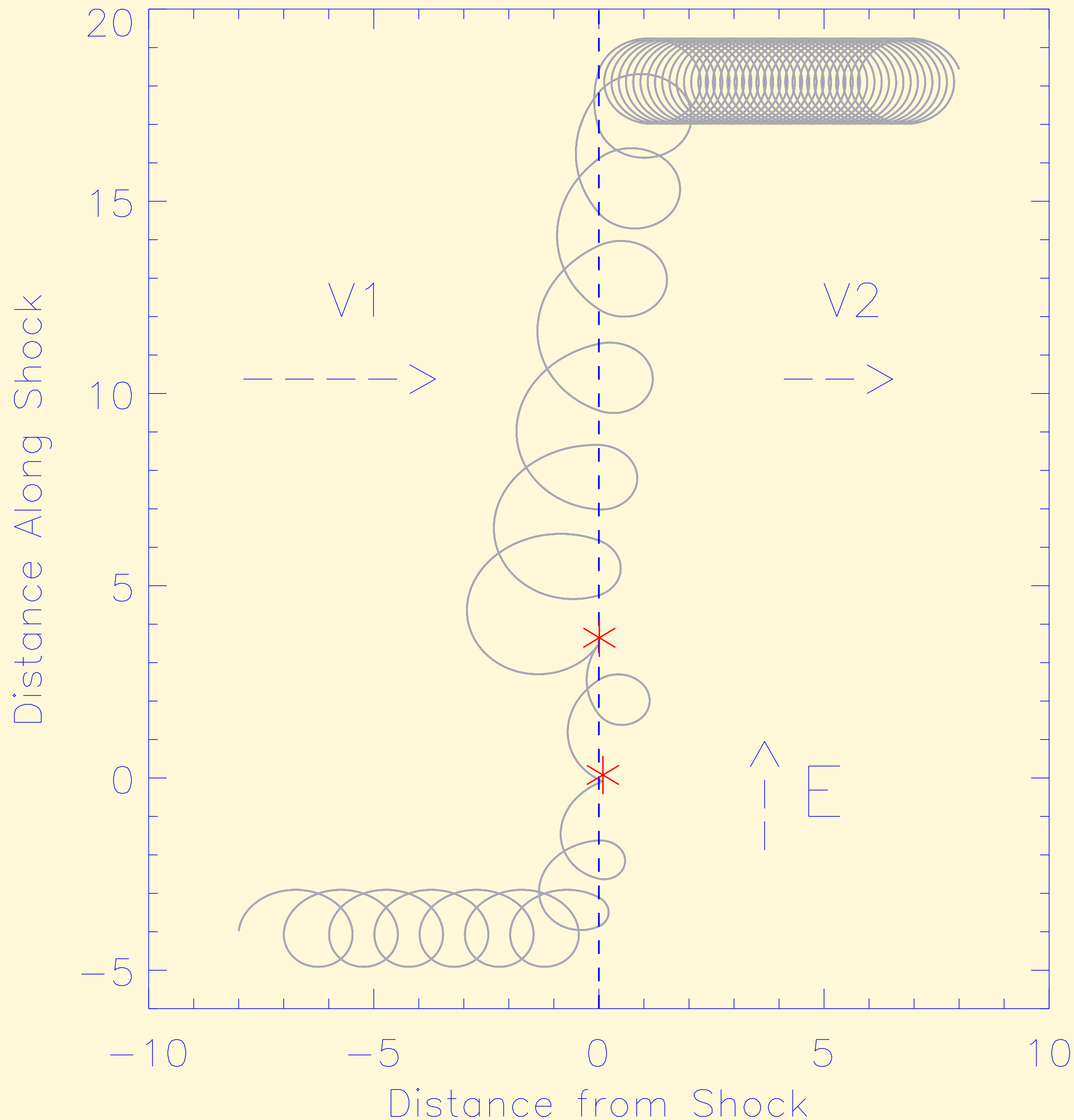
$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{U_{sh}^2}{4\kappa_\parallel} = \frac{3U_{sh}^2}{4\lambda_\parallel w}$$

Clearly, $\lambda_\parallel \stackrel{\sim}{>} r_g$

$$\Rightarrow \left(\frac{1}{p_c}\frac{dp_c}{dt}\right)_B = \frac{U_{sh}^2}{4r_g w}$$

which is known as the "Bohm limit". Applied to a typical SN blast wave this yields the Lagage and Cesarsky "upper limit" $\approx 10^{14}$ Z eV.


Shock
Nearly-Perpendicular Shock
U1
U2
Parallel Shock
U1
U2
Shock


20
15
10
5
0
−5
−10
−5
0
5
10
Distance Along Shock
Distance from Shock
V1
V2
E

Consider acceleration at a *perpendicular* shock.

$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{U_{sh}^2}{4\kappa_\perp}$$

In the simple case of "hard-sphere" scattering, this is

$$\frac{\kappa_\perp}{\kappa_\parallel} = \frac{1}{1 + \frac{\lambda_\parallel}{r_c}^2} \approx \frac{r_c}{\lambda_\parallel}^2$$

since typically $\lambda_\parallel / r_c \gg 1$. This yields

$$\frac{1}{p_c}\frac{dp_c}{dt} = \frac{U_{sh}^2}{\kappa_\parallel}\frac{\lambda_\parallel}{r_c}^2 = \left(\frac{1}{p_c}\frac{dp_c}{dt}\right)_B \frac{\lambda_\parallel}{r_c},$$

which is *larger* than the Bohm limit in all cases.

However, $\lambda_{\parallel}/r_c$ cannot become $arbitrarily$ large because the diffusion approximation becomes invalid. The anisotropy vector

$$\delta_i = \frac{3}{f}\kappa_{ij}\frac{\partial f}{\partial x_j}$$

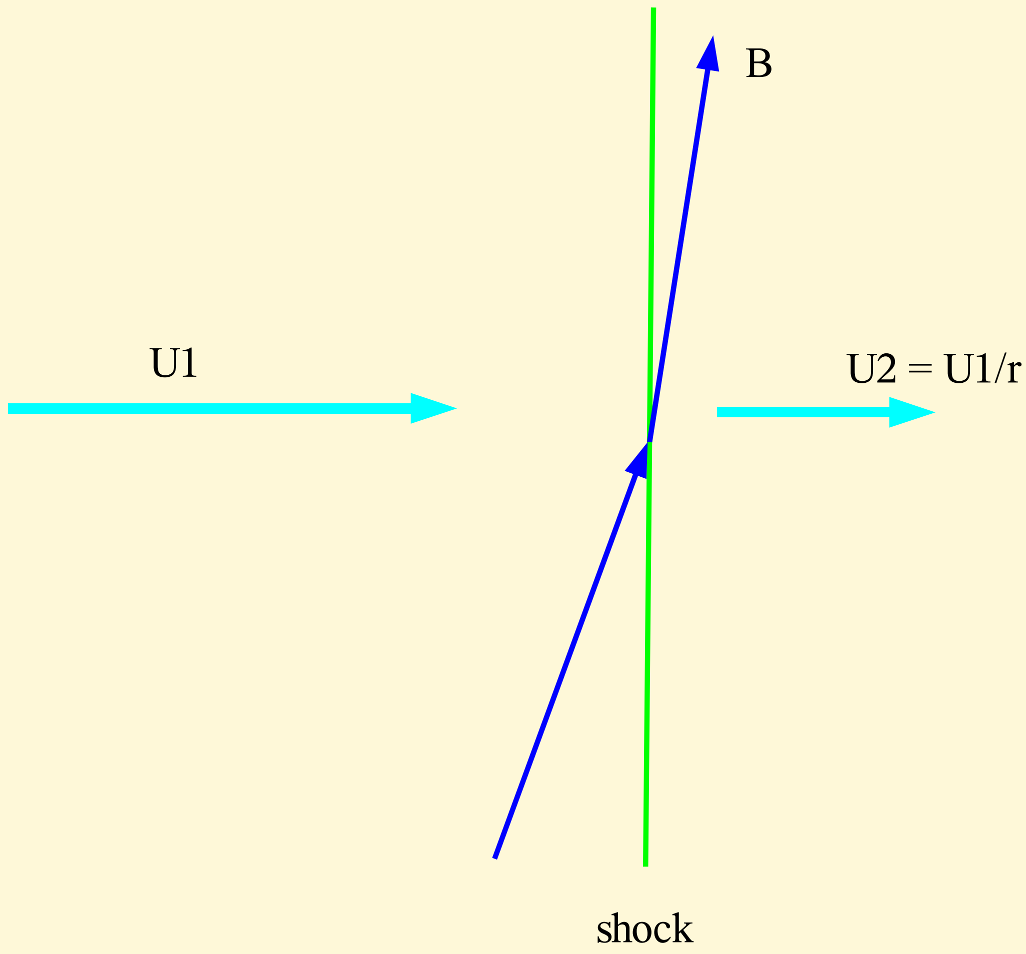
must be much less than unity. This requires

$$\kappa_{\perp} \tilde{>} U_{sh} r_c$$

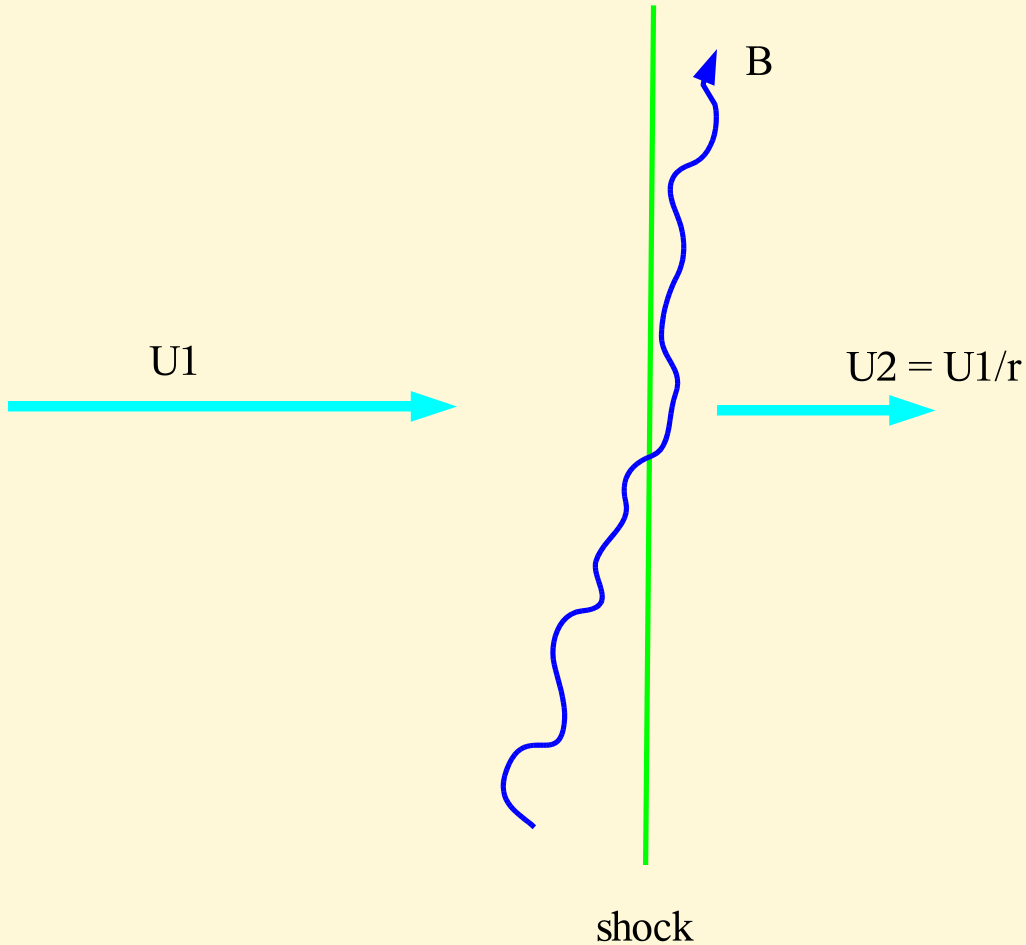
in the special case of hard sphere scattering, this gives

$$\frac{\lambda_{\parallel}}{r_c} \ll \frac{w}{U_{sh}.}$$

What about "injection" of low-energy particles



The magnetic field is turbulent.



The Lower Limit of Diffusive Shock Acceleration

Diffusive shock-acceleration theory is valid if the anisotropy is small. The general expression is:

$$|\delta_i| = \frac{3U_1}{w}\left\{1 + \frac{(\frac{\kappa_A}{\kappa_\parallel})^2 \sin^2\theta_{Bn} + (1 - \frac{\kappa_\perp}{\kappa_\parallel})^2 \sin^2\theta_{Bn}\cos^2\theta_{Bn}}{[(\frac{\kappa_\perp}{\kappa_\parallel})\sin^2\theta_{Bn} + \cos^2\theta_{Bn}]^2}\right\}^{\frac{1}{2}}$$

$$\ll 1 \quad \Rightarrow \quad \text{Diffusive Shock Acceleration is applicable}$$

The Lower Limit of Diffusive Shock Acceleration (cont.)

Case 1. Parallel shock ($\theta_{Bn} \rightarrow 0$)

$$\frac{3U_1}{w} \ll 1$$

Case 2. Perpendicular Shock ($\theta_{Bn} \rightarrow 90$)

$$\frac{3U_1}{w}[1 + (\frac{\kappa_A}{\kappa_\perp})^2]^{\frac{1}{2}} \ll 1$$

The Lower Limit of Diffusive Shock Acceleration (cont.)

Classical-scattering theory gives

$$\frac{\kappa_A}{\kappa_\perp} = \frac{\lambda_\parallel}{r_g} \gg 1 \qquad \text{(for most astrophysical applications)}$$

Thus, the classical-scattering theory predicts

$$w_{inj} \gg 3U_1(\lambda_\parallel / r_g)$$

The Lower Limit of Diffusive Shock Acceleration (cont.)

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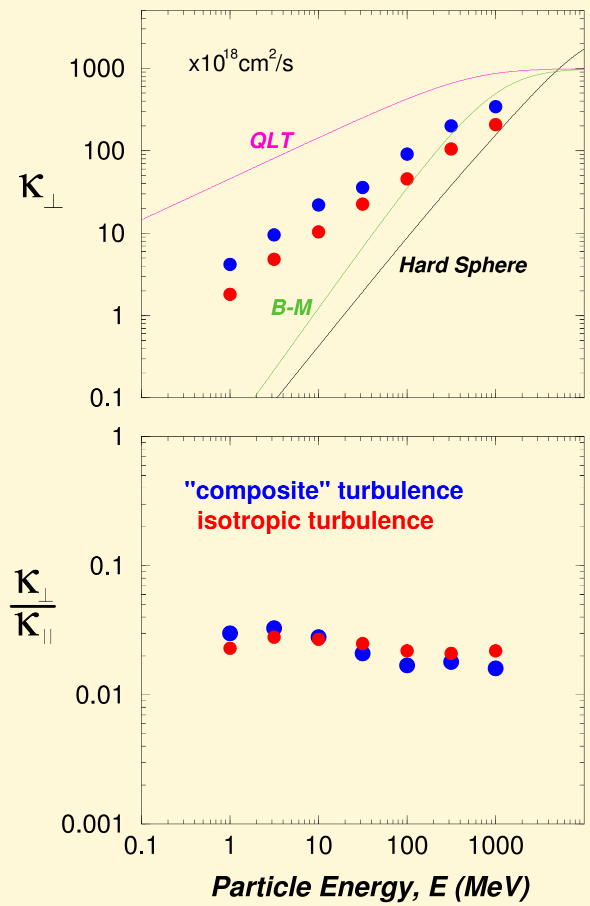
Thus, the classical-scattering theory predicts

$$w_{inj} \gg 3U_1(\lambda_\parallel / r_g)$$

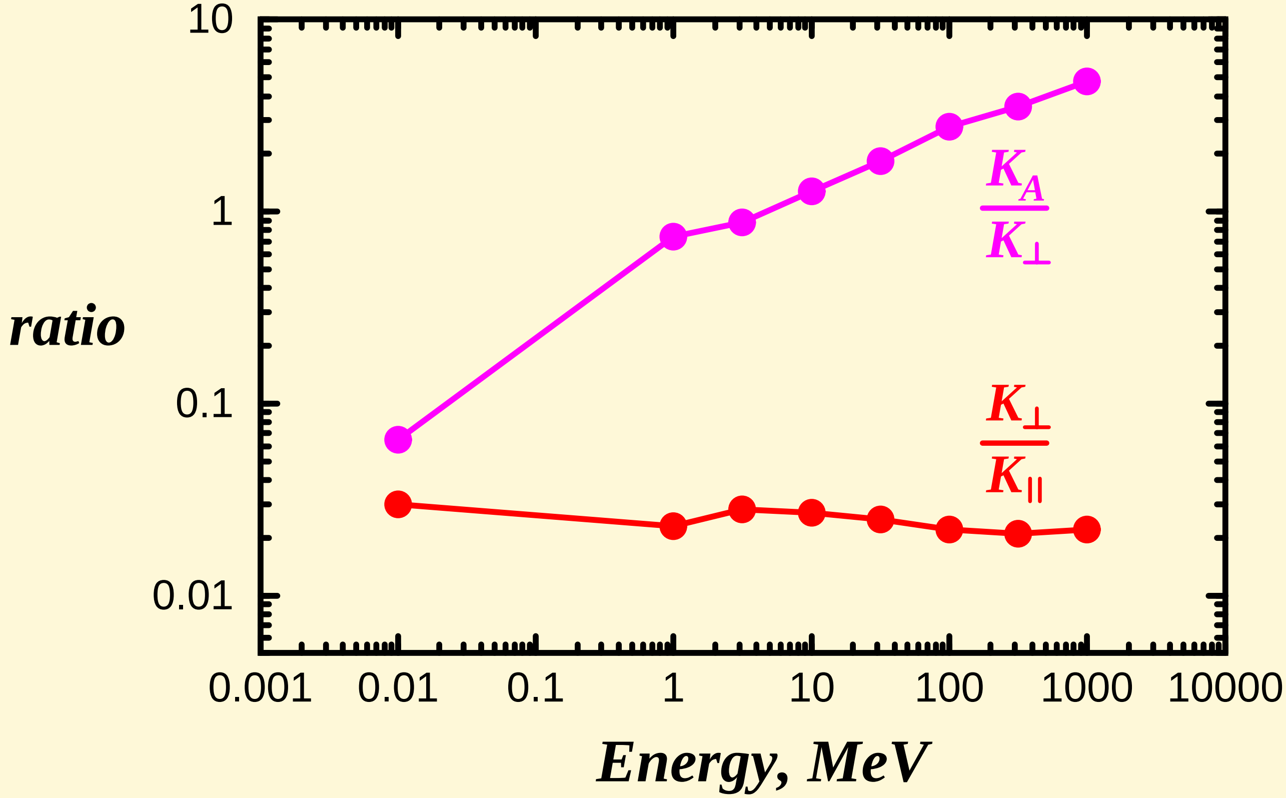
HOWEVER, classical-scattering theory is **NOT** a good approximation for perpendicular transport!

Brute force calculations of $\kappa_\perp$ (Giacalone and Jokipii, *ApJ*, 1999)



Values of $\kappa_\perp/\kappa_\parallel \approx .01 - .05$ fit GCr and ACR quite well.

Test-particle simulations using synthesized magnetic turbulence (Giacalone and Jokipii, ApJ, 1999 + one extra point)

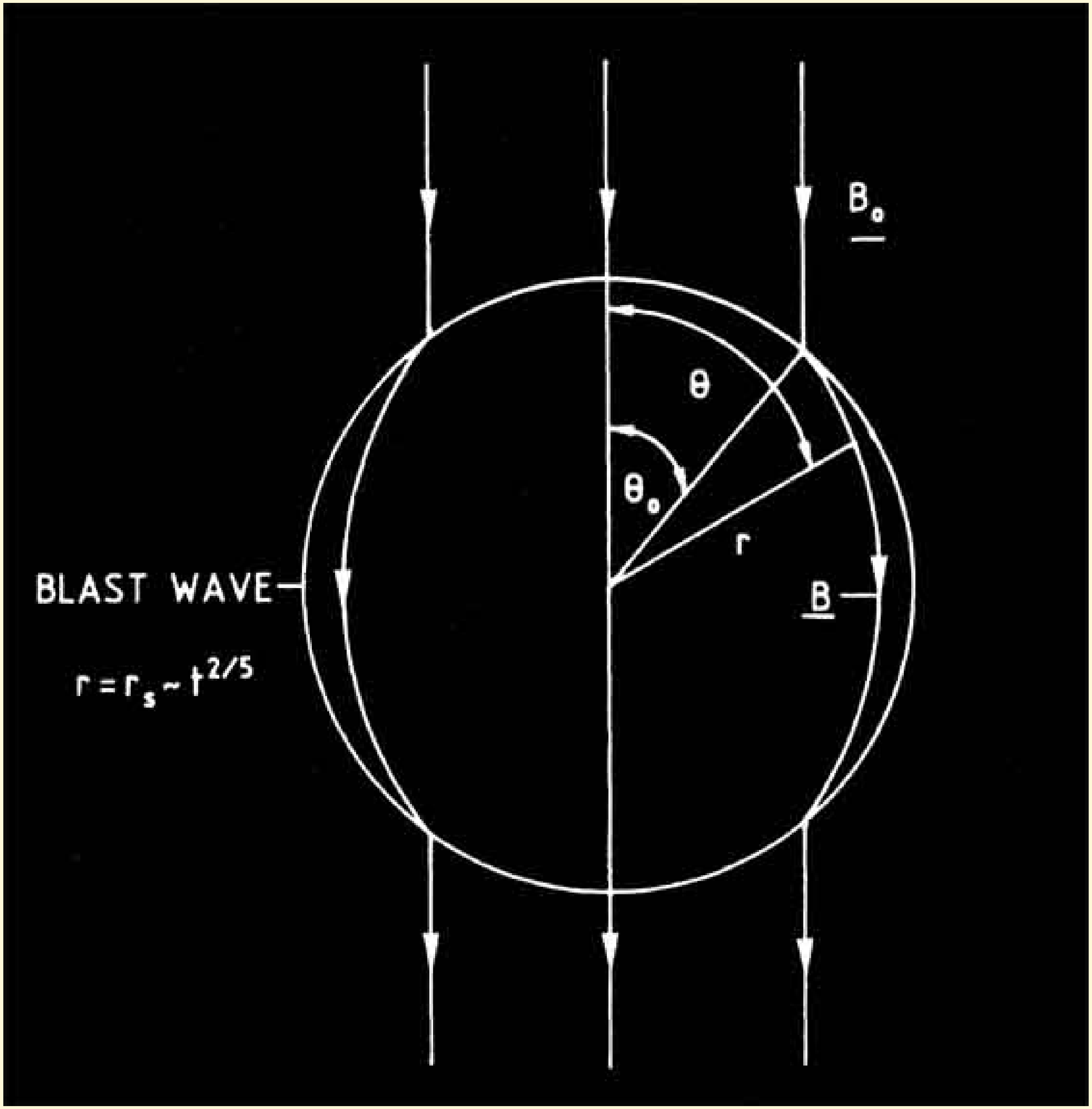


For a perpendicular shock, the injection velocity is given by

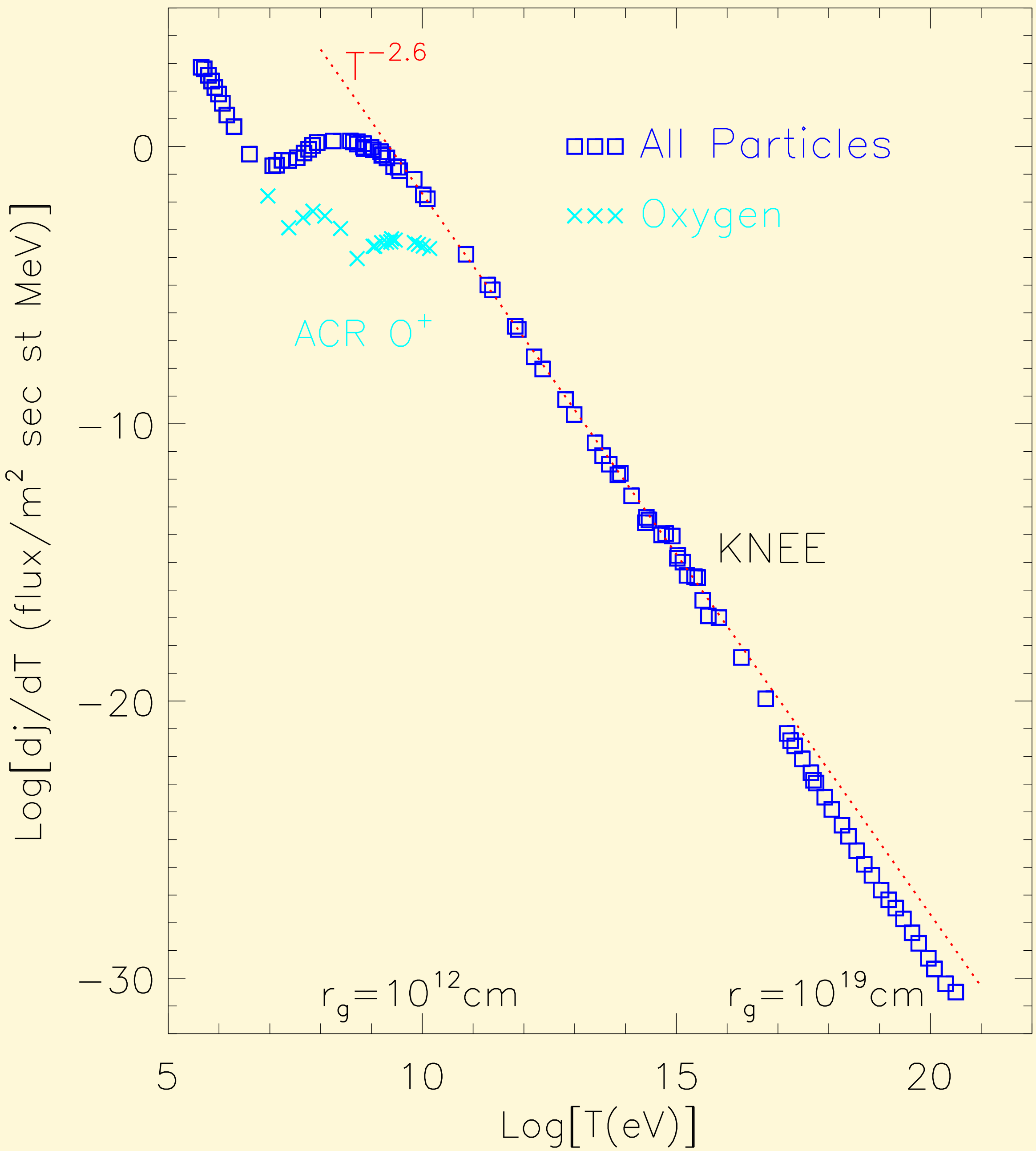
$$w_{inj} = 3U_1[1 + (\frac{\kappa_A}{\kappa_\perp})^2]^{\frac{1}{2}}$$

$$\approx 3U_1$$

$\Rightarrow$ The SAME as for a parallel shock.



B₀
θ
θ₀
r
BLAST WAVE
B
r = r_s ~ t^{2/5}



T^{-2.6}
All Particles
Oxygen
ACR O+
KNEE
r_g=10^{12}cm
r_g=10^{19}cm
0
-10
-20
-30
5
10
15
20
Log[dj/dT (flux/m^2 sec st MeV)]
Log[T(eV)]

$\Longrightarrow$ We need something other than shock acceleration.

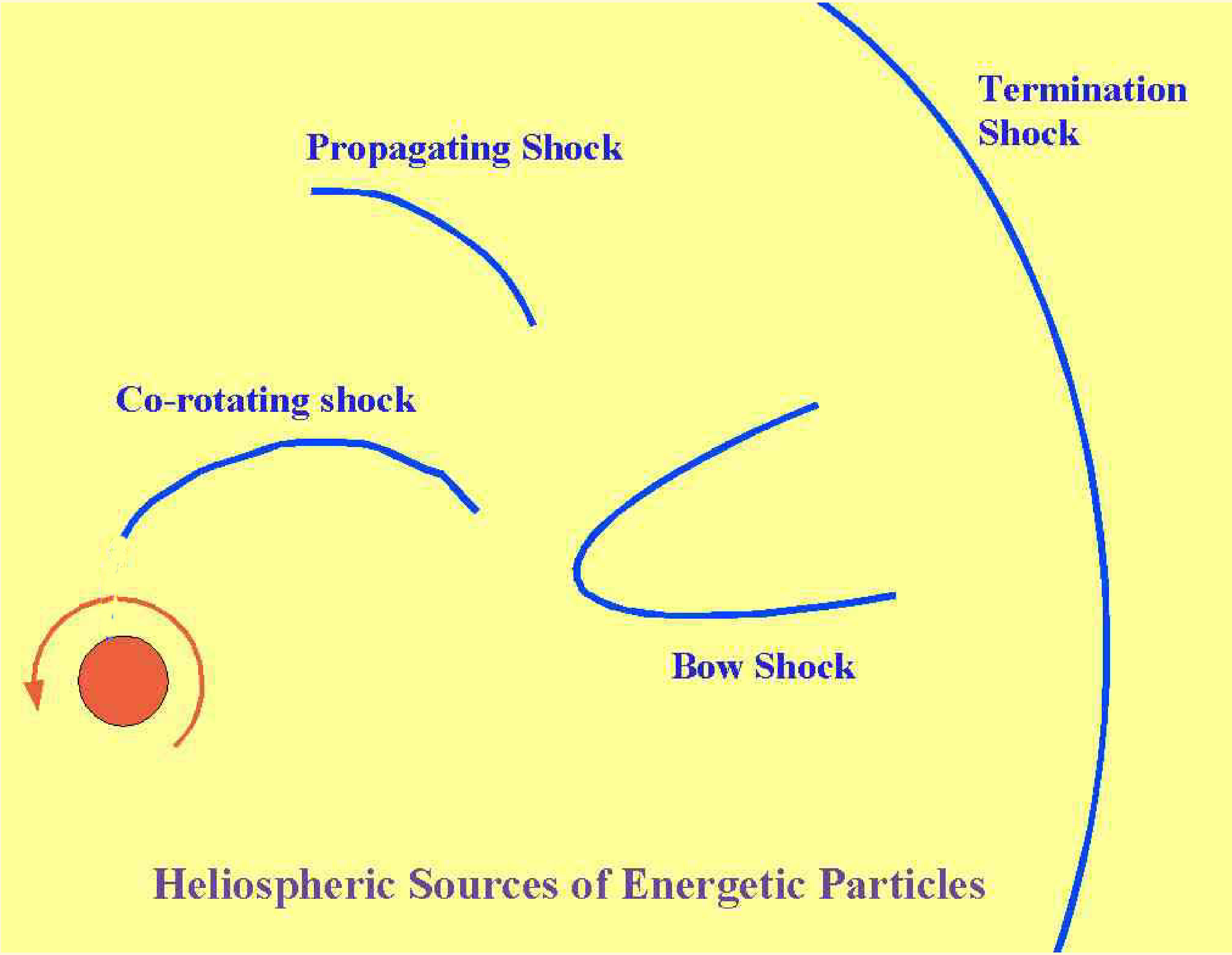
- Several accelerators have been suggested, some long before diffusive shock acceleration.

- They all have difficulties.

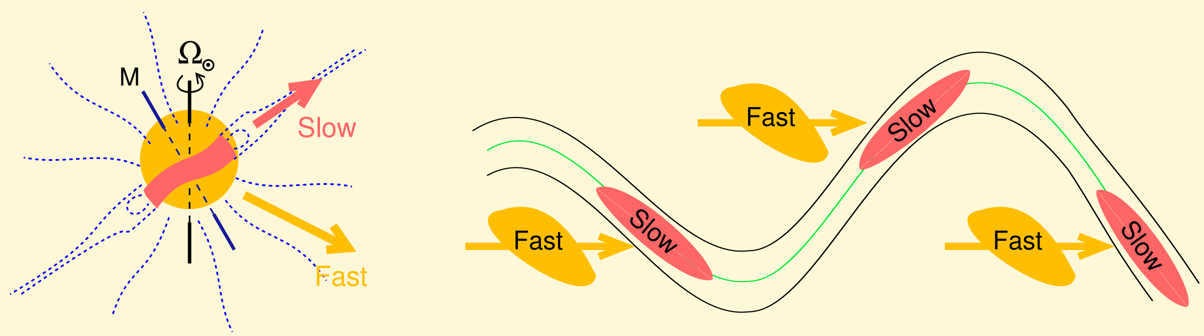
Here, I would like to suggest what seems to be a new mechanism, which provides a very good explanation for the heliospheric observations, and which may help at the energies beyond the knee in the galaxy.

It is closely related to diffusive shock acceleration, and therefore tends to give a similar energy spectrum.

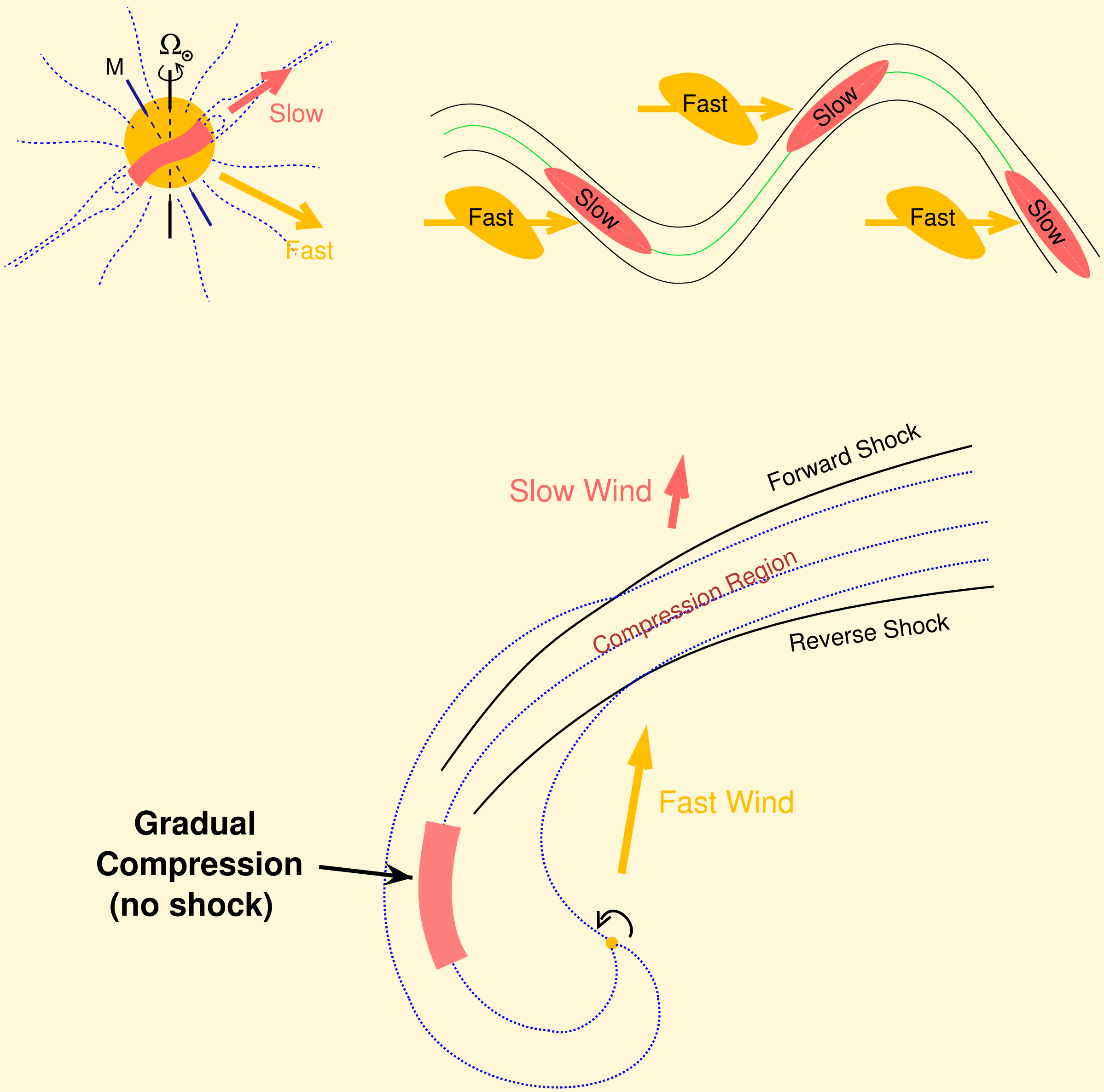
Begin with the heliosphere.


Termination
Shock
Propagating Shock
Co-rotating shock
Bow Shock
Heliospheric Sources of Energetic Particles

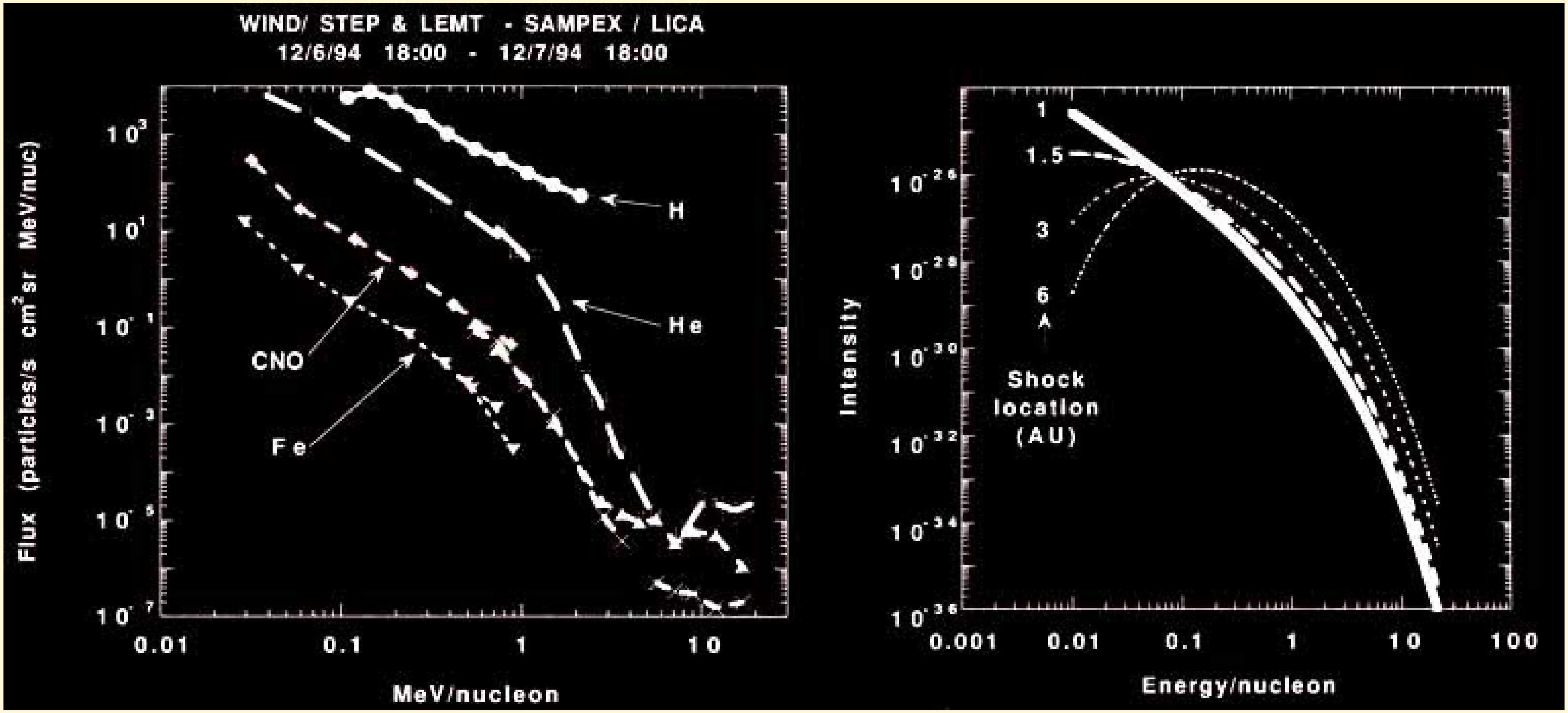
Co-rotating Interaction Regions



Co-rotating Interaction Regions



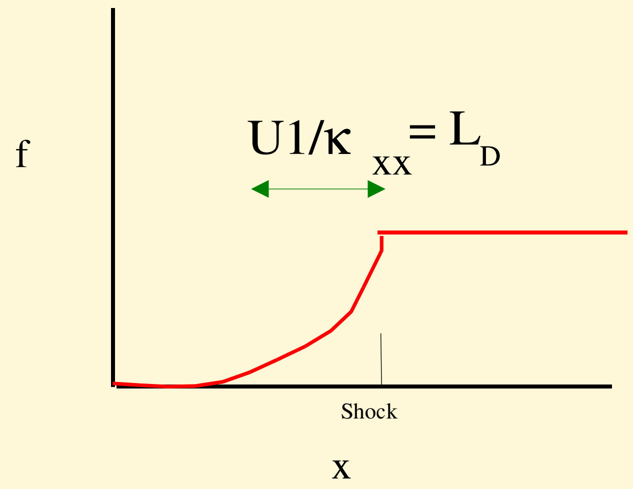
Observations of Particles observed near 1 AU in region of compression, but no shock. (Mason, 2000)

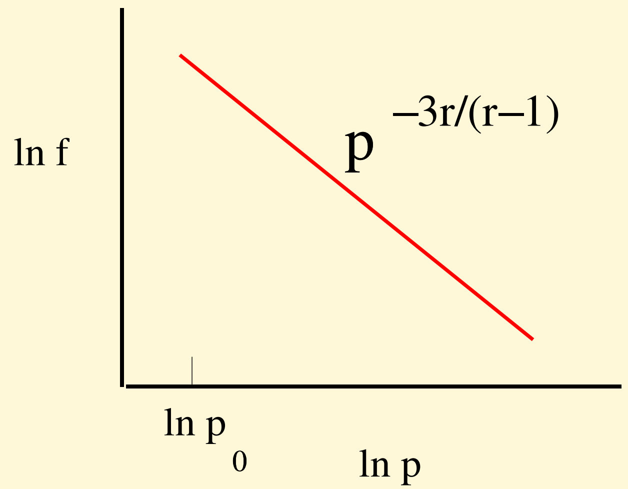


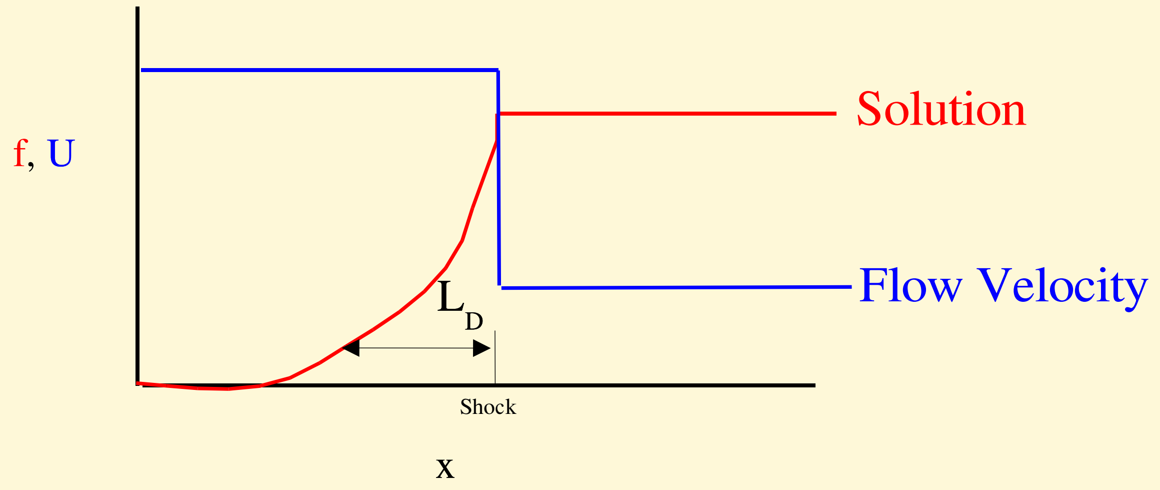
How are these particles accelerated?

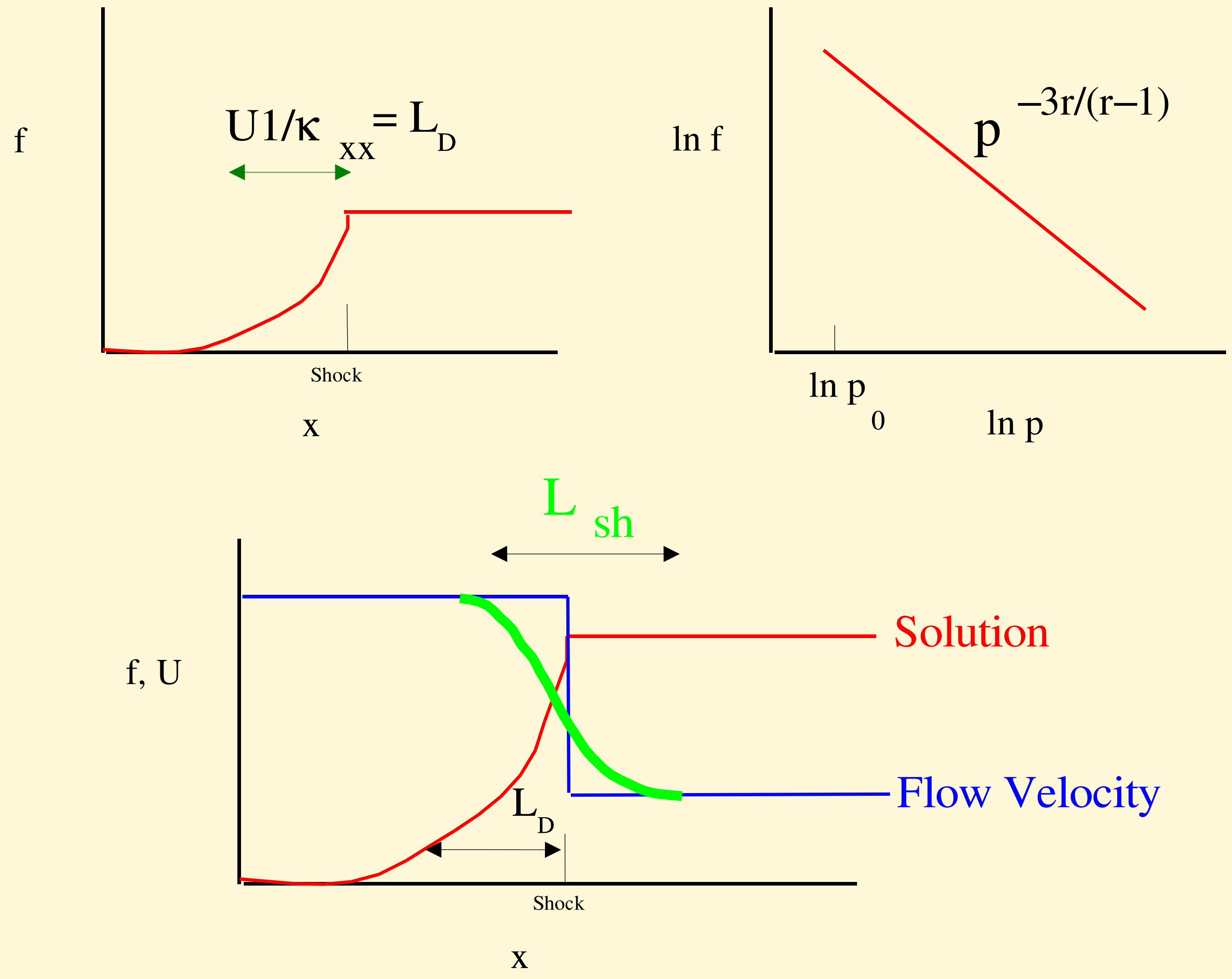
- Schwadron, Fisk and Gloeckler (GRL, 1996) have suggested statistical acceleration (essentially 2nd-order Fermi) in compression regions away from shocks. Lots of free parameters.

- We at Arizona recently (Giacalone, Kóta and Jokipii, 2002) have suggested an alternative, new mechanism: diffusive compression acceleration. The parameters are tightly constrained.


f
U1/κ xx = L_D
Shock
x


ln f
p −3r/(r−1)
ln p 0
ln p


f, U
Solution
Flow Velocity
L_D
Shock
x



f
U1/κ xx = L D
Shock
x
ln f
p −3r/(r−1)
ln p 0
ln p
L sh
f, U
Solution
Flow Velocity
L D
Shock
x

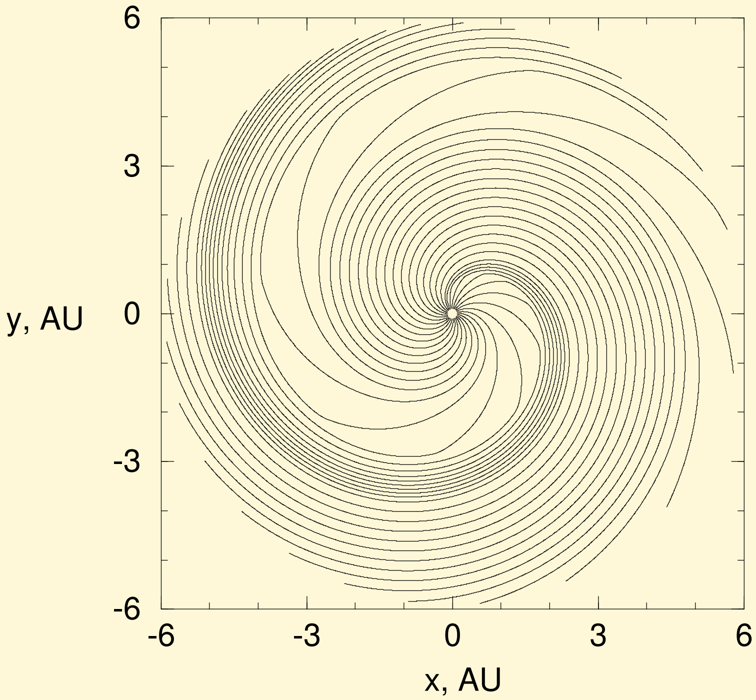
Actually a true discontinuity in U is not needed for acceleration. Consider a compression with scale L_c and let diffusive scale be $L_D = \kappa_{xx}/U1$, then we simply require

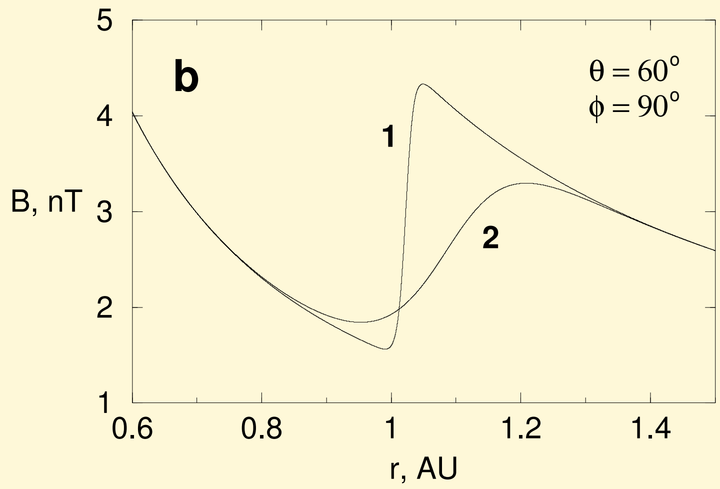
$$L_c \ll L_D$$

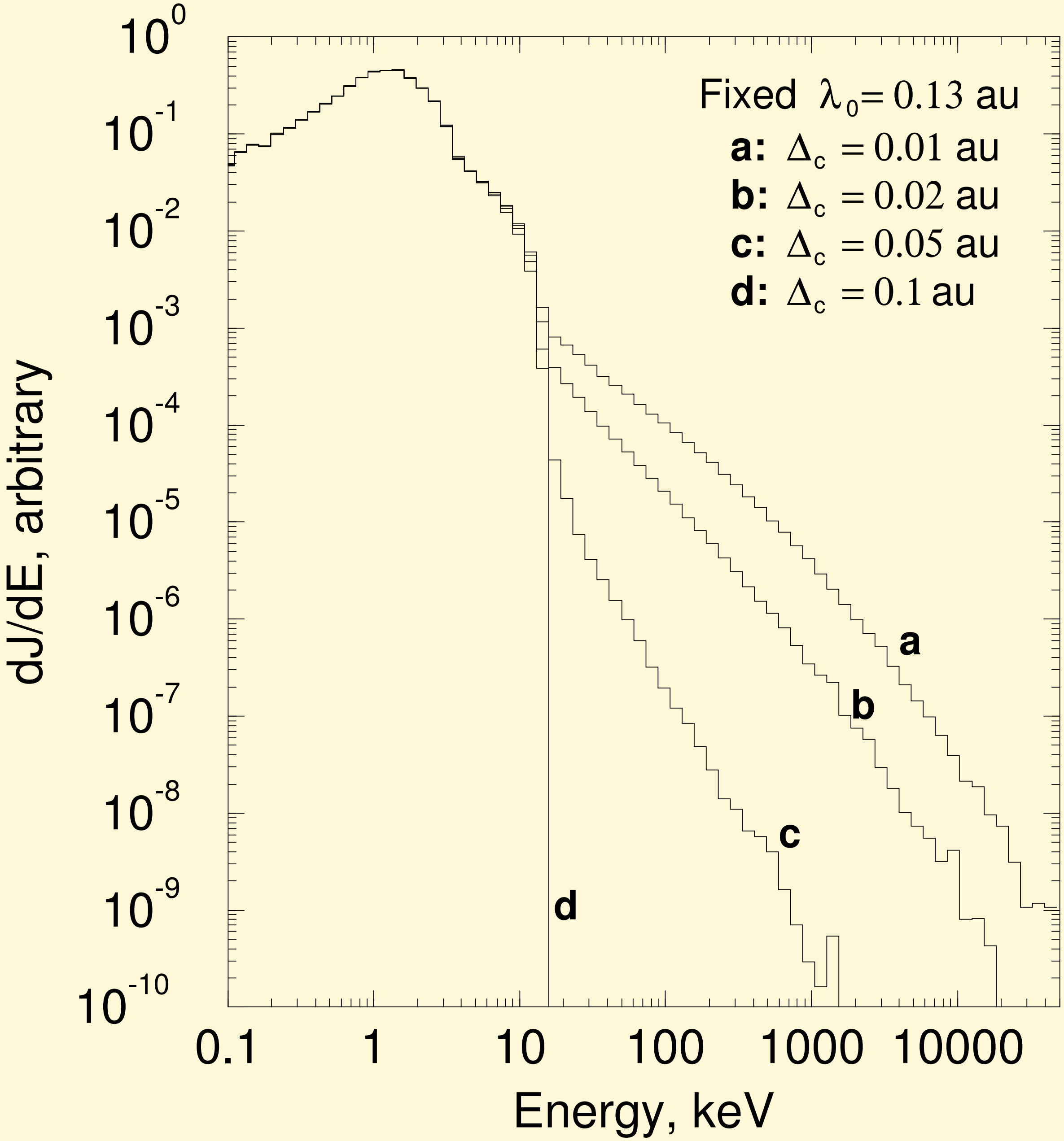
In fact, there are three regimes:

- $L_c \gg L_D$ .... no significant acceleration
- $L_c \approx L_D$ .... acceleration
- $L_c \ll L_d$ .... like shock acceleration

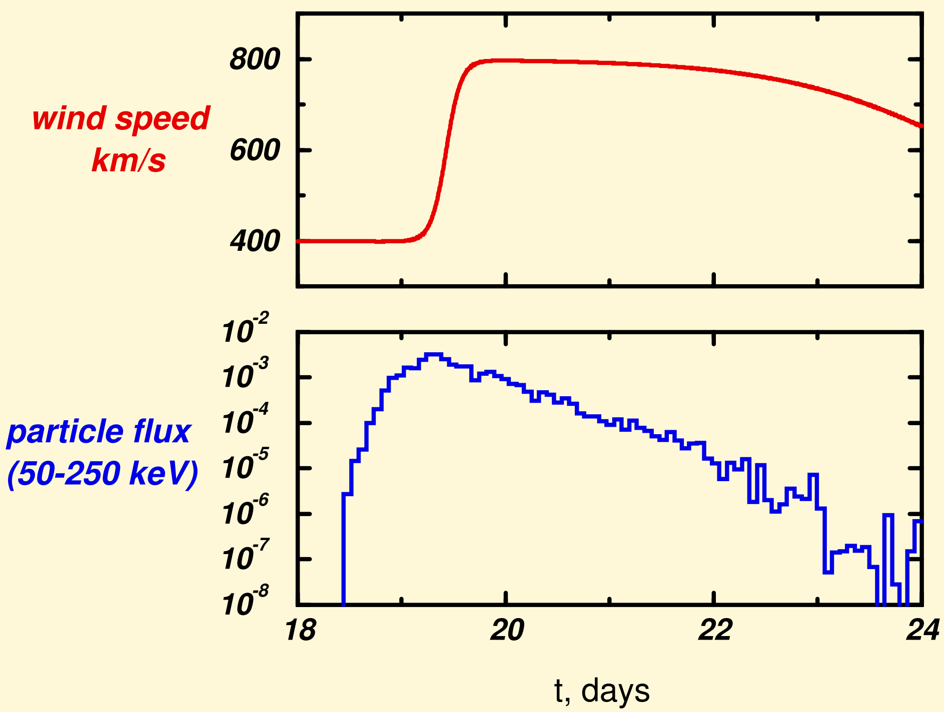
From Giacalone, Jokipii and Kóta (2002). Illustration of the interplanetary configuration used. Here, $\lambda_c \geq R$, so We used orbit simulations with $ad\ hoc$ scattering.



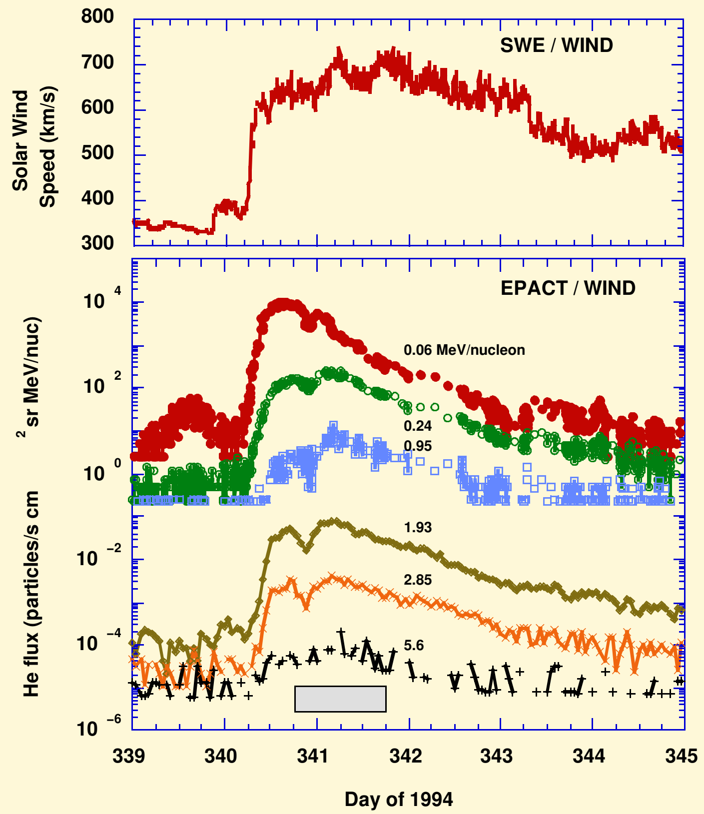



Fixed $\lambda_0 = 0.13$ au
a: $\Delta_c = 0.01$ au
b: $\Delta_c = 0.02$ au
c: $\Delta_c = 0.05$ au
d: $\Delta_c = 0.1$ au
dJ/dE, arbitrary
Energy, keV
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0.1
1
10
100
1000
10000
a
b
c
d

Simulations
(Giacalone et al., 2002)

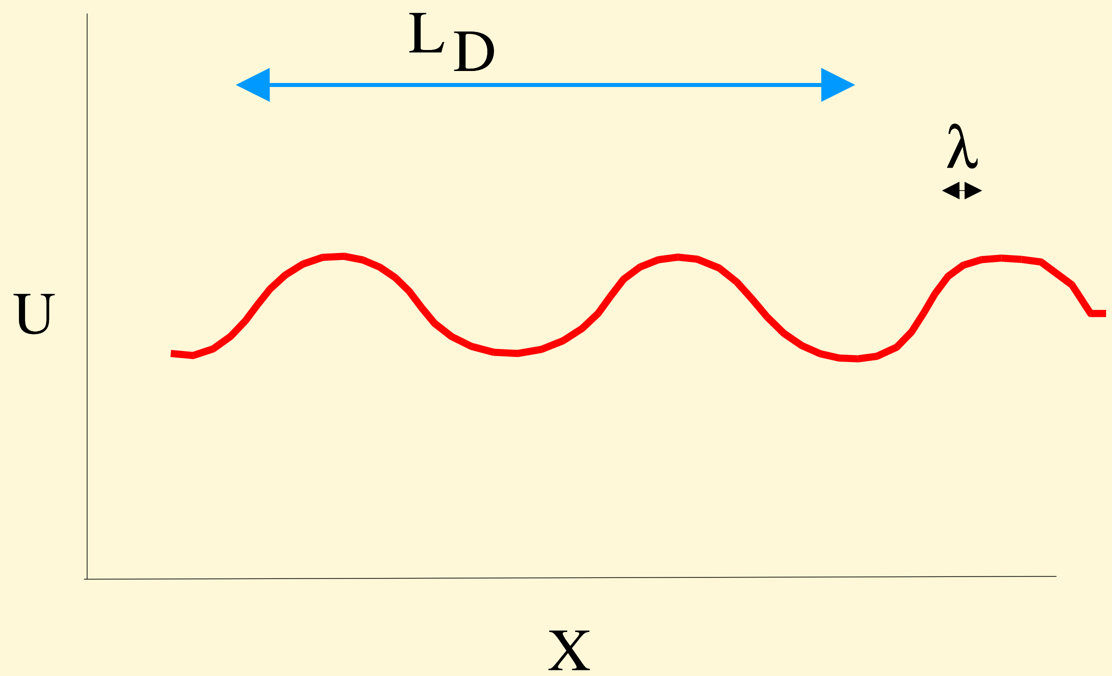


Observations
(Mason, 2000)



- This compressive acceleration has some very interesting properties.

- To see this, consider simple, sinusoidal compressions.



Clearly, L_D is larger than the scale of U

and λ is smaller, so diffusion applies.

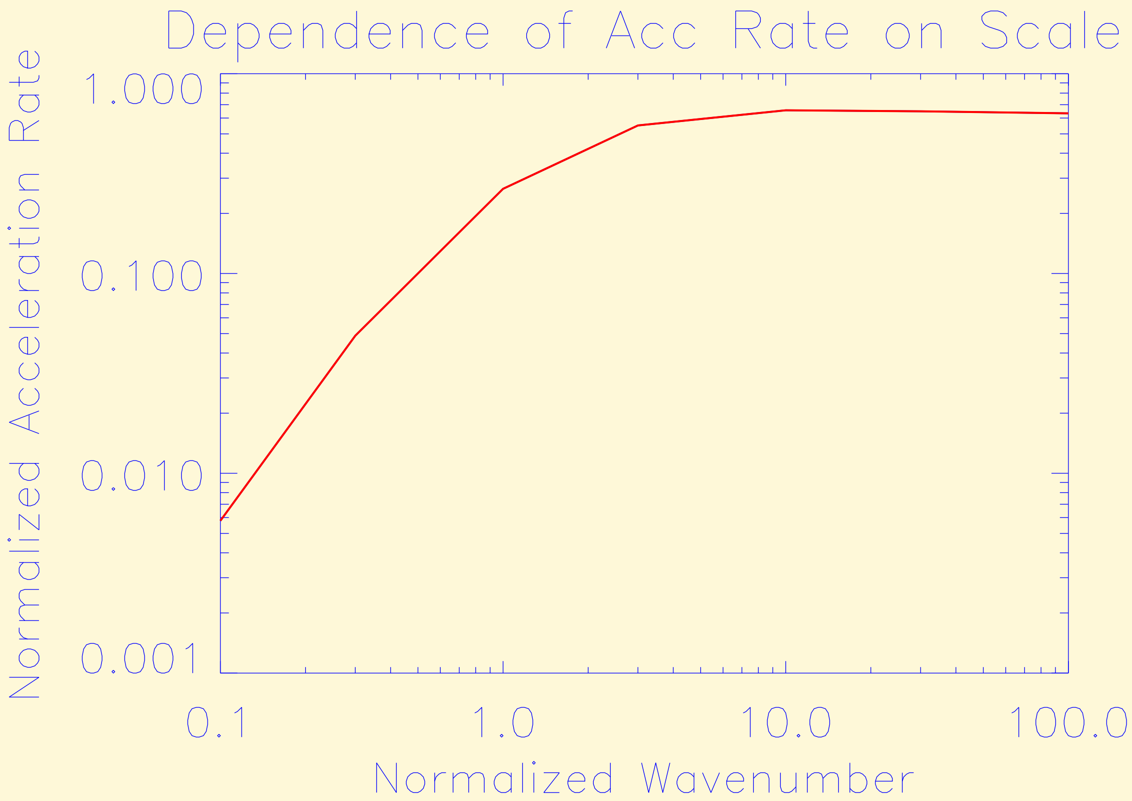
We expect that shock–like acceleration
will occur.

Set the flow speed in the one-dimensional Parker equation equal to a simple sine wave and kept κ_{xx} constant.

$$
\begin{aligned}
U(x) &= U_0(1 + \alpha\ sin(kx)) \\
\frac{\partial f}{\partial t} &= \kappa_{xx}\frac{\partial^2 f}{\partial x^2} - U_0\left[1 + \alpha\ sin(kx)\right]\frac{\partial f}{\partial x} + \\
&\quad \frac{1}{3}U_0\ \alpha\ k\ cos(kx)\left[\frac{\partial f}{\partial \ln p}\right] + Q(x,t)
\end{aligned}
$$

This has been studied to obtain acceleration rates and spectra.

Using dimensionless parameters - $\tau = tU_0^2/\kappa_{xx}, q = k(\kappa_{xx}/U_0)$, this has been solved numerically.



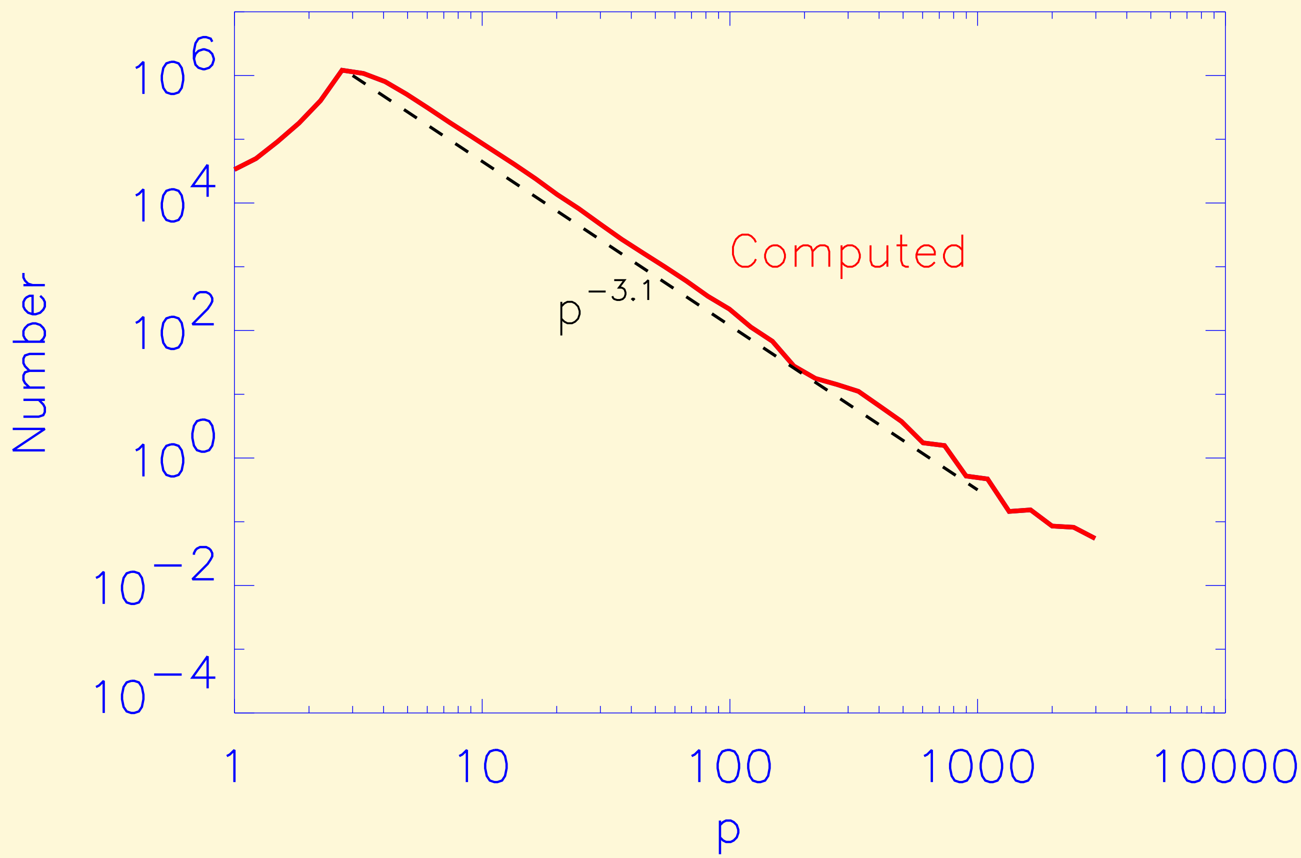
The acceleration time is of order κ_{xx}/U_0^2.

This clearly has similarities with $both$ shock acceleration and 2nd order Fermi acceleration.

However, it is signifiantly different from both.

For example, if the magnetic field is $normal$ to the x direction, the particle gradient drifts parallel to the $\mathbf{U} \times \mathbf{B}$ electric field in the magnetic field compressions is the main source of energy gain.

A simple steady-state model with an energy independent loss produces a reasonable power-law spectrum:



Ko and Webb (2002) and Ptuskin (2002, private communication) have shown that this problem may be studied analytically in the limit

$$L_c \ll L_D$$

Here, f nearly constant in space, and one may do a do multiple time-scale analysis. Upon defining $< f >= \int_{period} f dx$

$$\frac{\partial < f >}{\partial t} = -\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^4 \frac{U_0^2}{18\kappa_{xx}}\frac{\partial < f >}{\partial p}\right] - \frac{f}{\tau_{loss}} + Q$$

If $\kappa_{xx}/U_0^2 \ll \tau_{loss}$ we have, asymptotically, $< f > \rightarrow p^{-3}$, which an intriguing result. Similar equations in this limit have also been studied by Bykov and by Zank and Axford.

NUMBERS

For the solar wind near 1 AU, for a compression speed U_0 of some 70 km/sec, and $\kappa \approx 10^{19} cm^2/sec$

$$t_{acc} \approx 3 \times 10^5 sec$$

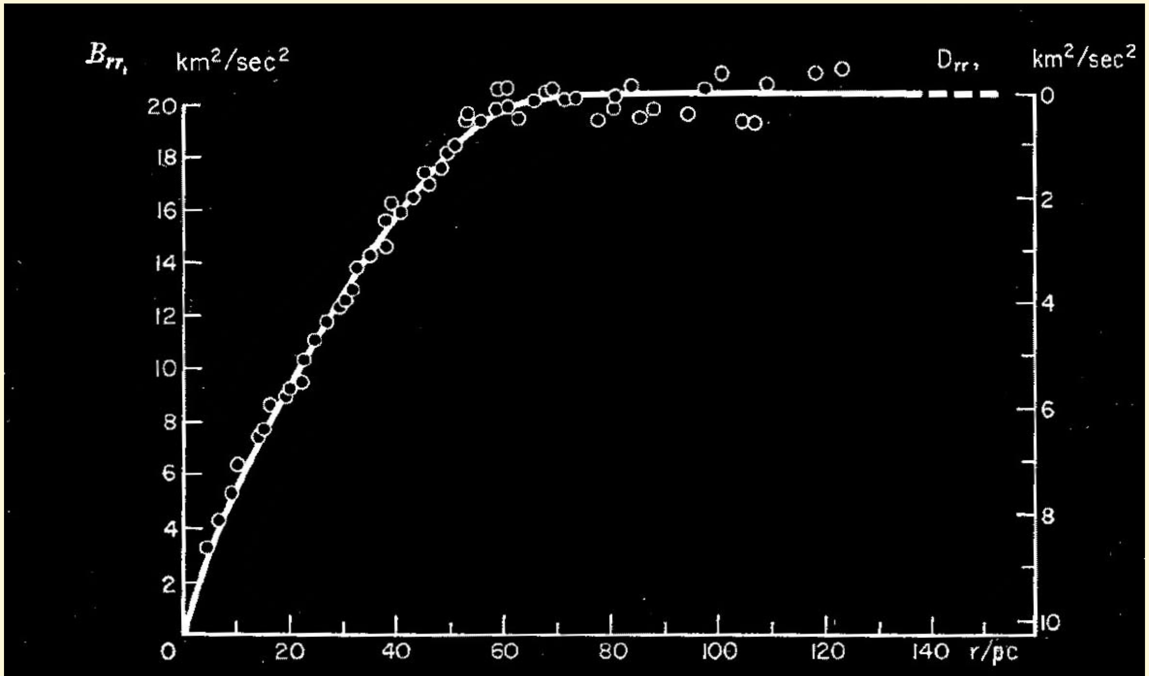
Comparable with the adiabatic cooling time.

In the interstellar medium , again taking a speed of 50 km/sec and $\kappa \approx 10^{28} cm^2/sec$,

$$t_{acc} \approx 3 \times 10^6 yrs,$$

or less than the loss time from the galaxy.

Velocity fluctuations occur in the interstellar medium.



Structural function B_{rr} and correlation function D_{rr} of interstellar turbulence.

Diffusive compression acceleration might help explain cosmic rays beyond the knee!.

SUMMARY and CONCLUSIONS

- Shocks (including perpendicular shocks) provide a natural explanation for most cosmic rays.

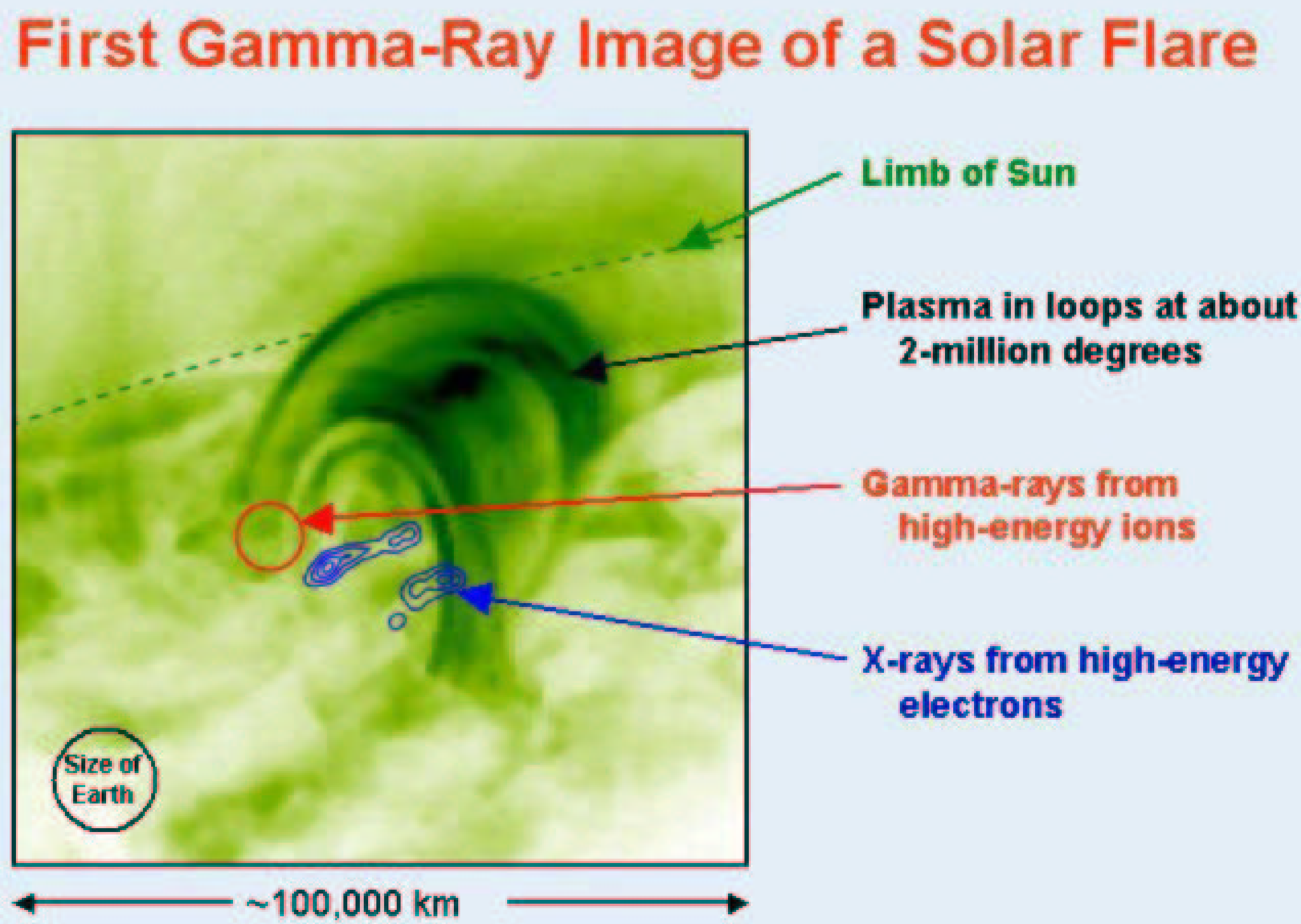
- Statistical acceleration is unattractive.

- Acceleration occurs where shocks apparently cannot do the job.

- Diffusive compression acceleration provides a very compelling explanation of energetic-particle observations in the heliosphere.

- his mechanism, In the interstellar medium it may produce the particles beyond the cosmic-ray "knee" at 3×10^{15} eV.

THE END



Note separation of e, p. Due to drifts?