Current Status of Shock Acceleration Theory

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Very quick introduction to shock acceleration. Basically rather simple!

Magnetic fields can easily change a charged particle's direction of motion, but not (as long as field is stationary) its energy

Almost isotropic distributions

$$f(\vec{p}) \approx f(p), \quad p = |\vec{p}|$$

Diffusive transport

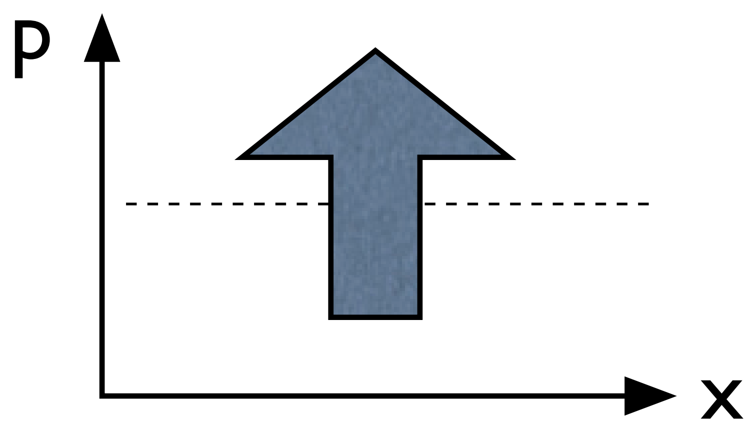
$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f = \nabla(\kappa \nabla f) + \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p}$$

Acceleration almost entirely from compression!

Useful to think in terms of the acceleration flux,

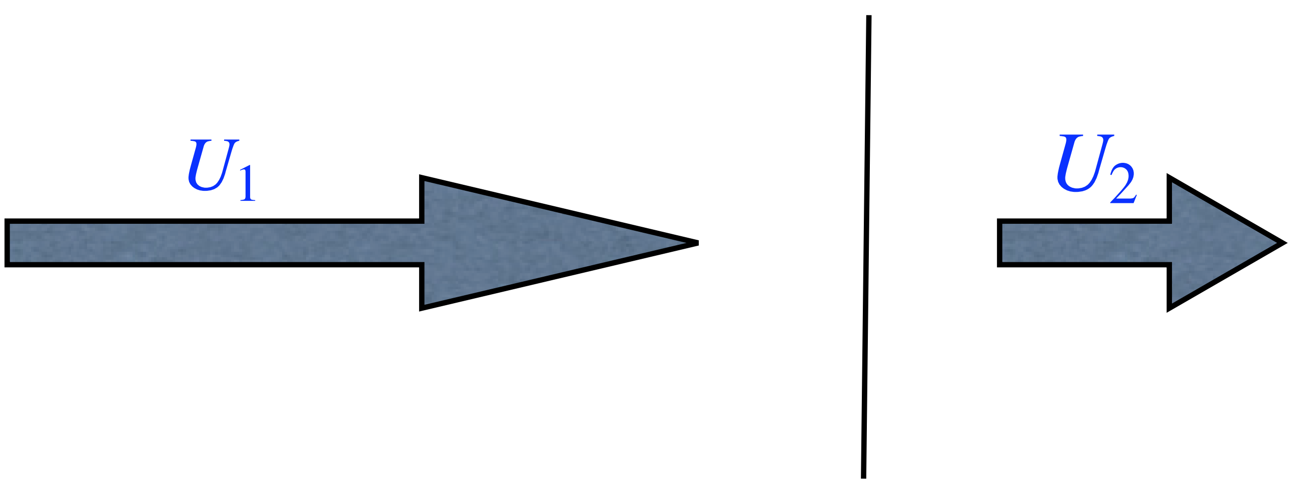
$$\Phi(p) = \int \frac{4\pi p^3}{3} f(p)(-\nabla \cdot \vec{U})\, d^3x$$

Rate at which particles are being accelerated through a given momentum (or energy) level.



Suppose compression occurs only at a shock, then

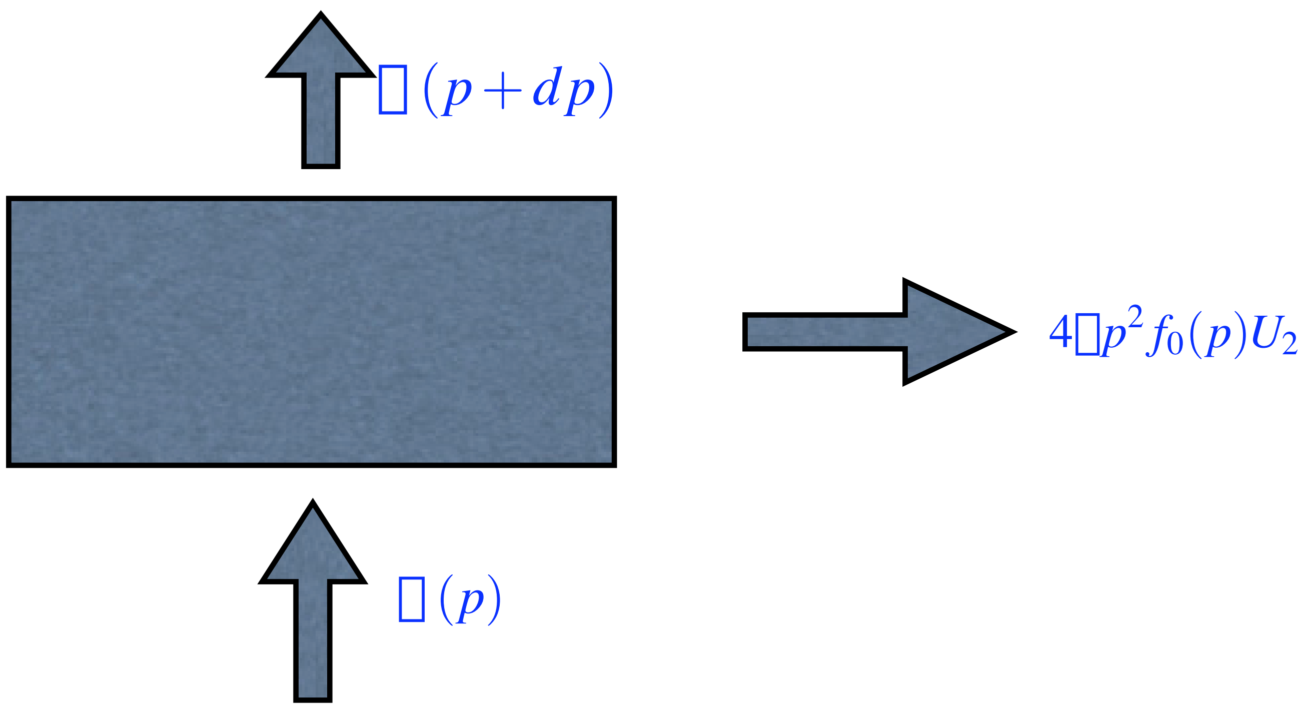
$$\square(p) = \frac{4\square p^3}{3} f_0(p)\,(U_1 - U_2)$$



and is localised at the shock.

Now write down particle conservation law for balance between rate of advection away from shock region and acceleration

$$\frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$



Can get all standard results from this simple "box" model including (with a little more work) time scales, in particular....

$$\frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$

$$\frac{\partial}{\partial p}\left(\frac{4\pi p^3}{3} f_0(p)(U_1 - U_2)\right) = -4\pi p^2 f_0(p) U_2$$

$$\frac{U_1 - U_2}{3} p \frac{\partial f_0}{\partial p} = -U_1 f_0$$

$$p \frac{\partial f_0}{\partial p} = \frac{-3U_1}{U_1 - U_2} f_0$$

Power-law spectrum with exponent fixed by shock compression!

Test-particle (linear) theory

- Shock is simple jump discontinuity

$$U(x) = \begin{cases} U_1, & x < 0, \\ U_2, & x > 0 \end{cases}$$

- Particles have power-law spectrum in momentum

$$f(p) \propto p^{-s}, \qquad s = \frac{3U_1}{U_1 - U_2}$$

- Acceleration time-scale is of order

$$t_{acc} = \frac{3}{U_1 - U_2}\left(\frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2}\right)$$

But easy to show that accelerated particle pressure can be significant, so must worry about reaction effects. Also, if process is to work with high efficiency, as appears to be required, eg, to explain the Galactic cosmic ray origin, we need a nonlinear theory.

In principle easy - we just have to solve the diffusive transport equation and the usual hydrodynamic equations with an additional cosmic ray pressure in the momentum equation!

$$P_C(x) = \int \frac{4\square p^3 v}{3} f(p,x)\, dp$$

In practice very hard!

Possible approaches

- Throw it at the computer
- Monte-Carlo approach
- Two-fluid approximation
- Semi-analytic theories

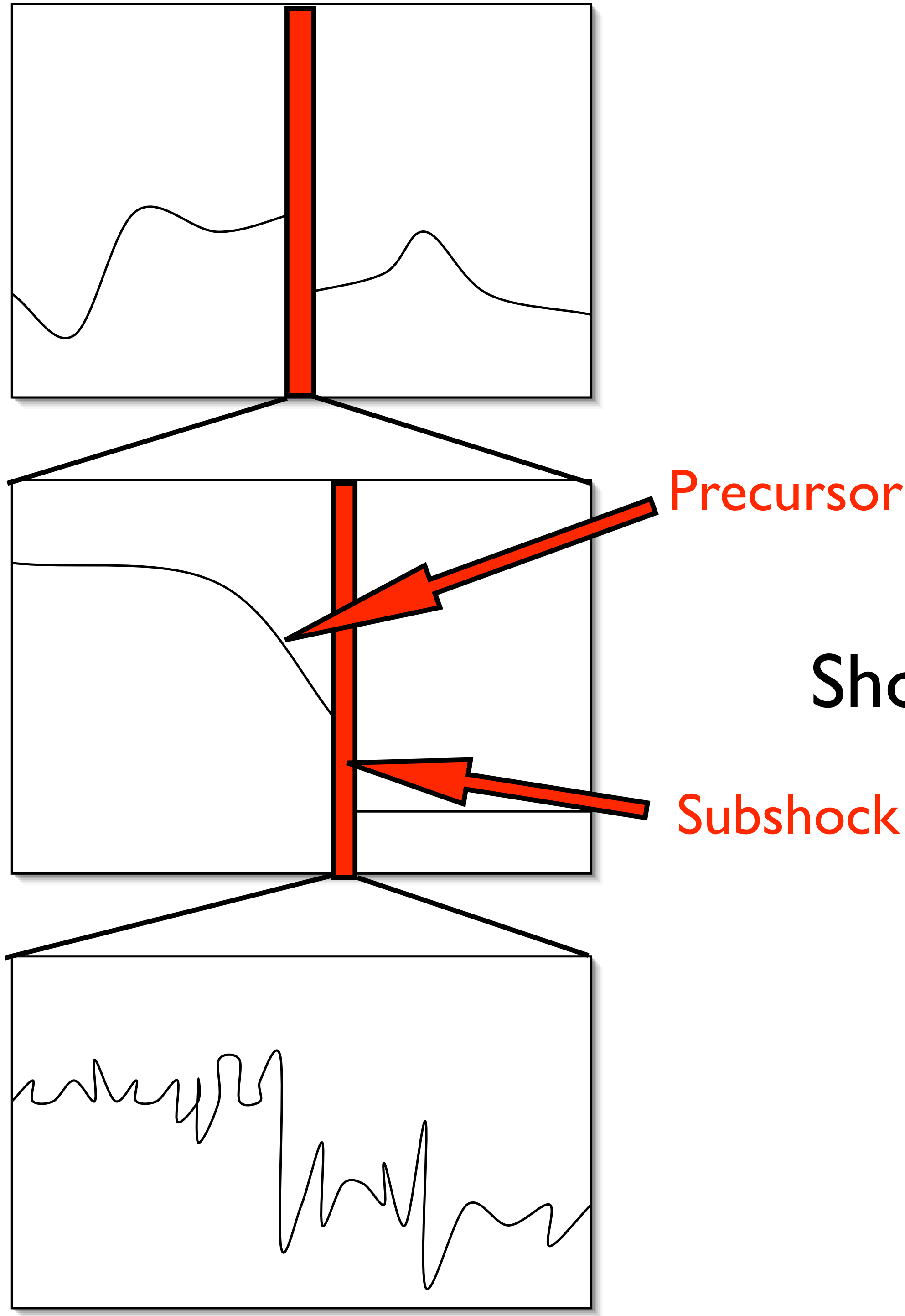
Good general agreement now between all approaches!

Very wide scale separation - numerical nightmare, but useful for analytic approaches. Can distinguish two extreme scales..

Outer scale of macroscopic system and maximum energies

Inner scale of injection processes and kinetic effects

Aim of analytic theory should be to bridge the gap between these two regimes, but not to try to be a complete theory.



Outer scale
Astrophysics

Intermediate scales
Shock acceleration theory

Inner scale
Plasma physics
Injection!

Much more tractable problem!

Can (hopefully) assume steady planar structure with fixed mass and momentum fluxes.

$$\square U = A$$

$$AU + P_G + P_C = B$$

and we still have the steady balance between acceleration and loss downstream...

$$\frac{\partial \square}{\partial p} = -4\square p^2 f_0(p) U_2$$

.. but the problem is that the acceleration flux now depends on the upstream velocity profile ***and*** the particle distribution.

However, if one makes an *Ansatz*

$$f_0(p) \rightarrow f(x,p)$$

the particle conservation equation and the momentum balance equation,

$$AU + P_G + P_C = B$$

$$\frac{\partial \square}{\partial p} = -4\square p^2 f_0(p) U_2$$

become two coupled equations for

$$U(x), \quad f_0(p)$$

An obvious *Ansatz* would be to assume a distribution similar to that familiar from the test-particle theory,

$$f(x,p) = f_0(p)\exp\int \frac{U(x)\,dx}{\square(x,p)}$$

This is actually close to Malkov's *Ansatz* who, however, uses

$$f(x,p) = f_0(p)\exp\int \left(-\frac{1}{3}\frac{\partial \ln f_0}{\partial \ln p}\right)\frac{U(x)\,dx}{\square(x,p)}$$

which he claims has some advantages.

Remarkably, the crudest *Ansatz*, which simply assumes the accelerated particles penetrate a fixed distance upstream and then abruptly stop, appears to work quite well and gives results very similar to Malkov's. This approximation, originally due to Eichler, is

$$f(x,p) = \begin{cases} f_0(p), & x > -L(p) \\ 0, & x < -L(p) \end{cases}$$

It leads to equations which can be heuristically derived in a nonlinear box model and which have been used by a number of authors, most recently P. Blasi.

Defining $U_p = U(-L(p))$

$$\square = \int \frac{4\square p^3}{3} f(x,p) \frac{du}{dx} = \frac{4\square p^3}{3} f_0 (U_p - U_2)$$

and thus

$$\frac{\partial \square}{\partial p} = -4\square p^2 f_0 U_2 = -\frac{3U_2}{U_p - U_2} \frac{\square}{p}$$

$$A\left(1 - M_p^{-2}\right) \frac{\partial U_p}{\partial p} = \frac{4\square p^3}{3} v f_0 = \frac{\square}{U_p - U_2} v$$

Can be written in many ways, but just two coupled ODEs!

Remarkably, if we ignore gas pressure and switch to particle kinetic energy, rather than momentum, as independent variable, the last equation can be written

$$\frac{\partial}{\partial T}\left(U_p - U_2\right)^2 = \frac{2\square}{A}$$

Thus if losses can be neglected and the acceleration flux is a constant

$$U_2 \approx 0, \quad U_p \approx \sqrt{\frac{2\square\, T}{A}}, \quad f_0 \quad p^{-3} T^{-1/2}$$

which is just Malkov’s “universal” spectrum

Can be thought of as the asymptotic attractor for all nonlinearly modified solutions at high energies.

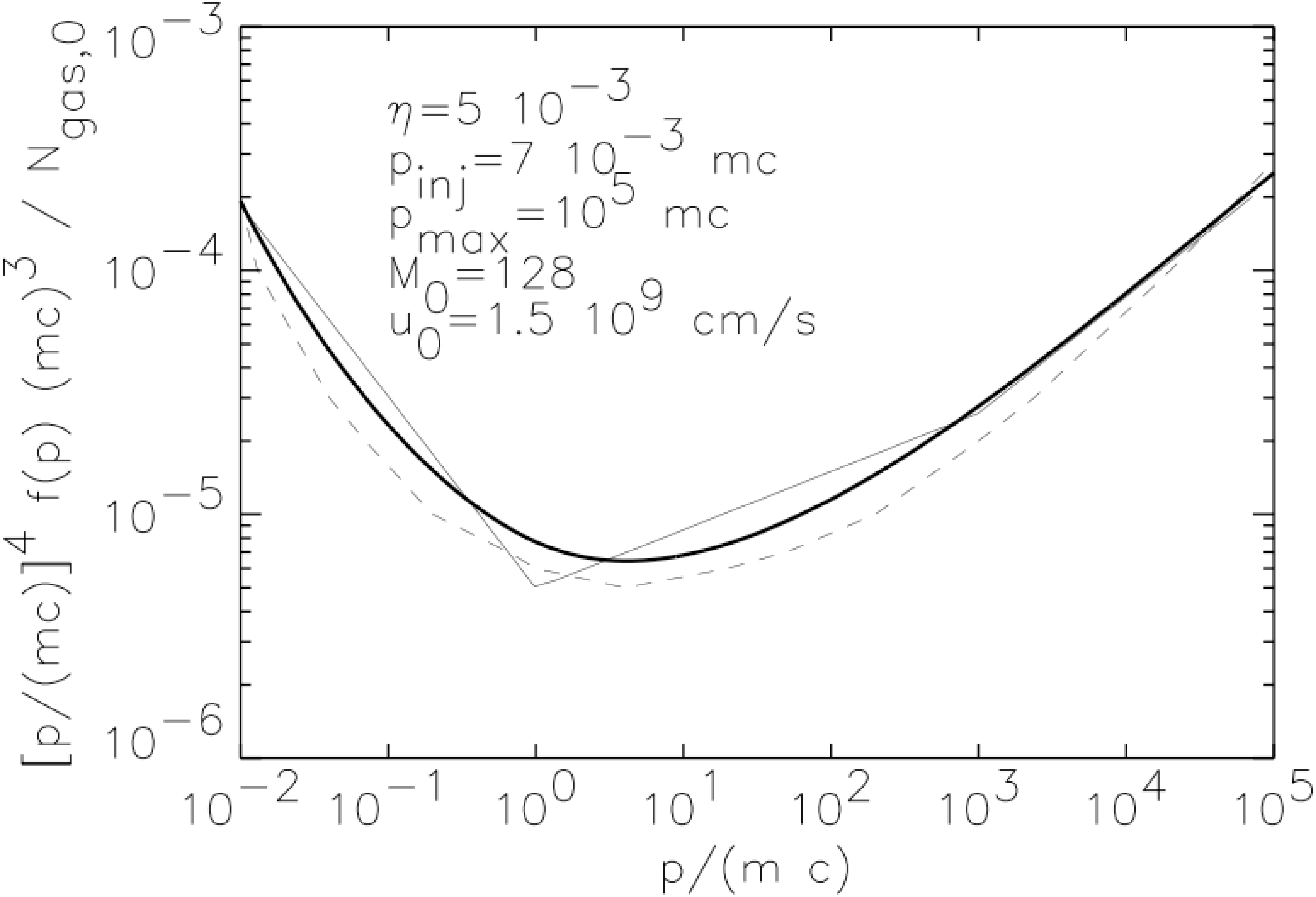
Power-law spectrum hardens to 3.5

Almost no particle escape!

Precursor velocity profile is linear.

At low energies spectrum has slope appropriate to the sub-shock compression

Smooth concave interpolation between these...



From P. Blasi, 2002

But does depend on absence of mesoscale instabilities - not obvious that this is the case. In fact can show generic instability of density fluctuations unless

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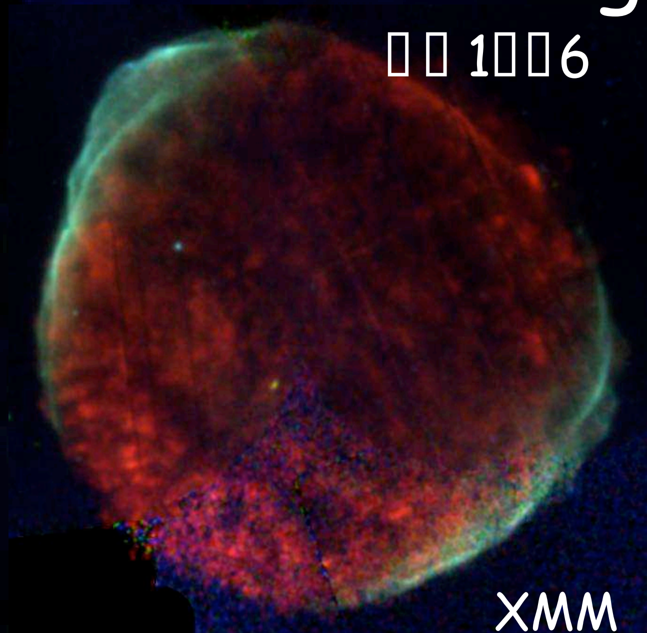
Also recently there has been some very interesting work on magnetic instabilities, mainly by A.Bell, but also by P. Diamond and others.

May lead to amplified magnetic fields and acceleration to significantly higher energies?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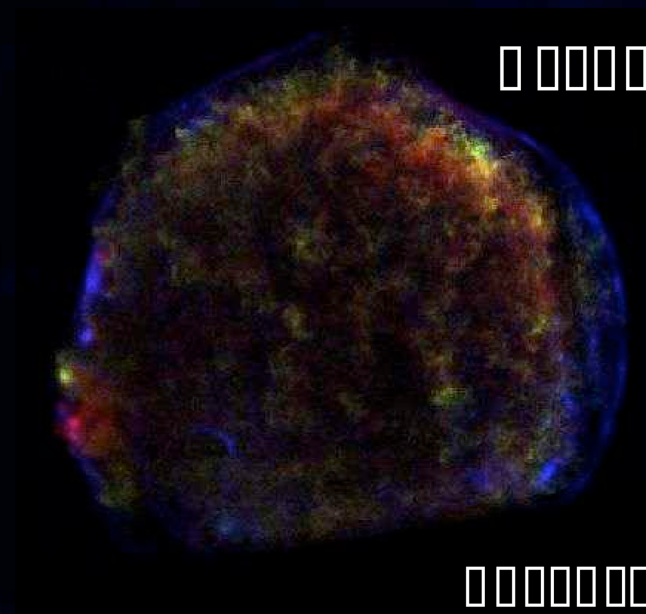

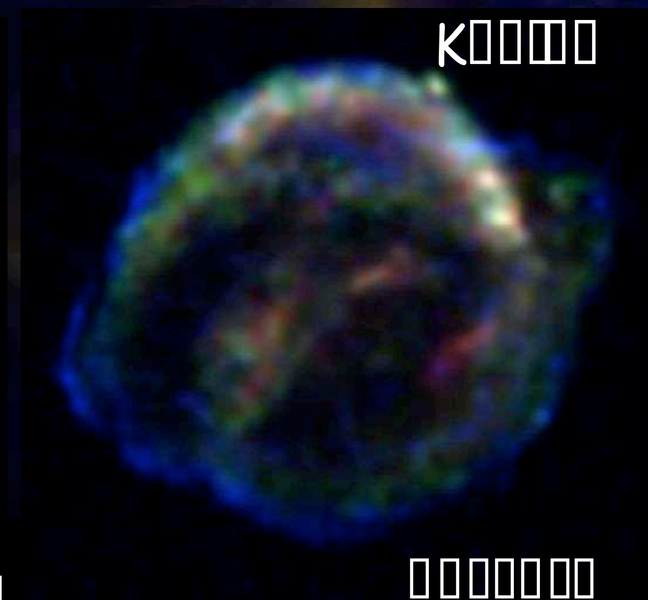

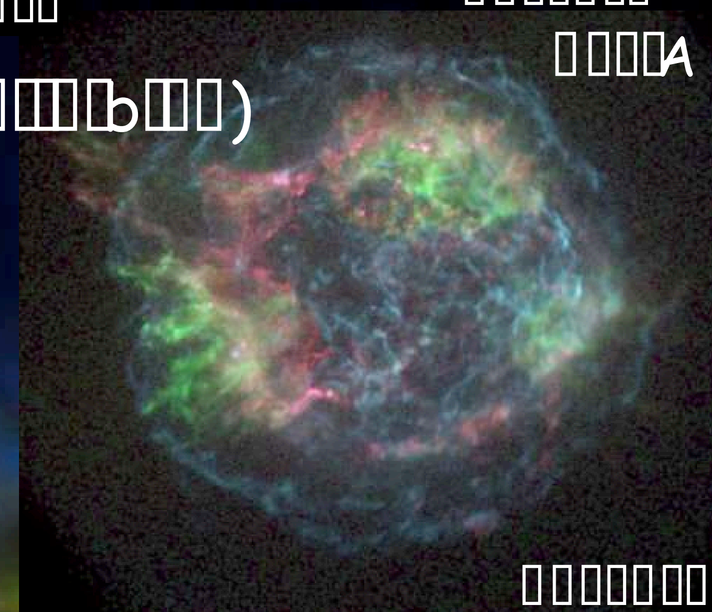

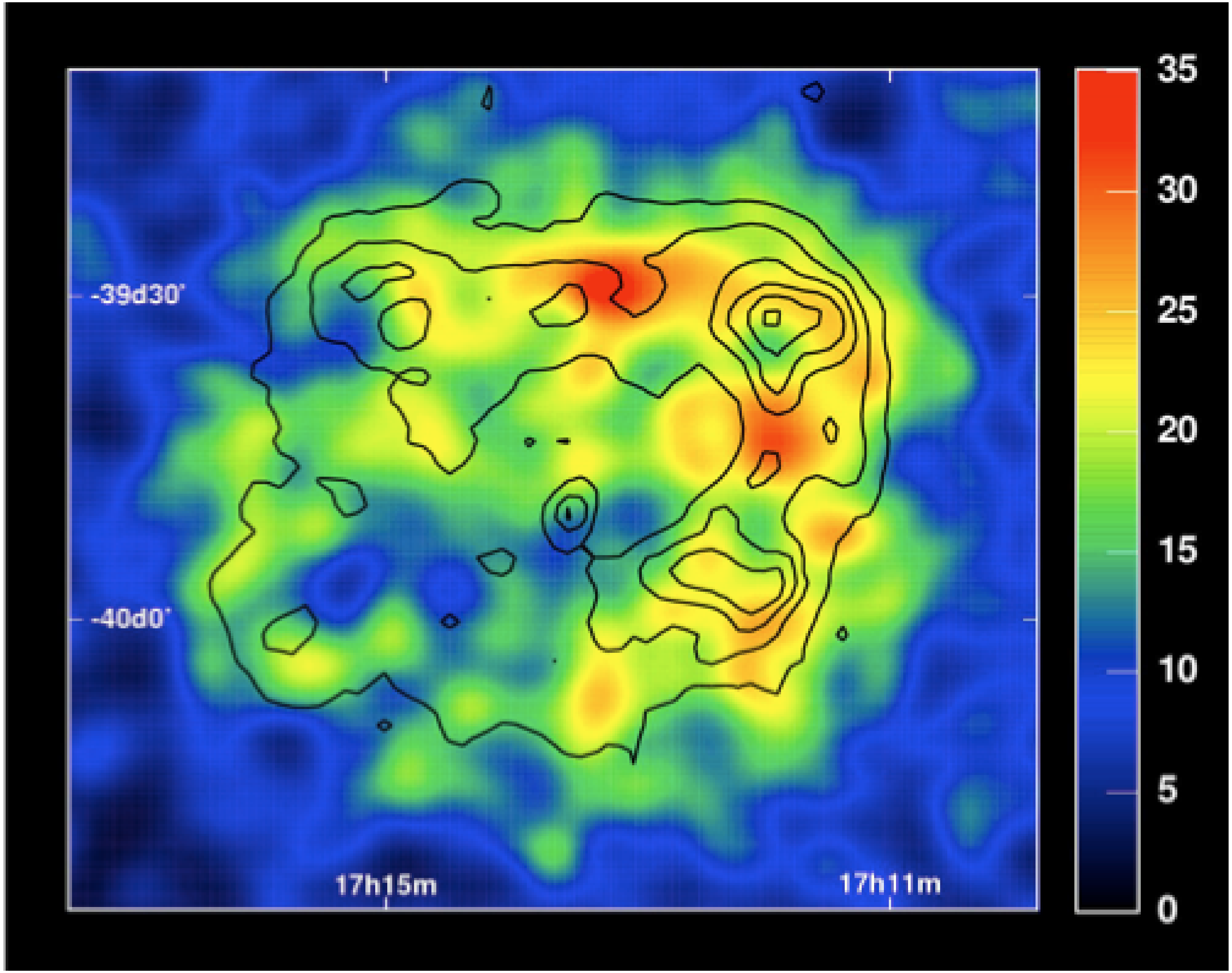

H.E.S.S. observations of RXJ1713.7-3946
(Nature, in press)