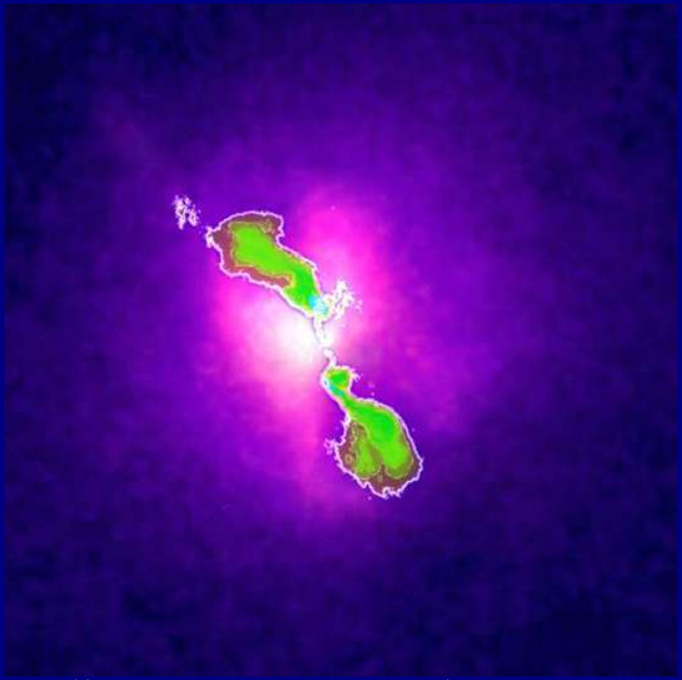
A Bayesian View on Faraday Rotation Maps

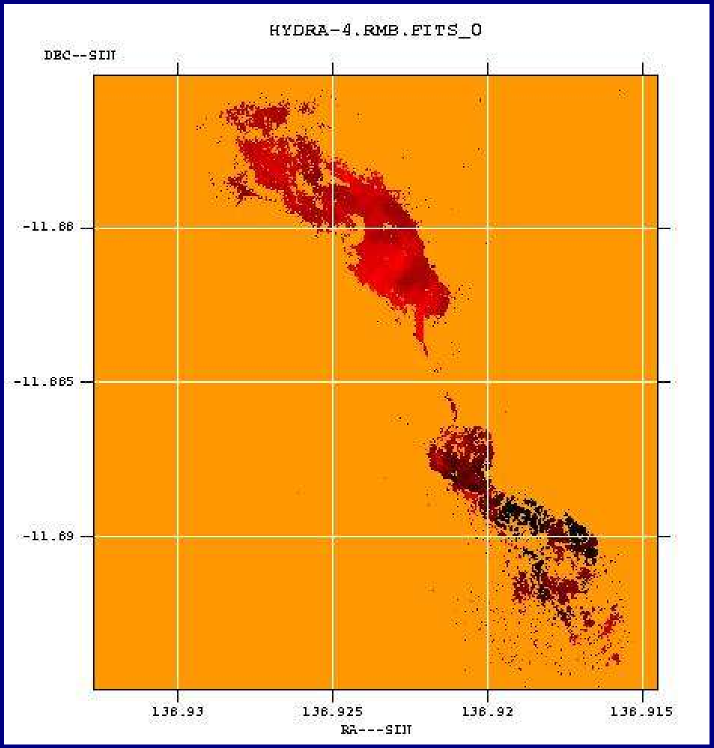
- Seeing the Magnetic Power Spectrum in Clusters of Galaxies



Radio: VLA, Greg Taylor
X-ray: Chandra

Corina Vogt
Torsten Enßlin

What is it all about?



Hydra A in Abell 780
(Taylor et al., 1993)

We measure:

$$RM = a_0 \int dl\, n_e\, B_z$$

$$a_0 = e^3/(2\pi m_e^2 c^4)$$

We want:

- Magnetic fluctuations -
  Magnetic power spectrum $\varepsilon_B(k)$
- Spectral index
- Central field strength B_0
- Field correlation length λ_B

From RM to B_0
A Statistical Approach

magnetic field autocorrelation tensor:

$$M_{ij}(\vec{r}) = \langle B_i(\vec{x})\, B_j(\vec{x}+\vec{r}) \rangle_{\vec{x}}$$

assumptions:

- statistically isotropical fields
- statistically homogeneous fields
- div ***B*** = 0

scalar magnetic autocorrelation function:

$$w(r) = \sum_i M_{ii}(\vec{r}) = \langle B_i(\vec{x}) \cdot B_i(\vec{x}+\vec{r}) \rangle_{\vec{x}}$$

$$w(0) = \frac{B_0^2}{8\pi}$$

Enßlin & Vogt, (2003)

A Straight Forward Approach

observational accessible RM autocorrelation

$$C_{RM}(r_p) \propto \int dx_p^2\, RM(\vec{x}_p)\, RM(\vec{x}_p + \vec{r}_p)$$

→

interesting magnetic field autocorrelation

$$C_{RM}(r_p) \propto \int_{-\infty}^{\infty} dr_z\, w\left(\sqrt{r_p^2 + r_z^2}\right)$$

$$\vec{r} = (\vec{r}_p, r_z)$$

Fourier Space – Fourier Analysis

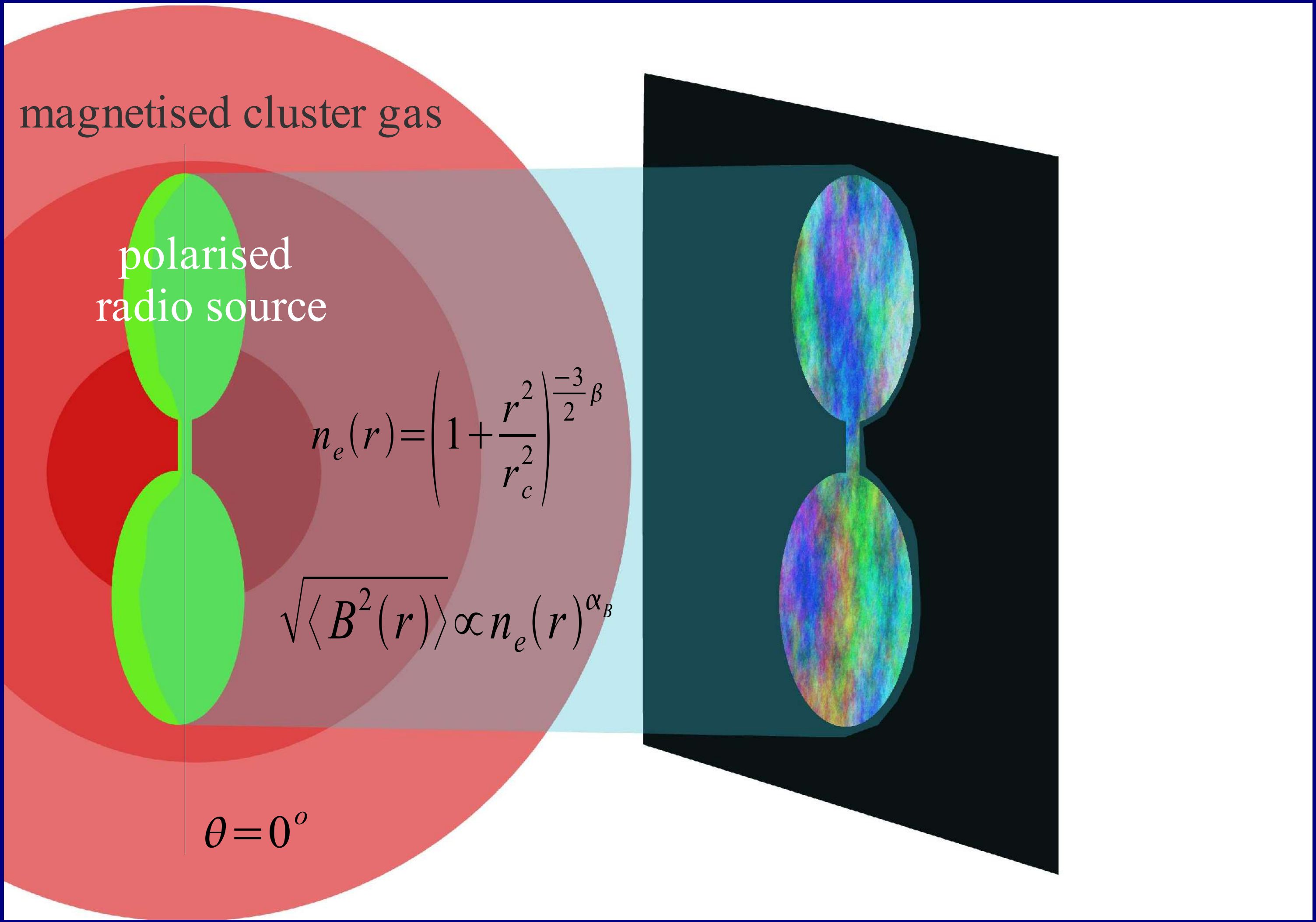
$$\hat{C}_{RM}(\vec{k}_p) \propto \langle |\hat{RM}(\vec{k}_p)|^2 \rangle$$

→

$$\hat{C}_{RM}(\vec{k}_p) = \frac{1}{2}\hat{w}(\vec{k}_p, 0)$$

$$\vec{k} = (\vec{k}_p, k_z)$$

The Window makes the Difference



global electron density distribution

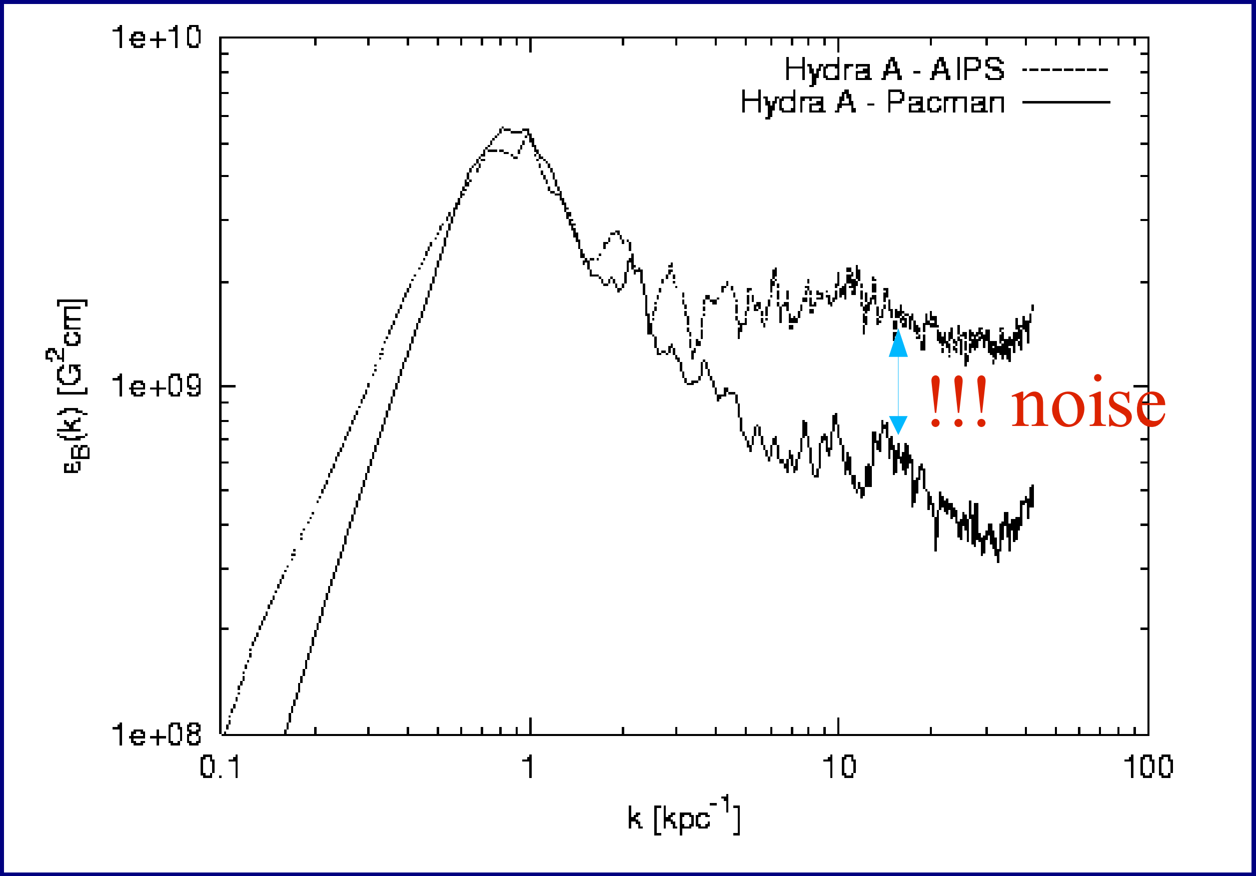
limited source size

↓

sampling volume

For the Fourier Analysis

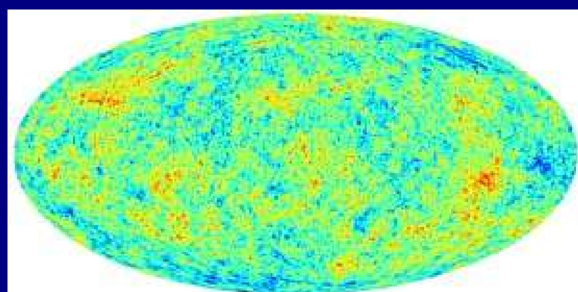
Window function: modifies spectral energy distribution
$\rightarrow$ NO reliable determination of spectral index possible
Suppression of power on small k-scales

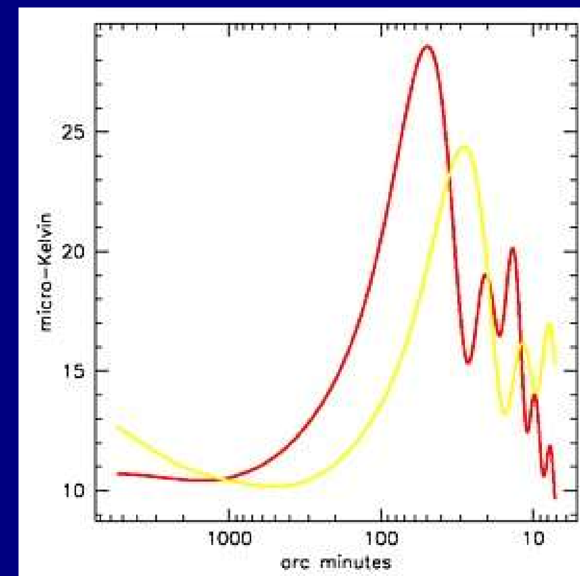


$\lambda_B = 1.2$ kpc
$B_0 = 9\ \mu$G

A Maximum Likelihood Estimator

Problem:
CMB



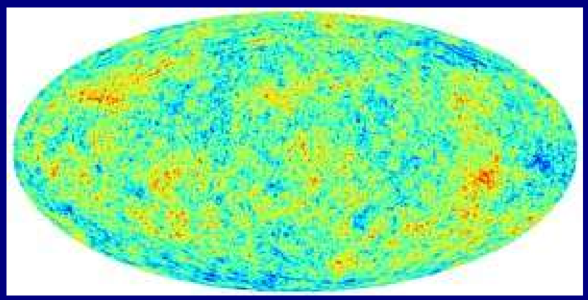


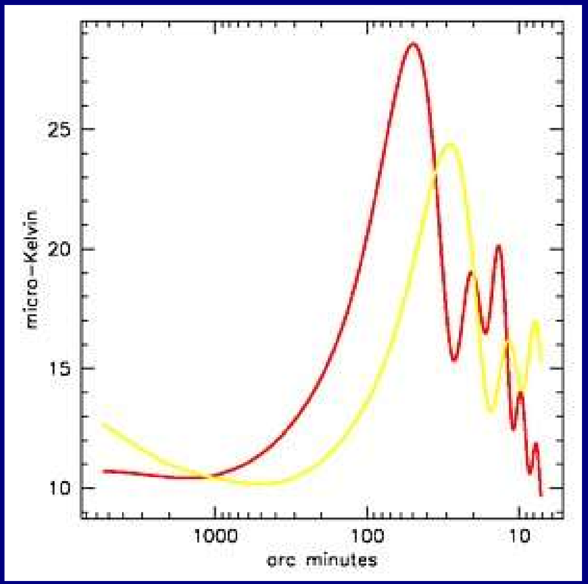
Even possible non-Gaussian magnetic fields seen in projection are close to Gaussian due to the central limit theorem

Observations tell us that RM is close to Gaussian (e.g. Feretti et al. 1999)

A Maximum Likelihood Estimator

Problem: CMB





Likelihood function:

$$L_{\Delta}(\varepsilon_{B_i}) = \frac{1}{(2\pi)^{2/N} |C_{RM}|^{1/2}} \exp\left(\frac{-1}{2} \Delta^T C_{RM}^{-1} \Delta\right)$$

$$C_{RM} = C_{RM}(\varepsilon_{B_i}) = \langle RM(\vec{x}) RM(\vec{y}) \rangle$$ - covariance matrix

Δ - RM data

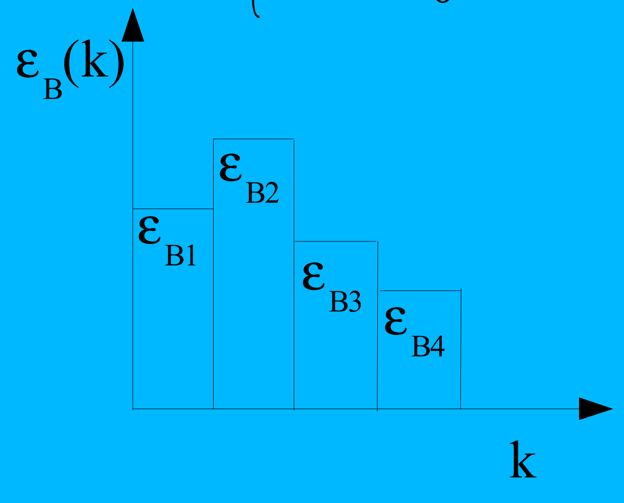
N - Number of Data

A Model for the Magnetic Fluctations

Covariance Matrix:

$$C_{RM}(\varepsilon_{B_i}) \propto \sum_i \varepsilon_{B_i} \int_z^{\infty} dz\, f(\vec{x})\, f(\vec{x}+\vec{r}) \int_{k_i}^{k_{i+1}} dk \frac{J_0(kr)}{k}$$

$$\varepsilon_B(k) = \sum_i \begin{Bmatrix} \varepsilon_{Bi} \; if \; k_i \le k \le k_{i+1} \\ 0 \end{Bmatrix}$$



f(x) describes the window

Algorithm to Maximise L: Bond et al., 1998

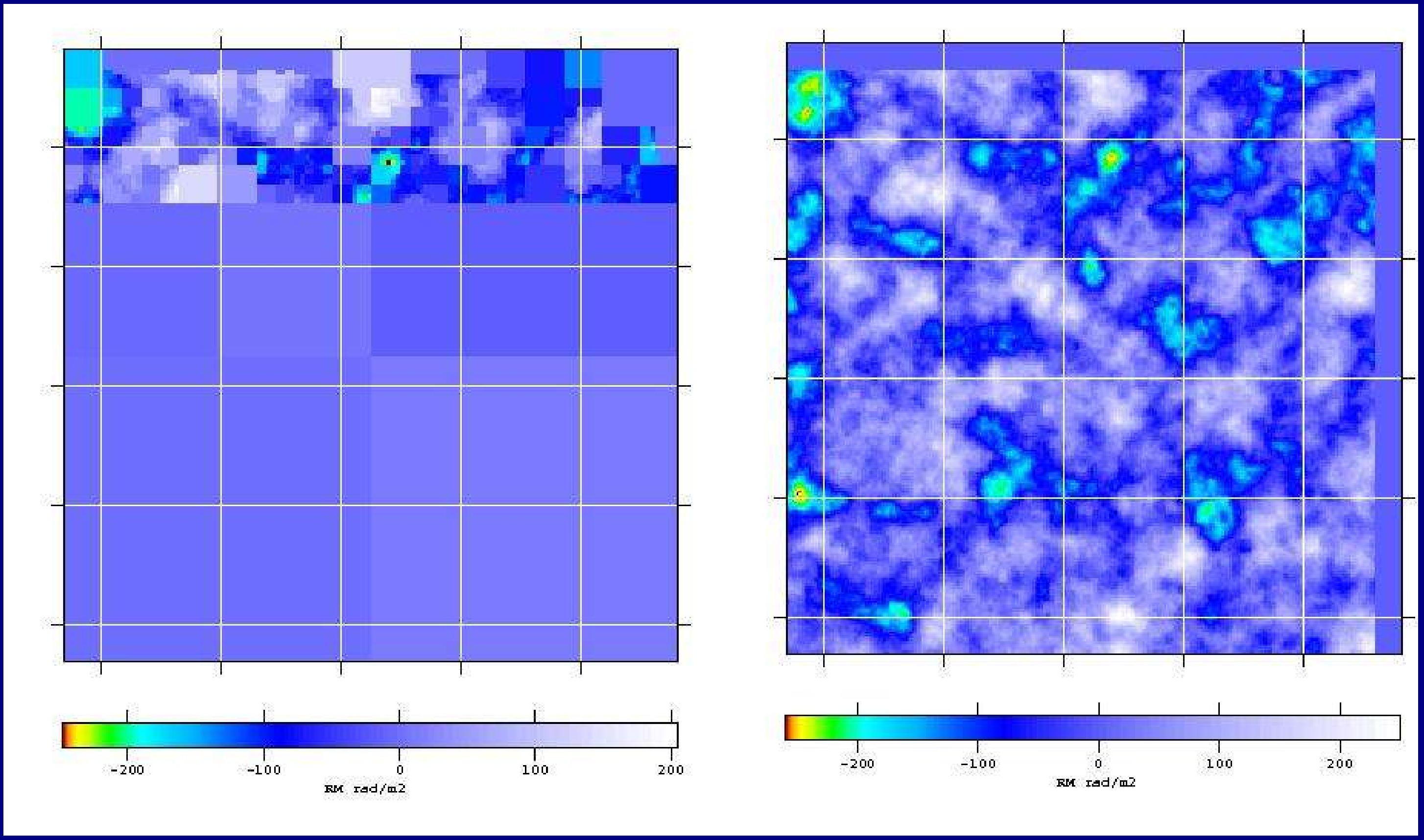
Magnetic Power Spectrum

+ Algorithm allows to calculate error bars & correlations between errors

Testing the Algorithm with simulated data

Magnetic Power spectrum:

- mimic Kolmogorov power spectrum
- energy injection scale $k_c = 0.8\ kpc^{-1}$



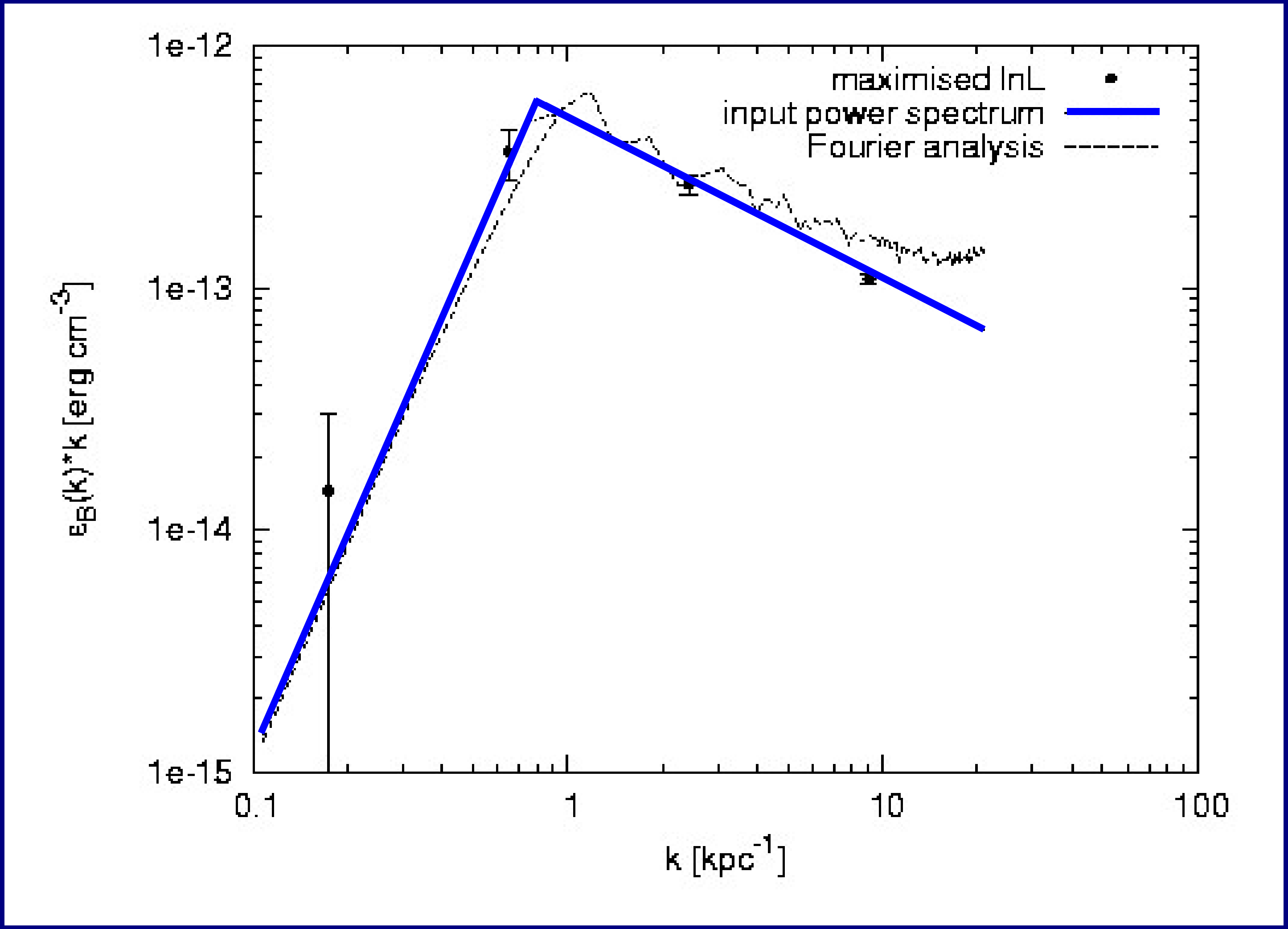
$\Delta \rightarrow N = 1500$
arbitrarily avaraged

generated RM map
(37 x 37) kpc

Window:

- $B_0 = 5\ \mu G$
- uniform $n_e = 0.001\ cm^{-3}$
- Box: (150 x 150)kpc
- Length 300 kpc

It works!!!



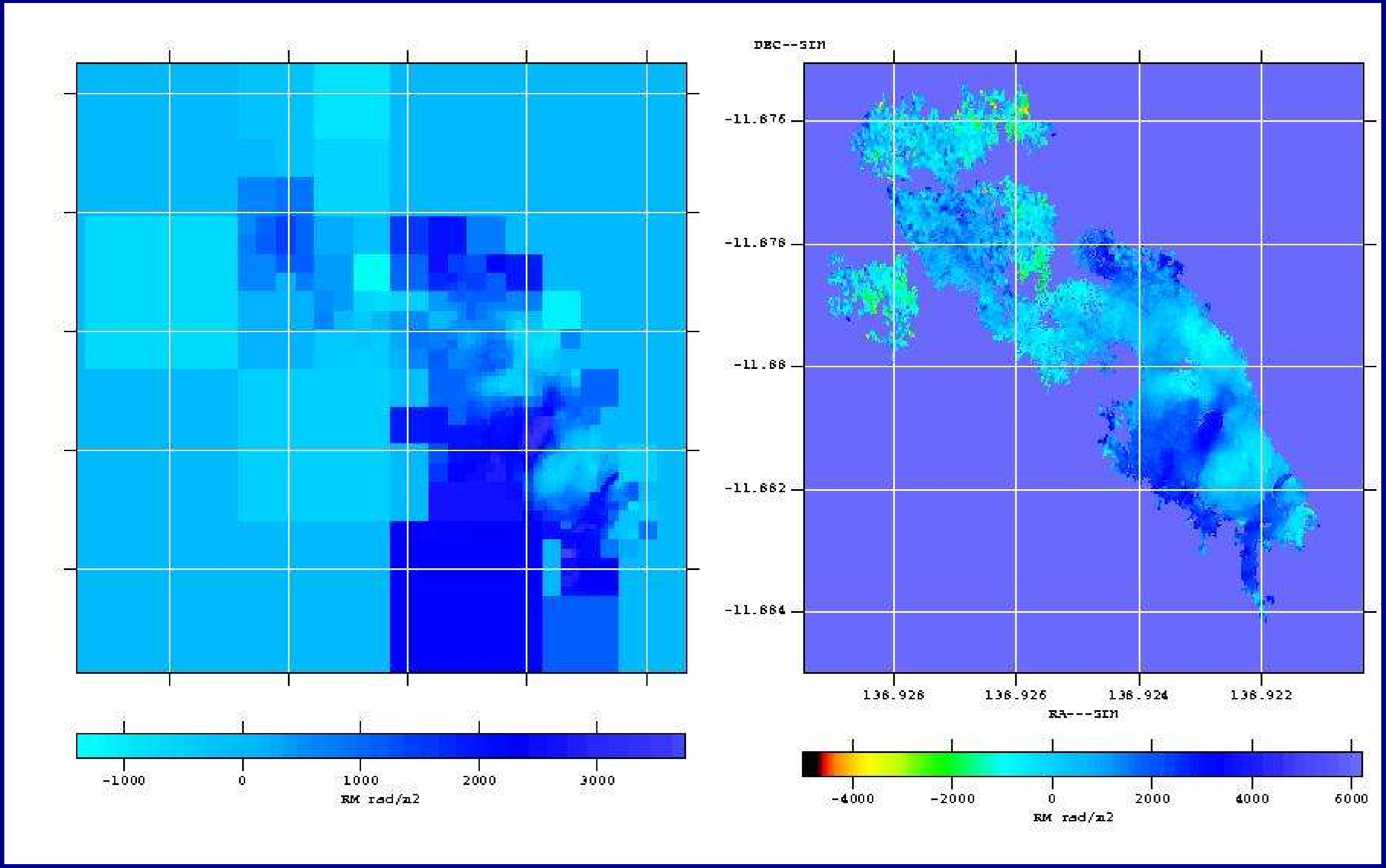
$B_0 = (4.7 \pm 0.3)\ \mu G$

The Real Data – Hydra North

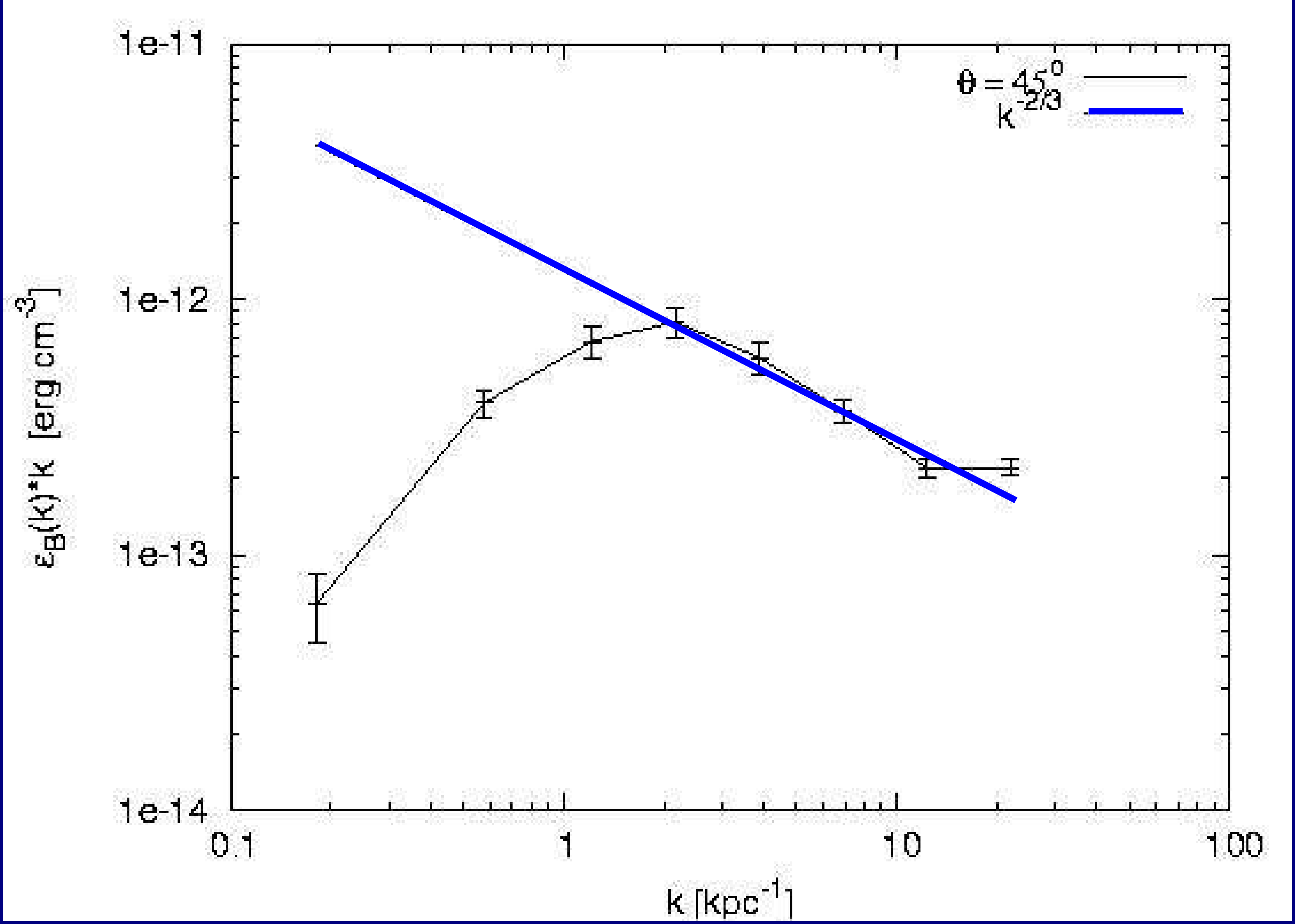
cooling flow cluster

Δ – used data points

Pacman – RM map



A Magnetic Power Spectra of Hydra A

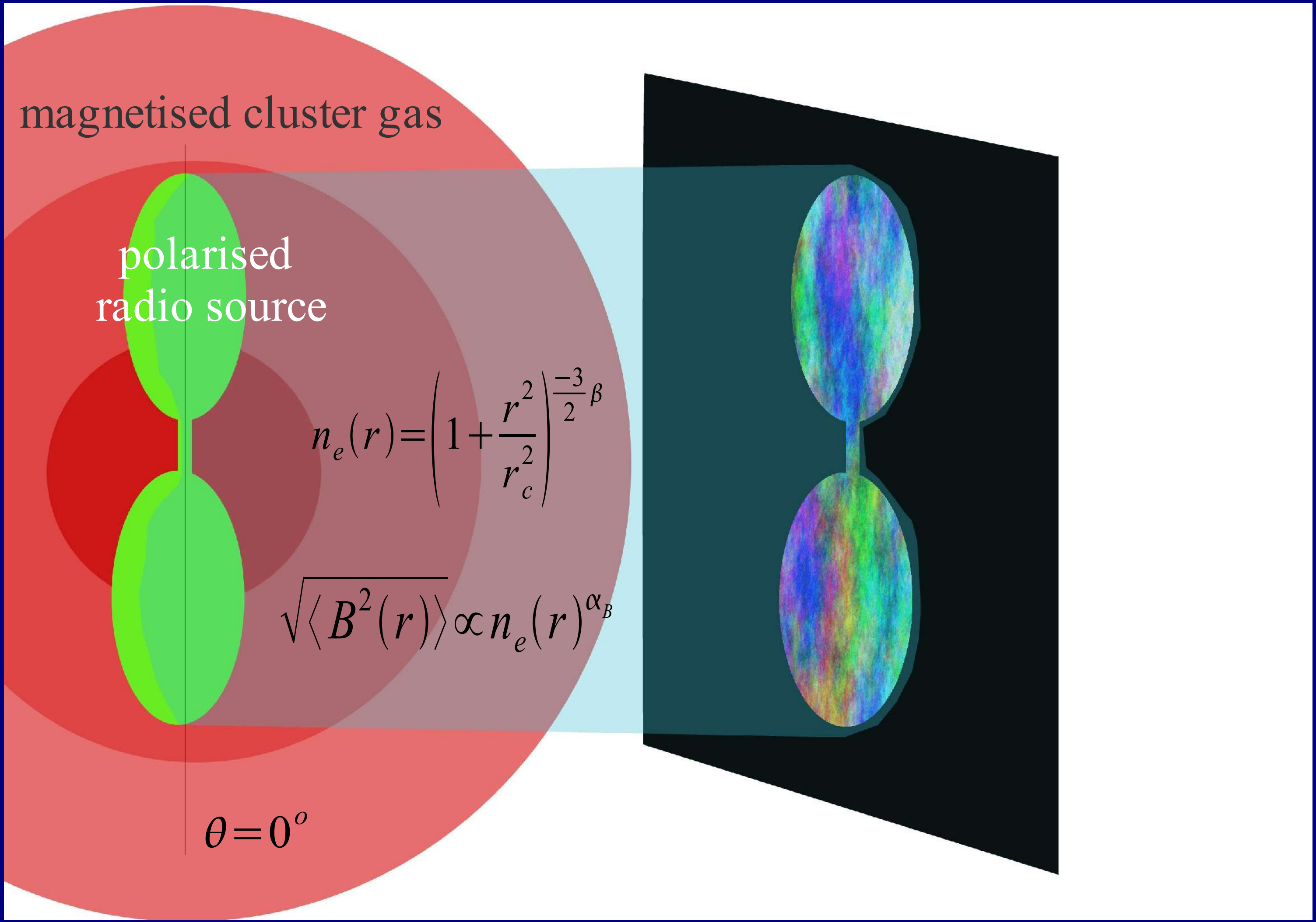


Hydra A:

$B_0 = (7.3 \pm 0.2)\ \mu G$

$\lambda_B = (2.8 \pm 0.2)$ kpc

The Window makes the Difference



global electron density distribution

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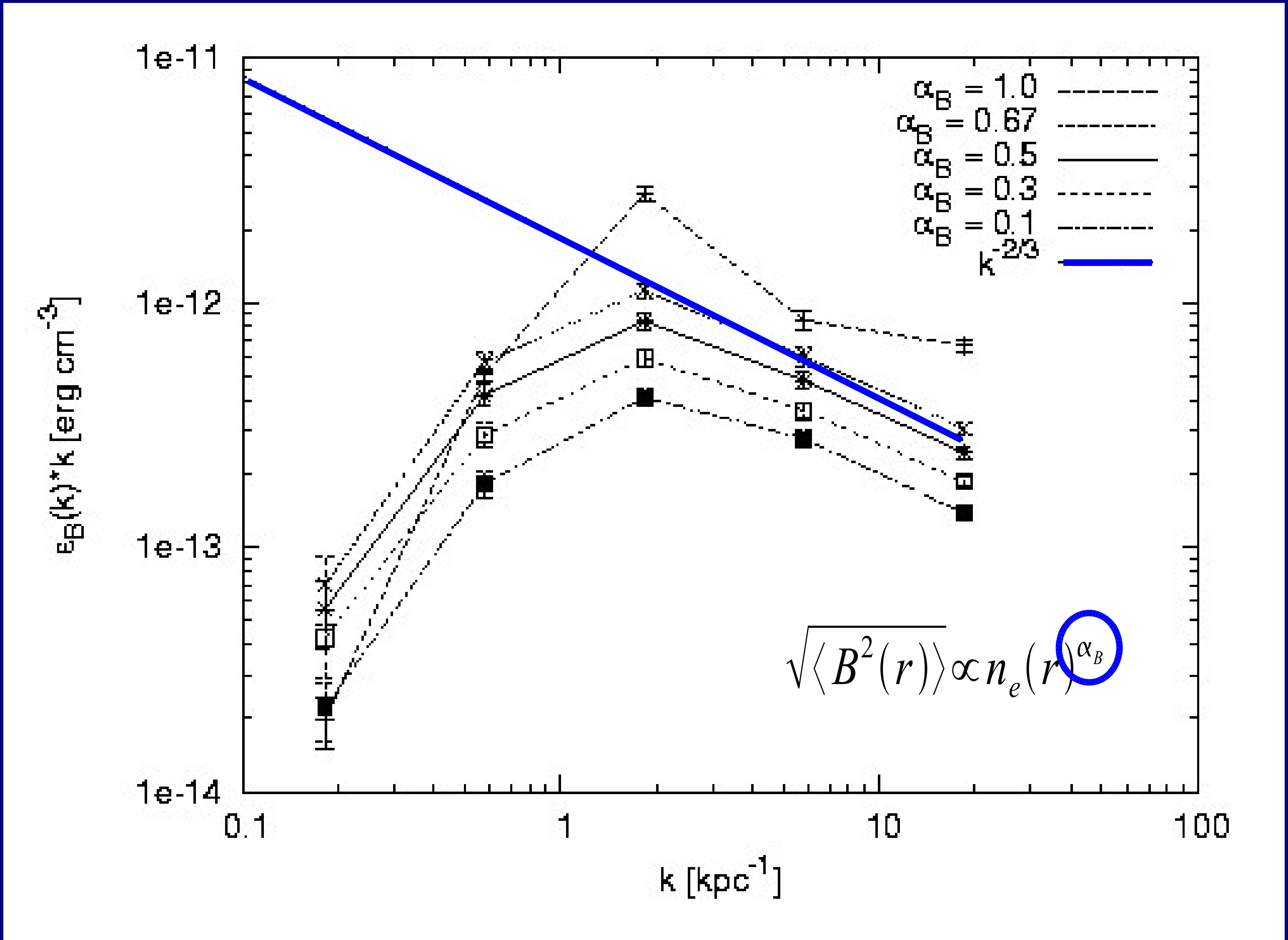
↓

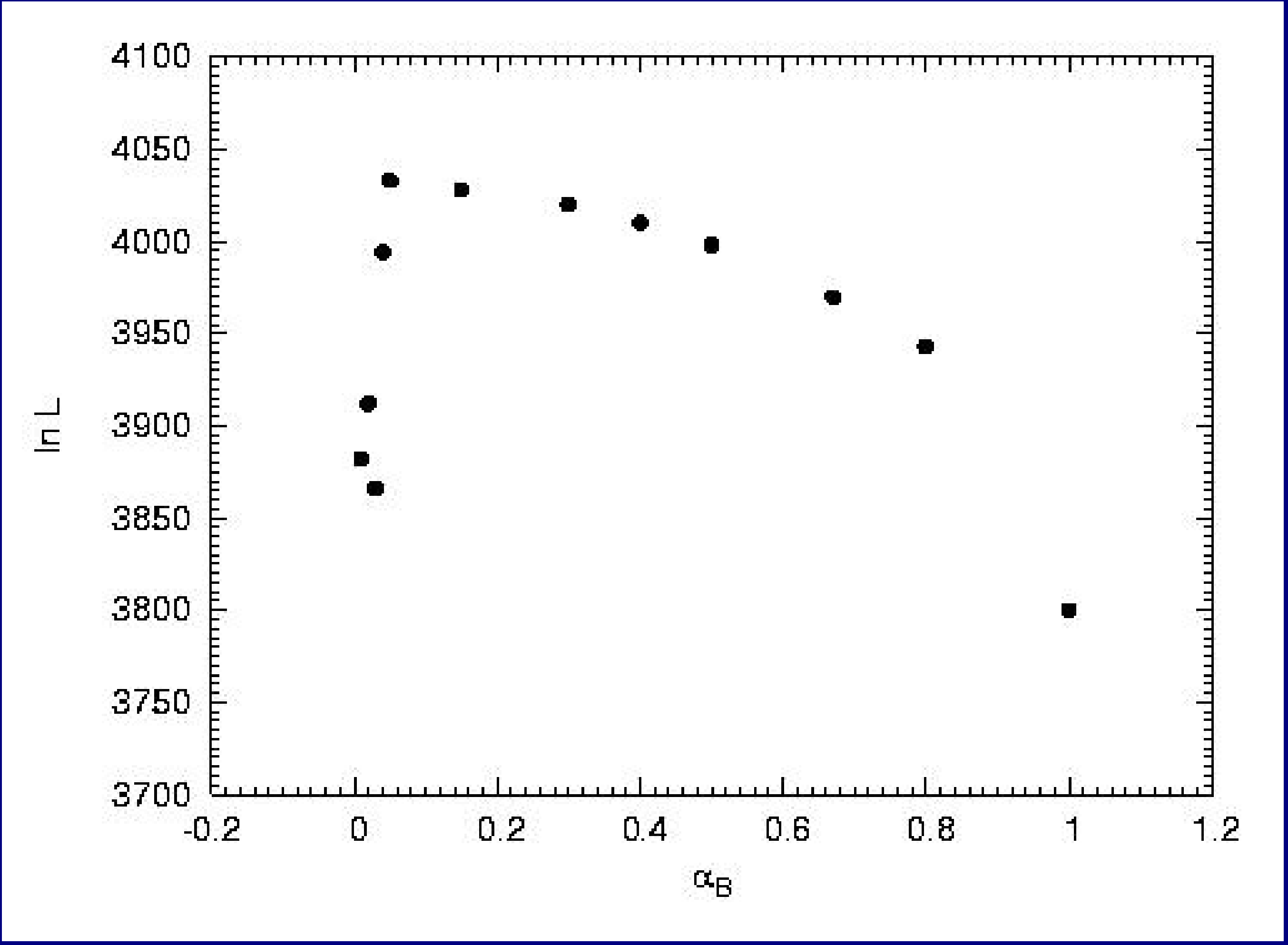
sampling volume

Power Spectra for Various Scalings



$\theta = 45°$

The likelihood of a_B



Hydra A:

$\alpha_B \sim 0.1 \ldots 0.8$

seems reasonable

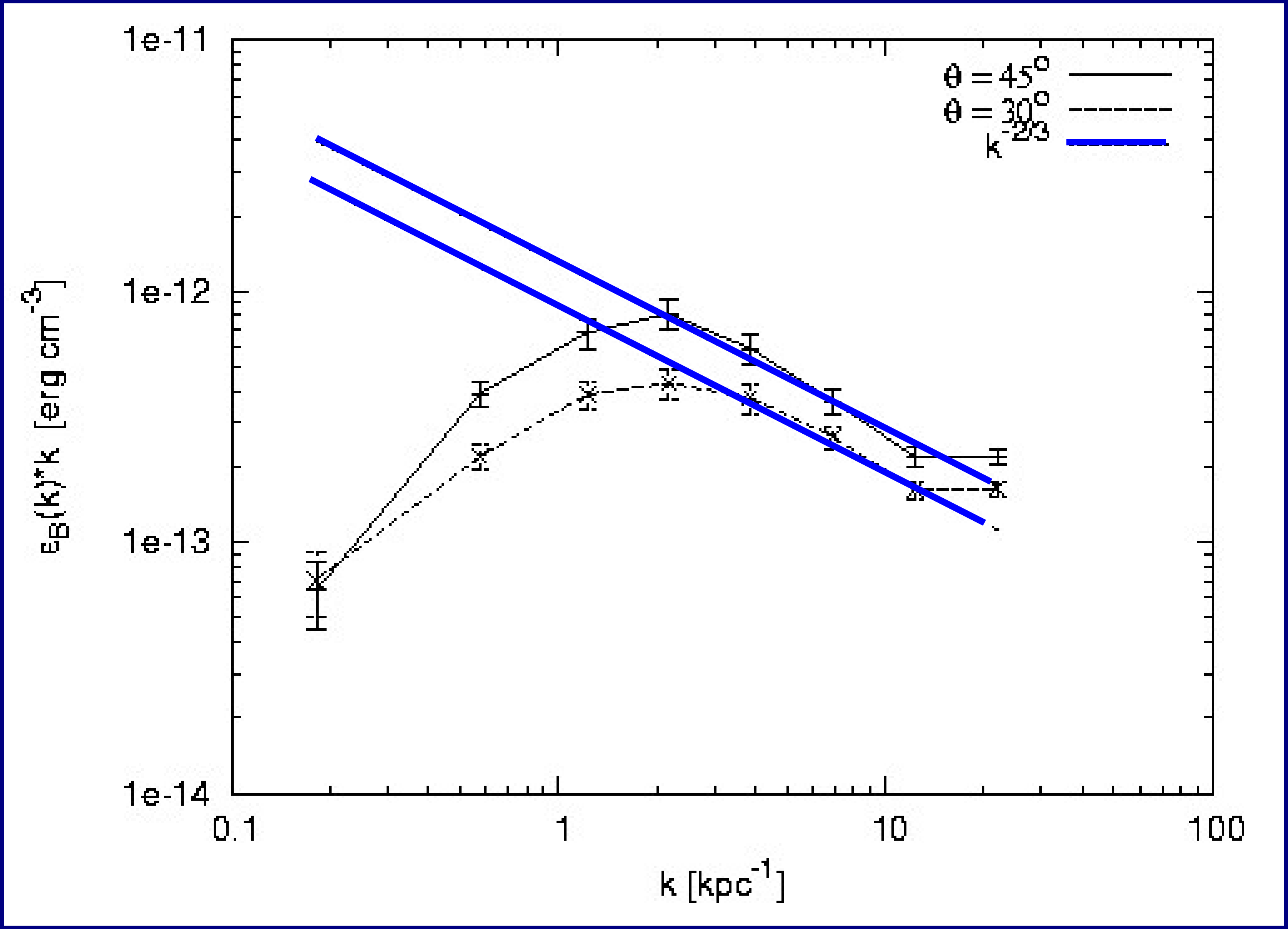
Conclusions

- Maximum-Likelihood-Method in order to determine the magnetic power spectrum in galaxy clusters
- Applied to the cooling flow cluster Hydra A
- Scaling parameter $\alpha_B \sim 0.1 \ldots 0.8$
- Spectral index Kolmogorov like over one order of magnitude
- Central field strength $B_0 = (7 \pm 0.2 \pm 2)\ \mu G$
- Field correlation length $\lambda_B = (3.0 \pm 0.2 \pm 0.5)$ kpc

  - uncertainty in window geometry
  - astrophysical interpretation (see Torstens talk)

Thank you!

Magnetic Power Spectra

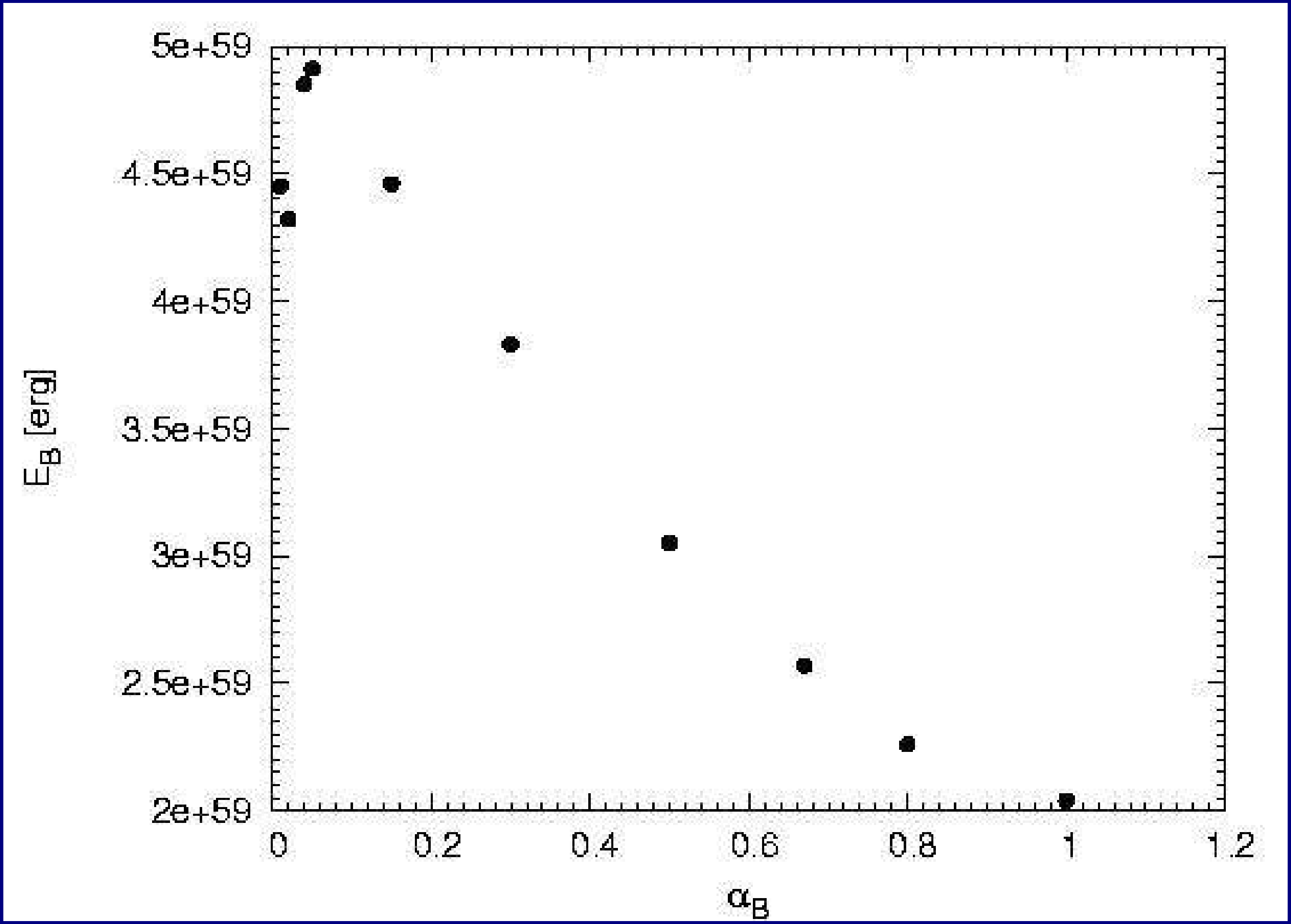


Hydra A:

$B \sim 6\ \mu G$

$\lambda_B \sim 3$ kpc

Magnetic Energy Density



RM dispersion via Window function