

THERMAL CONDUCTION IN MAGNETIZED TURBULENT GAS

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ABSTRACT

We discuss diffusion of particles in turbulent flows. In hydrodynamic turbulence, it is well known that distance between two particles imbedded in a turbulent flow exhibits a random walk behavior. The corresponding diffusion coefficient is $\sim v_{inj}l_{turb}$, where v_{inj} is the amplitude of the turbulent velocity and l_{turb} is the scale of the turbulent motions. It is not clear whether or not we can use a similar expression for magnetohydrodynamic turbulence. However, numerical simulations show that mixing motions perpendicular to the local magnetic field are, up to high degree, hydrodynamical. This suggests that turbulent heat transport in magnetized turbulent fluid should be similar to that in non-magnetized one, which should have a diffusion coefficient $\sim v_{inj}l_{turb}$. We review numerical simulations that support this conclusion. The application of this idea to thermal conductivity in clusters of galaxies shows that this mechanism may dominate the diffusion of heat and may be efficient enough to prevent cooling flow formation when turbulence is vigorous.

Key words : clusters of galaxies – ISM: general – MHD – turbulence

I. ASTROPHYSICAL MOTIVATION

It is well known that Astrophysical fluids are turbulent and that magnetic fields are dynamically important. One characteristic of the medium that magnetic fields and turbulence may substantially change is the heat transfer.

There are many instances when heat transfer through thermal conductivity is important. For instance, thermal conductivity is essential in rarefied gases where radiative heat transfer is suppressed. This is exactly the situation that is present in clusters of galaxies. It is widely accepted that ubiquitous X-ray emission due to hot gas in clusters of galaxies should cool significant amounts of the intracluster medium (ICM) and this must result in cooling flows (Fabian 1994). However, observations do not support the evidence for the cool gas (see Fabian et al. 2001) which is suggestive of the existence of heating that replenishes the energy lost via X-ray emission. Heat transfer from the outer hot regions can do the job, provided that the heat transfer is sufficiently efficient.

Gas in clusters of galaxies is magnetized and the conventional wisdom suggests that the magnetic fields strongly suppress thermal conduction perpendicular to their direction. Realistic magnetic fields are turbulent and the issue of the thermal conduction in such a situation has been long debated. A recent paper by Narayan & Medvedev (2001) obtained estimates for the thermal conductivity of turbulent magnetic fields, but those estimates happen to be too low to explain the absence of cooling flows for many of the clusters of galaxies (Za-

kamska & Narayan 2002).

Narayan & Medvedev (2001) treat the turbulent magnetic fields as static. In hydrodynamical turbulence it is possible to neglect plasma turbulent motions only when the diffusion of electrons which is the product of the electron thermal velocity v_{elect} and the electron mean free path in plasma l_{mfp} , i.e. $v_{elect}l_{mfp}$, is greater than the turbulent velocity v_{turb} times the turbulent injection scale l_{inj} , i.e. $v_{turb}l_{inj}$. If such scaling estimates are applicable to heat transport in magnetized plasma, the turbulent heat transport should be accounted for heat transfer within clusters of galaxies. Indeed, data for $v_{elect}l_{mfp}$ given in Zakamska & Narayan (2002; Narayan & Medvedev 2001) provide the classical Spitzer (1962) diffusion coefficient $\kappa_{Sp} \equiv v_{elect}l_{mfp} \sim 6.2 \times 10^{30} \text{ cm}^2 \text{ sec}^{-1}$ for the inner region of $R \sim 100\text{kpc}$ and $\kappa_{Sp} \equiv v_{elect}l_{mfp} \sim 3.6 \times 10^{29} \text{ cm}^2 \text{ sec}^{-1}$ for the very inner region of $R \sim 10\text{kpc}$ (for Hydra A). If turbulence in the cluster of galaxies is of the order of the velocity dispersion of galaxies, while the injection scale is of the order of 20 kpc, the diffusion coefficient is $\sim v_{turb}l_{inj} \sim 3.1 \times 10^{30} \text{ cm}^2 \text{ sec}^{-1}$, where we take $v_{turb} \sim 500 \text{ km/sec}$.

In this paper, we review numerical simulations of thermal diffusion in magnetized turbulence flows. This topic has broad astrophysical applications. Clusters of galaxies are just one of the examples where non-radiative heat transfer is essential. This process is also important for many regions within galactic interstellar medium, e.g. for supernova remnants.

II. SEPARATION OF TWO PARTICLES IN HYDRODYNAMIC TURBULENCE

To understand transport of a passive scalar in turbulent flows, let us consider two mass-less particles first. Let the distance between the particles be l . We assume that $l_d < l < l_{inj}$, where l_d is the dissipation range and l_{inj} is the energy injection scale.

Kolmogorov theory provides a scaling law for *incompressible non-magnetized* hydrodynamic turbulence. The beauty of the Kolmogorov theory that it does provide a simple scaling for hydrodynamic motions. If the velocity at a scale l from the inertial range is v_l , the Kolmogorov theory states that the kinetic energy ($\rho v_l^2 \sim v_l^2$ as the density is constant) is transferred to next scale within one eddy turnover time (l/v_l). Thus within the Kolmogorov theory the energy transfer rate ($v_l^2/(l/v_l)$) is scale-independent,

$$\frac{v_l^2}{t_{cas}} \sim \frac{v_l^2}{(l/v_l)} = \text{constant}, \quad (1)$$

and we get the famous Kolmogorov scaling

$$v_l \propto l^{1/3}. \quad (2)$$

An equivalent description is to express spectrum $E(k)$ as a function of wave number k ($\sim 1/l$). The two descriptions are related by $kE(k) \sim v_l^2$. The famous Kolmogorov spectrum is $E(k) \sim k^{-5/3}$.

We are interested in the time evolution of the separation l . Since the separation shows a random walk behavior with stance of $v_l dt$, we can write

$$\frac{dl^2}{dt} \sim \frac{(l + v_l dt)^2 - (l - v_l dt)^2}{dt} \sim lv_l, \quad (3)$$

where we ignore constants of order unity. Using $\epsilon \sim v_l^3/l$, we get

$$\frac{dl^2}{dt} \sim l(\epsilon l)^{1/3}, \quad (4)$$

where ϵ is the energy injection rate. This leads to

$$l^{2/3} - l_0^{2/3} = (C_R)^{1/3} \epsilon^{1/3} (t - t_0), \quad (5)$$

where C_R is Richardson constant. When $l \gg l_0$, we can write

$$l^2 = C_R \epsilon (t - t_0)^3, \quad (6)$$

which was first discovered by Richardson (1926). Recent direct numerical simulations suggest that $C_R \sim 1$. Boffetta & Sokolov (2002) obtained $C_R \sim 0.55$. Ishihara & Kaneda (2001) obtained $C_R \sim 0.7$.

The above argument suggests that the separation l shows a random-walk-like behavior. (However, the stance depends on the separation in this case.) Therefore the diffusion coefficient is given by

$$\kappa_{dynamic} \sim \begin{cases} lv_l & \text{when } l < l_{inj}, \\ l_{inj} v_{turb} & \text{when } l > l_{inj}. \end{cases} \quad (7)$$

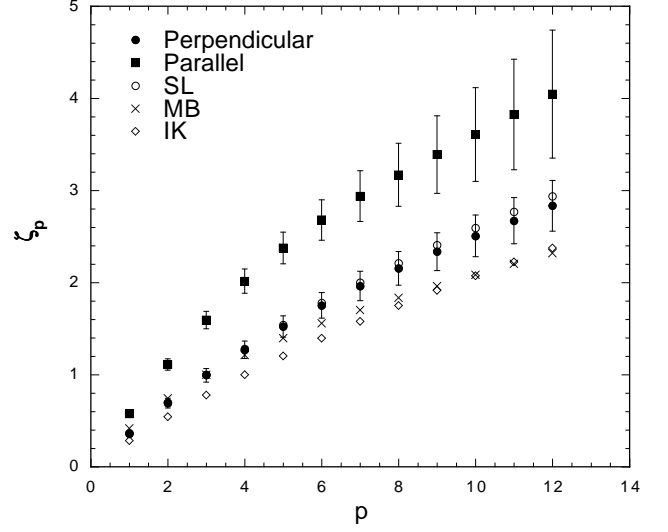


Fig. 1.— Intermittency. Our result (denoted by the filled circles) suggests that MHD turbulence looks like ordinary hydrodynamic turbulence when viewed across the local field lines. SL represents the original She-Leveque model for ordinary hydrodynamic turbulence. IK and MB stand for the IK theory (see Politano & Pouquet 1995) and the Müller-Biskamp model (2000) respectively. Error bars are for 1- σ level. From Cho, Lazarian, & Vishniac (2002).

Astrophysical flows are magnetized. Hall (1949) and Hiltner (1949) discovered magnetic fields in the interstellar medium and Kim et al. (1989) detected magnetic fields in the ICM. Therefore, we cannot use this above relations without justification.

Then, can we have expressions similar to those in equation (7) in MHD case? To answer this question, we must consider velocity statistics in MHD turbulence. We are particularly interested in the motions perpendicular to the local magnetic field direction. Earlier numerical studies by Cho, Lazarian & Vishniac (2002) revealed a good correspondence between hydrodynamic motions and motions of fluid perpendicular to the local direction of magnetic field (see Figure 1). The symbols represent scaling exponents, ζ_p , of the (longitudinal) velocity structure functions

$$S_p = \langle ([\mathbf{v}(\mathbf{x} + \mathbf{r}) - \mathbf{v}(\mathbf{x})] \cdot \hat{\mathbf{r}})^p \rangle \propto r^{\zeta_p}. \quad (8)$$

The open circles are for the hydrodynamic turbulence: She & Leveque (1994) proposed a scaling relation: $\zeta_p^{SL} = p/9 + 2[1 - (2/3)^{p/3}]$, which is in good agreement with experiments. The filled circles, which represent the scalings exponents for perpendicular directions in MHD turbulence, coincide well with the hydrodynamic ones.

This surprising result can be understood in terms of the eddies in the planes perpendicular to magnetic field lines (see discussion in Cho, Lazarian & Vishniac 2002). Lazarian & Vishniac (1999) showed that the eddies will not be forming magnetic knots if the reconnection is as fast as the stochastic reconnection scheme suggests. This means that the motions of the magnetized fluid

will be very similar to the hydrodynamic motions in the planes perpendicular to the local direction of magnetic field.

To what extent heat transfer in a turbulent medium is affected by a magnetic field is the subject of the present study. To solve this problem we shall systematically study the passive scalar diffusion in a magnetized turbulent medium, compare results of MHD and hydrodynamic calculations, and investigate the heat transfer perpendicular and parallel to the mean magnetic field for magnetic fields of different intensities.

III. NUMERICAL METHODS

We use a 3rd-order hybrid essentially non-oscillatory (ENO) upwind shock-capturing scheme to solve the ideal MHD equations. To reduce spurious oscillations near shocks, we combine two ENO schemes. When variables are sufficiently smooth, we use the 3rd-order Weighted ENO scheme (Jiang & Wu 1999) without characteristic mode decomposition. When the opposite is true, we use the 3rd-order Convex ENO scheme (Liu & Osher 1998). We use a three-stage Runge-Kutta method for time integration. We solve the ideal MHD equations in a periodic box:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (9)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{1}{\rho} \nabla (a^2 \rho) - \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho} = \mathbf{f}, \quad (10)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (11)$$

with $\nabla \cdot \mathbf{B} = 0$ and an isothermal equation of state. Here \mathbf{f} is a random large-scale isotropic driving force, ρ is density, \mathbf{v} is the velocity, and \mathbf{B} is magnetic field. We drive turbulence using 21 (solenoidal) Fourier components with $2 < k < \sqrt{12}$, where k is wavenumber. Each forcing component has correlation time of one. The resulting turbulence is statistically isotropic. The rms velocity v_{turb} is maintained to be approximately unity, so that \mathbf{v} can be viewed as the velocity measured in units of the r.m.s. velocity of the system and $\mathbf{B}/\sqrt{4\pi\rho}$ as the Alfvén velocity in the same units. The time t is roughly in units of the large eddy turnover time ($\sim l_{inj}/v_{turb}$) and the length in units of l_{inj} , the scale of the energy injection. The magnetic field consists of a uniform background field and a fluctuating field: $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$.

We use a passive scalar $\psi(\mathbf{x})$ to trace thermal particles. We inject a passive scalar with a Gaussian profile:

$$\psi(\mathbf{x}, t = t_0) \propto \exp^{-(\mathbf{x}-\mathbf{x}_0)^2/\sigma_0^2}, \quad (12)$$

where $\sigma_0 = 1/16$ of a side of the numerical box and \mathbf{x}_0 lies at the center of the computational box. The value of σ_0 ensures that the scalar is injected in the inertial range of turbulence. The energy injection scale (l_{inj})

is $\sim 1/2.5$ of a side of the numerical box. The scalar field follows the continuity equation

$$\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{v}) = 0. \quad (13)$$

We are mainly concerned with time evolution of σ_i ($i=x, y, \text{ and } z$):

$$\sigma_i^2 = \frac{\int (x_i - \bar{x}_i)^2 \psi(\mathbf{x}, t) d^3x}{\int \psi(\mathbf{x}, t) d^3x}, \quad (14)$$

where

$$\bar{x}_i = \left[\frac{\int x_i \psi(\mathbf{x}, t) d^3x}{\int \psi(\mathbf{x}, t) d^3x} \right]. \quad (15)$$

Common wisdom was that the mean magnetic field suppresses diffusion in the direction perpendicular to it. If this is the case, we expect to see $\sigma_{\perp} < \sigma_{\parallel}$. Otherwise, we will get $\sigma_{\perp} \sim \sigma_{\parallel}$.

We inject passive scalars after turbulence is fully developed. Fig. 2(a) shows when we inject the passive scalars. For the hydrodynamic run with $M_s = 0.3$, where M_s is the sonic Mach number, and 192^3 grid points (thick solid line), we inject passive scalars 5 times. The injection times are marked by arrows. We also mark the injection times for the MHD run with $V_A (= B_0/\sqrt{4\pi\rho}) = 1$, $M_s = 0.3$, and 192^3 grid points (thin solid line for $\langle V^2 \rangle$ and dashed line for $\langle b^2/(4\pi\rho) \rangle$).

IV. THEORETICAL CONSIDERATIONS

Let us consider two massless particles in *MHD turbulence*. As in the hydrodynamic case, we expect that the separation follows

$$\frac{dl^2}{dt} \sim \frac{(l + v_l dt)^2 - (l - v_l dt)^2}{dt} \sim lv_l. \quad (16)$$

Since perpendicular motions are hydrodynamic (Cho, Lazarian, & Vishniac 2002), we will arrive at the same Richardson's law:

$$l^{2/3} - l_0^{2/3} = (C_R)^{1/3} \epsilon^{1/3} (t - t_0), \quad (17)$$

where C_R is Richardson constant for MHD.

When we inject a passive scalar field as in equation (12), we may write

$$\begin{aligned} \sigma^{2/3} - \sigma_0^{2/3} &= (C_1)^{1/3} \epsilon^{1/3} (t - t_0), \\ &= C_2 v_{turb} (t - t_0), \end{aligned} \quad (18)$$

where $\sigma = (\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{1/2}$ and the dimensionless constant C_1 is not necessarily the same as C_R . The constant $C_2 \propto (C_1/l_{inj})^{1/3}$ has dimension. In this paper, we do not attempt to obtain C_1 or C_R . Instead, we investigate how C_2 behaves when we vary B_0 .

Usually it was considered that MHD turbulence is different from its hydrodynamic counterpart. However, as we discussed earlier, Cho, Lazarian, & Vishniac (2002) showed that motions perpendicular to the local mean fields are hydrodynamic to high order. This means that many turbulent processes are as efficient as hydrodynamics ones. For example, Cho et al. (2002) numerically showed that cascade timescale in MHD turbulence follows hydrodynamic scaling relations (see also Maron & Goldreich 2001). The similarity between magnetized and unmagnetized turbulent flows motivates us to speculate that turbulent mixing is also efficient in MHD turbulence. This is why we may use equation (18), which is derived from hydrodynamic turbulence. It is worth noting that these facts are consistent with a recent model of fast magnetic reconnection in turbulent medium (Lazarian & Vishniac 1999).

V. RESULTS

In Figure 2, we compare the time evolution of σ in hydrodynamic case and in MHD case. In the MHD cases, we vary the Alfvén velocity of the mean field ($V_A = B_0/\sqrt{4\pi\rho}$). When $V_A = 1$, the Alfvén velocity of the mean field is slightly larger than the rms fluid velocity ($v_{turb} \sim 0.7$). This is so-called subAlfvénic regime. The case of $V_A = 0.1$ corresponds to the super-Alfvénic regime. Our results show that $v_{turb} \sim 1$ in the hydrodynamic case and $v_{turb} \sim 0.7$ in the MHD case.

In Figure 2(a), we compared the diffusion rate of the passive scalar. The figure shows that there are good relations between $\sigma^{2/3}$ and $(t - t_0)$. The slopes correspond to the constant C_2 in equation (18). The slopes are very similar and this enables us to write

$$\kappa_{dynamic} = C_{dyn} l_{inj} v_{turb}, \quad (19)$$

where C_{dyn} is a constant of order unity. This is the effective diffusion by turbulent motions suitable for scales larger than l_{inj} . The value of C_{dyn} remains almost constant for B_0 's of up to equipartition value, $B_0/\sqrt{4\pi\rho} \sim v_{turb}$ ($\sim b/\sqrt{4\pi\rho}$). The exact value of C_{dyn} is uncertain. In hydrodynamic cases, C_{dyn} is of order of ~ 0.3 (see Lesieur 1990 chapter VIII and references therein).

Figure 2(b) shows that diffusion rate does not strongly depend on the direction of the mean field. When the mean magnetic fields are strong (as in Figure 2(b)), the local magnetic field at any given point in the computation box has a preferred direction (i.e. x-direction). On average, the angle between the magnetic field and x-axis is around $\tan^{-1}(b/B_0) \sim 30^\circ$, where we use $b \sim 0.6$ and $B_0 = 1$. Therefore, the parallel and perpendicular conductivity based on the mean field is statistically the same as that based on the local magnetic field.

VI. ASTROPHYSICAL IMPLICATIONS

We have shown that turbulence motions provide efficient mixing in MHD turbulence. In this section, we show that this process is as efficient as that proposed by Narayan & Medvedev (2001) for some clusters.

We summarize models of thermal diffusion in Fig. 3. In the classical picture, thermal diffusion is highly suppressed in the direction perpendicular to \mathbf{B}_0 . Transport of heat along wandering magnetic field lines (Narayan & Medvedev 2001) partially alleviates the problem. But the applicability of Narayan & Medvedev's model is a bit restricted - their model requires strong (i.e. $V_A \equiv B_0/\sqrt{4\pi\rho} \sim v_{turb}$) mean magnetic field. In the Galaxy, there are strong mean magnetic fields. But, in the ICM, this is unlikely. When the mean field is weak, the scales smaller than the characteristic magnetic field scale ($\equiv l_B$) may follow the Goldreich & Sridhar model (1995). However, this requires further studies. Our turbulent mixing model gives the same $\kappa_{dynamic}$ regardless of magnetic field geometry.

ICM — As we mentioned earlier, $\kappa_{Sp} \sim 1.3 \times 10^{30} (kT/keV)^{5/2} (n/10^{-3} cm^{-3})^{-1} cm^2 sec^{-1}$ and $\kappa_{dynamic} \sim 3.1 \times 10^{30} (v_{turb}/500 km/s)(l_{inj}/20 kpc) cm^2 sec^{-1}$. The ratio of the two is of order unity for the ICM:

$$f_{ICM} \equiv \kappa_{dynamic}/\kappa_{Sp} \sim O(1). \quad (20)$$

To be specific, for Hydra A, $f \sim 0.5$ for the inner region ($R \sim 100 kpc$) and $f \sim 8.6$ for the very inner region ($R \sim 10 kpc$). For 3C 295, $f \sim 0.34$ for the inner region and $f \sim 24$ for the very inner region.

Our model deals with thermal diffusion in *fully developed* MHD turbulence with $v_{turb} \sim 500 km/sec$. When turbulence is *not* fully developed or v_{turb} is *smaller*, we expect a lower thermal conductivity. The observed temperature inhomogeneities in several clusters may indicate that turbulence in the clusters is either under-developed or very weak.

Local Bubble and SNRs — The Local Bubble is a hot ($T \sim 10^6 K$; $kT \sim 100 eV$), tenuous ($n \sim 0.008/cm^3$) cavity immersed in the interstellar medium (Berghofer et al 1998; Smith & Cox 2001). Turbulence parameters are uncertain. We take typical interstellar medium values: $l_{inj} \sim 10 pc$ and $v_{turb} \sim 5 km/sec$. For these parameters, the ratio of $\kappa_{dynamic}$ to κ_{Sp} is

$$f_{in} = \kappa_{dynamic}/\kappa_{Sp} \sim 0.05, \quad (21)$$

for the inside of the Local Bubble. For the mixing layers, it is

$$f_{mix} = \kappa_{dynamic}/\kappa_{Sp} \sim 100, \quad (22)$$

where we take $\bar{T} \sim \sqrt{T_c T_h} \sim 10^5 K$, $\bar{n} \sim \sqrt{n_c n_h} \sim 0.1/cm^3$ (Begelman & Fabian 1990), $T_c \sim 10^4 K$, $n_c \sim 1/cm^3$, $T_h \sim 10^6 K$, and $n_h \sim 0.008/cm^3$. We expect similar results for supernova remnants since parameters are similar.

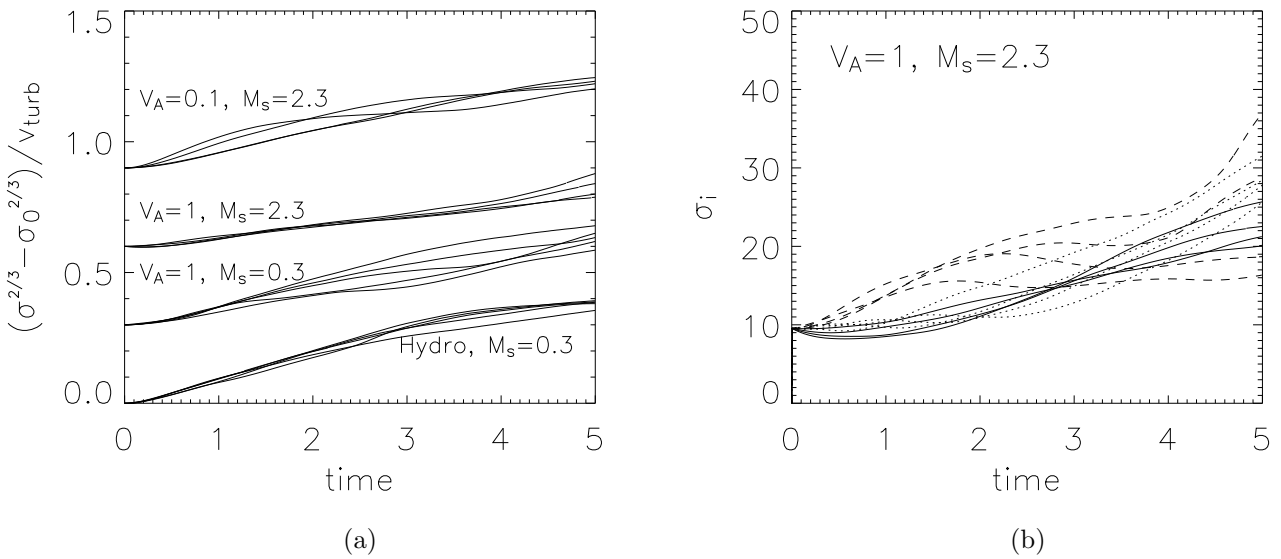


Fig. 2.— Time evolution of energy density and σ . Passive scalars are injected after turbulence is fully developed. (a) $(\sigma^{2/3} - \sigma_0^{2/3}) / v_{turb}$ vs. time. σ is the standard deviation of the passive scalar field. Y-axis is in the unit of box-size and Y-values are shifted by 0.3 units for convenience. Note that the slope does not strongly depend on the mean field strength $V_A (= B_0 / \sqrt{4\pi\rho})$ or sonic Mach number M_s . (b) MHD run with 216^3 grid points, $M_s \sim 2.3$, and $V_A = 1$. σ_i ($i=x, y,$ and z) vs. time. Solid lines=parallel to \mathbf{B}_0 ; dashed and dotted lines=perpendicular to \mathbf{B}_0 . From Cho et al. (2003).

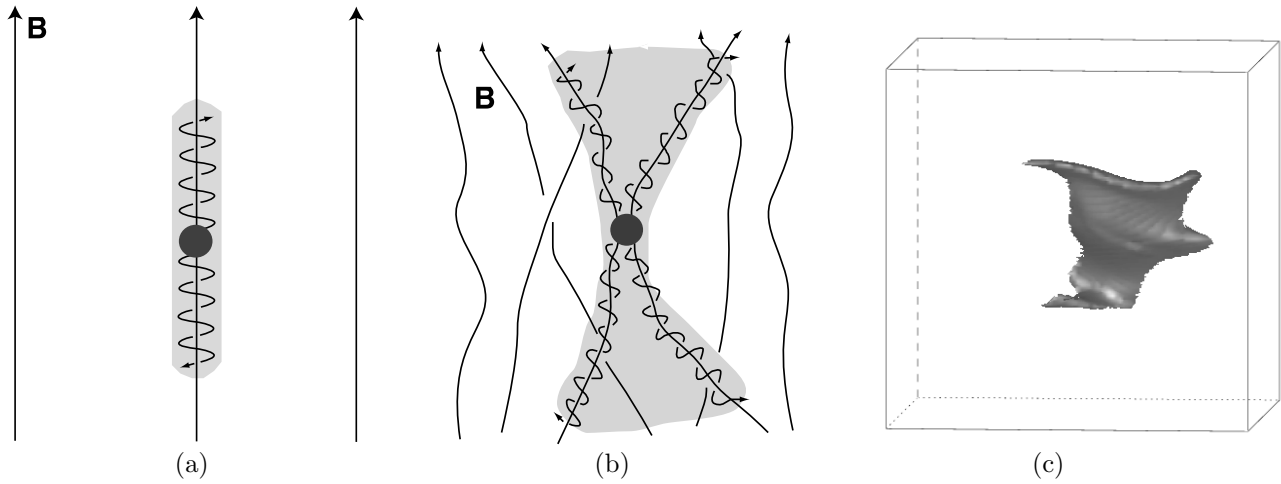


Fig. 3.— Models of thermal diffusion. (a) Classical picture. $\kappa_{\perp} \ll \kappa_{Sp}$. (b) Narayan & Medvedev (2001). Wandering of field lines provides efficient diffusion ($\kappa_{\perp} \sim \kappa_{Sp}/5$) in the direction perpendicular to \mathbf{B}_0 . But, the model assumes \mathbf{B}_0 of \sim equipartition value. (c) Turbulent diffusion model. Thermal electrons are mixed by turbulent motions, which leads to turbulent diffusion coefficient of $\kappa_{dynamic} \sim v_{turb} l_{inj}$. In many astrophysical situations, this coefficient is comparable with the Spitzer value. The figure is the snapshot of the passive scalar field at $t \sim 3$ from the MHD run described in Figure 1(c); 192^3 grid points, $M_s \sim 0.3$, and $V_A = 1$. In the case shown here, the mean field is strong and parallel to the dashed line. In general, mean magnetic fields, weak or moderately strong, do not strongly suppress turbulent motions/diffusion. From Cho et al. (2003).

VII. CONCLUSION

We have shown that magnetic fields (either random or mean magnetic field of up to equipartition value) do not suppress turbulent diffusion processes, which implies that turbulent diffusion coefficient has the form $\kappa_{dyn} \sim l_{inj} v_{turb}$ in MHD turbulence, as well as in hydrodynamic cases. This result has two important astrophysical implications. First, in the ICM, this turbulent diffusion coefficient is of the same order of the classical Spitzer value. Second, in the face of hot and cold media in the ISM (e.g. the boundary between the Local Bubble and surrounding warm media), this turbulent diffusion coefficient is much larger than the classical Spitzer value.

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