Radiative hydrodynamics (SPH-RT) & applications to circumstellar discs

Dimitris Stamatellos
School of Physics & Astronomy
Cardiff University, Wales, UK

Collaborator: A. Whitworth

Planet formation

Core accretion model

- a solid core forms first, and subsequently accretes an envelope of gas
- planet formation in a few Myr

Perri & Cameron 1974
Mizuno 1980
Lissauer 1993
Rafikov 2004
Goldreich et al. 2004

Gravitational fragmentation

Circumstellar discs may fragment to form giant planets if (i) they are massive enough (ii) they cool fast enough

Cameron 1978
Boss 1998, 2000
Rice et al. 2003, 2004
Mayer et al. 2002, 2006

Criteria for disc fragmentation

(i) Toomre criterion (Toomre 1964)

Disc must be massive enough

$$\ddot{r}_{\rm D} \equiv \frac{{\rm d}^2 r_{\rm D}}{{\rm d}t^2} \simeq -\pi G \Sigma(R) + \frac{a^2(R)}{r_{\rm D}} + \Omega^2(R) r_{\rm D}$$

$$\boxed{\Sigma(R) \gtrsim \frac{\alpha(R)\ \Omega(R)}{\pi\ G}}$$

(ii) Gammie criterion (Gammie 2001)

Disc must cool on a dynamical timescale

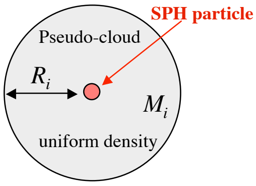
$$\boxed{t_{\rm COOL} \lesssim (3-12)\ \Omega^{-1}}$$

Can real discs fragment?

Gammie (2001) Rice, Lodato, Armitage et al. (2003) Mejia et al. (2005)	**MAYBE** Fragmentation when conditions are favourable [parameterized cooling]
Boss (1997-2006) Mayer, Quinn et al. (2004, 2006)	**YES** Fragmentation (cooling by convection)
Durisen, Mejia, Cai, Boley, Pickett et al. Gammie & Johnson (2003) Nelson et al. 2000, Nelson 2006	**NO** No Fragmentation (disc cannot cool fast)
Rafikov (2005, 2006) **Whitworth & Stamatellos (2006)**	**NO (close to the star)** No fragmentation close to the central star

Smoothed Particle Hydrodynamics (with radiative transfer)

Use each the gravitational potential of SPH particles to define a pseudo-cloud around each particle, which determines how the particle cools/heats.



SPH particle potential: $$\psi_i = -\frac{3\,G\,M_i}{5\,R_i}$$

SPH particle density: $$\rho_i = \frac{3\,M_i}{4\,\pi\,R_i^3}$$

Pseudo-cloud mass: $$M_i = c_{\rm M}\left[\frac{5\,\psi_i}{G}\right]^{3/2}\left[\frac{1}{36\,\pi\,\rho_i}\right]^{1/2}$$

Pseudo-cloud radius: $$R_i = c_{\rm R}\left[\frac{5\,\psi_i}{4\,\pi\,G\,\rho_i}\right]^{1/2}$$

Stamatellos, Whitworth et al., in preparation

Smoothed Particle Hydrodynamics (with radiative transfer)

Particle's column density: $$\Sigma_i = \frac{3\,M_i}{4\,\pi\,R_i^2} = \left[\frac{5\,\psi_i\,\rho_i}{4\,\pi\,G}\right]^{1/2}$$

Particle's optical depth: $$\tau_i = \kappa(\rho_i, T_i)\;\Sigma_i$$

Particle's cooling rate: $$\mathcal{L}_i = \frac{16\,\pi\,R_i^2\sigma_{\rm SB}\,T_i^4}{3\,\left[\bar{\tau}_i(\rho_i, T_i) + \bar{\tau}_i^{-1}(\rho_i, T_i)\right]}$$

Stamatellos, Whitworth et al., in preparation

Smoothed Particle Hydrodynamics (with radiative transfer)

ENERGY EQUATION

$$\frac{du_i}{dt} = \frac{1}{2}\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) \mathbf{v_{ij}} \cdot \nabla_i W\left(\frac{r_{ij}}{h_{ij}}\right) + \left.\frac{du_i}{dt}\right|_{\rm RAD}$$

Compressional heating ($\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}$)

Viscous heating (Π_{ij})

Radiative cooling/heating rate ($\left.\frac{du_i}{dt}\right|_{\rm RAD}$)

$$\left.\frac{du_i}{dt}\right|_{\rm RAD} = \mathcal{H}_0 - \left[\frac{4\,\sigma_{\rm SB}\,T_i^4}{\Sigma_i^2\,\bar{\kappa}(\rho_i,T_i)\,+\,\bar{\kappa}^{-1}(\rho_i,T_i)}\right]$$

Stamatellos, Whitworth et al., in preparation

SPH-RT: Equation of state & dust/gas opacities

Equation of state

- Vibrational & rotational degrees of freedom of H_2
- H_2 dissociation
- H ionisation
- Helium first and second ionisation

Black & Bodenheimer 1975

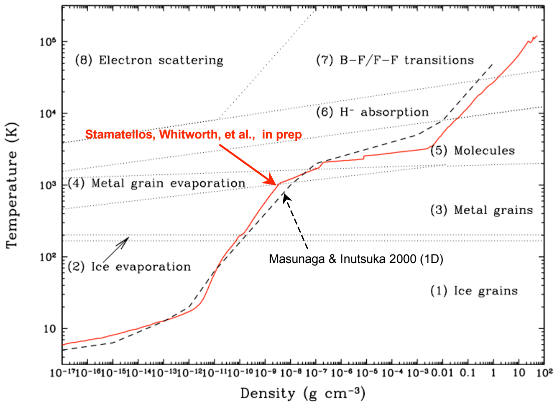
Dust & gas opacities

- Ice mantle melting
- Dust sublimation
- Molecular opacity
- H^- absorption
- B-F/F-F transitions

Bell & Lin 1994

The collapse of a 1-$M_\odot$ molecular cloud

- 200,000 SPH particles (UKAFF)
- 20 orders of magnitude in density
- 4 orders of magnitude in temperature

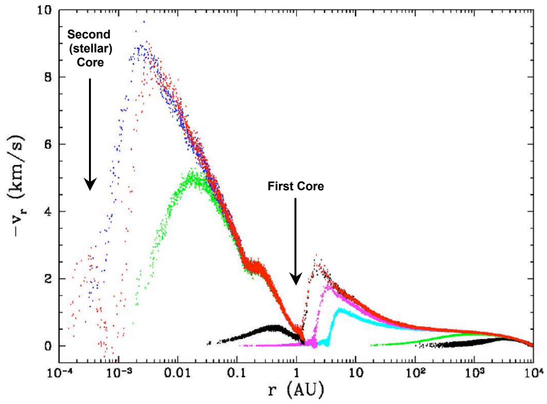


Stamatellos, Whitworth, et al. in preparation

… at minimal computational cost !

The collapse of a 1-$M_\odot$ molecular cloud

- 200,000 SPH particles (UKAFF)
- 20 orders of magnitude in density
- 4 orders of magnitude in temperature



Stamatellos, Whitworth, et al. in preparation

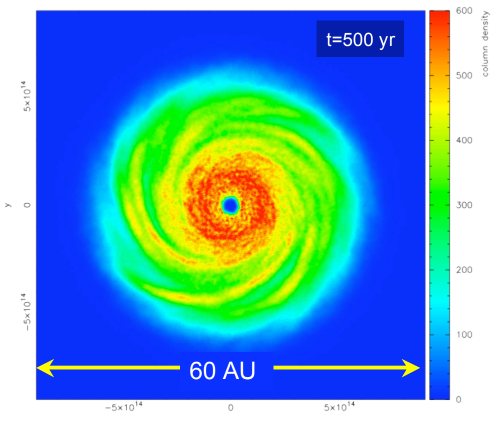
… at minimal computational cost !

SPH with radiative transfer: Disc simulations

- SPH simulations (200,000 particles) using UKAFF
- Realistic EOS
- Realistic dust opacities
- Realistic cooling (+ heating)

Simulations of disc evolution - **Inner disc (2-40 AU)**

Ambient heating: T_{ext}=10K [low]



$$\Sigma(R) \sim R^{-0.5}$$

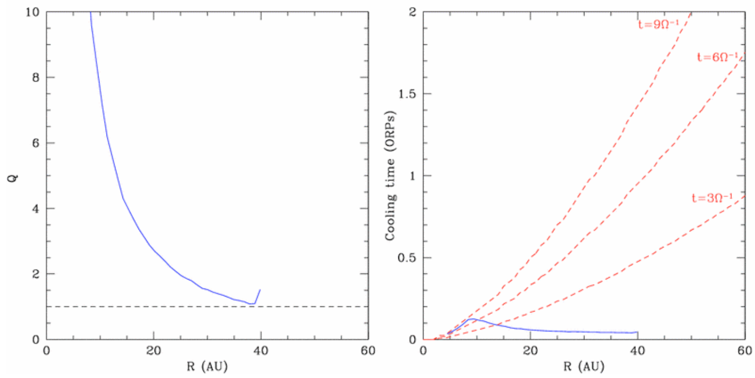
$$T(R) \sim R^{-1}$$

$$M_{\rm DISC} = 0.07 M_\odot$$

$$M_\star = 0.5 M_\odot$$

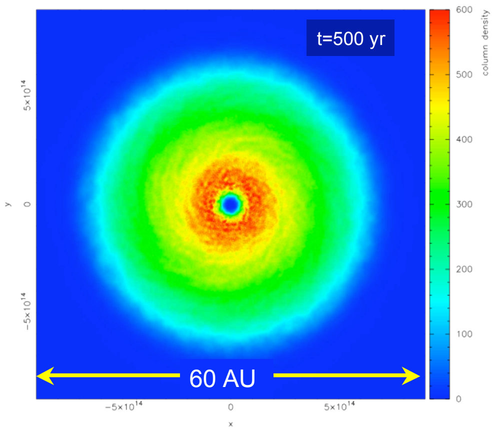
- Disc does not fragment because it cannot cool fast enough ($t_{cool} > 3\,\Omega^{-1}$)

Simulations of disc evolution - Inner disc (2-40 AU)



Simulations of disc evolution - Inner disc (2-40 AU)

Stellar heating: T_{ext}~R^{-1} [high]



$$\Sigma(R) \sim R^{-0.5}$$

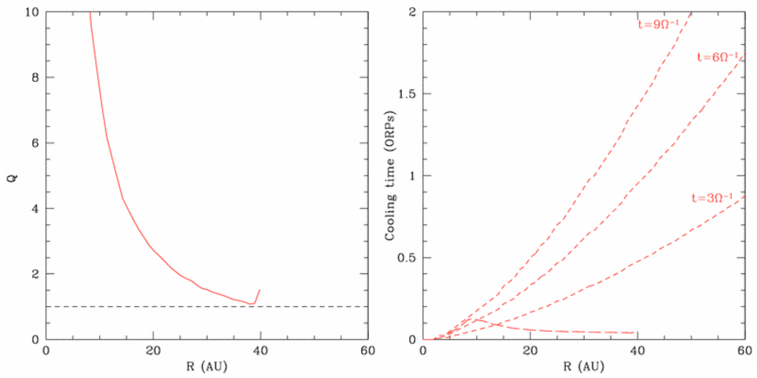
$$T(R) \sim R^{-1}$$

$$M_{\rm DISC} = 0.07 M_{\odot}$$

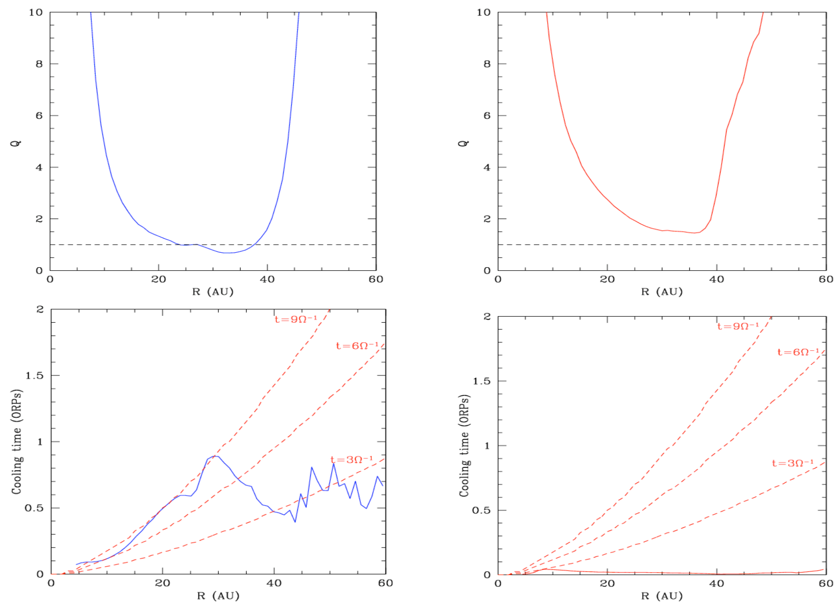
$$M_{\star} = 0.5 M_{\odot}$$

- **Disc does not fragment because it is too hot (Q > 1)**

Simulations of disc evolution - Inner disc (2-40 AU)



-- Low external heating -- Comparison -- High external heating --



Can real discs fragment?

Whitworth & Stamatellos, A&A, 2006

$$\mathcal{H}_{\rm D}(R) \sim a(R)^5 \sim L_\star^{5/8}\, R^{-5/4}$$

$$\mathcal{L}_{\rm D}(R) \sim r^2_{\rm FRAG\,D}(R)\; T^4(R)/\tau_{\rm FRAG\,D}(R) \sim L_\star^{5/8}\, R^{13/4} M_\star^{-3/2}$$

$$\mathcal{L}_{\rm D}(\mathcal{R}) \gtrsim \mathcal{H}_{\rm D}(\mathcal{R})$$

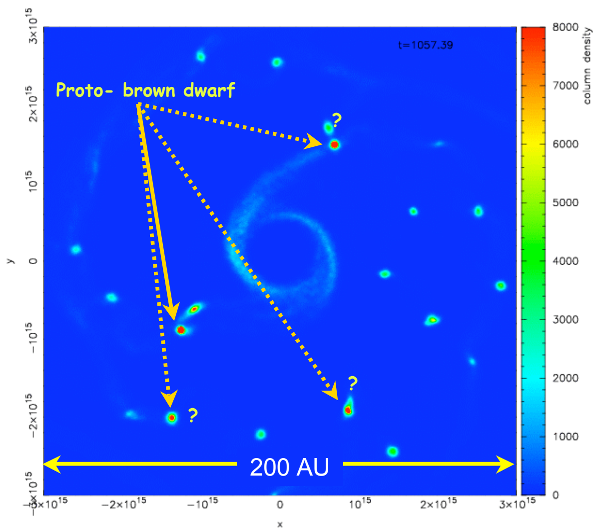
$$R \gtrsim R_{\rm MIN\,D} \sim 150\ {\rm AU} \left[\frac{M_\star}{\rm M_\odot}\right]^{1/3}$$

Discs cannot fragment to produce planets close (<100AU) to the star

Whitworth & Stamatellos, A&A, 2006; Rafikov 2005

Simulations of disc evolution - Outer disc (50-500 AU)

$$\Sigma(R) \sim R^{-7/4}$$
$$T(R) \sim R^{-1/2}$$
$$M_\star = 1{\rm M}_\odot$$
$$M_{\rm DISC} = 1{\rm M}_\odot$$



CONCLUSIONS

-SPH simulations of real discs (realistic opacities, cooling+heating, EOS)

-Simulations of the inner disc region **(2-50 AU)**

-Results are similar to those of Boley, Durisen et al. 2006

-Discs under (i) low or (ii) high external heating **do not fragment** as (i) they cannot cool fast enough or (ii) they are not Toomre unstable

-Simulations of the outer disc region **(50-500 AU)**

-Discs cool fast enough and **can fragment** (if they are Toomre unstable)

Formation of giant planets close to the star (R<60 AU) by disc fragmentation is unlikely, but…

… discs can fragment at R>100 AU to form brown dwarfs (and/or planets?) **[Whitworth & Stamatellos, A&A, 2006]**