Simulation of relativistic outflows in astrophysics

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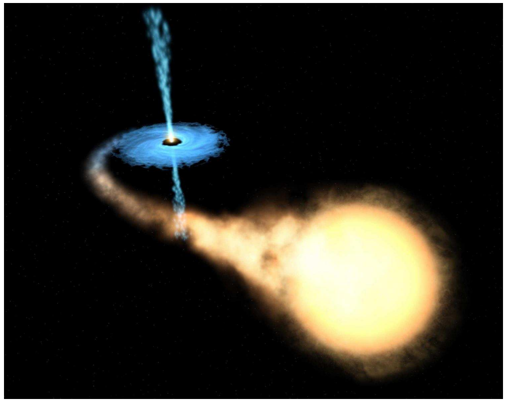
Plan of Talk



- A short introduction on relativistic flow & Equation of State.
- A brief look on the code.
- Relativistic outflows.
- Concluding Remarks.

Introduction

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- Examples: (a) Accretion on to BH: At $r \rightarrow$large, $v \sim 0$ & $T_p \sim$small ; At $r \sim$few$10 r_g$, $v \gtrsim 0.1c$, $T_p \sim 10^{12} K$; The adiabatic index (c_p/c_v) $\gamma \rightarrow 5/3 \rightarrow 4/3$.

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- (b) Jets: Originates at $r \sim \mathrm{few} 10 r_g$ with $v \sim 0$, and $T_p \sim 10^{12} K$ ($\gamma \sim 4/3$) ; at $r \sim 1000 r_g$ $v \gtrsim 0.9c$, $T_p \sim$moderate ($5/3 \gtrsim \gamma \gtrsim 4/3$) : As the jet hits ambient medium, $v \sim$ small, but T_p increases ($\gamma \sim 4/3$).

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- **It is important to use correct γ, influences the value of $c_s \Rightarrow$ would affect T distribution, as well as structure propagations:** Are we equipped to do so?

A brief account on EoS

- The equations of motion are ($c = 1$):

$$(\rho u^{\nu})_{;\nu} = 0, \qquad T^{\mu\nu}_{;\nu} = 0$$

ρ, u^{μ} are proper density and 4-velocity, $T^{\mu\nu} = \rho h u^{\mu} u^{\nu} + p g^{\mu\nu}$; $h =$specific enthalpy $= (e + p)/\rho$, e total proper energy density and p pressure.

- Number of eqs $= 5$; Number of variables $= 6$.

A brief account on EoS

- To resolve, one assumes an EoS *i.e.*, $h \equiv h(p,\rho)$ or $e \equiv e(p,\rho)$.
- In non-relativistic regime the internal energy density is given by

$$e_{int} = \frac{p}{(\gamma - 1)}$$

- the most favoured EoS in RHD is (hereafter ID) is

$$h = 1 + \frac{\gamma \Theta}{\gamma - 1}, \qquad \text{or} \qquad e = \rho + \frac{p}{\gamma - 1}$$

$\Theta = p/\rho$ a temperature like variable.

A brief account on EoS

- The general definition of polytropic index and sound speed:

$$n = \rho \frac{\partial h}{\partial p} - 1, \qquad c_s^2 = -\frac{\rho}{nh}\frac{\partial h}{\partial \rho}.$$

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- For ID, n & c_s:

$$n = \frac{1}{\gamma - 1}, \qquad c_s^2 = \frac{\gamma\Theta(\gamma-1)}{\gamma\Theta + \gamma - 1},$$

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- **(a) In ID γ or n has no temperature dependence**

  (b) Gives unphysical wave speeds !
- Are we free to use any other **arbitrary EoS**?

A brief account on EoS

- Relativistic kinetic theory imposes a strong constrain on EoS, known as Taub's inequality,

$$(h - \Theta)(h - 4\Theta) \geq 1$$

- ID do not satisfy TAUB's inequality (TI) for all values of Θ and $\gamma \Rightarrow$ ID is relativistically wrong!

A brief account on EoS

- Chandrashekhar (1934), Synge (1957) calculated the exact EoS for simple gas (RP)

$$h = \frac{K_3(1/\Theta)}{K_2(1/\Theta)},$$

K_2 and K_3 are modified Bessels function of second kind & of order 2 & 3.

- This accuracy comes at expense of extra computational cost as shown by Falle & Komissarov (1996)! $\Rightarrow$ Not suitable for Numerical RHD!
- **What do we do?** Try and propose a new equation of state

New EoS

- New EoS must satisfy:
    - (a) Taub's Inequality for all Θ!
    - (b) Should give $\gamma \rightarrow 5/3$ (or $n \rightarrow 3/2$) for $\Theta \rightarrow 0$ & $\gamma \rightarrow 4/3$ (or $n \rightarrow 3$) $\Theta \rightarrow \infty$!
    - (c) Good fit to RP!

New EoS

- Our proposed EoS (RC) is,

$$\frac{p}{e-\rho} = \frac{3p+2\rho}{9p+3\rho}; \qquad \text{or} \qquad h = 2\frac{6\Theta^2+4\Theta+1}{3\Theta+2}. \qquad (iv)$$

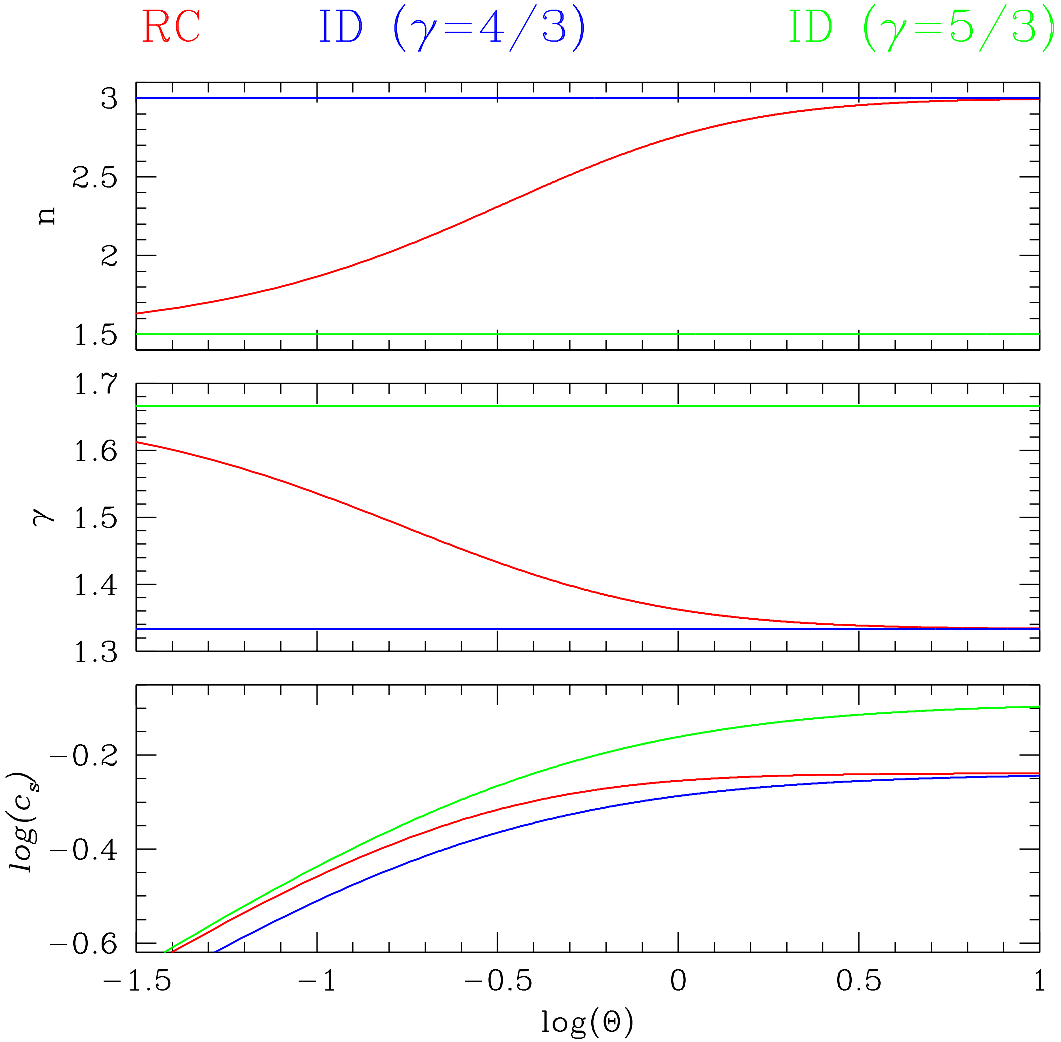
- Expression of n & c_s with RC EoS:

$$n = 3\frac{9\Theta^2+12\Theta+2}{(3\Theta+2)^2}, \qquad c_s^2 = \frac{\Theta(3\Theta+2)(18\Theta^2+24\Theta+5)}{3(6\Theta^2+4\Theta+1)(9\Theta^2+12\Theta+2)}.$$

- RC is extremely accurate!

$$\frac{|h_{\mathrm{RP}} - h_{\mathrm{RC}}|}{h_{\mathrm{RP}}} \lesssim 0.8\%$$

New EoS



- Comparing various quantities top to bottom (i) n, (ii) γ, (iii) c_s; for RC, ID($\gamma = 4/3$), ID($\gamma = 5/3$).

Highlights of the new code

- We have implemented RC and developed a new code (based on TVD scheme)!!
- The full eigen structure has been derived
- The **Jacobian matrix** and the **eigen structure is derived without assuming** any exact form of h (EoS)
- This makes the code ideal to compare the effect of various EoS
- For the details of the code and its test runs please see Ryu, Chattopadhyay, Choi (2006)

Highlights of the new code

- The equations of motions in conserved form **[**$(\rho u^{\nu})_{;\nu} = 0,\ T^{\mu\nu}_{;\nu} = 0$**]**:

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{F}^j}{\partial x^j} = 0, \qquad (v)$$

- The conserved quantities are:

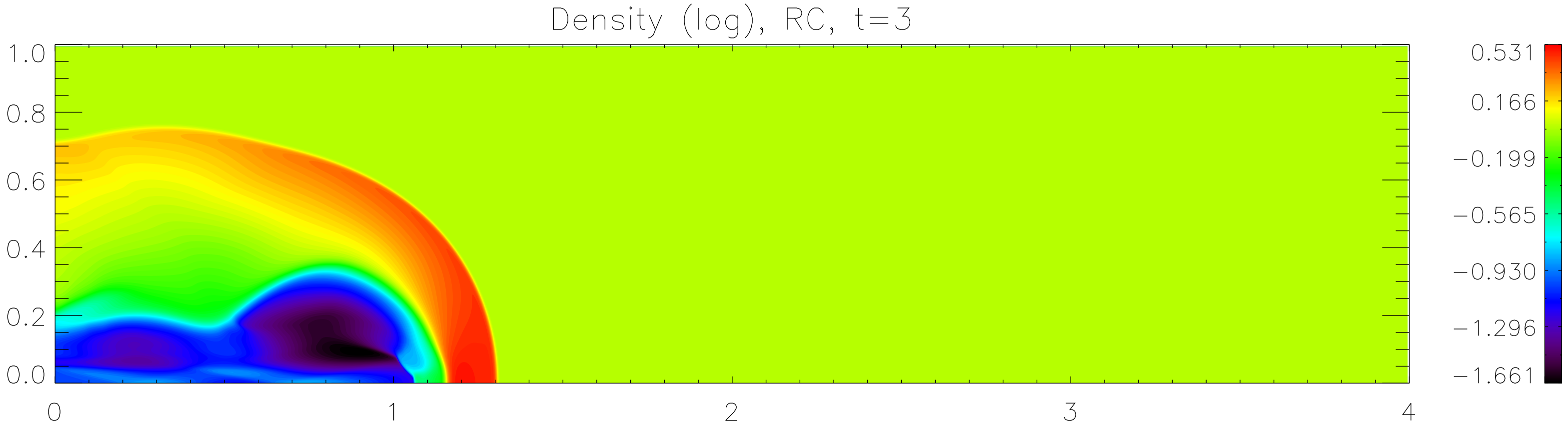
$$\vec{q} = \left[D \quad M^i \quad E\right]^T, \qquad \vec{F}^j = \left[Dv^j \quad M^i v^j + p\delta^{ij} \quad (E+p)\,v^j\right]^T.$$

- And the transformation relation bet^n observer frame to fluid rest frame are:

$$D = \Gamma\rho, \quad M^i = \Gamma^2 \rho h v^i, \quad E = \Gamma^2 \rho h - p, \qquad (vi)$$

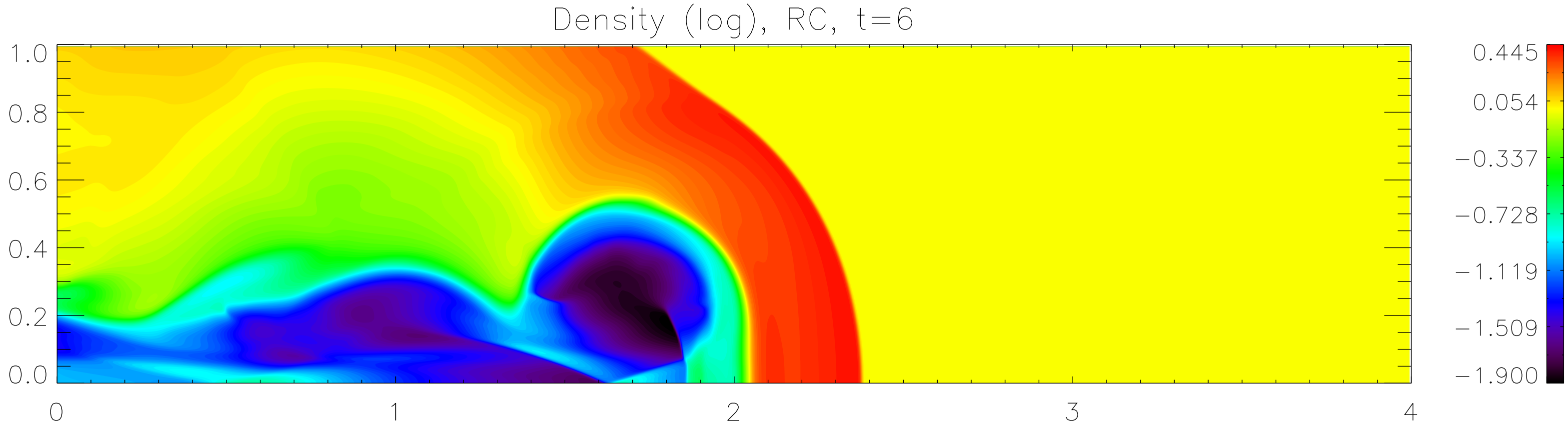
$$\Gamma = (1 + u_i u^i)^{1/2} = 1/(1 - v^2)^{1/2}$$

Relativistic JETS



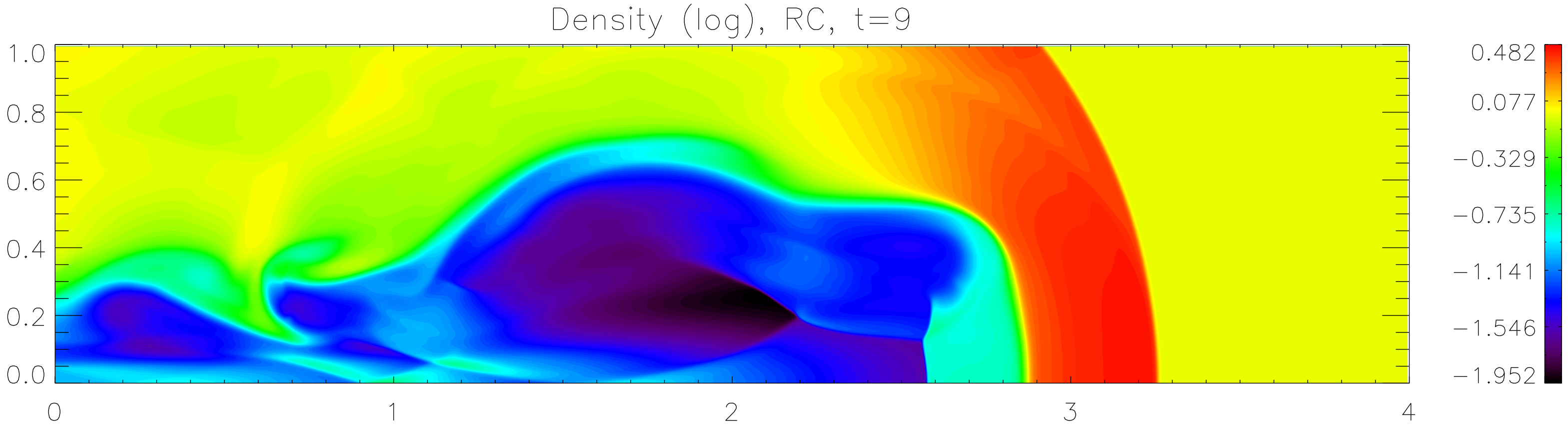
- Jet RC— $log(\rho)$: t=3; Res:1024×256; $Y = 4X$; $X = 21.3\times$jet radii(r_d); Init cond: $v_x = 0.9$, $\rho_j = 0.1$, $\rho_a = 1$, $p = 0.01$; Initial jet length$= r_d$.

Relativistic JETS



- Jet RC— $log(\rho)$: t=6;

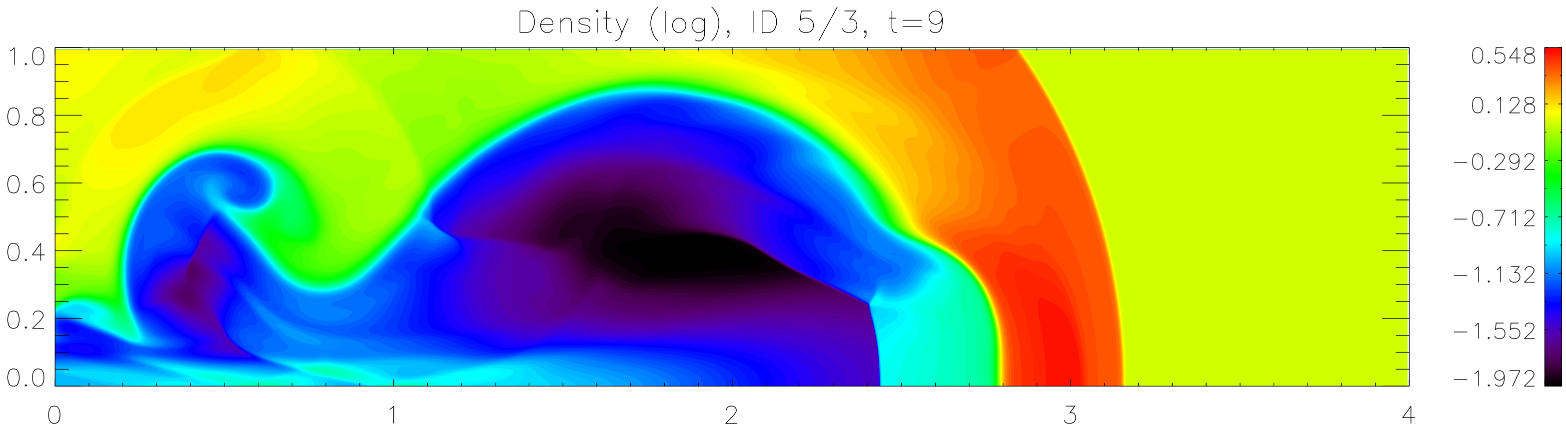
Relativistic JETS



- Jet RC— $log(\rho)$: t=9;

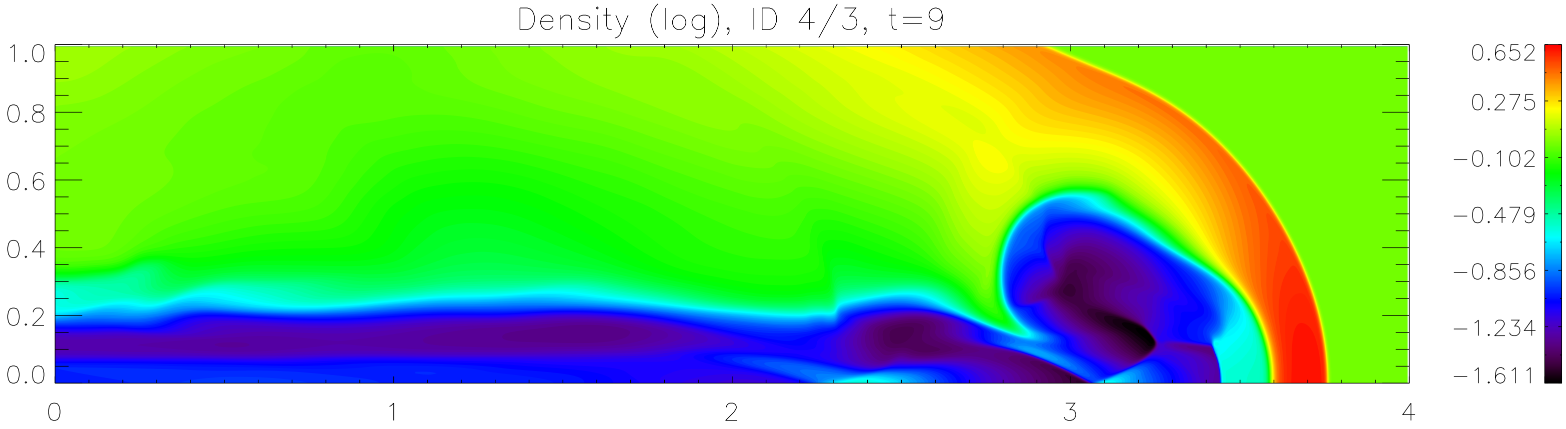
  How does solutions with ID EoS look like at this time?

Relativistic JETS



- Jet ID $\gamma = 5/3$ — $log(\rho)$: t=9; Res:1024×256; $Y = 4X$; $X = 21.3 \times$jet radii(r_d); Init cond: $v_x = 0.9$, $\rho_j = 0.1$, $\rho_a = 1$, $p = 0.01$; Initial jet length= r_d.

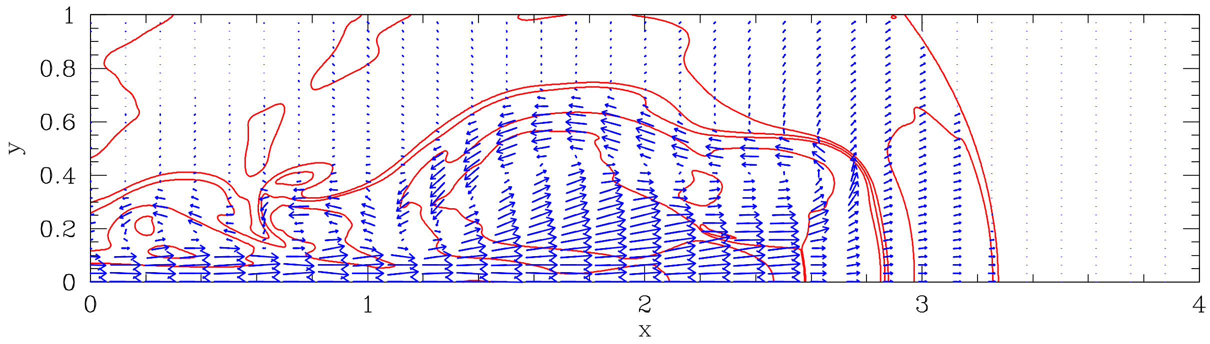
Relativistic JETS

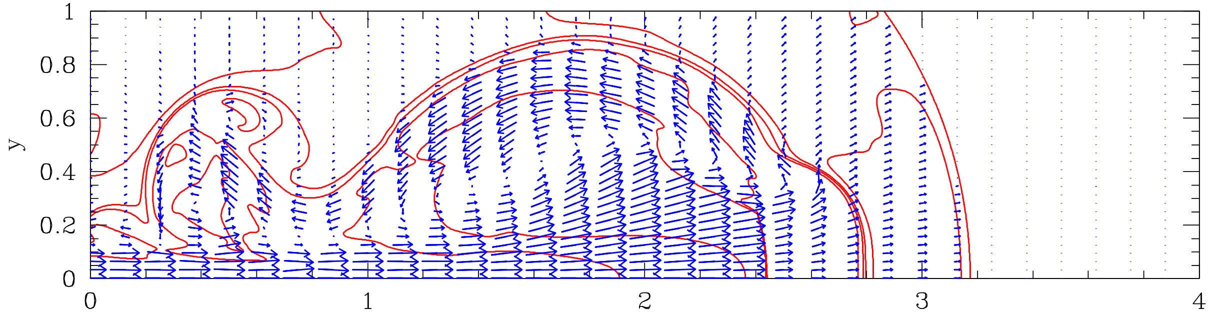


- Jet ID $\gamma = 4/3$— $log(\rho)$: t=9; Res:1024×256; $Y = 4X$; $X = 21.3 \times$jet radii(r_d); Init cond: $v_x = 0.9$, $\rho_j = 0.1$, $\rho_a = 1$, $p = 0.01$; Initial jet length= r_d.

  ID $\gamma = 4/3$ definitely is quite different, but why?

Relativistic JETS


RC; $v_{xi}=0.9$; time=9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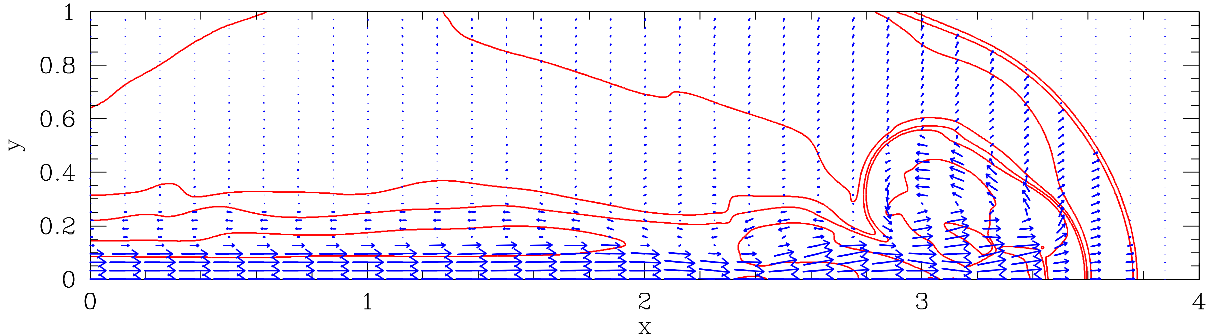
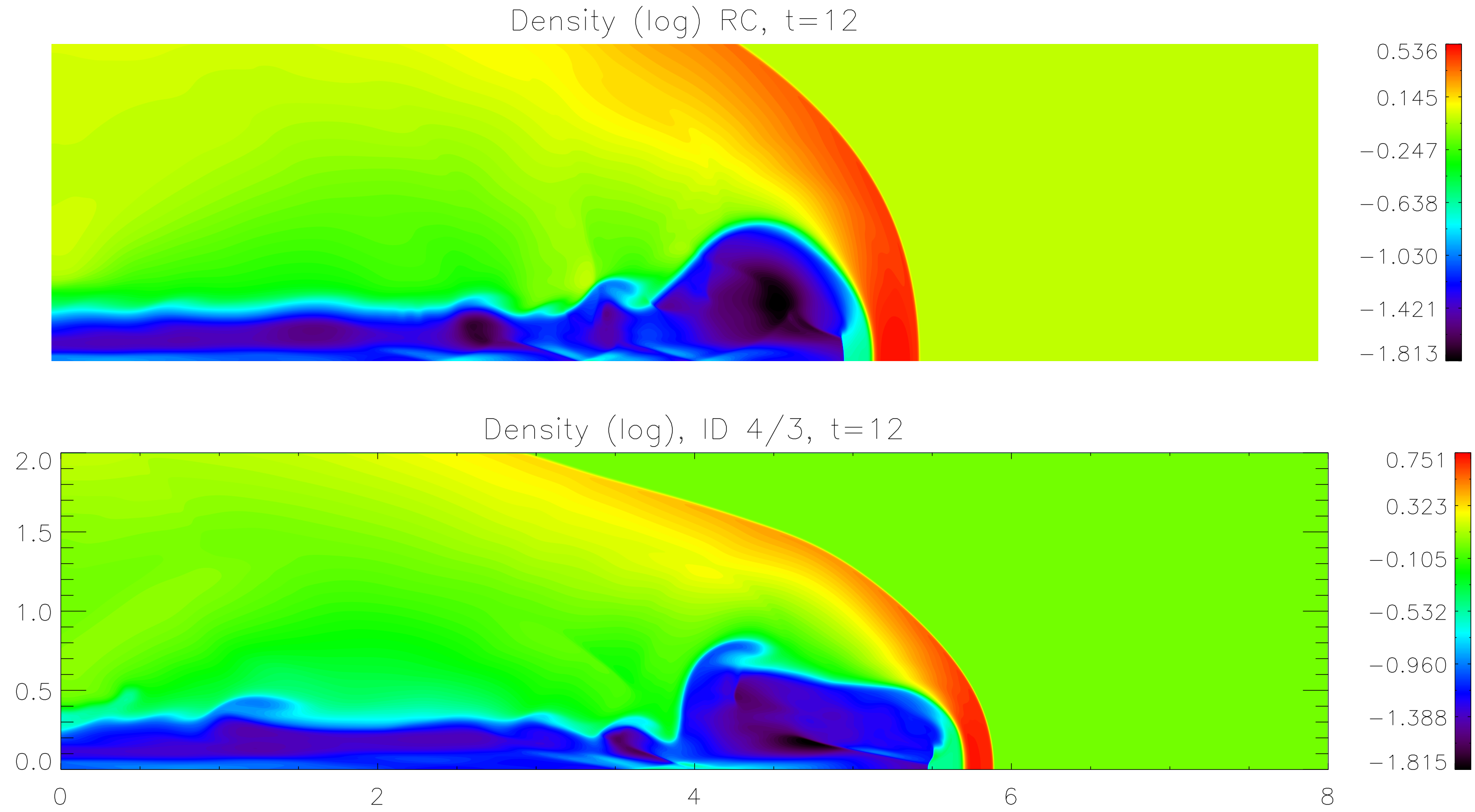

ID $\gamma=4/3$; $v_{xi}=0.9$; time=9

Relativistic JETS

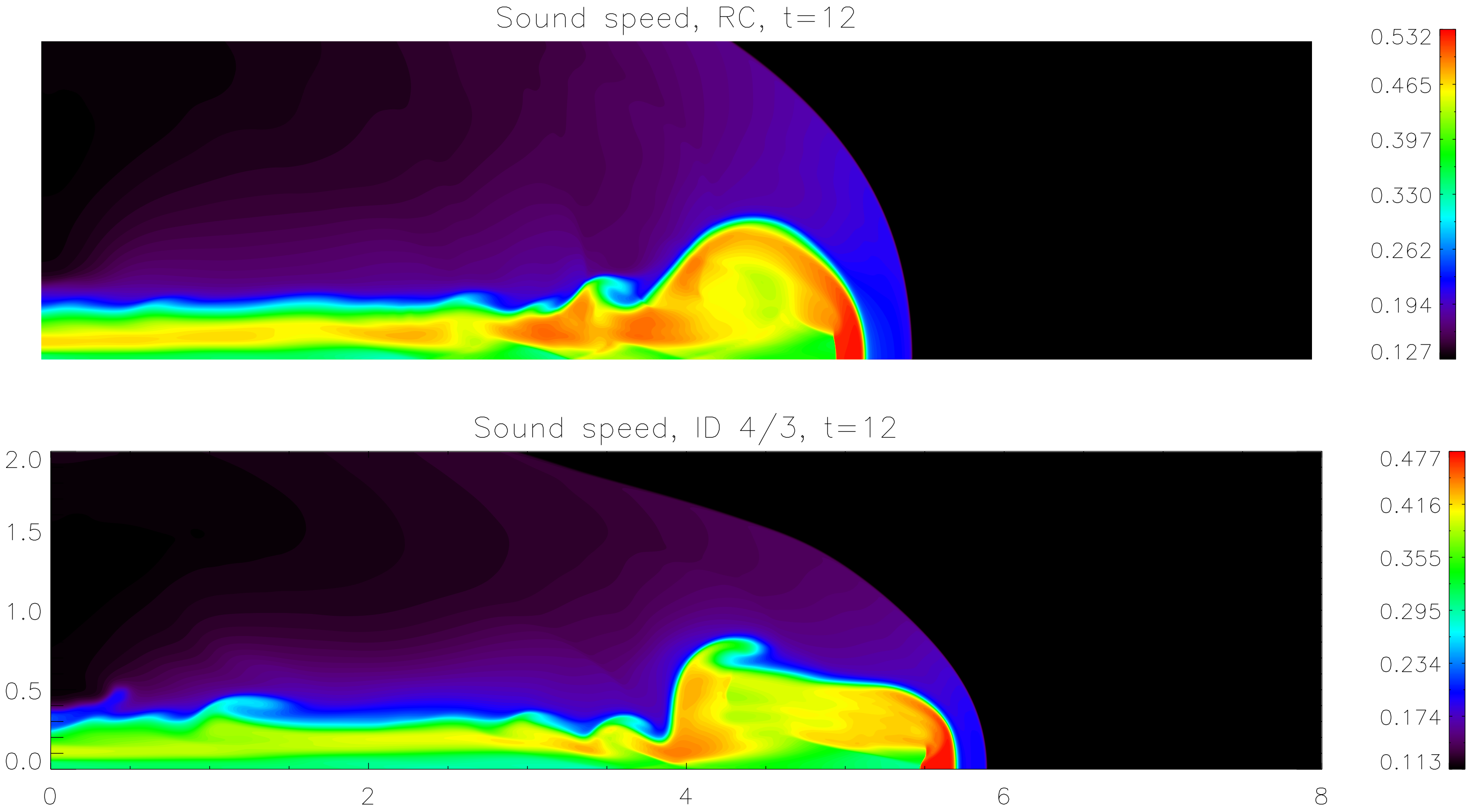
- As jet hits ambient medium it heats up
- A part of it flows back.
- With ID $\gamma = 5/3$ c_s is higher, it puffs up more, interacts with back flowing material, instabilities sets in
- With $\gamma = 4/3$, remains thin, limited back reaction, beam structure almost intact!
- With RC correct c_s gives a correct mode of interaction
- The bow shock is in median position between ID $\gamma = 5/3$ and $\gamma = 4/3$
- **With $\gamma = 4/3$, even moderate injection Γ, produces near perfect beam structure**
- Let us jack up injection velocity

Relativistic JETS



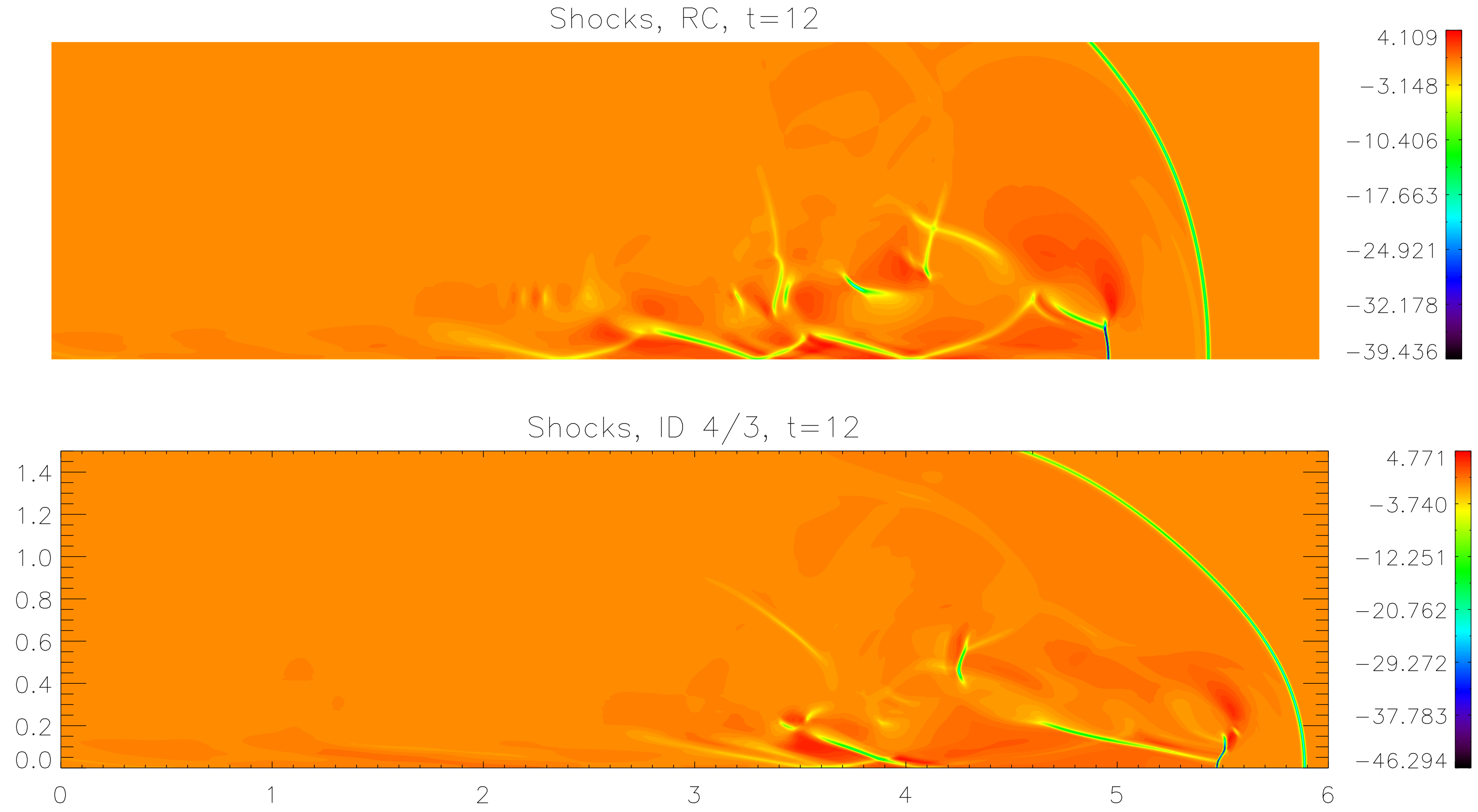
- Density contours at $t = 12$. Upper panel RC, lower panel ID ($\gamma = 4/3$), $v_{xi} = 0.95$.

Relativistic JETS



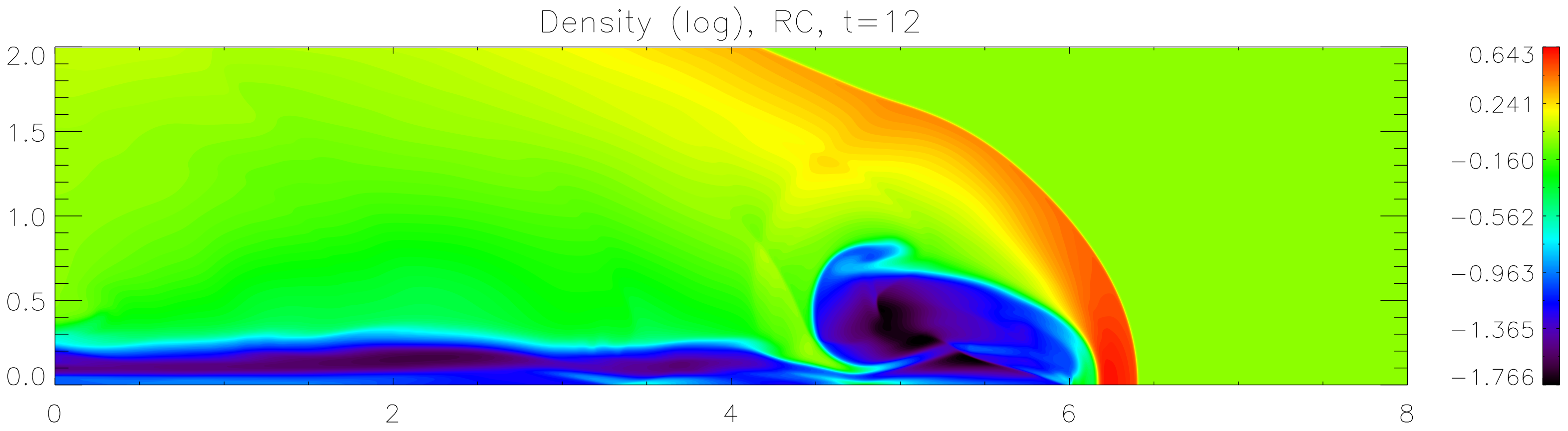
- Sound speed at $t = 12$. Upper panel RC, lower panel ID ($\gamma = 4/3$), $v_{xi} = 0.95$.

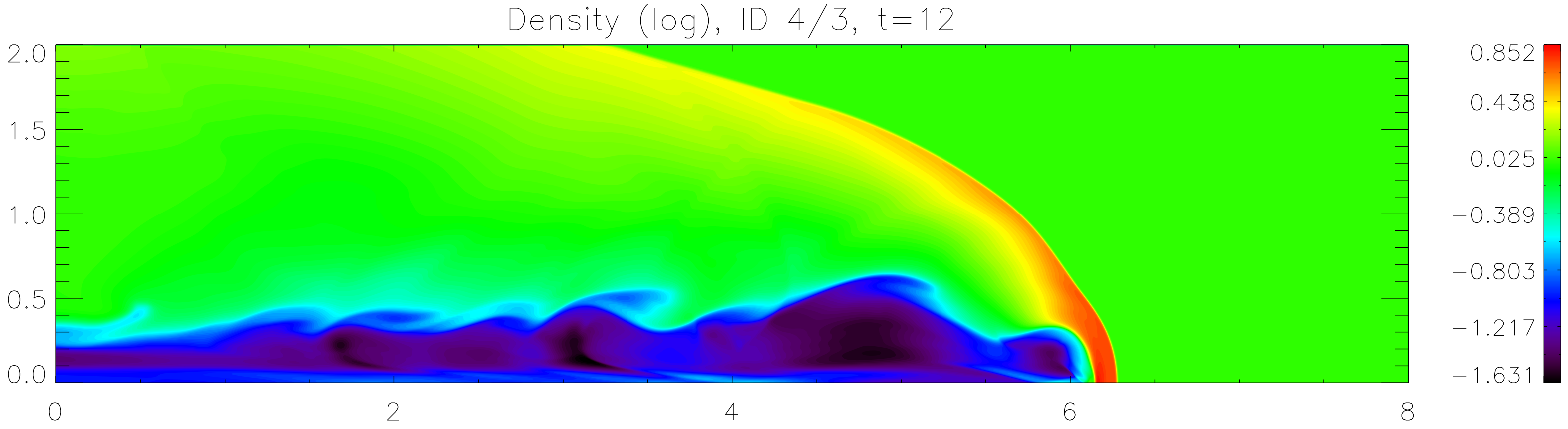
Relativistic JETS



- Shocks at $t = 12$. Upper panel RC, lower panel ID ($\gamma = 4/3$), $v_{xi} = 0.95$. Should affect emitted radiation.

Relativistic JETS

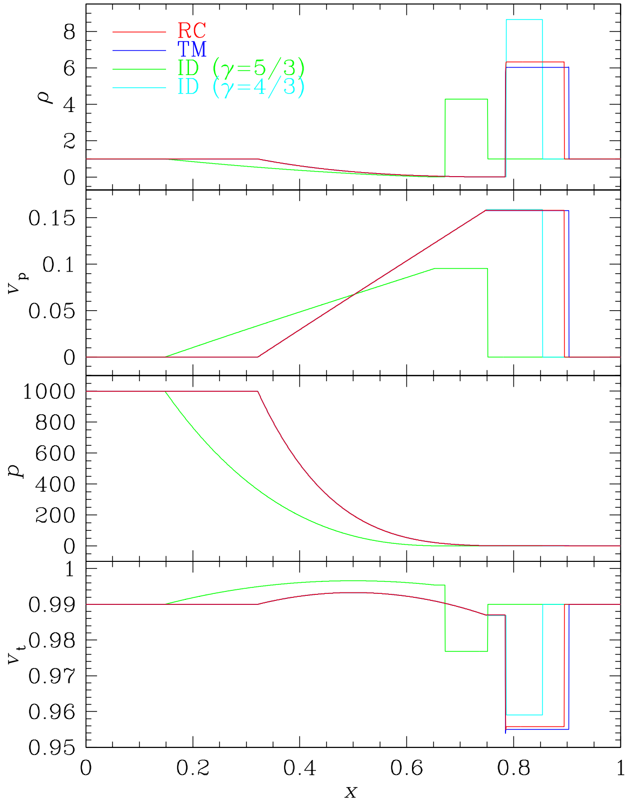

Density (log), RC, t=12


Density (log), ID 4/3, t=12

- $t = 12$; Density: $v_{xi} = 0.96$;

solution with RC is faster!!!

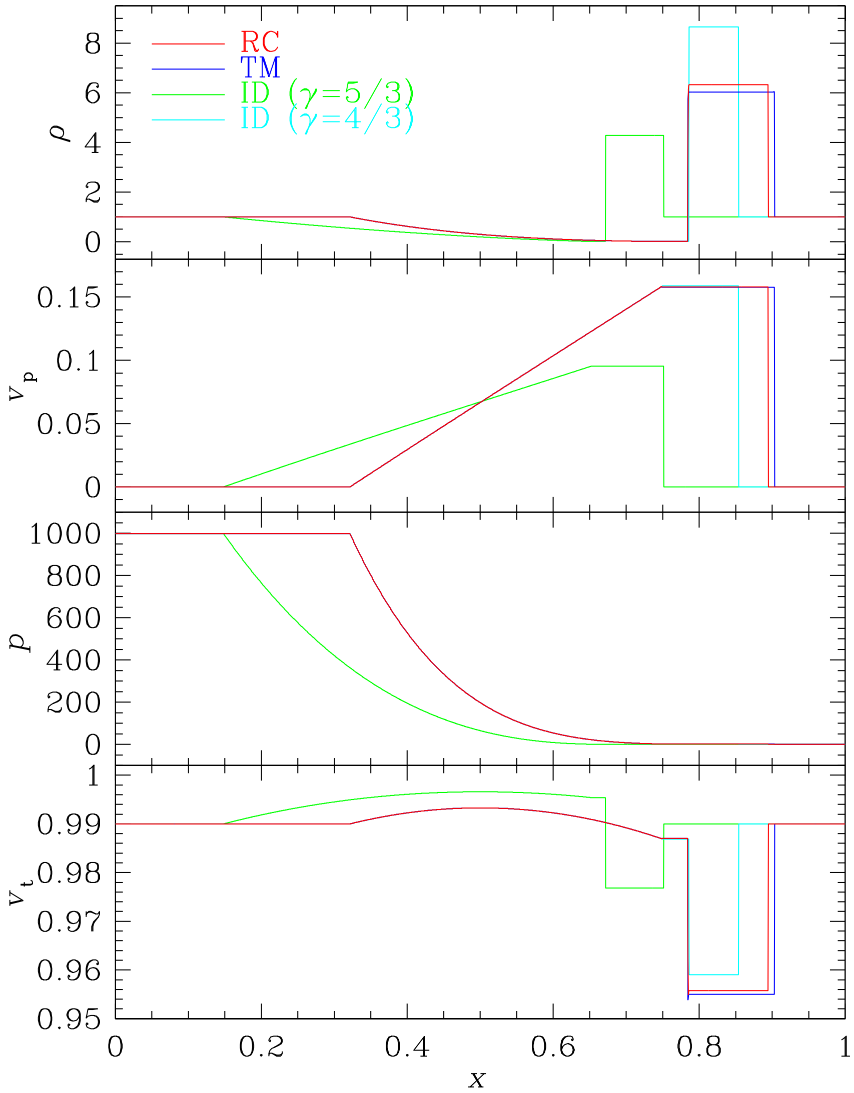
Relativistic JETS



- Strong-Shock Tube — t=0.18; Init cond:

$$\begin{aligned}(\rho, v^x, v^y, v^z, p) &= (1, 0, 0.99, 0, 10^3); 0 \le x \le 0.5 \\ &= (1, 0, 0.99, 0, 10^{-2}); 0.5 < x \le 1.\end{aligned}$$

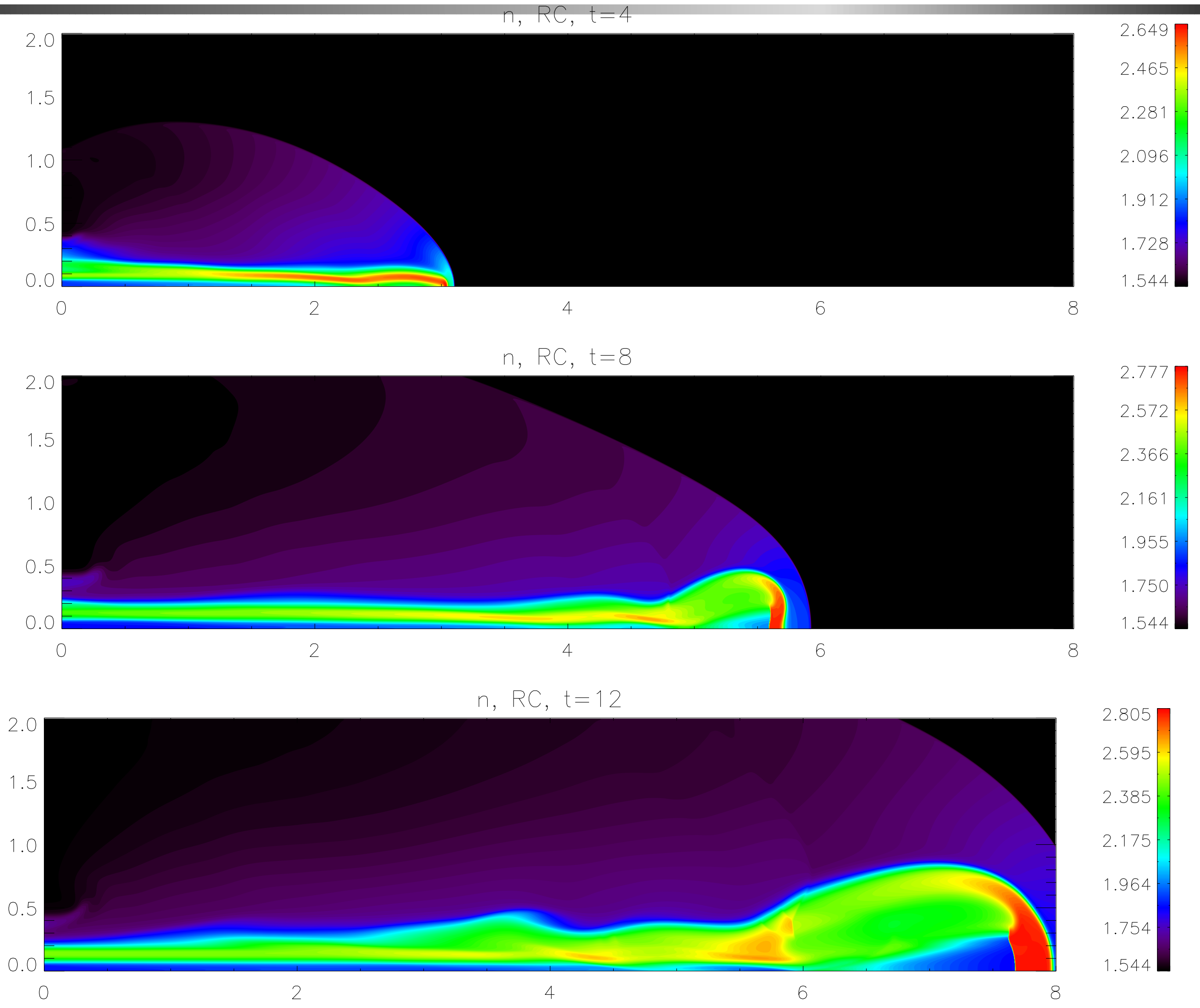
Relativistic JETS



Relativistic JETS

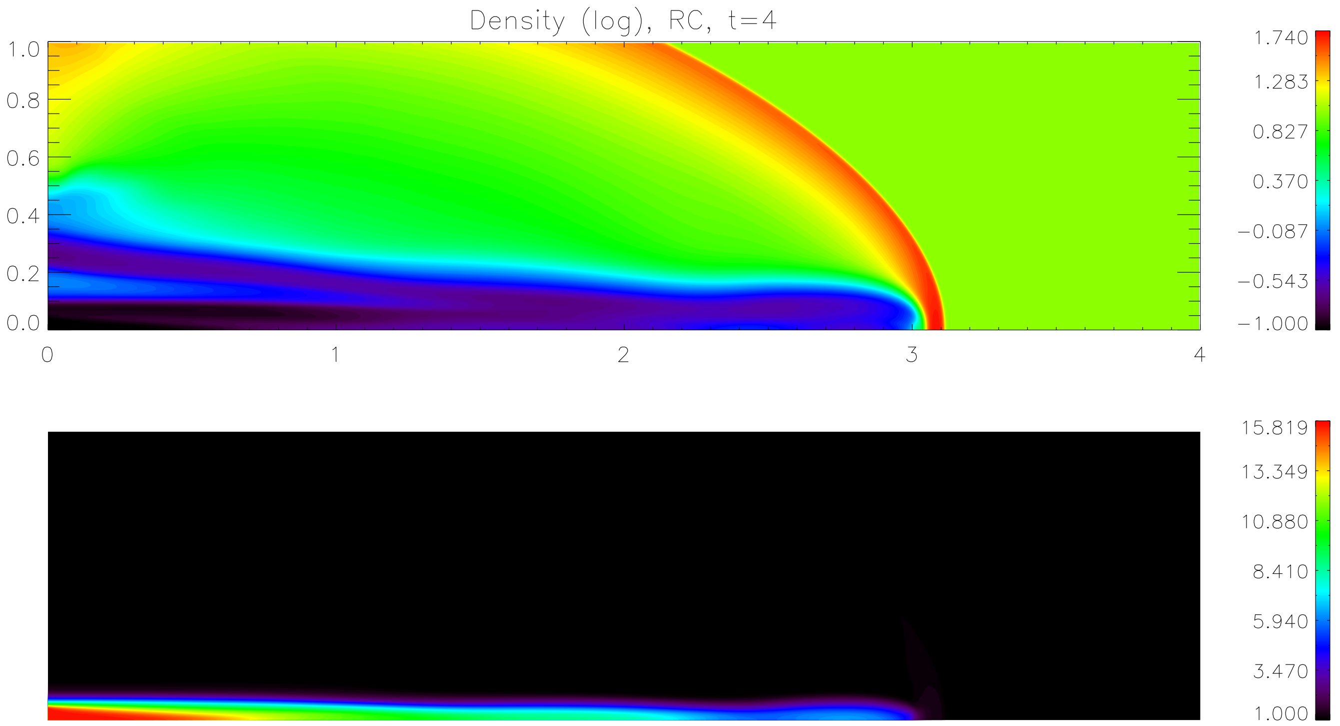
- Using ID EoS and some intermediate values of γ is not good, since γ generated by the interaction of the outflow with the the ambient medium

Relativistic JETS



- RC; $v_{xj} = 0.99$; n is variable and is neither 3 nor $3/2$

Relativistic JETS



- RC $v_{xi} = 0.998$, upper panel density and lower panel Γ

Concluding Remarks

- ID EoS is relativistically wrong.
- Considering temperature dependent adiabatic/polytropic index is important.
- Internal structure produced by RC is different.
- These internal shocks will produce a different spectrum.
- Injection variables derived by analyzing observational data will also be faulty if wrong EoS is chosen.
- The jet head moves slower or faster than that modeled by ID EoS.
- Adiabatic index depends on the thermodynamic properties of the jet, which in turn is determined by the complicated interaction with the ambient medium, thus cannot be assigned apriori.
- Our preliminary investigation suggests, it will be interesting to investigate propagation of jets through non uniform medium.