Hydrodynamics of molten droplet in millimeter scale

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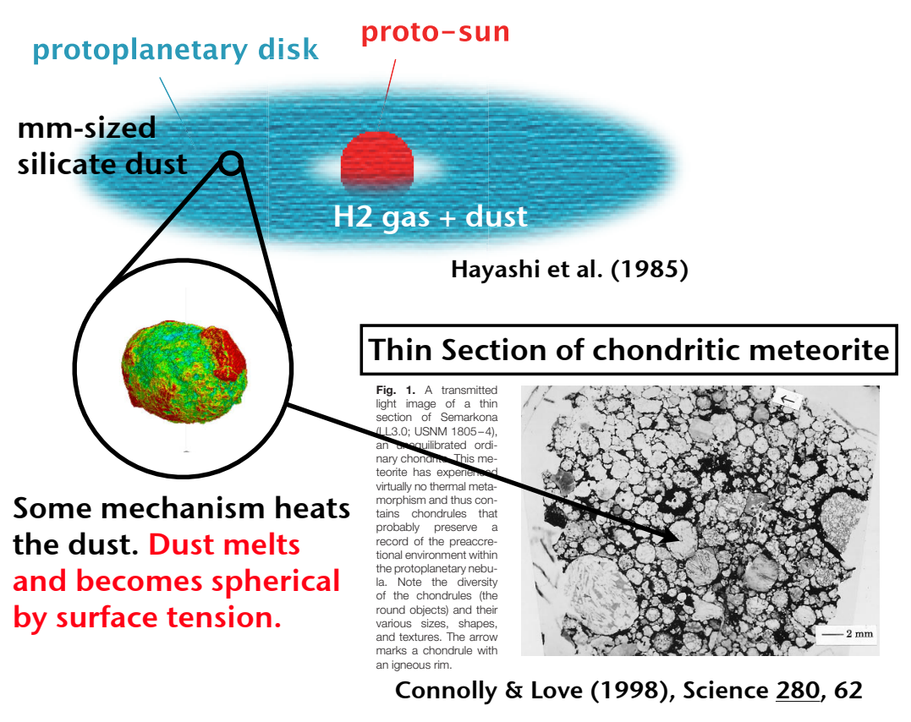
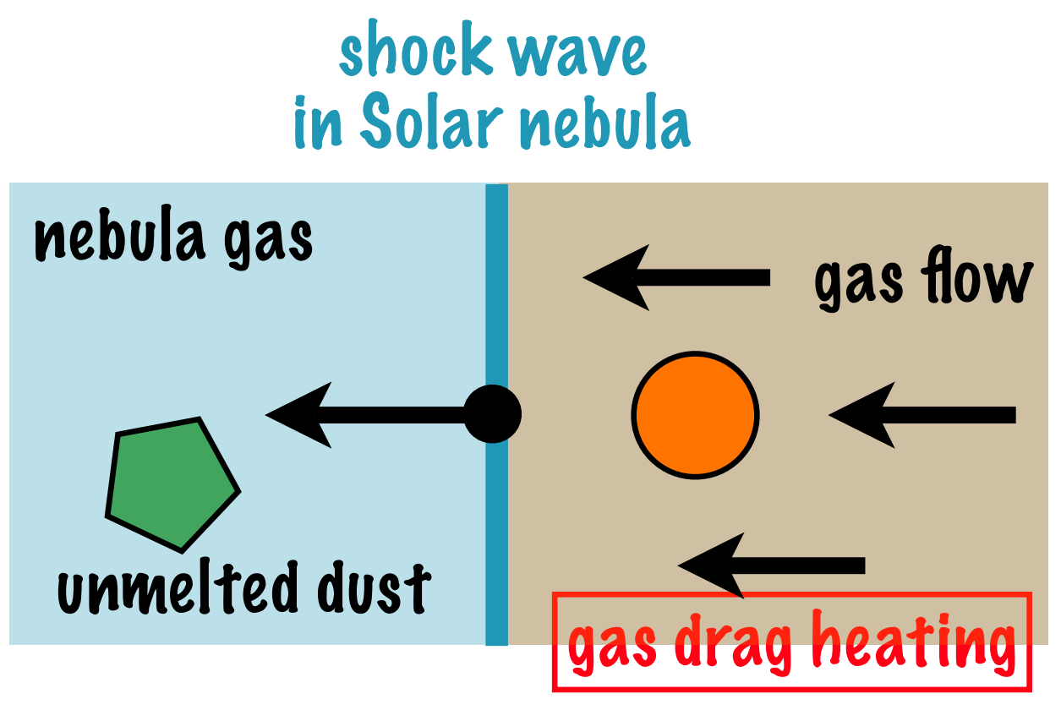
Abstract: Millimeter-sized, spherical silicate grains abundant in chondritic meteorites, which are called as chondrules, are considered to be a strong evidence of the melting event of silicate dust particles in the proto-planetary disk. One of the most plausible scenarios is that the chondrule precursor dust particles are heated and melt in the high-velocity rarefied gas flow (shock-wave heating model). The hydrodynamics of the molten silicate dust particles in the gas flow are very attractive issues: the internal flow tends to homogenize the chemical/isotopic abundances in the droplet, the surface deformation would relate to the external shapes of chondrules, the fragmentation by the gas drag force might determine the maximum sizes of chondrules, and so forth. The notable feature that is not seen in general astrophysical hydrodynamics is the incompressibility of the molten droplet. Therefore, we have to consider the multi-phase fluids (incompressible droplet and compressible ambient region). We developed the three-dimensional hydrodynamic code and simulated the hydrodynamics of millimeter-sized and incompressible molten silicate dust particles in the rarefied nebula gas flow. In the meeting, we plan to introduce the shock-wave heating scenario for chondrule formation and how interest the molten droplet hydrodynamics are.

Introduction

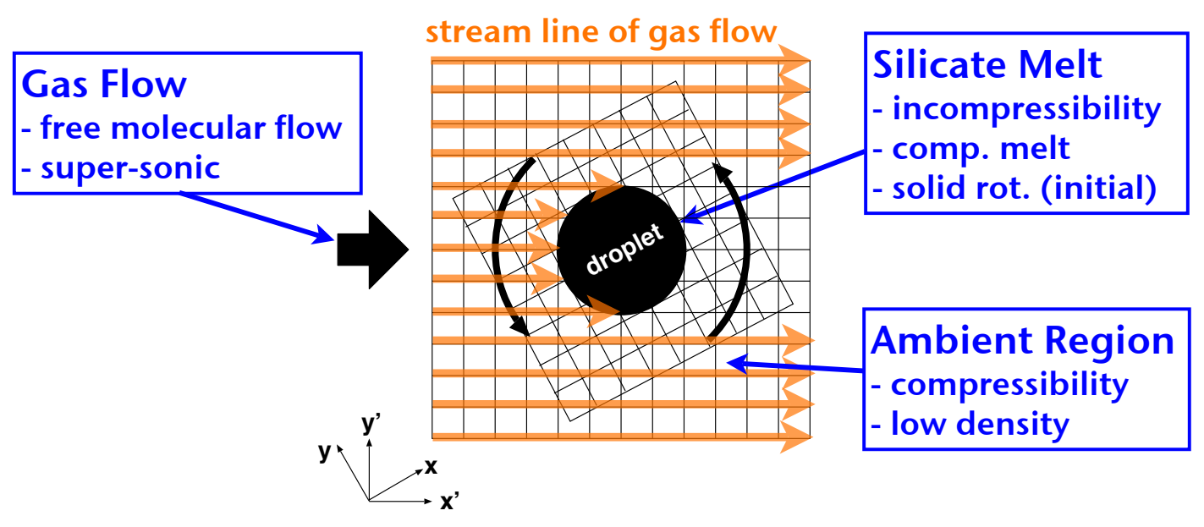
Chondrule?: Chondrules are millimeter-sized, once-molten, spherical-shaped grains mainly composed of silicate material. They are abundant in chondritic meteorites, which are the ma jority of meteorites falling onto the Earth. They are considered to have formed from chondrule precursor dust particles about 4.56 × 10^9 yr ago in the solar nebula (Amelin et al. 2002); they were heated and melted through flash heating events in the solar nebula and cooled again to solidify in a short period of time (e.g., Jones et al. 2000 and references therein).



Shock-Wave Heating Model: Let us suppose there is a gas medium containing dust particles with a dynamical equilibrium, i.e. they do not have a relative velocity initially. And let us suppose a shock wave passes the medium. Then, the gas is accelerated by the gas pressure and obtains some amount of velocity, while dust particles tend to remain the initial position. This causes the relative velocity between the dust particles and the gas. When the relative velocity is present, the frictional heating works on and melts the dust particles (e.g., Hood & Horanyi 1991, Iida et al. 2001, Desch & Connolly 2002, Ciesla & Hood 2002, Miura & Nakamoto 2006).



Hydrodynamics of mm-sized Droplet: In the shock-wave heating model, the molten droplet is exposed to the high-velocity rarefied gas flow. The ram pressure of the gas flow causes some hydrodynamical phenomena on a mm-sized incompressible droplet; deformation, internal flow, fragmentation, and so forth. These hydrodynamical behaviors should affect the physical properties of final products (chondrules). We developed the three-dimensional hydrodynamic code to simulate the hydrodynamics of a molten droplet exposed to a high-velocity rarefied gas flow.



Numerical Scheme

Basic Equations: Advection phase of Eq. of motion is solved by using the CIP scheme, which is one of the high-accurate advection solver (e.g., Yabe et al. 2001). It is difficult to solve the non-advection phase because the pressure p is a strong function of the density ρ because of large sound speed. The C-CUP scheme is one of the useful schemes to overcome this problem (Yabe & Wang 1991, see right side of this poster). Eq. of continuity is solved by the CIP-CSL2 scheme, which is the recent version of the CIP scheme and guarantees a mass conservation (e.g., Nakamura et al. 2001).

Eq. of continuity
$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0$$

Eq. of motion
$$\frac{\partial\boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=\frac{-\nabla p+\mu\nabla^2\boldsymbol{u}+\underbrace{\boldsymbol{F}_\mathrm{s}}_{\text{surface tension}}+\underbrace{\boldsymbol{F}_\mathrm{g}}_{\text{gas ram pressure}}}{\rho}+\underbrace{\boldsymbol{g}}_{\text{apparent gravity}}+\underbrace{\frac{1}{2}\nabla|\boldsymbol{\Omega}\times\boldsymbol{r}|^2}_{\text{centrifugal force}}+\underbrace{2\boldsymbol{u}\times\boldsymbol{\Omega}}_{\text{Coriolis force}}$$

Eq. of state
$$\frac{\partial p}{\partial\rho}=\underbrace{c_\mathrm{s}^2}_{\text{very large inside a droplet (incompressibility)}}$$

Incompressibility: In the incompressible fluid with large sound speed, a negative value of velocity divergence (compression) results into the extremely large increase of pressure. It makes the numerical solver unstable. To avoid this problem, we obtain a poisson equation for p(n+1) by combining eq. of motion and eq. of state (Yabe & Wang 1991). First, we solve the combined equation by appropriate method to obtain pressure p(n+1), and then, substituting p(n+1) to eq. of motion to obtain u(n+1).

(1) Non-advection phase
$$\boxed{\begin{aligned}\frac{\partial\boldsymbol{u}}{\partial t}&=-\frac{1}{\rho}\nabla p\\ \frac{\partial p}{\partial t}&=-\rho c_\mathrm{s}^2\nabla\cdot\boldsymbol{u}\end{aligned}}$$

▶

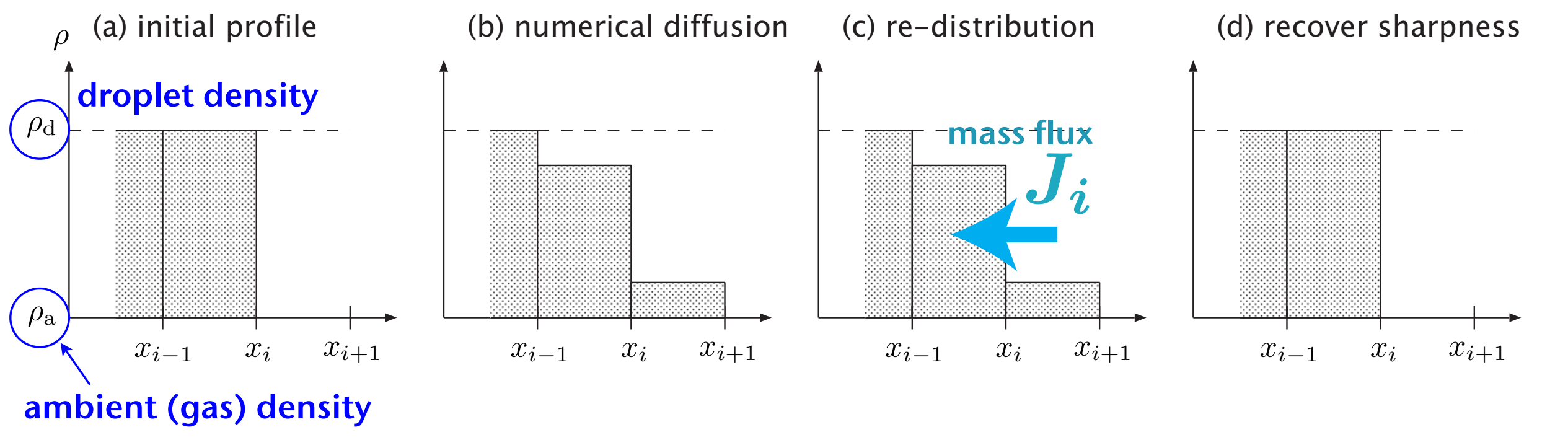
(2) after finite difference method
$$\boxed{\begin{aligned}\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^n}{\Delta t}&=-\frac{1}{\rho^n}\nabla p^{n+1}\\ \frac{p^{n+1}-p^n}{\Delta t}&=-\rho^n c_\mathrm{s}^2\nabla\cdot\boldsymbol{u}^{n+1}\end{aligned}}$$

▶

(3) combine two eqs.
$$\boxed{\nabla\cdot\left(\frac{\nabla p^{n+1}}{\rho^n}\right)=\frac{p^{n+1}-p^n}{\rho^n c_\mathrm{s}^2\Delta t^2}+\frac{\nabla\cdot\boldsymbol{u}^n}{\Delta t}}$$

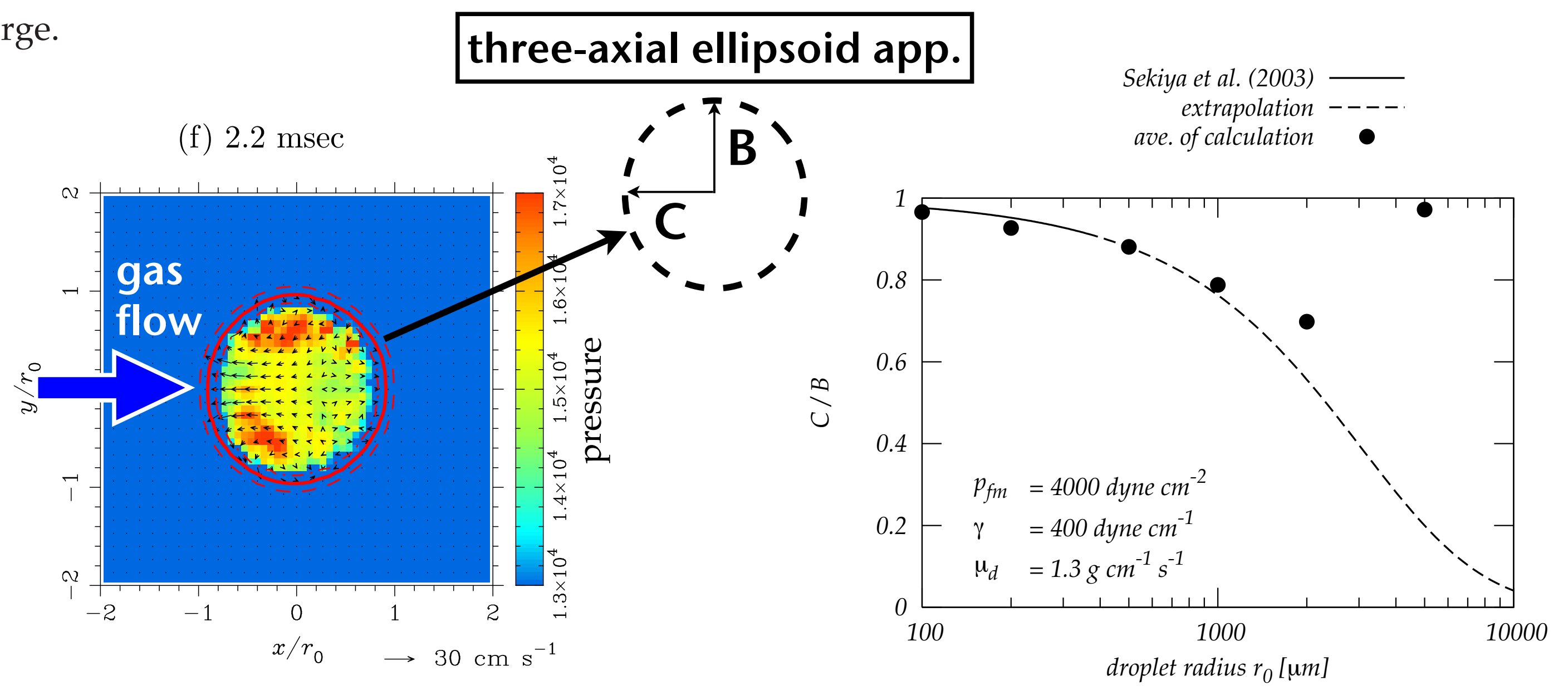
Solving above poisson equation for p^{n+1}, we obtain pressure for incompressible fluid.

Anti-diffusion: Discontinuity of the density profile at the interface between droplet and ambient region should be kept sharply (a). However, in general, the numerical advection on fixed Eulerian grids results into the numerical diffusion (b). To recover sharp discontinuity, we re-distribute the mass inside the cell from fewer one to more one (c). This anti-diffusion technique can prevent the numerical diffusion as the time step proceeds (d).


(a) initial profile (b) numerical diffusion (c) re-distribution (d) recover sharpness

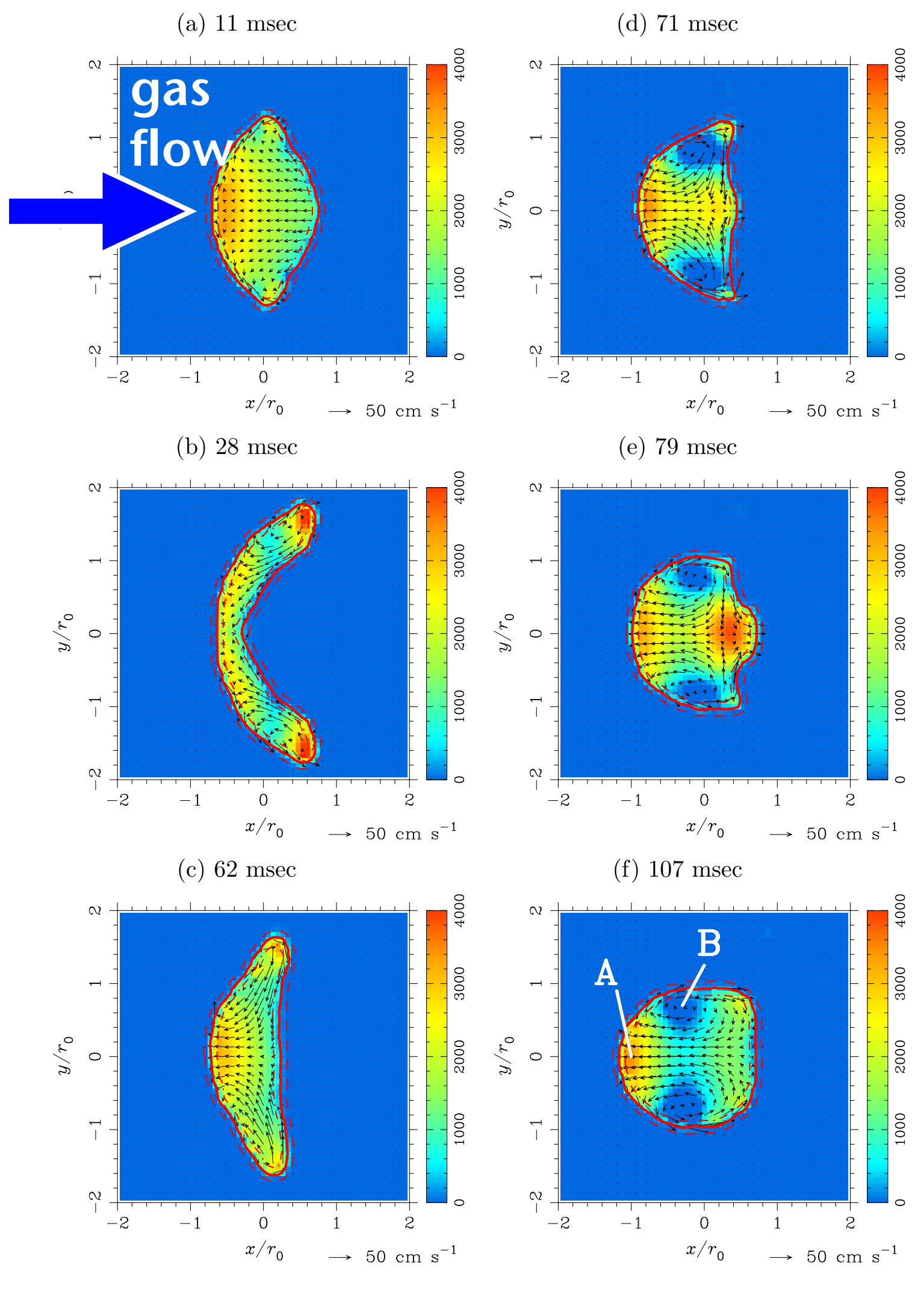
Results

Deformation: When the gas flow meets the molten droplet, the gas pressure deforms the external shape of droplet. We approximate the droplet shape as three-axial ellipsoid with radii of A, B, and C (A ≥ B ≥ C). In this case, it is found that A = B from symmetry. To confirm the accuracy of our results, we compare the axial ratio C/B with the analytic solution (Sekiya et al. 2003). They analytically derived the deformation and the internal flow of the droplet assuming that the non-linear terms of the hydrodynamical equations as well as the surface deformation are sufficiently small so that linearized equations are appropriate. We found that our simulation results match well with the analytic solution when the deformation is not so large.

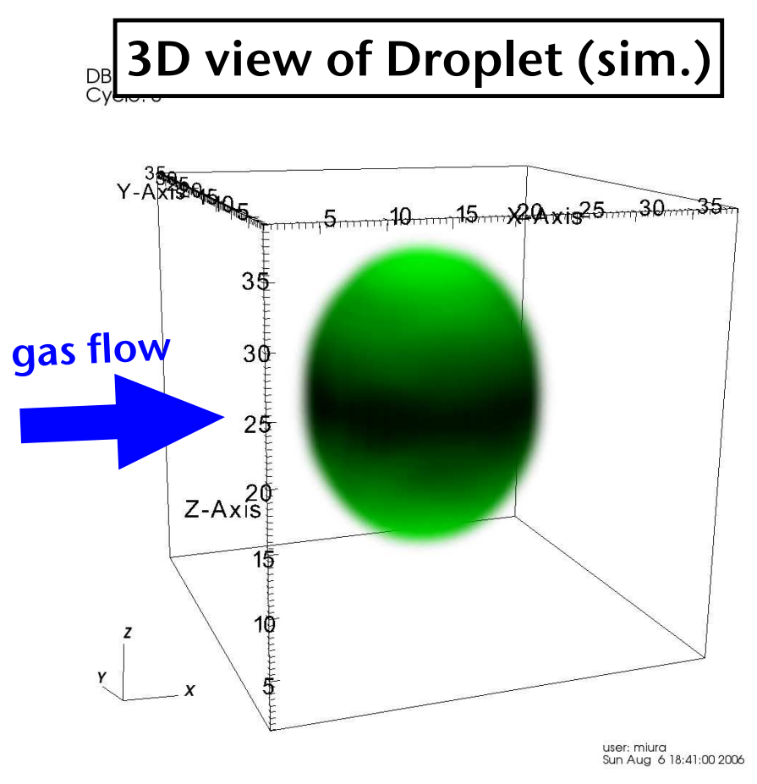

three-axial ellipsoid app.

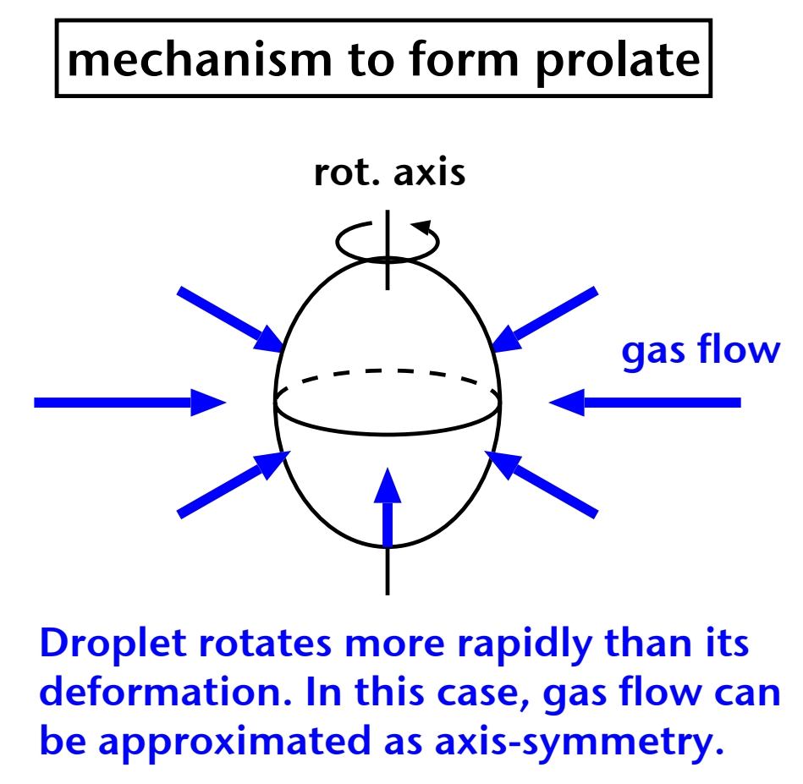
(f) 2.2 msec

Negative Pressure: The hydrostatic pressure is high at the part which is directly facing the gas flow (pointed by A) and extremely low around the center of eddies of the circulating internal fluid motion (pointed by B). Generally, it is considered that the boiling (or vaporization) would take place in any liquids where the vapor pressure of the liquid exceeds its hydrostatic pressure. Since the hydrostatic pressure at the center of eddies is almost zero, the boiling might occur at the center of eddies. It is well known as the cavitation.

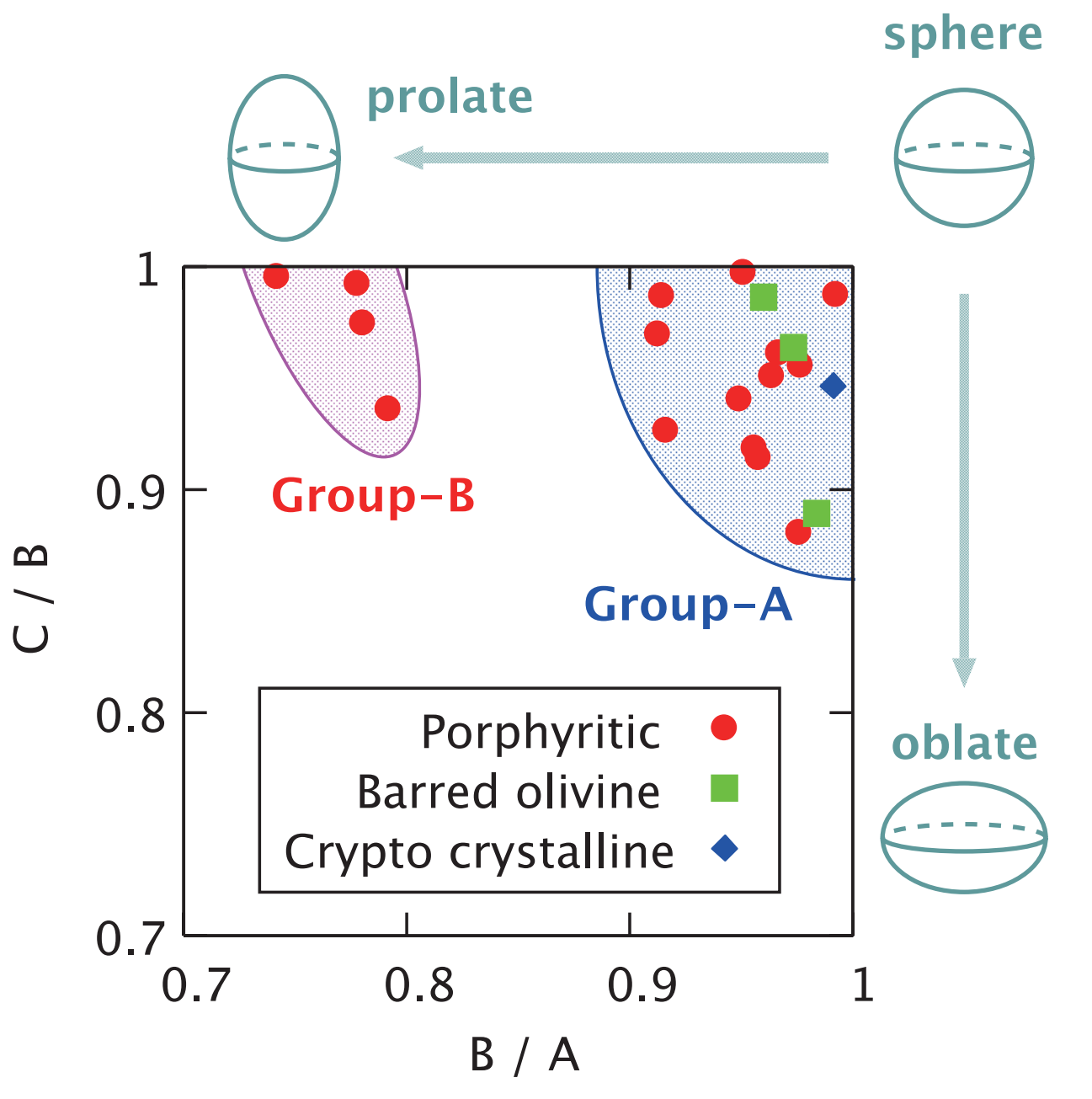

(a) 11 msec (d) 71 msec (b) 28 msec (e) 79 msec (c) 62 msec (f) 107 msec

Effect of Droplet Rotation: We consider that the droplet rotates around the z-axis, which is perpendicular to the direction of the gas flow. We found that the droplet shape becomes prolate when the rotation speed is much larger than the droplet deformation (high-viscosity) and is not large as the centrifugas force does not dominate. Actually, such prolate chondrules were observed (Tsuchiyama et al. 2003).

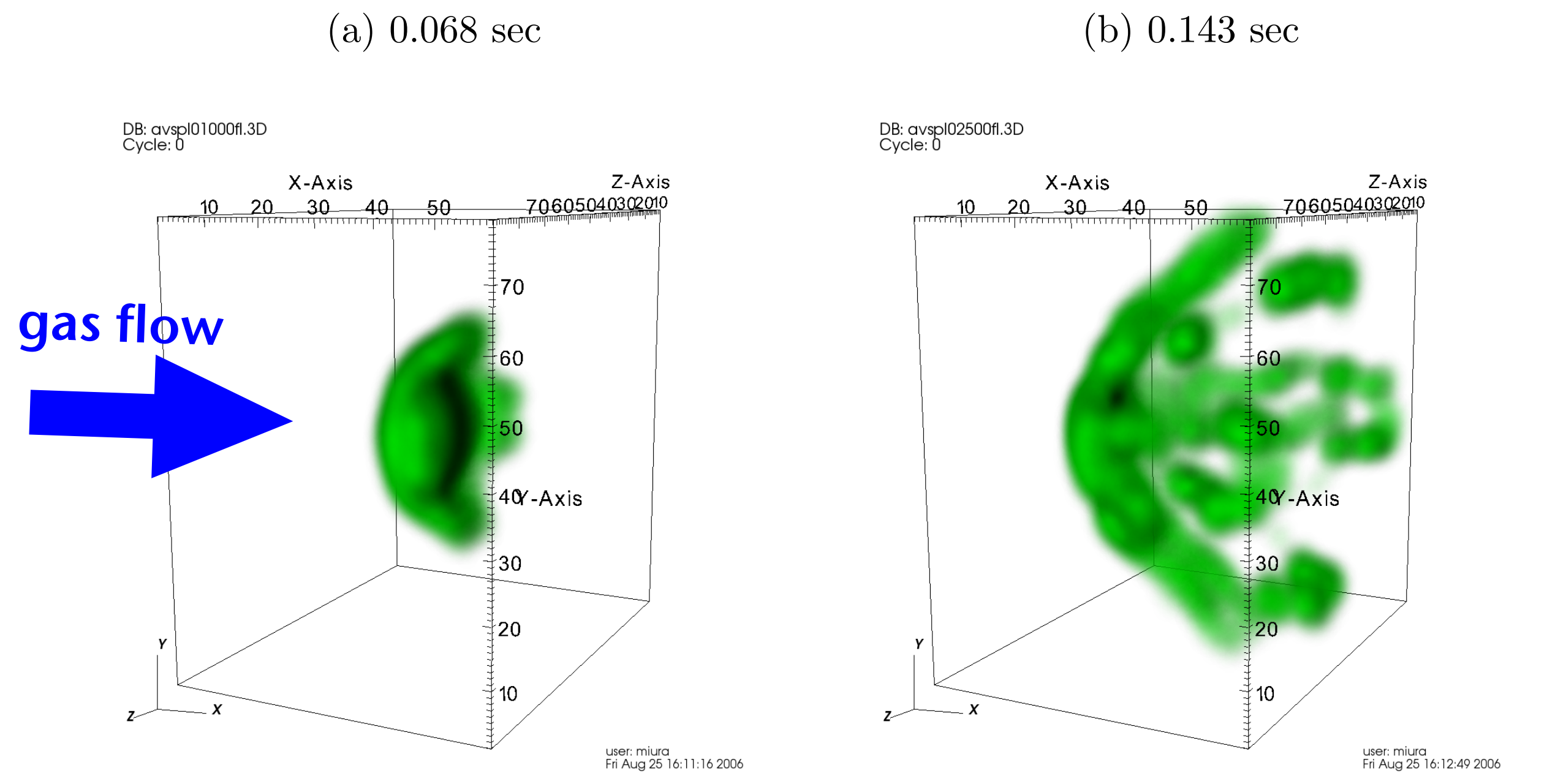

3D view of Droplet (sim.)


mechanism to form prolate

Droplet rotates more rapidly than its deformation. In this case, gas flow can be approximated as axis-symmetry.



Fragmentation: For the droplet with larger size in which the surface tension cannot keep the droplet shape against the gas ram pressure, the fragmentation will occur. The panel (a) shows the droplet shape just before the fragmentation. It is found that the droplet surface which is directly facing to the gas flow is stripped off backward. The panel (b) is just after the fragmentation. We found that the parent droplet breaks up to many smaller pieces. An upper limit of chondrule sizes might be determined by the fragmentation (Susa & Nakamoto 2002).


(a) 0.068 sec (b) 0.143 sec

Summary: We developed three-dimensional hydrodynamic code for multi-phase analysis (incompressible and compressible). We applied this code to the shock-wave heating model for chondrule formation and showed the importance of mm-sized hydrodynamics in the planetary formation scenario. In the future, we are planing to reveal the chondrule formation histories and physical properties by using the hydrodynamic simulation.

References: [1] Amelin et al. 2002, Science 297, 1678. [2] Jones et al. 2000, In Protostars and Planets IV, 927. [3] Hayashi et al. 1985, In Protostars and Planets II, 1100. [4] Hood & Horanyi 1991, Icarus 93, 259. [5] Iida et al. 2001, Icarus 153, 430. [6] Desch & Connolly 2001, Meteoritics and Planetary Science 37, 183. [7] Ciesla & Hood 2002, Icarus 158, 281. [8] Miura & Nakamoto 2006, ApJ 651, 1272. [9] Yabe et al. 2001, Journal of Computational Physics 169, 556. [10] Yabe & Wang 1991, Journal of Physical Society of Japan 60, 2105. [11] Nakamura et al. 2001, Journal of Computational Physics 174, 171. [12] Sekiya et al. 2003, Progress of Theoretical Physics 109, 717. [13] Tsuchiyama et al. 2003, Lunar and Planetary Science Conference, abstract #1271. [14] Susa & Nakamoto 2002, ApJ 564, L57.

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