Optimization of SPH

for Numerical Simulation of Subcluster Acquisition in Formation of Galaxy Clusters

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Abstract

We have investigated thermal evolutions of intracluster hot gas from a view point of the core size. In the observed core-size distribution there exist huge cores of >0.4 Mpc which deviate strongly from the self-similar relation. A possible origin of such huge cores is acquisitions of subgroups of galaxies or subclusters. In order to investigate the effects of the acquisitions on core sizes, we are preparing a combined SPH + N-body code. Critical physical processes of interest are the propagation of shock and its dissipation. Thus, first of all, we examine the detailed behavior of numerical viscosity in SPH, and optimize our code. We considered the propagation of spherical Taylor-Sedov blast wave and the acquisition of the subcluster. We confirmed that the standard Monaghan-Gingold viscosity tends to exaggerate the viscous heating, while the signal velocity form provides relatively reasonable results. The viscosity limiter do not always improve the viscosity, especially for limited particle simulations.

Introduction

According to the standard scenario of hierarchical structure formation in the universe, galaxy clusters are formed through a number of **mergers** of subclusters and/or **acquisitions** of subgroup of galaxies. X-ray observations have founded evidences of mergers (e.g., Maughan et al. 2003**[6]**; Belsole et al. 2004**[2]**): 1E0657-56, for example, is a typical cluster in which we can see the clear bow shock (Vikhlinin et al. 2001**[14]**).

It is, on the other hand, also known that the overall structure of the intracluster gas can be roughly explained by the beta-model, which is an isothermal hydrostatic gas model. This may suggest the natutre of clusters that the drastic major merger is not always occuring in the recent past, and clusters are roughly relaxed. It is actually rare to be found catastrophic cluster-cluster mergers. On the basis of the beta-model, in the observed core-size distribution **there exist huge cores of >0.4 Mpc which deviate strongly from the self-similar relation between the core and virial radii** (Akahori, Masai 2006**[1]**). A possible origin of such huge cores is the mergers or acquisitions. It is thus important to understand effects of minor mergers on the dynamical and thermal properties of the clusters as well as that of major mergers.

Hydrodynamical simulations of intracluster gas including cold dark matter have revealed formation processes of clusters in detail (e.g., Evrard 1990**[4]**). Compared with dark matter which interact only through the gravity, the calculation of intracluster gas is somewhat complex. In 1999, the Santa Barbara Cluster Comparison Project (Frenk et al. 1999**[5]**) discussed the systematic differences in 12 codes including 7 various algolisms such as smoothed particle hydrodynamics **(SPH)** or grid-base, from the experiment of the formation of Coma-like cluster from nearly the same initial condition. They found that **SPH systematically exhibits less central entropy floor compared with the grid-base codes.** A primary cause for this disagreement may be the inconservative of entropy in SPH (Hernquist 1993**[9]**), and which can be improved by the optimization of the momentum equation: **full conservative formulation** (Springel & Hernquist (2002)**[11]**; see also Springel 2005**[12]** (GADJET-II)).

Critical physical processes in the mergers or acquisitions are thought to be the accreation (Vilialization) shock and viscosity. Monaghan (1997)**[8]** noted that classical Monaghan & Ginglod (1983)**[7]** viscosity sometimes **overestimates the shear viscosity**, and proposed the Signal velocity form of viscosity, when some authors customize the viscosity by adding the **viscosity limiter** (e.g., Steinmetz 1996**[13]**). Dolag et al. (2004)**[3]** showed that **the strength of turbulences in the simulated clusters depends strongly on the viscosity form**.

In the present paper, for the future study about the effects of subcluster acquisition by using a combined **N-body**+**SPH** code, we calculate the propagation of spherical Taylor-Sedov blast waves and the acquisition of subclusters, and examine the detailed behavior of the numerical viscosities in SPH.

Viscosity in SPH

SPH is a grid free Lagrangean code. Density at the position r_i is expressed as:

$$\rho(r_i) = \sum_{j=1}^{N} m_j \overline{W}_{ij}$$

where W is the smoothing kernel:

$$\overline{W}_{ij} = \frac{1}{2}\left[W(r_{ij},h_i) + W(r_{ij},h_j)\right]$$

$$W(r,h) = \frac{8}{\pi h^3}\begin{cases} 1-6(r/h)^2+6(r/h)^3 & : 0 \le r/h < 1/2 \\ 2[1-(r/h)]^3 & : 1/2 \le r/h < 1 \\ 0 & : 1 \le r/h \end{cases}$$

averaged by particle i and j (symmetrized expression). m_j is the mass of i's particle with

$$m_i = \eta \rho_i h_i^3$$

(η is a constant). The momentum equation for i's particle is given by

$$\frac{dv_i}{dt} = -\frac{1}{\rho_i}\nabla_i P_i + a_i^{visc}$$

where the first and second terms give the pressure gradient and artificial viscosity, respectively (we skiped the notation of the gravity term). In SPH, we have

$$\frac{dv_i}{dt} = -\sum_{j=1}^{N} m_j\left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right)\nabla_i \overline{W}_{ij} - \sum_{j=1}^{N} m_j \Pi_{ij}\nabla_i \overline{W}_{ij}$$

or the full conservative formulation:

$$\frac{dv_i}{dt} = -\sum_{j=1}^{N} m_j\left(f_i \frac{P_i}{\rho_i^2}\nabla_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2}\nabla_i W_{ij}(h_j)\right) - \sum_{j=1}^{N} m_j \Pi_{ij}\nabla_i \overline{W}_{ij}$$

$$f_i = \left(1 + \frac{h_i}{3\rho_i}\frac{\partial \rho_i}{\partial h_i}\right)^{-1}$$

The Monaghan-Gingold (MG) value is the standard numerical viscosity:

$$\Pi_{ij} = \frac{-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2}{\rho_{ij}} \qquad \mu_{ij} = \frac{\overline{h}_{ij} v_{ij}\cdot r_{ij}}{r_{ij}^2 + \varepsilon \overline{h}_{ij}^2}$$

where the α term is proportional to the velocity differential, being consistent with the shear and bulk viscosity in the Navier-Stokes (Watkins et al. 1996[15]; see also Meglicki et al. 1993[10]). Quadratic β term is corresponding to the von Neuman Richtmyer viscosity, which works in high-M flow.

Bulk viscosity is primary produced by the α term, and this sometimes oversmooth the post-shock turbulence according to its overestimation. Monaghan (1997) 【7】 proposed the modified viscosity as

$$\Pi_{ij} = \frac{-\alpha v_{ij}^{sig} w_{ij}}{2\overline{\rho}_{ij}} \qquad w_{ij} = \frac{v_{ij}\cdot r_{ij}}{|r_{ij}|}$$

where v^{sig} is the signal velocity (S):

$$v_{ij}^{sig} = c_i + c_j - 3w_{ij}$$

The difference compared with the MG viscosity is that this viscosity is expressed in the first order of h/r, which is better for small pair separation.

To modify the overestimation of the shear viscosity, some authors apply the empirical efficiency, the viscosity limiter (L):

$$f_i = \frac{|\nabla_i \cdot v_i|}{|\nabla_i \cdot v_i| + |\nabla_i \times v_i|}$$

We multiply the viscos tensor by the geometrical mean of i and j's particle as

$$f = (f_i + f_j)/2$$

Finally, to avold the irregular viscous cooling, we set the conditions to work:

$$v_{ij}\cdot r_{ij} < 0 \qquad \dot{\rho}_i > 0$$

(particles approaches with each other, and density increases).

The energy equation of i's particle is given by

$$\frac{du_i}{dt} = -\frac{1}{2}\sum_{j=1}^{N} m_j\left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) v_{ij}\cdot\nabla_i \overline{W}_{ij}$$

In the full conservative formulation, instead of the above we use the entropy equation as

$$\frac{da_i}{dt} = -\frac{1}{2}\frac{\gamma-1}{\rho_i^{\gamma-1}}\sum_{j=1}^{N} m_j \Pi_{ij} v_{ij}\cdot\nabla_i \overline{W}_{ij}$$

where

$$P = a(s)\rho^{\gamma}$$

$$u = a(s)\rho^{\gamma-1}/(\gamma-1)$$

Exp.1 Point-like Energy Injection

- ★Uniform gas, negligible temperature, no gravity
- ★Thermal energy (E=1 in norm. unit) is injected at the center
- ★Comparison with the Taylor-Sedov solution
- ★N~381,000 (h=0.050@ρ=1 in norm. unit)
- ★Runge-Kutta Gill (4th.), double precision
- ★Fixed timestep dt=2.2E-5 (in norm. unit) < physical timescales
- ★Barnes-Hut Tree for neighbor searching

Physical timescales
Free-fall time
2-body relaxation time
Sound crossing time for r_{ij}
Sound crossing time for h
v_{ij} crossing time for r_{ij}
v^{sig}_{ij} crossing time for r_{ij}
Int. energy / power of work
Int. energy / power of viscous heat

Tab.1: Physical timescales.

Labels
MG: Monaghan-Gingold, α=1.0, β=2α
S: Signal velocity form, α=1.0
L: viscosity limiter
T: α=3.0

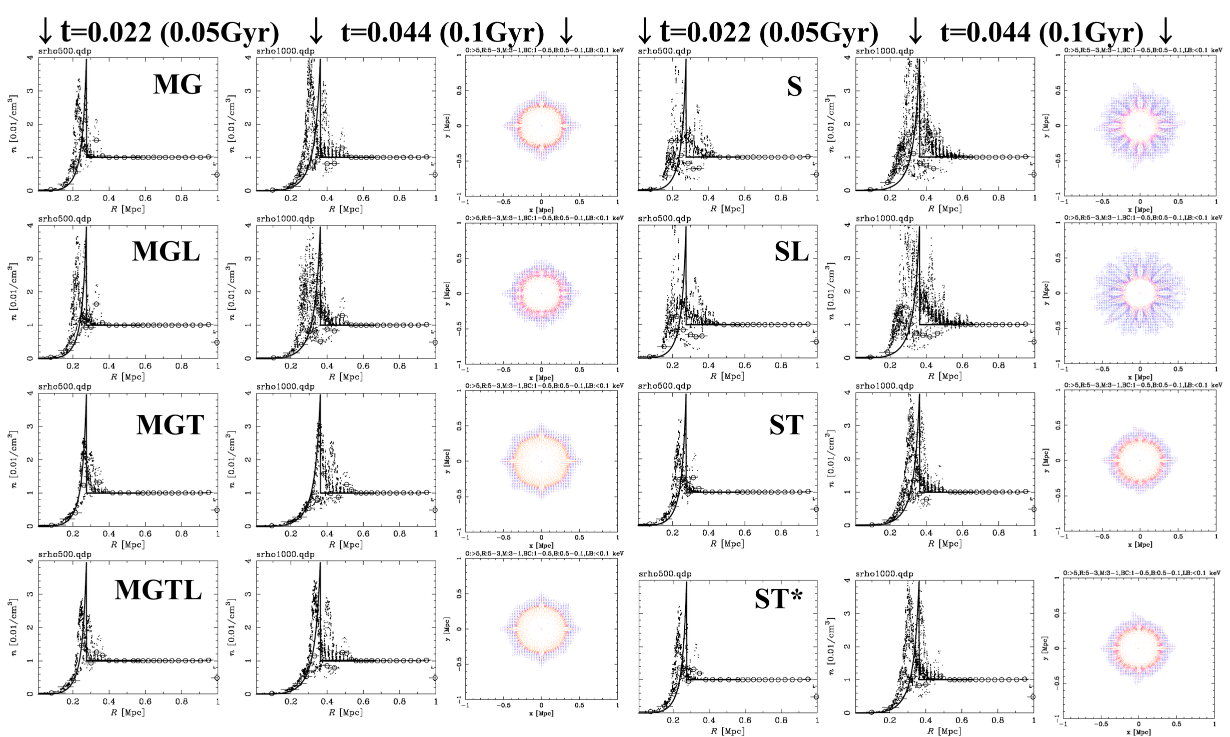

Fig.1: Particles distribution in 0.1× 0.1 box, and 2-D temperature map.

ST*: Leapflog+Runge-Kutta 2nd.

Result The strong blast wave with the Mach number M~1.5 (t=0.022) and M~2-3 (t=0.044) arises. Some ghosts in front of the the shock are peculiar waves which propagate on-axis. As a result, the viscosities with the standard value (α=1.0) can hardly reproduce the propagation of the shock front. MGT viscosity (α=3.0) describes the accurate propagation of the shock front in time, as well as the density structure behind the wave. The broadening of ~ 2-3 h of the shock front is typical in SPH simulations. As for ST viscosity, the propagation in time is marginally described; it may be improved if we apply a larger α. The viscosity limiter, L, produces ~10% higher maximum density at the shock. But penetrations of some particles and/or the delay of the propagation are also seen. Note that we have nearly the same result in the following cases: (1) when we use the timestep twice as large as the present test, (2) when we use the second-order time integration scheme (Leapflog + Runge-Kutta 2nd.).

Exp. 2 Subcluster Acquisition

- ★Main cluster: R_{vir} =3 Mpc, $M_{vir}=10^{15}\ M_{\odot}$, DM: King model, Gas: beta-model (β=2/3), $\rho_0=0.02\ cm^{-3}$, $R_c=0.2$ Mpc, Mass ratio 6:1 ($f_{gas}=0.143$)
- ★DM=100,000, SPH=100,000 (h=0.039 Mpc@ρ_0)
- ★Sub cluster: similar shape, mass 1/60, radius 1/2
- ★Init. Infall v=433km/s, from d=4 Mpc
- ★Variable timestep determined by 1/2 of the minimum physical timescales
- ★BH Tree (θ=1.0, quadrupole), systematic error < 1%
- ★Gravitational softening $\nabla\Phi \propto 1/(r^2 + (0.1h)^2)$

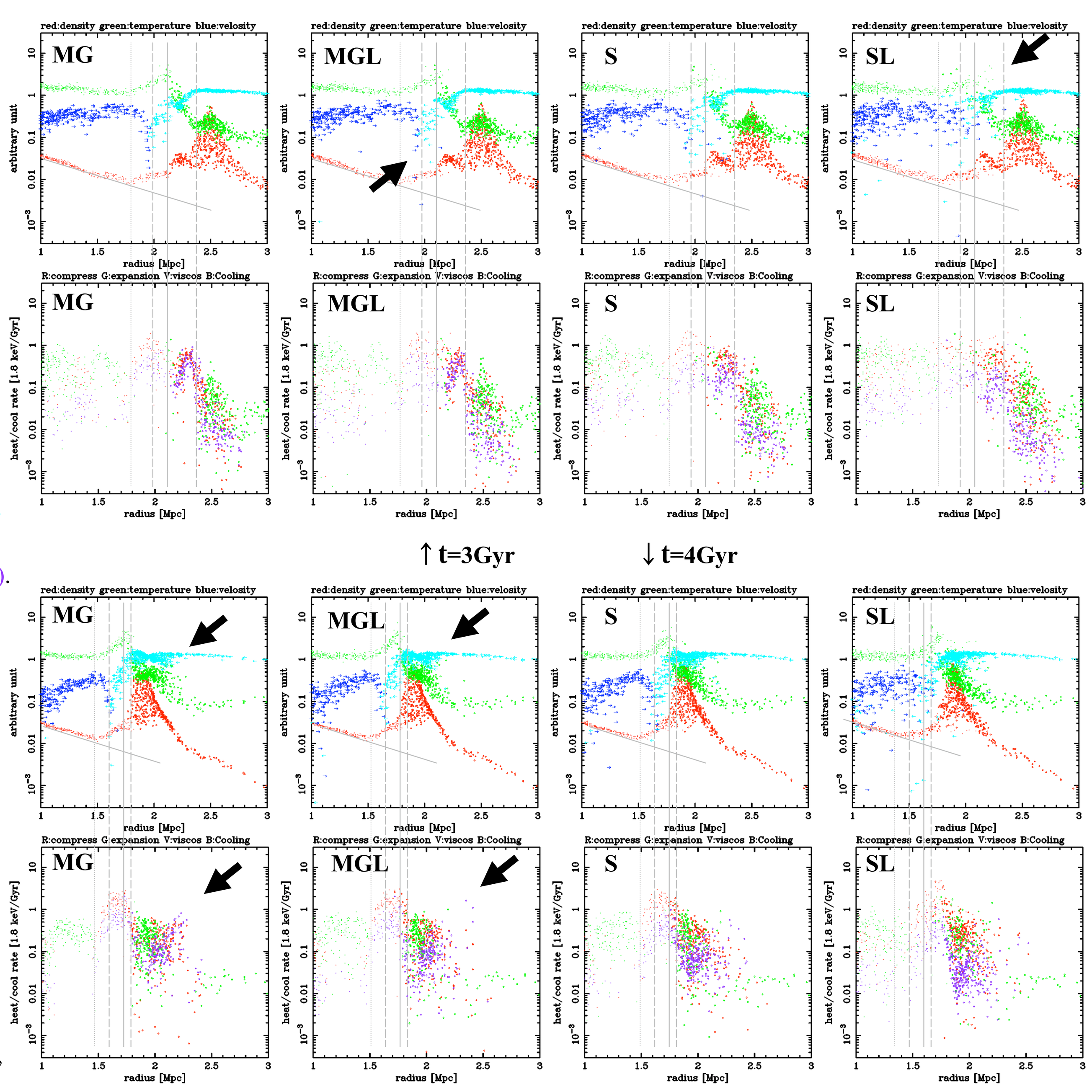

Fig.2: Properties of particles in 0.2×0.2 Mpc box at t=3.0 (2 tops) and 4.0 Gyr (2 bottoms). The upper panels represent the density (red), temperature (green), radial outflow velocity (blue), and inflow veocity (aqua). The lower panels represent the adiabatic compression heating (red), adiabatic expansion cooling (green), and viscous heating (violet). The additional lines are rough measures for the contact interface (solid), shock front (dashed), and smoothed front area (dotted).

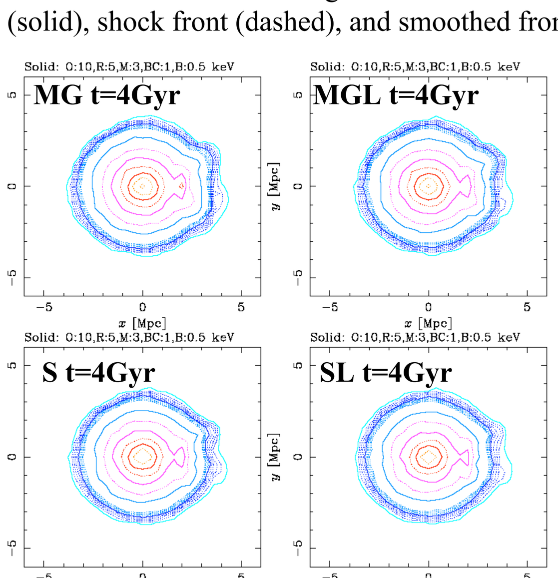

Fig.3: Emission-weighted temperature map at t=3.0 and 4.0 Gyr. The solid red, magenta, and blue lines represent the contour of 5, 3, 1 keV, respectively.

Result MG viscosity held the good convergence of particles. But at 4Gyr, the velocity of particles at ~2.2 Mpc decreased due likely to the viscous heating about ten times as large as that in case S. It is expected for the small bulk velocity differential in the inflow region, so that the heating may be caused by **the overestimation of the shear viscosity**. **MGL viscosity** produced the relatively scattering distribution in the velocity and temperature compared with case MG. This can be understood that **for the limited particle simulation it is enphasized the demerit of scattering rather than the merit of sharpening.** **S viscosity** showed somewhat unclear interfaces and penetrations of some particles. But at 4Gyr, **the sharp interfaces are reproduced and there is no overheating at ~2.2 Mpc. SL viscosity produced scattering distributions and penetrations.**

The above varieties of individual properties of particles significantly affect the smoothed values that we refer to for the comparison with other simulations or observations. For example, Fig. 2 represents the emission-weighted temperature maps for the simulated clusters. **The difference of the viscosities produces the difference of several tens percent in the temperature,** in the simulation of the subcluster acqusition.

Conclusion

We confirmed the visible difference in the properties of the merging gas due to the difference of numerical viscosities. The standard Monaghan-Gingold viscosity provides solid viscous heating with the convergence of particles, but the viscosity causes the deceleration of some particles, overestimating the shear viscosity. The viscousity limiter causes scattering of the particle distribution, and does not improve the viscosity at least our limited particle simulations. We conclude that the signal velocity form without the viscosity limiter is a better choice for the simulation of the subcluster acquisition.

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