# Discriminating Primordial Isocurvature Perturbations using 21 cm Observations

Toyokazu Sekiguchi
Nagoya Univ.

Ref: M.Kawasaki (ICRR), TS, T.Takahashi (Saga Univ.)
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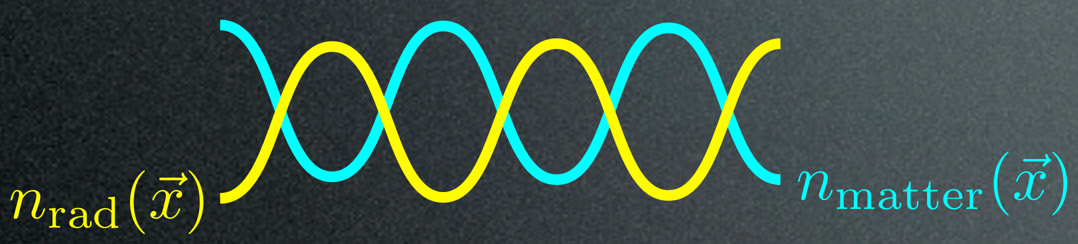
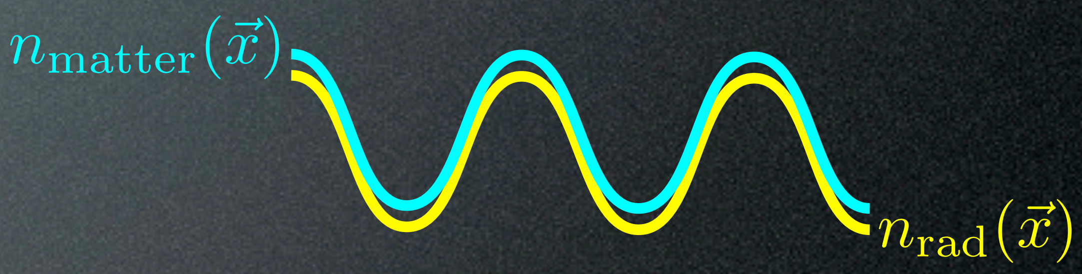
# Isocurvature perturbations

- Initial perturbation for structure formation

isocurvature (entropy) perturbation




adiabatic (curvature) perturbation




- CDM isocurvature (CI) $S_{\rm CI} = \delta_{\rm cdm} - \frac{3}{4}\delta_\gamma$
- baryon isocurvature (BI): $S_{\rm BI} = \delta_{\rm b} - \frac{3}{4}\delta_\gamma$

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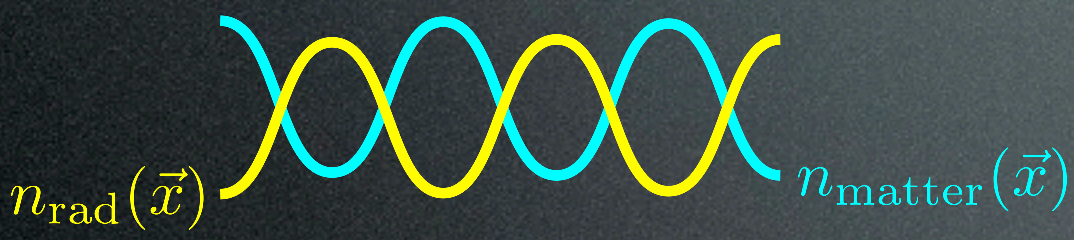


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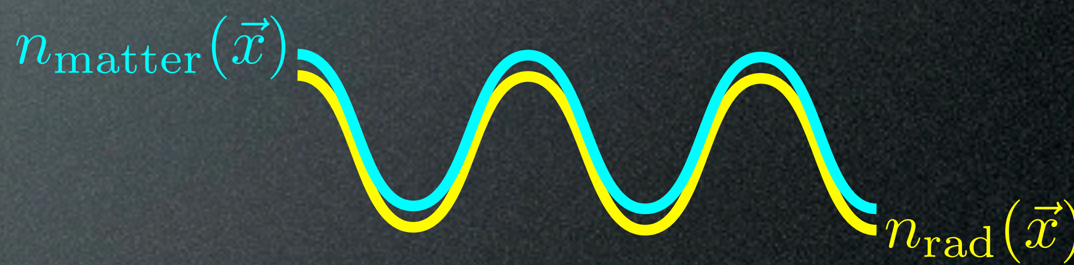


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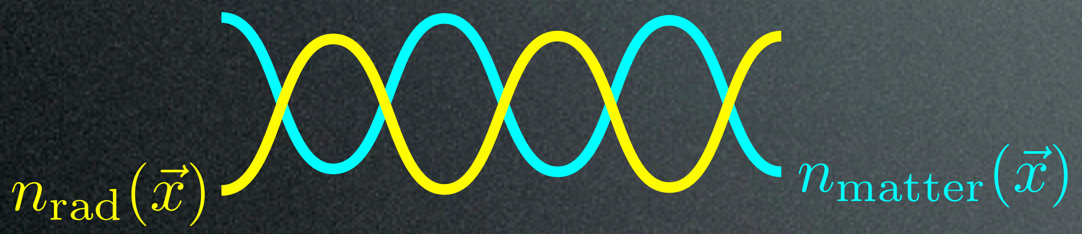


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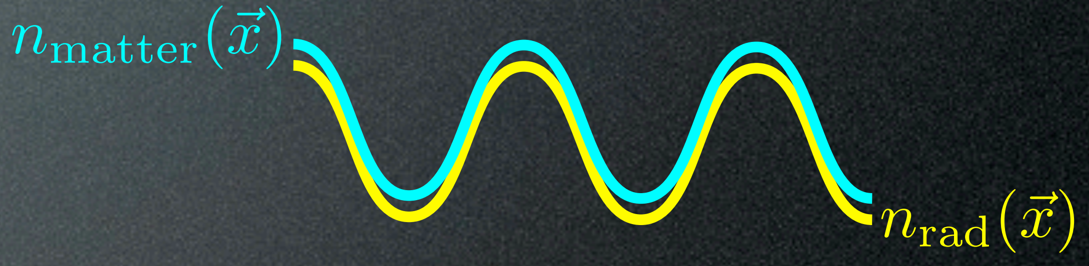


- Models for isocurvature perturbations
  - axion model » CI
  - Affleck-Dine mechanism » BI
  - curvaton scenario » CI, BI

→ probe for generation mechanisms of CDM/baryon

# Can we distinguish CI/BI?

- CMB and other observations of total matter or metric perturbations are only sensitive to the total isocurvature perturbation $S_m$.

$$S_m = \frac{\Omega_{\rm cdm}}{\Omega_m} S_{\rm CI} + \frac{\Omega_b}{\Omega_m} S_{\rm BI}$$

- We need observations that are sensitive to $\delta_b$ in the form other than $\delta_m$.

- 21 cm line observation is promising!

- We investigate to what extent CI/BI can be distinguished with future 21 cm surveys.

target redshifts: z>30

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- 21 cm line observation is promising!
- We investigate to what extent CI/BI can be distinguished with future 21 cm surveys.

target redshifts: z>30 ← At lower redshifts

- $\delta_b \approx \delta_{\rm cdm}$
- nonlinear gas physics

# Redshifted 21 cm line

- 21 cm line emission/absorption

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- Spin temperature $T_s$

$$\frac{n_{\rm triplet}}{n_{\rm singlet}} = 3\exp\left[-\frac{T_{\rm 21cm}}{T_s}\right]$$

$$T_{\rm 21cm} \equiv E_{\rm 21cm}/k_B \simeq 68{\rm mK}$$

# Redshifted 21 cm line

- 21 cm line emission/absorption


Observed at
$\nu = \nu_0/(1+z)$

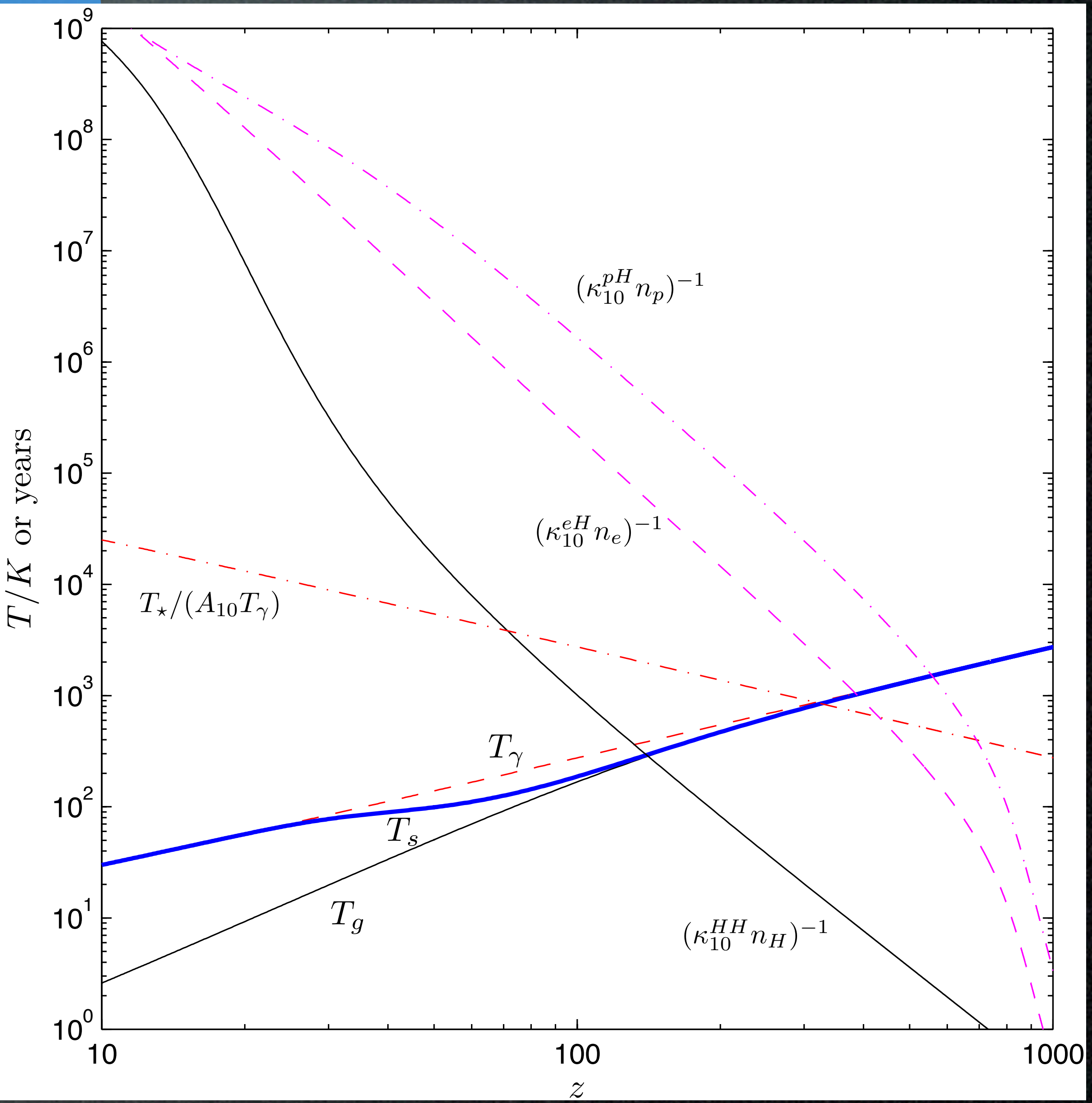
- Spin temperature $T_s$

$$\frac{n_{\rm triplet}}{n_{\rm singlet}} = 3\exp\left[-\frac{T_{21\rm cm}}{T_s}\right]$$

$$T_{21\rm cm} \equiv E_{21\rm cm}/k_B \simeq 68\text{mK}$$

Evolution is determined by:

- atomic collision (H-H, H-p, H-e)
- Thomson scattering with CMB
- ~~scattering of UV photons~~




Lewis & Challinor 2007

# Redshifted 21 cm line

- 21 cm line emission/absorption

HI gas
at redshift z

Observed at
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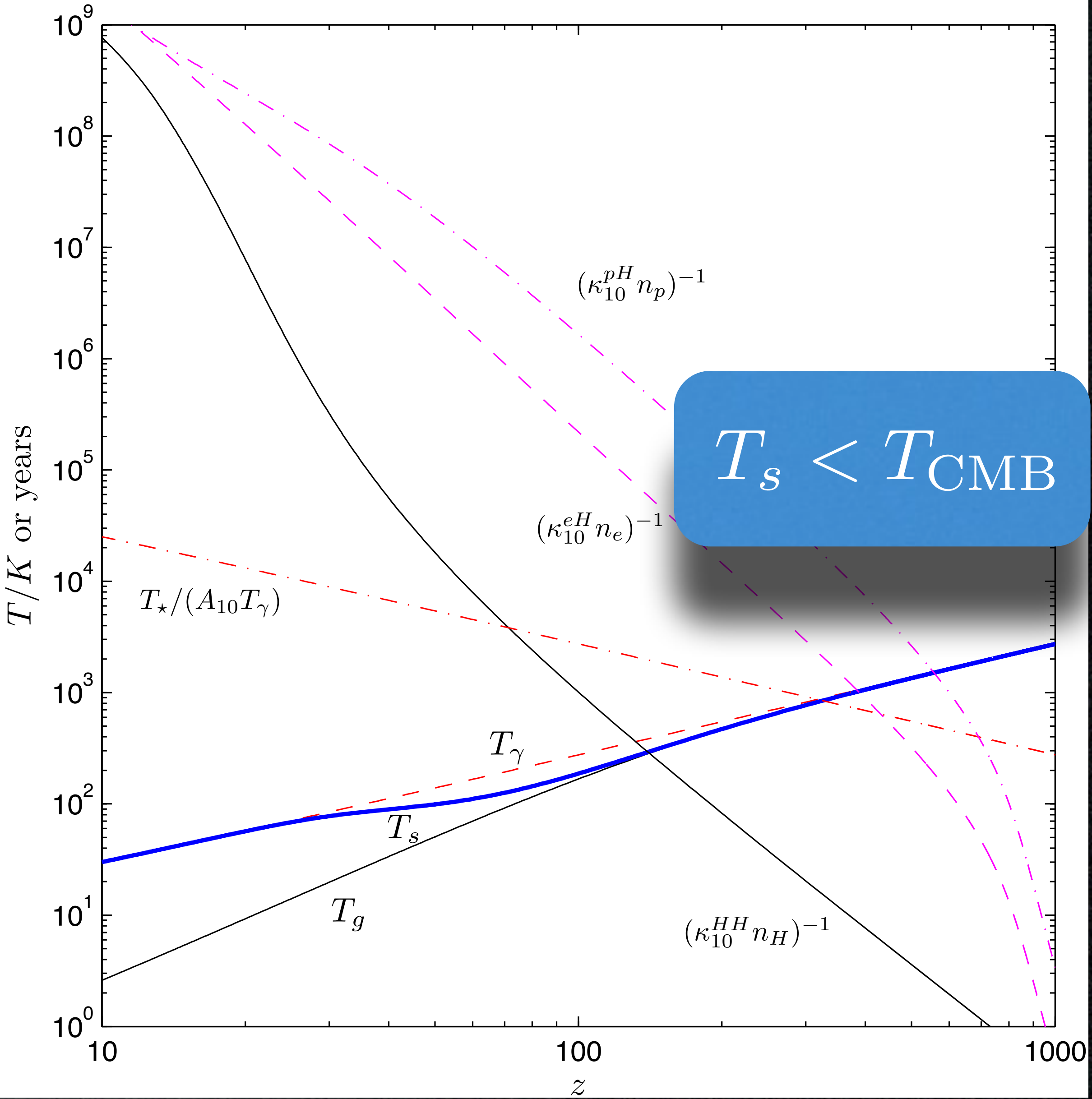


$T_s < T_{\rm CMB}$

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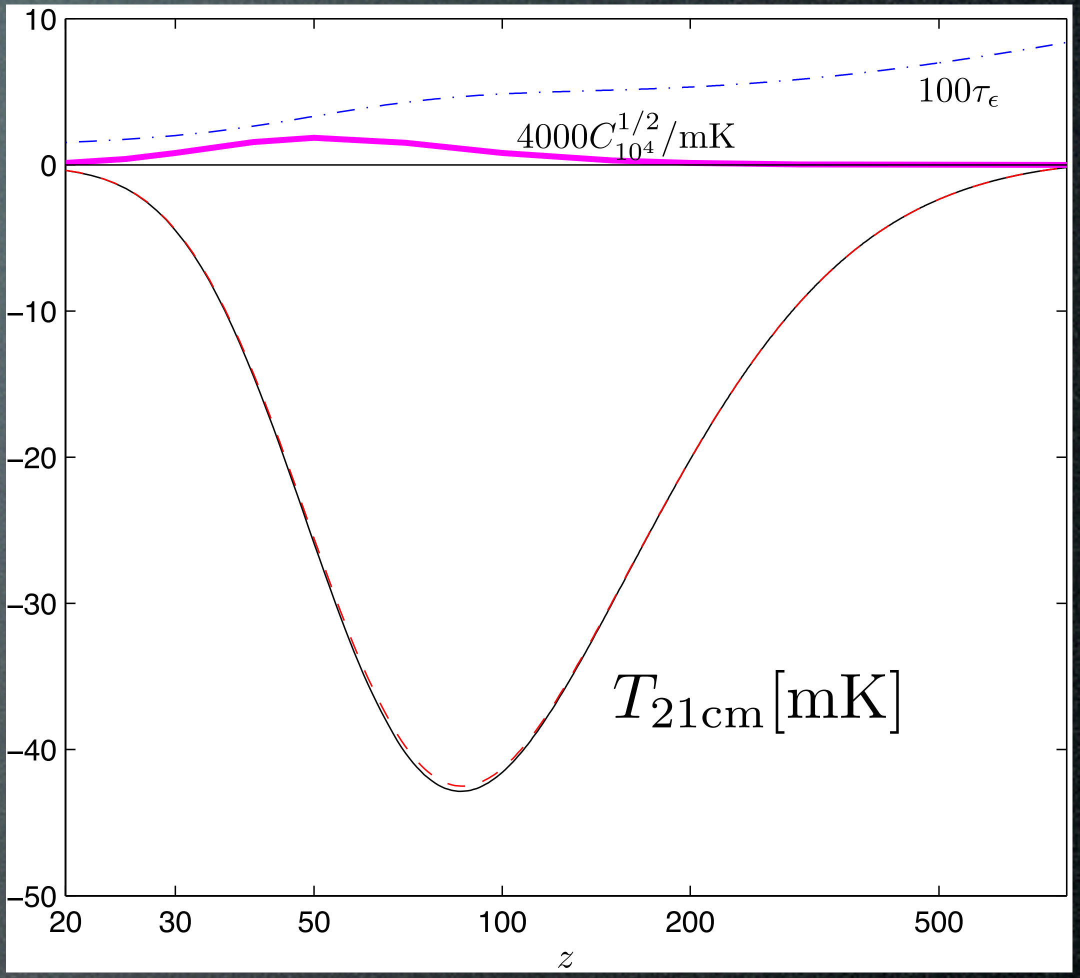
# 21 cm brightness temperature

- Brightness temperature

$$T_{21\mathrm{cm}} \equiv \frac{T_s - T_{\mathrm{CMB}}}{1+z}\tau_{21\mathrm{cm}}$$

optical depth

$$\tau_{21\mathrm{cm}}(z) = \frac{3\lambda_0^2 hcA_{10}n_{\mathrm{HI}}(z)}{32\pi k_B T_s(z)H(z)}$$

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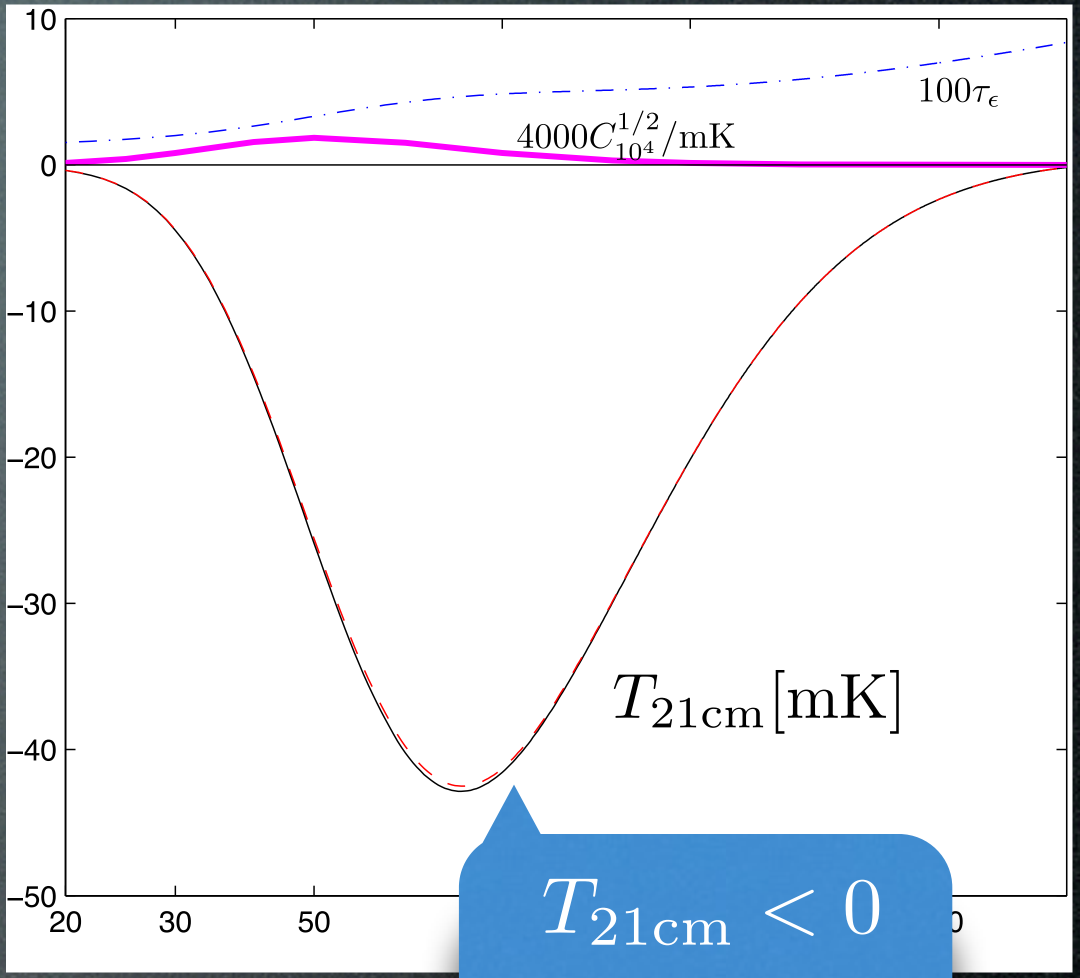


$T_{21\mathrm{cm}} < 0$
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Lewi[...] 07

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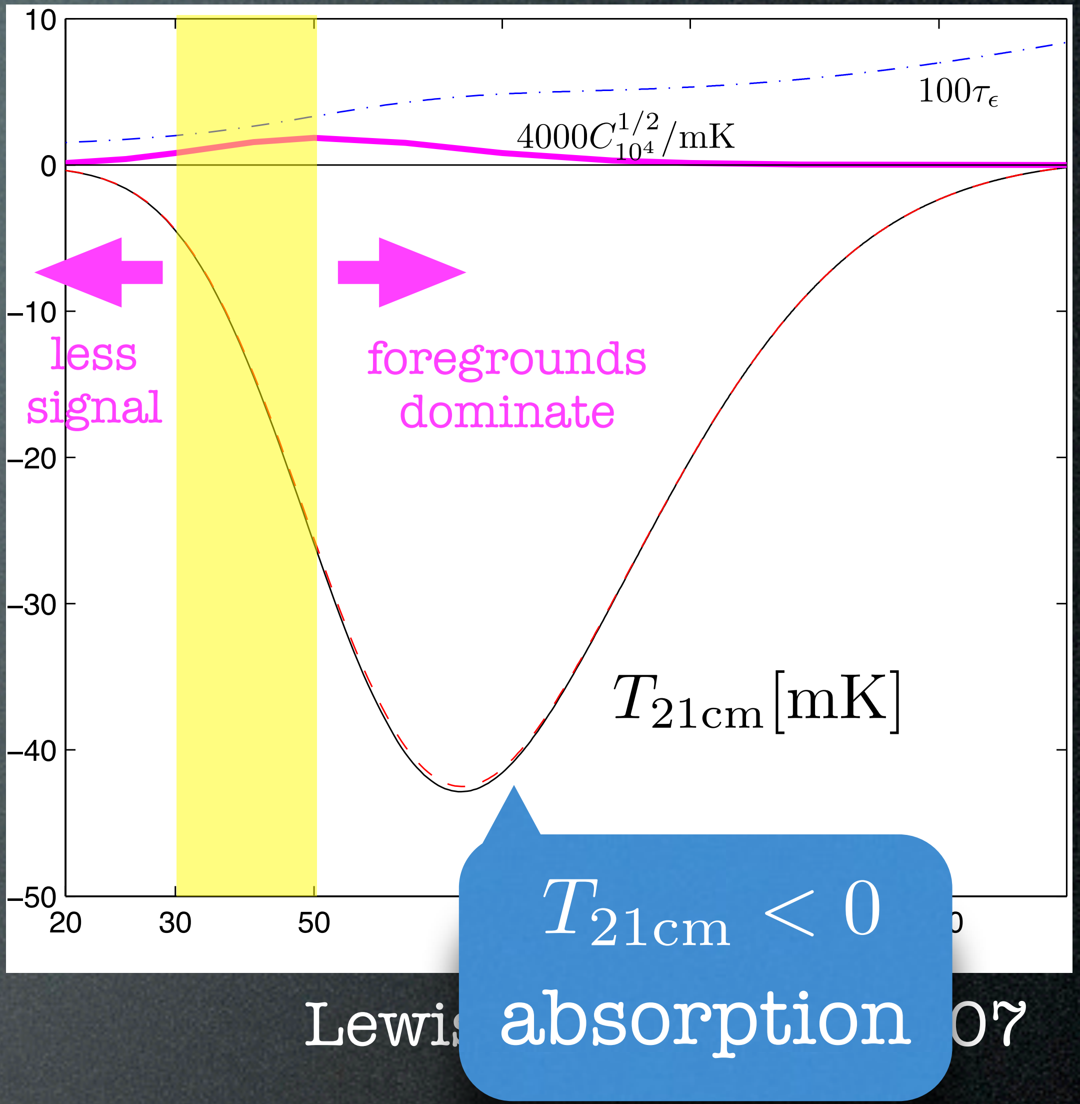


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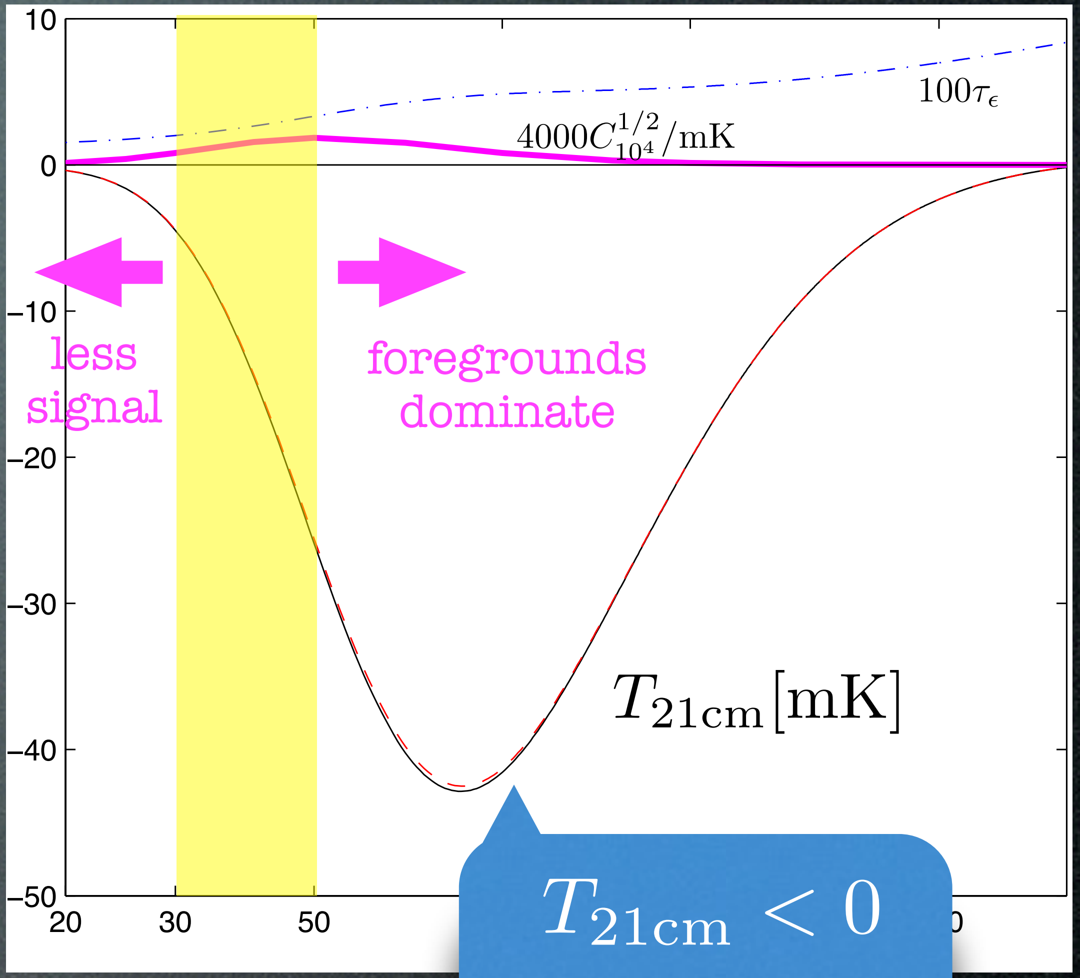


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$\mu = \hat{k}\cdot\hat{n}$

$\hat{n}$: line-of-sight direction

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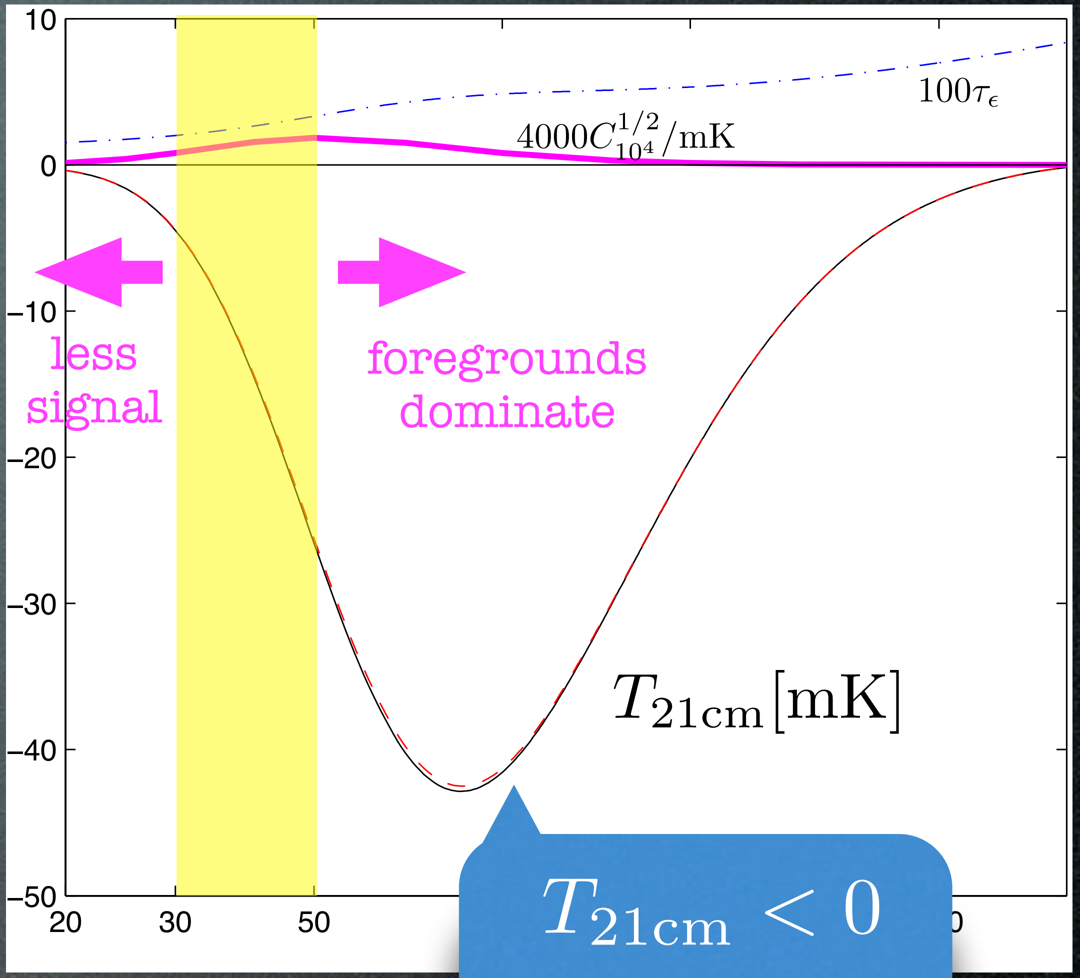


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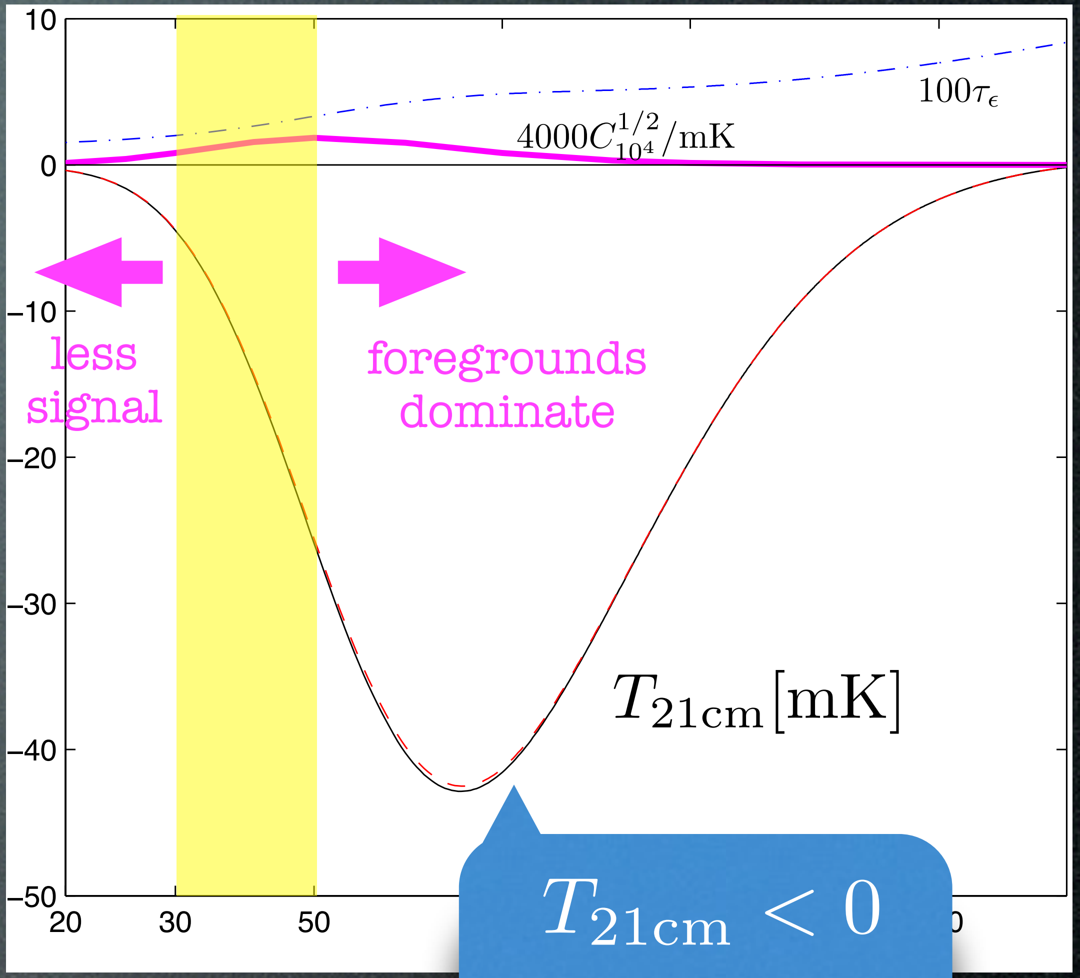


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21 cm can probe $\delta_b$, separately from $\delta_m$ .

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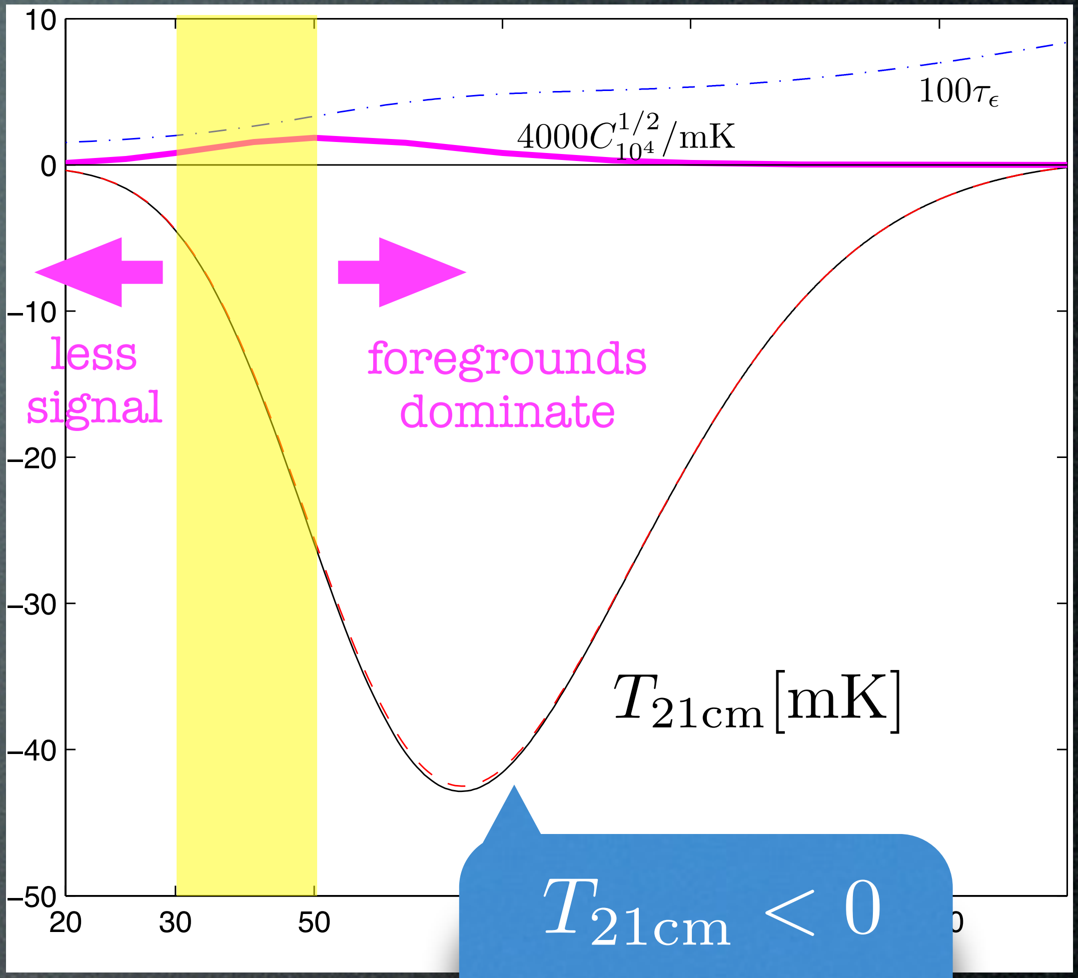


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isotropic

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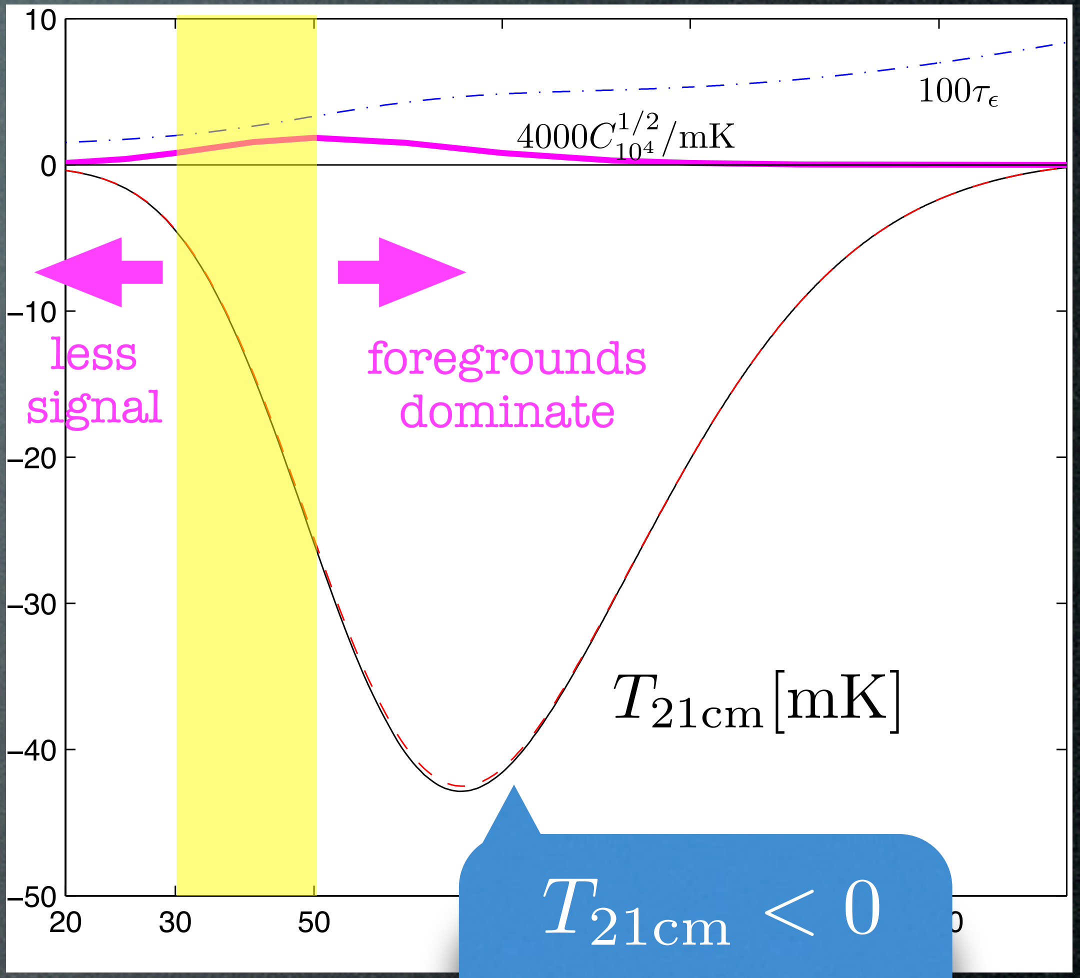


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depends on $\delta_b$ — isotropic

depends only on $\delta_m$ — $\propto \mu^2$

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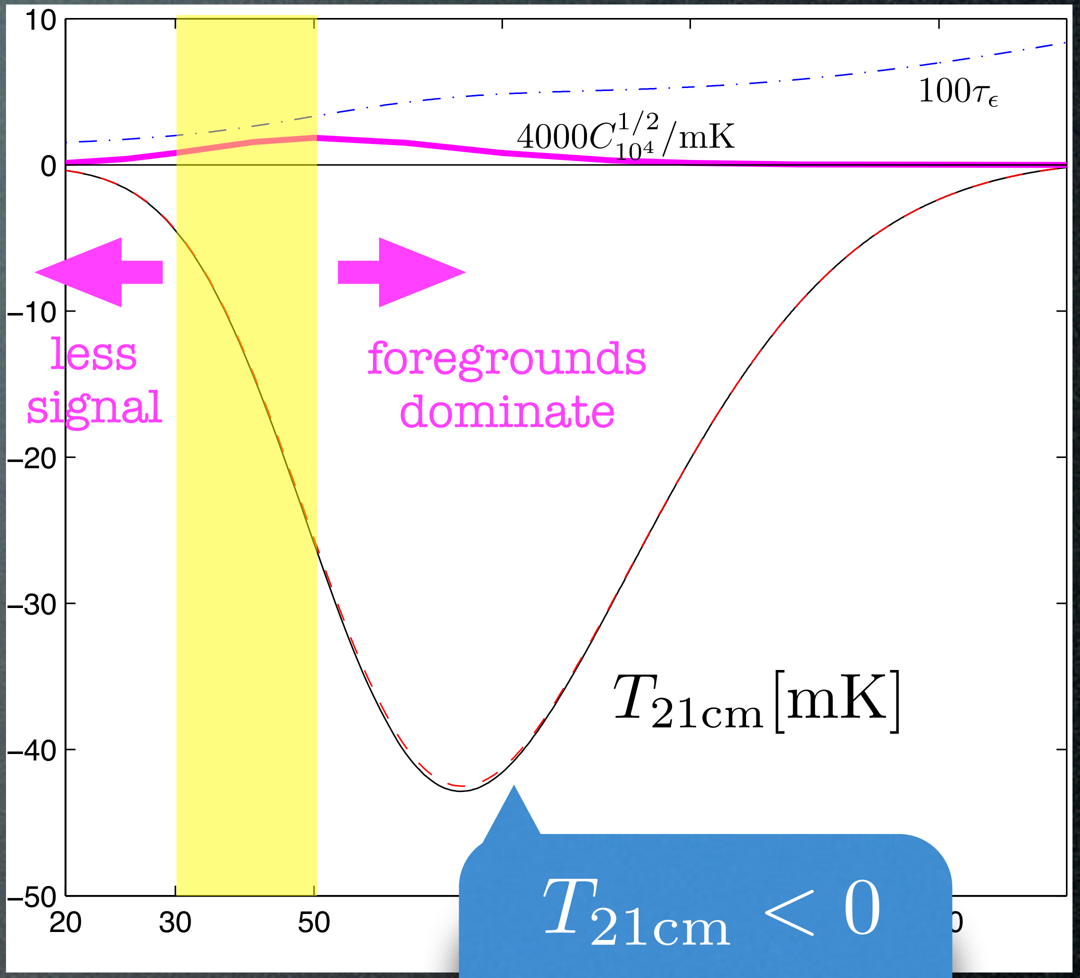


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depends on $\delta_b$ | depends only on $\delta_m$

$\mu = \hat{k}\cdot\hat{n}$

$\hat{n}$: line-of-sight direction

isotropic

$\propto \mu^2$

Tomography helps the separation!

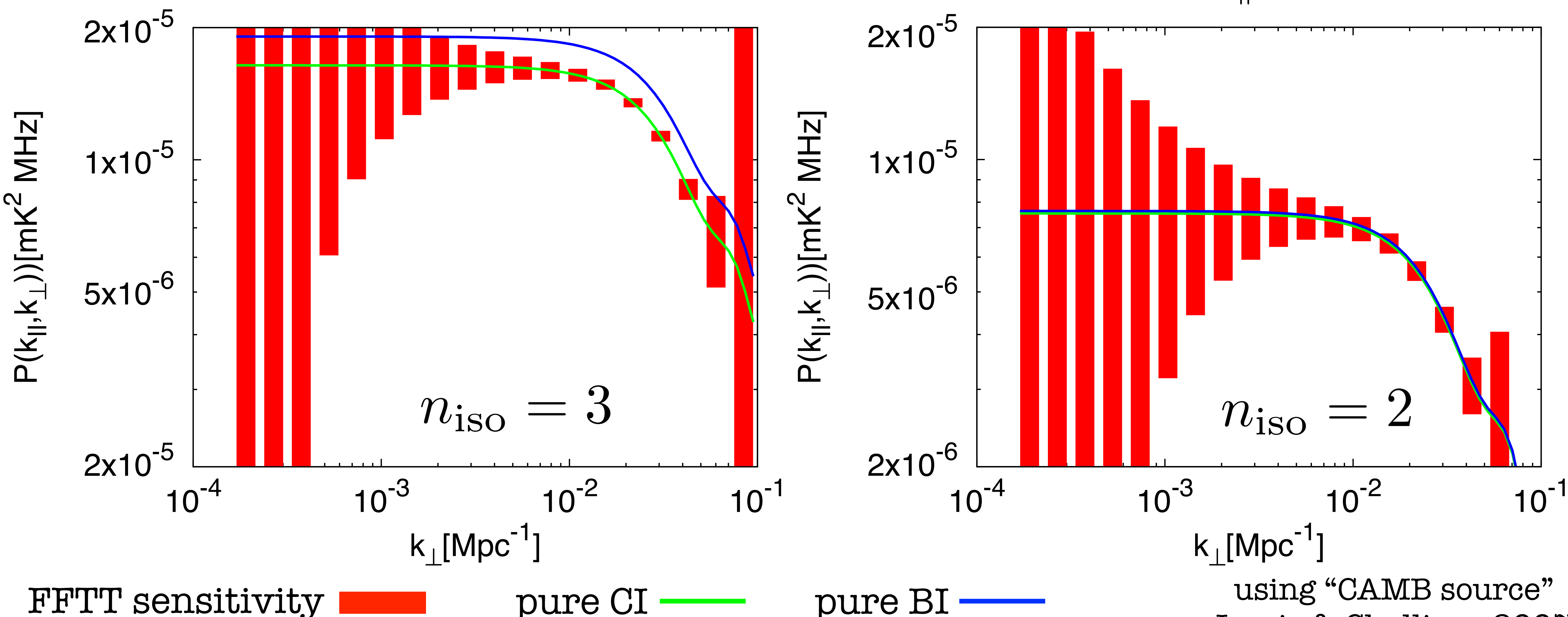
21 cm can probe $\delta_b$, separately from $\delta_m$.

# 21 cm power spectrum

- 2d anisotropic power spectrum $P(k_\parallel, k_\perp)$

$$\langle \delta T_{21\mathrm{cm}}(\vec{k}) \delta T_{21\mathrm{cm}}(\vec{k}') \rangle \equiv (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P(k_\parallel, k_\perp)$$

- $P_{S_m}(k_0)/P_{\mathrm{AD}}(k_0) = 0.1$ at $k_0 = 0.0002\mathrm{Mpc}^{-1}$ $\quad z = 50,\ k_\parallel = 0.06\mathrm{Mpc}^{-1}$




using "CAMB source"
Lewis & Challinor 2007

CI & BI can be in principle distinguished by 21 cm observations.

# Fisher matrix analysis

- Fisher matrix for 3d survey [Tegmark 1997]

$$F_{ij}^{(21\mathrm{cm})} = \int \frac{d^3u}{(2\pi)^3} \frac{V_\Theta}{[P_{\Delta T_b}^{(\mathrm{tot})}(\vec{u})]^2} \left( \frac{\partial P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u})}{\partial \lambda_i} \right) \left( \frac{\partial P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u})}{\partial \lambda_j} \right).$$

$\lambda_i$ cosmological parameter

$V_\Theta = \Omega_{\mathrm{FoV}} B$ : survey volume

$B$ : band width

k-space ⇔ u-space

$$k_\parallel = u_\parallel / y(z)$$
$$\vec{k}_\perp = \vec{u}_\perp / d_A(z)$$

$$y(z) = \frac{(1+z)^2}{\nu_0 H(z)}$$

signal+noise covariance:

$$P_{\Delta T_b}^{(\mathrm{tot})}(\vec{u}) = P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u}) + P_{\Delta T_b}^{(\mathrm{noise})}(\vec{u})$$

- limitation in k-space volume

$k_\parallel > 1/y(z)B$ : foreground removal

$k < k_{nl} \simeq 0.1\mathrm{Mpc}^{-1}$ : linear evolution


# Survey specification

- Fast Fourier Transform Telescope [Tegmark & Zaldarriaga 2009]

$$P^{\rm noise}_{\Delta T_b}(\vec{u}) = \frac{4\pi f_{sky}\lambda_\nu^2 T_{\rm sys}^2}{A\Omega_{\rm FoV} f_{\rm cover} t_{\rm obs}} W(\vec{u}_\perp)^2$$

$$W(\vec{u}_\perp) = \exp\left[-\frac{\lambda_\nu^2}{A}u_\perp^2\right]$$

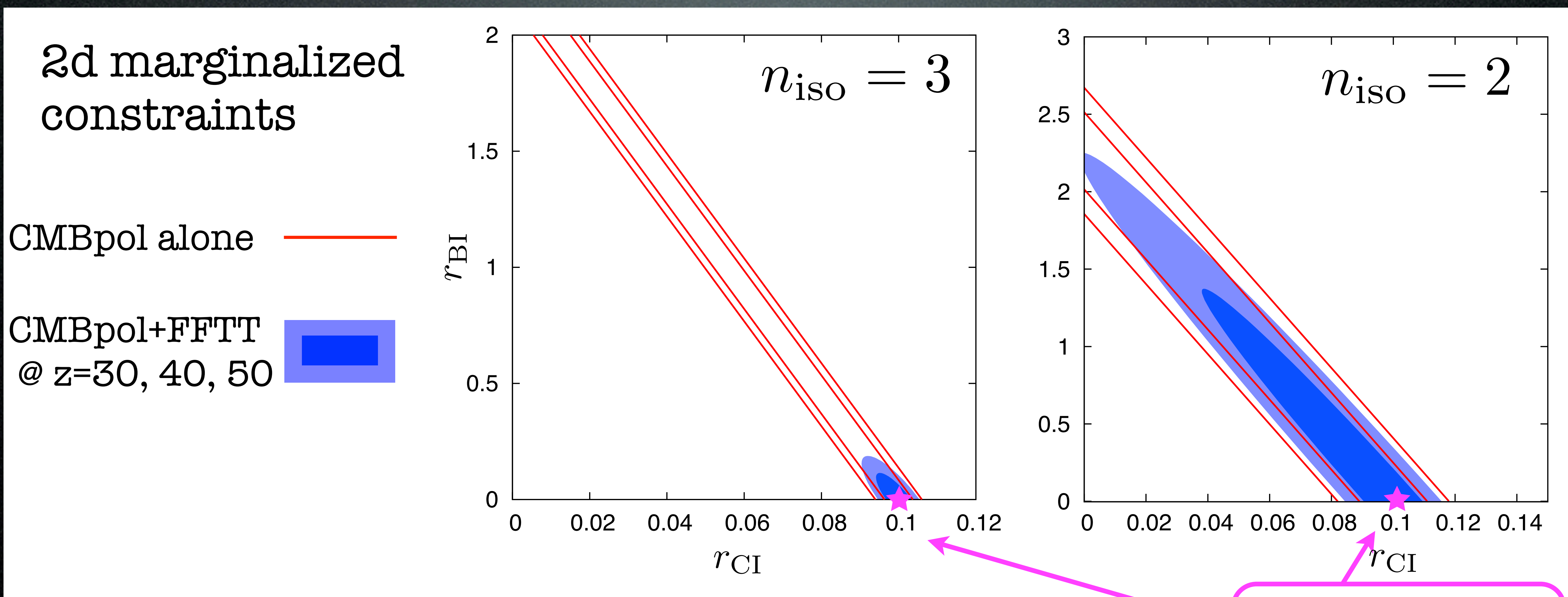
Gaussian window function

| | | |
|---|---|---|
| sky coverage | $f_{\rm sky}$ | 1 |
| array area | $A$ | $20\,{\rm km}^2$ |
| antenna coverage | $f_{\rm cover}$ | 1 |
| Field of View | $\Omega_{\rm FoV}$ | $\pi$ |
| system temperature (Galactic synchrotron) | $T_{\rm sys}$ | $220{\rm K}\left[\frac{(1+z)}{10}\right]^{2.8}$ |
| observation time | $t_{\rm obs}$ | 1 year |
| Band width | $B$ | 8 MHz |

# Future constraint

- Fisher matrix analysis with $(\omega_b, \omega_c, H_0, \tau_{\rm reion}, n_{\rm adi}, A_{\rm adi}, r_{\rm CI}, r_{\rm BI}, n_{\rm iso})$

$$r_{\rm CI} = P_{\rm CI}(k_0)/P_{\rm adi}(k_0),\ \ r_{\rm BI} = P_{\rm BI}(k_0)/P_{\rm adi}(k_0) \ \text{ at } k_0 = 0.0002{\rm Mpc}^{-1}$$




★ true model: pure CI
$(r_{\rm CI}, r_{\rm BI}) = (0.1, 0)$

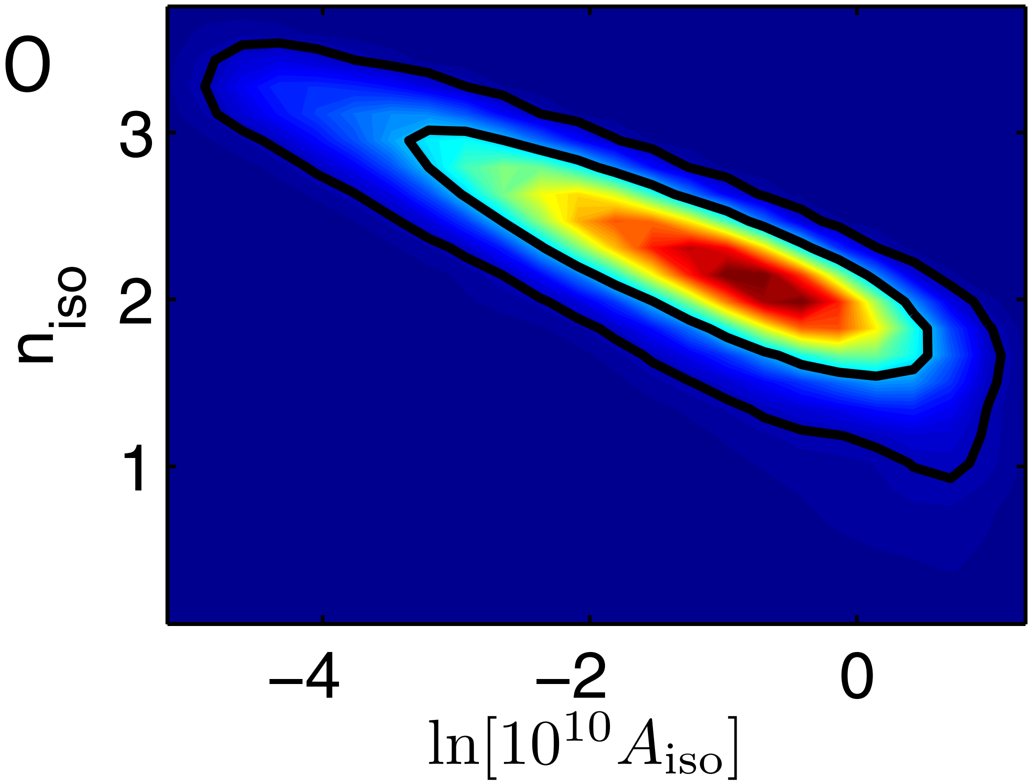
Future 21 cm surveys can distinguish CI/BI if $n_{\rm iso} \gtrsim 2$.

# Blue-tilted isocurvature power spectrum?

- Preference from recent observations

Bean+ 06, Keskitalo+ 06, Sollom+ 09, Li+ 10, etc.

Li+ 10




Data={WMAP7, ACBAR, CBI, Boomerang, SDSS LRG, Union SNeIa}

$\Rightarrow n_{\rm iso} \simeq 2 \sim 3$

| | $\log[10^{10}A_s^{\rm adi}]$ | $n_s^{\rm adi}$ | $\log[10^{10}A_s^{\rm iso}]$ | $n_s^{\rm iso}$ | $\cos\Delta$ | $\chi^2_{\rm min}$ |
|---|---|---|---|---|---|---|
| Adiabatic | $3.204 \pm 0.036$ | $0.964 \pm 0.010$ | – | – | – | 8217.0 |
| Mixed | $3.165 \pm 0.044$ | $0.972 \pm 0.014$ | $-1.375^{+1.233}_{-1.306}$ | $2.246^{+0.494}_{-0.428}$ | $0.094^{+0.075}_{-0.095}$ | 8213.5 |

- Some theoretical models

Kasuya & Kawasaki 09, etc.

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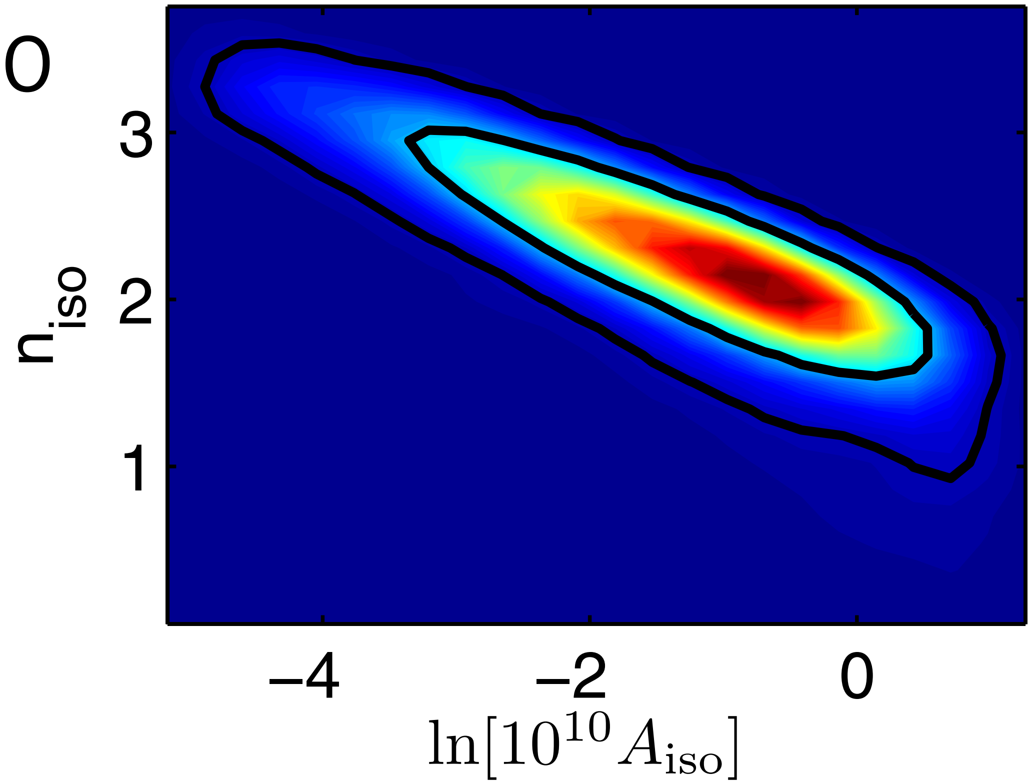


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➡ To be tested by Planck

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# Summary

- We investigated possibility for distinguishing primordial CDM and baryon isocurvature perturbations.
- 21 cm observations can measure the fluctuation of baryon separately from total matter perturbations, and in principle distinguish CI/BI, which is difficult from other cosmological observations, such as CMB.
- Future 21 cm tomography surveys can distinguish these two isocurvature modes, if their power spectra are blue-tilted.

Thank you for your attention!

# Can we distinguish CI/BI?

- $\delta_{\rm cdm}$ and $\delta_b$ in linear perturbation eqs.

gauge:
conformal Newtonian

$a$ : scale factor

$\mathcal{H} = \dot{a}/a$

$\dot{\tau} = a n_e \sigma_T$

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  - Poisson eq.

$$k^2\Phi = 4\pi G a^2 \bar{\rho}_{\rm tot}\left[\delta_{\rm tot} + (1+w_{\rm tot})v_{\rm tot}\right]$$

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  - photon & baryon Euler eqs. (anisotropic stress neglected)

$$\dot{v}_\gamma = \frac{k}{4}\delta_\gamma + k\Psi - \dot{\tau}\left[v_\gamma - v_b\right]$$

$$\dot{v}_b = -\mathcal{H}v_b + k\Psi + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b}\dot{\tau}\left[v_\gamma - v_b\right]$$

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➡ No pressure gradient

# Can we distinguish CI/BI?

- $\delta_{\rm cdm}$ and $\delta_b$ in linear perturbation eqs.

gauge:
conformal Newtonian

$a$: scale factor

$\mathcal{H} = \dot{a}/a$

$\dot{\tau} = a n_e \sigma_T$

  - Poisson eq.

$$k^2\Phi = 4\pi G a^2 \bar{\rho}_{\rm tot}\left[\delta_{\rm tot} + (1+w_{\rm tot})v_{\rm tot}\right]$$

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➡ CI/BI cannot be discriminated by observations of CMB, total matter and metric perturbations.

These are only sensitive to total matter isocurvature $S_m = \frac{\Omega_{\rm cdm}}{\Omega_m}S_{\rm CI} + \frac{\Omega_b}{\Omega_m}S_{\rm BI}$.

# Full set of perturbation eqs.
# (in synchronous gauge)

- photon

$$\dot{\delta}_\gamma = -\frac{4}{3}v_\gamma - \frac{2}{3}\dot{h}_L$$

$$\dot{v}_\gamma = k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - a\sigma_T n_e(v_\gamma - v_b)$$

- massless neutrino

$$\dot{\delta}_\nu = -\frac{4}{3}v_\nu - \frac{2}{3}\dot{h}_L$$

$$\dot{v}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right).$$

pressure gradient

- baryon

$$\dot{\delta}_b = -v_b - \frac{1}{2}\dot{h}_L$$

$$\dot{v}_b = \mathcal{H}v_b - \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b}a\sigma_T n_e(v_\gamma - v_b)$$

- CDM

$$\dot{\delta}_c = -v_c - \frac{1}{2}\dot{h}_L$$

$$\dot{v}_c = \mathcal{H}v_c$$

Thomson scatter

- Einstein eq.

$$k^2\eta_T - \frac{1}{2}\mathcal{H}\dot{h}_L = -4\pi G a^2\bar{\rho}_{\rm tot}\delta_{\rm tot}$$

$$k^2\dot{\eta}_T = -4\pi G a^2(\bar{\rho}_{\rm tot} + \bar{p}_{\rm tot})v_{\rm tot}$$

$\delta_b$ and $\delta_{\rm cdm}$
only appear in the form of $\delta_m$.

# Redshifted 21 cm line (2)

- Differential brightness temperature

$$\Delta T_b = T_b - T_{\rm CMB} \simeq \frac{T_s - T_{\rm CMB}}{1+z}\tau_{\rm 21cm}$$

At high redshift z>30
(prior to formation of first objects), $T_s < T_{\rm CMB}$ . → $\Delta T_b < 0$ : absorption

$\mu = \hat{k} \cdot \hat{n}$

$\hat{n}$: line-of-sight direction

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isotropic

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depends only on $\delta_m$

isotropic

$\propto \mu^2$

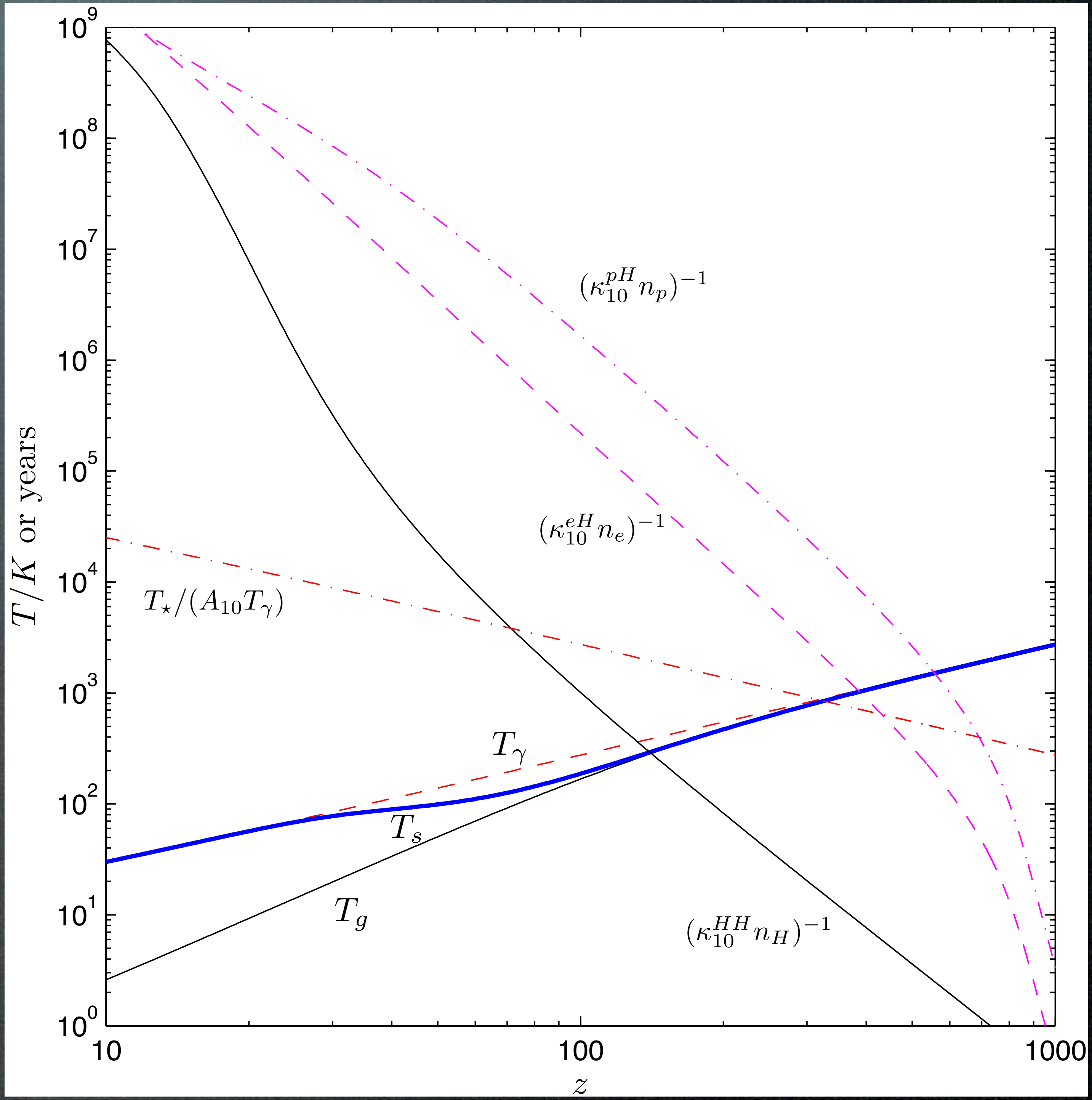
Tomography helps the separation!

21 cm can probe $\delta_b$, separately from $\delta_m$ .

# Evolution of spin temperature

$T_s$ is affected by

- Absorption of CMB photons
- Atomic (H-H, H-p, H-e) collision
- ~~Scattering of UV photons (after formation of first objects)~~




Lewis & Challinor 2007

# Evolution of brightness temperature

- optical depth of 21 cm line emission/absorption

$$\tau_{21\mathrm{cm}}(z) = \frac{3\lambda_0 hc A_{10}\bar{n}_{\mathrm{HI}}}{32\pi k_B \bar{T}_s H}(z)$$

$$\simeq 1.08\times 10^{-2} x_{\mathrm{HI}} \left[\frac{T_s}{T_{\mathrm{CMB}}}(z)\right]^{-1} \left[\frac{\omega_b}{0.023}\right] \left[\frac{\omega_m}{0.13}\right]^{-1/2} \left[\frac{1+z}{10}\right]^{1/2}$$

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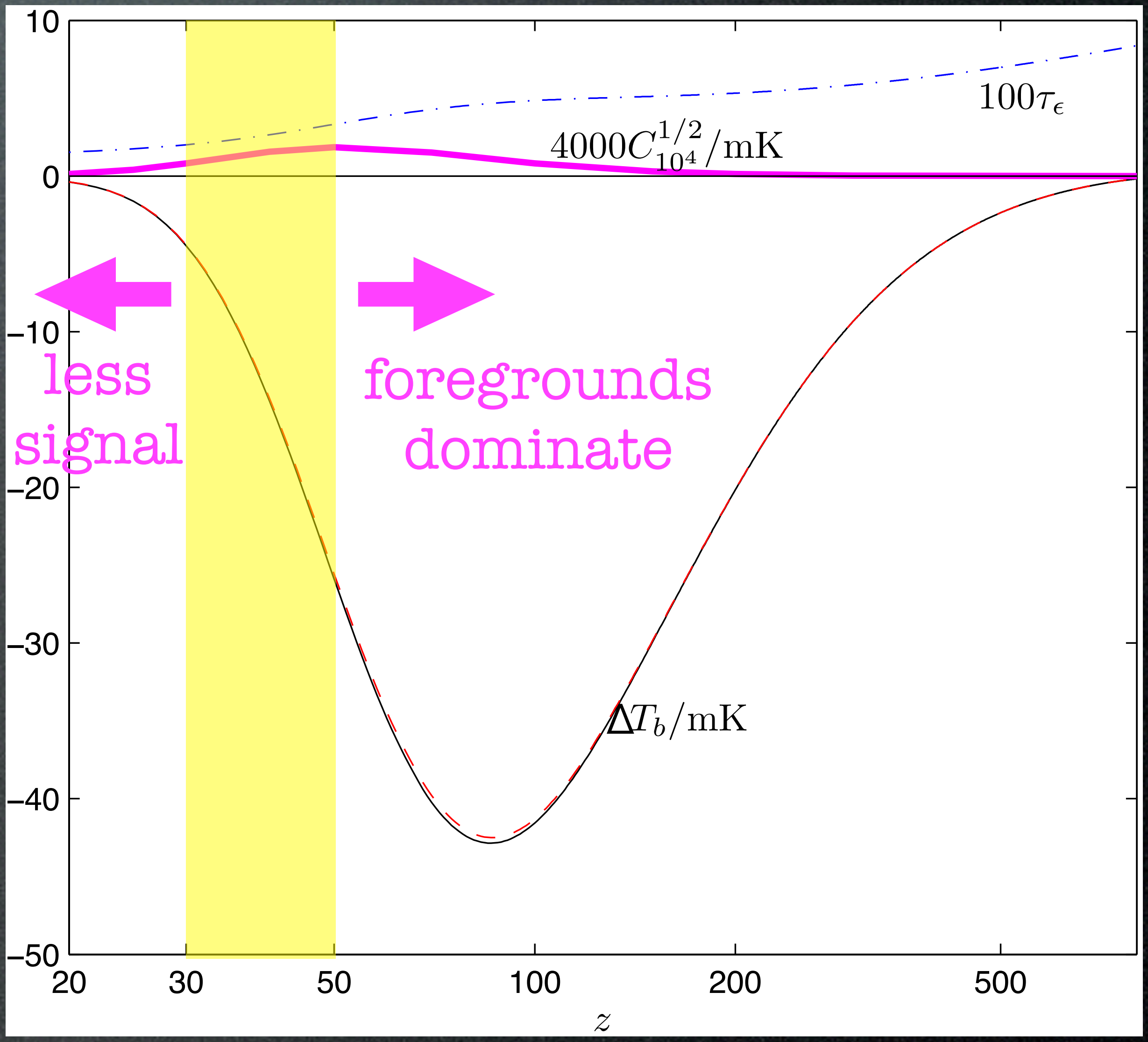
$$\Delta T_b = \frac{T_s - T_{\mathrm{CMB}}}{1+z}\tau_{21\mathrm{cm}}$$




Lewis & Challinor 2007

# 21 cm tomography

Consider an observation at around frequency $\nu$;

↘ central redshift: $z_\nu = \dfrac{\nu_0}{\nu} - 1$

- frequency difference ⟷ distance in line-of-sight direction

$$r_\parallel = y(z_\nu)\Delta\nu \qquad y(z) = \frac{(1+z)^2}{\nu_0 H(z)}$$

- sky-position ⟷ distance in transverse direction

$$\vec{r}_\perp = d_A(z_\nu)\vec{\Theta} \qquad d_A(z) = \int_0^z \frac{dz'}{H(z')}$$

- correspondence in Fourier space

$$k_\parallel = u_\parallel / y(z_\nu)$$

$$\vec{k}_\perp = \vec{u}_\perp / d_A(z_\nu)$$

$$f(\vec{u}) = \int d\Delta\nu d^2\Theta\, f(\Delta\nu, \vec{\Theta}) \exp\left[i(\Delta\nu u_\parallel + \vec{\Theta}\cdot\vec{u}_\perp)\right]$$

$$f(\vec{k}) = \int d^3r\, f(\vec{r}) \exp\left[i\vec{r}\cdot\vec{k}\right]$$

Observed data resides in u-space

# Fisher matrix of 21 cm tomography

- Fisher matrix [Tegmark 1997]

$$F_{ij}^{(21\mathrm{cm})} = \int \frac{d^3u}{(2\pi)^3} \frac{V_\Theta}{[P_{\Delta T_b}^{(\mathrm{tot})}(\vec{u})]^2} \left( \frac{\partial P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u})}{\partial \lambda_i} \right) \left( \frac{\partial P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u})}{\partial \lambda_j} \right).$$

$\lambda_i$ cosmological parameter

$V_\Theta = \Omega_{\mathrm{FoV}} B$ : survey volume

$B$ : band width

- limitation in k-space volume

$k < k_{nl} \simeq 0.1\mathrm{Mpc}^{-1}$ : linear evolution

$k_{||} > 1/y(z)B$ : foreground removal

- signal+noise covariance

$$P_{\Delta T_b}^{(\mathrm{tot})}(\vec{u}) = P_{\Delta T_b}^{(\mathrm{signal})}(\vec{u}) + P_{\Delta T_b}^{(\mathrm{noise})}(\vec{u})$$

# FFTT parameters

- FFTT noise power spectrum [Tegmark & Zaldarriaga 2009]

$$P^{\rm noise}_{\Delta T_b}(\vec{u}) = \frac{4\pi f_{sky} \lambda_\nu^2 T_{\rm sys}^2}{A \Omega_{\rm FoV} f_{\rm cover} t_{\rm obs}} W(\vec{u}_\perp)^2$$

- survey volume

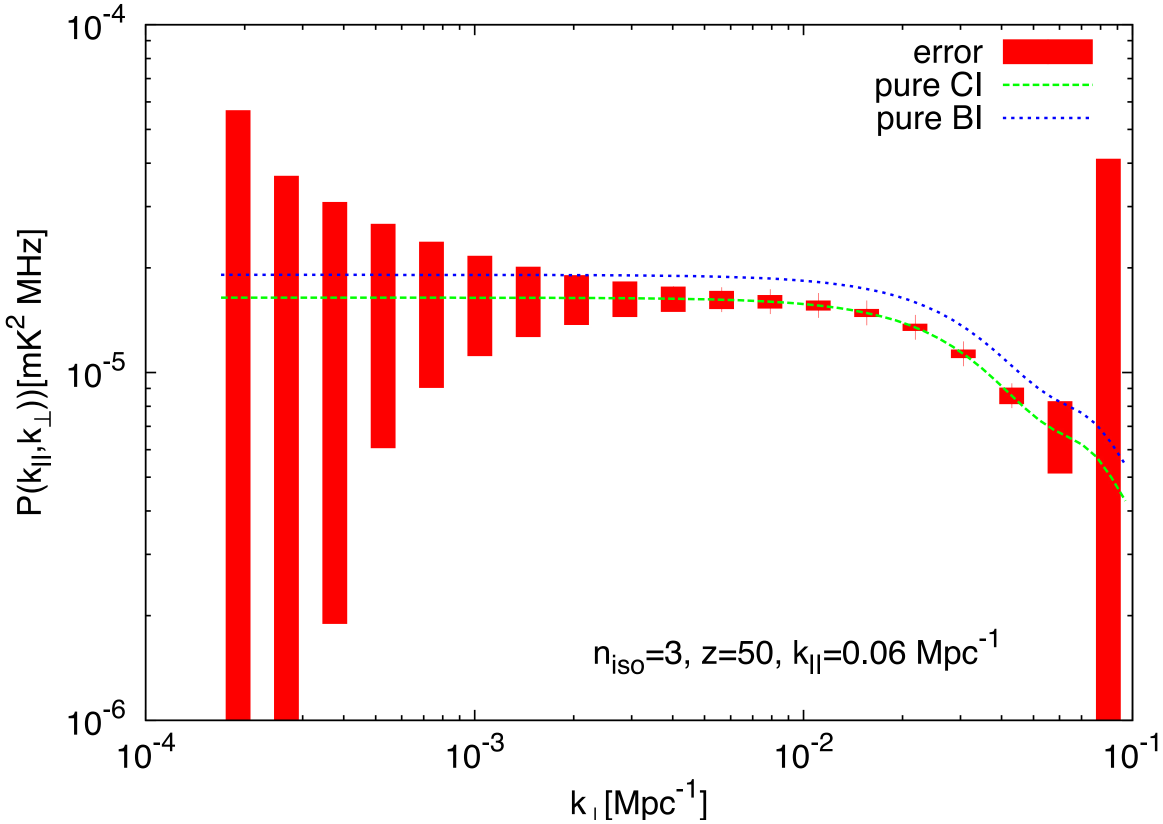
$$V_\Theta = \Omega_{\rm FoV} B$$

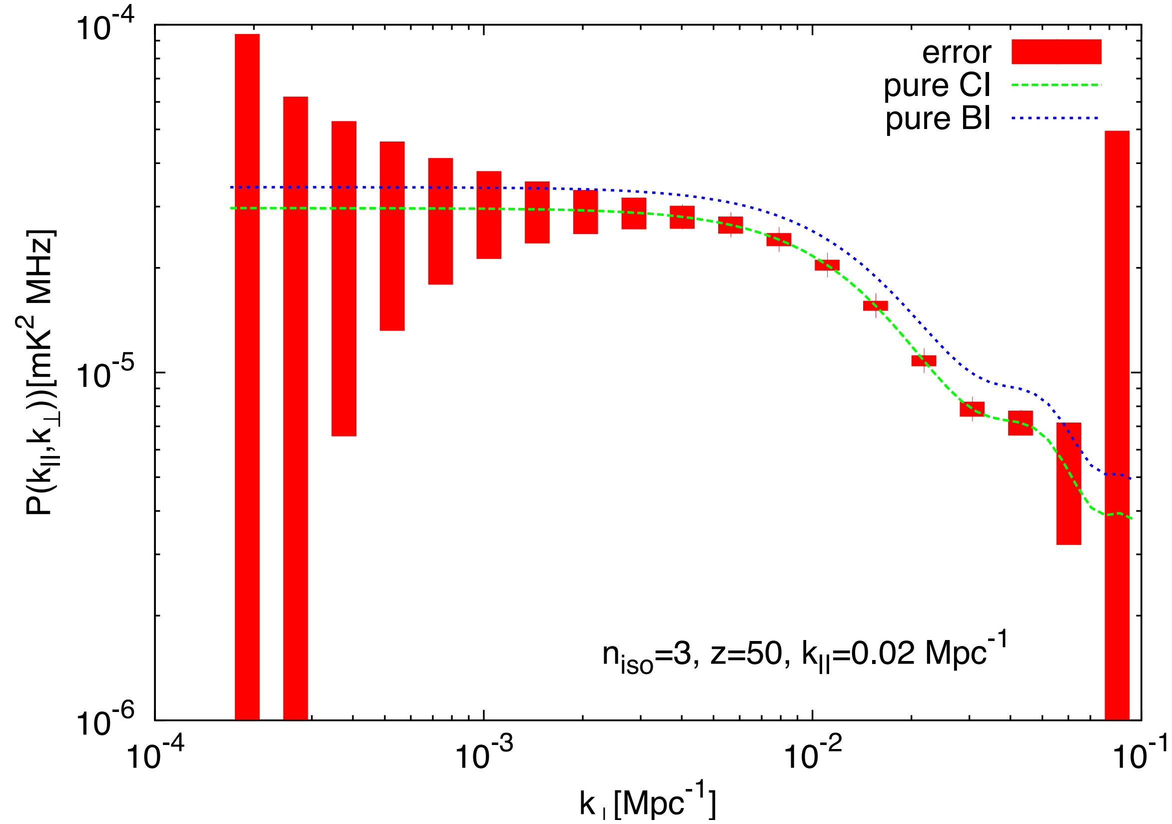
- Gaussian window function

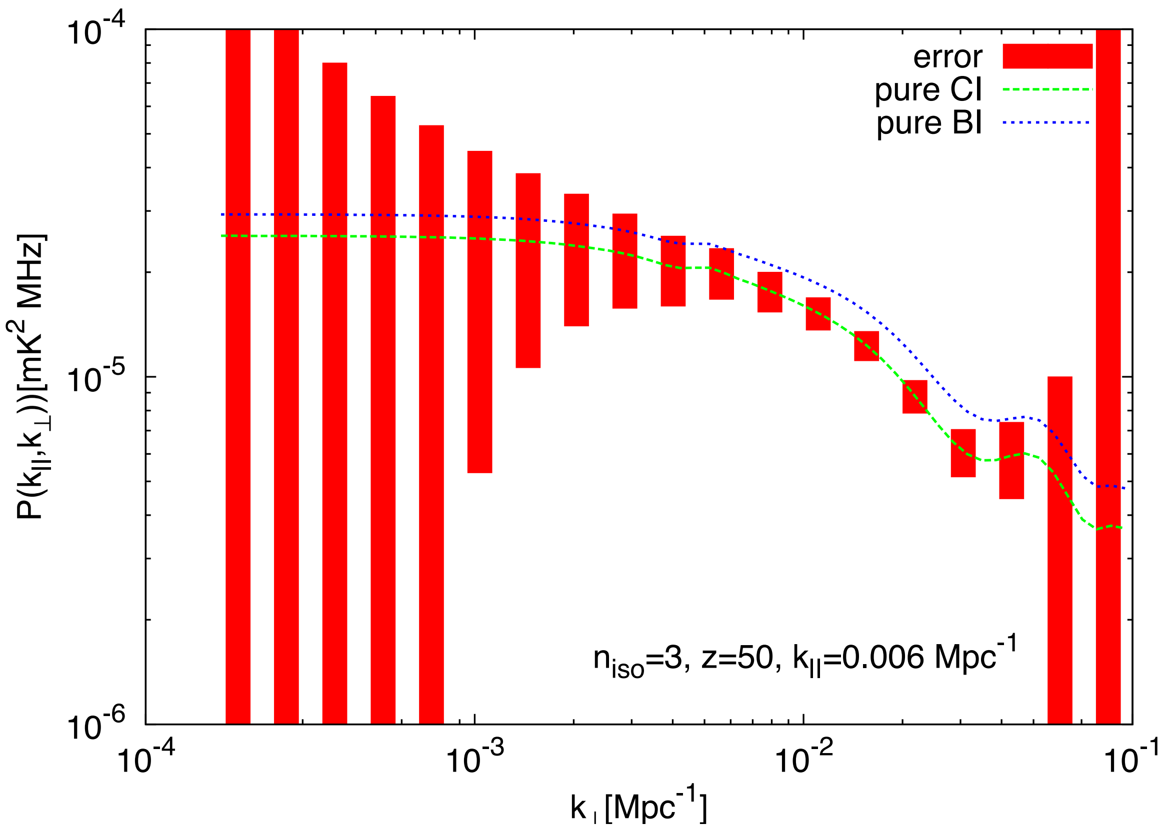
$$W(\vec{u}_\perp) = \exp\left[-\frac{\lambda_\nu^2}{A} u_\perp^2\right]$$

| | | |
|---|---|---|
| sky coverage | $f_{\rm sky}$ | $1$ |
| array area | $A$ | $20\,{\rm km}^2$ |
| antenna coverage | $f_{\rm cover}$ | $1$ |
| Field of View | $\Omega_{\rm FoV}$ | $\pi$ |
| system temperature (Galactic synchrotron) | $T_{\rm sys}$ | $220{\rm K}\left[\frac{(1+z)}{10}\right]^{2.8}$ |
| observation time | $t_{\rm obs}$ | $1$ year |
| Band width | $B$ | $8\,{\rm MHz}$ |

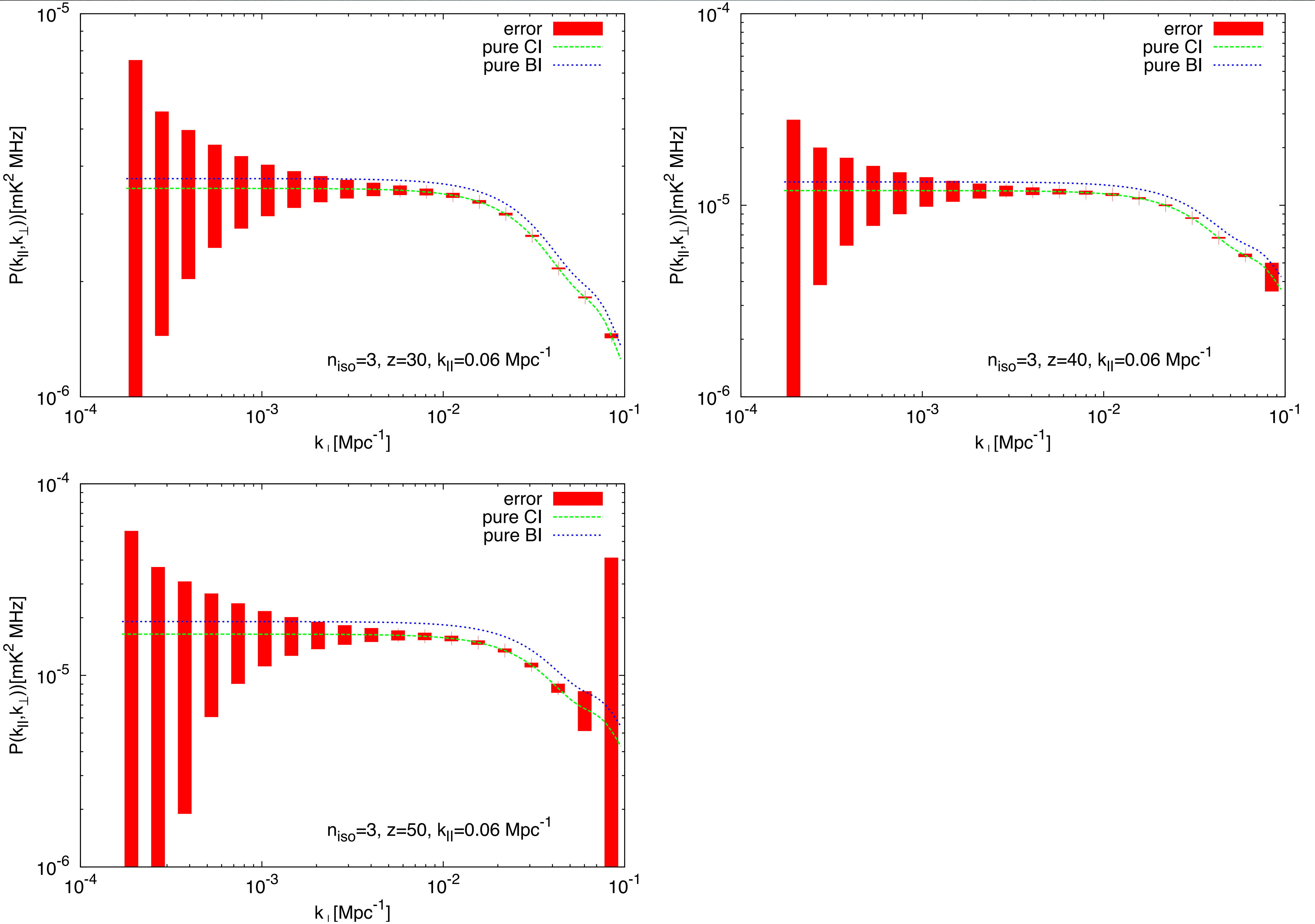
# $k_{||}$-dependence



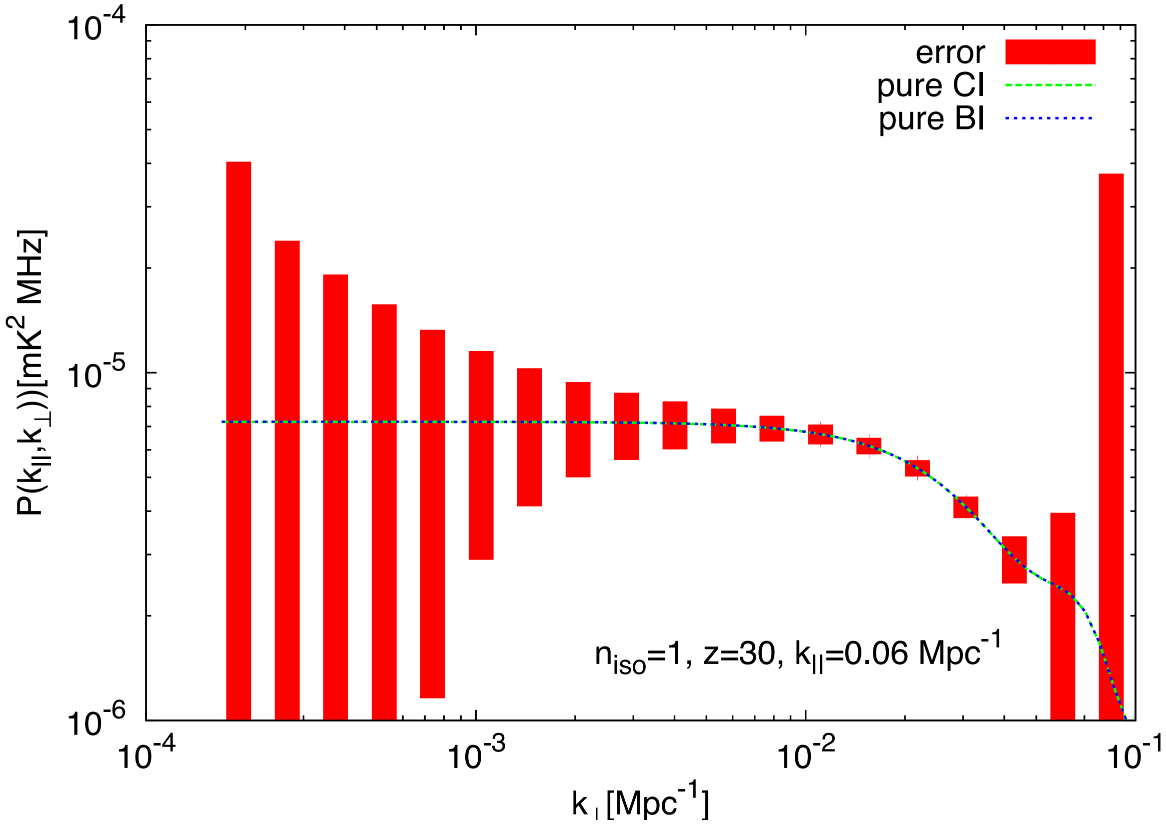



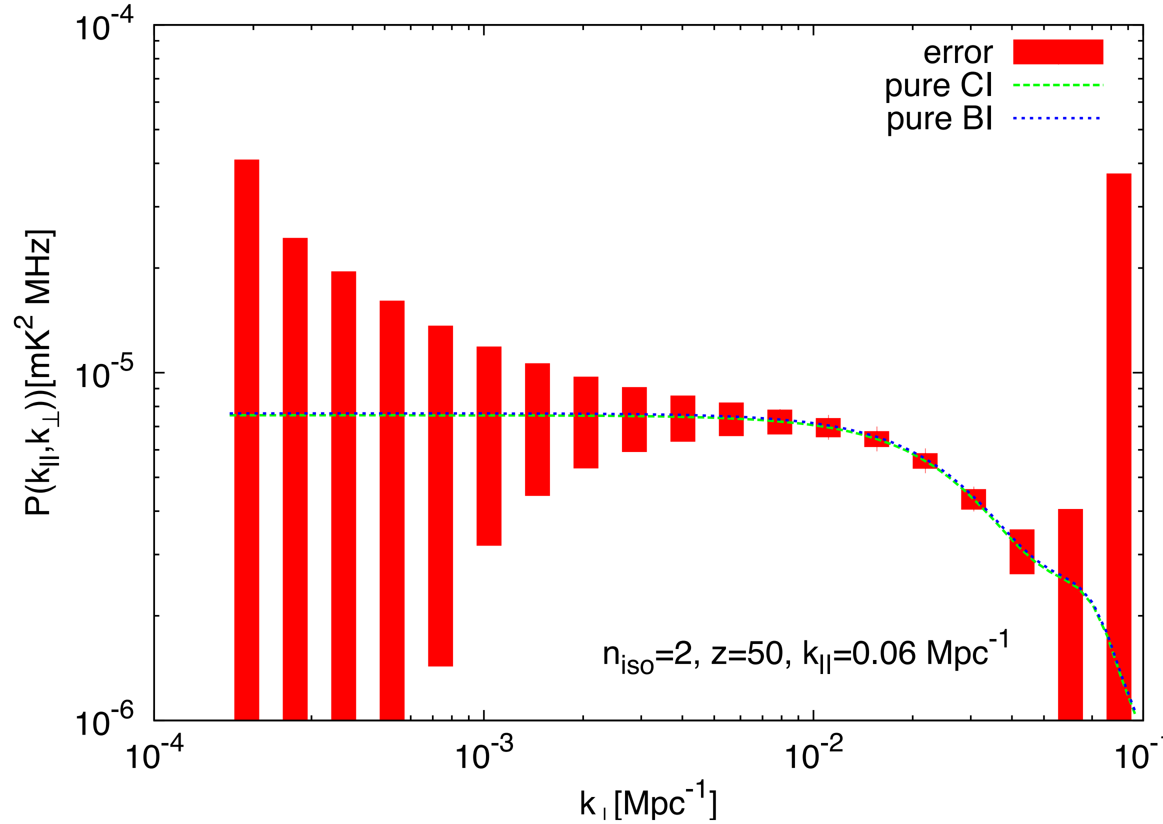



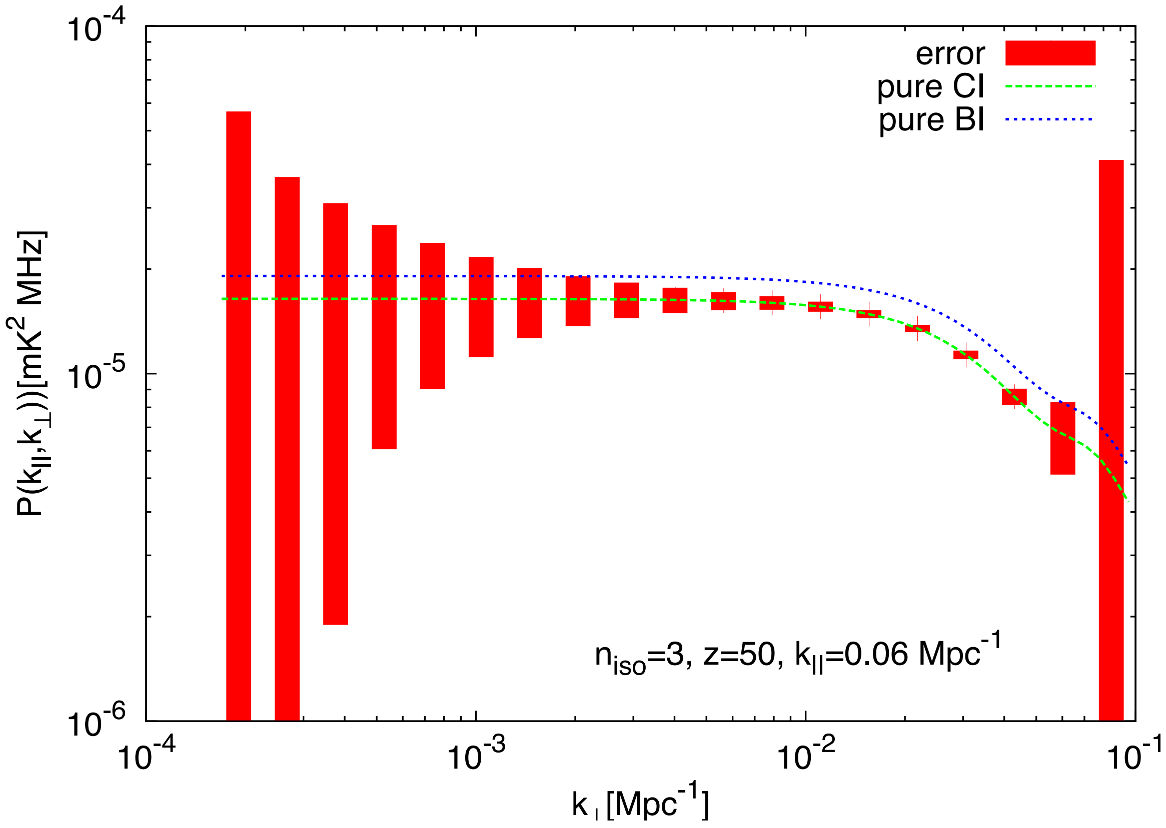
# redshift-dependence



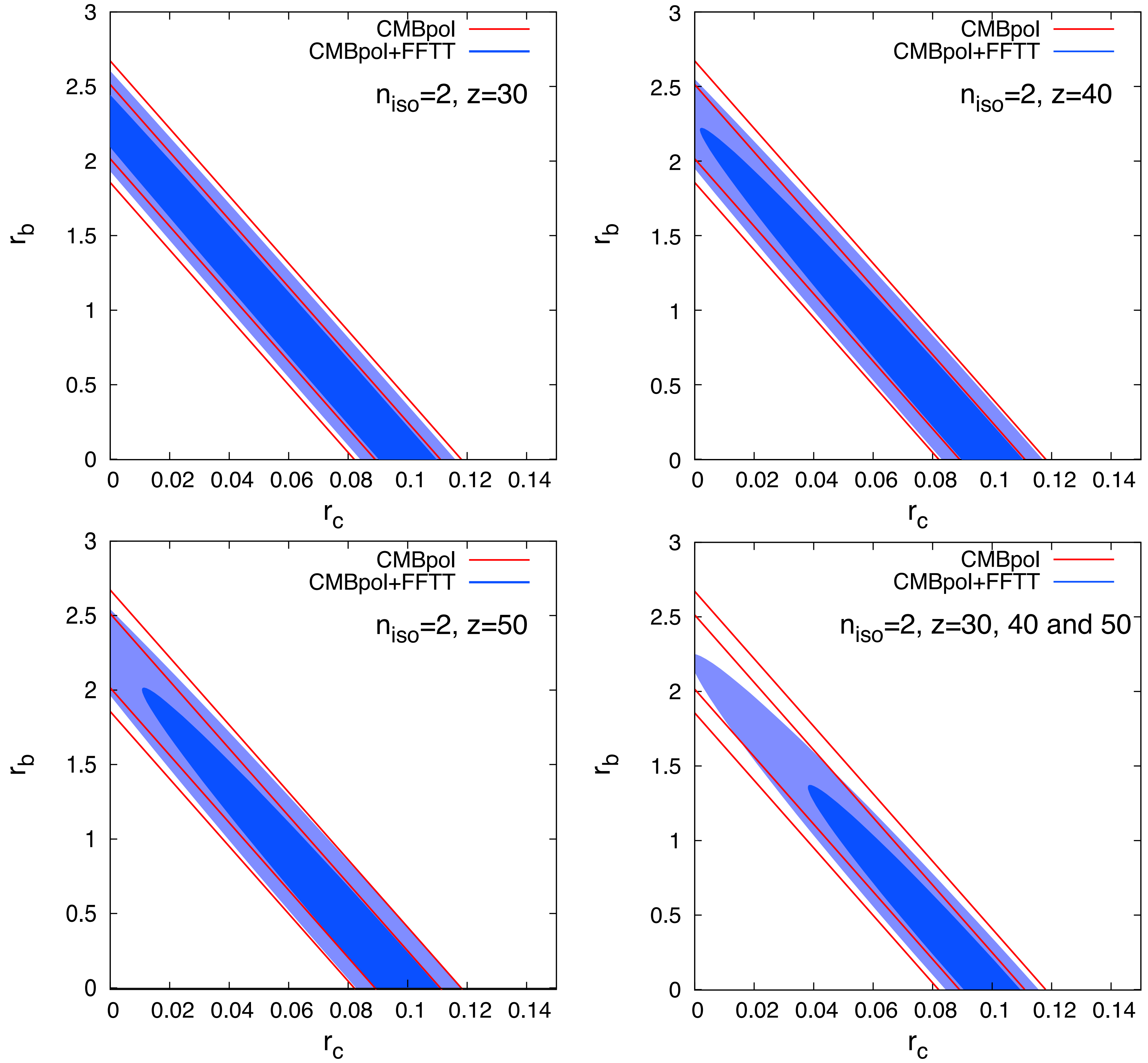
# n_iso-dependence



# constraint for n_iso=2



# constraint for n_iso=3