Removing Large-scale Variations in Astronomical Observations

Jungyeon Cho (조정연)

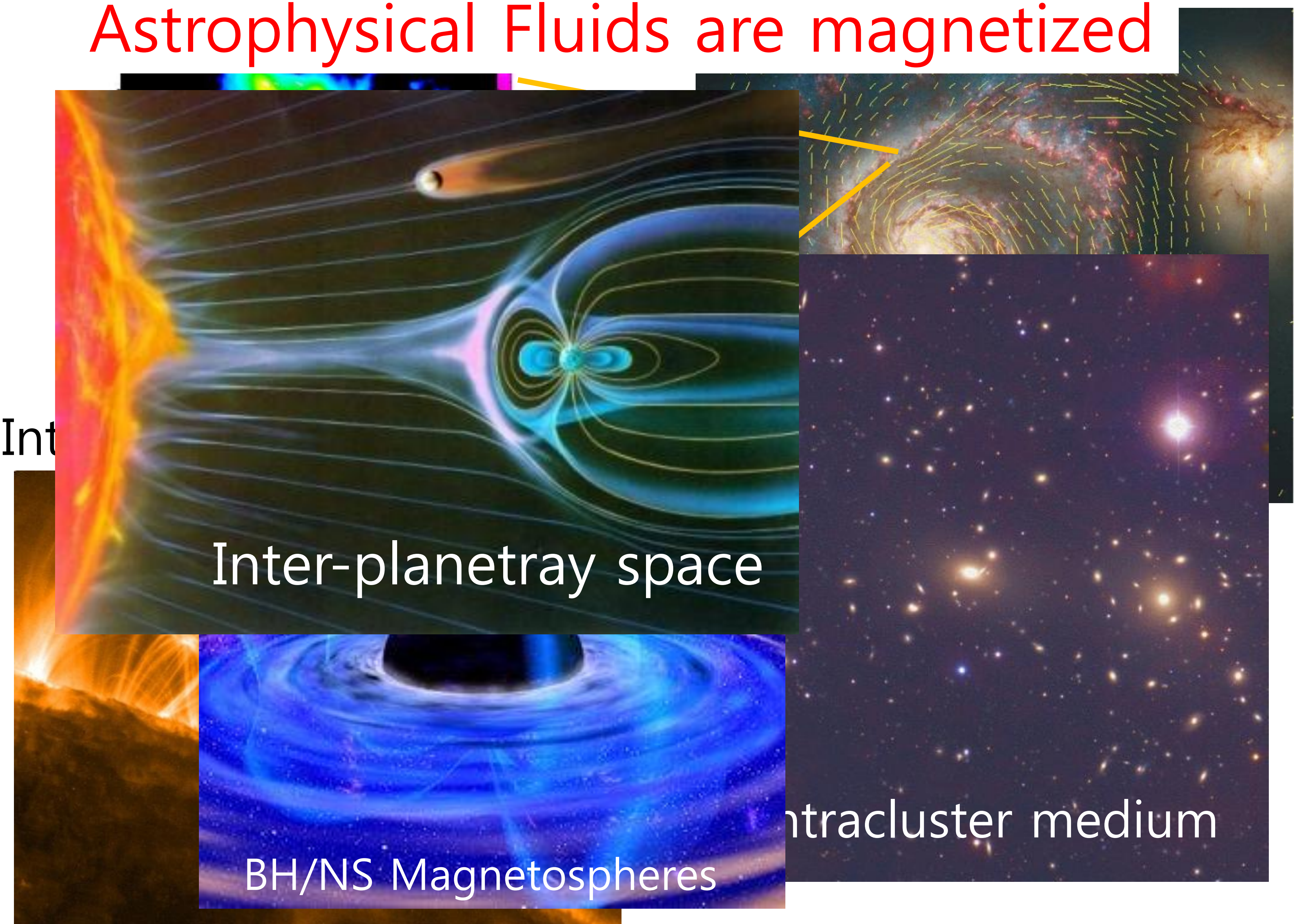
Chungnam National University

Cho & Yoo (2016)
Cho (2017; submitted)

Outline

- Long Motivation/Introduction
- A Technique of removing large-scale variations

Astrophysical Fluids are magnetized



Measurement of B

-Synchrotron emission
-Faraday rotation
-Zeeman effects
-Emission/extinction by dust
  (thought Chandrasekhar-Fermi method)
-Goldreich-Kylafis effect
-…

Motivation: Chandrasekhar-Fermi method

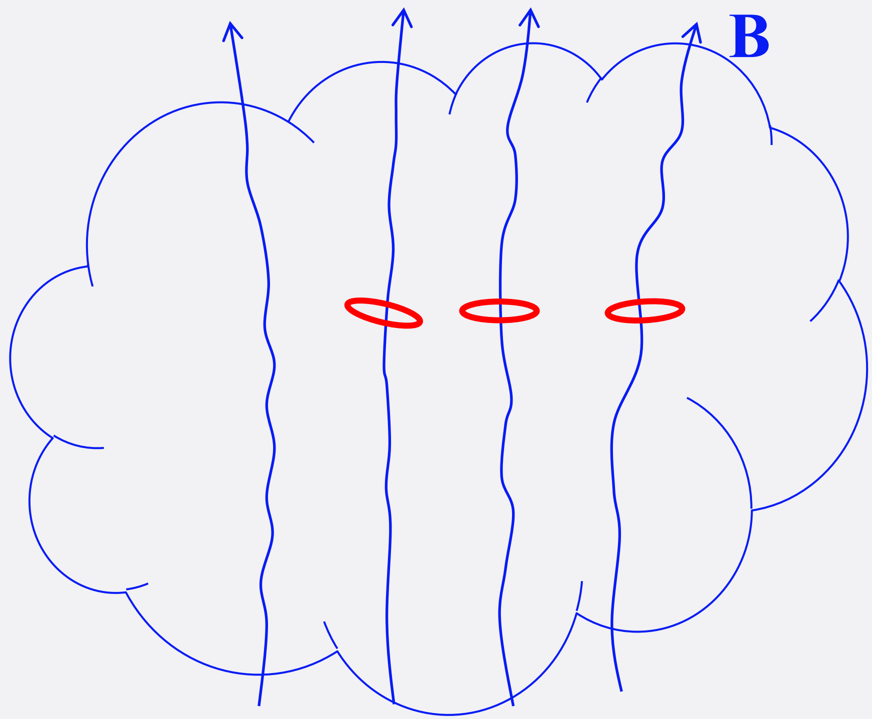
Magnetic field plays important roles in star formation

➔ Strength of B?

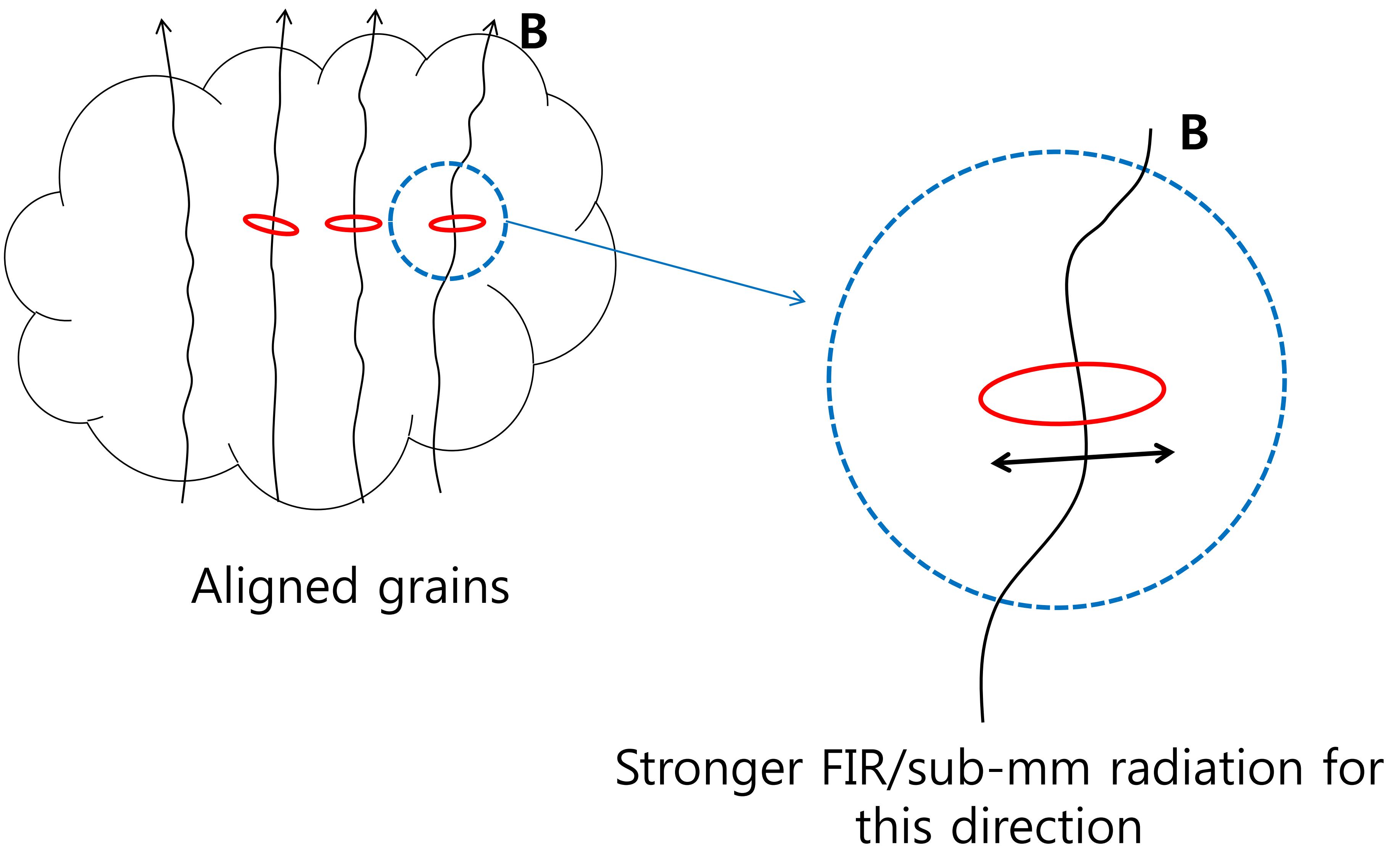
Observation of dust polarization

➔ Strength of the mean field on the plane of the sky ($B_{0,sky}$)

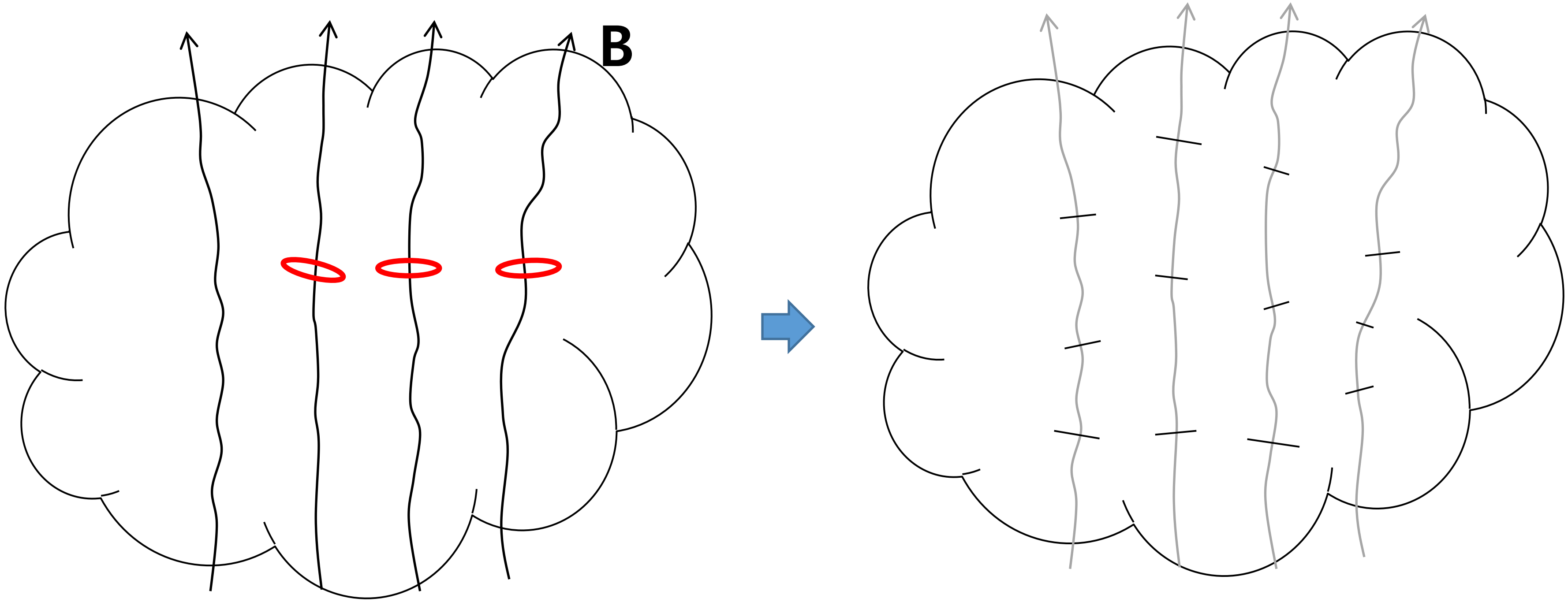
Grains are aligned ← long axis ⊥ **B**



Radiation from elongated grains



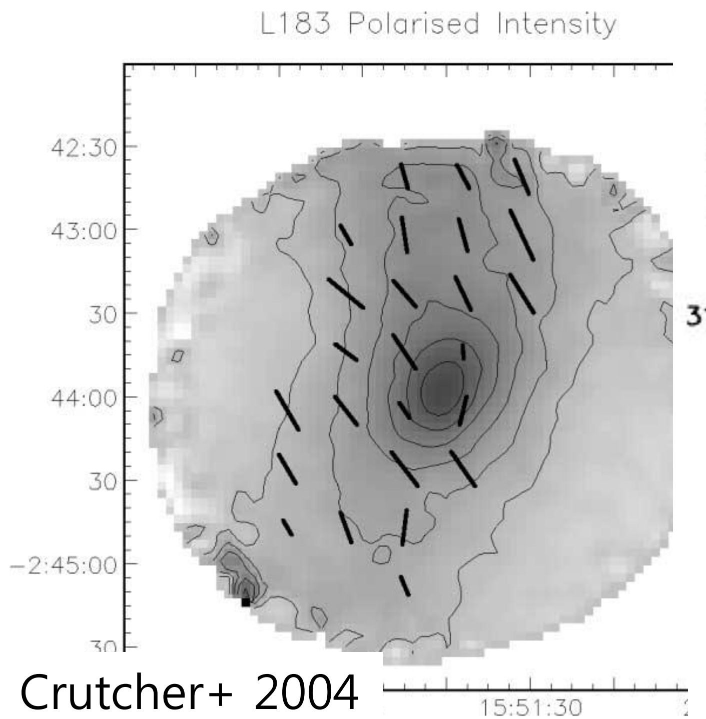
Observations of polarized FIR/sub-mm emission:



Aligned grains

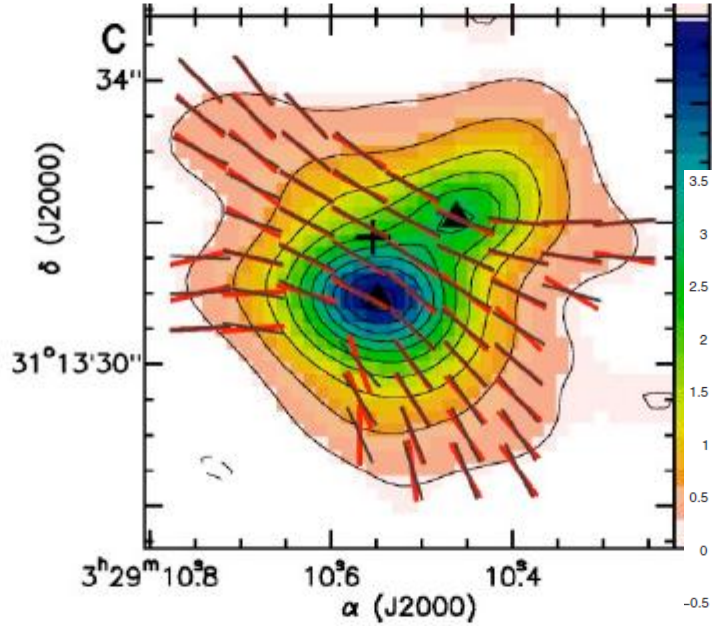
Direction of polarization
(Direction of pol $\perp$ **B**)

Polarization → Info about B

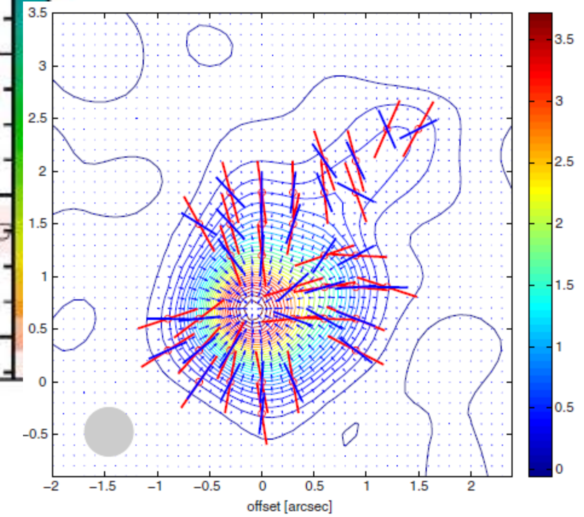


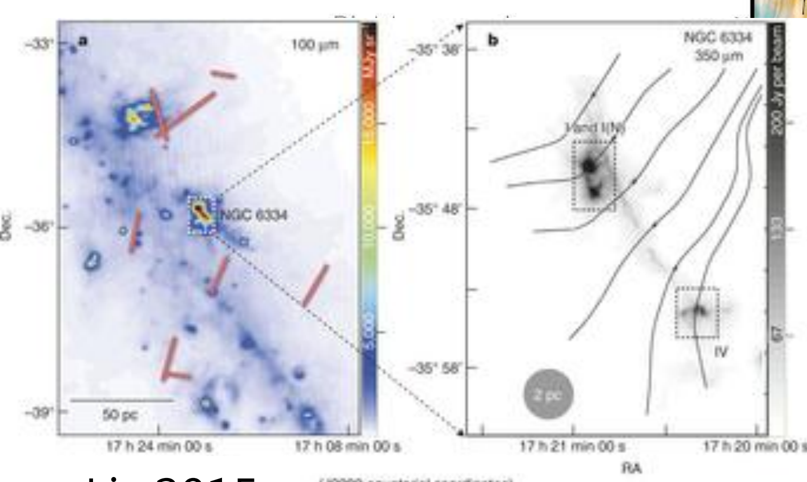
Crutcher+ 2004

Girart+ 2006

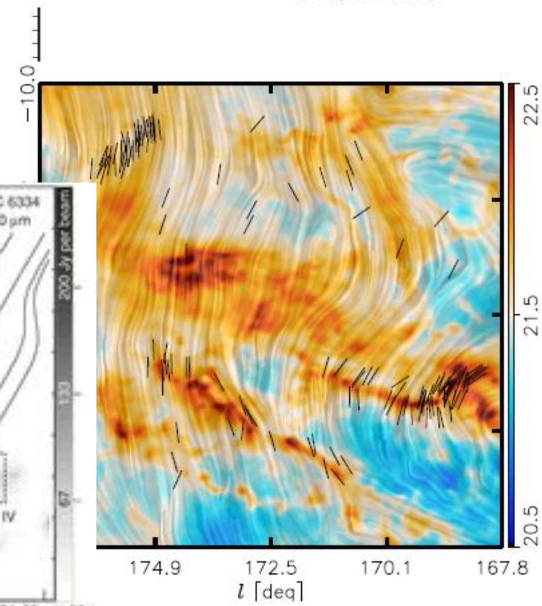


Tang+2009
Koch+2012

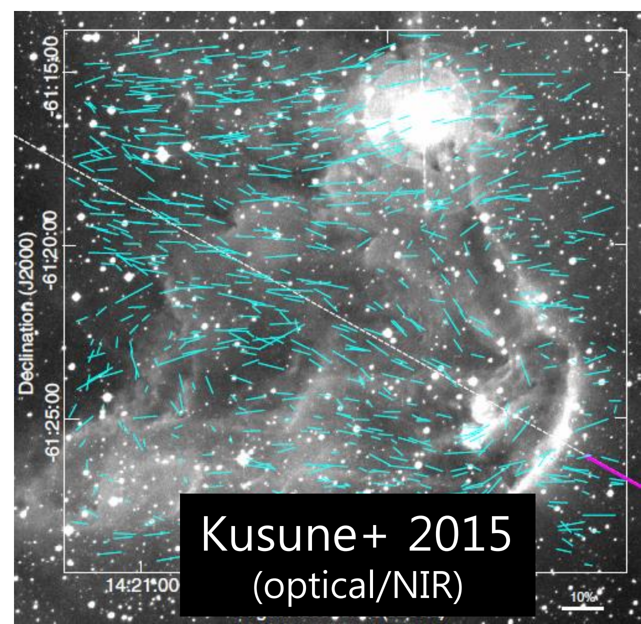




Li+2015



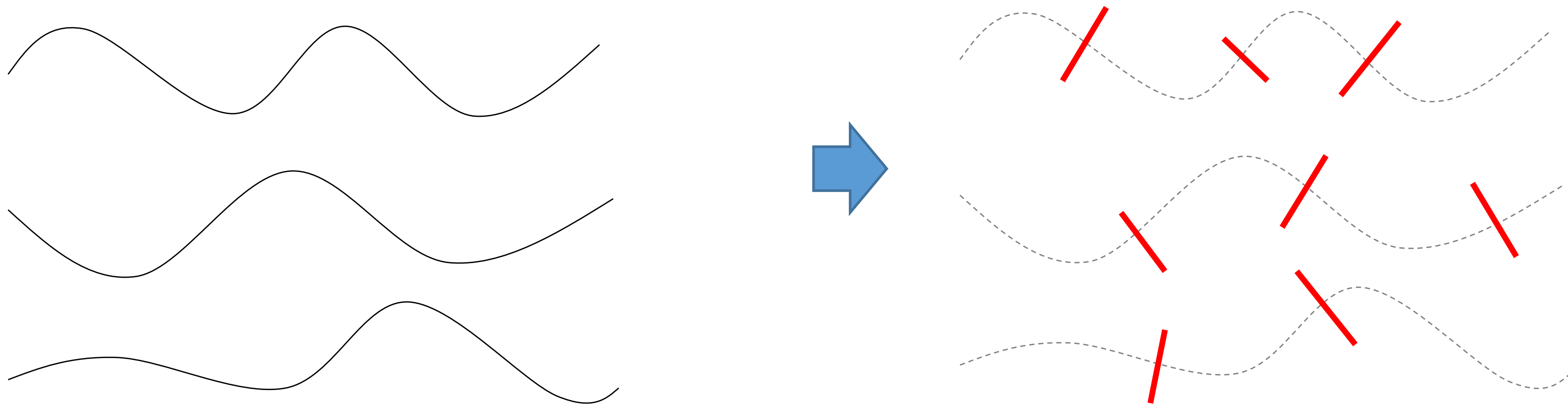
Soler+ 2016



Kusune+ 2015
(optical/NIR)

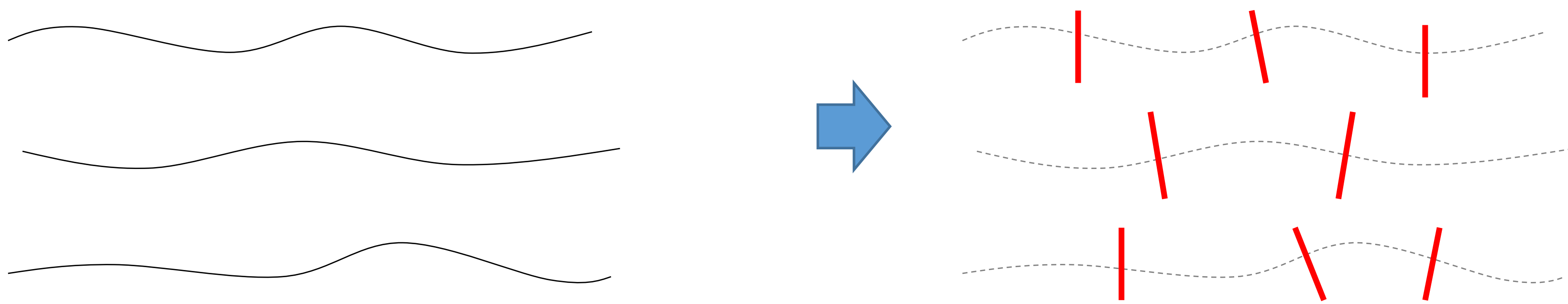
The CF method is based on the following fact:

If the mean field B_0 is weak:



Large $\delta\phi$

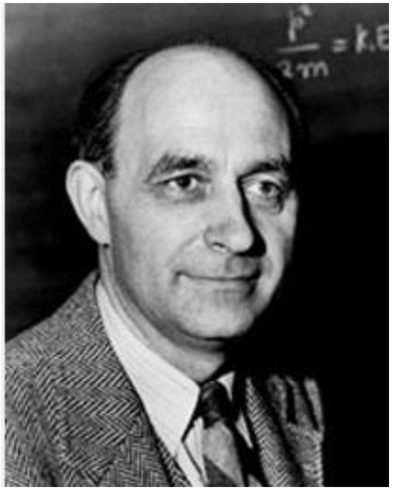
If the mean field B_0 is strong:



Small $\delta\phi$

Original CF method (C & F 1953):

$$B_{0,sky} \sim \sqrt{4\pi\rho}\,\frac{\delta V_{los}}{\delta\phi}$$

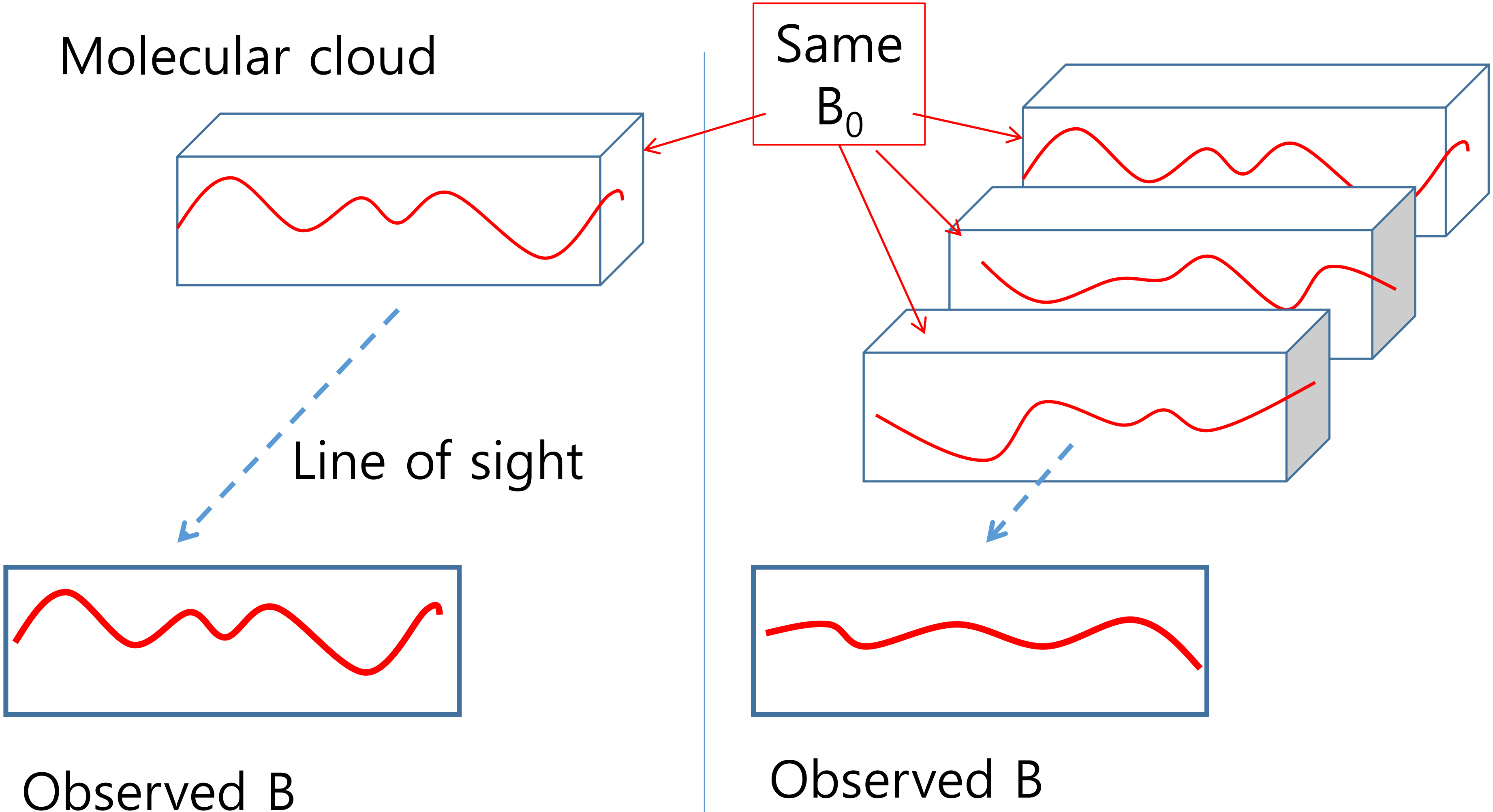


Numerical Simulations (e.g. Ostriker et al. 2001):

$$B_{0,sky} \approx \frac{1}{2}\sqrt{4\pi\rho}\,\frac{\delta V_{los}}{\delta\phi}$$

≈ average velocity

But, the CF-method needs modification…



We should take into account the number of eddies along the line of sight!

If there are N independent eddies along the LOS, $\delta\phi$ will be reduced by $N^{1/2}$

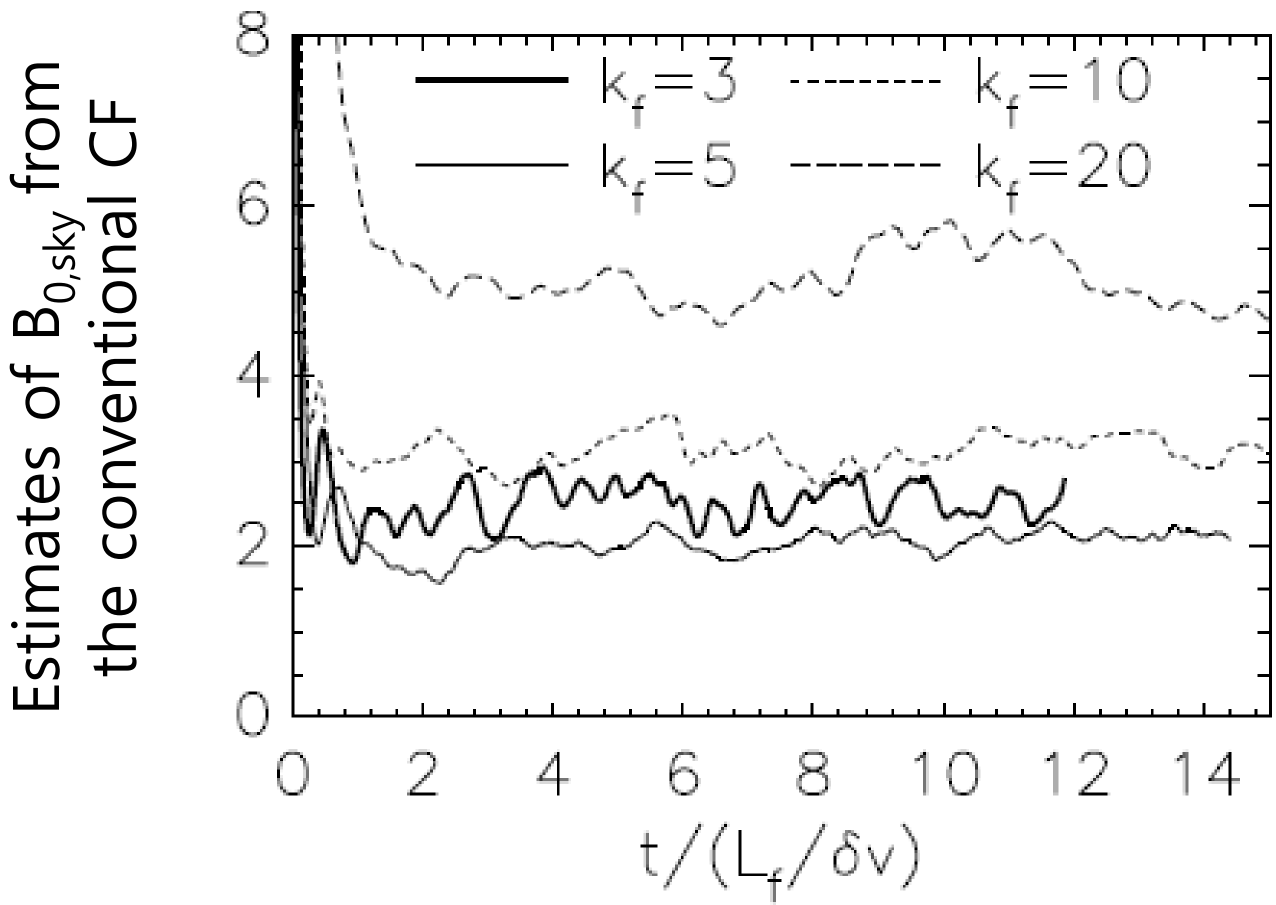
↓

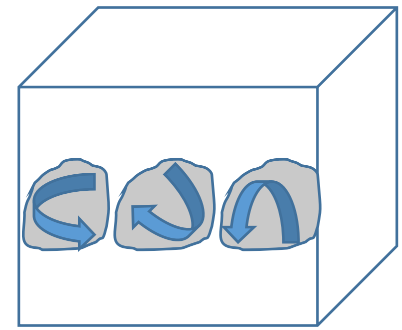
The conventional CF-method overestimates the strength of magnetic field by $N^{1/2}$

$$B_{0,sky} \approx \sqrt{4\pi\rho}\,\frac{\delta V_{los}}{\delta\phi}$$

Cho & Yoo (2016)

Conventional CF indeed overestimates $B_{0,sky}$





* $N = 1/k_f$

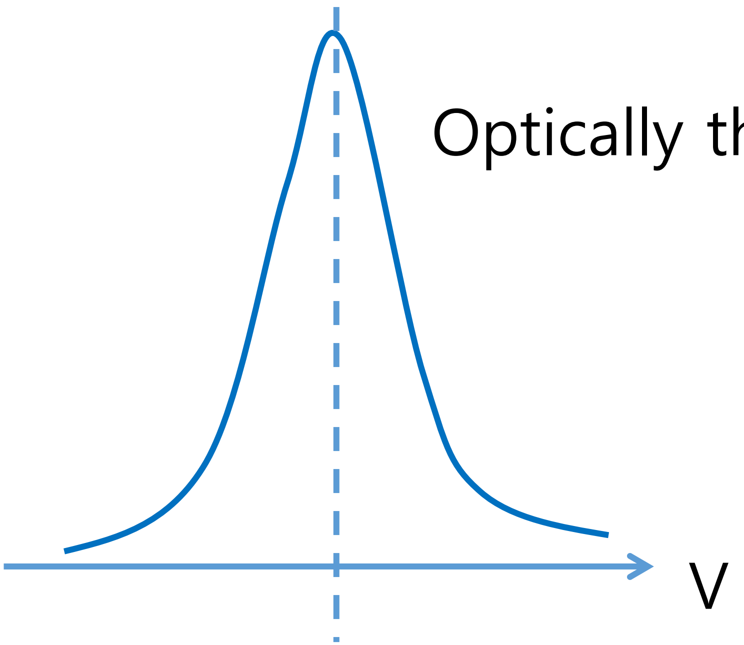
* In our simulations, $B_{0,sky} = 1$

Cho & Yoo (2016)

Then, how to find N?

➔We need to know N!
(N=number of independent eddies along the LOS)

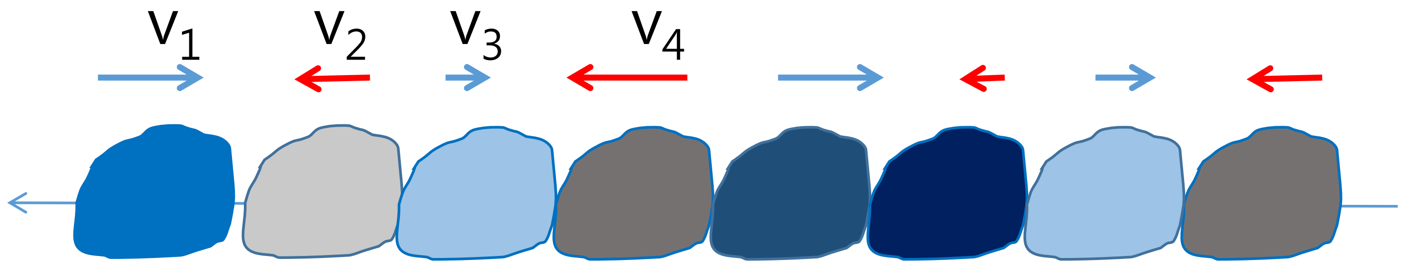
➔ We can get N from the standard deviation of centroid velocities



$$V_c = \frac{\int \rho v \ dz}{\int \rho \ dz}$$

Centroid velocity = average velocity

δV_C has something to do with $N^{1/2}$



V shows a random walk ➔ Let's consider $(V_1+V_2+V_3+\ldots)$

St Dev of $(V_1+V_2+V_3+\ldots) \sim N^{1/2}\,|V|$

➔ St Dev of $(V_1+V_2+V_3+\ldots)/N \sim N^{-1/2}\,|V|$

δV

➔ $\delta V_C \sim N^{-1/2}\,|V|$

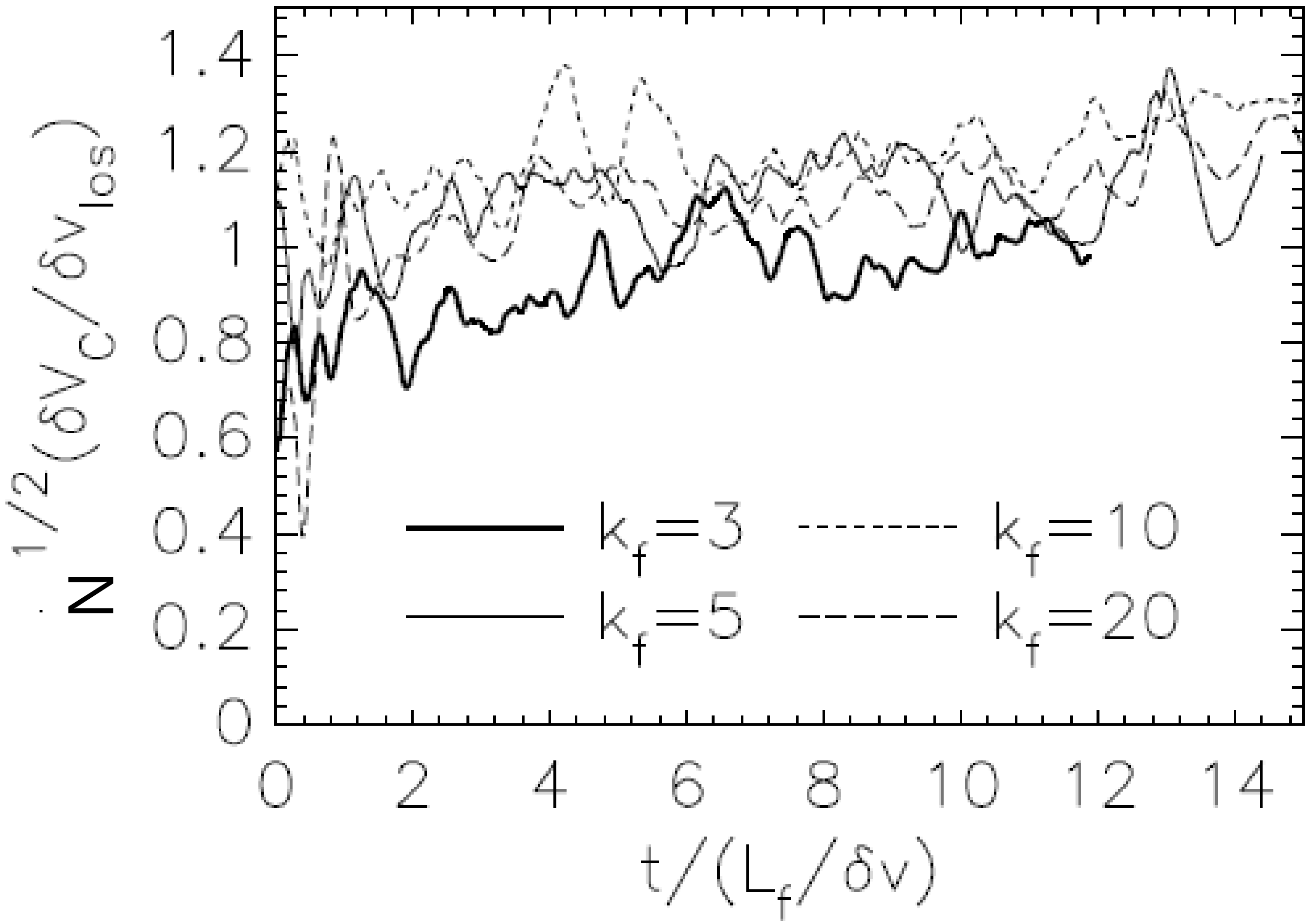
➔ $\delta V_C\,/|V| \sim 1/N^{1/2}$

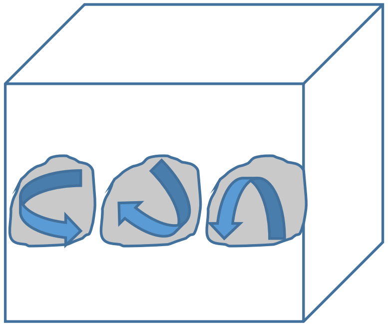
$$\boxed{\delta V_C\,/\delta V_{los} \propto 1/N^{1/2}}$$

Standard deviation of cent. vel.

Average width of an optically thin line

$$\delta V_C / \delta V_{los} \propto 1/N^{1/2} \rightarrow (\delta V_C / \delta V_{los}) N^{1/2} \sim \text{const}$$





$* \ N = 1/k_f$

$* \ N = 1/k_f$

Cho & Yoo (2016)

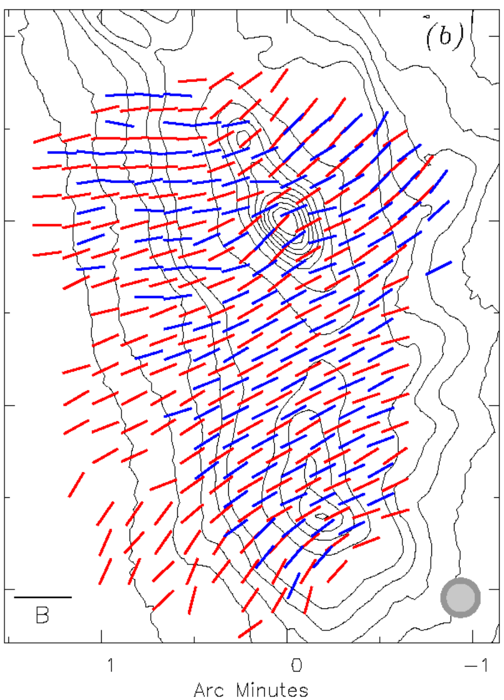
Is N really large?

Table 1
Results from Our Fit of Equation (53) to the Dispersion Data for OMC-1

Fit Result			Derived Quantities		
δ[a] (mpc)	$b^2(0)$[b]	a_2' (arcmin^{-2})	N[c]	$\langle B_t^2\rangle / \langle B_0^2\rangle$[d]	$\langle B_0^2\rangle^{1/2}$[e] ($\mu$G)
16.0 ± 0.4	0.0134 ± 0.001	0.059 ± 0.001	20.7	0.28 ± 0.01	760

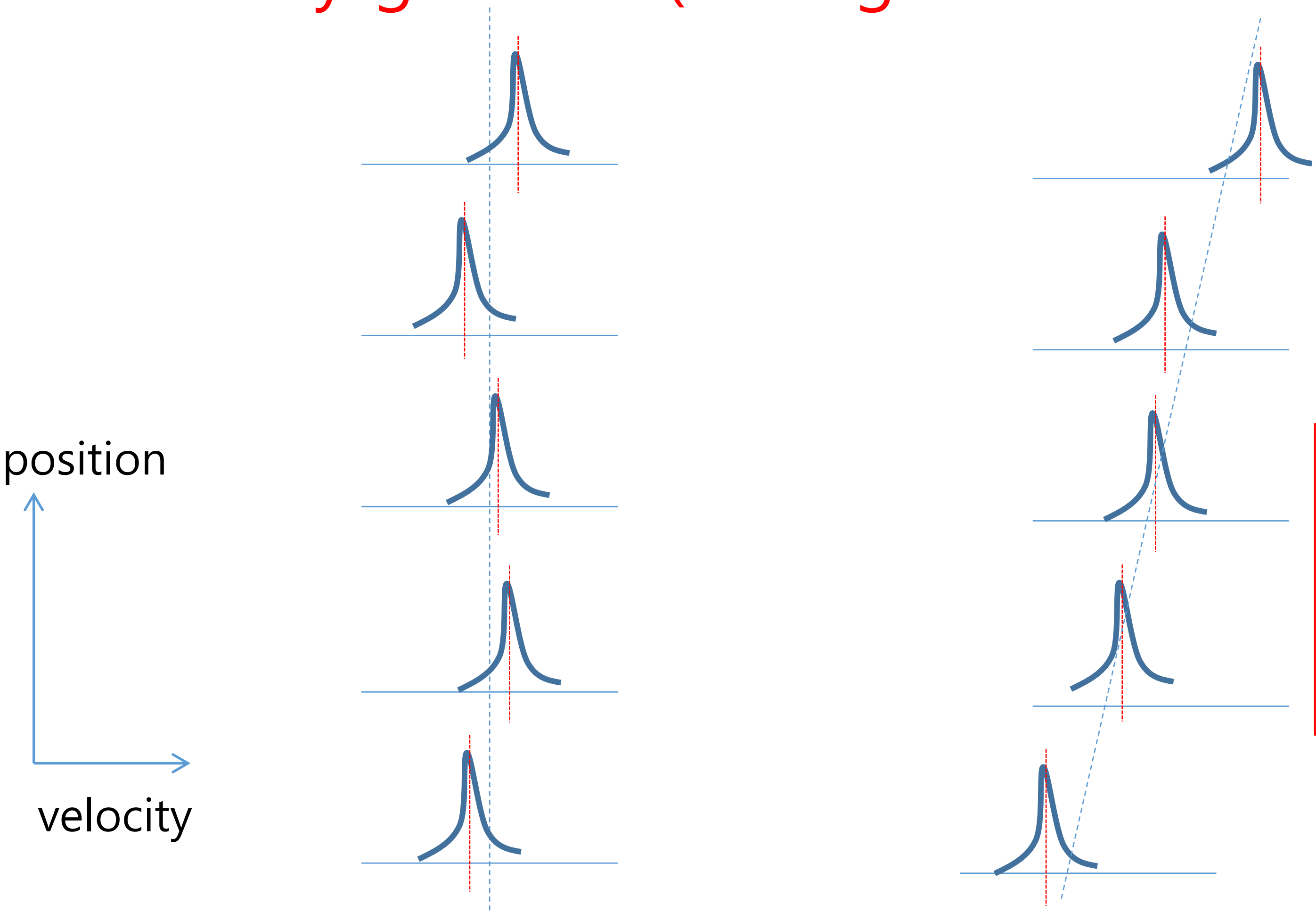
350μm

450μm



Houde + (2009)

A possible problem with our technique:
it doesn't work well if there is a large-scale velocity gradient (or large-scale variation; LSV)



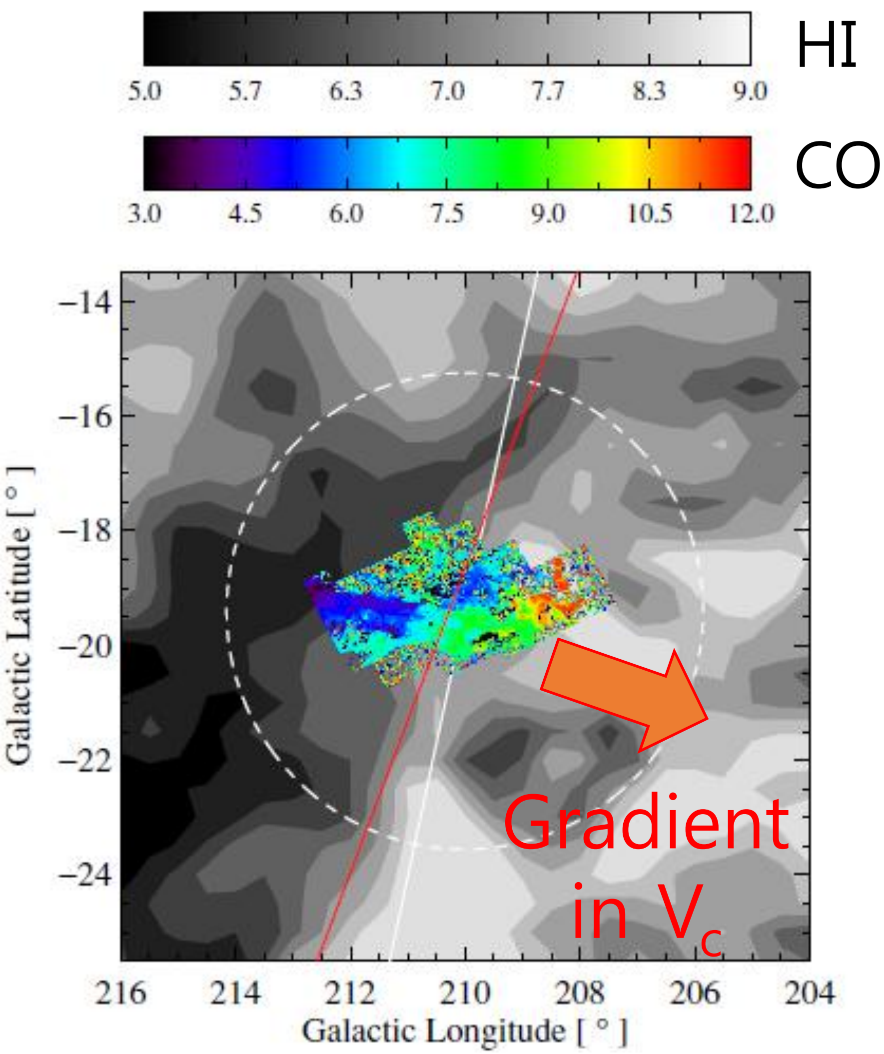
Without LVG

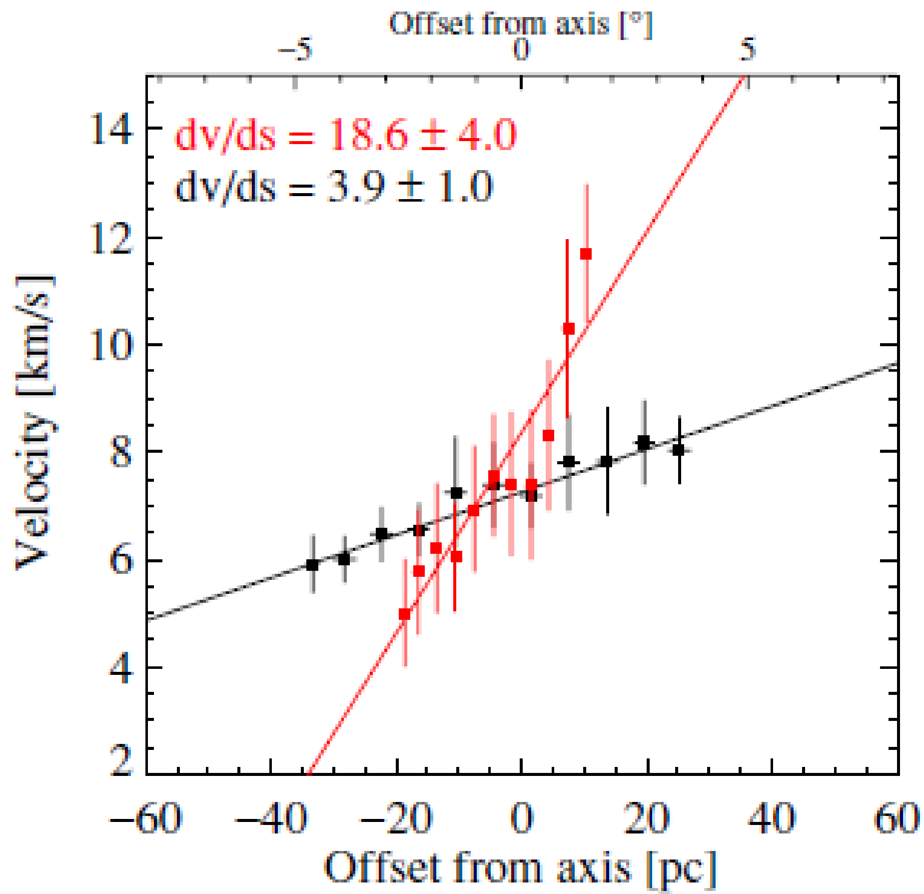
With LSV

Calculation of δV_C can be contaminated by LSV

Large-scale variations are commonly observed

e.g.) Large-scale velocity gradients in giant molecular clouds
➔ Make it difficult to measure δV_c

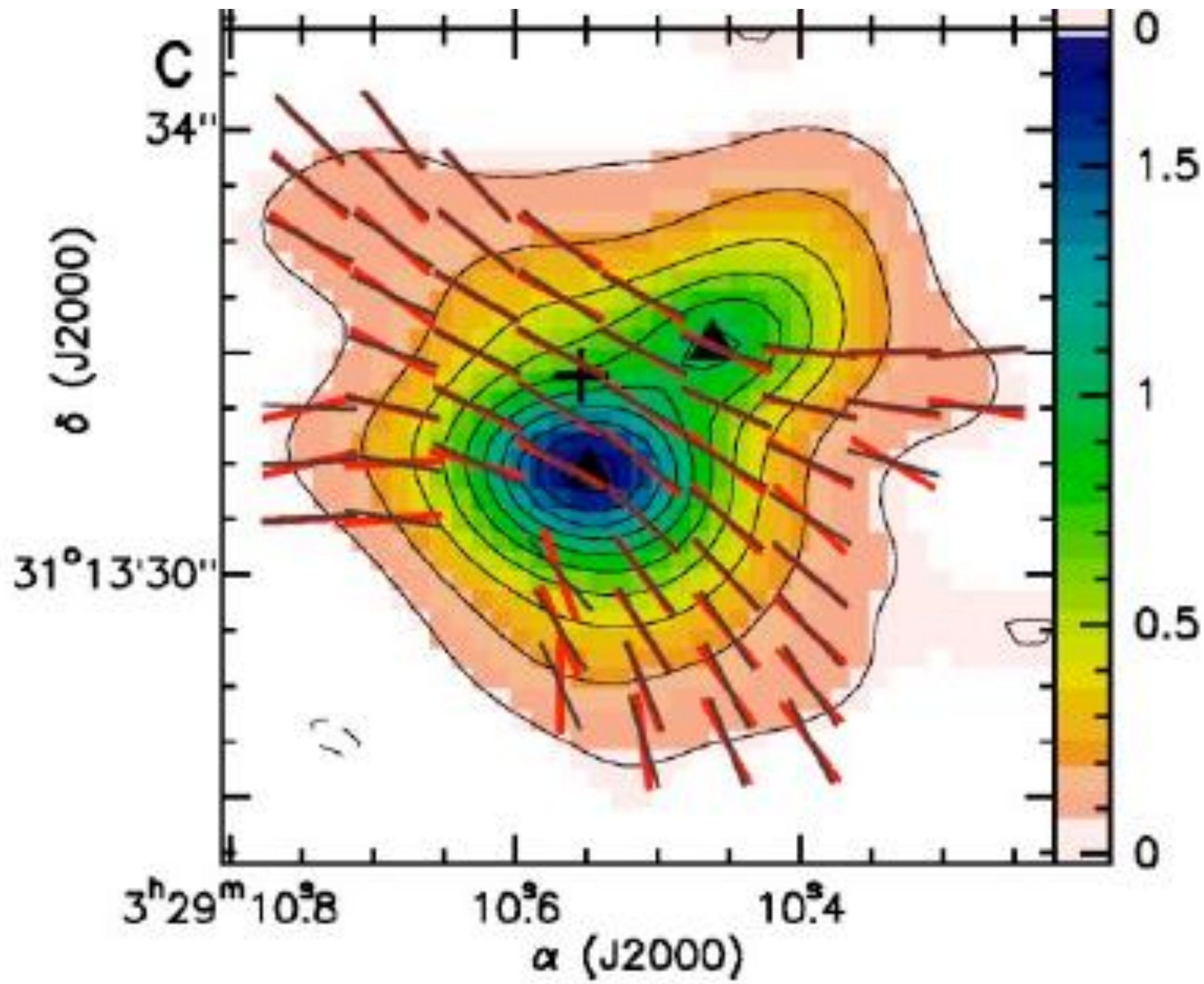




Large-scale variations are commonly observed

e.g.) Hour-glass morphology of B

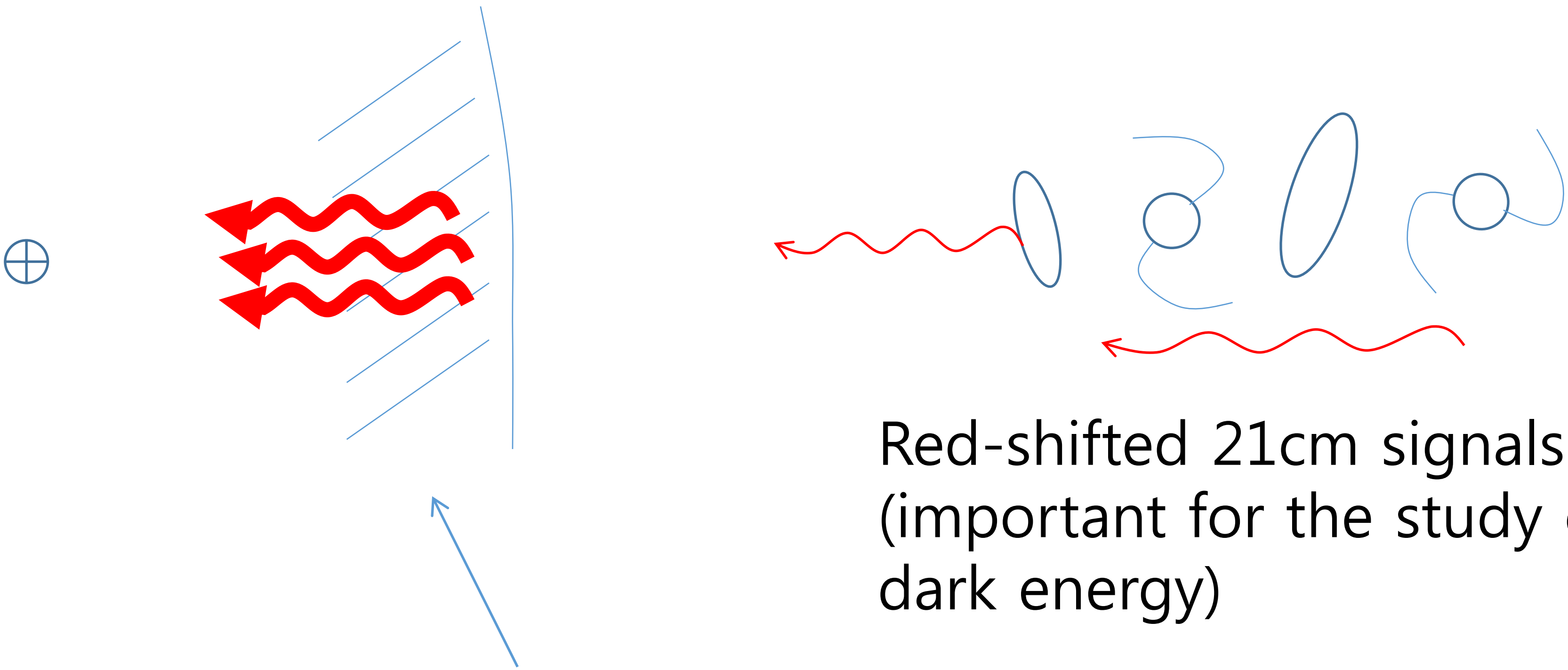
➔ Makes it difficult to accurately measure $\delta\phi$



Girart+(2006)

Large-scale variations are commonly observed

e.g.) red-shifted 21cm observations



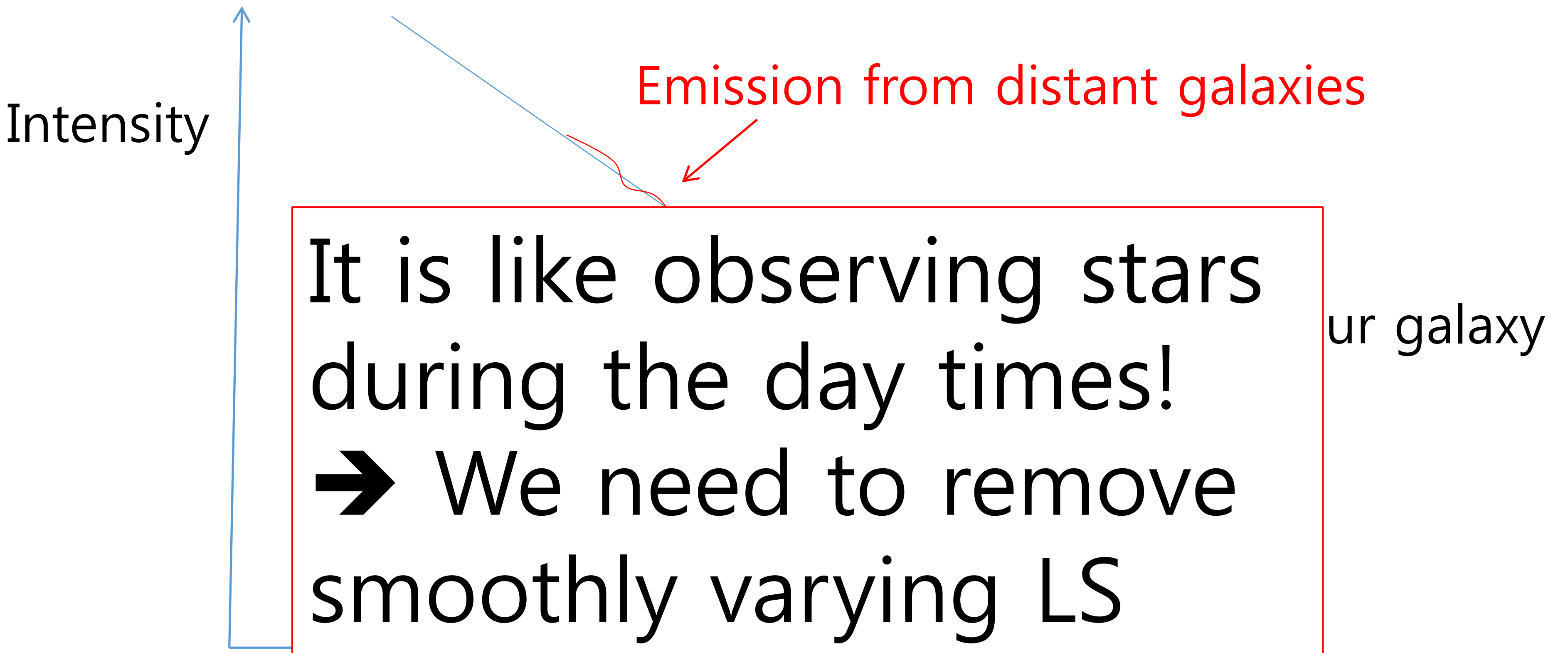
Red-shifted 21cm signals (important for the study of dark energy)

Galactic foreground emissions
(mostly synchrotron)

➔ Much stronger than cosmological signals

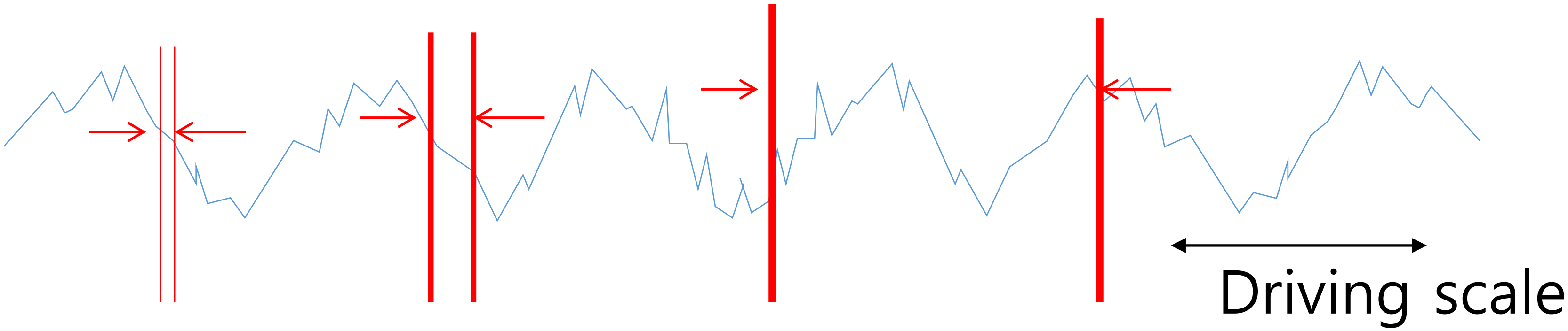
Large-scale variations are commonly observed

e.g.) red-shifted 21cm observations



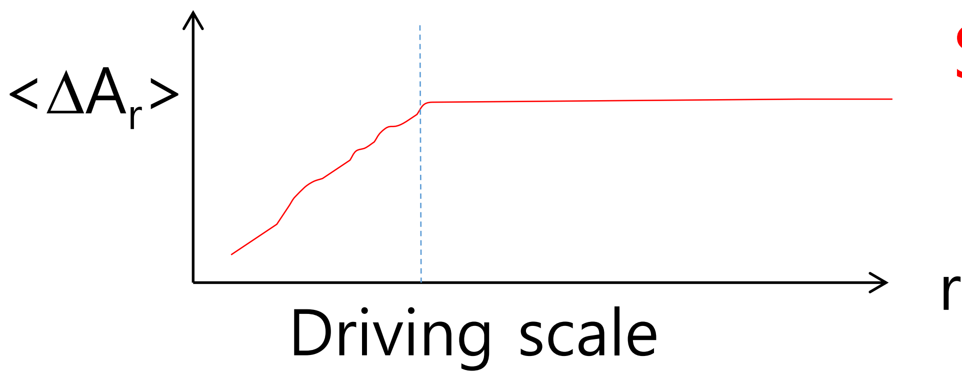
It is like observing stars during the day times!
➔ We need to remove smoothly varying LS variations

Then, is it possible to remove LSV?



Let's consider a quantity $\Delta A_r = |\ A(x+r) - A(x)\ |$.

➔ How does ΔA_r change as r increases?



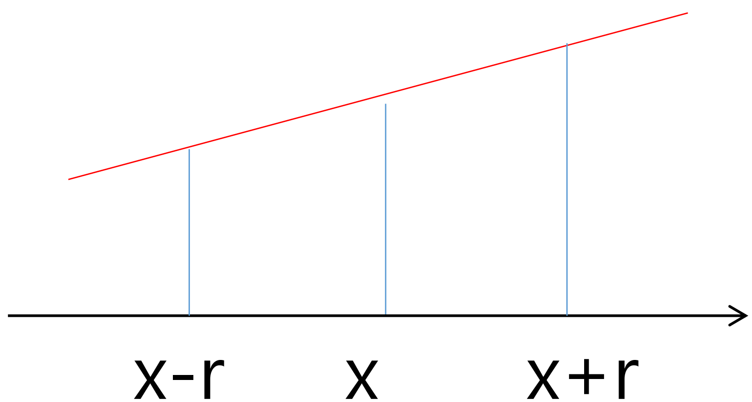
Structure Function: typical fluctuation as a function of separation

➔ (2-point) 2nd-order SF: $SF_2(r) = \langle \Delta A_r^2 \rangle = \langle |A(\mathbf{x}+\mathbf{r}) - A(\mathbf{x})|^2 \rangle$

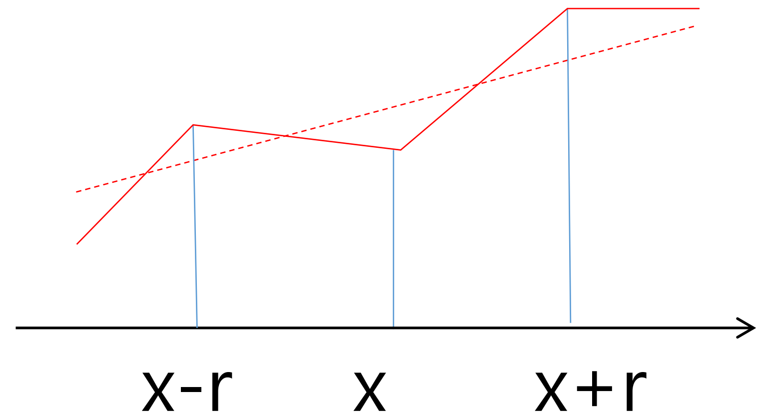
Our method of removing LSV: Multi-point SF!

Example: 3-point SF

$$\mathrm{SF}_2(r) = \langle |A(\mathbf{x}+\mathbf{r}) - 2A(\mathbf{x}) + A(\mathbf{x}-\mathbf{r})|^2 \rangle ,$$

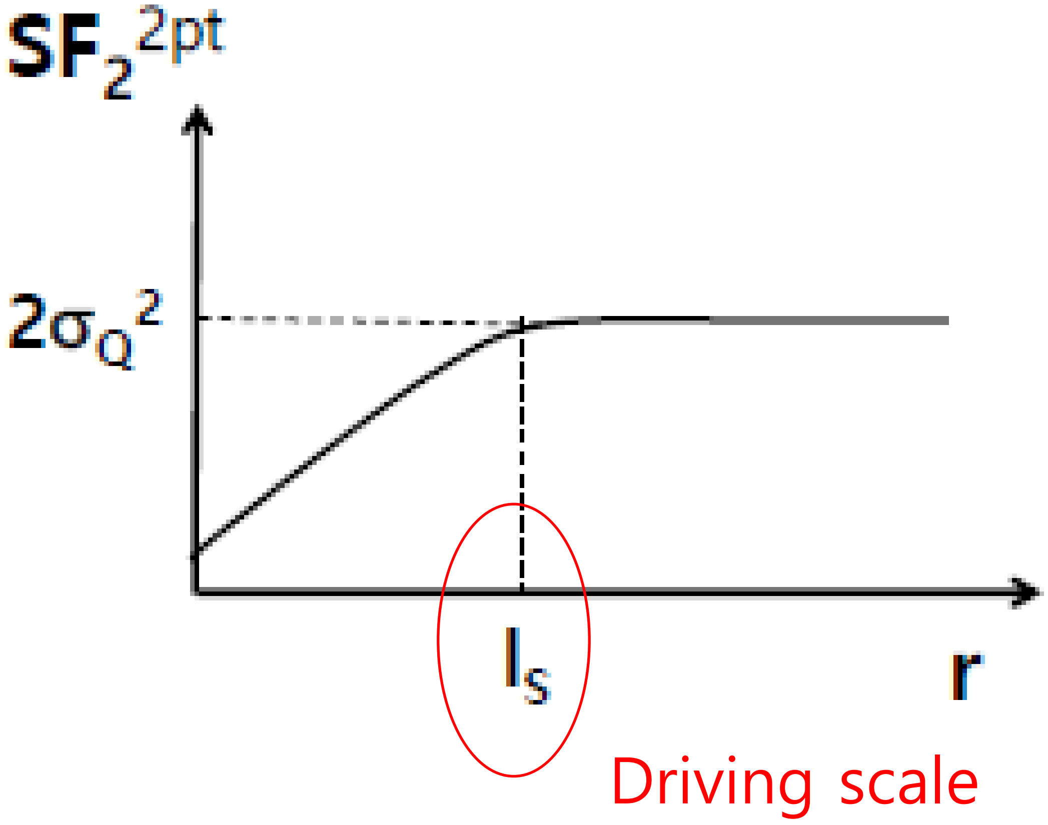


If there is a constant gradient, the gradient can be completely removed by the SF!



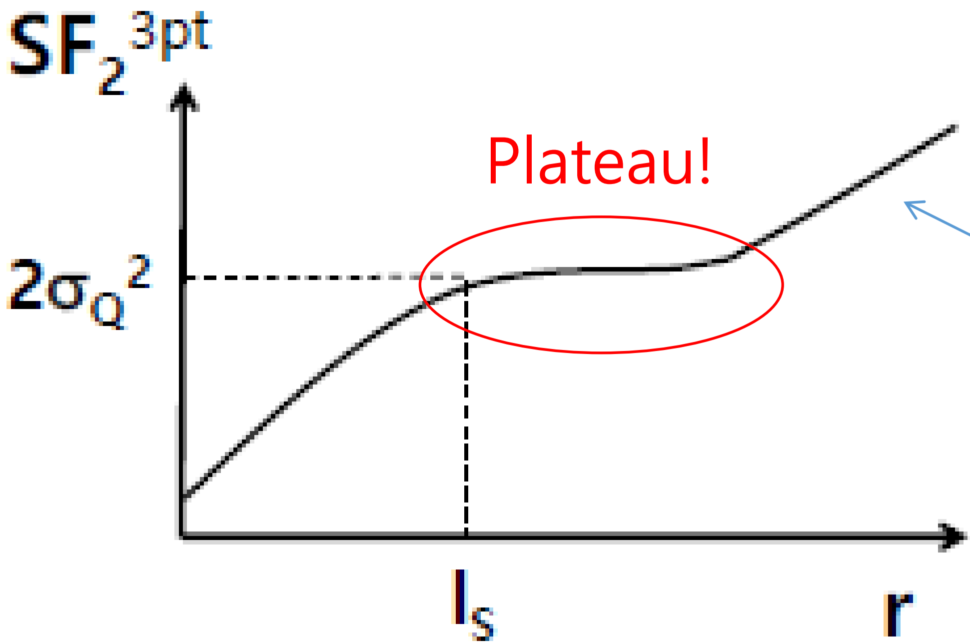
Nevertheless, the SF can still capture fluctuations by turbulence!

Behavior of Multi-point SF



If there are small-scale fluctuations only, …

Behavior of Multi-point SF



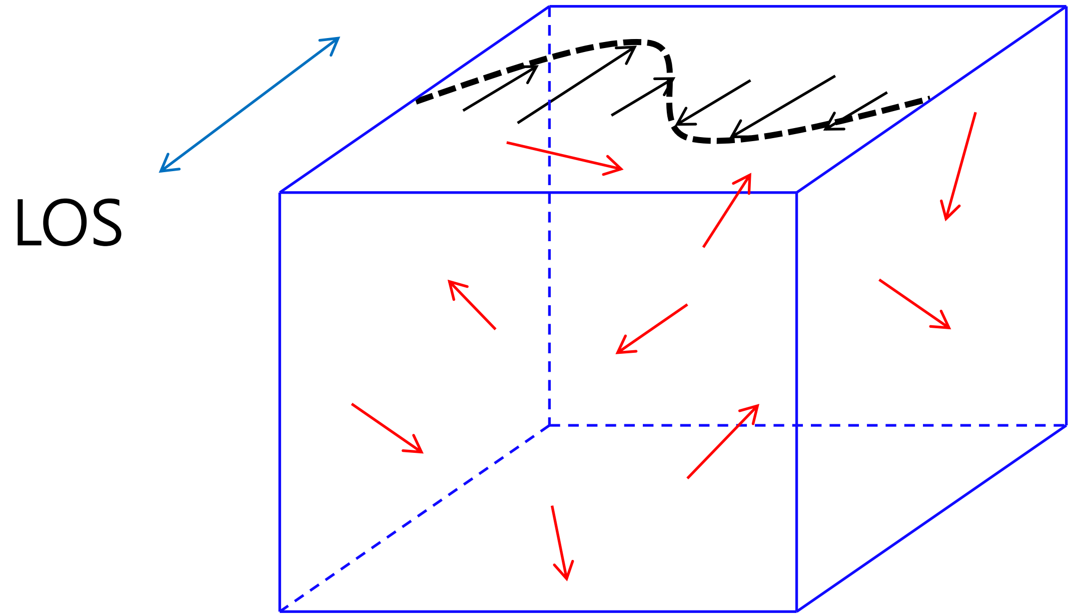
Ultimately, effects of large-scale variations emerge

If we remove most of the large-scale variations,…

So, is it possible to remove LSV?

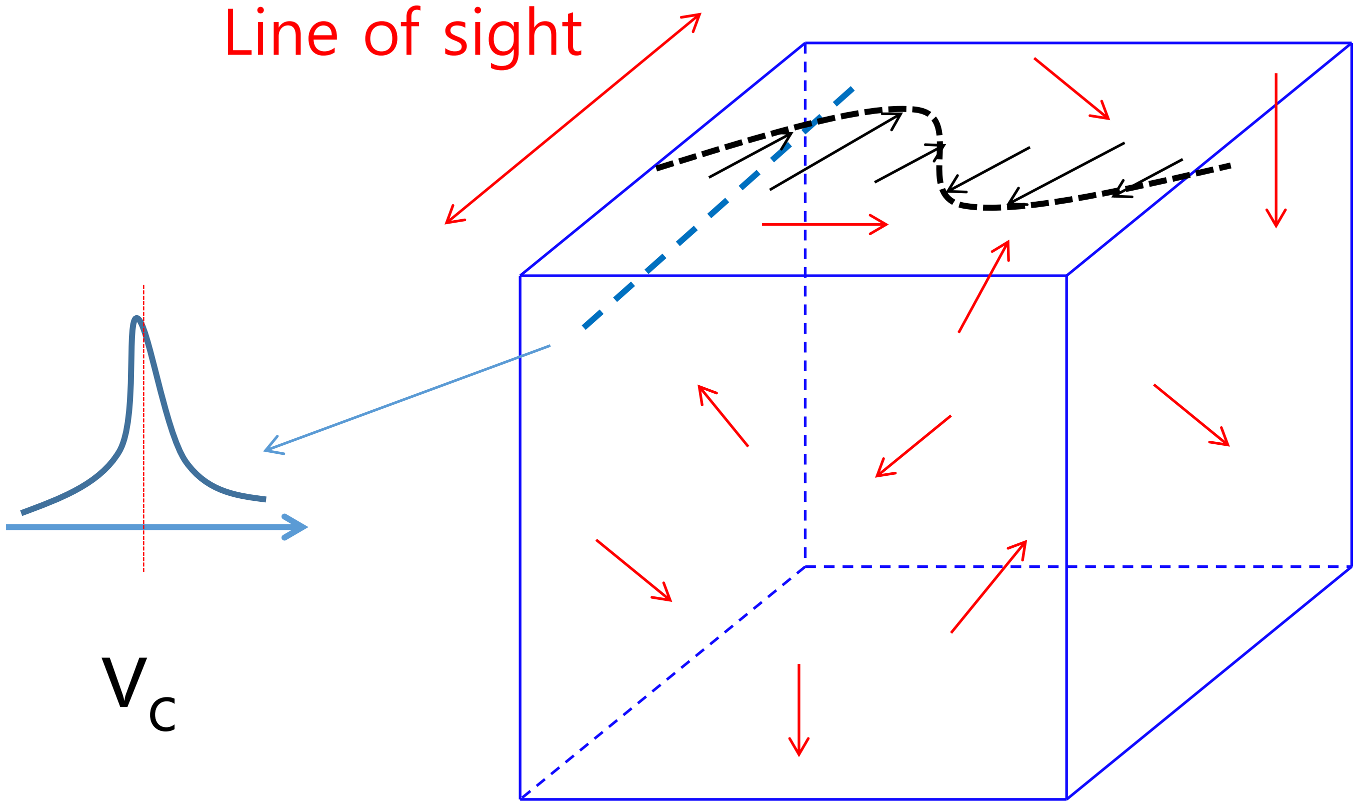
We test the possibility using numerical simulations.

Turbulence data + large-scale V of a sinusoidal form



* The sinusoidal wave mimics a large-scale velocity

Note: $\delta V_{turb} \sim 0.5$,
$\delta V_{sine\ wave} \sim 2$,

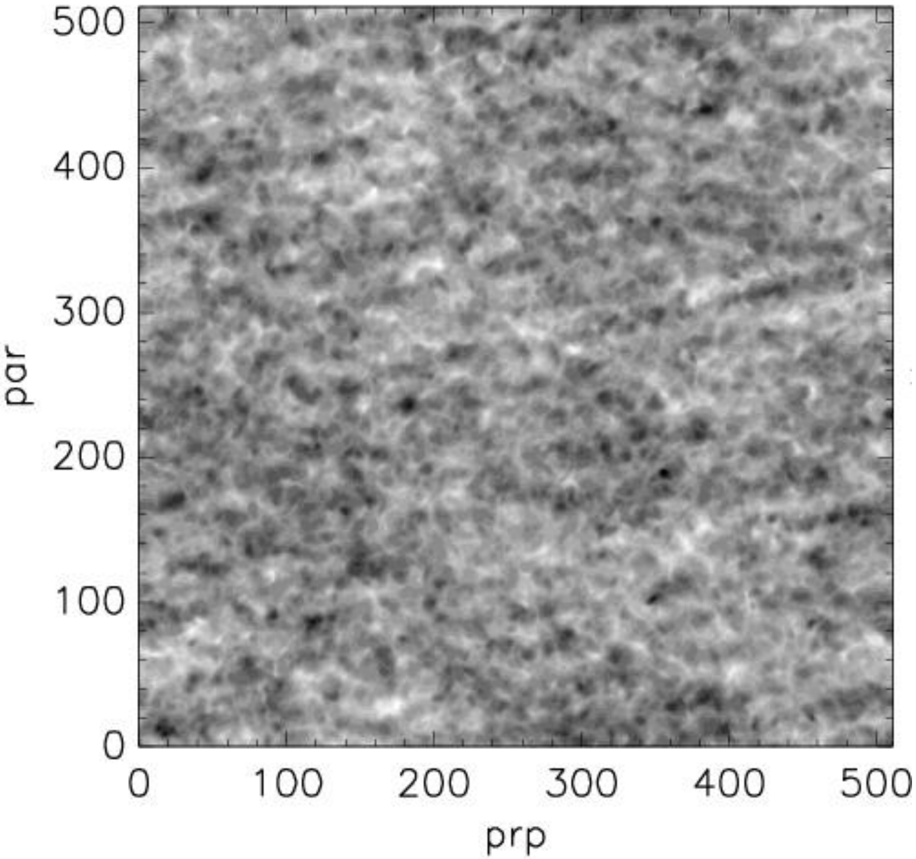


Resolution: 512^3

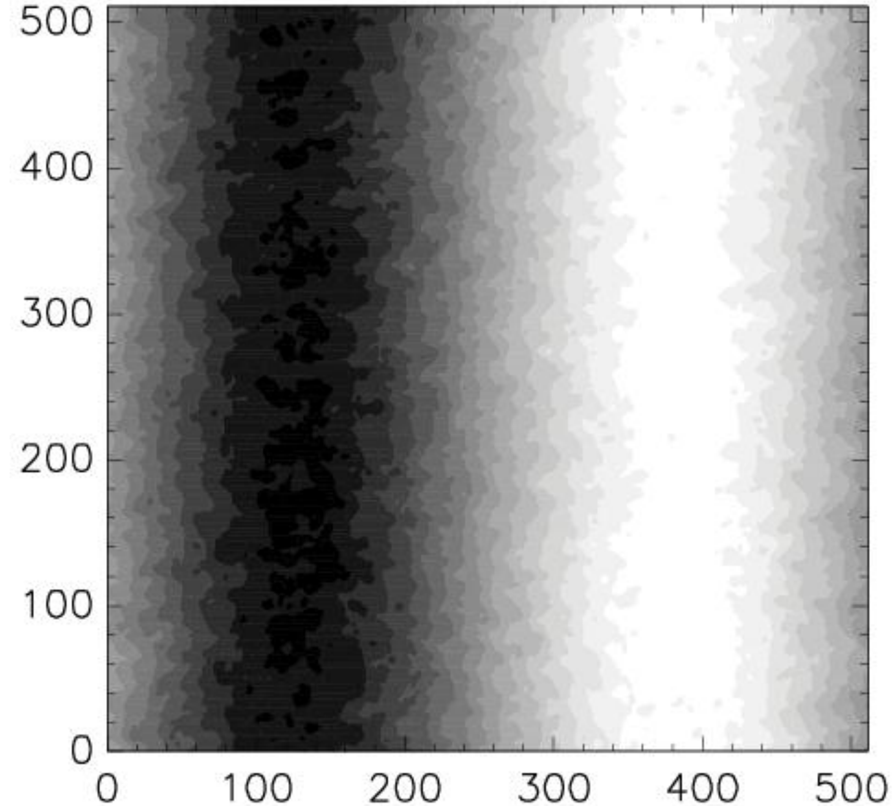
➔ We calculate V_c for 512^2 LOS's.

$$V_c = \frac{\int \rho v \ dz}{\int \rho \ dz}$$

Centroid Velocity V_c



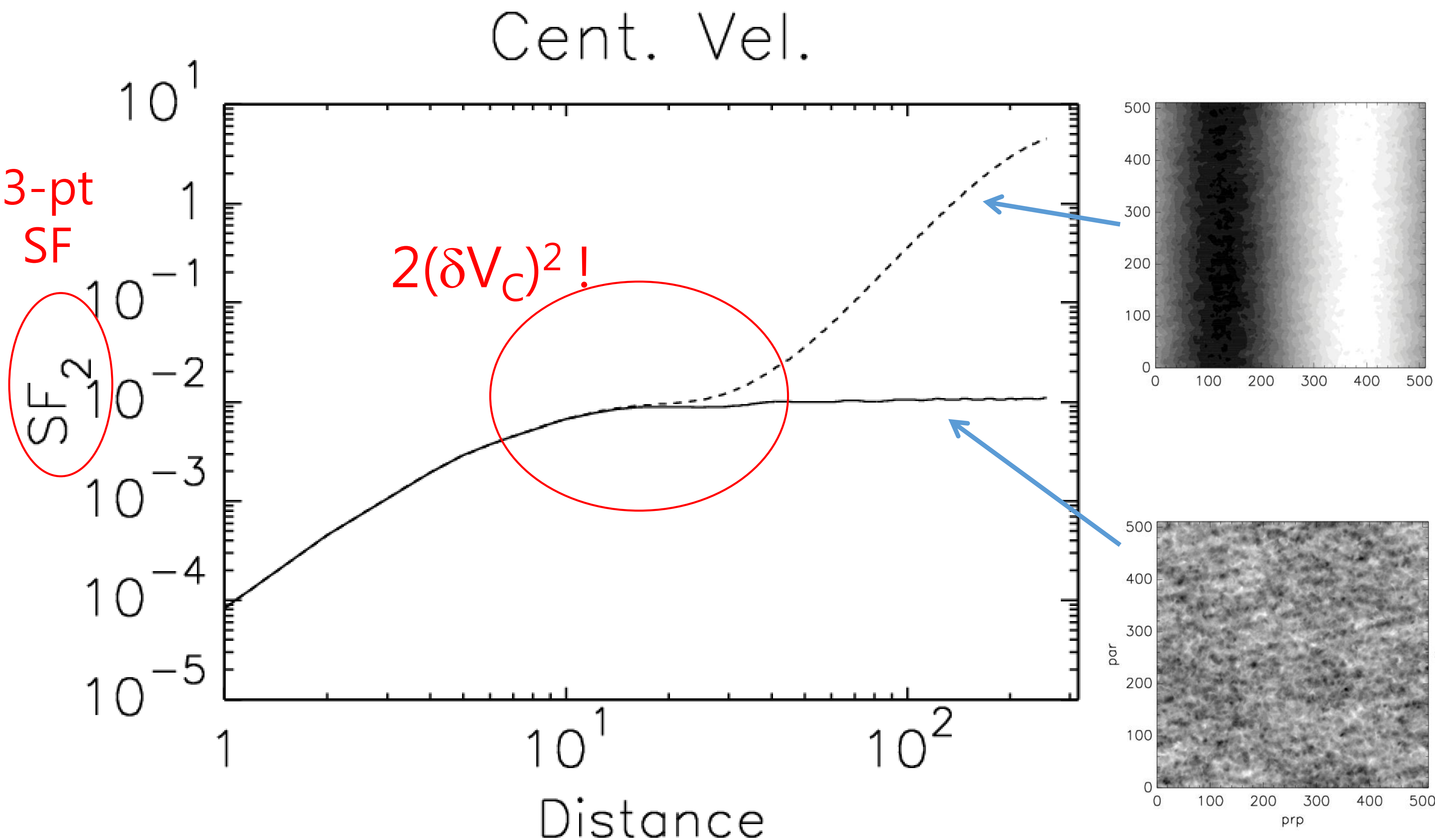
Small-scale turbulence data (Turbulence is driven at small scales)



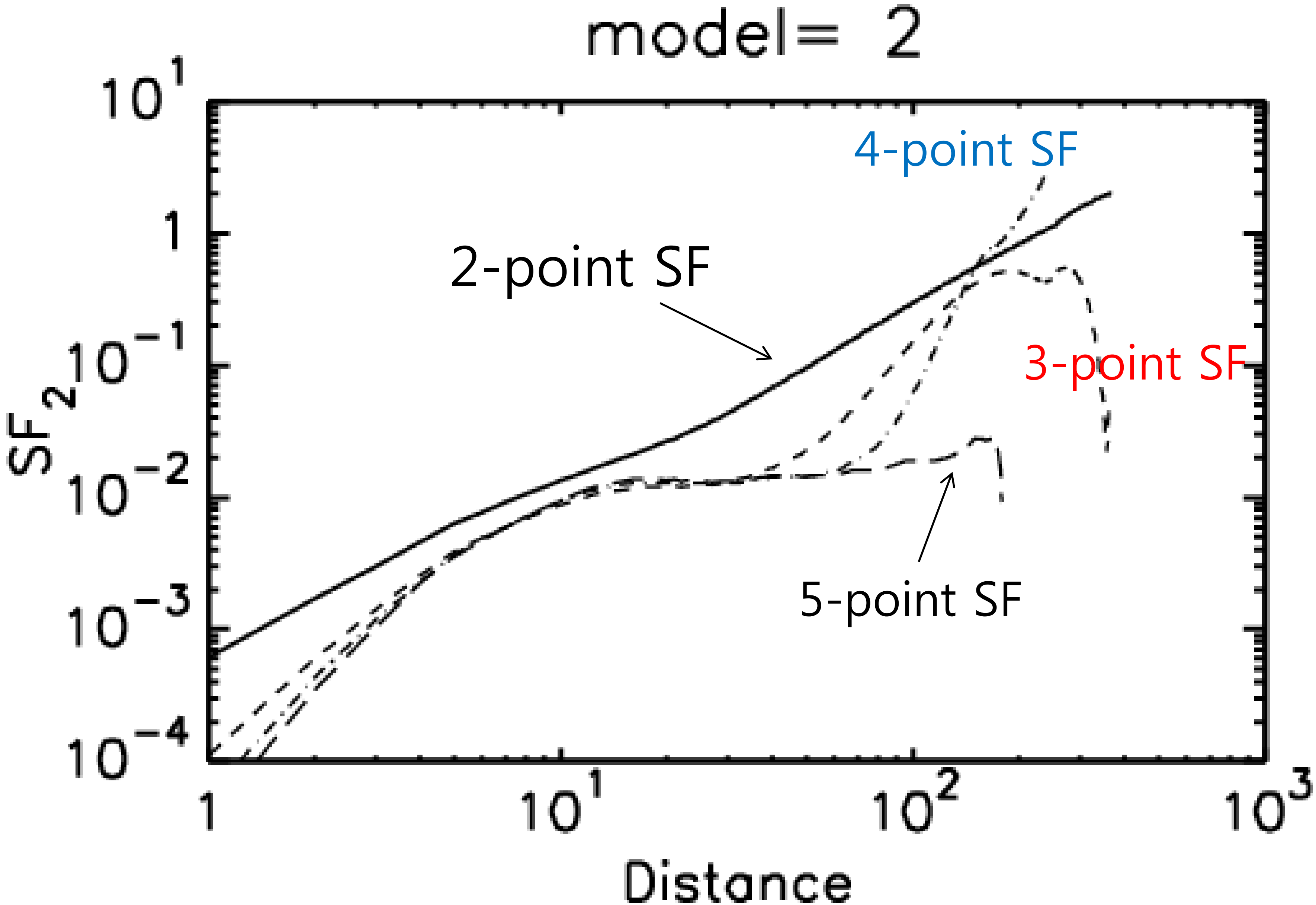
Small-scale turbulence data +sine wave

➔ Can we remove LSV?

3-Pt Structure function tells us about $\delta V_{C,\,turb}$!



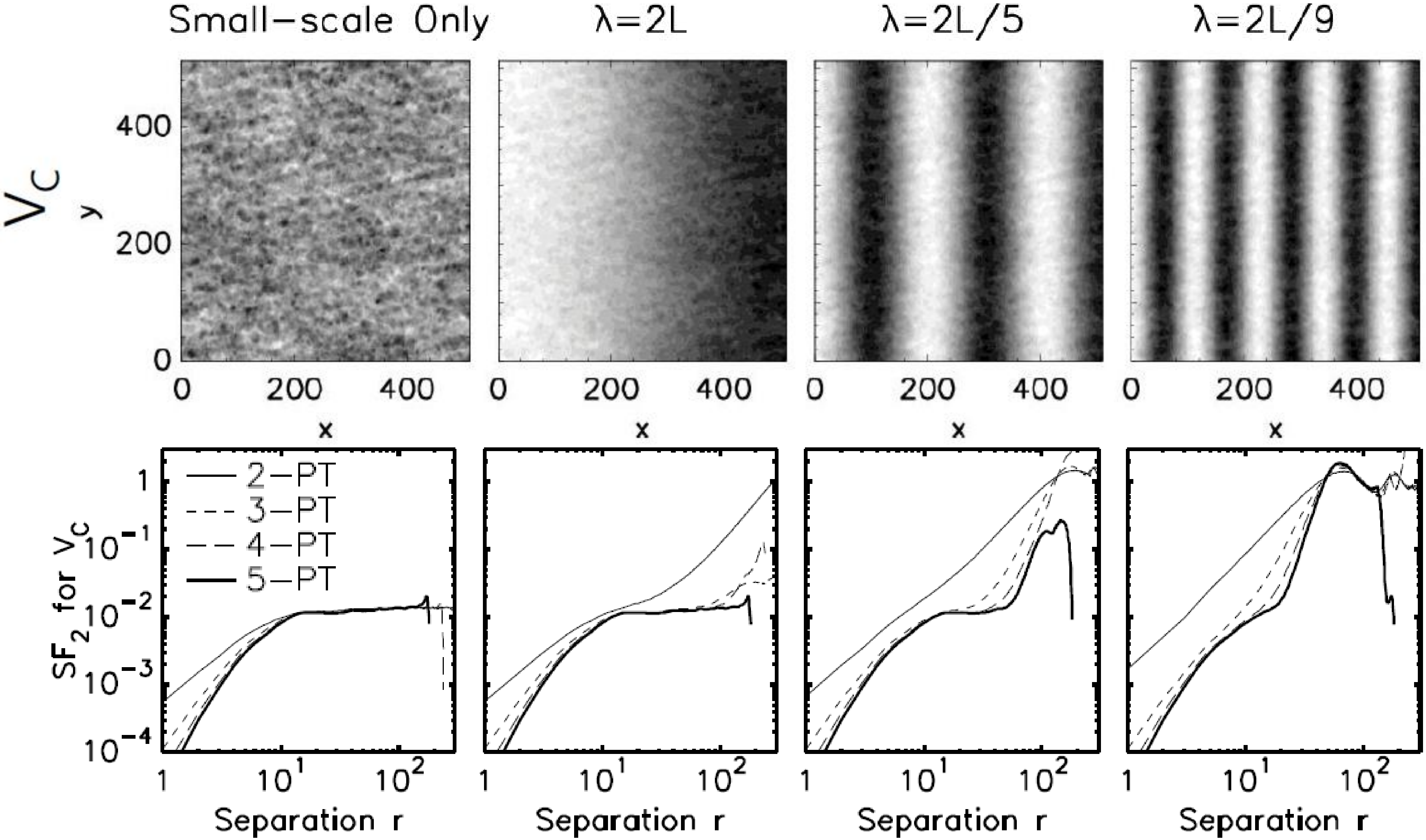
Cho (2017)



$$SF_2 = \langle |A(x+2r) - 3A(A+r) + 3A(x) - A(x-r)|^2 \rangle$$

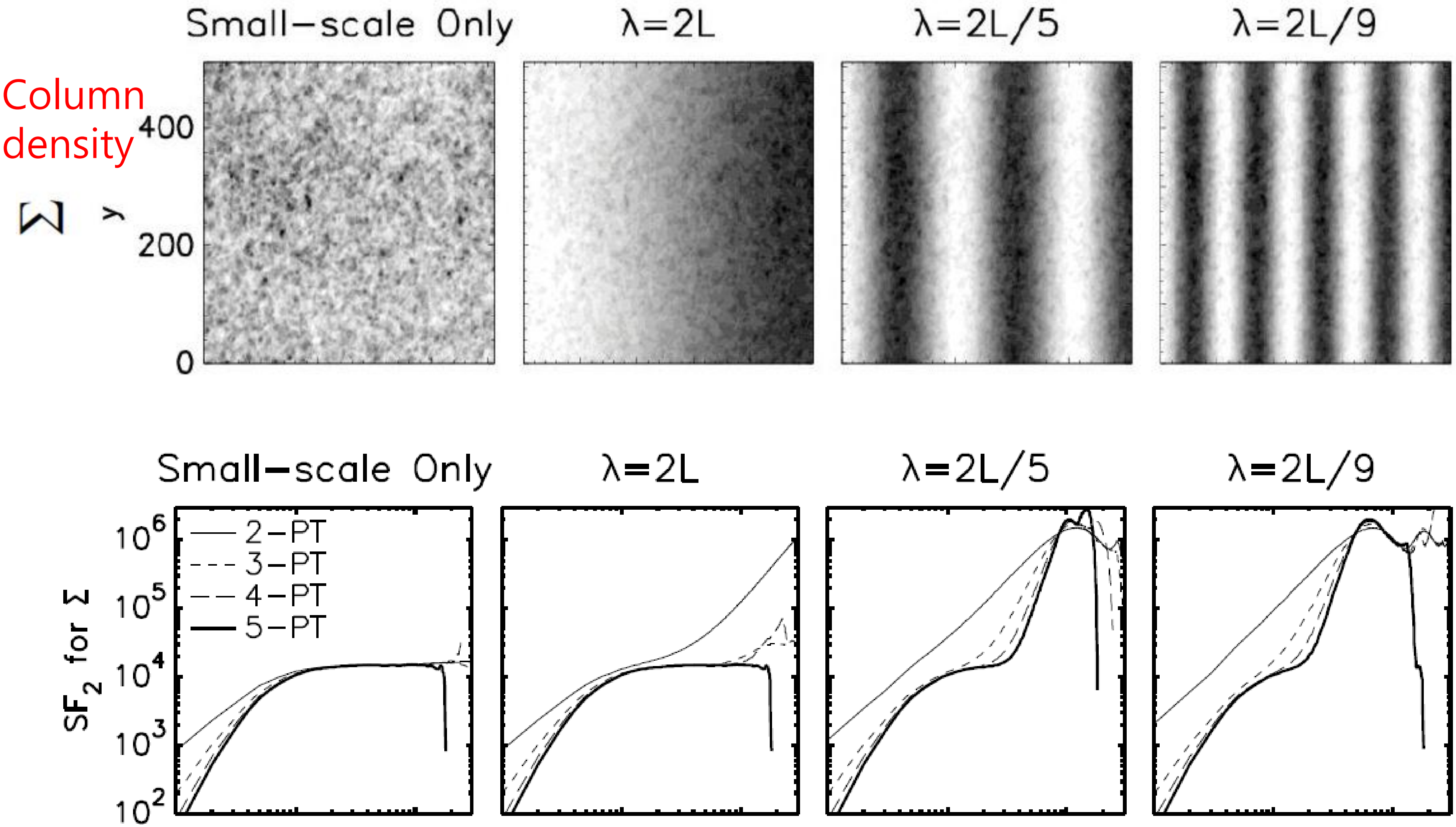
$$SF_2 = \langle |A(x+2r) - 4A(A+r) + 6A(x) - 4A(x-r) + A(x-2r)|^2 \rangle$$

More results

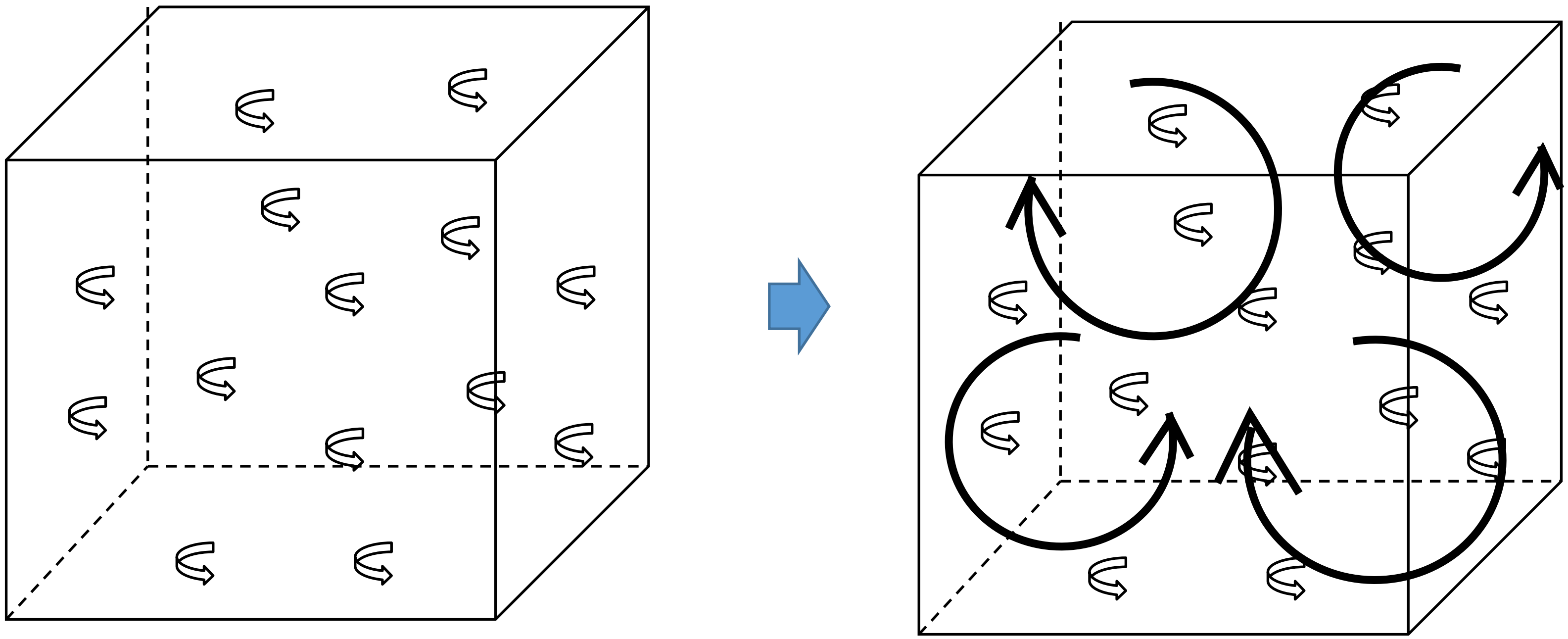


Cho (2017)

More results



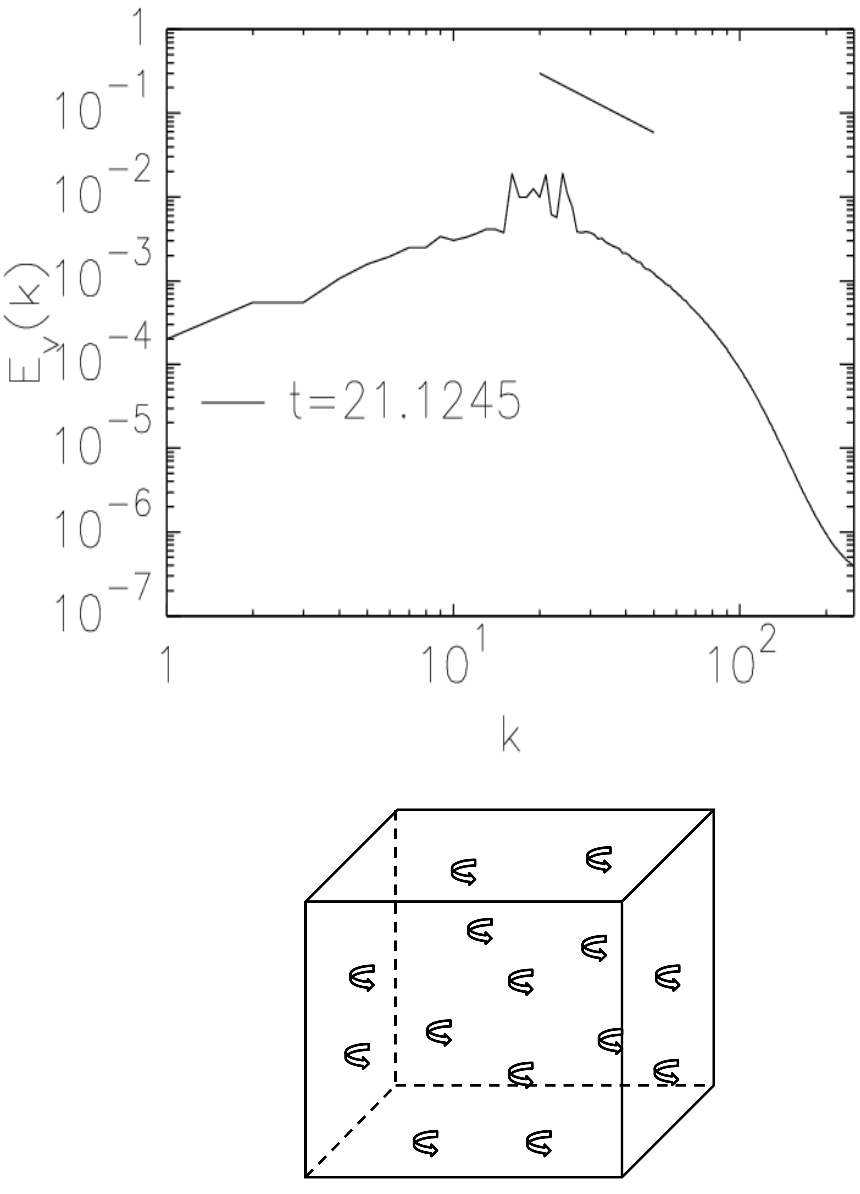
What if the large-scale V is complicated?

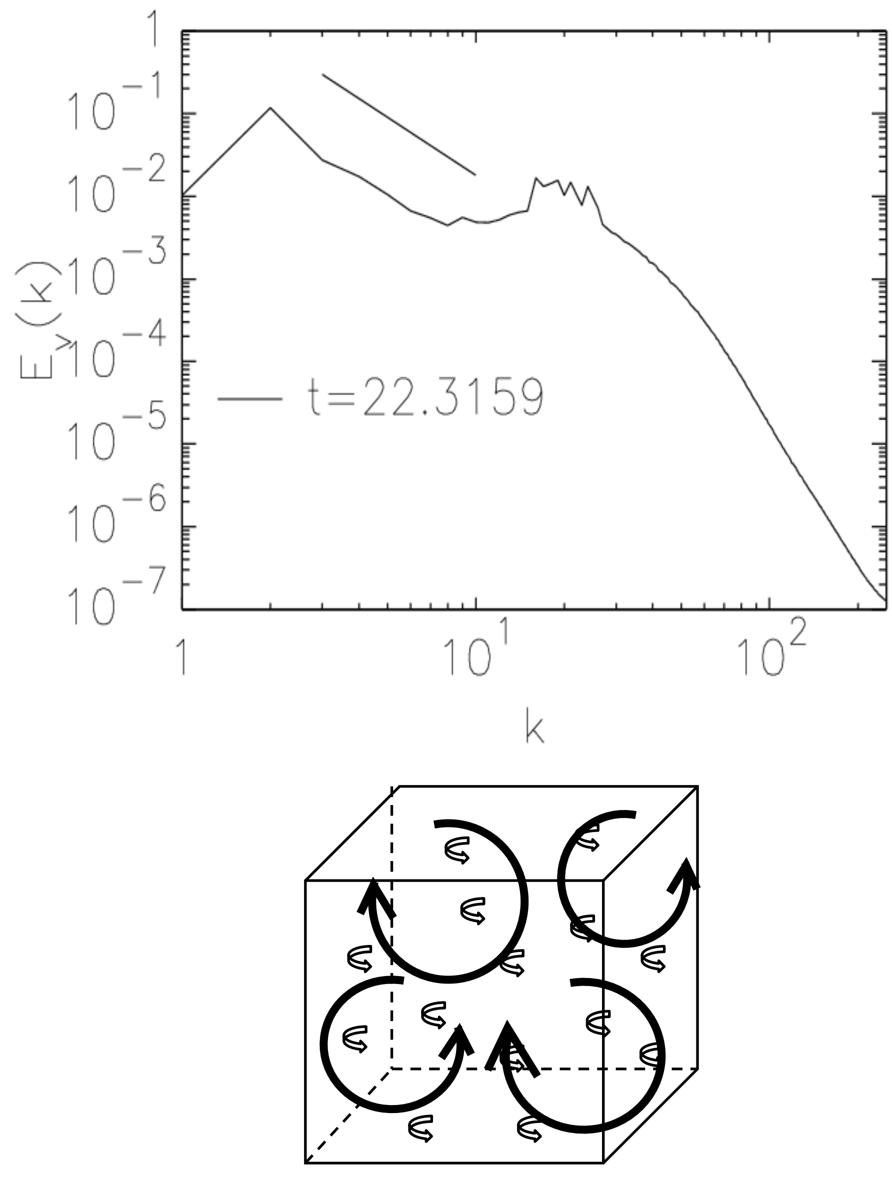


Turbulence with small-scale driving

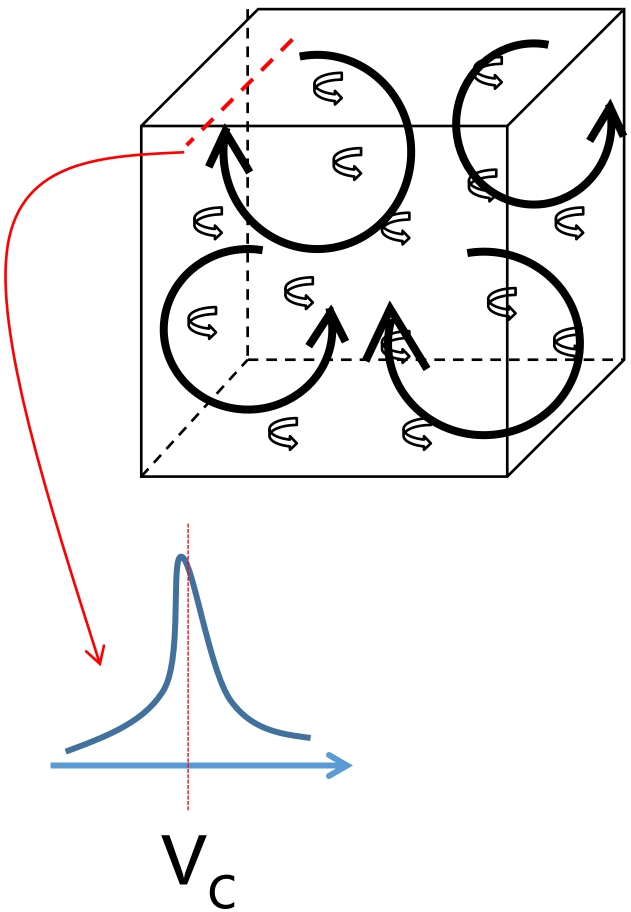
We apply an additional large-scale driving, which generates LVG

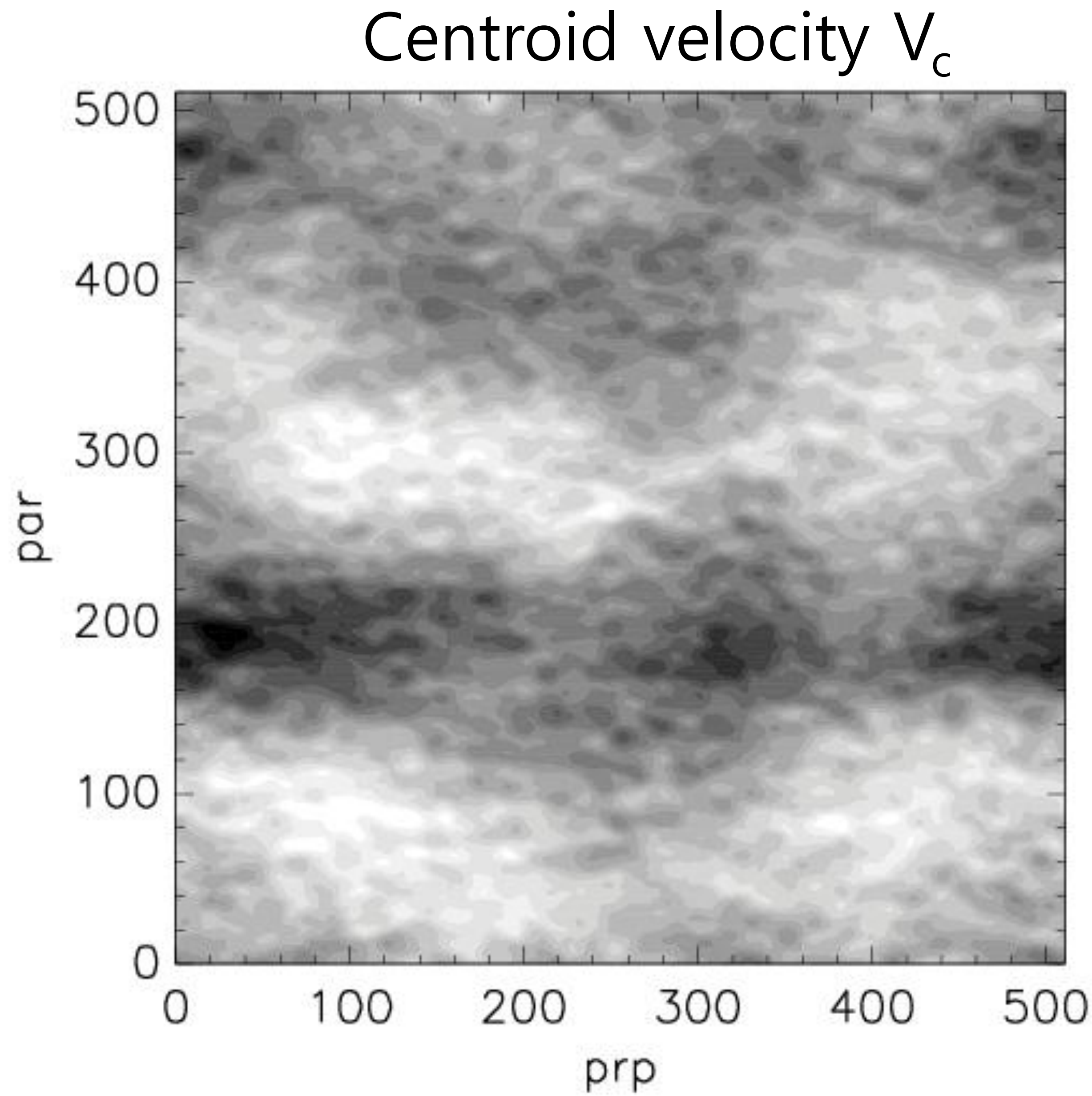
What if the large-scale V is complicated?

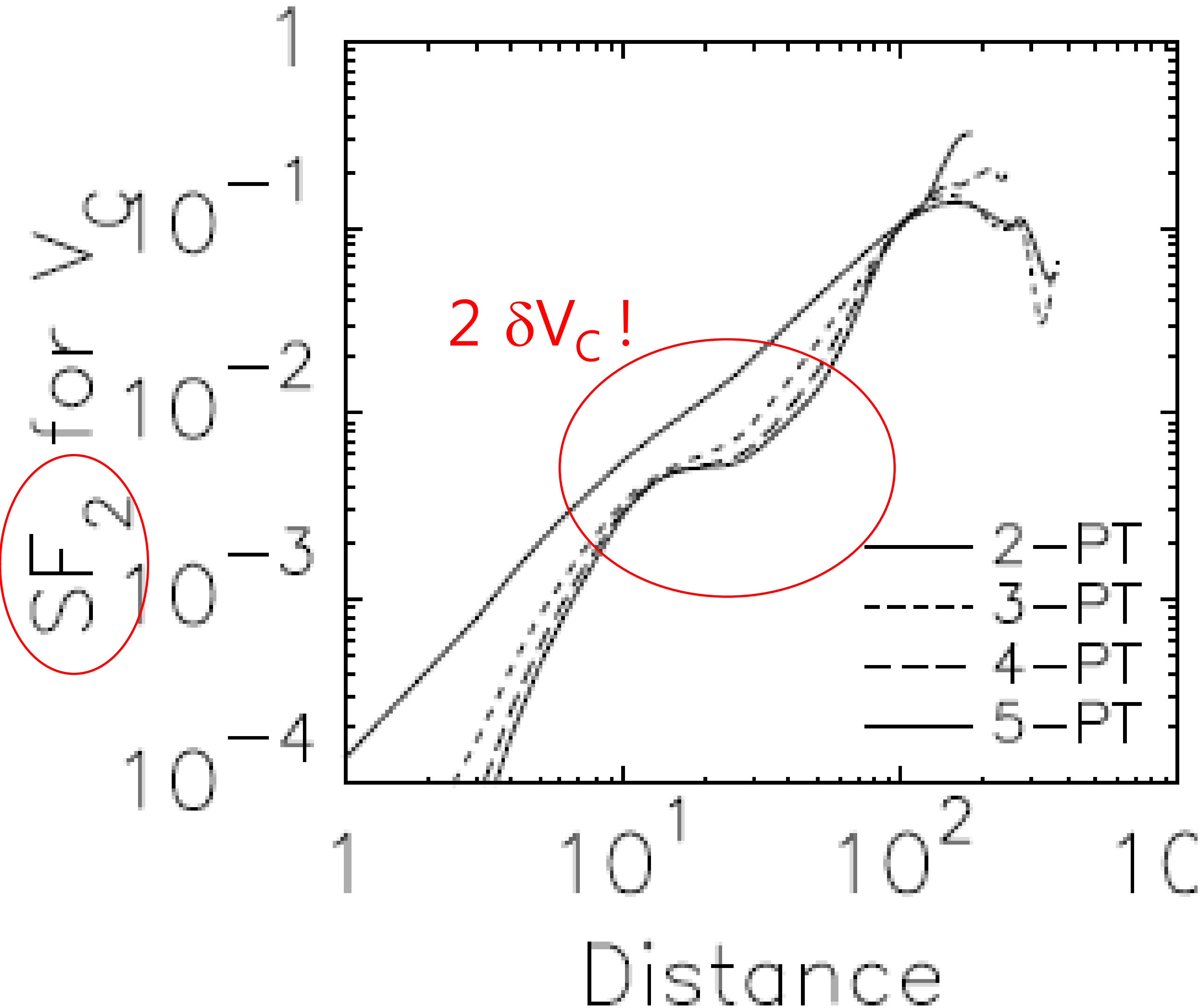


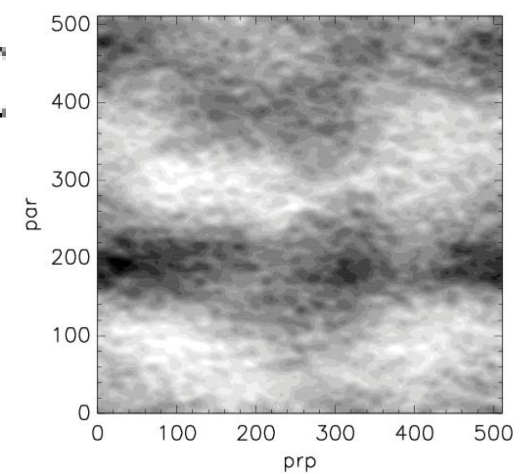


Can we remove the effect of LSV?









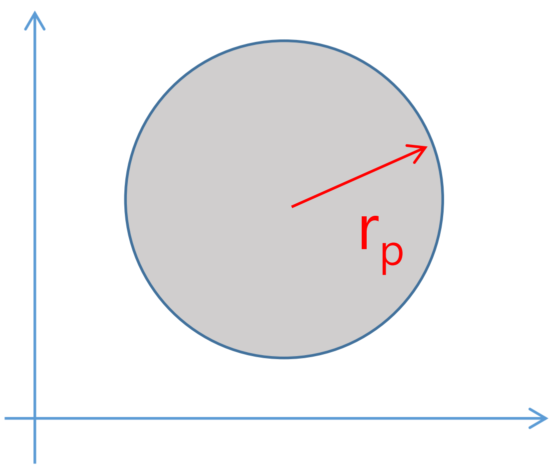
Cho (2017)

Can we obtain small-scale maps?

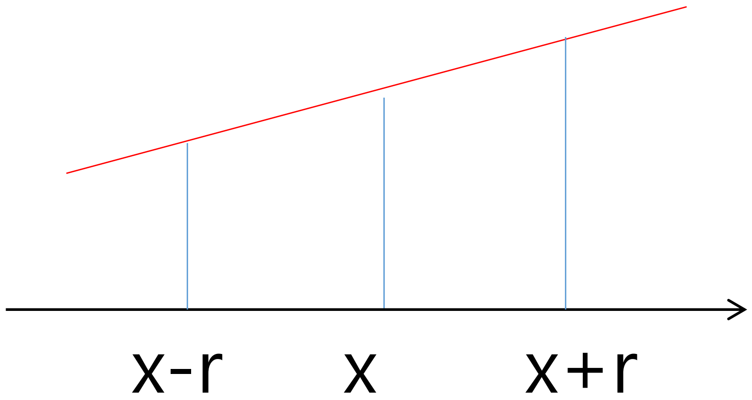
Since we can remove large-scale variations using multi-point SF, we may be able to obtain small-scale maps….

Our approach: multi-point average technique!

Usual (1-point) average: $$\bar{Q}(\mathbf{x}) = \sum_{|\mathbf{x}-\mathbf{x}'|<r_p} Q_L(\mathbf{x}')/N,$$



Can we obtain small-scale maps?



Suppose that a large-scale variation has a constant slope

$$\Rightarrow Q(\mathbf{x}-\mathbf{r}) - 2Q(\mathbf{x}) + Q(\mathbf{x}+\mathbf{r}) = 0$$

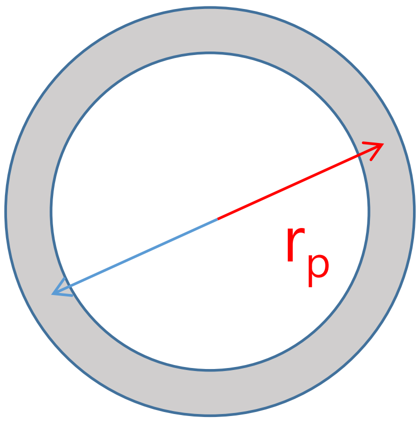
$$\Rightarrow Q_L(\mathbf{x}) = [Q_L(\mathbf{x}+\mathbf{r}) + Q_L(\mathbf{x}-\mathbf{r})]/2$$

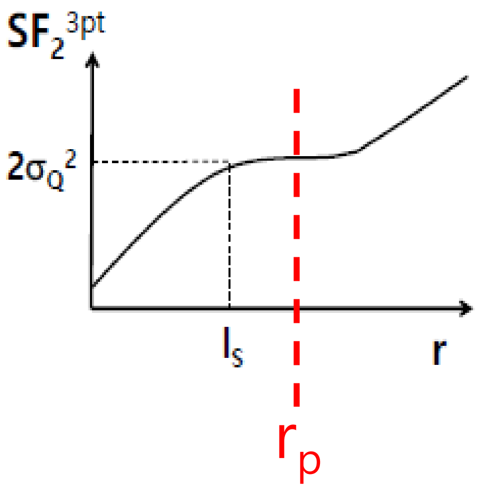
Obtaining small-scale maps

Our approach: multi-point average technique!

2-point average:

$$\bar{Q}(\mathbf{x}) = \sum_{r_p - \Delta < |\mathbf{r}| < r_p + \Delta} \left[Q_L(\mathbf{x}+\mathbf{r}) + Q_L(\mathbf{x}-\mathbf{r}) \right] / 2N$$



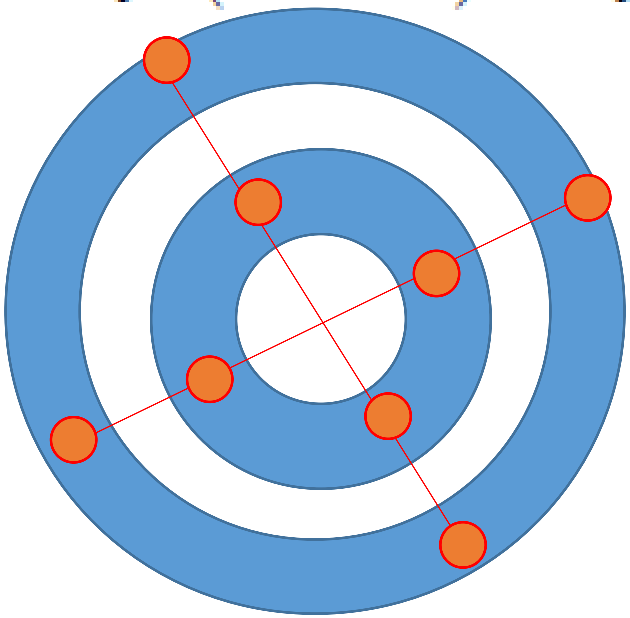


Obtaining small-scale maps

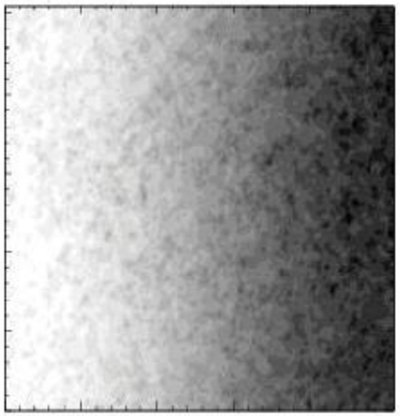
Our approach: multi-point average technique!

4-point average:

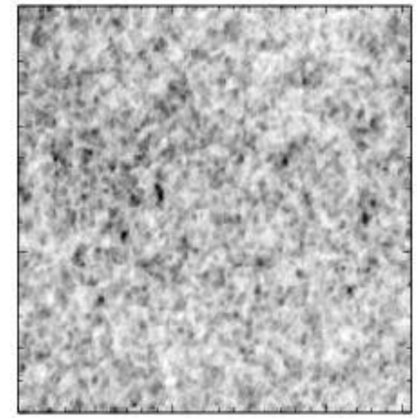
$$\bar{Q}(\mathbf{x}) = \sum_{r_p-\Delta<|\mathbf{r}|<r_p+\Delta} [4Q_L(\mathbf{x}+\mathbf{r}) + 4Q_L(\mathbf{x}-\mathbf{r}) \\ -Q_L(\mathbf{x}+2\mathbf{r}) - Q_L(\mathbf{x}-2\mathbf{r})] /6N$$



Obtaining small-scale maps

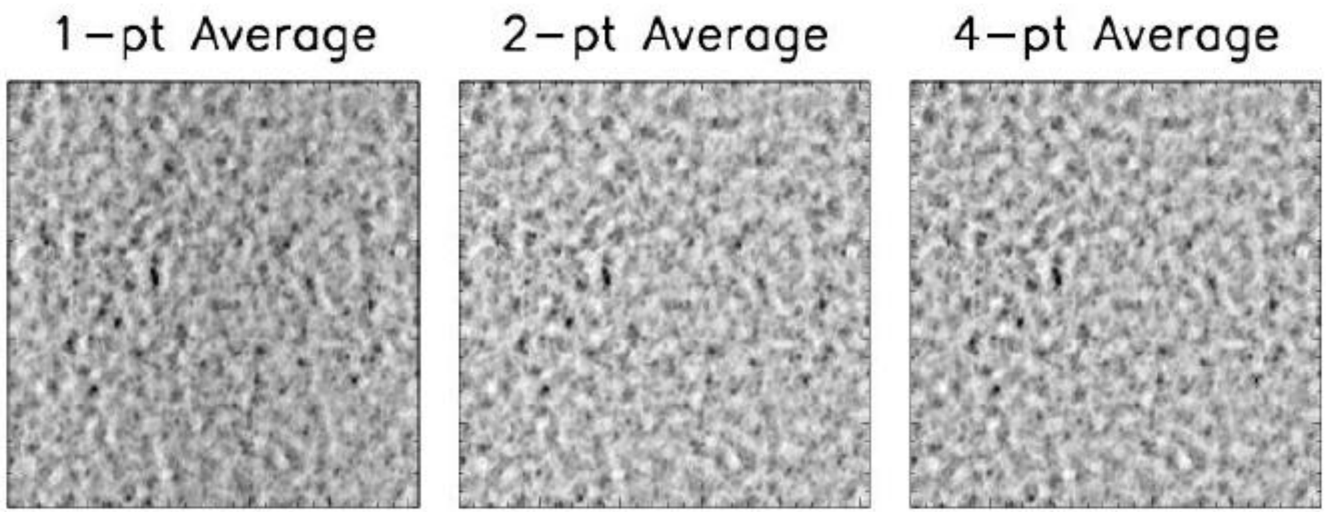


Can we remove the large-scale variation and retrieve the small-scale map?

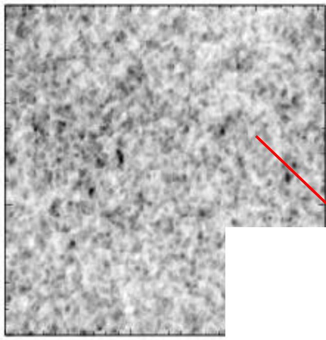


Large+small-scale map

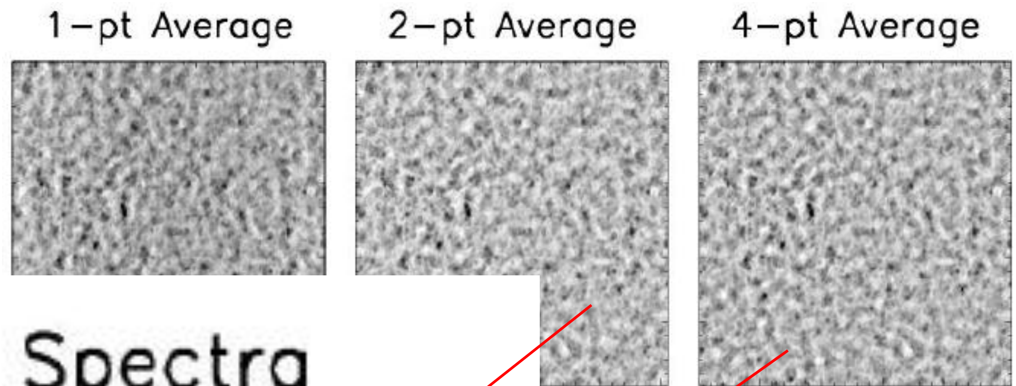
small-scale map

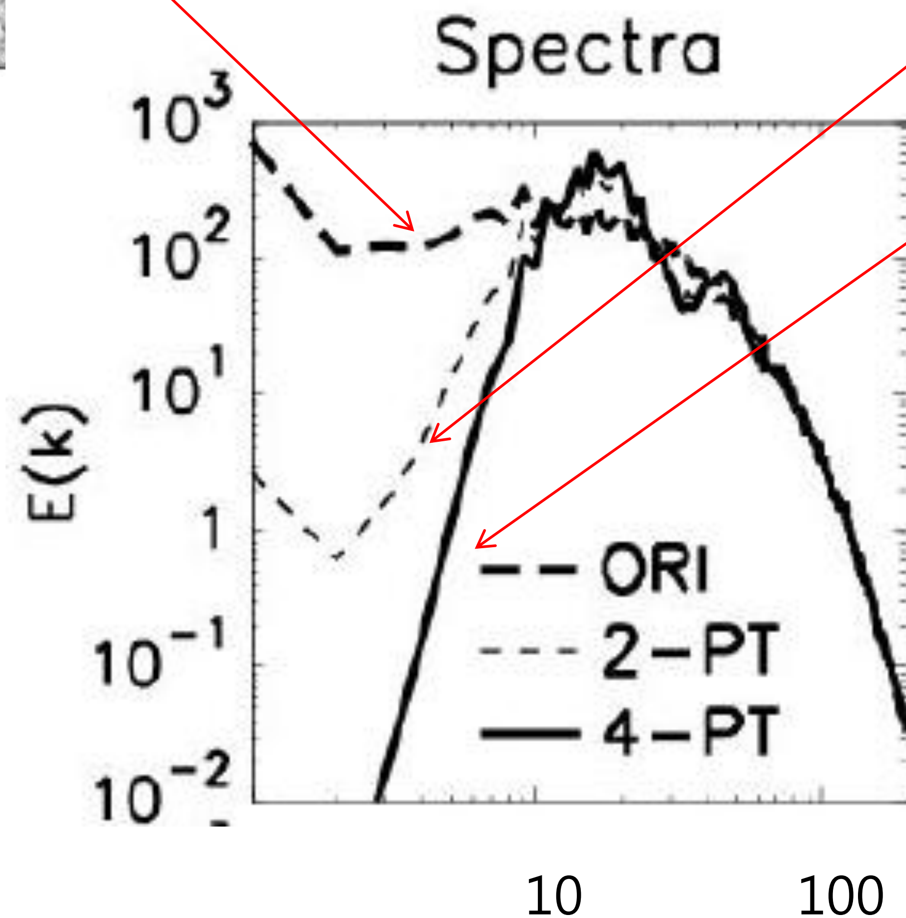


Obtaining small-scale maps

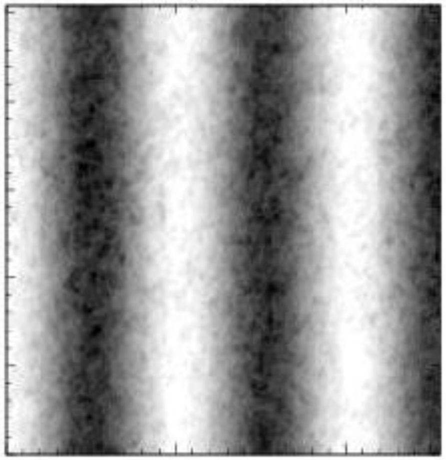


small-scal

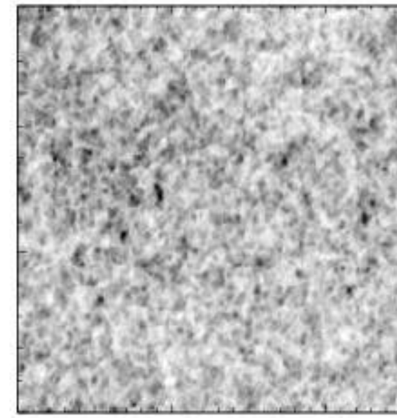




Obtaining small-scale maps

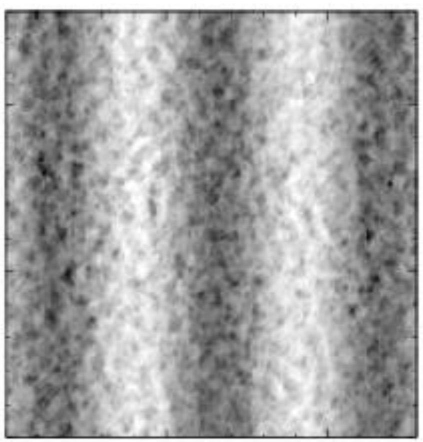


Can we remove the large-scale variation and retrieve the small-scale map?

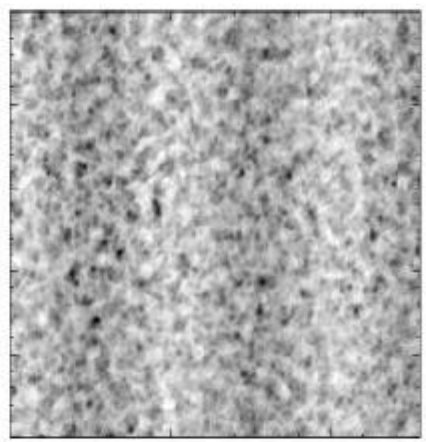


Large+small-scale map

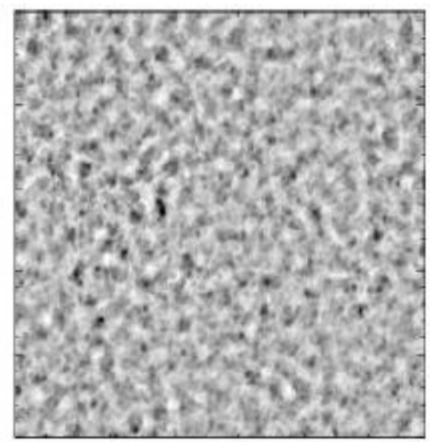
small-scale map



1-pt average

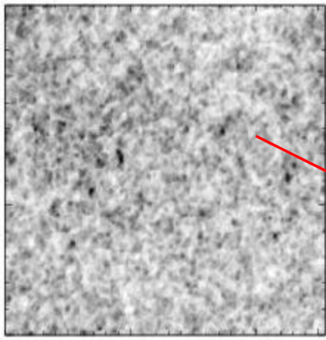


2-pt average

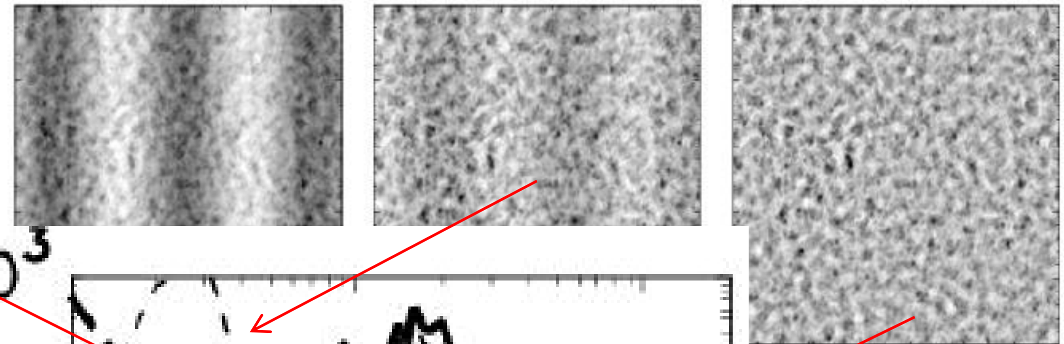


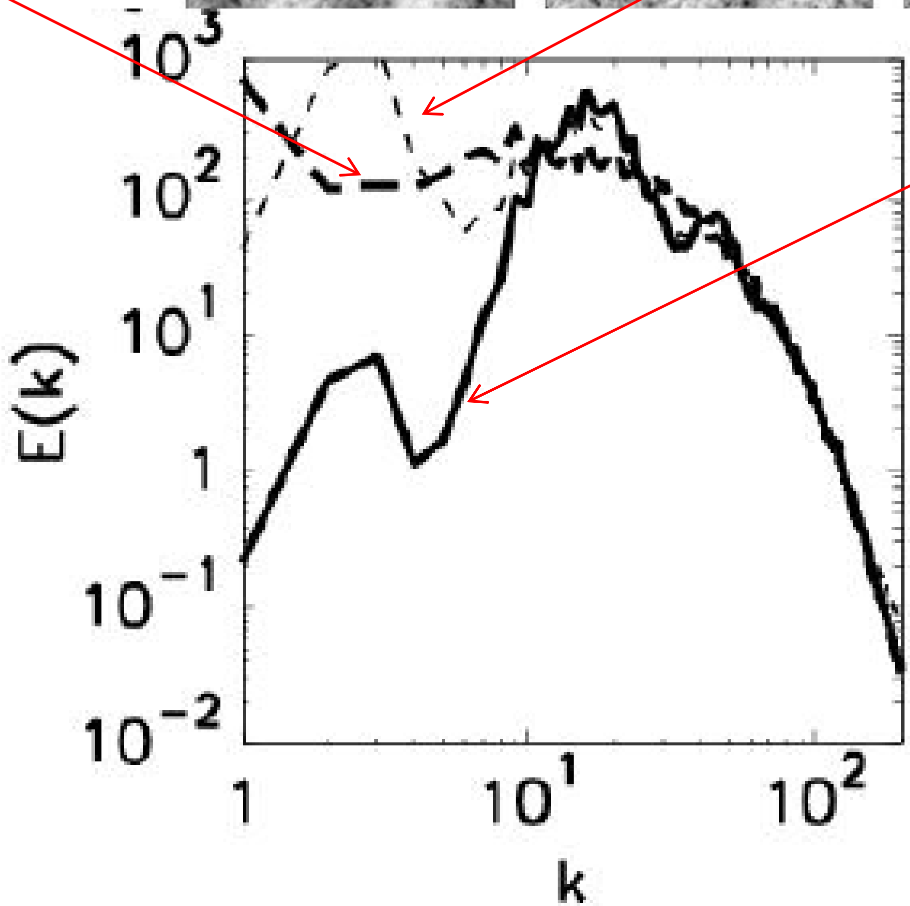
4-pt average

Obtaining small-scale maps



small-scale ma

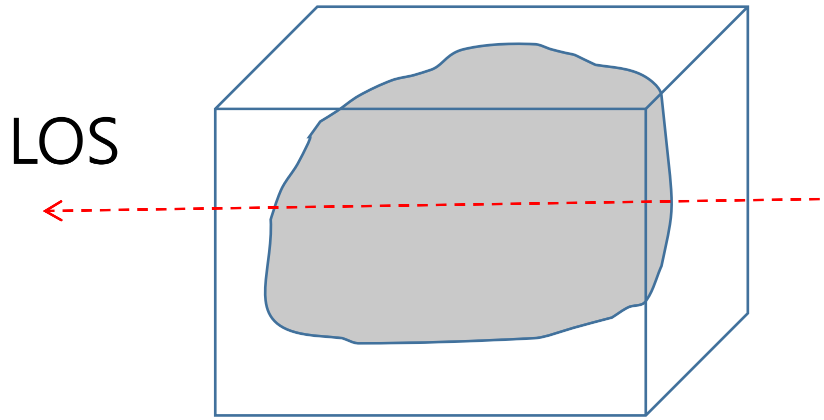




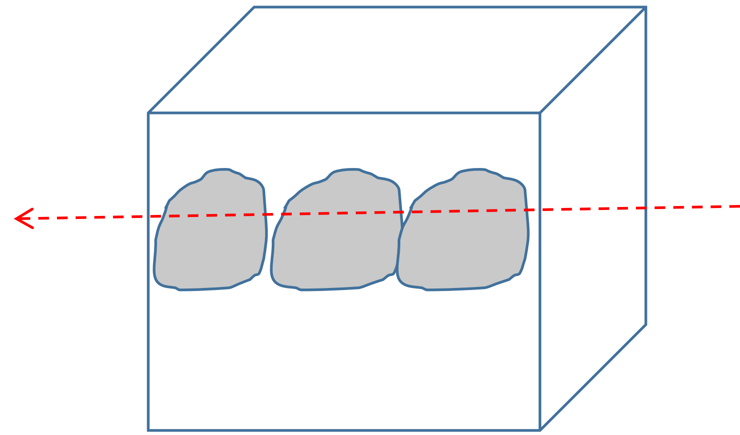
4-pt average
works best

Summary

Introduction: Measurement of δV_c is important



If N=1, the CF method returns correct B_0



If N>1, correct B_0 = CF / $N^{1/2}$

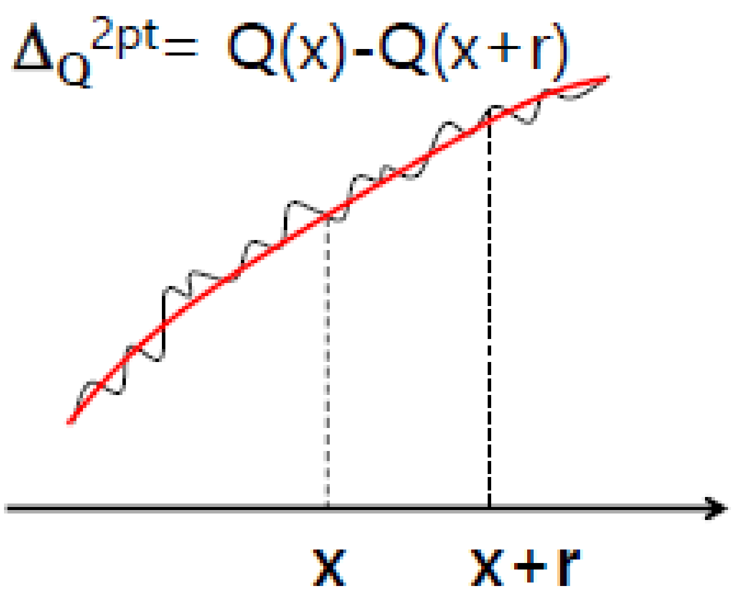
- We found a way to get $N^{1/2}$

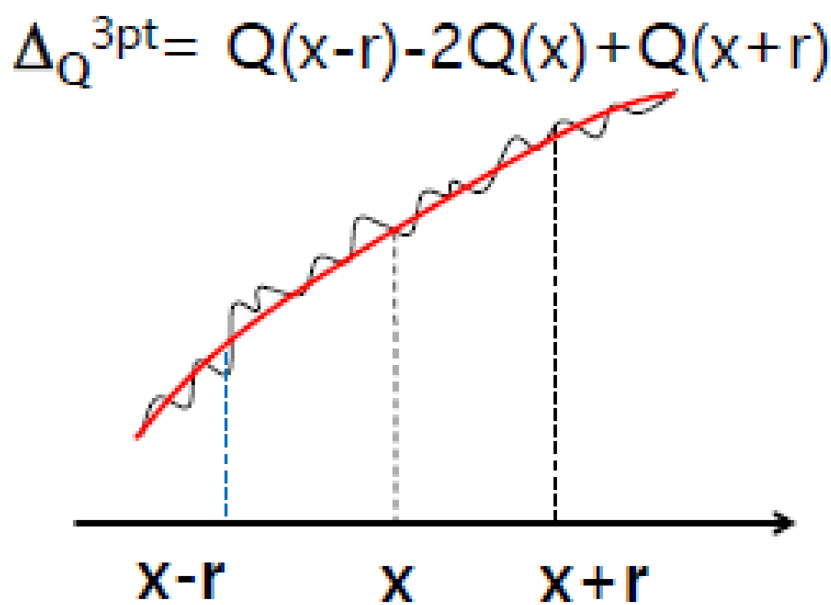
  [← δV_c (St dev of centroid velocity)!]
- Our technique fails when there is a large-scale velocity gradient ➔ We can fix it!

Summary

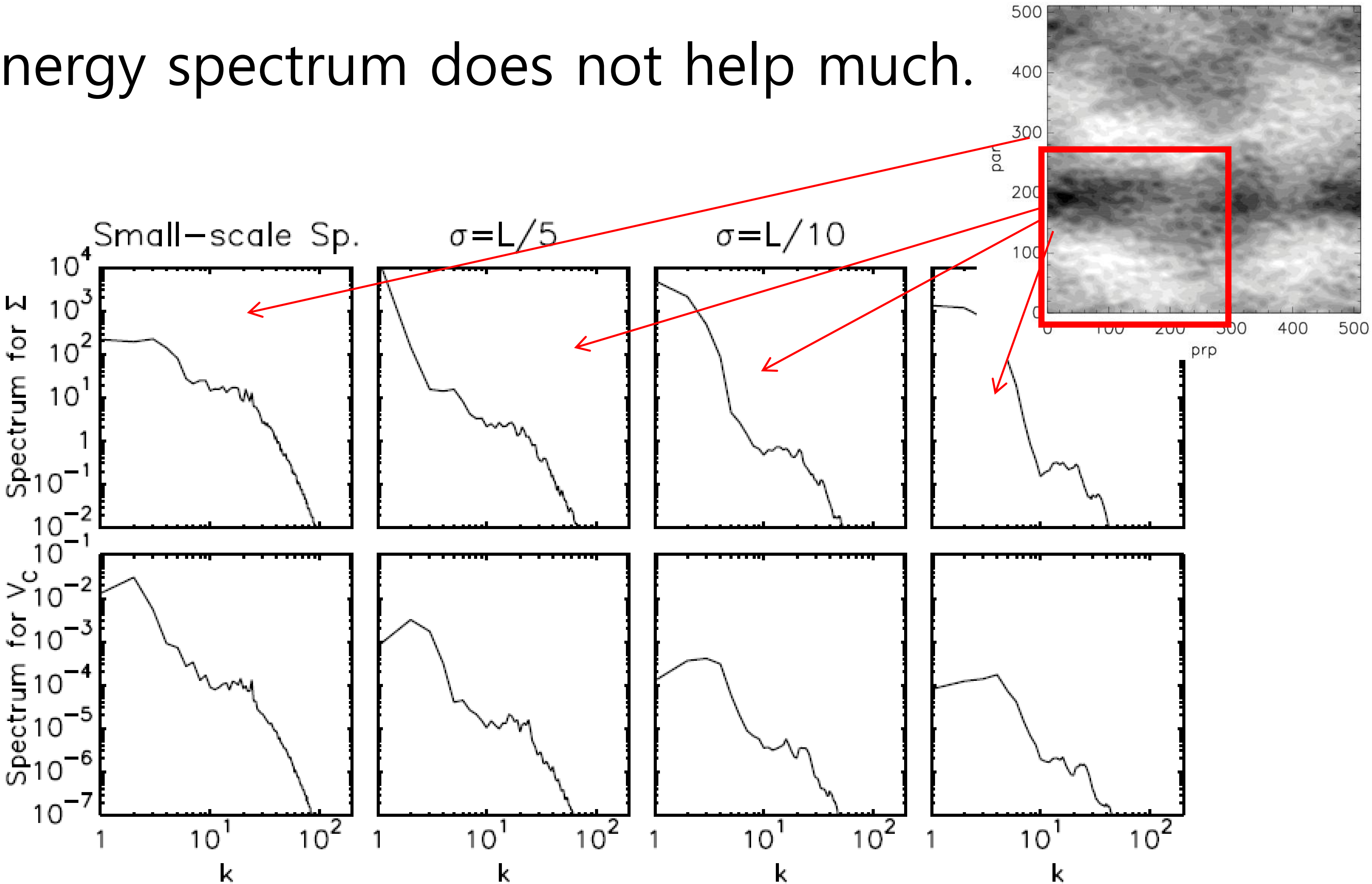
Our multi-point structure function technique can

- Obtain the magnitudes of small-scale fluctuations
- Help us to obtain small-scale maps





Energy spectrum does not help much.



This is due to the edge effect…