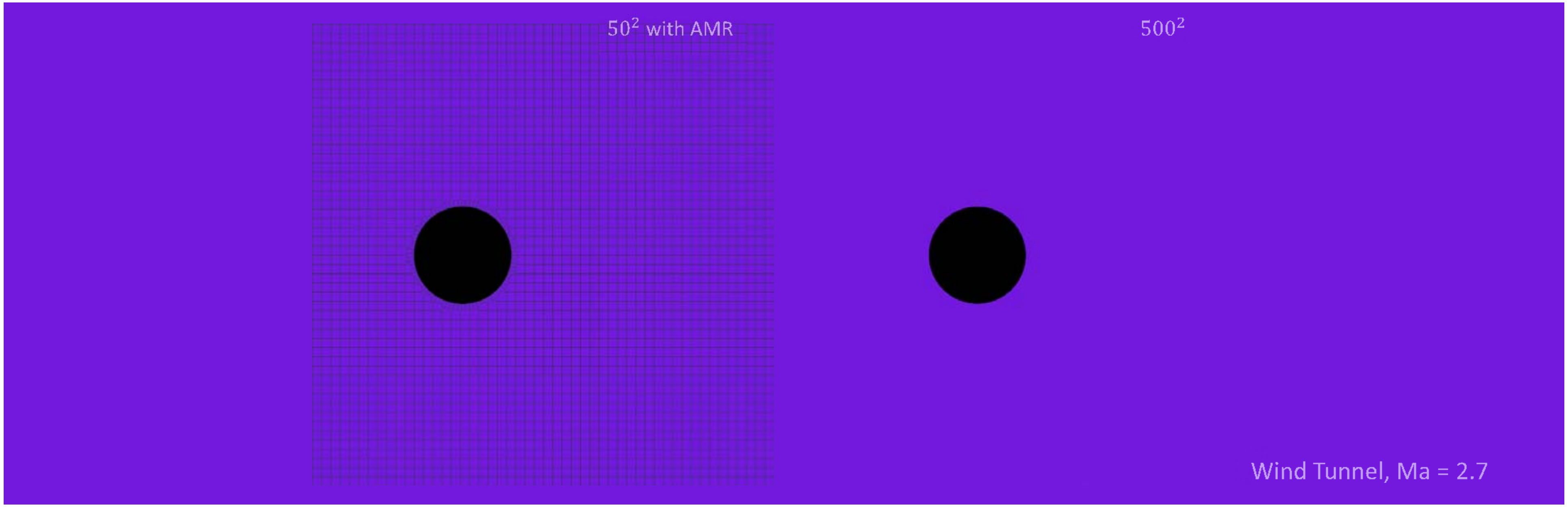
# YUMMy

## Yonsei Unstructured Moving Mesh Hydrodynamic Simulation code





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# Outline

- Introduction to the Unstructured Moving Mesh
    - Unstructured
    - Moving
    - Mesh
- Test
    - 1D Shock tube test
    - Interacting blast wave test
    - Gresho Vertex
    - Kelvin-Helmholtz Instability
    - Rayleigh-Taylor Instability
    - Noh Shock Test
    - Moving Boundary Test
    - Wind Tunnel

# 2 Hydrodynamic descriptions

- **Lagrangian**
  - A way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. (Wikipedia)
  - Smoothed-**particle** hydrodynamics(SPH); GASOLINE, GADGET-2,...
  - pro. : easy to use, analyze, Galilean invariance / con. : sticky, instability, dissipation
- **Eulerian**
  - A way of looking at fluid motion that focused on specific locations in the space through which the fluid flows as time passes. (Wikipedia)
  - Cartesian **Grid**, Adaptive Mesh Refinement(AMR) ; ENZO, FLASH, ATHENA, RAMSES, PLUTO,...
  - pro. : accurate, instability / con. : Galilean invariance, coordinate
- **Hybird : Lagrangian + Eulerian**
  - AREPO, GIZMO, YUMMy, ...
  - pro. : ?? / con. : ??

YUMMy is

Lagrangian Eulerian

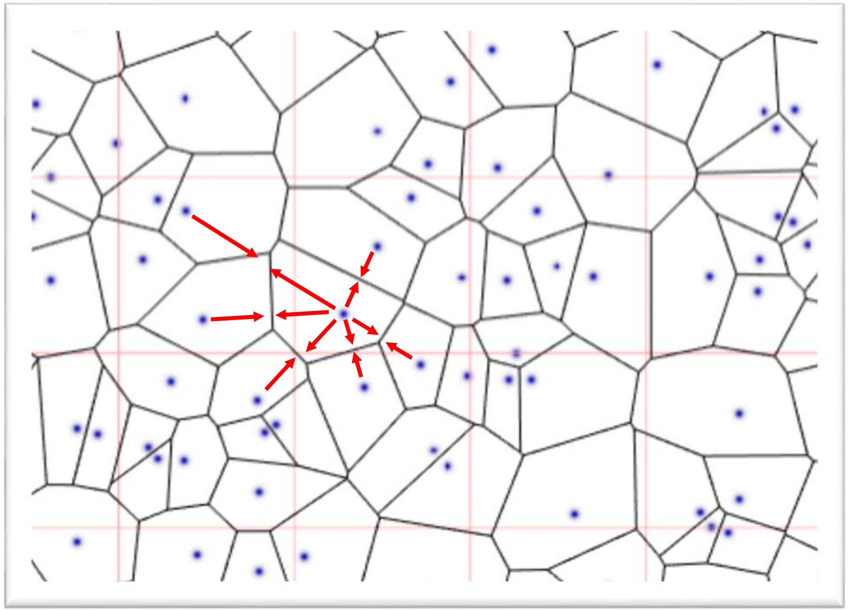
# Unstructured Moving Mesh

Based on both of Lagrangian and Eulerian

# Unstructured Moving Mesh

- Irregular shape; Voronoi Cell

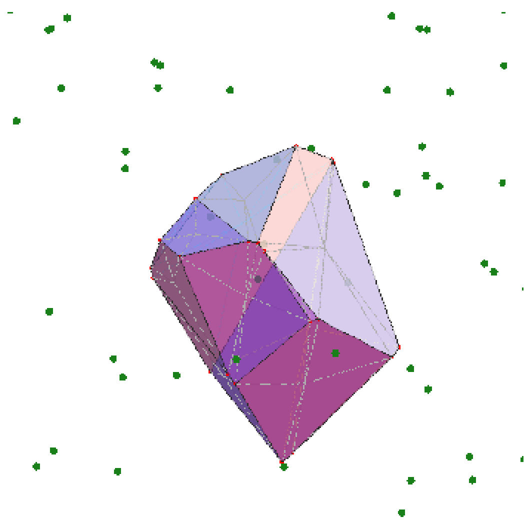
$$R_k = \{x \in X \mid d(x, P_k) \leq d(x, P_j) \; for \; all \; j \neq k\}$$



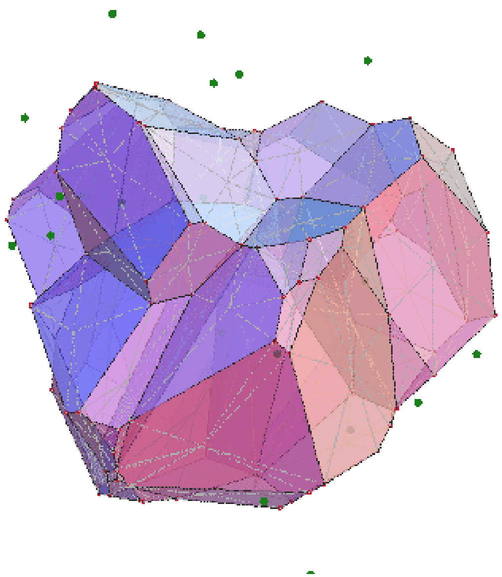
# Unstructured Moving Mesh

- Irregular shape; Voronoi Cell

Each mesh has more surfaces(>5 in 2D, >10 in 3D) which face arbitrary direction than the cube used in conventional code → more accurate hydro-quantities evolution.



Voronoi Cell



Voronoi Cells

# Unstructured Moving Mesh

- **Voronoi tessellation**
  - Incremental **E**xpanding **M**ethod(IEM)
    - Intuitive, fast, extendable to the high dimension(>3)
  - Hybrid-Neighbor Searching Method
    - Tree + 'TreeMap' : reduce required total number of searching cubes
    - ~2 times faster than just used in tree algorithm

Speed $\sim O(N)$

- We propose the robust solution to deal with the degeneracy problem
  - $\min(\vec{a} \cdot \overrightarrow{\boldsymbol{o\alpha_i}})$

# Unstructured Moving Mesh

- Meshes move according to the speed → Galilean-invariance
  - of flow mostly derived by Euler equations
  - for regularizing the shape mesh

# Unstructured Moving Mesh

- Eulerian Description

$$U_t + \nabla \cdot F = 0$$

$$U = \begin{pmatrix} \rho \\ \rho v \\ \rho e \end{pmatrix}, F(U) = \begin{pmatrix} \rho v \\ \rho v v^T + P \\ (\rho e + P) v \end{pmatrix}$$

- Godunov's 2nd order method
- MUSCL-Hancock Scheme
- For the 2nd order accuracy
  - Barth and Jesperson TVD for unstructured mesh
  - Q-R Factorization : the slope in the mesh
- HLLC Approximate Riemann solver
  - Adaptive Noniterative Riemann Solver
    - Two Rarefaction Riemann Solver(TRRS)
    - Two Shock Riemann Solver(TSRS)
    - Primitive Variable Riemann Solver(PVRS)

# TEST

- 1D
  1. 1D Riemann Problem : Rarefication-Shock Test
  2. Interacting Blast Waves

- 2D
  3. Gresh Vertex
  4. Kelvin-Helmholtz Instability
  5. Ray-Taylor Instability
  6. Noh Shock Test
  7. Moving Boundary Test
  8. Wind Tunnel Test

AREPO

$t = 5.0$

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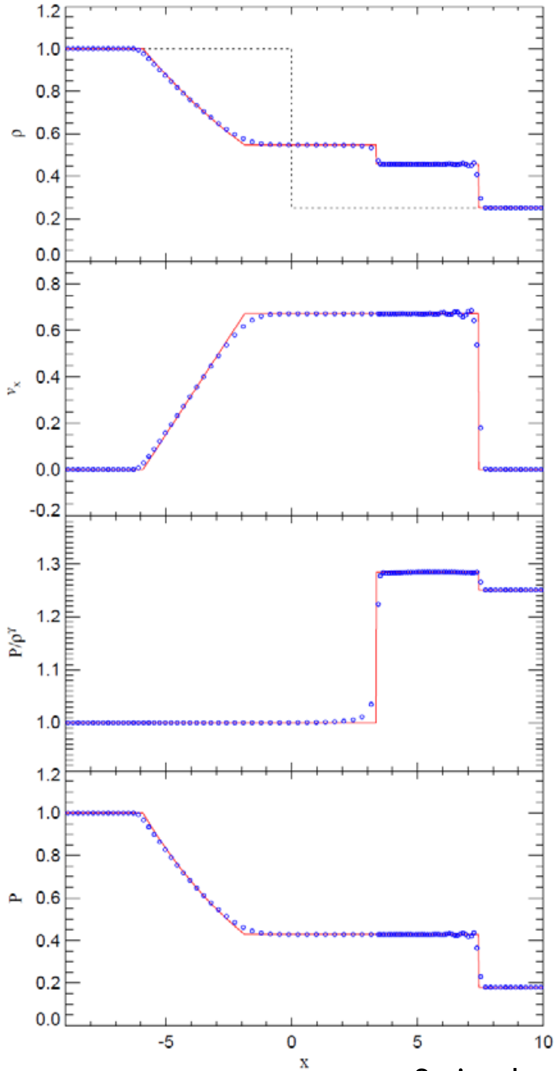
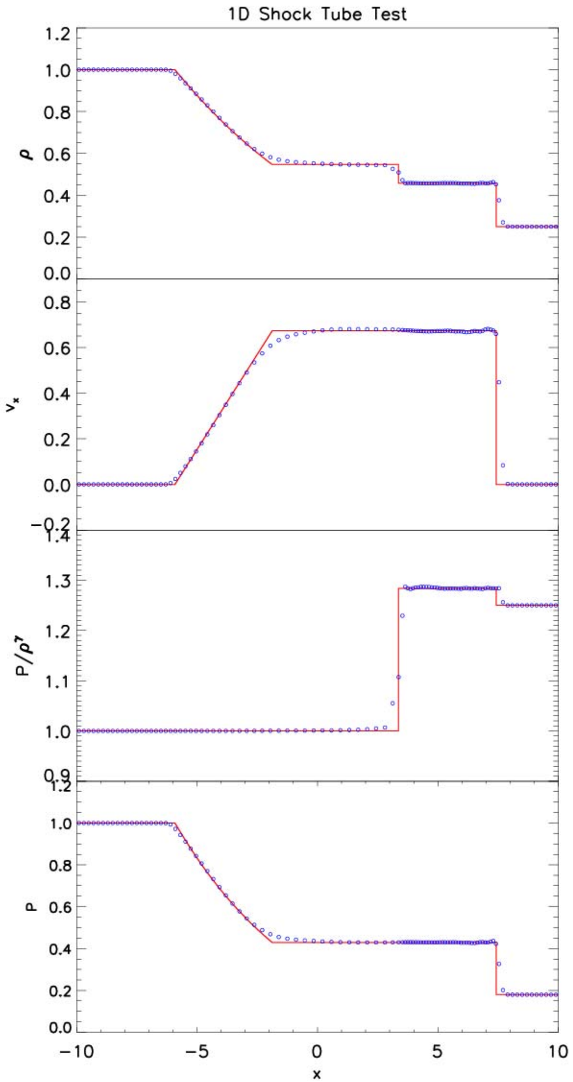



Springel, 2009




Initial Conditions

- $x < 0: \; P_1 = 1, \rho_1 = 1, v_1 = 0$
- $x > 0: P_2 = 0.1795, \rho_2 = 0.25, v_2 = 0$
- $\gamma = 1.4$

# Interacting blast waves(Woodward & Collella, 1984) Moving Mesh



Initial Conditions

- $\rho = 1, \gamma = 1.4, x \in [0, 1]$
- $P = 1000$ for $x < 0.1$, $P = 100$ for $x > 0.9$, $P = 0.01$ for elsewhere

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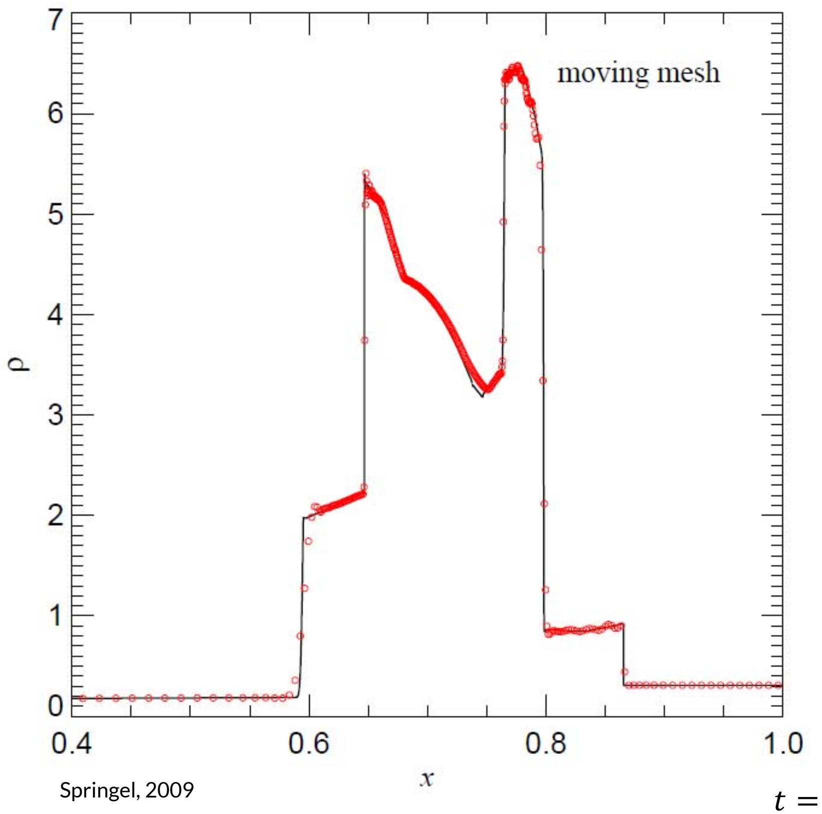


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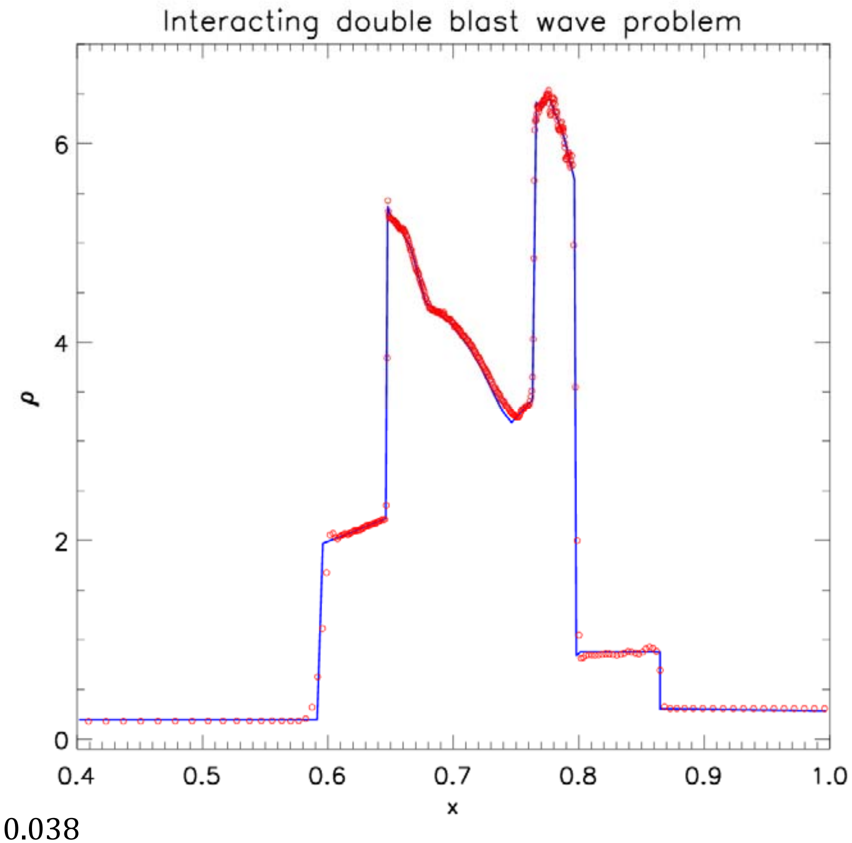


$t = 0.038$

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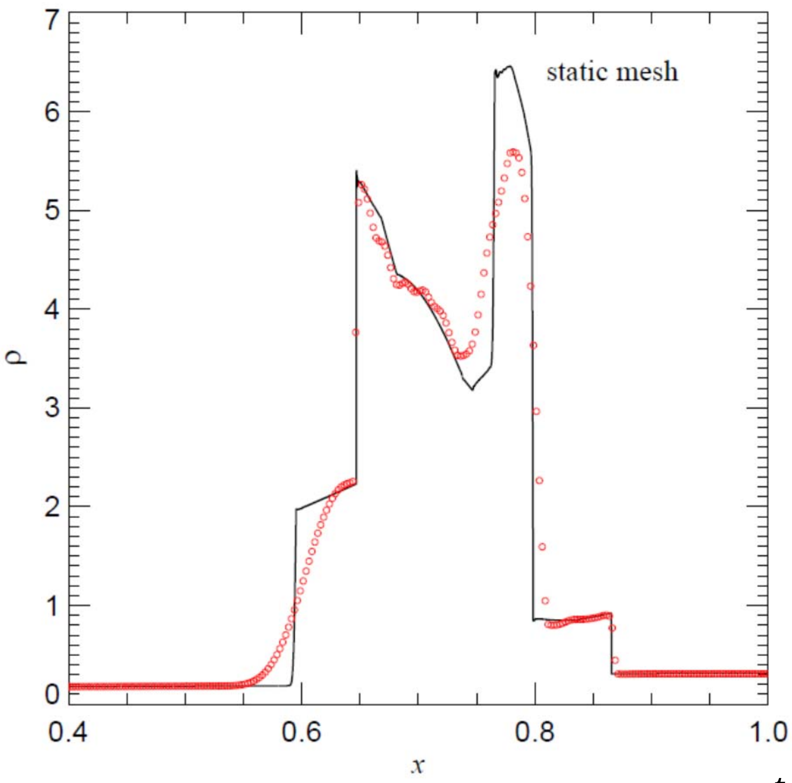


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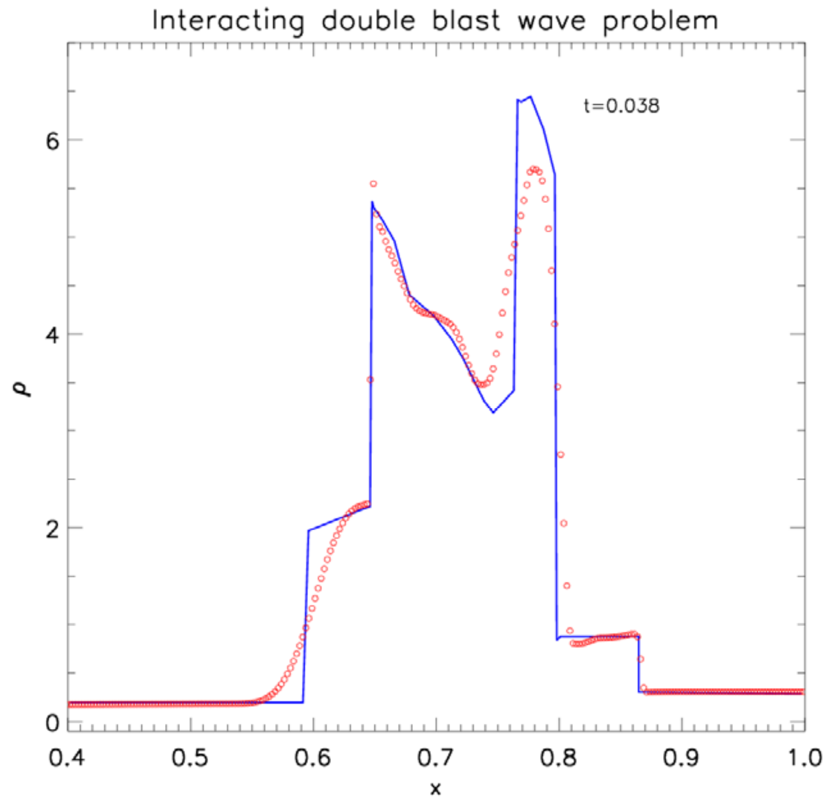


$t = 0.038$

➢ Comparison between moving and static mesh with same resolution

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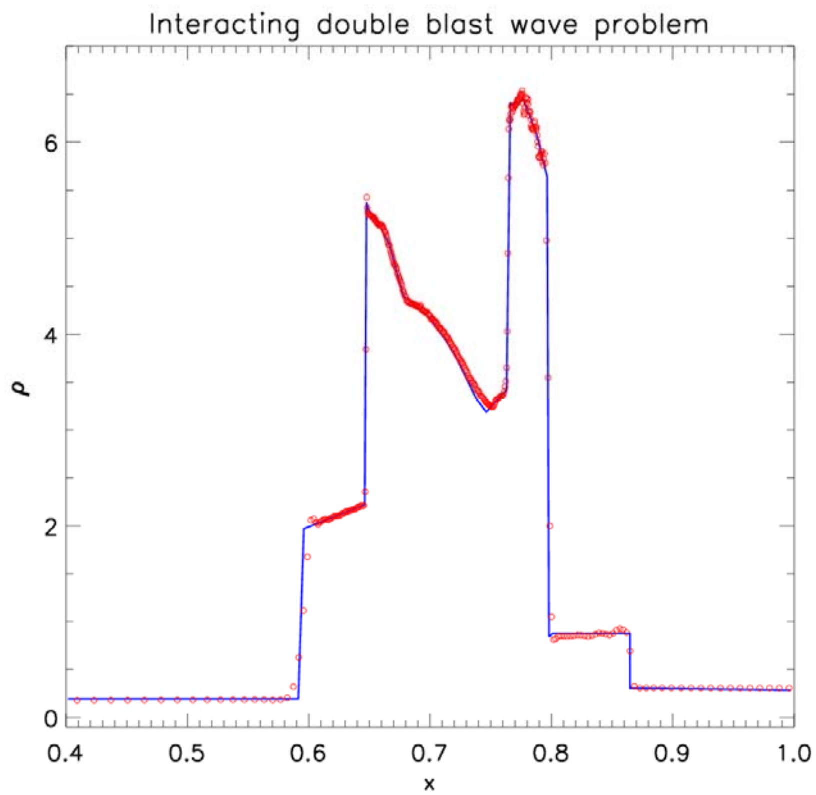
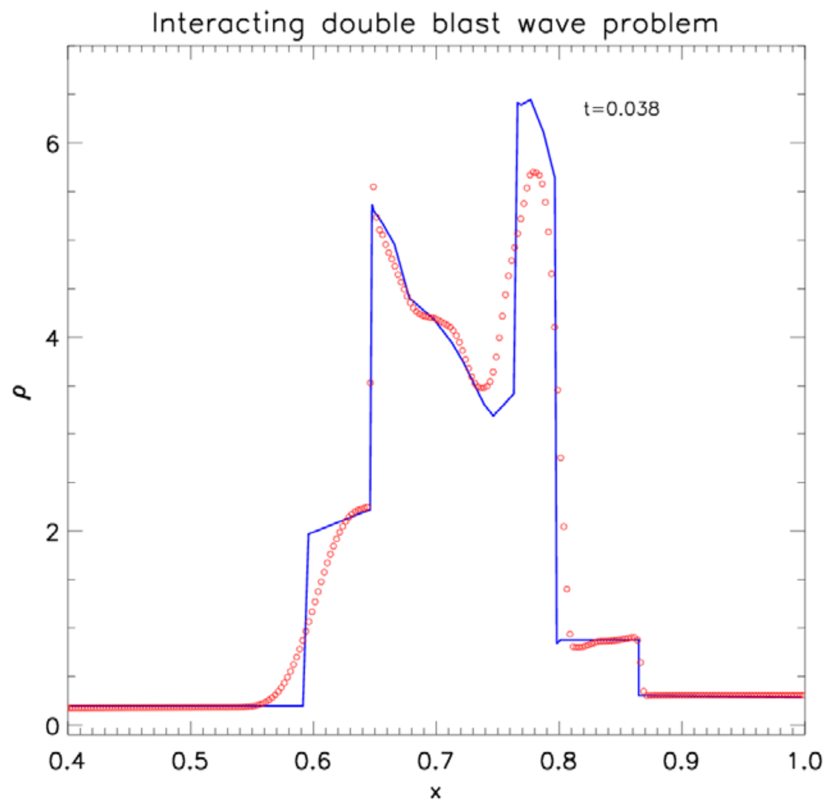


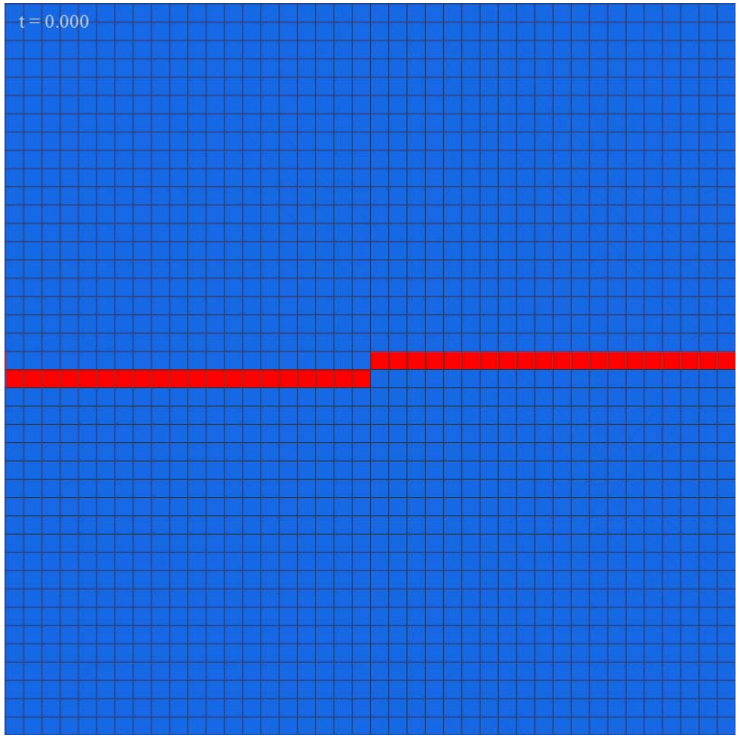




$t = 0.038$

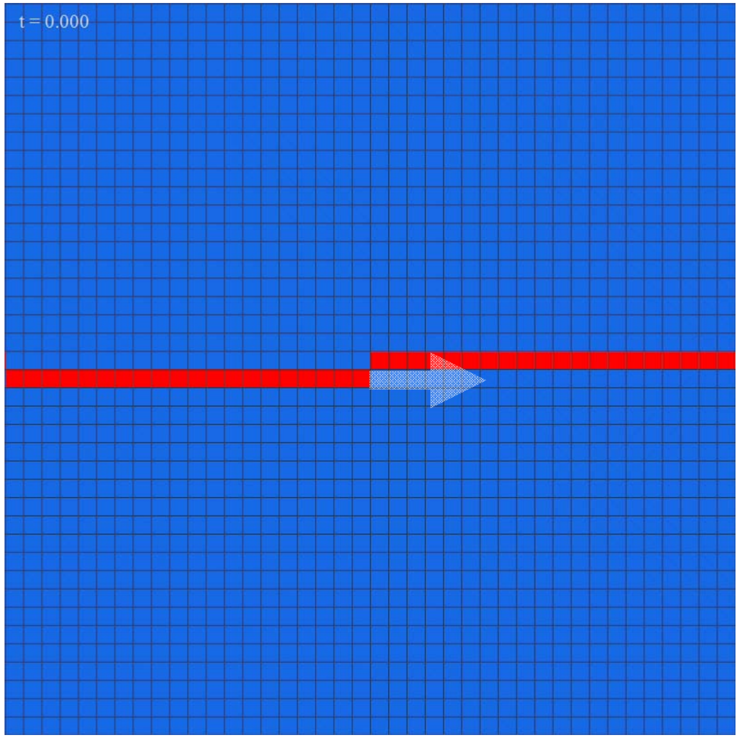
Initial Conditions

- Azimuthal velocity $v_\phi(r) = \begin{cases} 5r & for \quad 0 \le r < 0.2 \\ 2 - 5r & for \quad 0.2 \le r < 0.4 \\ 0 & for \quad r \ge 0.4 \end{cases}$

- $P(r) = \begin{cases} 5 + 25/2r^2 & for \quad 0 \le r < 0.2 \\ 9 + 25/2r^2 - 20r + 4\ln(r/0.2) & for \quad 0.2 \le r < 0.4 \\ 3 + 4\ln 2 & for \quad r \ge 0.4 \end{cases}, \quad \rho = 1$




$v_x = 0$




Frame velocity, $v_x = 1$

# Gresho Vortex, $v_x = 0$ with Moving Mesh

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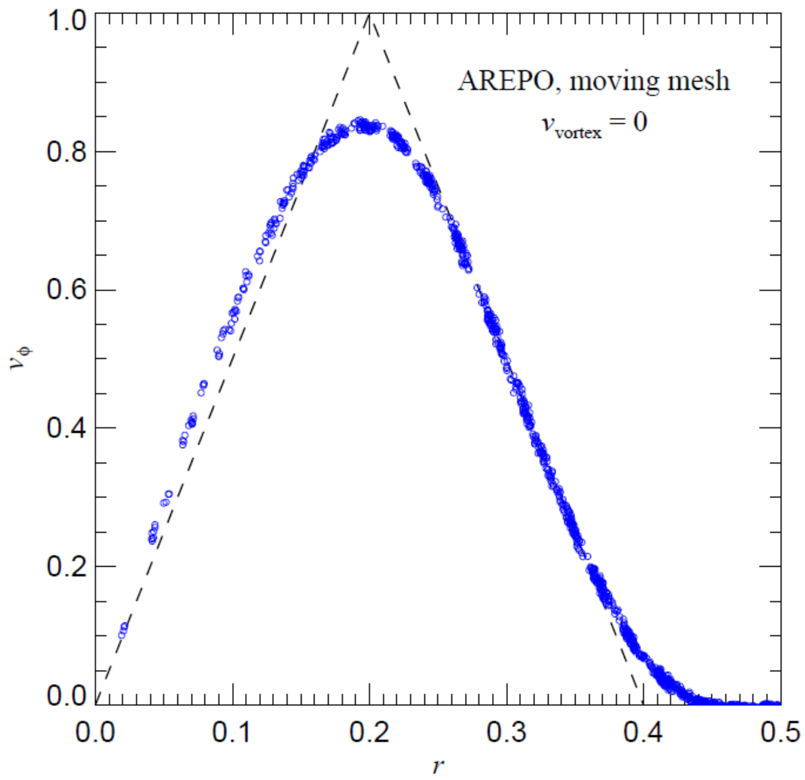


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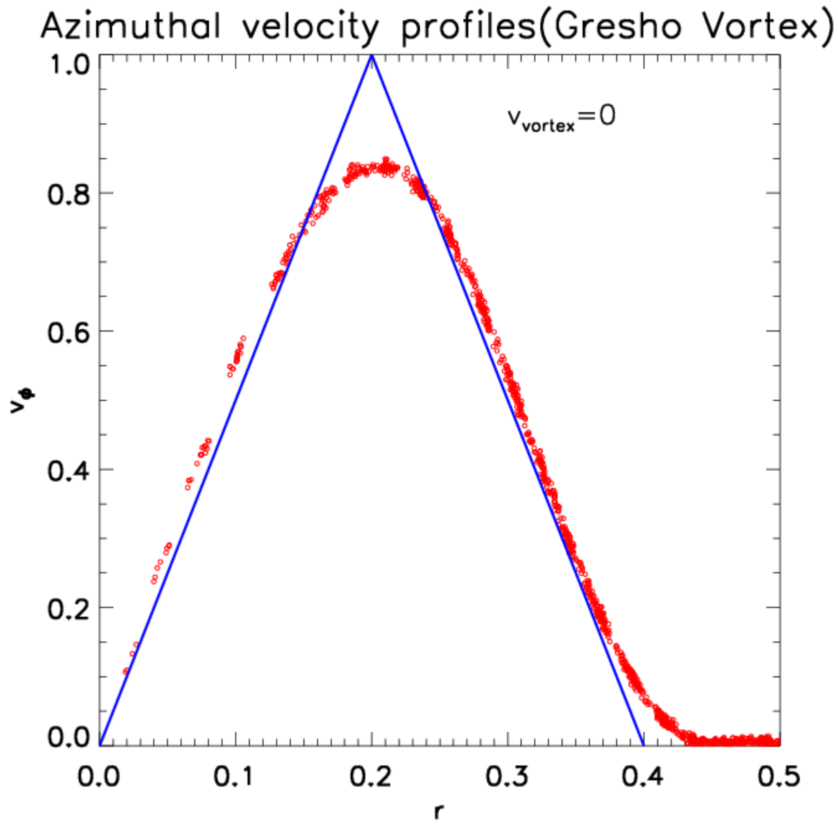
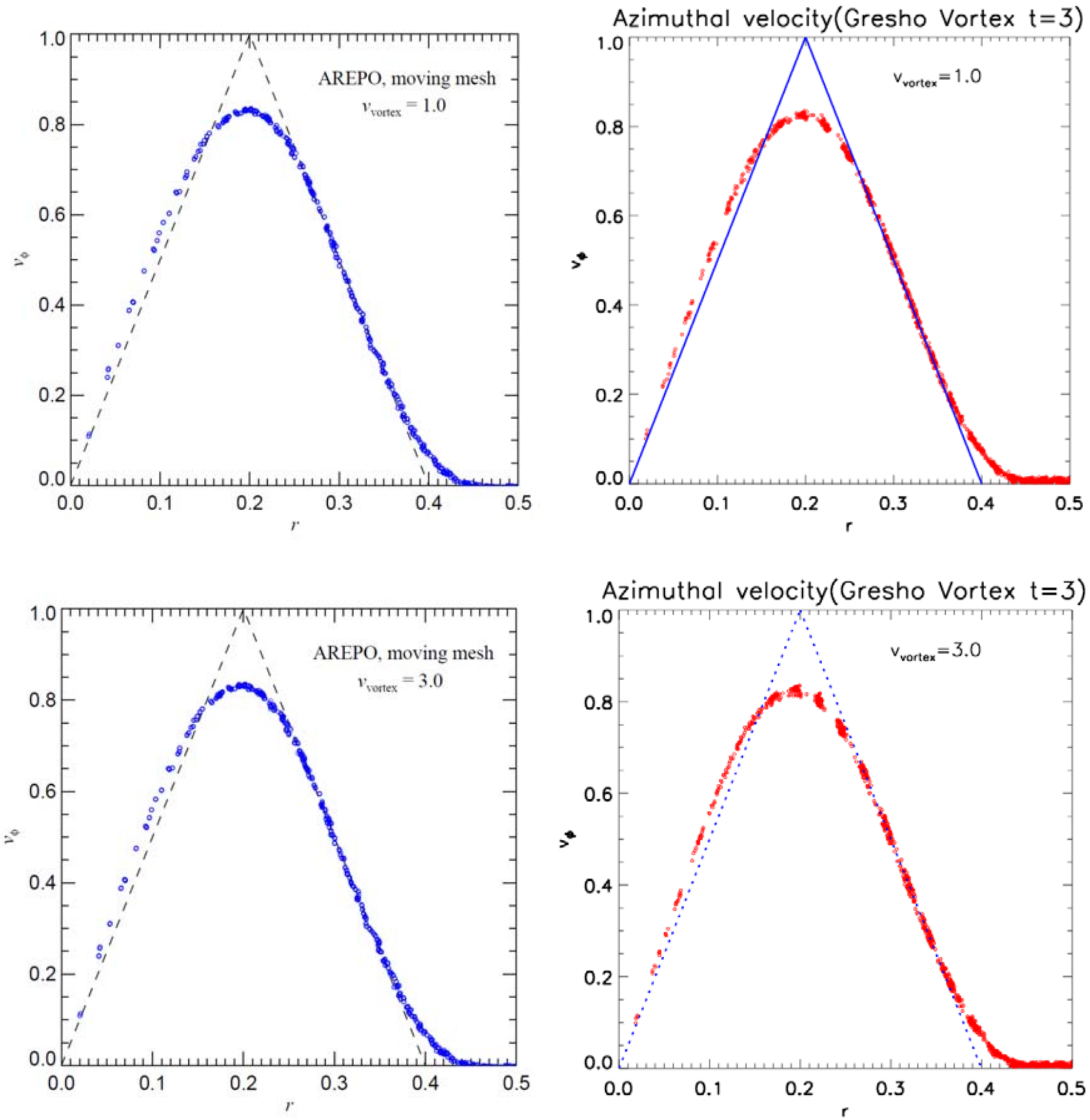


# Gresho Vortex, $v_x = 1, 3 > 0$ with Moving Mesh

AREPO

YUMMy

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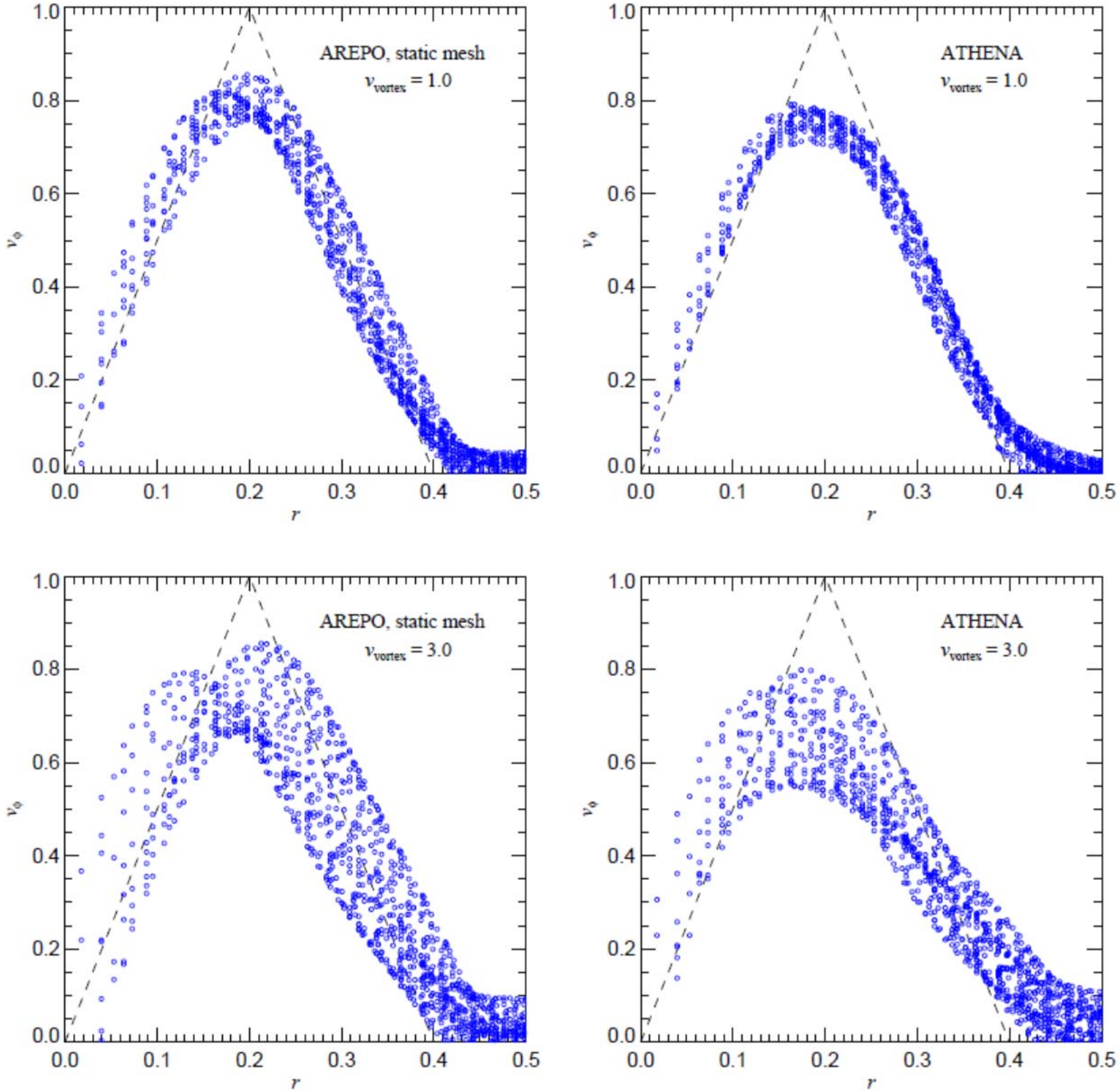
# Gresho Vortex, $v_x = 1, 3 > 0$ with Static Mesh

AREPO
(static mesh)

ATHENA

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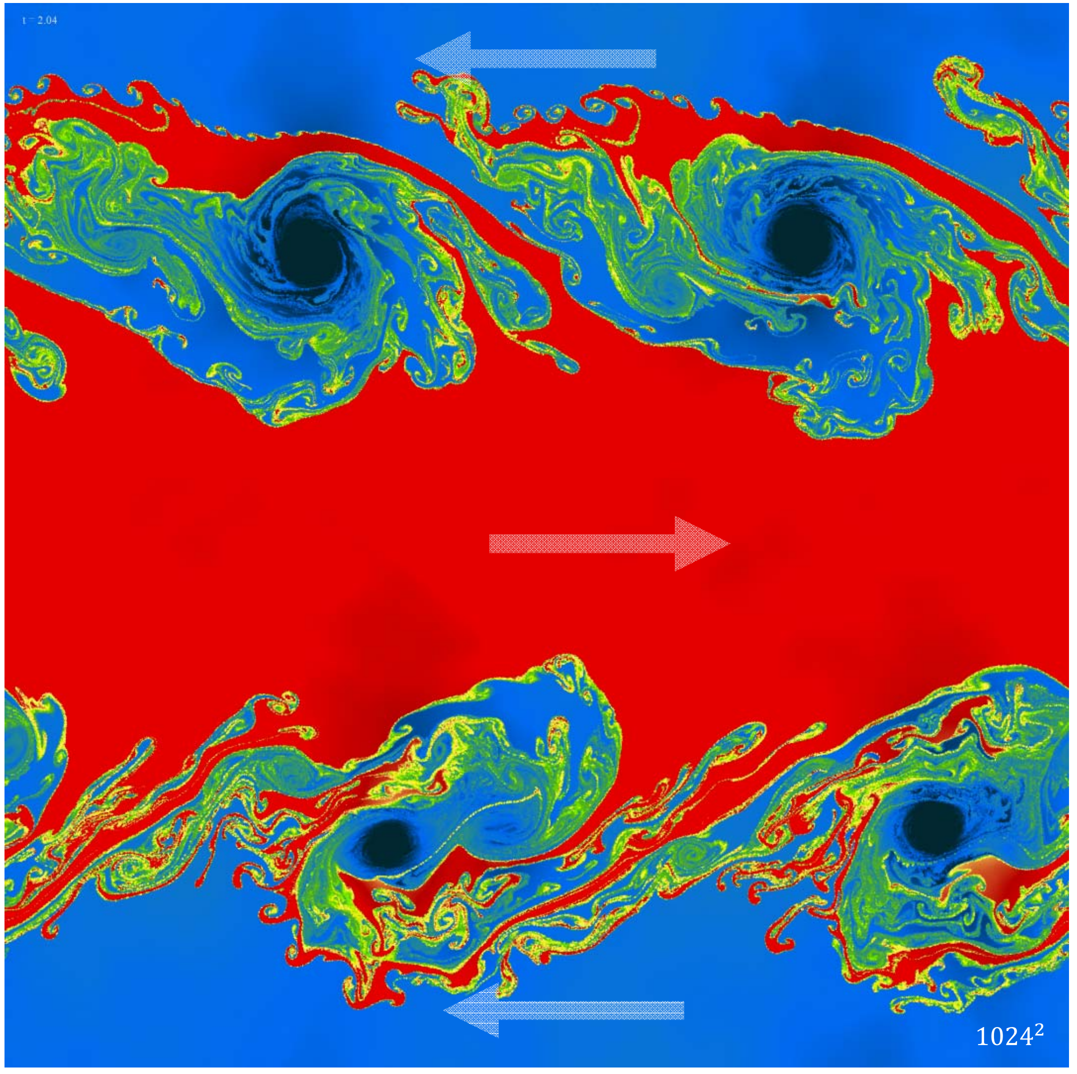
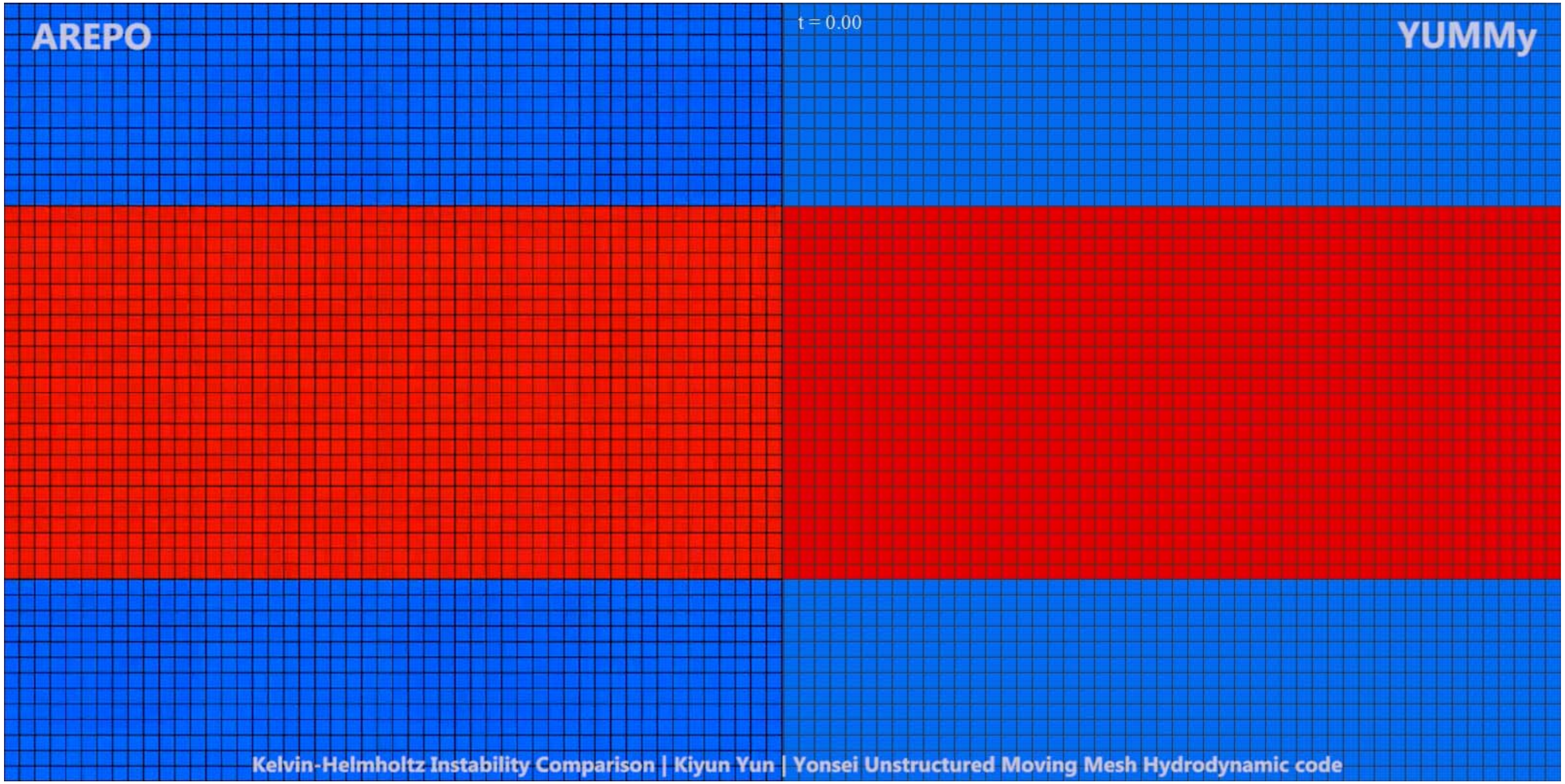



YUMMy

## Initial Conditions

- Periodic boundary
- $x \in [0,1], y \in [0,1]$
- $\rho = 2, v_x = 0.5$ for $|y - 0.5| < 0.25$
- $\rho = 1, v_x = -0.5$ for Elsewhere
- $P = 2.5, \gamma = 5/3$
- $v_y(x,y) = \omega_0 \sin(4\pi x) \left\{ \exp\left[-\frac{(y-0.25)^2}{2\sigma^2}\right] + \exp\left[-\frac{(y-0.75)^2}{2\sigma^2}\right] \right\}$

# Side-by-Side Comparison



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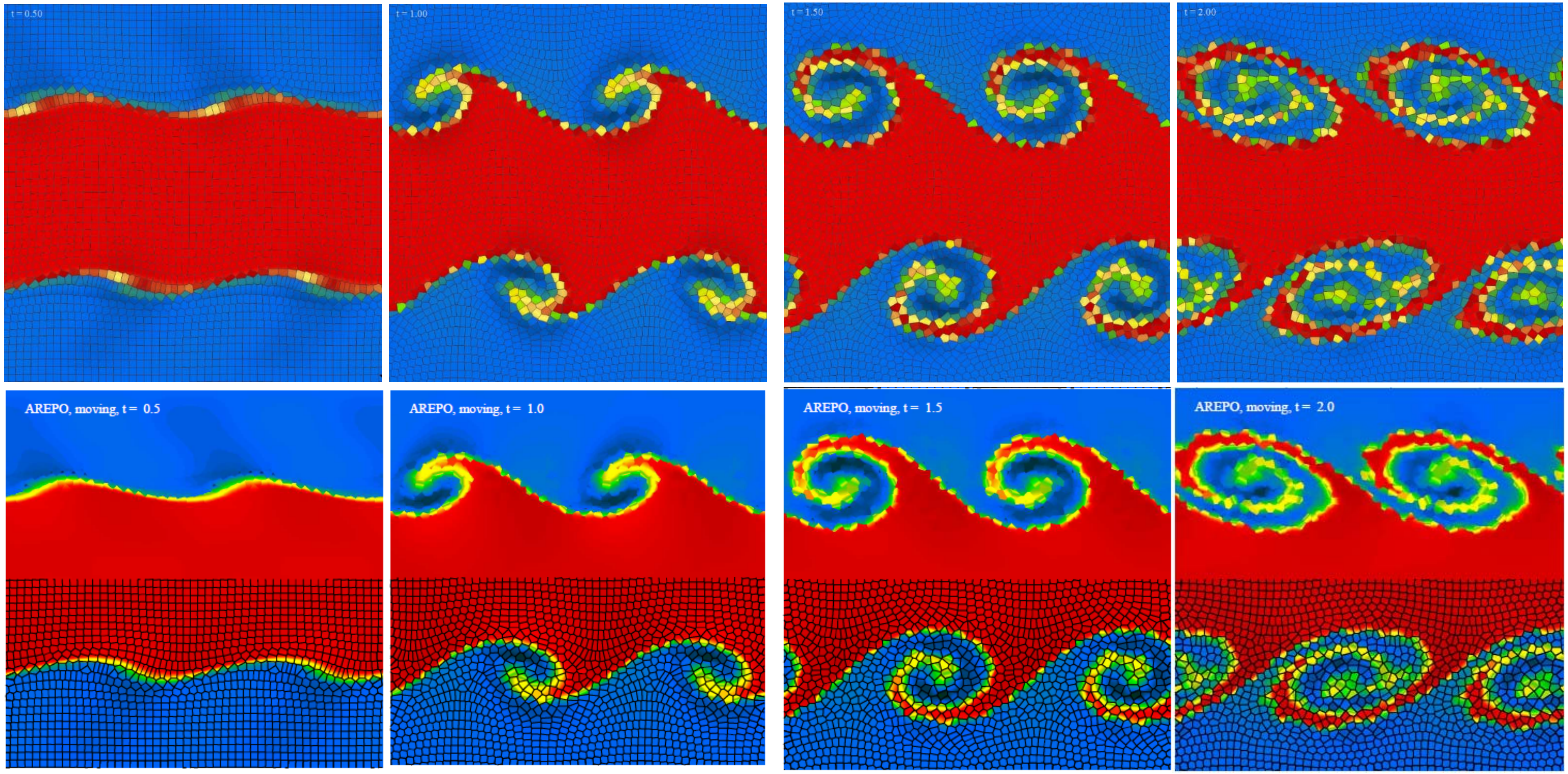


AREPO

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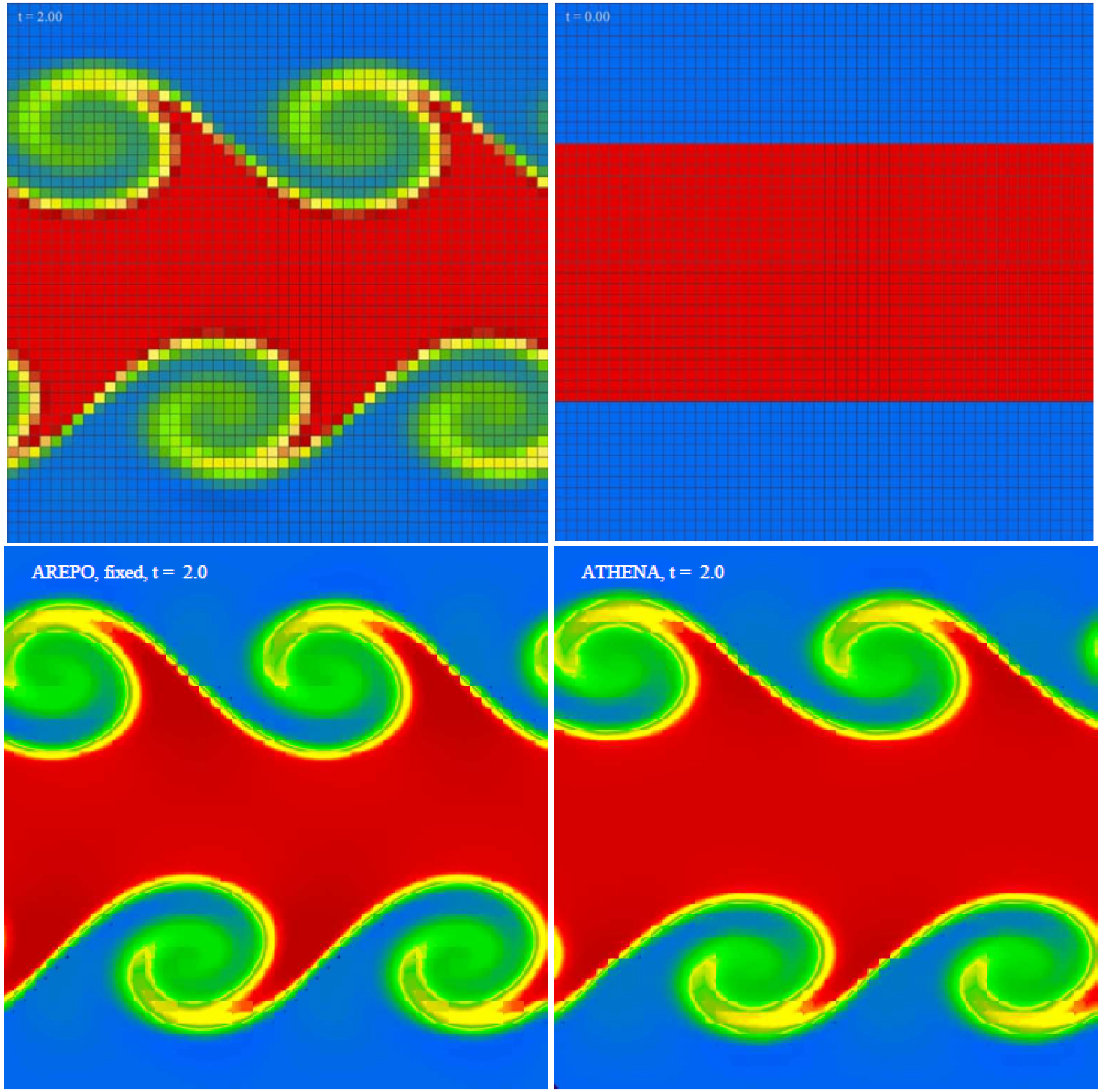
# Kelvin-Helmholtz Instability **Static Mesh**



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AREPO

ATHENA

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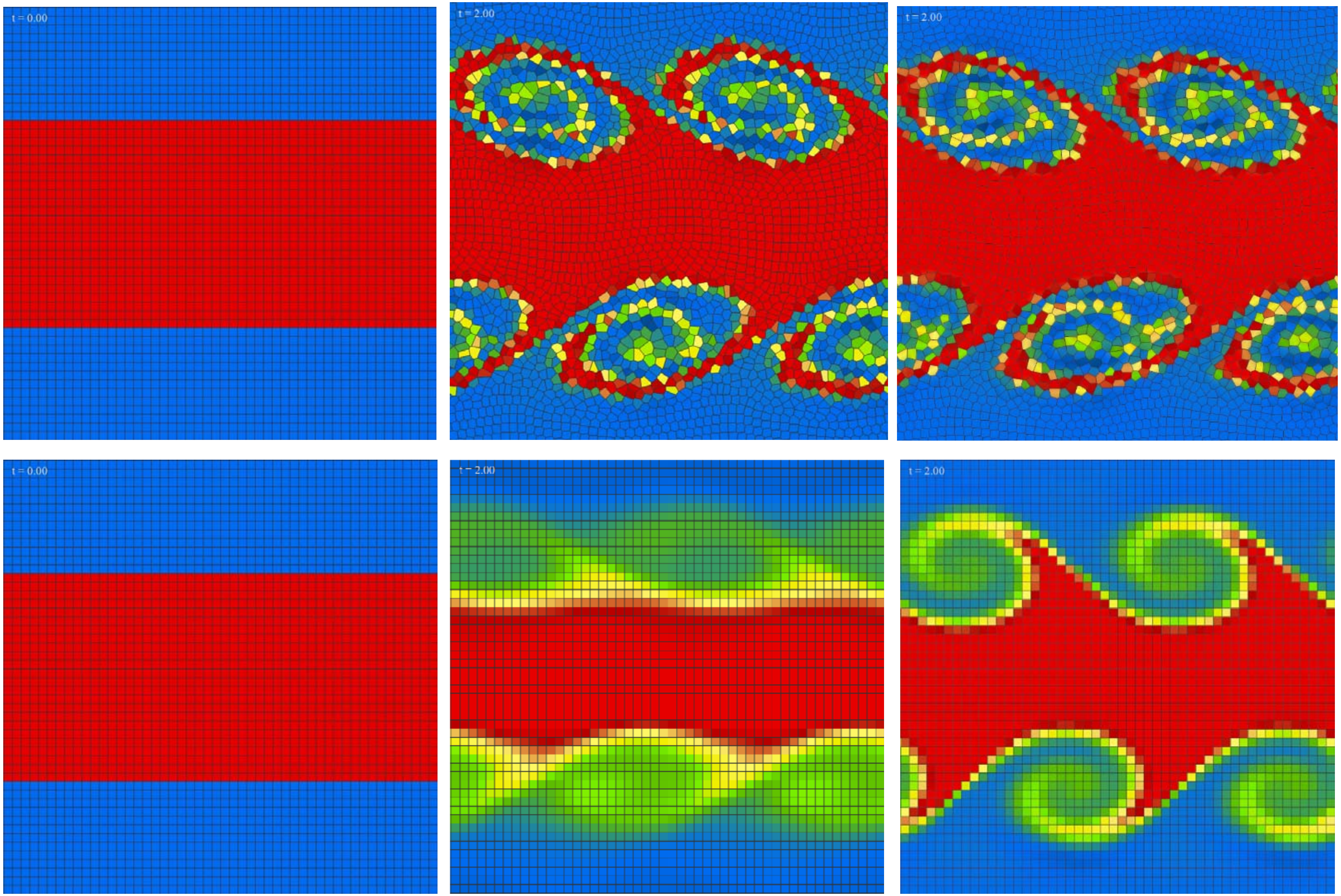
## Kelvin-Helmholtz Instability **Galilean Invariance** $v_x = 1, v_y = 1$

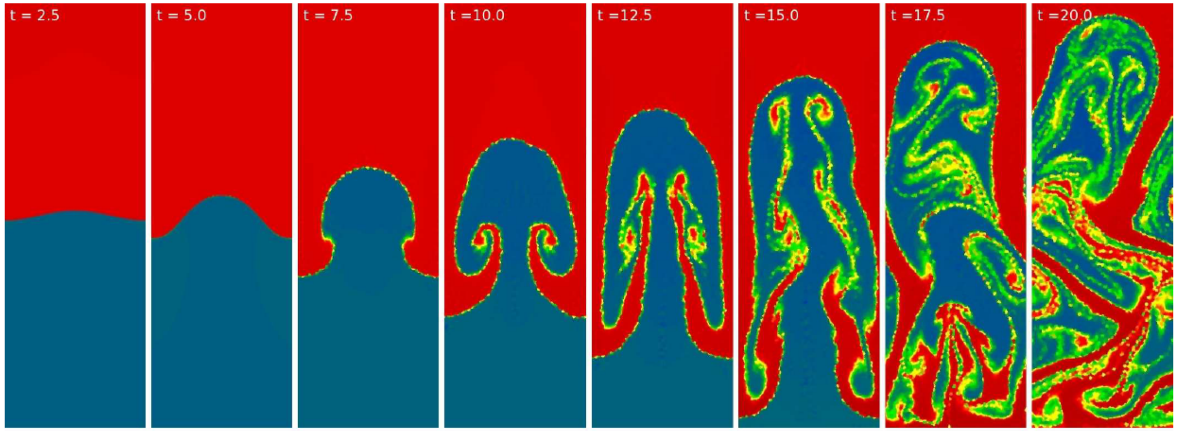





$(v_x, v_y) = (1,1)$ $(v_x, v_y) = (0,0)$

## Initial Conditions

- $x \in [0,0.5], y \in [0,1.5]$
- Periodic boundary at x-boundaries, reflecting wall at top and bottom
- TOP : $\rho = 2$ BOTTOM : $\rho = 1$
- $P_0 = 2.5, \gamma = 1.4$
- $P(y) = P_0 + g(y - 0.75)\rho, g = -0.1$
- $v = 0$, everywhere $v_y(x, y) = \omega_0[1 - \cos(4\pi x)][1 - \cos(4\pi y/3)], \omega_0 = 0.0025$

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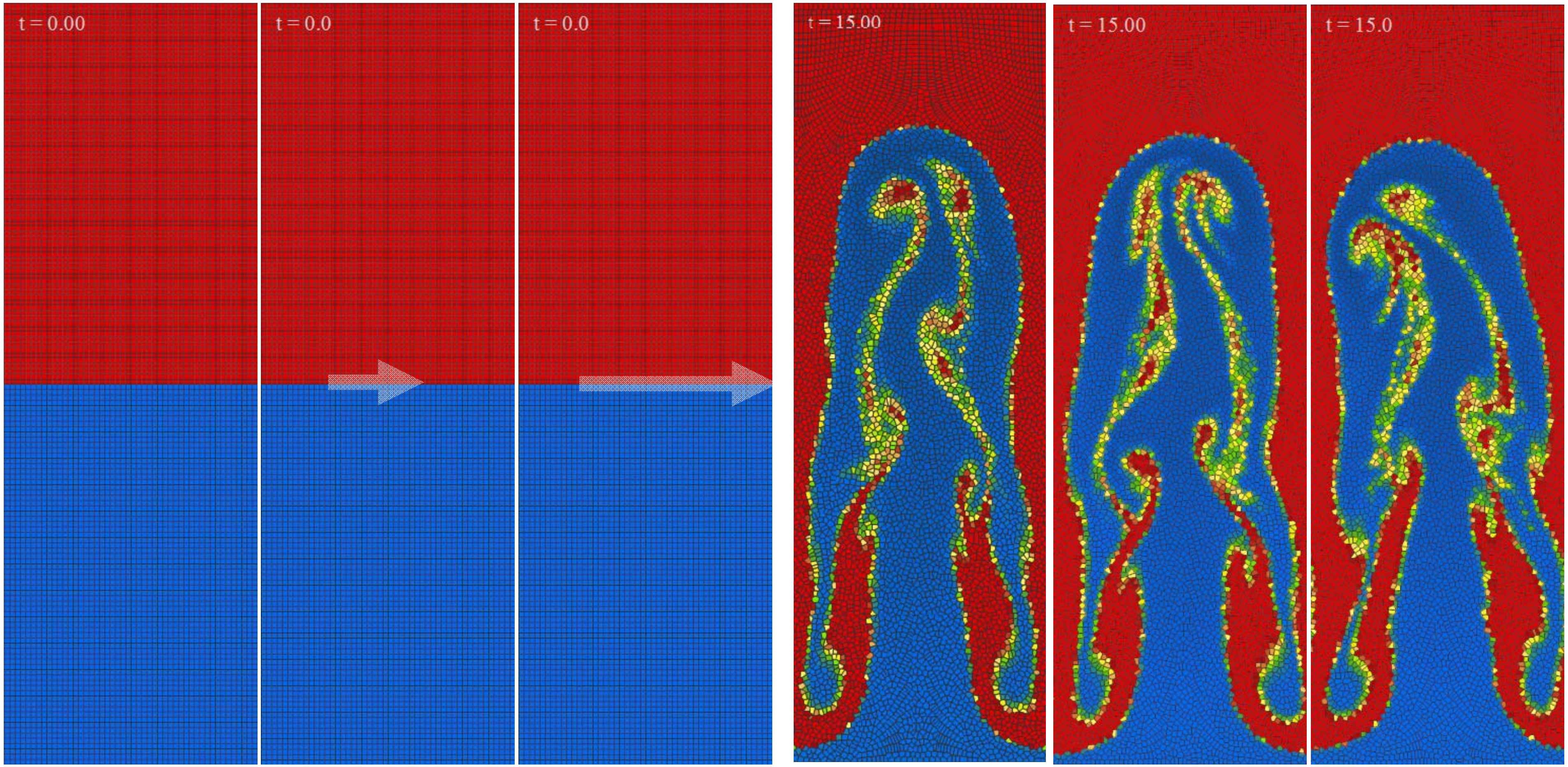
## Various Frame Velocities, Moving Mesh

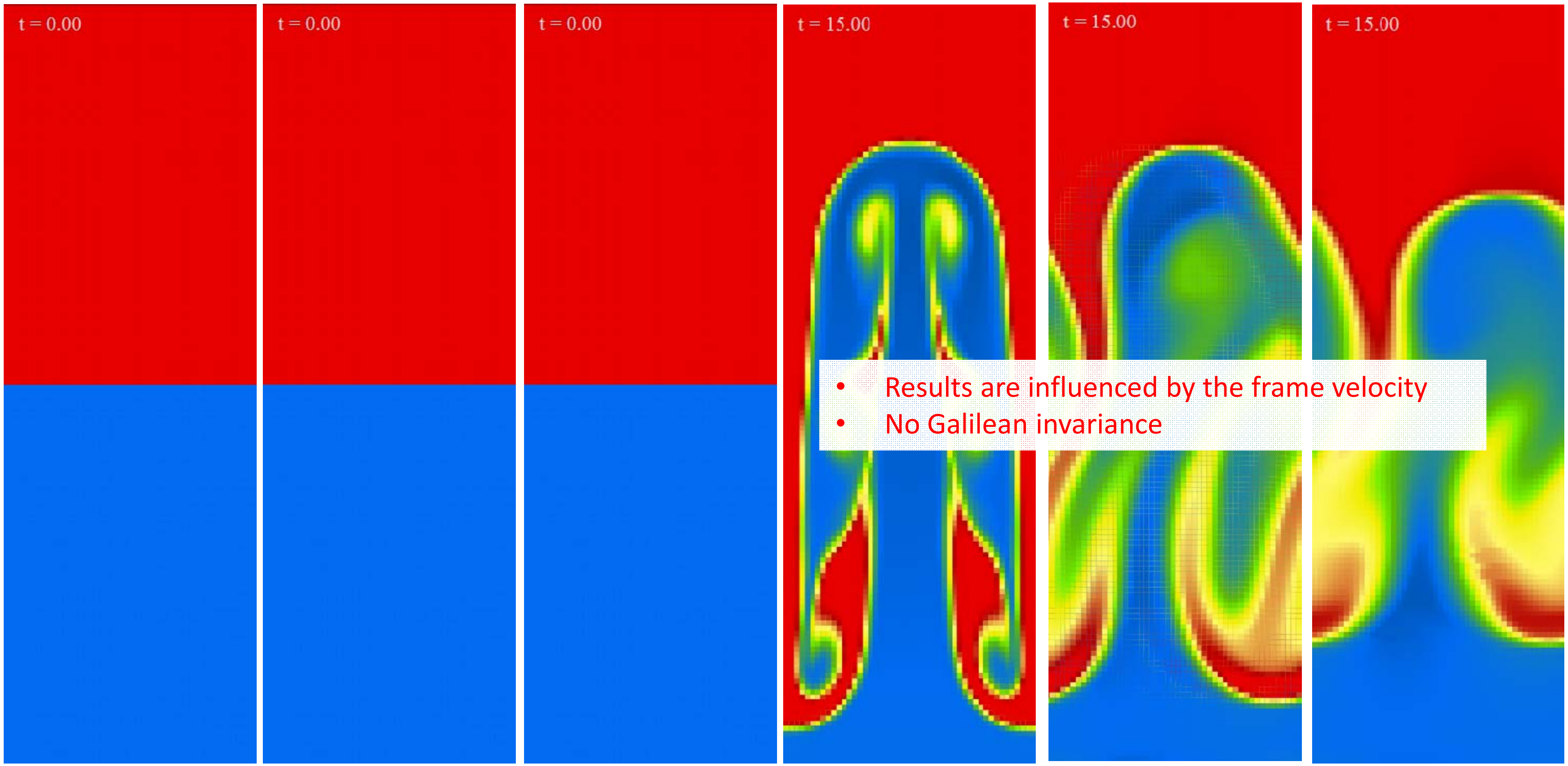
$48 \times 144$ $t = 15.0$




$v_x = 0$ $v_x = 1$ $v_x = 10$ $v_x = 0$ $v_x = 1$ $v_x = 10$

## Various Frame Velocities, Static Mesh

$48 \times 144$ $t = 15.0$

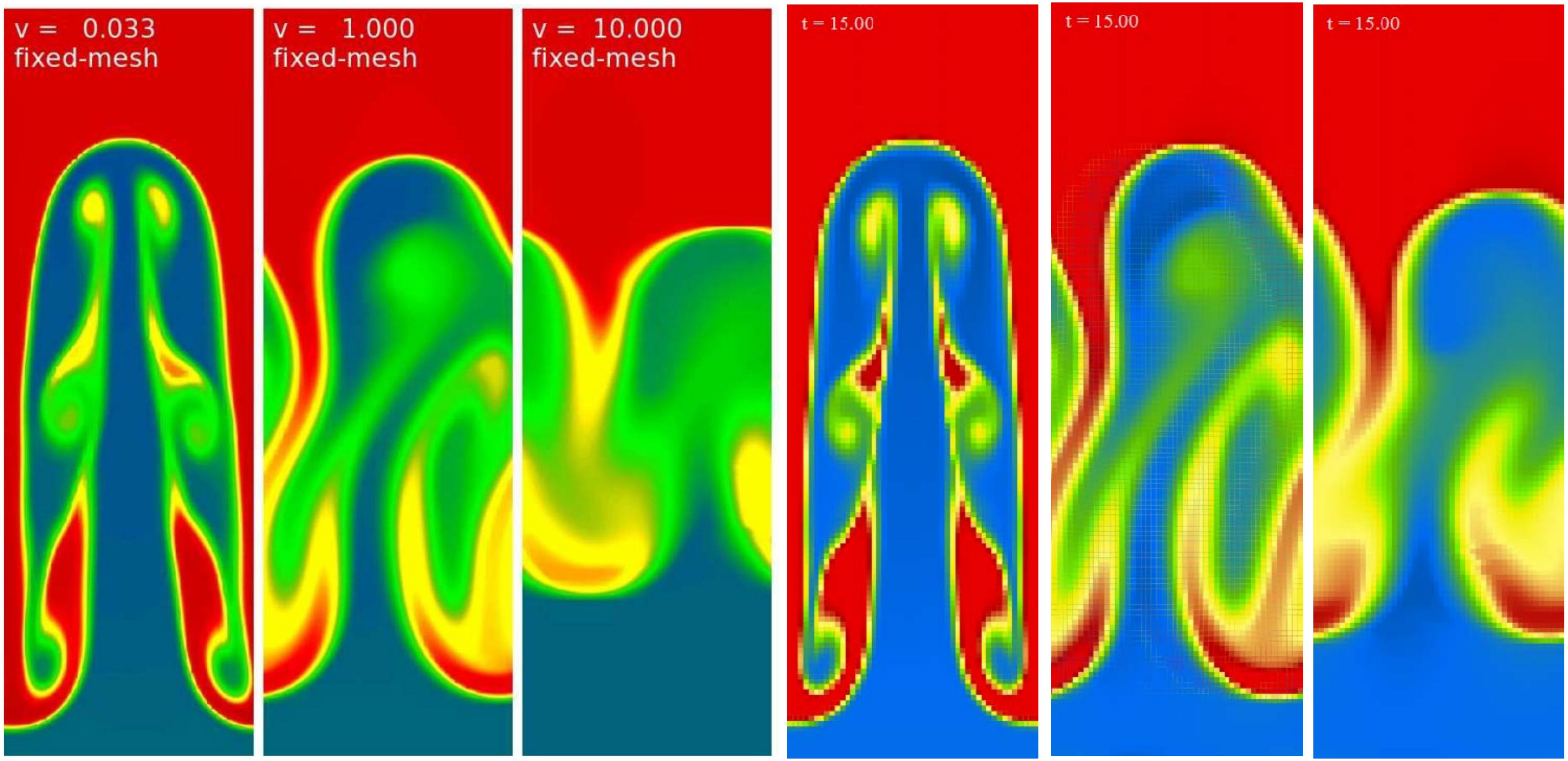



$v_x = 0$ $v_x = 1$ $v_x = 10$ $v_x = 0$ $v_x = 1$ $v_x = 10$

- Results are influenced by the frame velocity
- No Galilean invariance

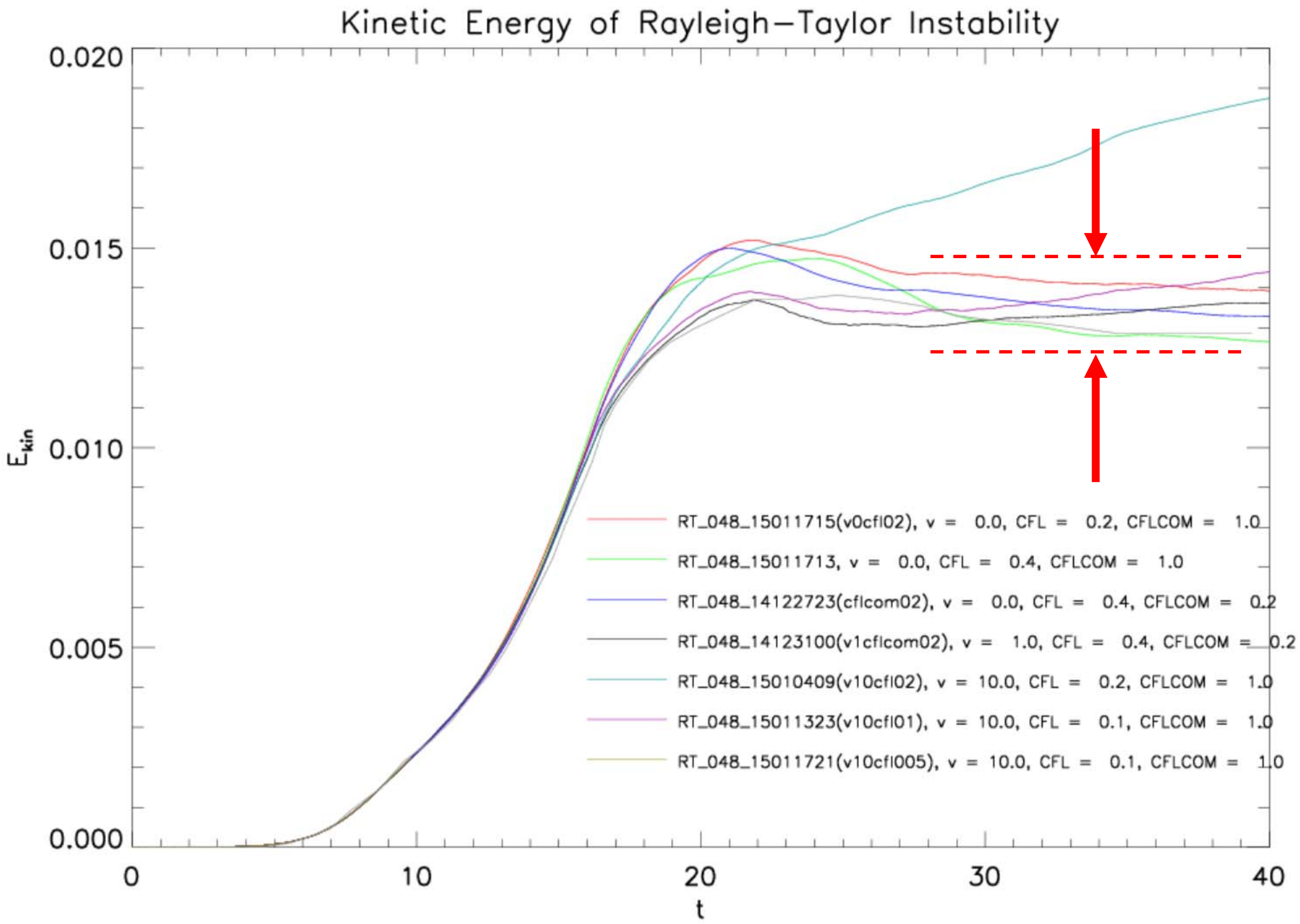
## AREPO vs YUMMy for various frame velocities and static mesh

AREPO YUMMy

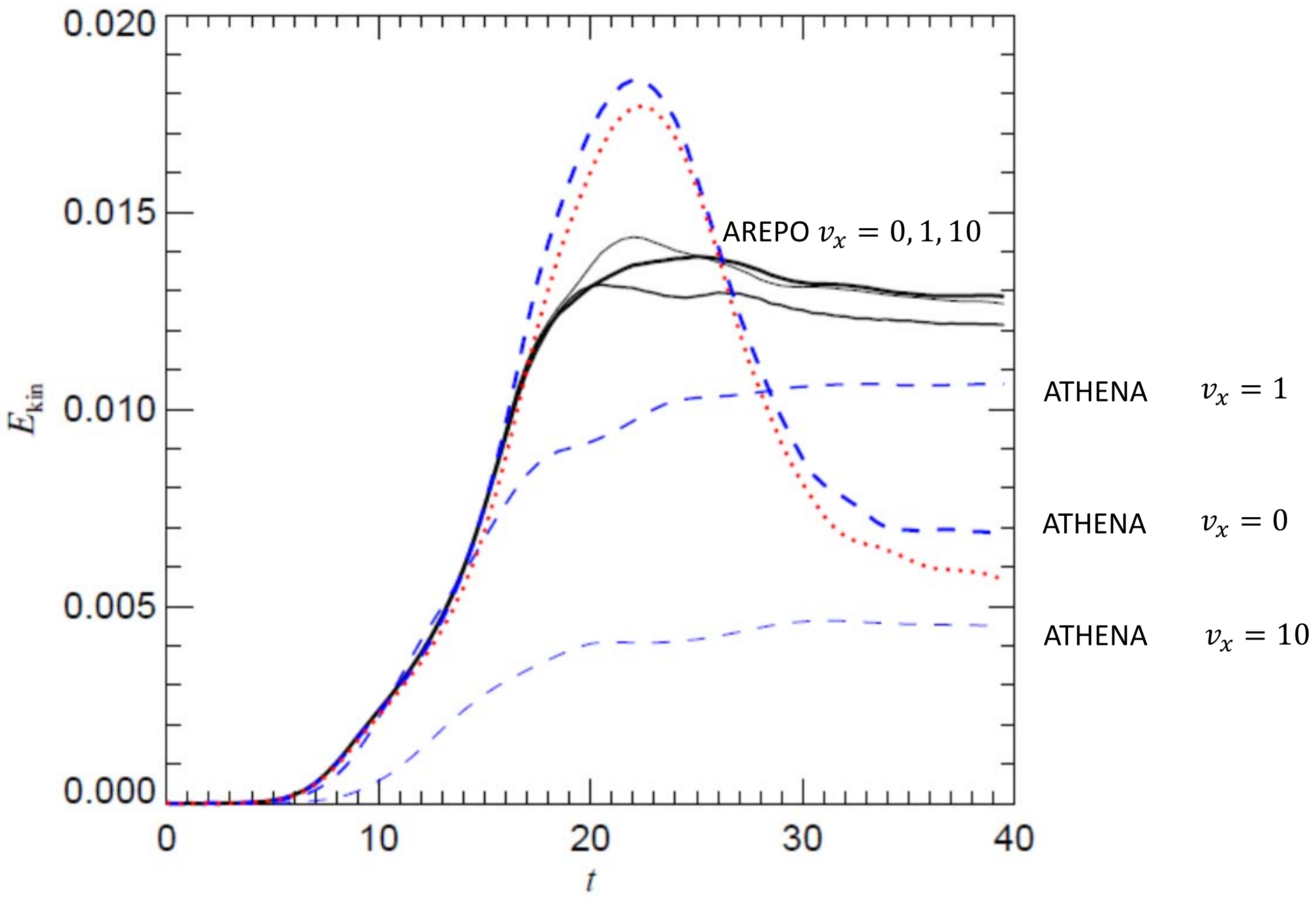



$v_x = 0$ $v_x = 1$ $v_x = 10$ $v_x = 0$ $v_x = 1$ $v_x = 10$

# Galilean invariance



# Galilean invariance

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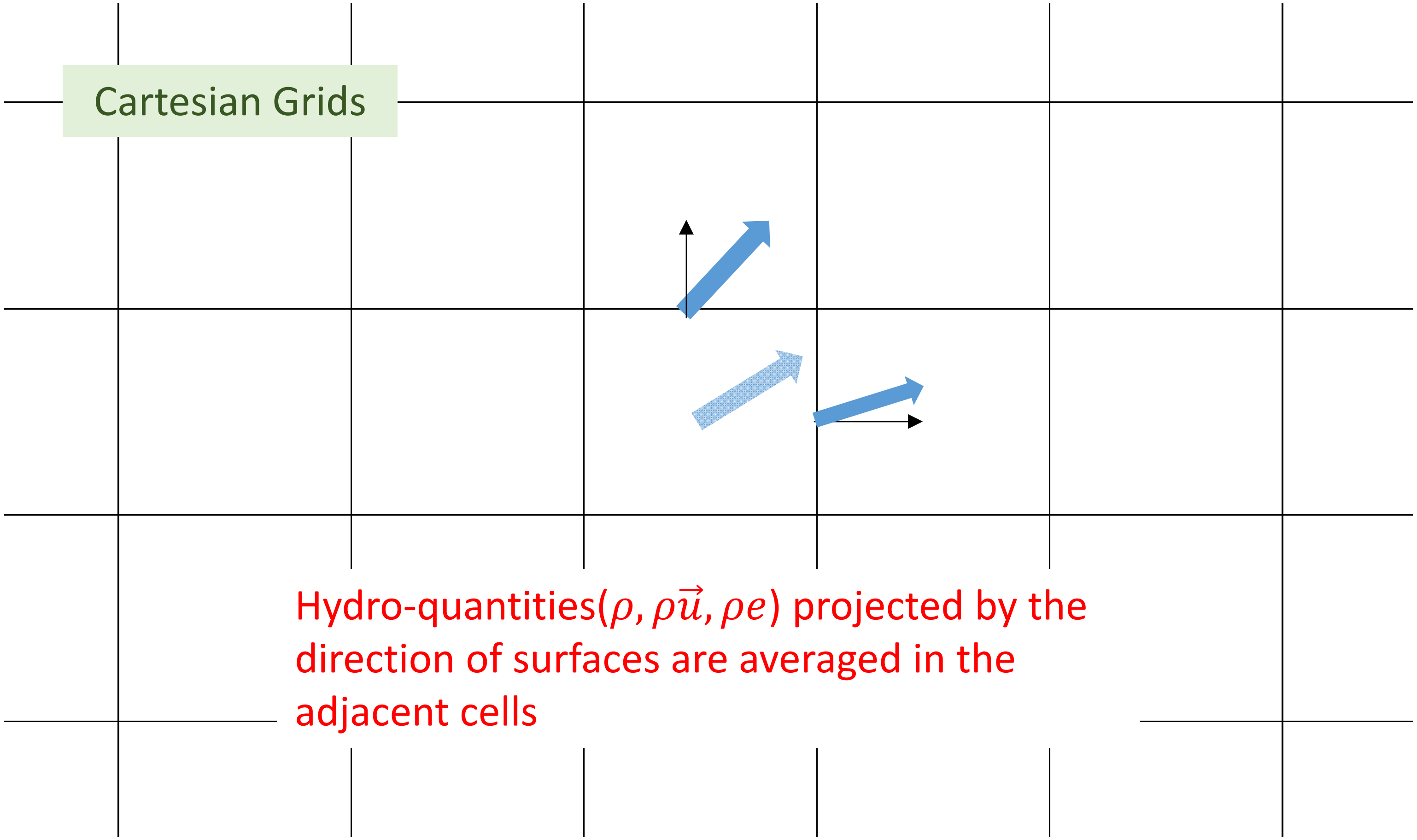
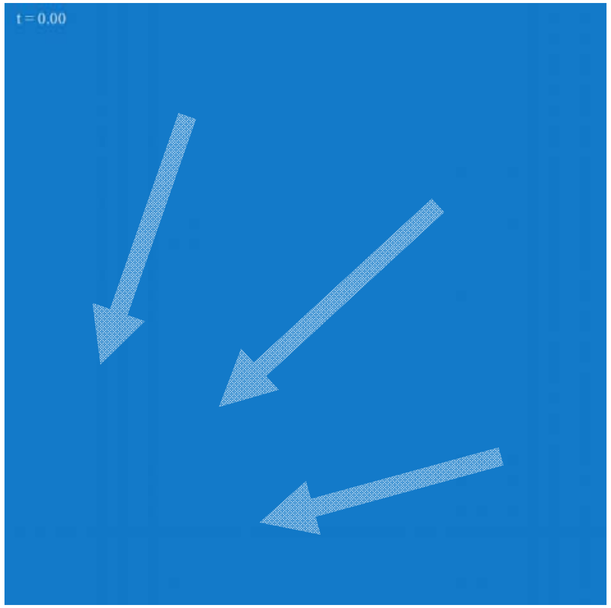
# Galilean invariance

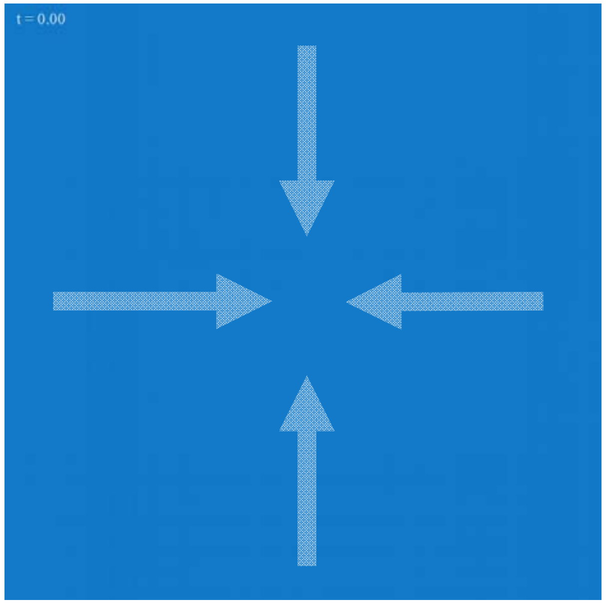
- The laws of motion are the *same* in all inertial frames.
- Any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary.(wiki)

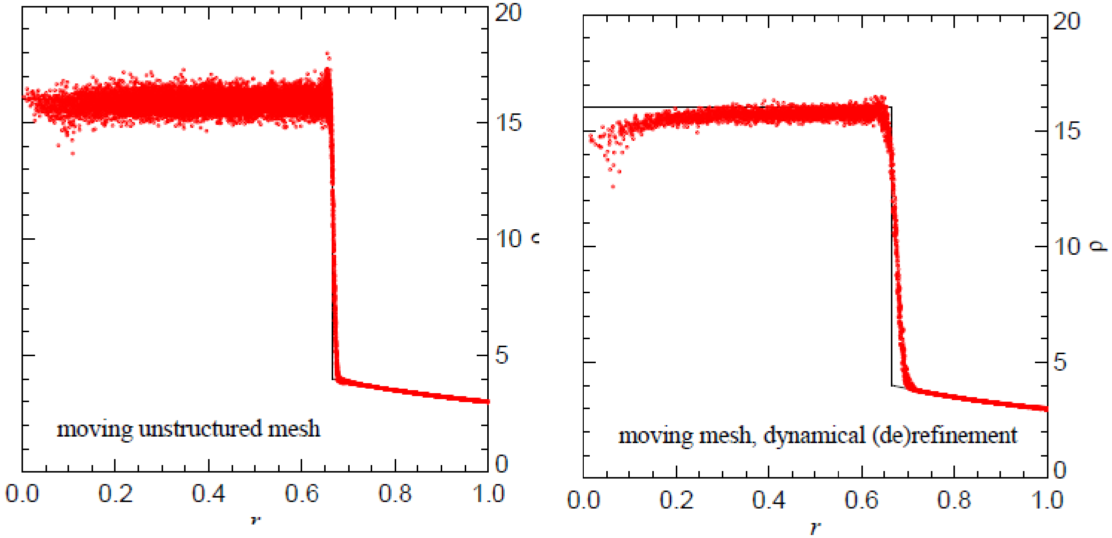
# Galilean invariance



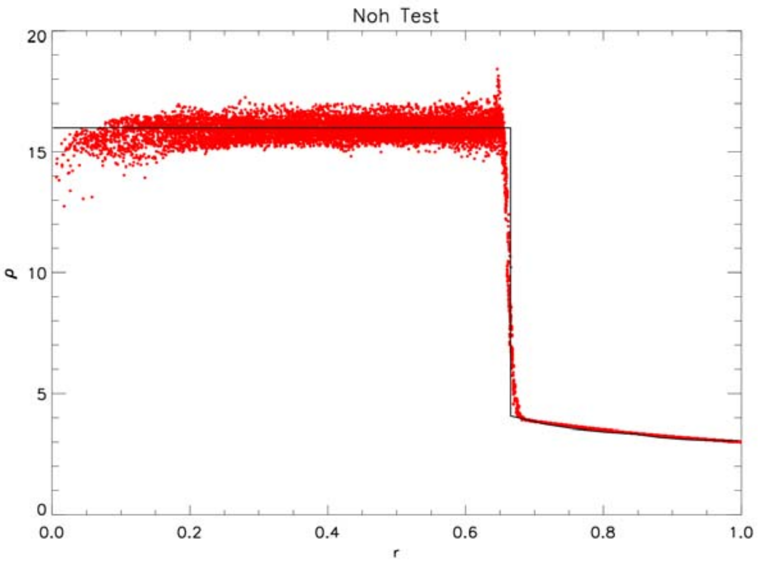
$t = 2.0$



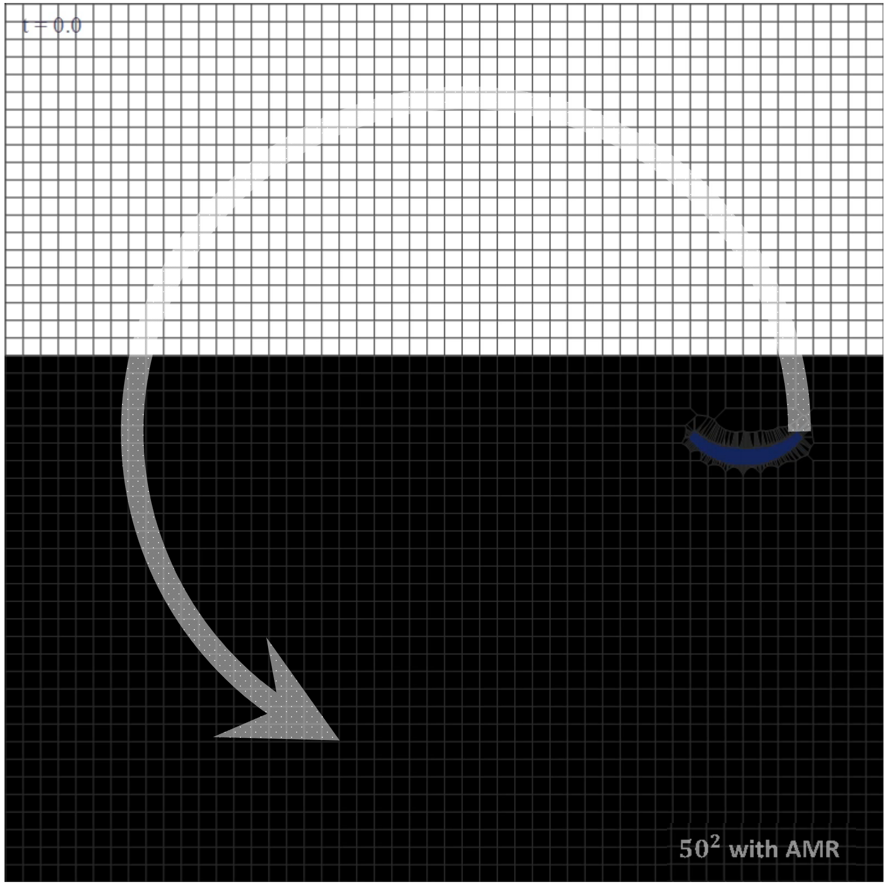

AREPO



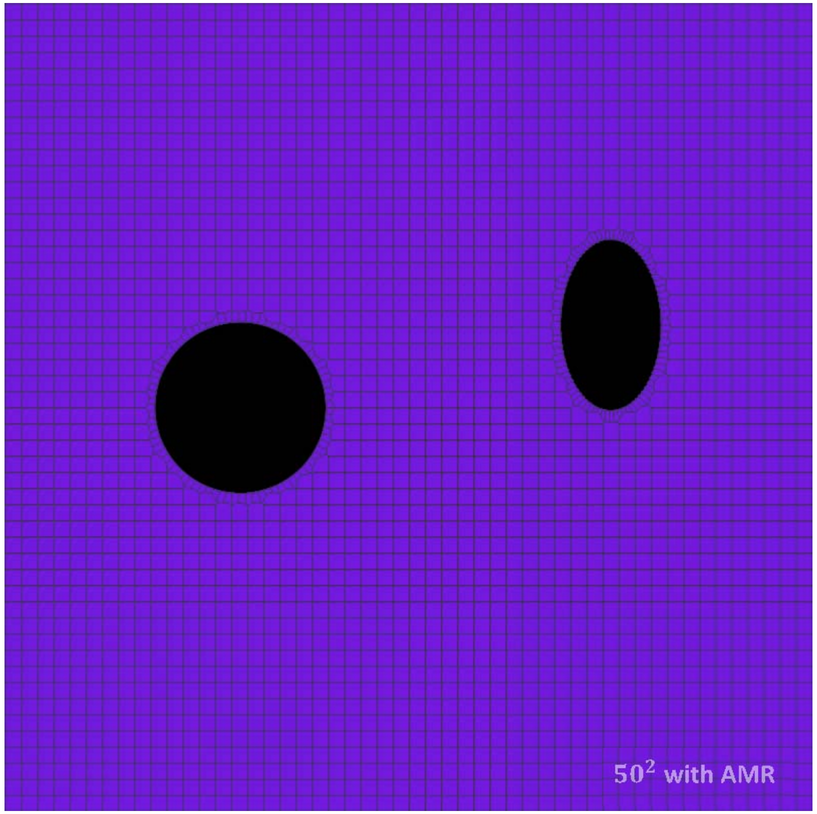

YUMMy

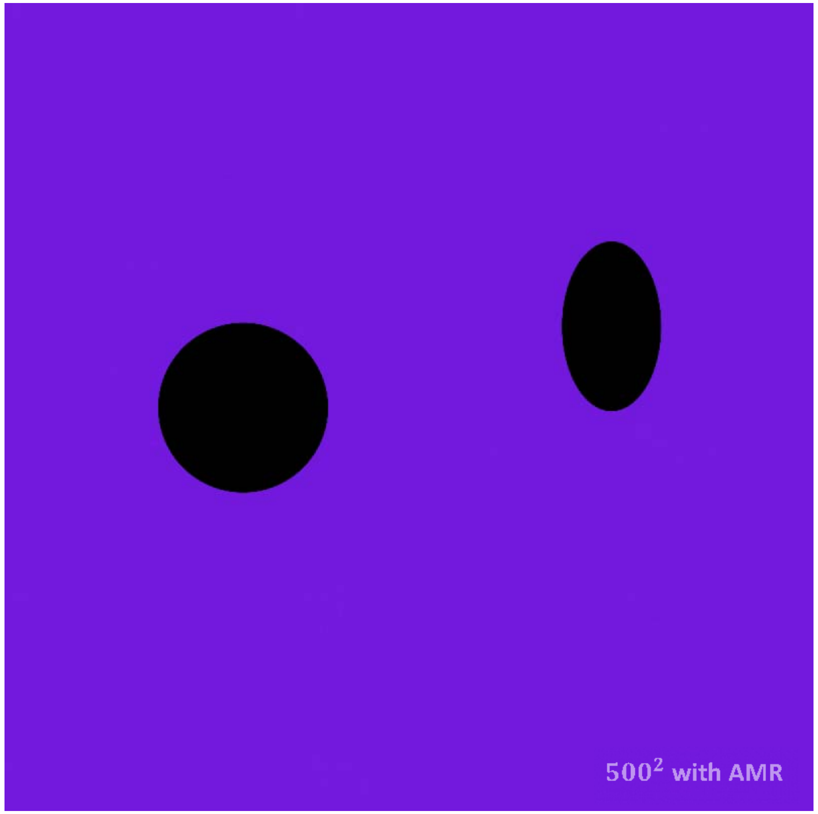
# Moving Boundary Test





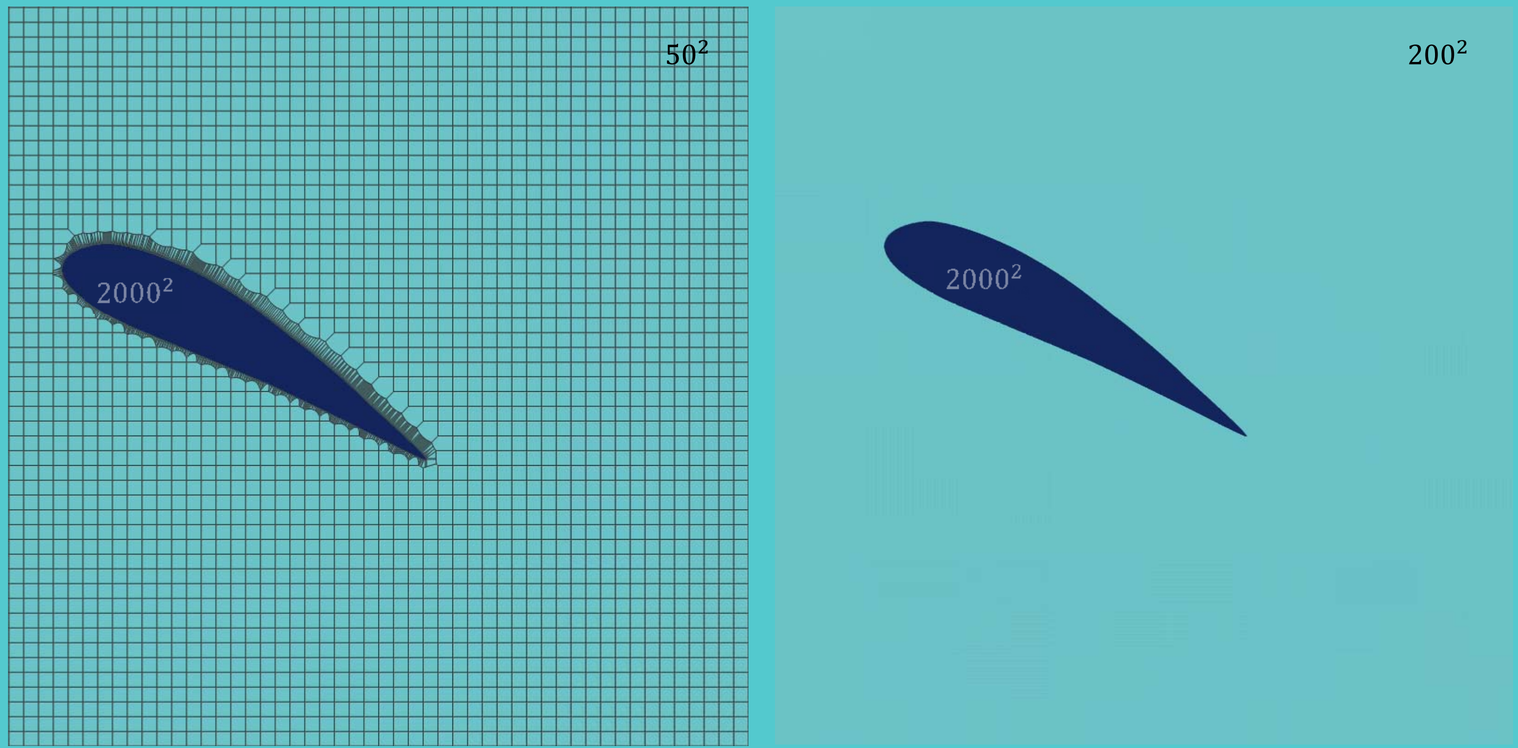

# 2(multiple) objects




Ma = 2.7

# Wing

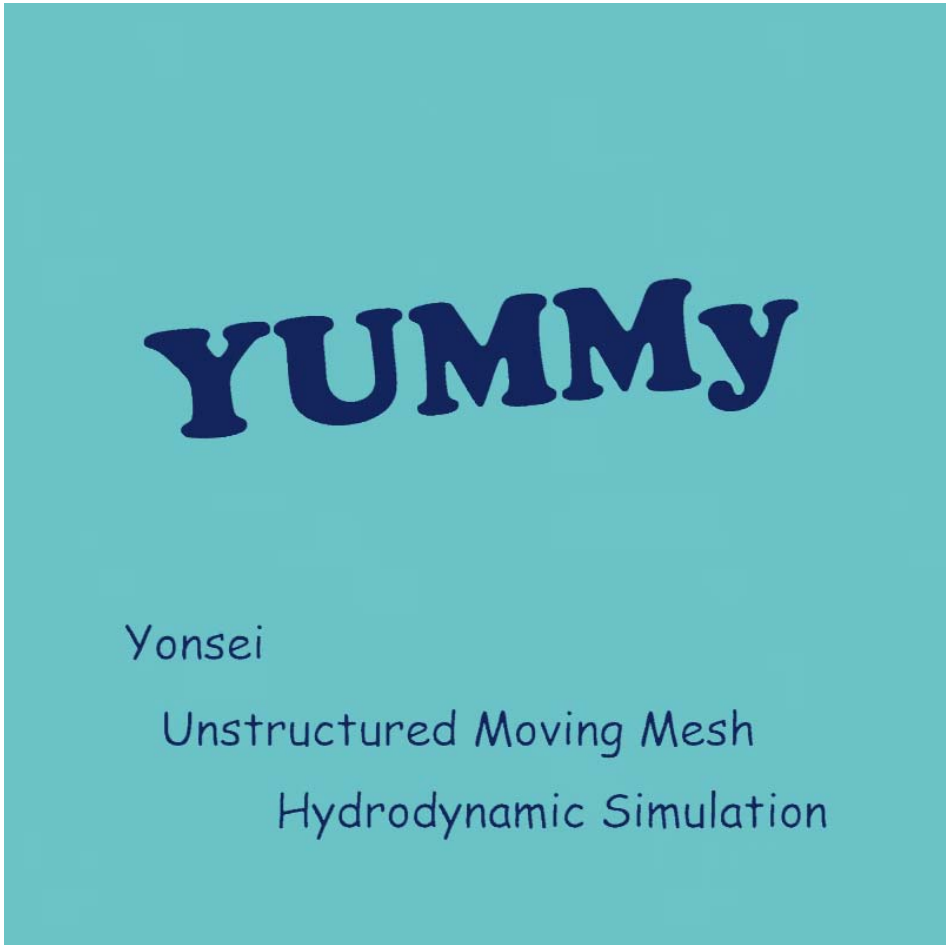



- Easy/Possible to set arbitrary shape of boundary condition
- Cartesian coordinate, without needing different coordinates (polar, cylindrical)

$50^2$, $200^2$ for medium, $2000^2$ for boundary condition

Ma = 1.5

# Characters(very complex boundary)




- Karman vertox street
- Turbulences
- Bow shock

Ma = 1.5

$600^2$ for medium, $2000^2$ for boundary condition

# Summary

- YUMMy; Yonsei Unstructured Moving Mesh Hydrodynamic Simulation code
- Both of Lagrangian and Eulerian description are implemented.
- The Voronoi Tessellation is adapted for unstructured mesh.
  - The IEM, Hybrid-Neighbor Searching and  schemes are implemented → high performance
  - Figuring out the robust solution to generate voronoi tessellation → in process  of publishing paper

- YUMMy is the Galilean-invariance code.
  - Suitable to implement for cosmological simulation and interaction between multiple objects
- The results of both static and moving mesh tests are consistent with that of another moving mesh code, AREPO.
- It's possible to use arbitrary shape of objects without needing different coordinates (polar, cylindrical)