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Effects of multiple scale driving on turbulence statistics

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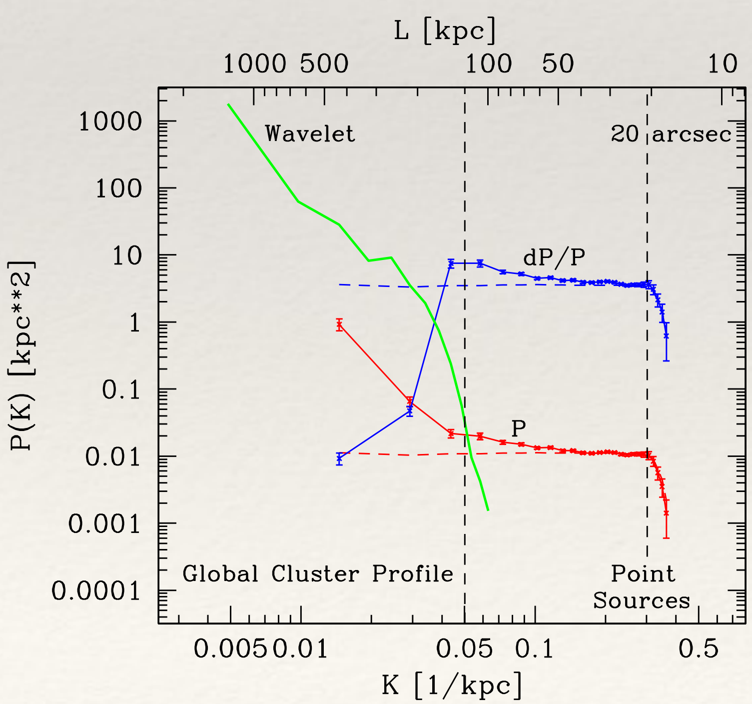
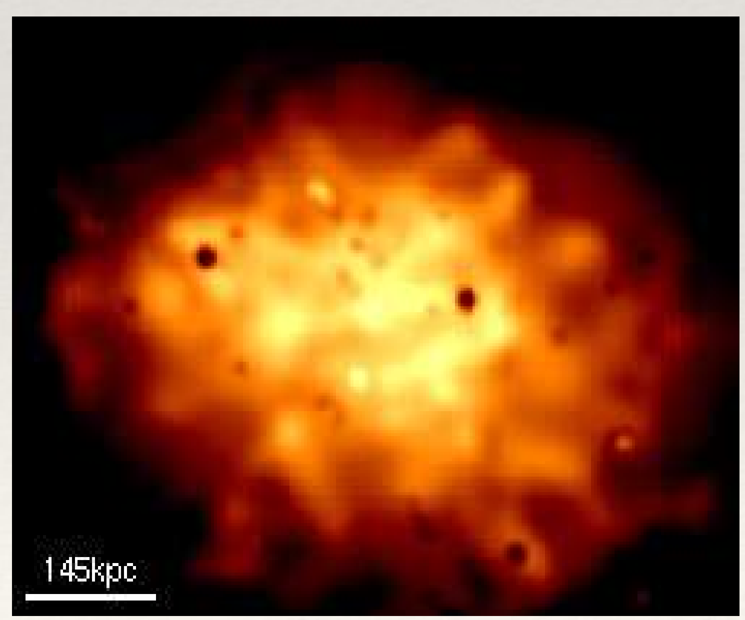
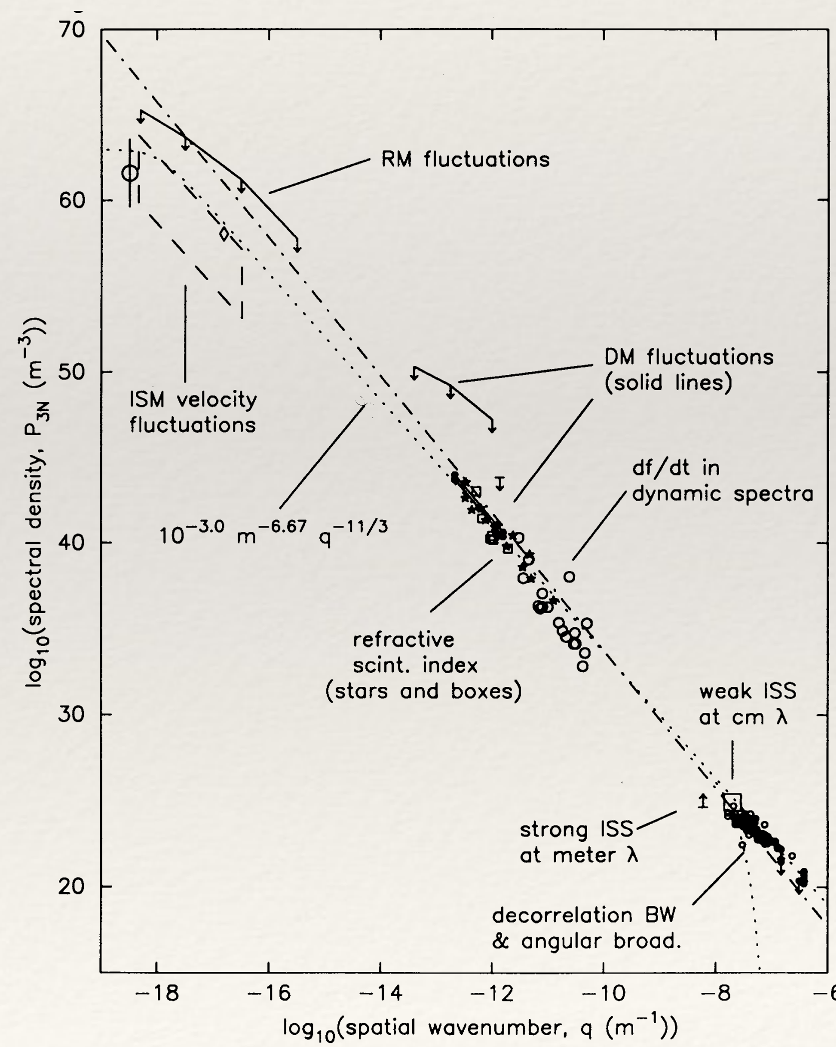
Introduction

* Astrophysical fluids
  - Magnetized & Turbulent
    (e.g Interstellar medium, intracluster medium, solar winds ..)
  ⇒ Numerical simulation of driven MHD turbulence





Schuecker et al. (2004)



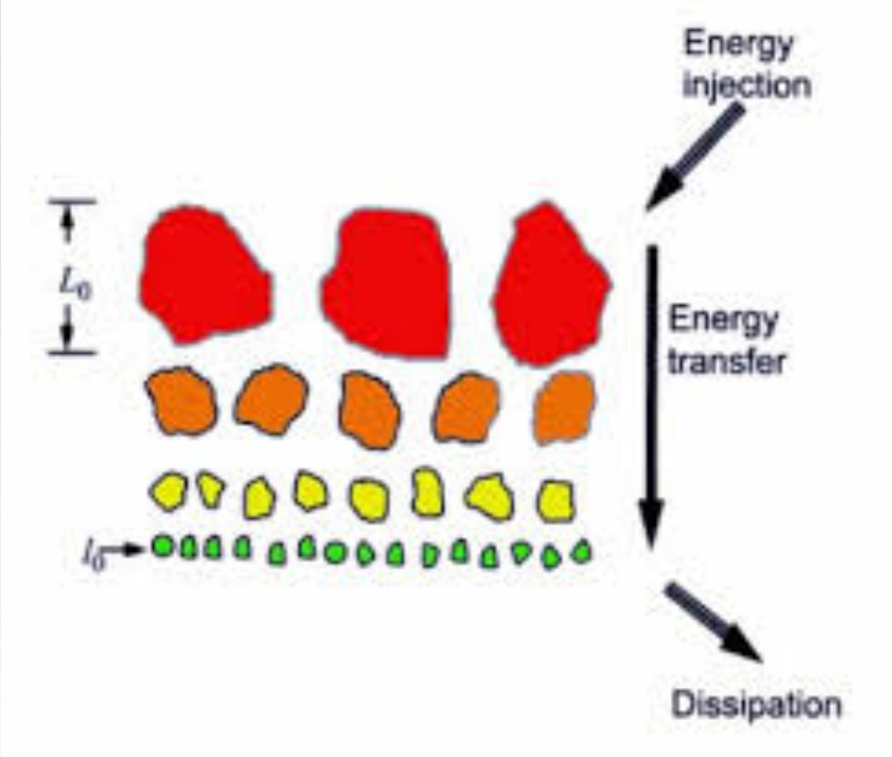
Armstrong et al. (1995)

Introduction

- Astrophysical fluids
  - Magnetized & Turbulent
    (e.g Interstellar medium, intracluster medium, solar winds ..)
  ⇒ Numerical simulation of driven MHD turbulence

- Energy cascade of turbulence
  → Energy injection (driving) is required !

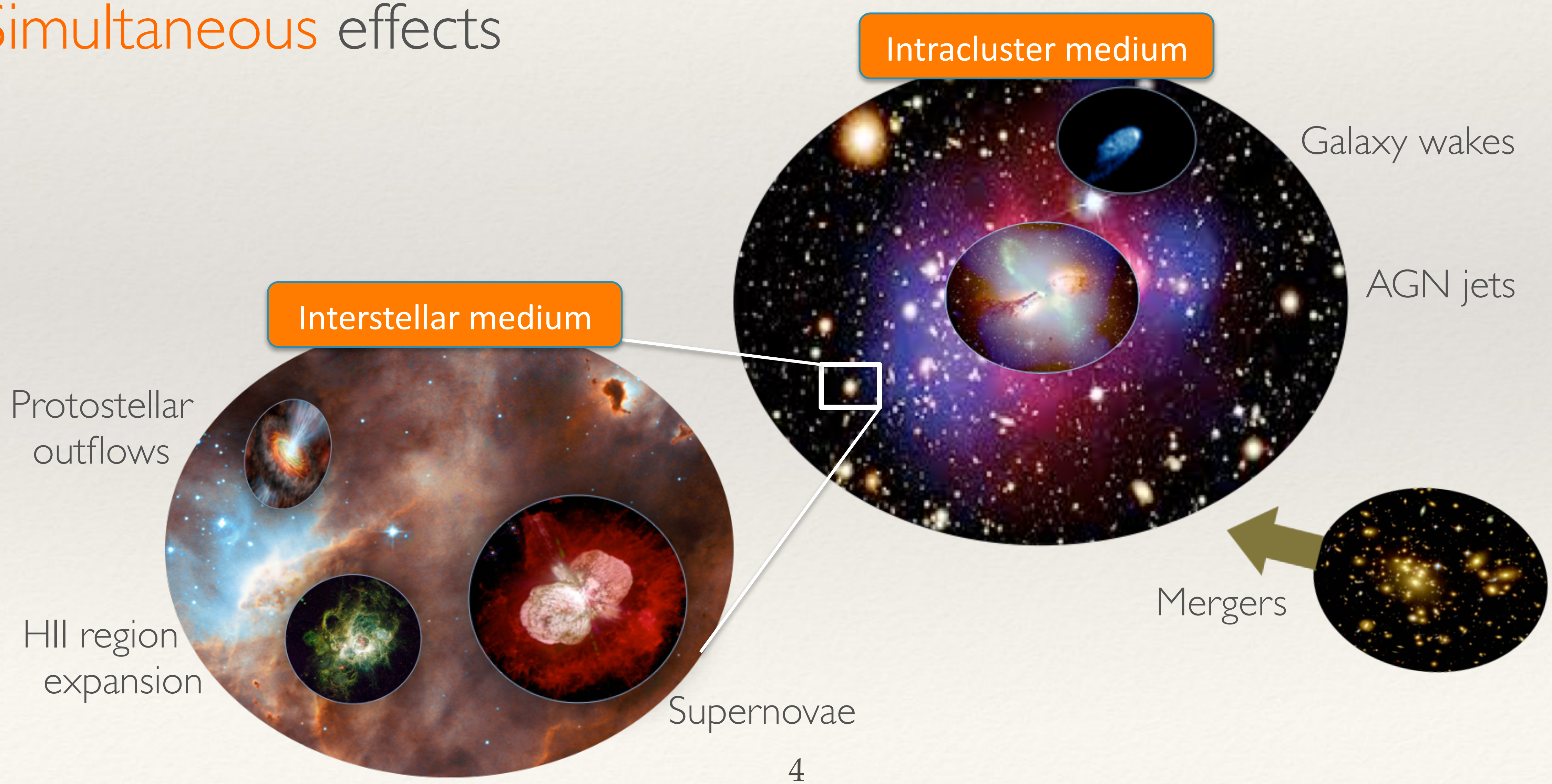

Leonardo Da Vinci's drawing



Introduction

* Variety of driving mechanisms on various scales
  - Interstellar medium : few pc ~ hundreds of pc
  - Intracluster medium : tens of kpc ~ hundreds of kpc

* Simultaneous effects



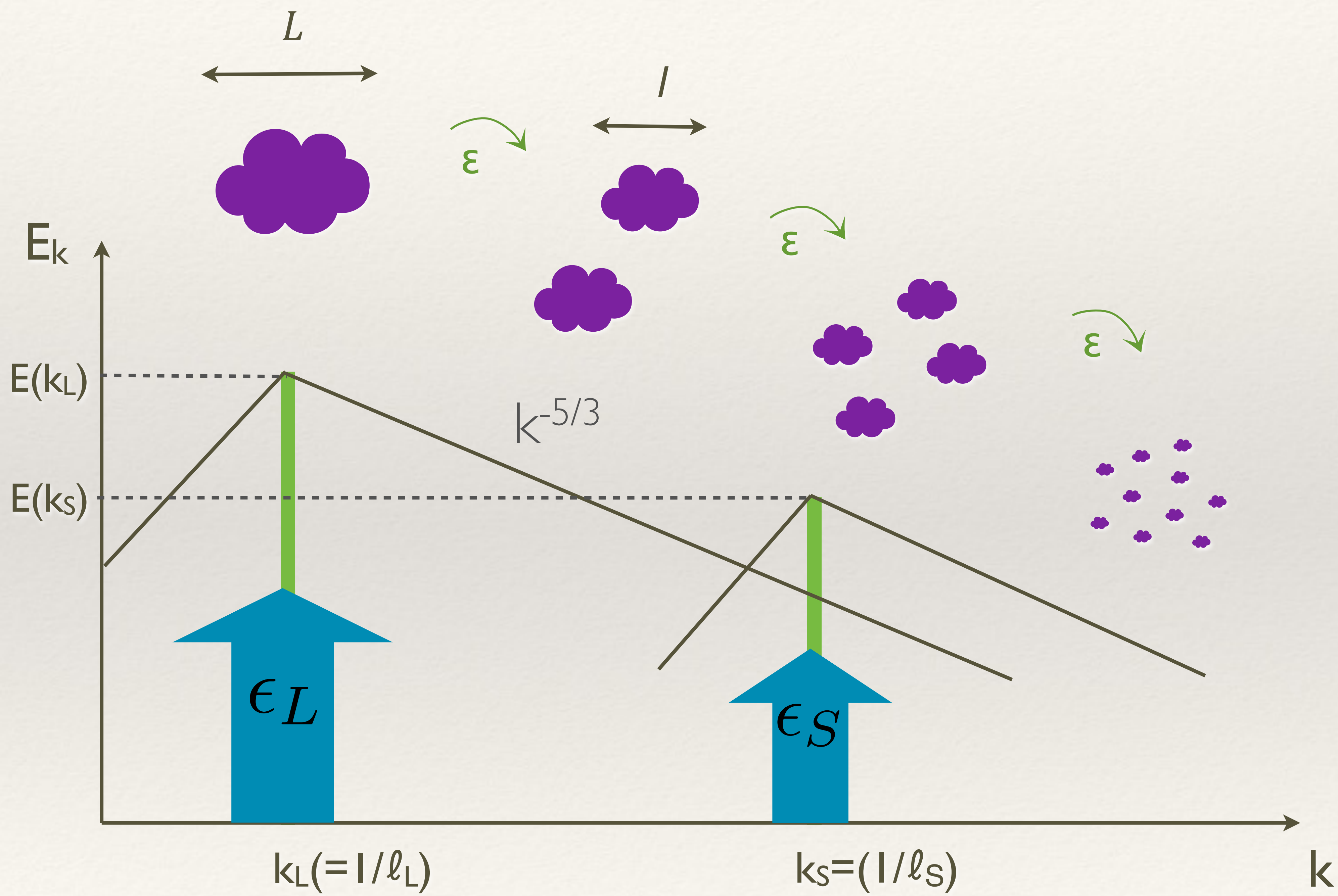
Numerical method

* Incompressible/compressible MHD turbulence simulations
  - Pseudo-spectral code for incompressible MHD simulations
  - Essentially Non-Oscillatory scheme for isothermal compressible MHD simulations
  - Resolution : 256^3 grids
  - Average velocity ~ O(1)
  - External magnetic field B_0=0.001(weak) or 1.0 (strong) (in the same unit as the Alfven speed)

* Forcing
  - Solenoidal forcing (divergence-free)
  - Driven at two ranges in Fourier space
  - large-scale random forcing in $2<k<\sqrt{12}$
  - small-scale random forcing in $15<k<26$

Numerical method

* Spectrum of turbulence model driven at two different scales



Analytic expectation

* Expected scaling relations

According to Kolmogorov's theory (Kolmogorov 1941)

$$v = (\epsilon l)^{\frac{1}{3}} \ \& \ k \sim 1/l \quad \Longrightarrow \quad \frac{v_L}{v_S} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{1}{3}} \left(\frac{l_L}{l_S}\right)^{\frac{1}{3}} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{1}{3}} \left(\frac{k_S}{k_L}\right)^{\frac{1}{3}}$$

An approximation,

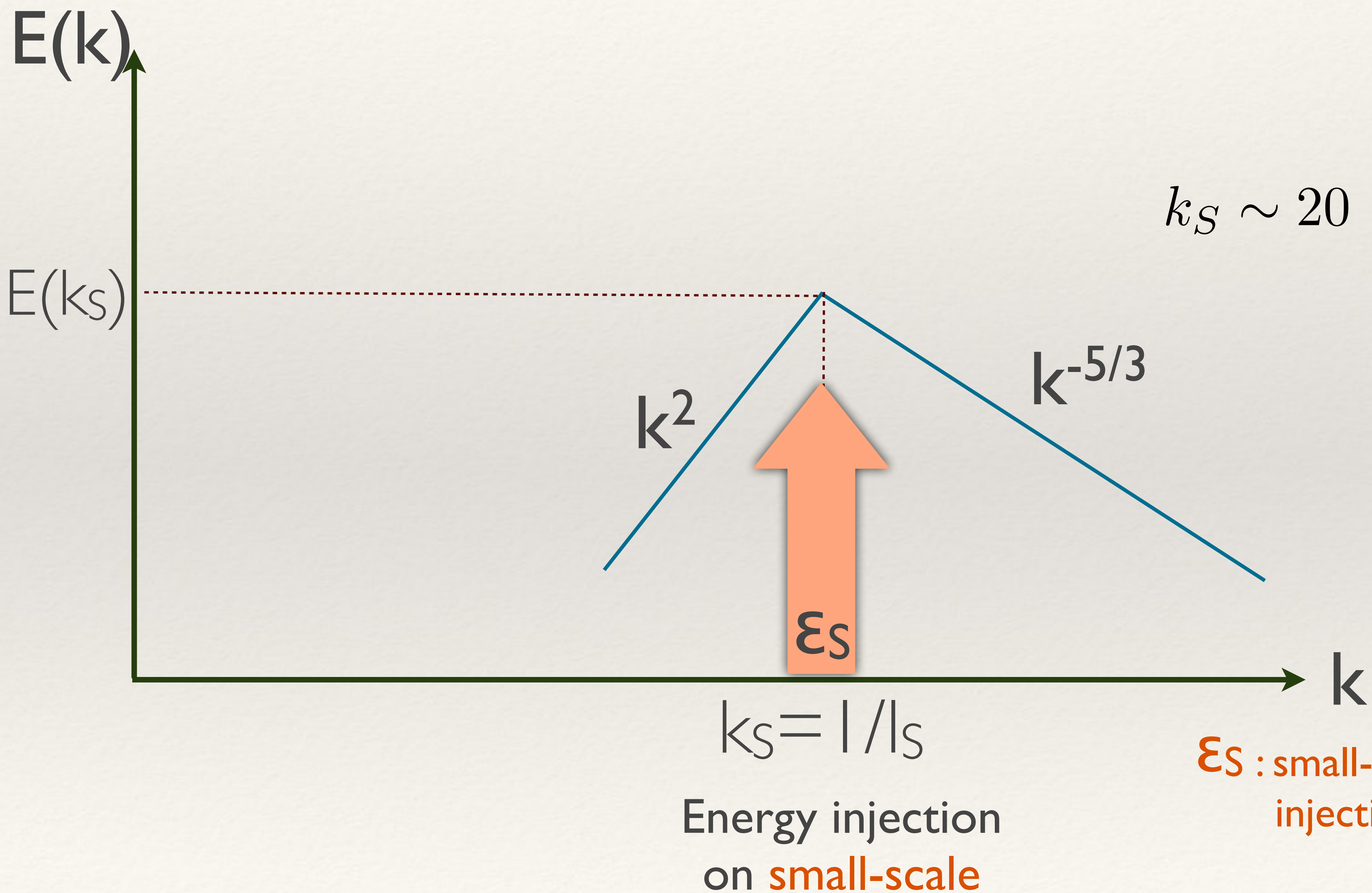
$$v \approx \sqrt{kE(k)} \quad \Longrightarrow \quad \frac{v_L}{v_S} \approx \frac{\sqrt{k_L E(k_L)}}{\sqrt{k_S E(k_S)}} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{1}{3}} \left(\frac{k_S}{k_L}\right)^{\frac{1}{3}}$$

$$\frac{E(k_L)}{E(k_S)} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}} \left(\frac{k_S}{k_L}\right)^{\frac{5}{3}} \quad \Longrightarrow \quad k_S / k_L = 8$$ for our simulations

$$\Longrightarrow \quad \frac{E(k_L)}{E(k_S)} = 32 \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}}$$

Numerical method

* Single-scale driving

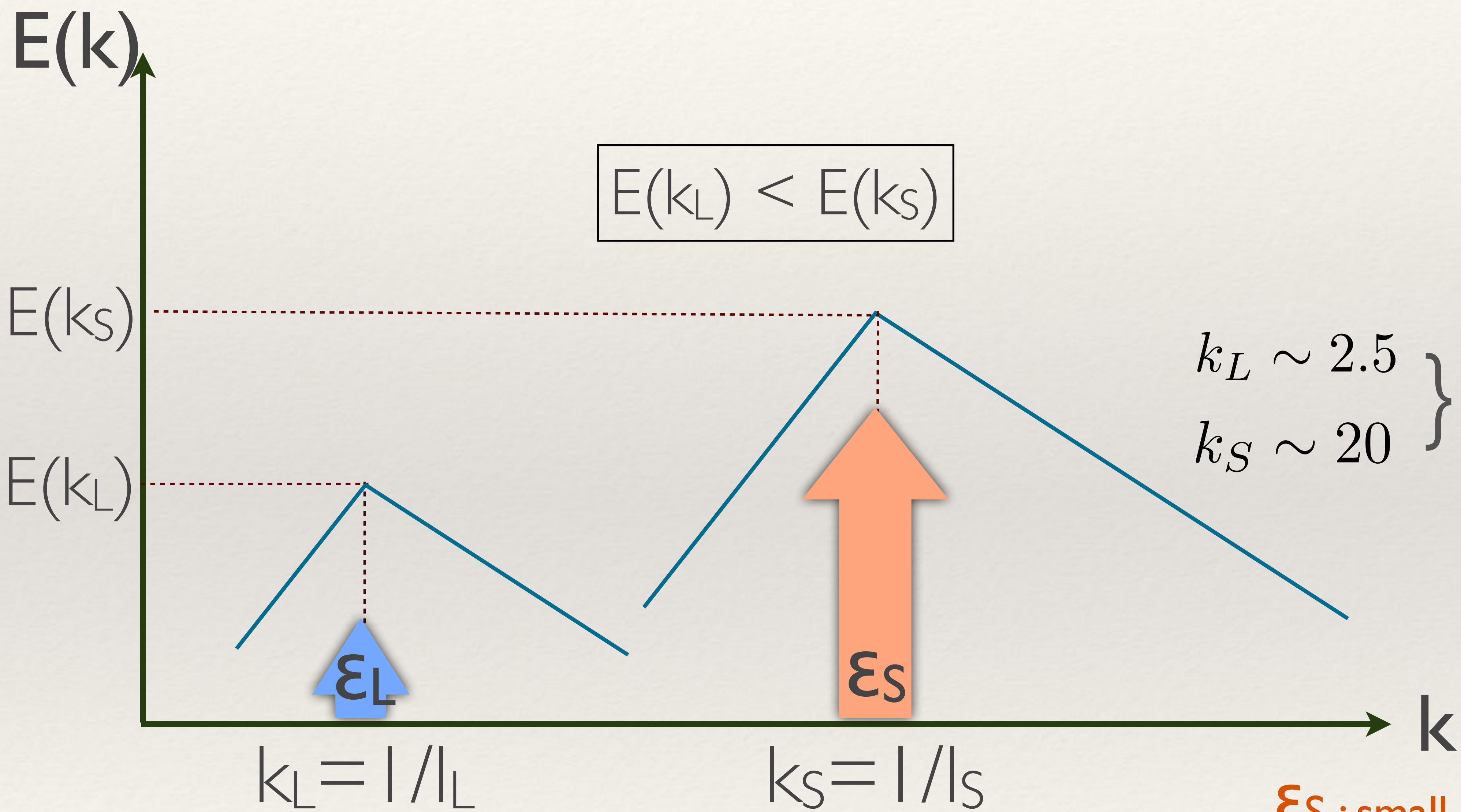


$k_S \sim 20$ for our simulations

εS : small-scale energy injection rate

Numerical method

* Multiple-scale driving ($\varepsilon_L \lll \varepsilon_S$)



Additional energy injection on large-scale

Energy injection on small-scale

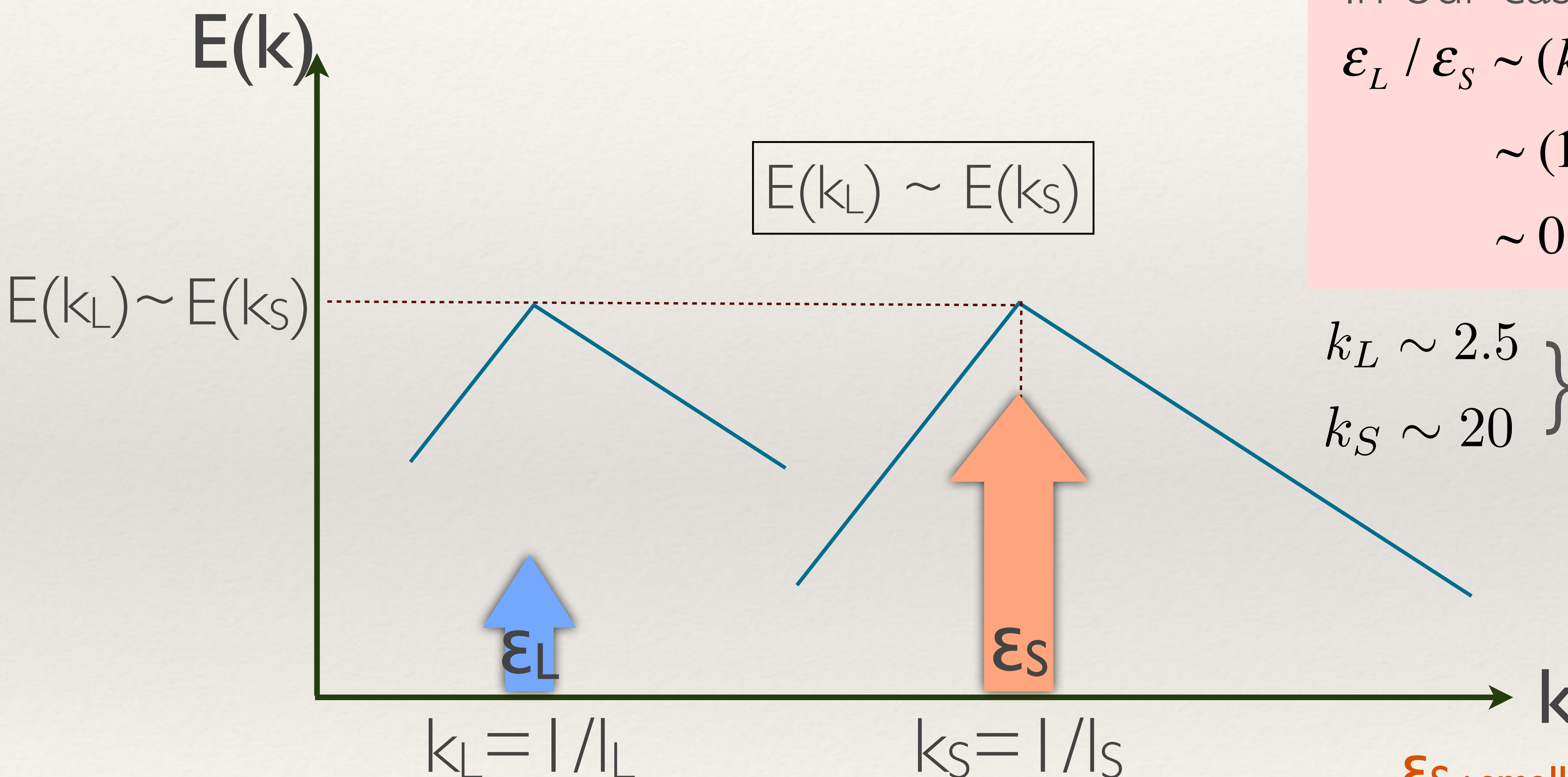
$k_L \sim 2.5$
$k_S \sim 20$
} for our simulations

ε_S : small-scale energy injection rate

ε_L : large-scale energy injection rate

Numerical method

* Multiple-scale driving ($\varepsilon_L \ll \varepsilon_S$)



Additional energy injection on large-scale

Energy injection on small-scale

In our cases,

$$\varepsilon_L / \varepsilon_S \sim (k_L / k_S)^{2.5}$$
$$\sim (1/180)$$
$$\sim 0.0056$$

$k_L \sim 2.5$, $k_S \sim 20$ } for our simulations

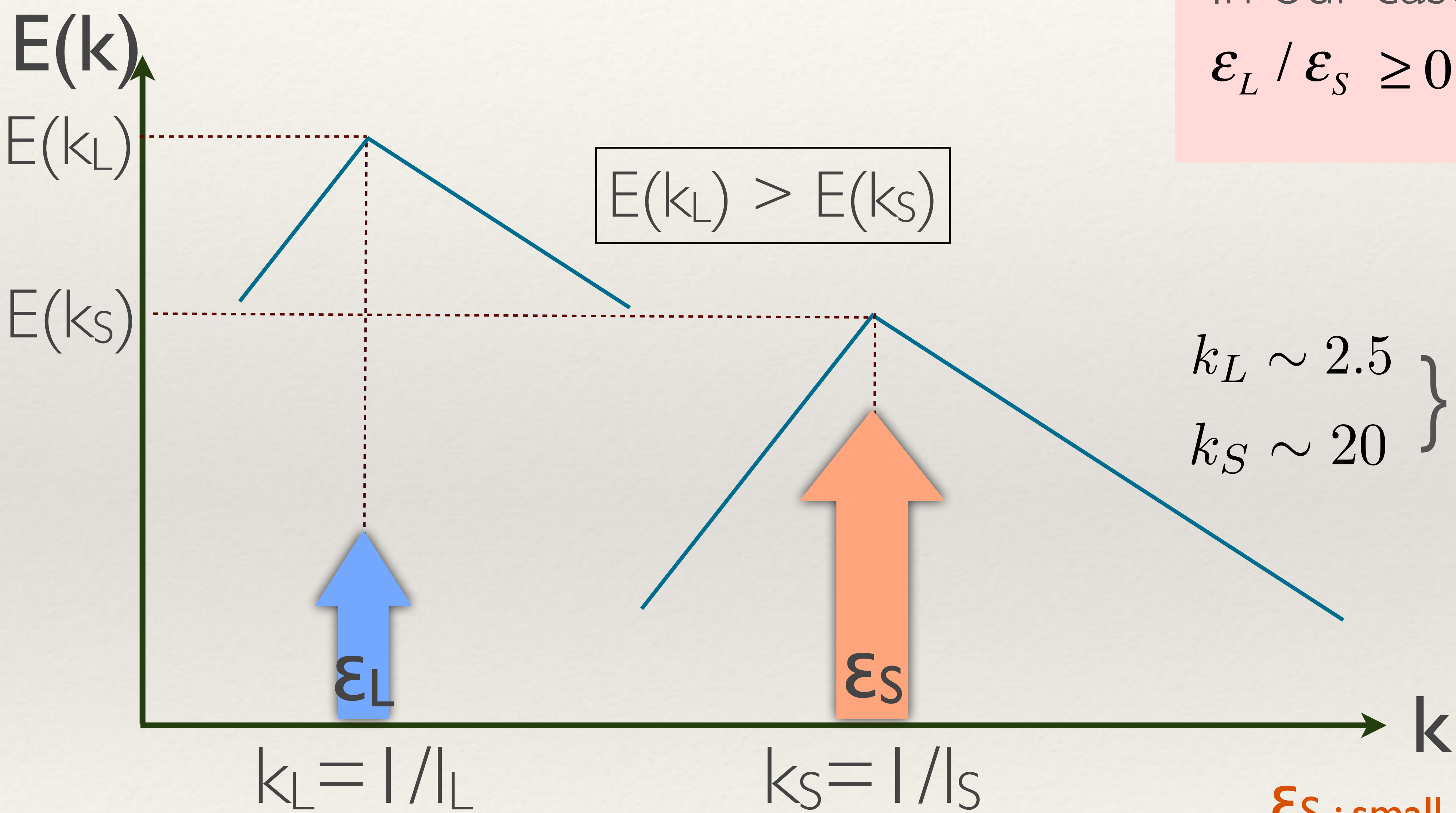
ε_S : small-scale energy injection rate

ε_L : large-scale energy injection rate

Numerical method

* Multiple-scale driving ($\varepsilon_L \ll \varepsilon_S$)

In our cases,
$\varepsilon_L / \varepsilon_S \geq 0.0056$



$k_L \sim 2.5$
$k_S \sim 20$
} for our simulations

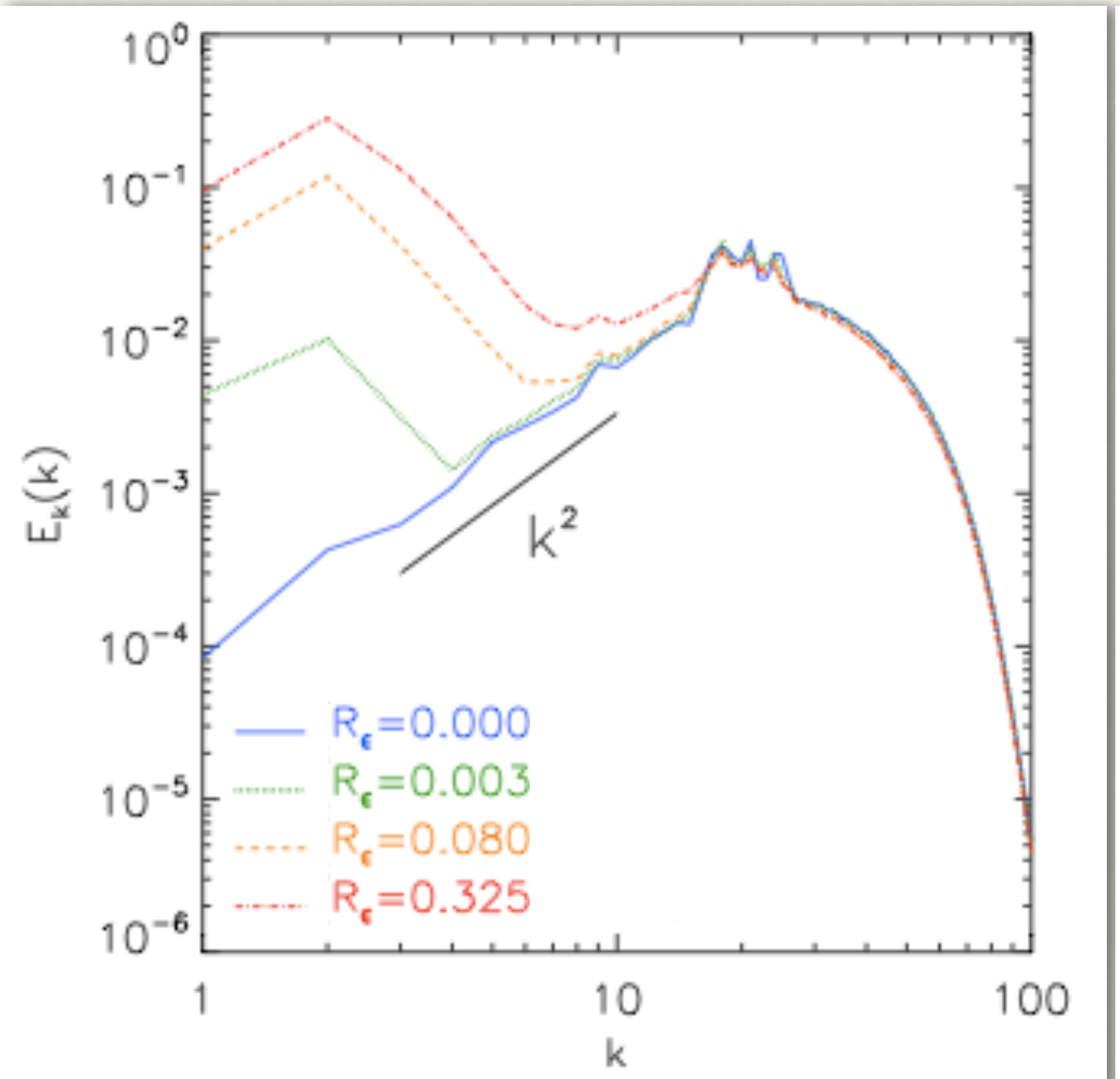
Additional energy injection on large-scale

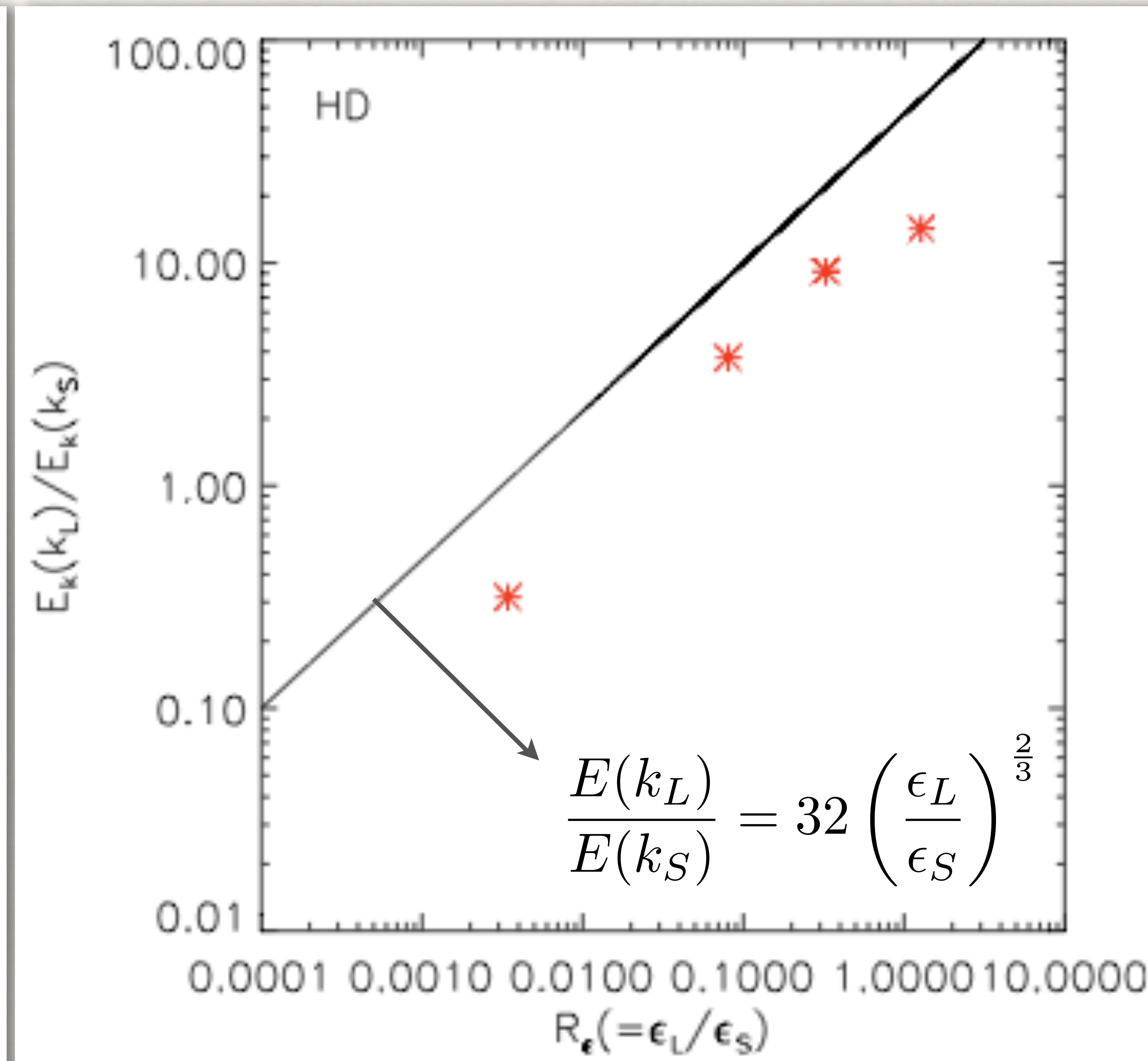
Energy injection on small-scale

ε_S : small-scale energy injection rate
ε_L : large-scale energy injection rate

Result - Incompressible HD test

* Kinetic energy spectrum & Scaling relation

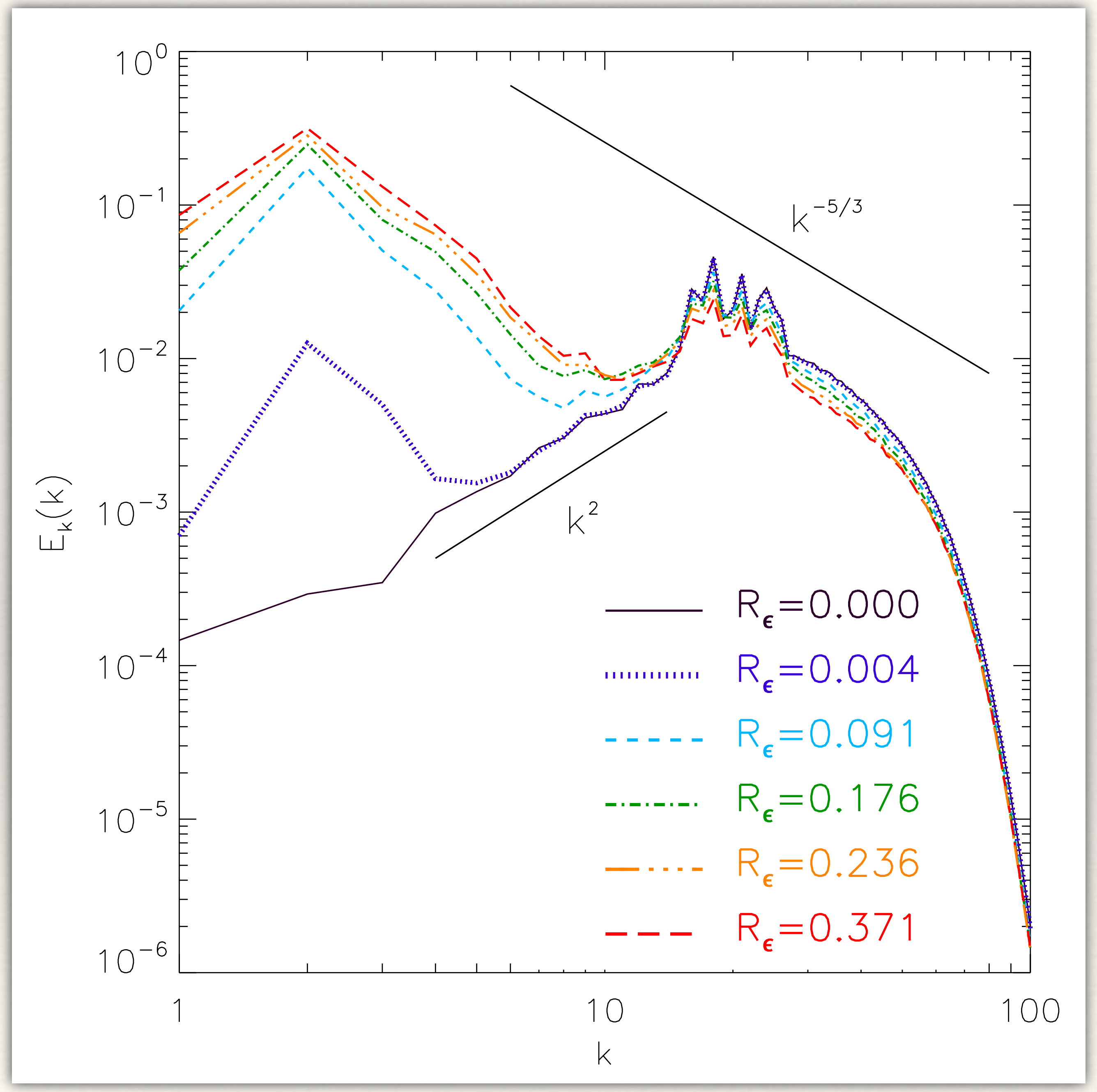


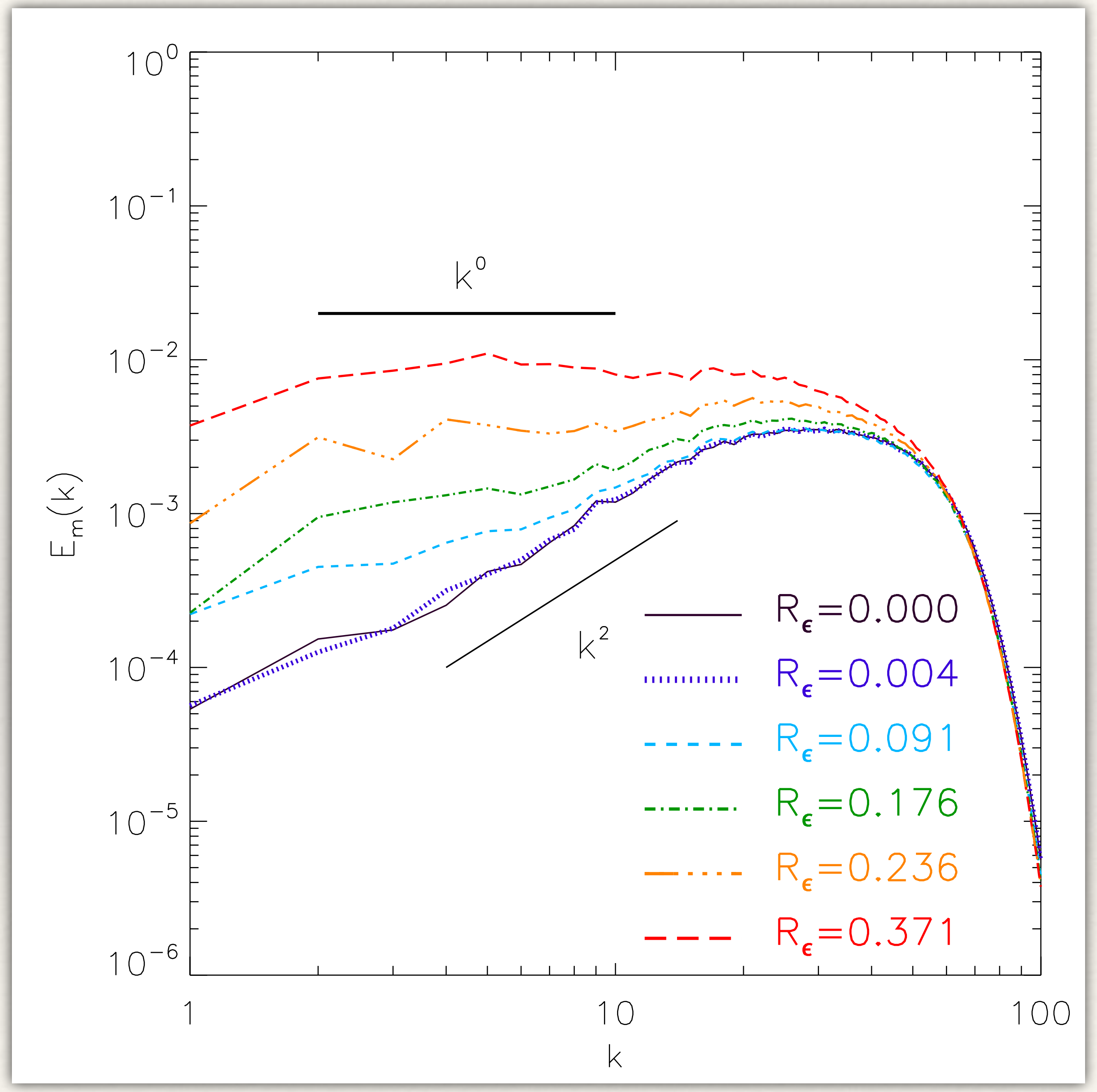


$R_\varepsilon = \varepsilon_L/\varepsilon_S$

Result - Incompressible MHD w/ weak B0

* Kinetic and magnetic energy spectrum

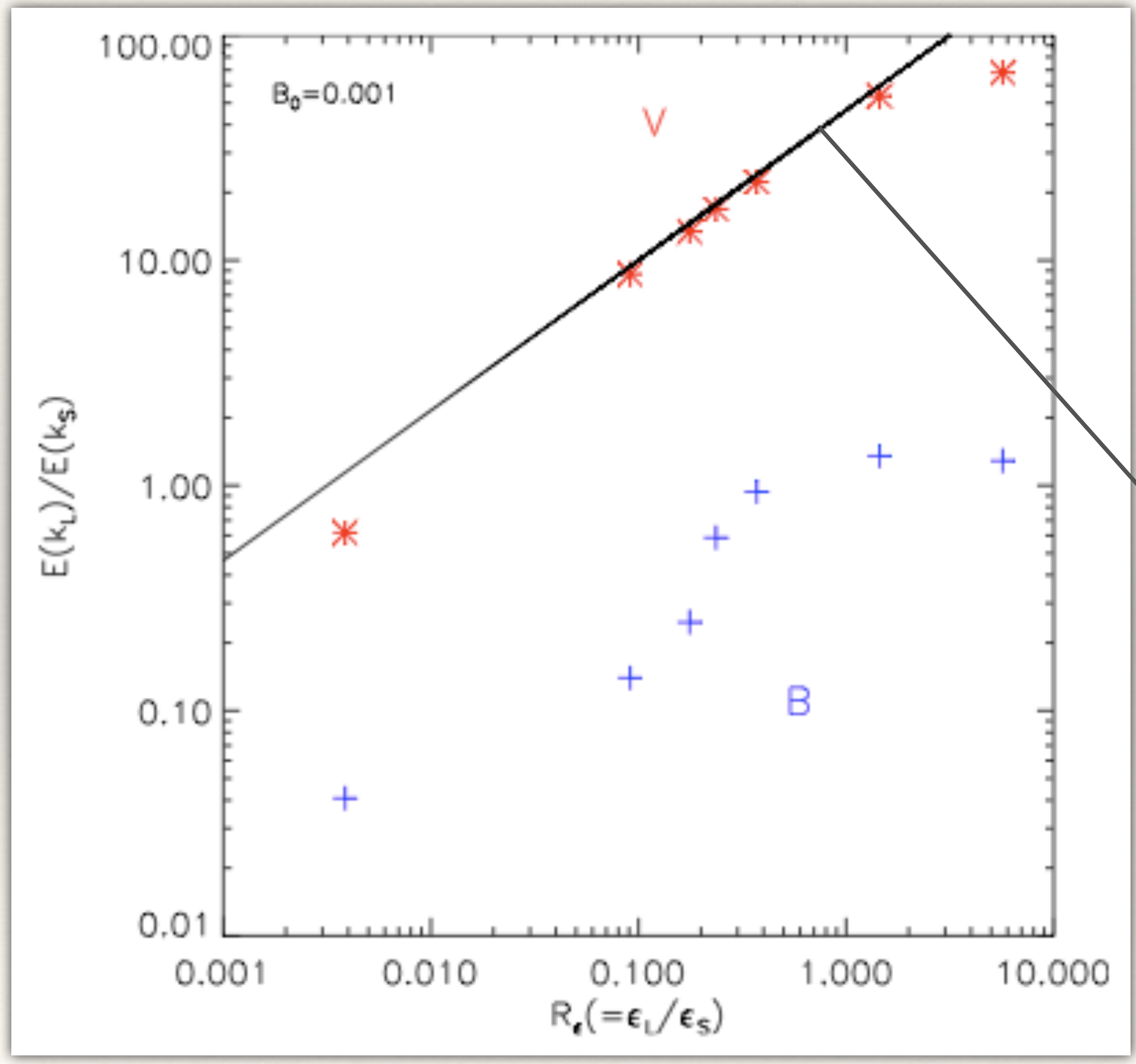




$R_\varepsilon = \varepsilon_L / \varepsilon_S$

Result - Incompressible MHD w/ weak B0

* Scaling relations



$$\frac{E(k_L)}{E(k_S)} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}} \left(\frac{k_S}{k_L}\right)^{\frac{5}{3}}$$

$k_L \sim 2.5$
$k_S \sim 20$
} for our simulations

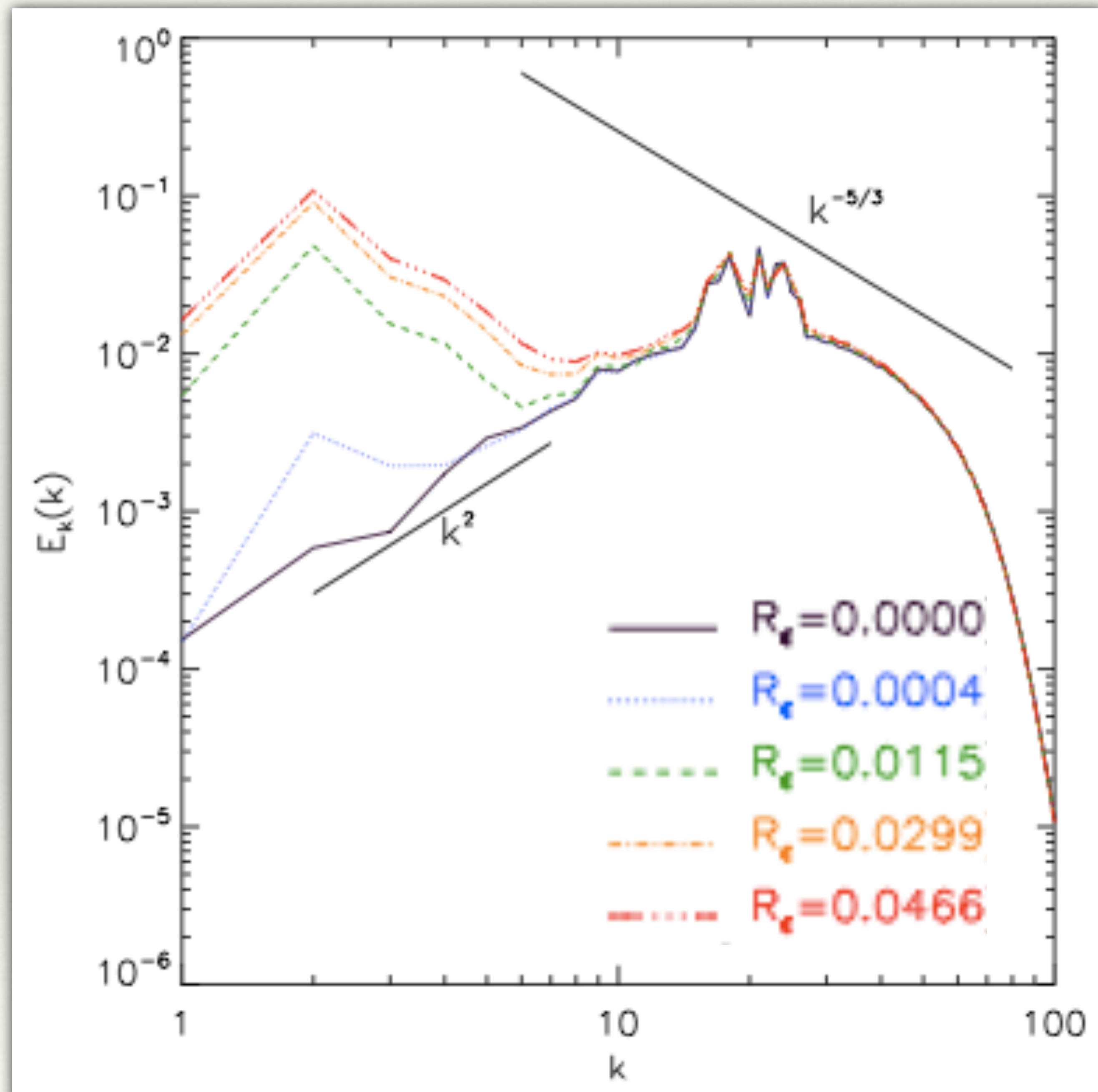
$$\frac{E(k_L)}{E(k_S)} = 32 \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}}$$

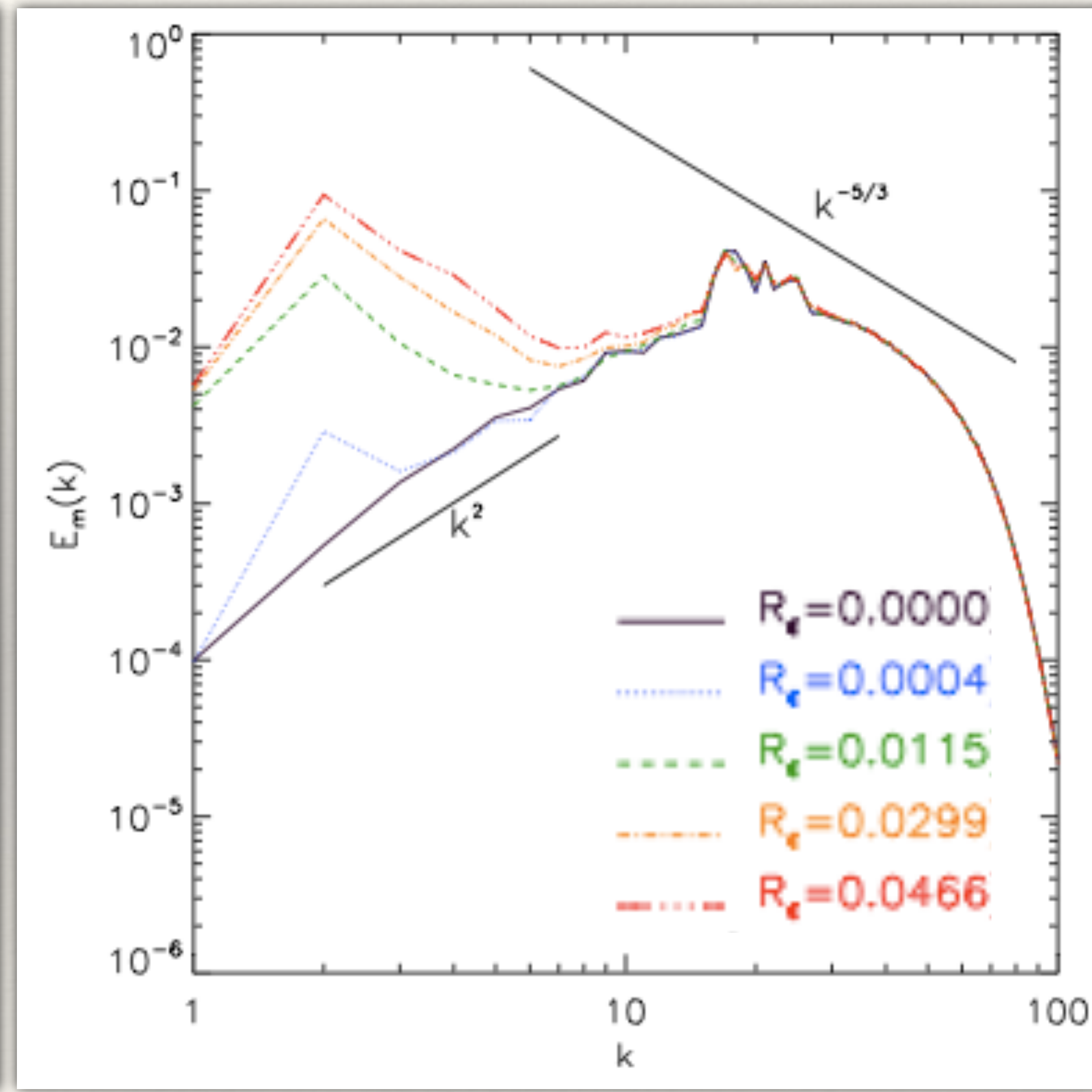
∗ : for kinetic energy spectrum

$+$: for magnetic energy spectrum

Result - Incompressible MHD w/ strong B0

* Kinetic and magnetic energy spectrum

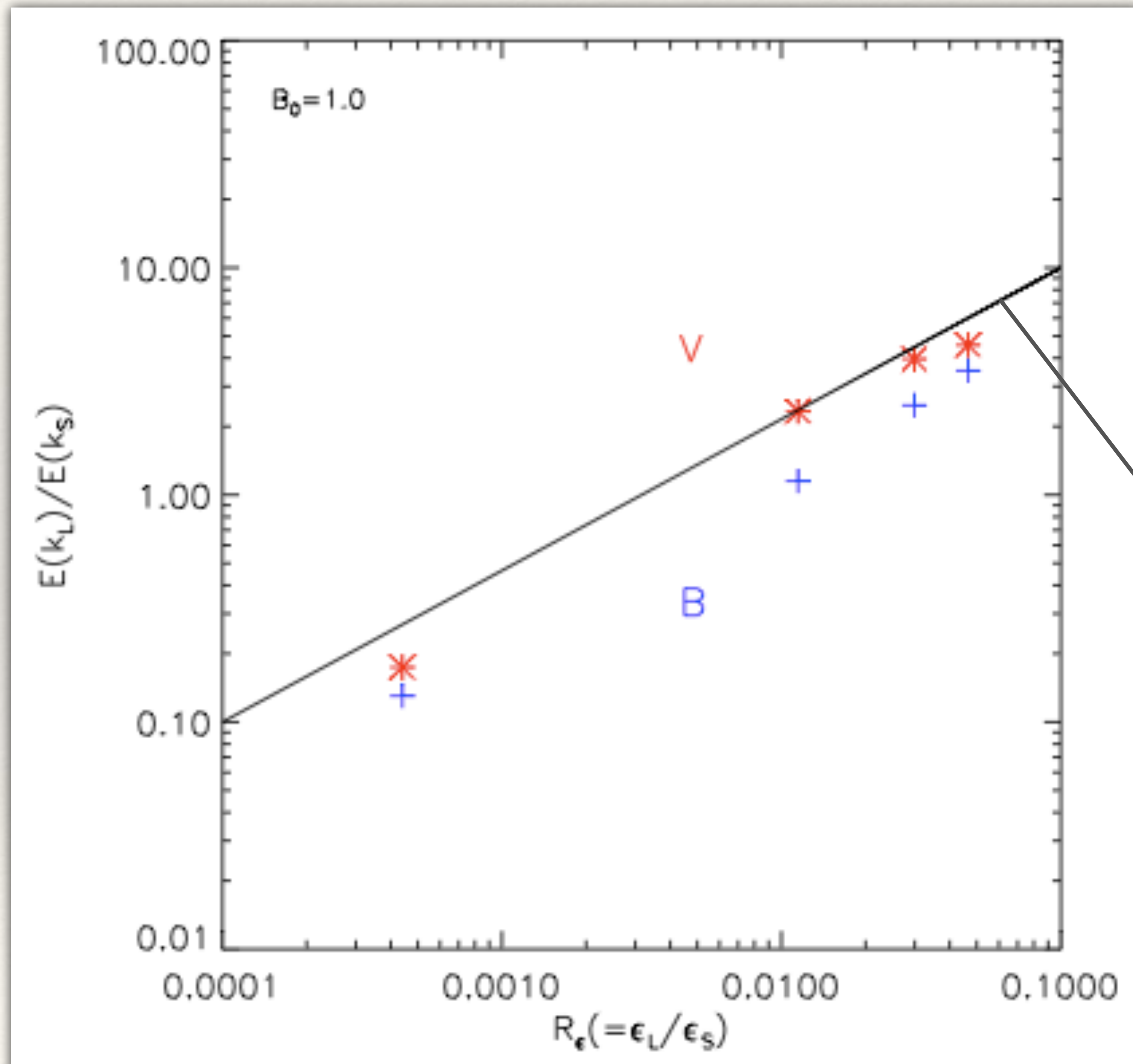




$R_\varepsilon = \varepsilon_L/\varepsilon_S$

Result - Incompressible MHD w/ strong B0

* Scaling relations



$$\frac{E(k_L)}{E(k_S)} = \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}} \left(\frac{k_S}{k_L}\right)^{\frac{5}{3}}$$

$$k_L \sim 2.5$$
$$k_S \sim 20$$
} for our simulations

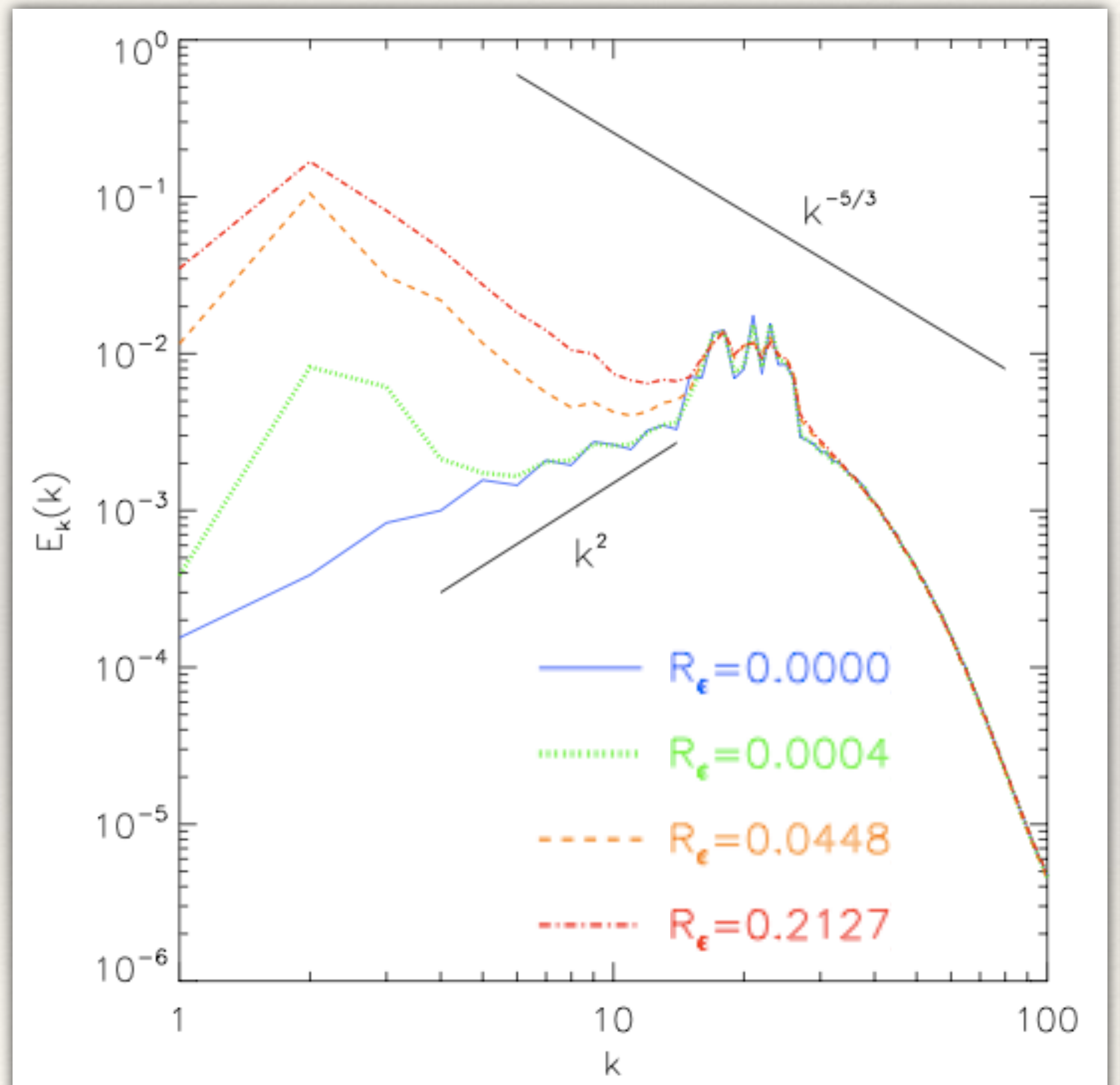
$$\frac{E(k_L)}{E(k_S)} = 32 \left(\frac{\epsilon_L}{\epsilon_S}\right)^{\frac{2}{3}}$$

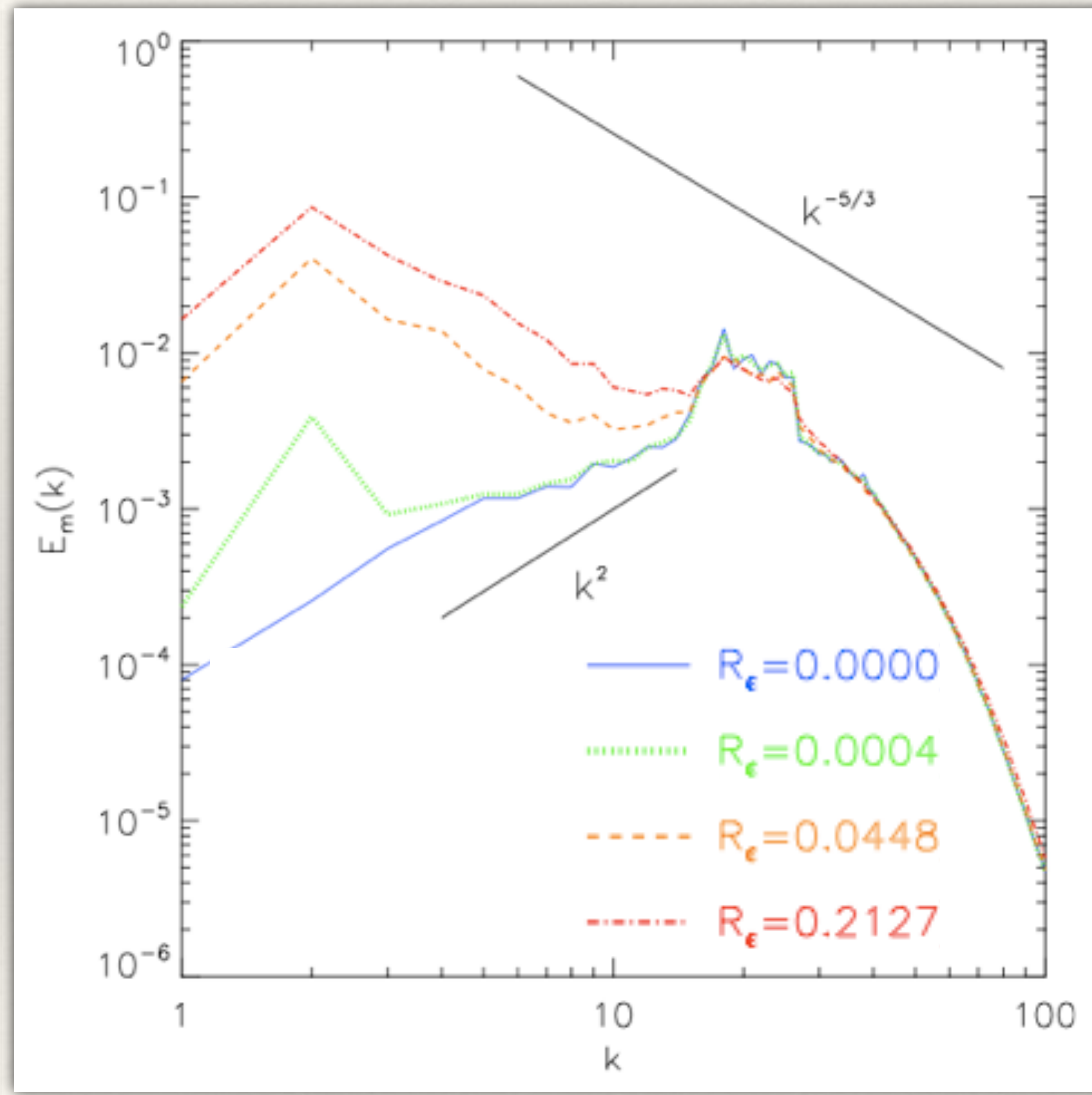
$*$: for kinetic energy spectrum

$+$: for magnetic energy spectrum

Result - Compressible MHD w/ strong B0

* Kinetic and magnetic energy spectrum

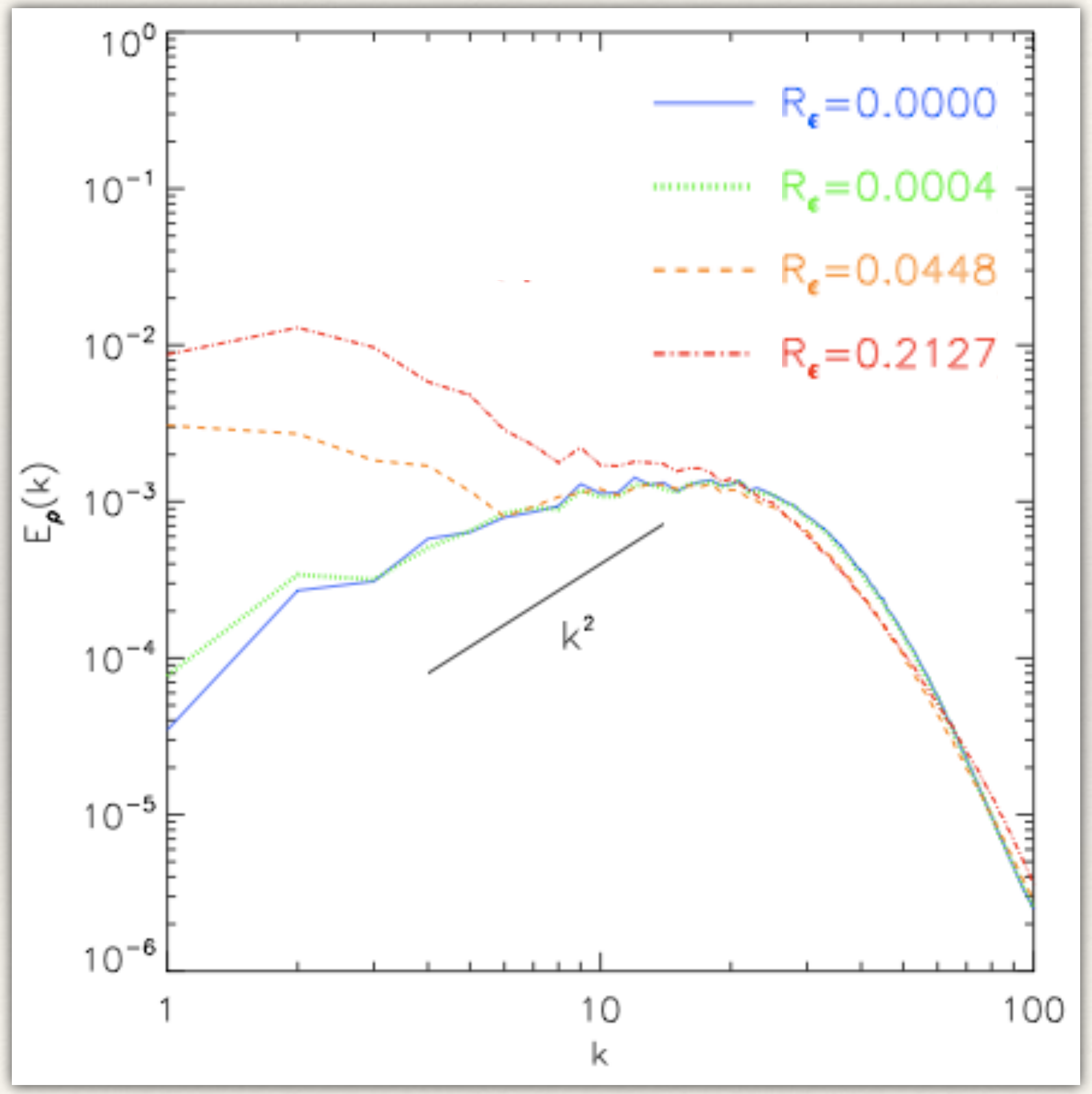




$R_\varepsilon = \varepsilon_L / \varepsilon_S$

Result - Compressible MHD w/ strong B0

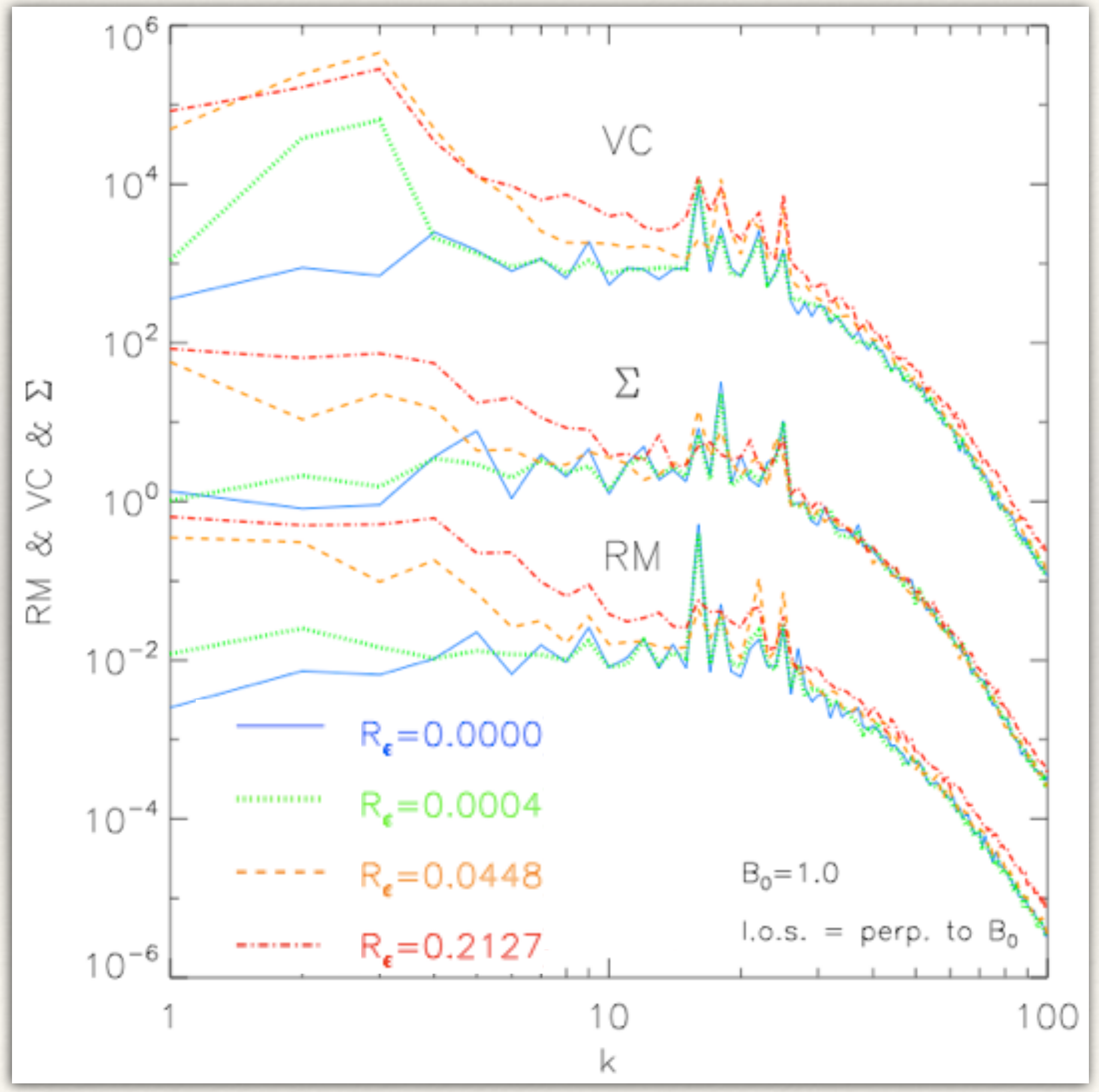
* Density spectrum



$R_\varepsilon = \varepsilon_L/\varepsilon_S$

Result - Compressible MHD w/ strong B0

* Observational implication



* Observable quantities
  - velocity centroids

$$VC = \int \rho v_y dy$$

  - column density

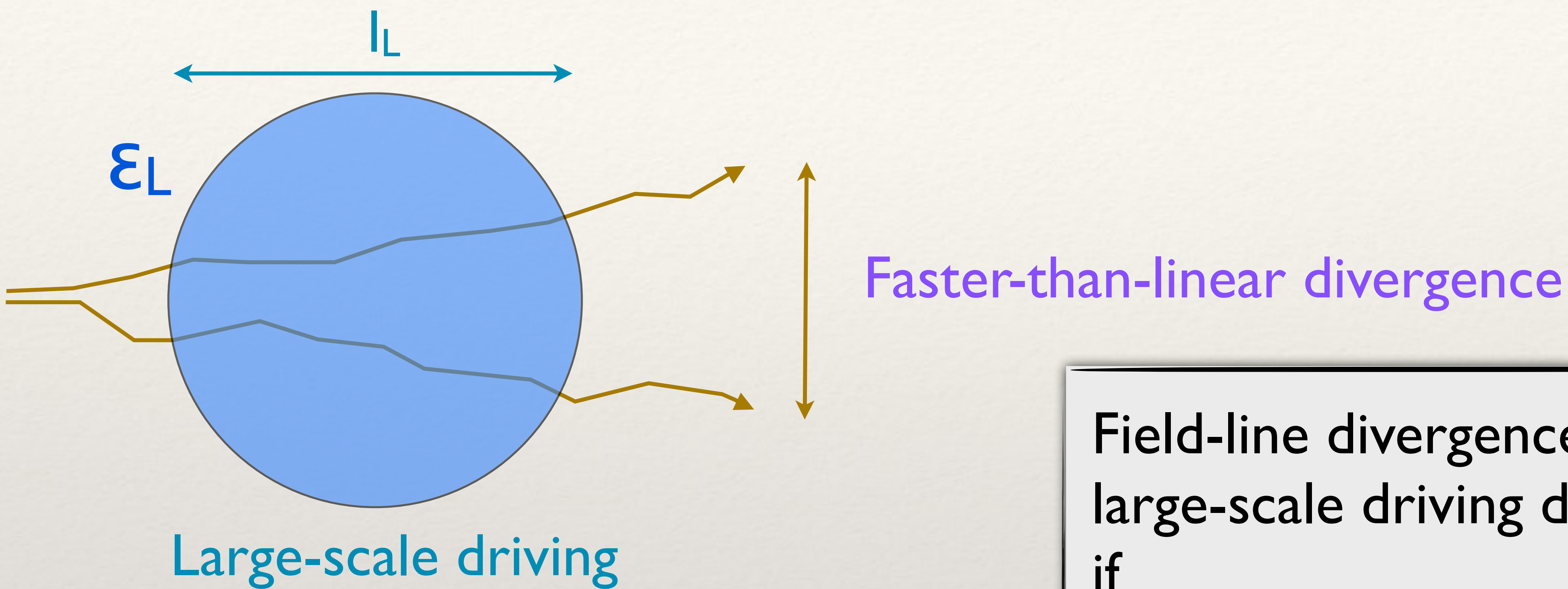
$$\Sigma = \int \rho dy$$

  - rotation measure

$$RM = \int \rho B_y dy$$

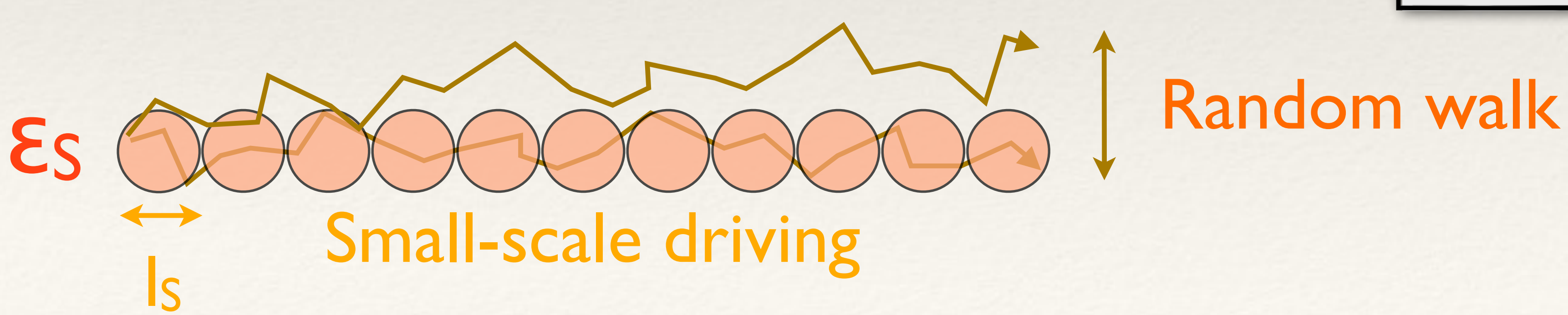
Discussion

* Magnetic field-line divergence



Field-line divergence by large-scale driving dominant if

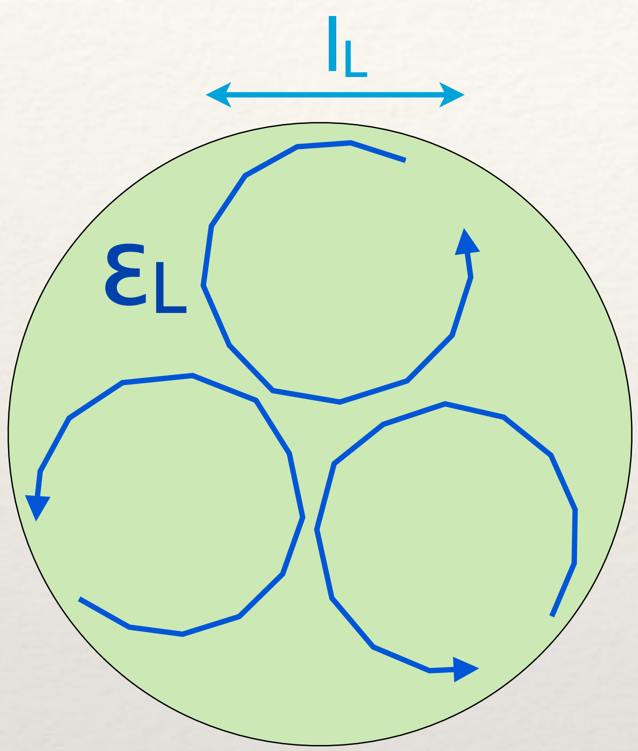
$$\frac{E(k_L)}{E(k_S)} \geq 1 \quad \text{or} \quad \frac{\varepsilon_L}{\varepsilon_S} \geq \left(\frac{l_S}{l_L}\right)^{5/2}$$



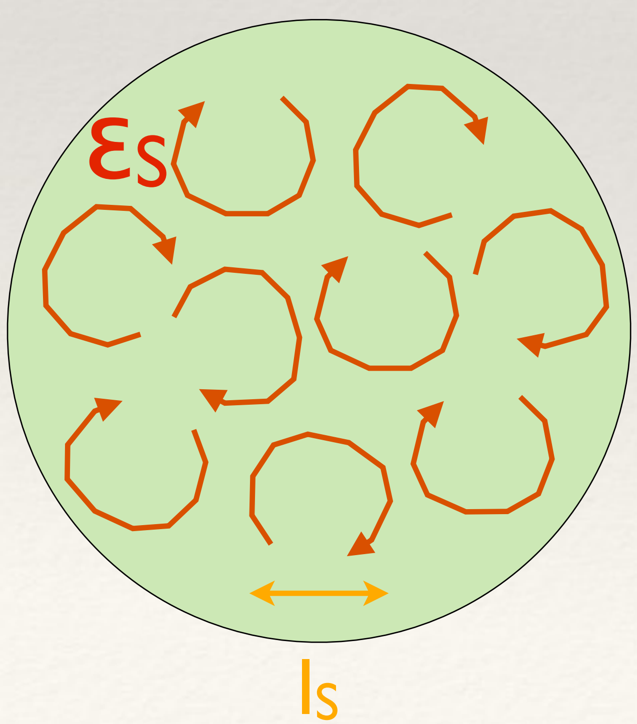
In our cases,
$\varepsilon_L / \varepsilon_S \geq 0.006$

Discussion

* Turbulence diffusion



Large-scale driving



Small-scale driving

Turbulence diffusion by large-scale motion will dominate when

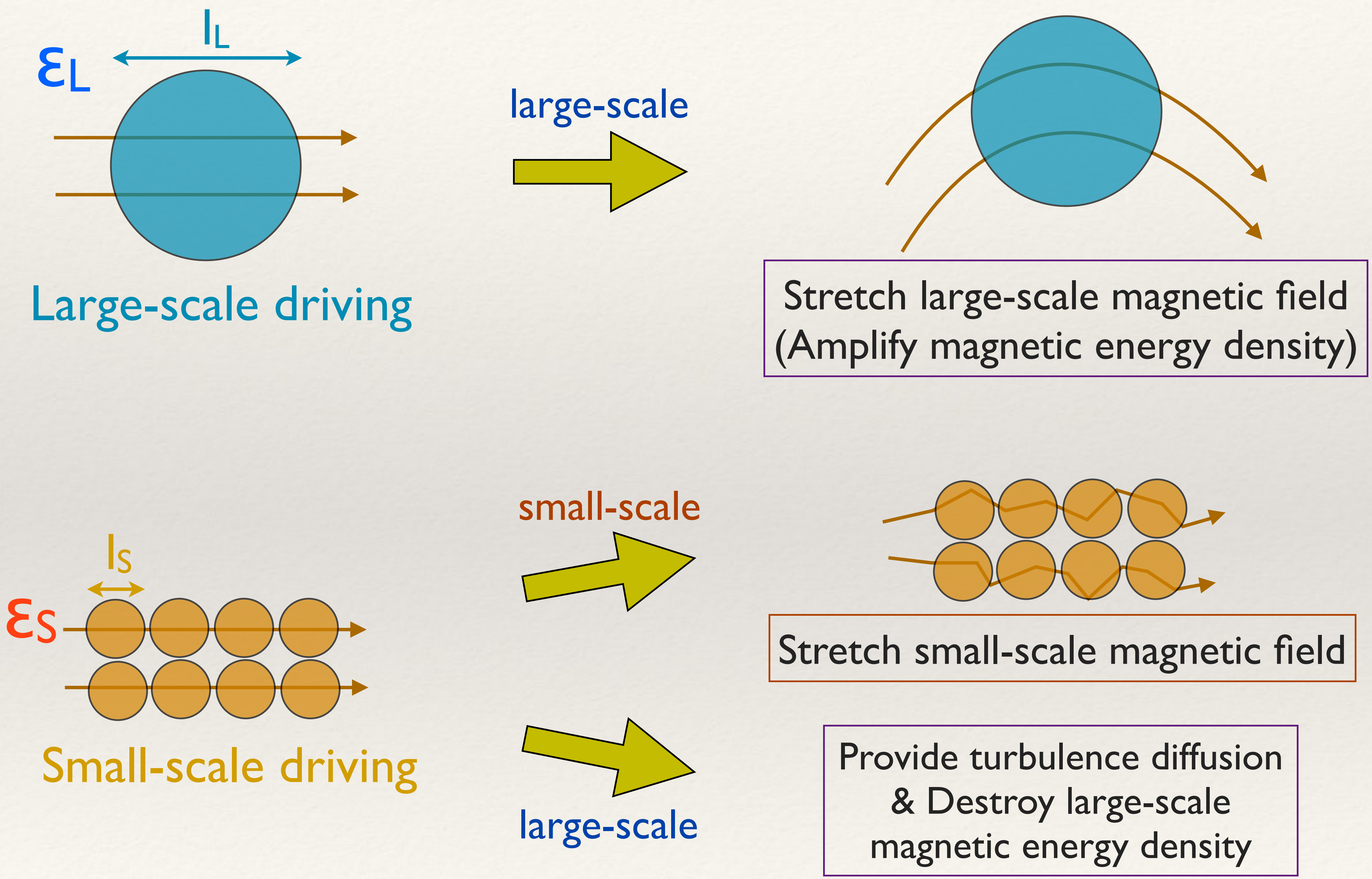
$$\frac{\varepsilon_L}{\varepsilon_S} \geq \left(\frac{l_S}{l_L}\right)^4$$

In our cases,

$\varepsilon_L / \varepsilon_S \geq 0.0002$

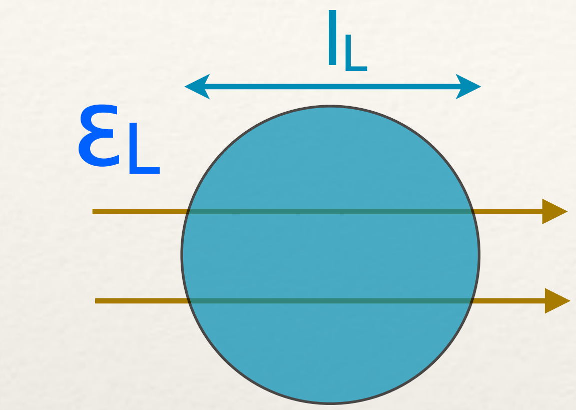
Discussion

* Turbulence dynamo

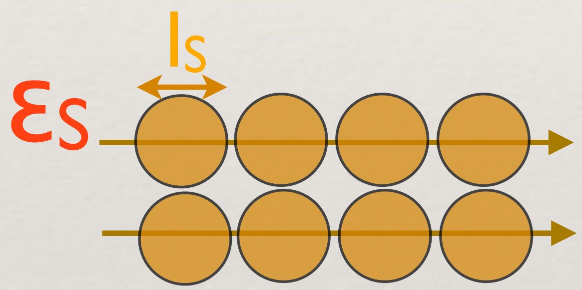


Discussion

* Turbulence dynamo



Large-scale driving



Small-scale driving

magnetic energy density amplification rate by large-scale driving

$$\sim C v_L b_L^2 / l_L$$

C : a small number

destroy rate of large-scale magnetic energy density by small-scale driving

$$\sim (l_S v_S) b_L^2 / l_L^2$$

large-scale magnetic field density will grow

if $\frac{v_L}{v_S} \geq \frac{l_S}{l_L} C^{-1}$ or $\frac{\varepsilon_L}{\varepsilon_S} \geq \left(\frac{l_S}{l_L}\right)^4 C^{-3}$

In our cases,

$\varepsilon_L / \varepsilon_S \geq 0.0002 C^{-3}$

Conclusion

* Astrophysical turbulence have many kinds of driving mechanisms on various scales
* We perform incompressible/compressible MHD turbulence simulation with two-scale driving
* We derived analytically expected relation assuming that there are two peaks in spectrum.
* In small-scale driving dominant system, we are able to distinguish peaks in large- and small-scale, even ϵ_L is much smaller than ϵ_S .
* Two-scale driving affect several physical properties such as magnetic field-line divergence, turbulence diffusion, turbulence dynamo.

(Yoo & Cho 2014, ApJ, 178,99 in detail)

* We are currently performing data sets for investigating effect of two scale driving in the system with high Mach number ($M_s = v/c_S \gtrsim 1$)

Thank you :)