Numerical Simulations for Magnetohydrodynamics based on Upwind Schemes

Energy vs Entropy

Hanbyul Jang, Dongsu Ryu

Department of Physics, School of Natural Sciences, UNIST, Korea

ULSAN NATIONAL INSTITUTE OF SCIENCE AND TECHNOLOGY

Astrophysics-Plasma Laboratory

hanbyul@sirius.unist.ac.kr

Upwind schemes

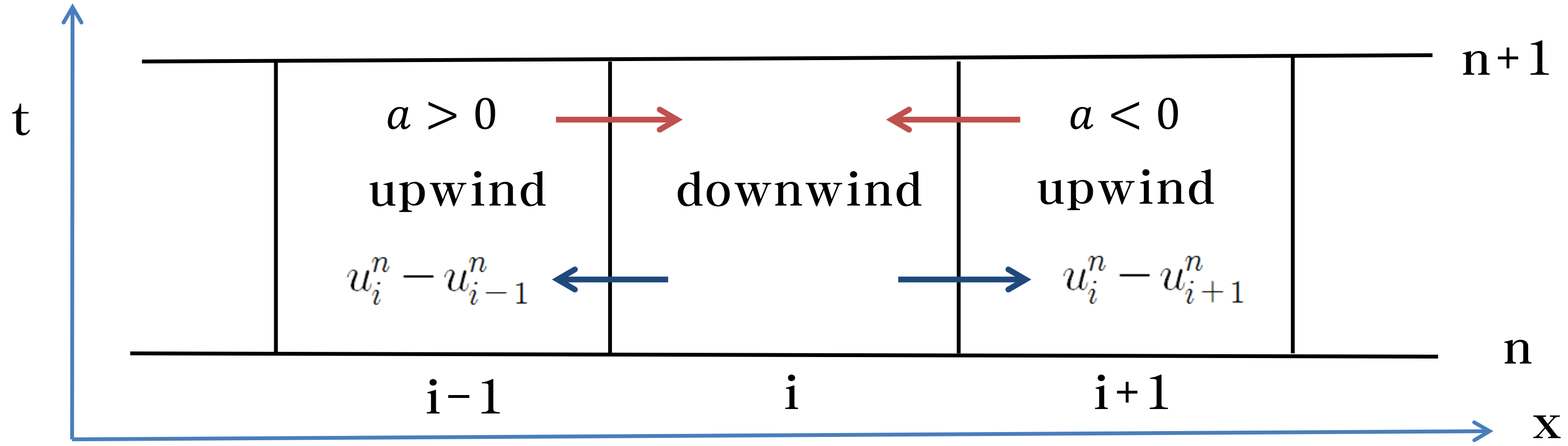
Numerical methods for solving hyperbolic equations using propagation information

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

one-dimensional linear advection equation

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\frac{u_i^n - u_{i-1}^n}{\Delta x} = 0 \text{ for } a > 0$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} - a\frac{u_i^n - u_{i+1}^n}{\Delta x} = 0 \text{ for } a < 0$$



Robust and reliable schemes for gas dynamics

MHD-E

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{F}}{\partial x} = 0$$

$$\vec{q} = \begin{bmatrix} D \\ M_i \\ E \\ B_y \\ B_z \end{bmatrix}$$

state vector

- D → Mass
- M_i → Momentums
- E → Total energy
- B_y, B_z → Magnetic fields

$$\vec{F} = \begin{bmatrix} Dv_x \\ M_i v_x + p^* \delta_{ij} - B_x B_i \\ E v_x + p^* v_x - B_x (\vec{v} \cdot \vec{B}) \\ v_x B_y - v_y B_x \\ v_x B_z - v_z B_x \end{bmatrix}$$

flux vector

$$D = \rho$$

$$M_i = \rho v_i$$

$$E = \frac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} + \frac{1}{2}B^2$$

ρ proper rest mass density

v_i fluid velocities

$p^* = p + \frac{1}{2}B^2$ total pressure

MHD-S

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{F}}{\partial x} = 0$$

$$\vec{q} = \begin{bmatrix} D \\ M_i \\ S \\ B_y \\ B_z \end{bmatrix}$$

state vector

- D → Mass
- M_i → Momentums
- S → Entropy
- B_y, B_z → Magnetic fields

$$\vec{F} = \begin{bmatrix} D v_x \\ M_i v_x + p^* \delta_{ij} - B_x B_i \\ S v_x \\ v_x B_y - v_y B_x \\ v_x B_z - v_z B_x \end{bmatrix}$$

flux vector

$D = \rho$

$M_i = \rho v_i$

$S = p \rho^{1-\gamma}$

ρ proper rest mass density

v_i fluid velocities

$p^* = p + \frac{1}{2} B^2$ total pressure

Jacobian matrix

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{F}}{\partial x} = \frac{\partial \vec{q}}{\partial t} + A\frac{\partial \vec{q}}{\partial x} = 0$$

$$A = \frac{\partial \vec{F}}{\partial \vec{q}} = \frac{\partial \vec{F}}{\partial \vec{u}}\frac{\partial \vec{u}}{\partial \vec{q}}$$

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} \\ A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} \end{pmatrix}$$

$$\vec{u} = \begin{bmatrix} \rho \\ v_x \\ v_y \\ v_z \\ p \\ B_y \\ B_z \end{bmatrix}$$

parameter vector

Jacobian matrix

Eigenvalues $\det(A - a_m I) = 0 \quad (m = 1,2,3,4,5,6,7)$

Right Eigenvectors $A\vec{R}_m = a_m \vec{R}_m$

Left Eigenvectors $\vec{L}_m \cdot A = a_m \vec{L}_m$

$$LAR = \Lambda = \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_7 \end{bmatrix}$$

$$L = \begin{bmatrix} \vec{L}_1 \\ \vec{L}_2 \\ \vdots \\ \vec{L}_m \end{bmatrix} \qquad R = [\vec{R}_1 | \vec{R}_2 | \ldots | \vec{R}_m]$$

$$L\left[\frac{\partial \vec{q}}{\partial t} + A\frac{\partial \vec{q}}{\partial x}\right] = 0$$

$$L\frac{\partial \vec{q}}{\partial t} + L(R\Lambda L)\frac{\partial \vec{q}}{\partial x} = 0 \quad (LR = I)$$

$$L\frac{\partial \vec{q}}{\partial t} + \Lambda L\frac{\partial \vec{q}}{\partial x} = 0$$

$$\frac{\partial \vec{w}}{\partial t} + \Lambda\frac{\partial \vec{w}}{\partial x} = 0 \quad (\vec{w} \equiv L\vec{q})$$

The system decouples into m independent advection equations ⇒ $\dfrac{\partial w_m}{\partial t} + a_m \dfrac{\partial w_m}{\partial x} = 0$

Eigenvalues

MHD $c_s^2 = \frac{\gamma p}{\rho}$ $b_i = \frac{B_i}{\sqrt{\rho}}$

$$a_1 = v_x - \sqrt{\frac{1}{2}[c_s^2 + b^2 + \sqrt{(c_s^2 + b^2)^2 - 4c_s^2 b_x^2}\,]},$$ -fast magnetosonic mode

$$a_2 = v_x - |b_x|,$$ -Alfven mode

$$a_3 = v_x - \sqrt{\frac{1}{2}[c_s^2 + b^2 - \sqrt{(c_s^2 + b^2)^2 - 4c_s^2 b_x^2}\,]},$$ -slow magnetosonic mode

$$a_4 = v_x,$$ entropy mode

$$a_5 = v_x + \sqrt{\frac{1}{2}[c_s^2 + b^2 - \sqrt{(c_s^2 + b^2)^2 - 4c_s^2 b_x^2}\,]},$$ +slow magnetosonic mode

$$a_6 = v_x + |b_x|,$$ +Alfven mode

$$a_7 = v_x + \sqrt{\frac{1}{2}[c_s^2 + b^2 + \sqrt{(c_s^2 + b^2)^2 - 4c_s^2 b_x^2}\,]}.$$ +fast magnetosonic mode

$$a_1(^-_{fast}) \le a_2(^-_{Alfven}) \le a_3(^-_{slow}) \le a_4(= v_x) \le a_5(^+_{slow}) \le a_6(^+_{Alfven}) \le a_7(^+_{fast})$$

Eigenvectors

MHD-E

$$\beta_y = \frac{B_y}{\sqrt{B_y^2+B_z^2}},\quad \beta_z = \frac{B_z}{\sqrt{B_y^2+B_z^2}},\quad \alpha_s = \frac{\sqrt{\lambda_f^2-c_s^2}}{\sqrt{\lambda_f^2-\lambda_s^2}},\qquad \alpha_f = \frac{\sqrt{c_s^2-\lambda_s^2}}{\sqrt{\lambda_f^2-\lambda_s^2}}$$

for fast mode

$$C=\alpha_f,\qquad Y_y = c_s\alpha_s\beta_y,\qquad Y_z = c_s\alpha_s\beta_z,\qquad D=\frac{b_x}{\lambda_f}=\pm\frac{\lambda_s}{c_s}sgn(B_x)$$

for slow mode

$$C=\alpha_s,\qquad Y_y = -c_s\alpha_f\beta_y,\qquad Y_z = -c_s\alpha_f\beta_z,\qquad D=\frac{b_x}{\lambda_s}=\pm\frac{\lambda_f}{c_s}sgn(B_x)$$

for Alfven mode

$$D=\pm\, sgn(B_x)$$

1. compressible mode

$$\vec{R} = \begin{pmatrix} C \\ (\lambda+v_x)C \\ v_y C - Y_y D \\ v_z C - Y_z D \\ Y_y/\sqrt{\rho} \\ Y_z/\sqrt{\rho} \\ (\lambda^2+\lambda v_x+\frac{1}{2}v^2-\frac{\gamma-2}{\gamma-1}c_s^2)C-(Y_y v_y+Y_z v_z)D \end{pmatrix}$$

$$\vec{L} = \frac{1}{2c_s^2}\begin{pmatrix} [\frac{1}{2}(\gamma-1)v^2-\lambda v_x]C+(Y_y v_y+Y_z v_z)D \\ [\lambda-(\gamma-1)v_x]C \\ (1-\gamma)v_y C - Y_y D \\ (1-\gamma)v_z C - Y_z D \\ B_y(1-\gamma)C+Y_y\sqrt{\rho} \\ B_z(1-\gamma)C+Y_z\sqrt{\rho} \\ (\gamma-1)C \end{pmatrix}^T$$

2. Alfven mode

$$\vec{R} = [0,\ 0,\ -\beta_z,\ \beta_y,\ \beta_z D/\sqrt{\rho},\ -\beta_z D/\sqrt{\rho},\ \beta_y v_z-\beta_z v_y]^T$$

$$\vec{L} = \frac{1}{2}[\beta_z v_y-\beta_y v_z,\ 0,\ -\beta_z,\ \beta_y,\ \beta_z D\sqrt{\rho},\ -\beta_y D\sqrt{\rho},\ 0]$$

3. Entropy mode

$$\vec{R} = \left[1, v_x, v_y, v_z, 0, 0, \frac{1}{2}v^2\right]^T,\qquad \vec{L} = \frac{(\gamma-1)}{c_s^2}\left[\frac{c_s^2}{(\gamma-1)}-\frac{1}{2}v^2, v_x, v_y, v_z, B_y, B_z, -1\right]$$

Eigenvectors

MHD-S

$$\beta_y = \frac{B_y}{\sqrt{B_y^2+B_z^2}},\quad \beta_z = \frac{B_z}{\sqrt{B_y^2+B_z^2}},\quad \alpha_s = \frac{\sqrt{\lambda_f^2-c_s^2}}{\sqrt{\lambda_f^2-\lambda_s^2}},\quad \alpha_f = \frac{\sqrt{c_s^2-\lambda_s^2}}{\sqrt{\lambda_f^2-\lambda_s^2}}$$

for fast mode

$$C=\alpha_f,\quad Y_y = c_s\alpha_s\beta_y,\quad Y_z = c_s\alpha_s\beta_z,\quad D=\frac{b_x}{\lambda_f}=\pm\frac{\lambda_s}{c_s}sgn(B_x)$$

for slow mode

$$C=\alpha_s,\quad Y_y = -c_s\alpha_f\beta_y,\quad Y_z = -c_s\alpha_f\beta_z,\quad D=\frac{b_x}{\lambda_s}=\pm\frac{\lambda_f}{c_s}sgn(B_x)$$

for Alfven mode

$D=\pm\, sgn(B_x)$

1. compressible mode

$$\vec{R} = \begin{pmatrix} C \\ (\lambda+v_x)C \\ v_yC-Y_yD \\ v_zC-Y_zD \\ Y_y/\sqrt{\rho} \\ Y_z/\sqrt{\rho} \\ c_s^2C/\gamma \end{pmatrix}\qquad \vec{L} = \frac{1}{2c_s^2}\begin{pmatrix} [c_s^2(\gamma-1)/\gamma-\lambda v_x]C+(Y_yv_y+Y_zv_z)D \\ \lambda C \\ -Y_yD \\ -Y_zD \\ Y_y\sqrt{\rho} \\ Y_z\sqrt{\rho} \\ C \end{pmatrix}^T$$

2. Alfven mode

$$\vec{R} = [0,\ 0,\ -\beta_z,\ \beta_y,\ \beta_zD/\sqrt{\rho},\ -\beta_zD/\sqrt{\rho},\ 0]^T$$

$$\vec{L} = \frac{1}{2}[\beta_zv_y-\beta_yv_z,\ 0,\ -\beta_z,\ \beta_y,\ \beta_zD\sqrt{\rho},\ -\beta_yD\sqrt{\rho},\ 0]$$

3. Entropy mode

$$\vec{R} = [1, v_x, v_y, v_z, 0, 0, c_s^2(1-\gamma)/\gamma]^T,\quad \vec{L} = [1/\gamma, 0, 0, 0, 0, 0, -1/c_s^2]$$

Numerical Simulations

Total Variation Diminishing (TVD)

Harten 1983 (HD), Ryu et al 1995 (MHD)

Weighted Essentially Non-Oscillatory (WENO)

Jiang & Shu 1996 (HD), Jiang & Wu 1999 (MHD)

TVD (Space2-Time2)
WENO3-RK3 (Space3-Time3)
WENO5-RK4 (Space5-Time4)

Numerical Simulations

MHD-E

TVD_E

WENO3-RK3_E

WENO5-RK4_E

MHD-S

TVD_S

WENO3-RK3_S

WENO5-RK4_S

MHD 1D Alfven wave tests

Initially

$$\rho = \rho_0 + \delta\rho \qquad \rho_0 = 1,\ \delta\rho = 0$$

$$v_x = v_{x0} + \delta v_x \qquad v_{x0} = 0,\ \delta v_x = 0$$

$$v_y = v_{y0} + \delta v_y \qquad v_{y0} = 0,\ \delta v_y = 0$$

$$v_z = v_{z0} + \delta v_z \qquad v_{z0} = 0,\ \delta v_z = 10^{-4}\cos(kx)$$

$$p = p_0 + \delta p \qquad p_0 = 1/\gamma,\ \delta p = 0$$

$$B_x = B_{x0} \qquad B_{x0} = 1$$

$$B_y = B_{y0} + \delta B_y \qquad B_{y0} = 0.5,\ \delta B_y = 0$$

$$B_z = B_{z0} + \delta B_z \qquad B_{z0} = 0,\ \delta B_z = -\delta v_z \sqrt{\rho_0}$$

+Alfven wave

Periodic boundary

$$k = \frac{2\pi}{L}$$

$$\gamma = 5/3$$

$$c_{s0} = \sqrt{\gamma p_0/\rho_0} = 1$$

$$\frac{B_{x0}}{\sqrt{\rho_0}} = 1$$ Alfven speed

$t = 100$ 100 times

Numerical dissipation

$$\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)} = e^{-t\tau} \qquad \tau = -\frac{1}{t} ln\left(\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)}\right)$$

Decay rate

MHD 1D Alfven wave tests

Initially

Periodic boundary

$$\rho = \rho_0 + \delta$$

$$v_x = v_{x0} + \delta$$

$$v_y = v_{y0} + \delta$$

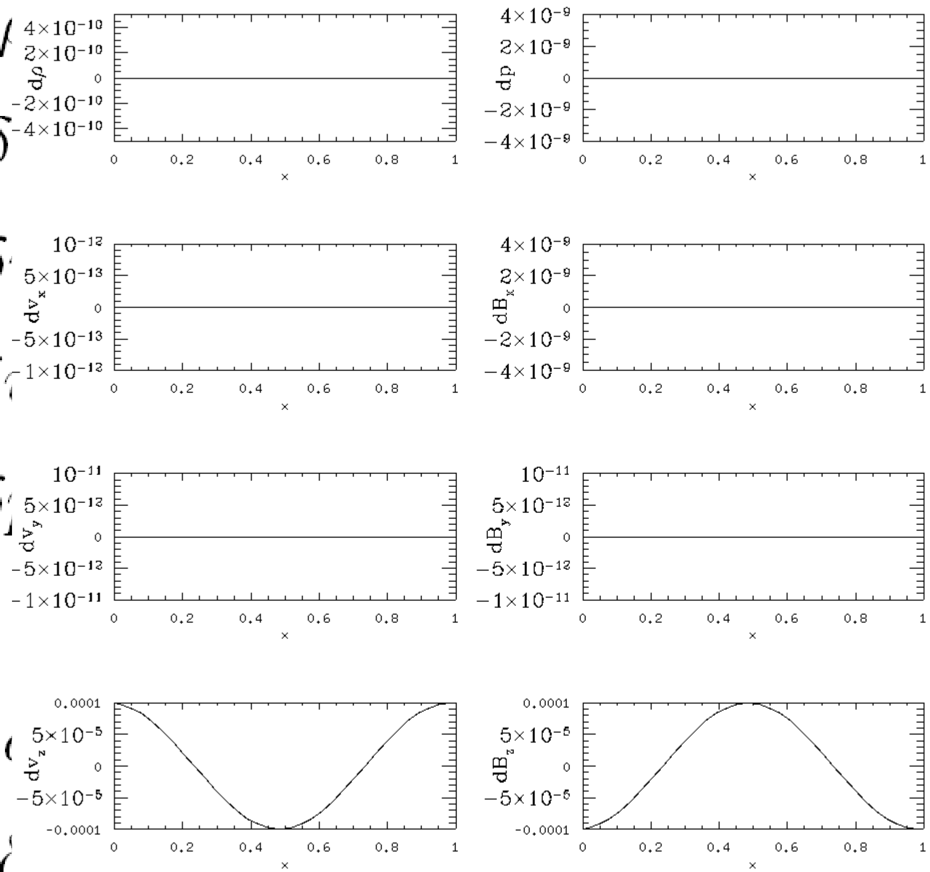
$$v_z = v_{z0} + \delta$$

$$p = p_0 + \delta$$

$$B_x = B_{x0}$$

$$B_y = B_{y0} +$$

$$B_z = B_{z0} +$$



$$k = \frac{2\pi}{L}$$

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$$c_{s0} = \sqrt{\gamma p_0 / \rho_0} = 1$$

$$\frac{B_{x0}}{\sqrt{\rho_0}} = 1$$ Alfven speed

$t = 100$ 100 times

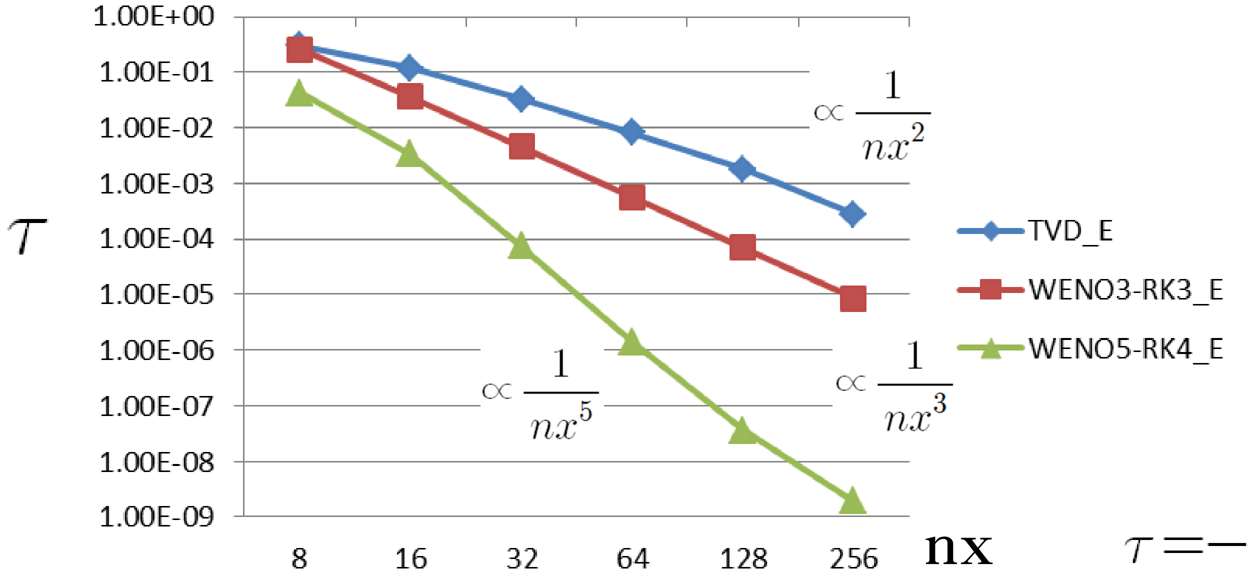
Alfven wave

Numerical dissipation

$$\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)} = e^{-t\tau}$$

$$\tau = -\frac{1}{t} ln\left(\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)}\right)$$ Decay rate

MHD 1D Alfven wave tests

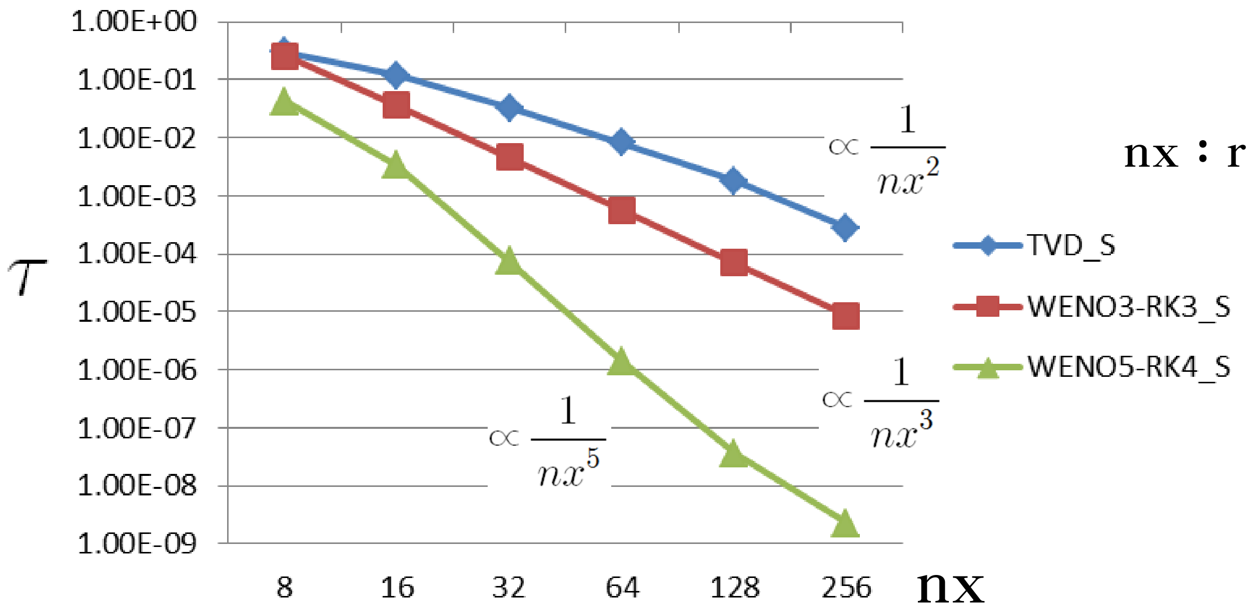


MHD-E

$$\tau = -\frac{1}{t} ln\left(\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)}\right)$$

Decay rate

nx : resolution



MHD-S

MHD 2,3D Alfven wave tests

Initially

Periodic boundary

$$\rho = \rho_0 + \delta\rho \qquad \rho_0 = 1, \ \delta\rho = 0$$

$$v'_x = v'_{x0} + \delta v'_x \qquad v'_{x0} = 0, \ \delta v'_x = 0$$

$$v'_y = v'_{y0} + \delta v'_y \qquad v'_{y0} = 0, \ \delta v'_y = 0$$

$$v'_z = v'_{z0} + \delta v'_z \qquad v'_{z0} = 0, \ \delta v'_z = 10^{-4}\cos(k_x x + k_y y + k_z z)$$

$$p = p_0 + \delta p \qquad p_0 = 1/\gamma, \ \delta p = 0$$

$$B'_x = B'_{x0} \qquad B'_{x0} = 1$$

$$B'_y = B'_{y0} + \delta B'_y \qquad B'_{y0} = 0.5, \ \delta B'_y = 0$$

$$B'_z = B'_{z0} + \delta B'_z \qquad B'_{z0} = 0, \ \delta B'_z = -\delta v'_z \sqrt{\rho_0}$$

+Alfven wave

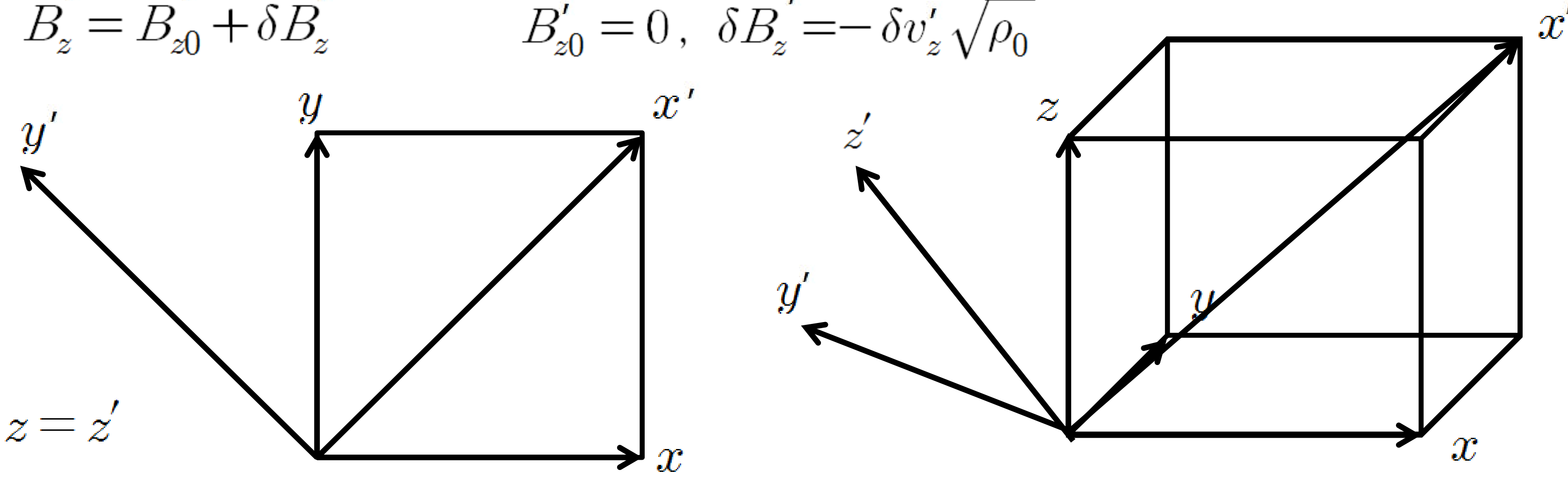
$$k = \frac{2\pi}{L}$$

$$\gamma = 5/3$$

$$c_{s0} = \sqrt{\gamma p_0/\rho_0} = 1$$

$$\frac{B'_{x0}}{\sqrt{\rho_0}} = 1$$ Alfven speed

$$t = 100$$ 100 times



Flux CT scheme $\vec{\nabla} \cdot \vec{B} = 0$

A DIVERGENCE-FREE UPWIND CODE FOR MULTIDIMENSIONAL MAGNETOHYDRODYNAMIC FLOWS

DONGSU RYU,[1] FRANCESCO MINIATI,[2] T. W. JONES,[2] AND ADAM FRANK[3]

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$$\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial y}(B_x v_y - B_y v_x) = 0\,, \qquad (1)$$

and

$$\frac{\partial B_y}{\partial t} + \frac{\partial}{\partial x}(B_y v_x - B_x v_y) = 0\,. \qquad (2)$$

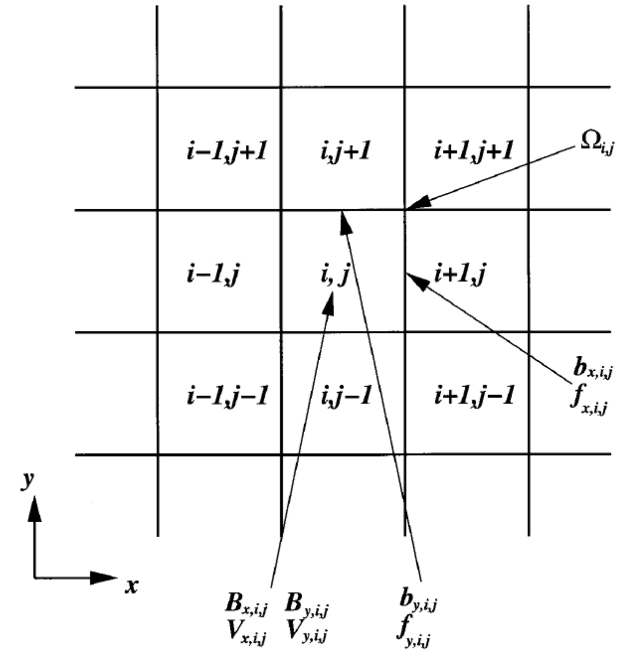
$$B_{x,i,j} = \tfrac{1}{2}(b_{x,i,j} + b_{x,i-1,j}) \qquad (3)$$

and

$$B_{y,i,j} = \tfrac{1}{2}(b_{y,i,j} + b_{y,i,j-1})\,. \qquad (4)$$

$$\bar{f}_{x,i,j} = \frac{1}{2}(B^n_{y,i,j} v^n_{x,i,j} + B^n_{y,i+1,j} v^n_{x,i+1,j}) - \frac{\Delta x}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i+1/2,j} R^n_{k,i+1/2,j}(5)\,, \qquad (6)$$

$$\bar{f}_{y,i,j} = \frac{1}{2}(B^n_{x,i,j} v^n_{y,i,j} + B^n_{x,i,j+1} v^n_{y,i,j+1}) - \frac{\Delta y}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i,j+1/2,j} R^n_{k,i,j+1/2}(5)\,. \qquad (7)$$



$$\bar{\Omega}_{i,j} = \tfrac{1}{2}(\bar{f}_{y,i+1,j} + \bar{f}_{y,i,j}) - \tfrac{1}{2}(\bar{f}_{x,i,j+1} + \bar{f}_{x,i,j})\,. \qquad (15)$$

$$b^{n+1}_{x,i,j} = b^n_{x,i,j} - \frac{\Delta t^n}{\Delta y}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i,j-1}) \qquad (16)$$

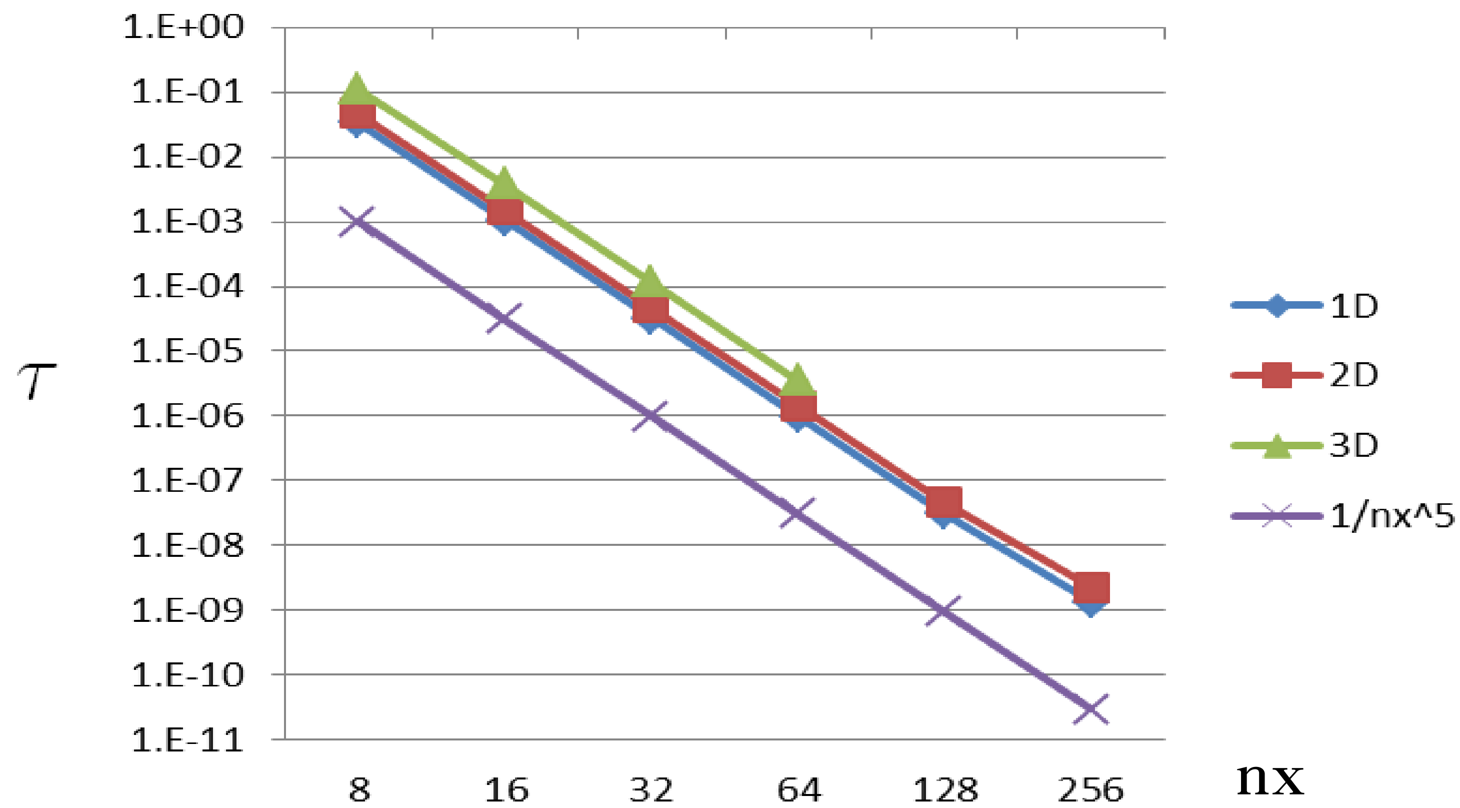
and

$$b^{n+1}_{y,i,j} = b^n_{y,i,j} + \frac{\Delta t^n}{\Delta x}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i-1,j})\,. \qquad (17)$$

$$\oint_S \boldsymbol{b}^{n+1} \cdot d\boldsymbol{S} = (b^{n+1}_{x,i,j} - b^{n+1}_{x,i-1,j})\Delta y + (b^{n+1}_{y,i,j} - b^{n+1}_{y,i,j-1})\Delta x = 0\,, \qquad (18)$$

MHD Alfven wave tests

$$\tau = -\frac{1}{t} ln\left(\frac{v_{z,rms}(t)}{v_{z,rms}(t=0)}\right)$$ Decay rate



WENO5-RK4_E

MHD 2D Fast wave test

Periodic boundary

$CFL = 0.8$

$tend = 10$

$\gamma = 5/3 = 1.66666666667$

$k_x = 2\pi / xsize$

$k_y = 2\pi / ysize$

$n_x = n_y = 32$ (2D)

$\rho = \rho_0 + \delta\rho$

$v_\parallel = v_{\parallel 0} + \delta v_\parallel$

$v_\perp = v_{\perp 0} + \delta v_\perp$

$v_z = v_{z0} + \delta v_z$

$B_\parallel = B_{\parallel 0}$

$B_\perp = B_{\perp 0} + \delta B_\perp$

$B_z = B_{z0} + \delta B_z$

$p = p_0 + \delta p$

$\rho_0 = 1$

$v_{\parallel 0} = v_{\perp 0} = v_{z0} = 0$

$B_{\parallel 0} = 10$

$B_{\perp 0} = 5$

$B_{z0} = 0$

$p_0 = 1/\gamma = 0.6$

$c_{s0}^2 = \gamma p_0 / \rho_0 = 1$

$\lambda = \lambda_{fast}^+$

$\delta v_\parallel = 10^{-4} \cos(k_x x + k_y y)$

$\delta\rho = \frac{\rho_0}{\lambda} \delta v_\parallel$

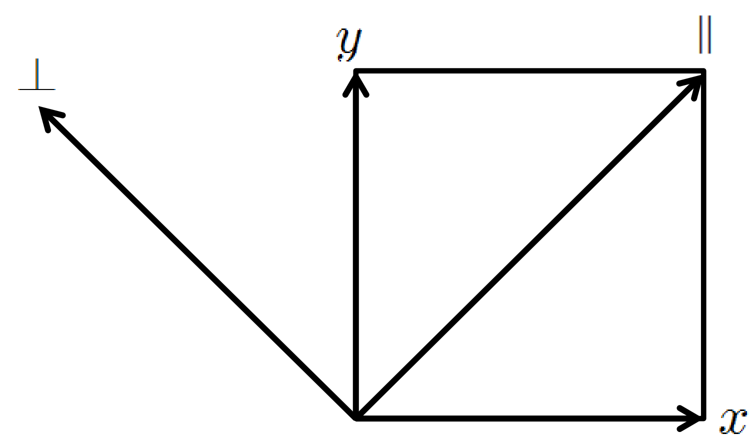
$\delta v_\perp = \frac{b_{\parallel 0} b_{\perp 0}}{b_{\parallel 0}^2 - \lambda^2} \delta v_\parallel$

$\delta v_z = 0$

$\delta B_\perp = -\frac{\lambda}{b_{\parallel 0}} \sqrt{\rho_0}\, \delta v_\perp$

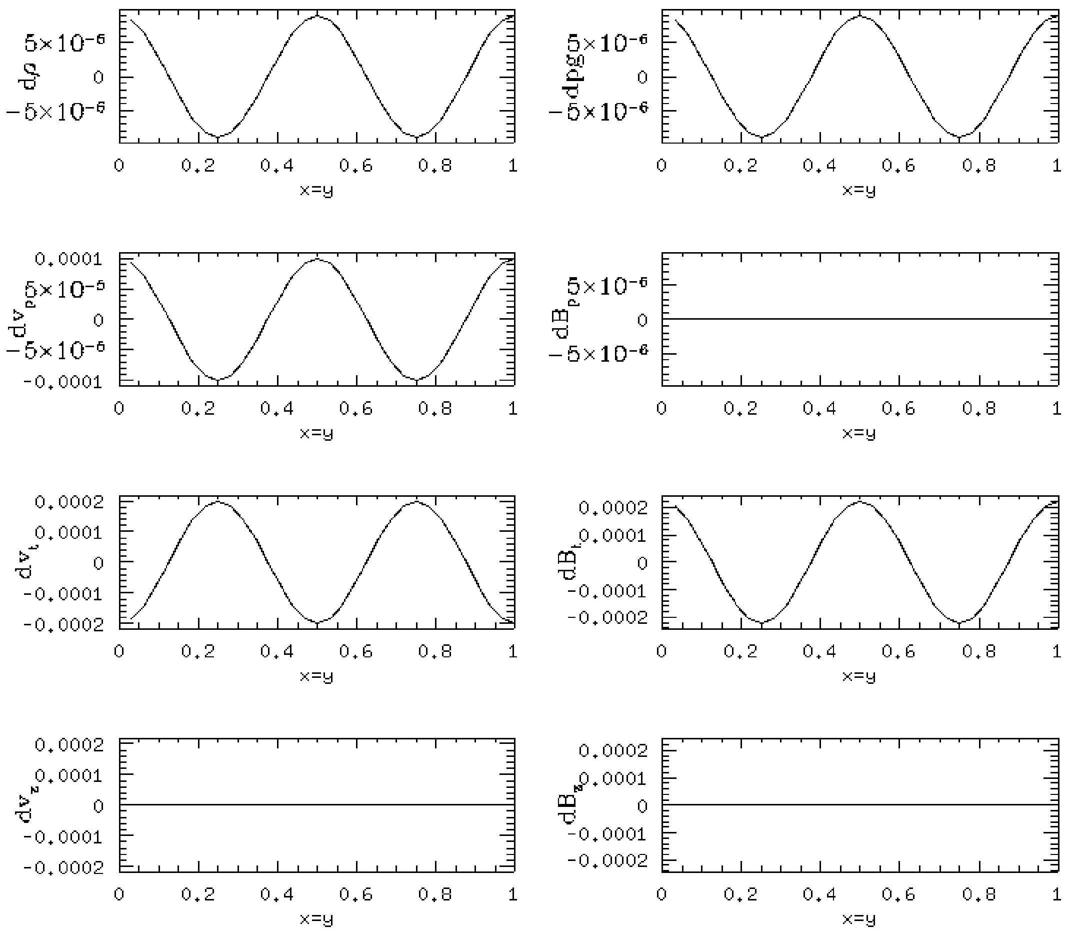
$\delta B_z = 0$

$\delta p = c_{s0}^2 \delta\rho$



MHD 2D Fast wave test

Initially



$$\delta v_{\parallel} = 10^{-4}\cos(k_x x + k_y y)$$

$$\delta\rho = \frac{\rho_0}{\lambda}\delta v_{\parallel}$$

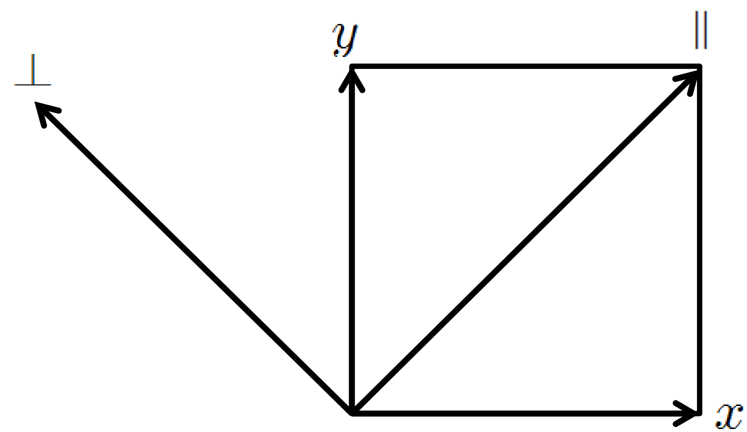
$$\delta v_{\perp} = \frac{b_{\parallel 0} b_{\perp 0}}{b_{\parallel 0}^2 - \lambda^2}\delta v_{\parallel}$$

$$\delta v_z = 0$$

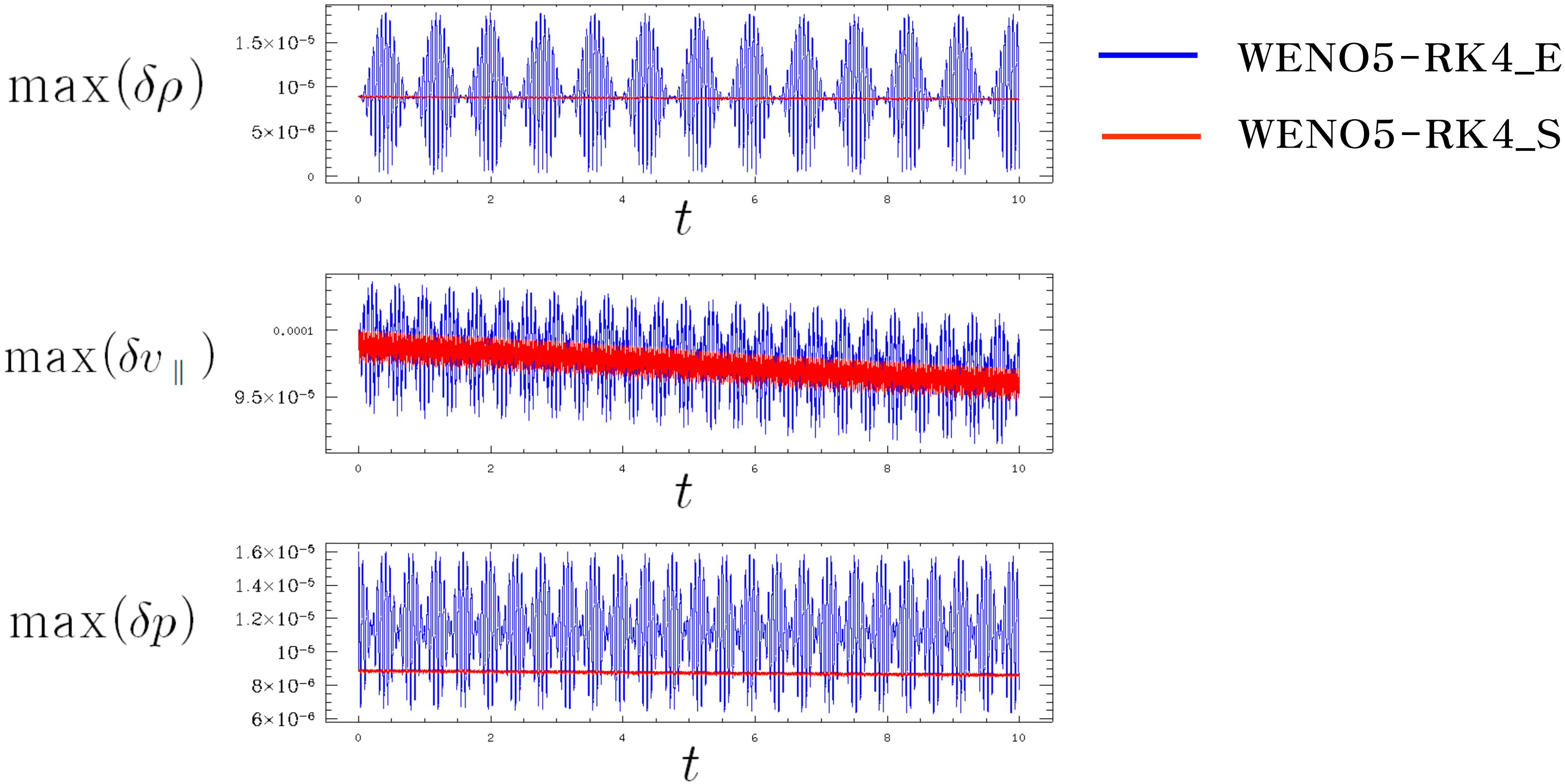
$$\delta B_{\perp} = -\frac{\lambda}{b_{\parallel 0}}\sqrt{\rho_0}\,\delta v_{\perp}$$

$$\delta B_z = 0$$

$$\delta p = c_{s0}^2 \delta\rho$$



MHD 2D Fast wave test



$$E = \frac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} + \frac{1}{2}B^2 \longrightarrow p = (\gamma - 1)[E - \frac{1}{2}(\rho v^2 + B^2)]$$

$$S = p\,\rho^{1-\gamma} \longrightarrow p = S\rho^{\gamma - 1}$$

Flux CT scheme $\vec{\nabla} \cdot \vec{B} = 0$

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and

$$\frac{\partial B_y}{\partial t} + \frac{\partial}{\partial x}(B_y v_x - B_x v_y) = 0 \,. \qquad (2)$$

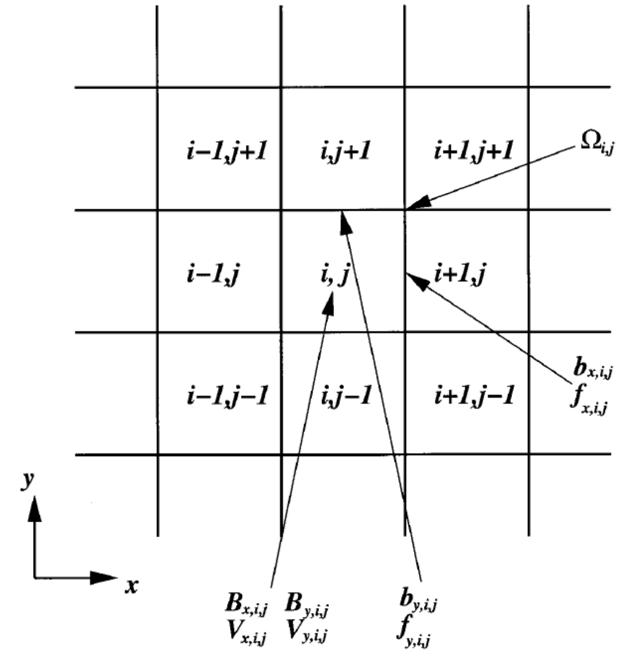
$$B_{x,i,j} = \tfrac{1}{2}(b_{x,i,j} + b_{x,i-1,j}) \qquad (3)$$

and

$$B_{y,i,j} = \tfrac{1}{2}(b_{y,i,j} + b_{y,i,j-1}) \,. \qquad (4)$$

$$\bar{f}_{x,i,j} = \frac{1}{2}(B^n_{y,i,j} v^n_{x,i,j} + B^n_{y,i+1,j} v^n_{x,i+1,j}) - \frac{\Delta x}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i+1/2,j} R^n_{k,i+1/2,j}(5) \,, \qquad (6)$$

$$\bar{f}_{y,i,j} = \frac{1}{2}(B^n_{x,i,j} v^n_{y,i,j} + B^n_{x,i,j+1} v^n_{y,i,j+1}) - \frac{\Delta y}{2\,\Delta t^n} \sum_{k=1}^{7} \beta^n_{k,i,j+1/2,j} R^n_{k,i,j+1/2}(5) \,. \qquad (7)$$



$$\bar{\Omega}_{i,j} = \tfrac{1}{2}(\bar{f}_{y,i+1,j} + \bar{f}_{y,i,j}) - \tfrac{1}{2}(\bar{f}_{x,i,j+1} + \bar{f}_{x,i,j}) \,. \qquad (15)$$

$$b^{n+1}_{x,i,j} = b^n_{x,i,j} - \frac{\Delta t^n}{\Delta y}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i,j-1}) \qquad (16)$$

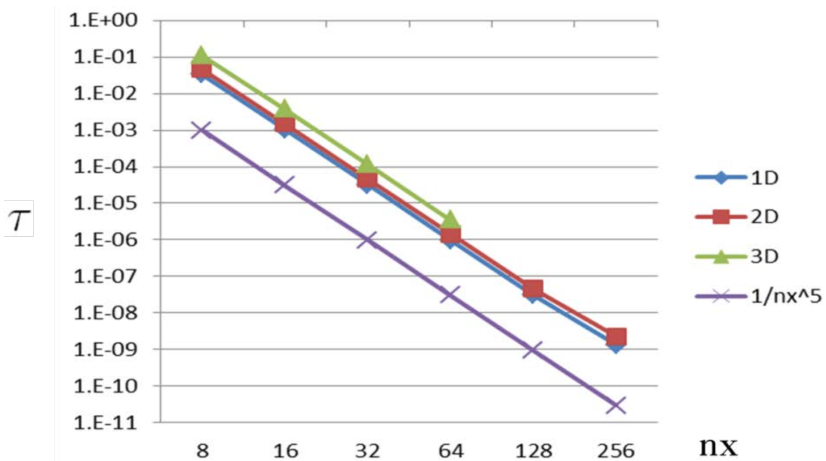
and

$$b^{n+1}_{y,i,j} = b^n_{y,i,j} + \frac{\Delta t^n}{\Delta x}(\bar{\Omega}_{i,j} - \bar{\Omega}_{i-1,j}) \,. \qquad (17)$$

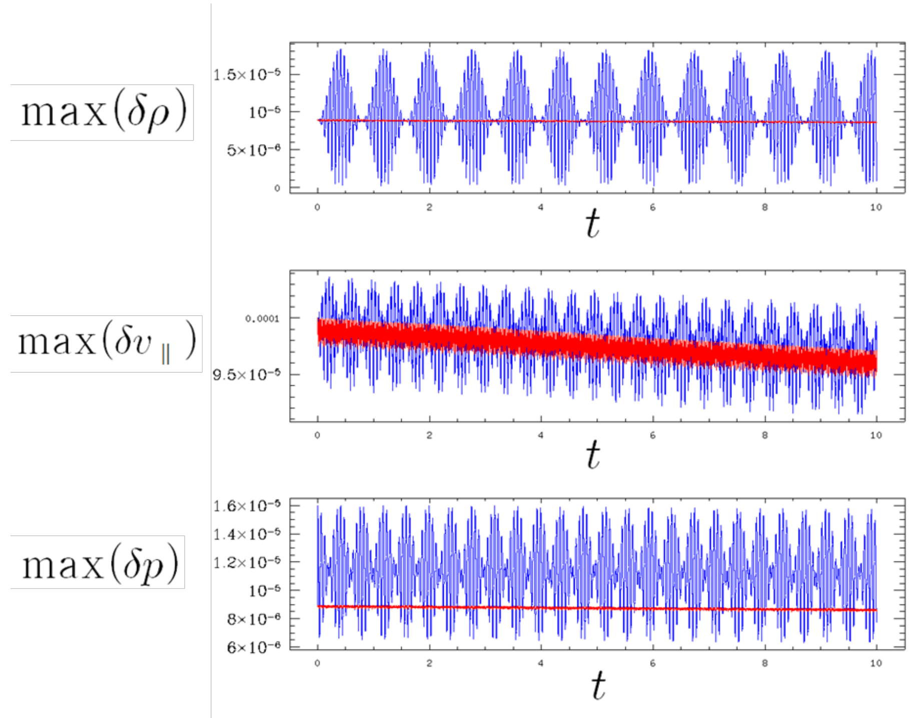
$$\oint_S \boldsymbol{b}^{n+1} \cdot d\boldsymbol{S} = (b^{n+1}_{x,i,j} - b^{n+1}_{x,i-1,j})\Delta y + (b^{n+1}_{y,i,j} - b^{n+1}_{y,i,j-1})\Delta x = 0 \,, \qquad (18)$$

Conclusions

Flux CT scheme $\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$



Does not effect on the order of schemes



Induces numerical errors to B->E->p

Serious oscillation is generated for strong B

Conclusions

MHD-E	MHD-S
Shocks	No shocks
p~E	p<<E
high β ($p_g \sim p_m$)	low β ($p_g \ll p_m$)

In the shock regimes → MHD-E

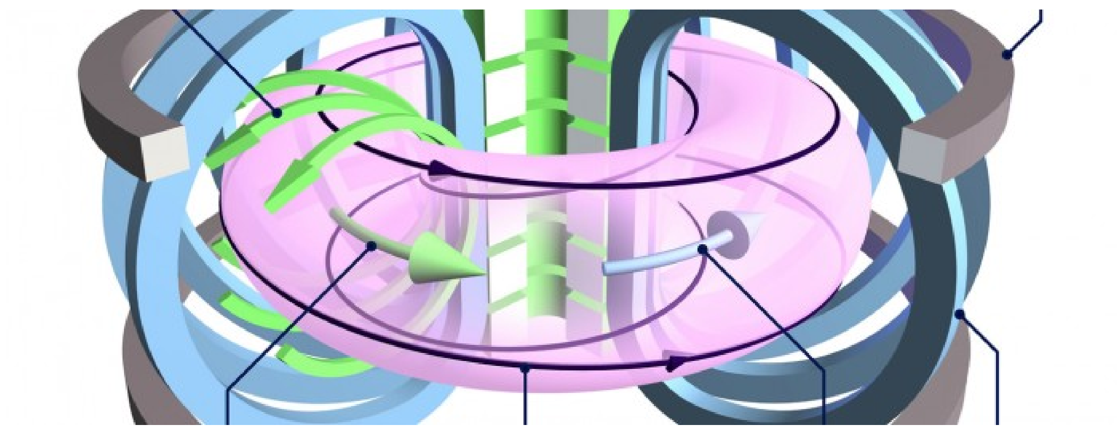
Else where → MHD-S

using TVD, WENO3, WENO5

Conclusions

Application to Nuclear Fusion

: in tokamok



No shocks, p<<E, low β ($p_g \ll p_m$) ➡ MHD-S

Propagate waves many times
→ small numerical dissipation ➡ WENO

Develop Korea tokamak simulations!

Thank you :)