Gravitational Radiation Capture of Black Holes

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3 Approaches
    - Post-Newtonian
    - Numerical Relativity
4 Results
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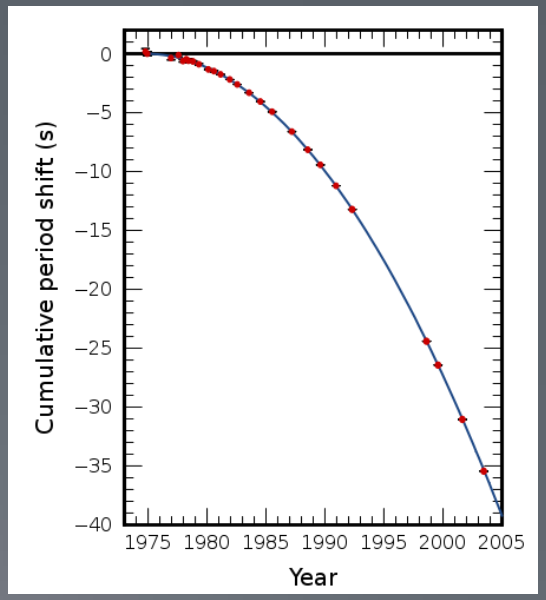
Introduction

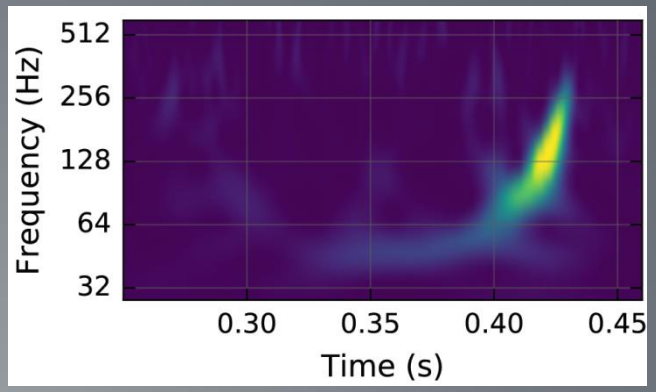
- GW & GR capture

01

Gravitational Waves (GWs)

- Ripples of spacetime curvature that progagate as waves
- Indirect detection
  - PSR 1913+16 (Hulse & Taylor 1974, Weisberg & Taylor 2005)
- Direct detections
  - aLIGO (advanced Laser Interferometer Gravitational-wave Observatory) on 14 Sep. and 26 Dec. 2015
- New window - GW astronomy has started!





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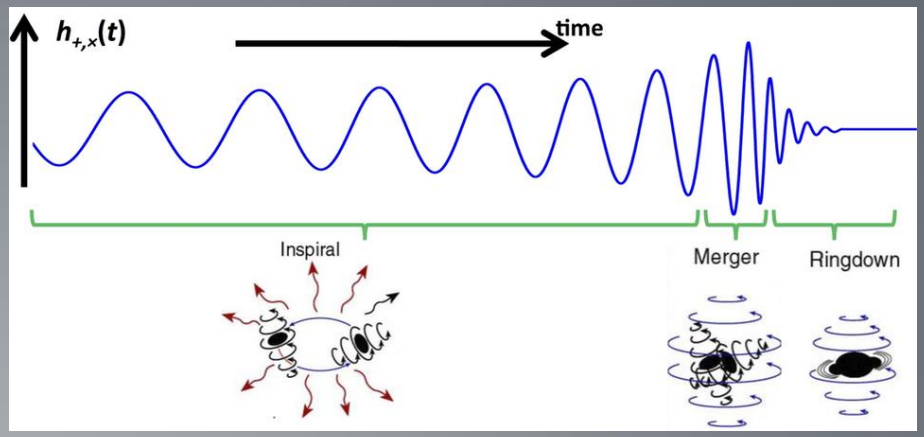
GW sources

- GRB/Supernova, Spinning Neutron star, Cosmological Sources, …

- Compact Binary Coalescence (CBC)
  - Strongest source (Abadie et al. 2010)
  - Predictable wave forms
  - Detectable frequency & strength for ground-based detectors



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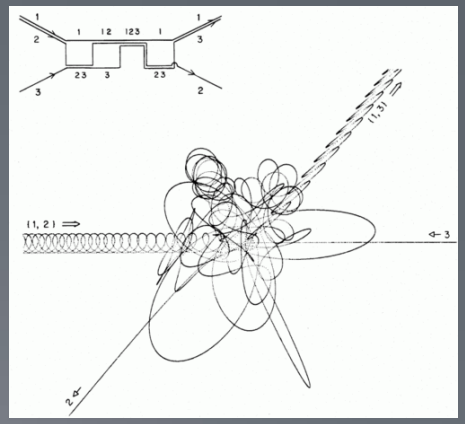
Formation of Compact Binaries

- From stellar binary (Postnov & Yungelson 2006)
  - Survived binary even after the stellar evolution
- Three-body process (Aarseth & Heggie 1976, Bae et al. 2014)
  - Encounter of three bodies at the same time
- Gravitational Radiation capture (GR capture, Quinlan & Shapiro 1987, 1989, Mouri & Taniguchi 2002)
  - Energy loss through the emission of GWs



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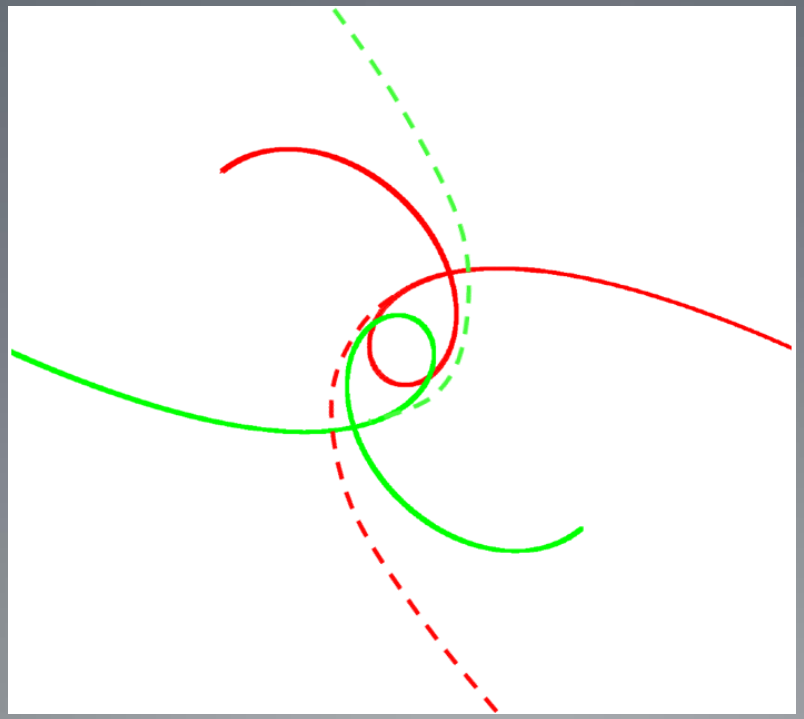
GR capture

- Energy radiation > initial orbital energy
- Unbound (E>0) → bound (E<0)
- Hyperbolic orbit (e>1) → Elliptic orbit (e<1)



01

Astrophysical Applications

- Eccentric BH binary merging from GR capture
- BHs in dense region
  - Center of globular clusters
  - Galactic center near the supermassive BH
    - Over 10^4 BHs within 1pc in the center of Milky Way (Morris 1993, Miralda-Escude & Gould 2000, Freitag et al. 2006, Hopman & Alexander 2006)
- Primordial BHs (Bird et al. 2016, Clesse et al. 2016)

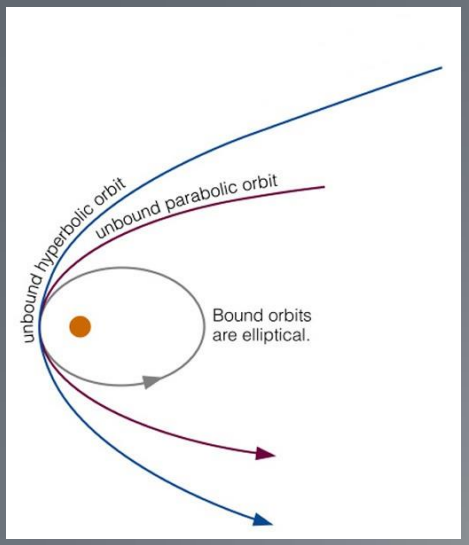
Assumption

- Parabolic approximation

02

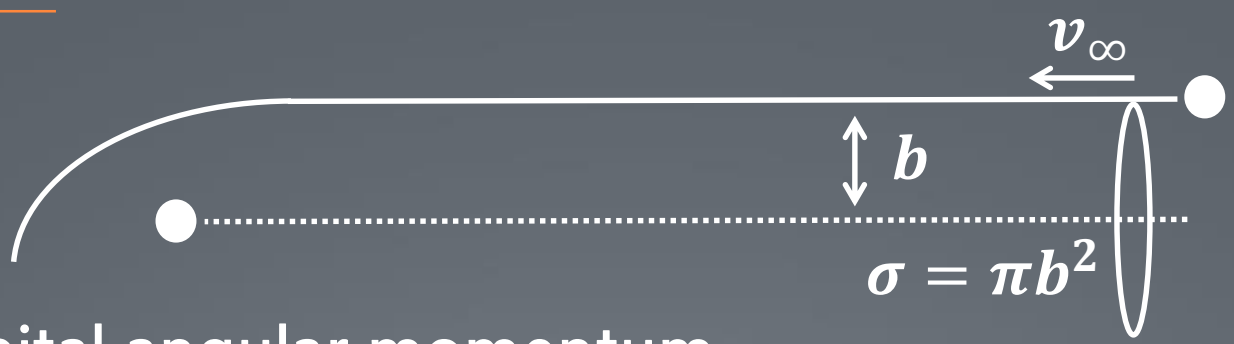
Parabolic approximation

- Difficulties in finding marginally capturing hyperbolic orbit
  - Full orbit simulation is necessary.
- Parabolic approximation
  - Hyperbolic orbit is similar with parabolic orbit at pericenter.
  - Assume that GW radiation from hyperbolic is same with parabolic.
  - Use parabolic orbit instead of hyperbolic orbit.
  - Parabolic orbit simulations → E_{rad} → Hyperbolic with $E \approx E_{rad}$ can be captured.



02

Critical Impact parameter



- Orbital angular momentum

$$L = b\mu v_\infty$$

- Energy

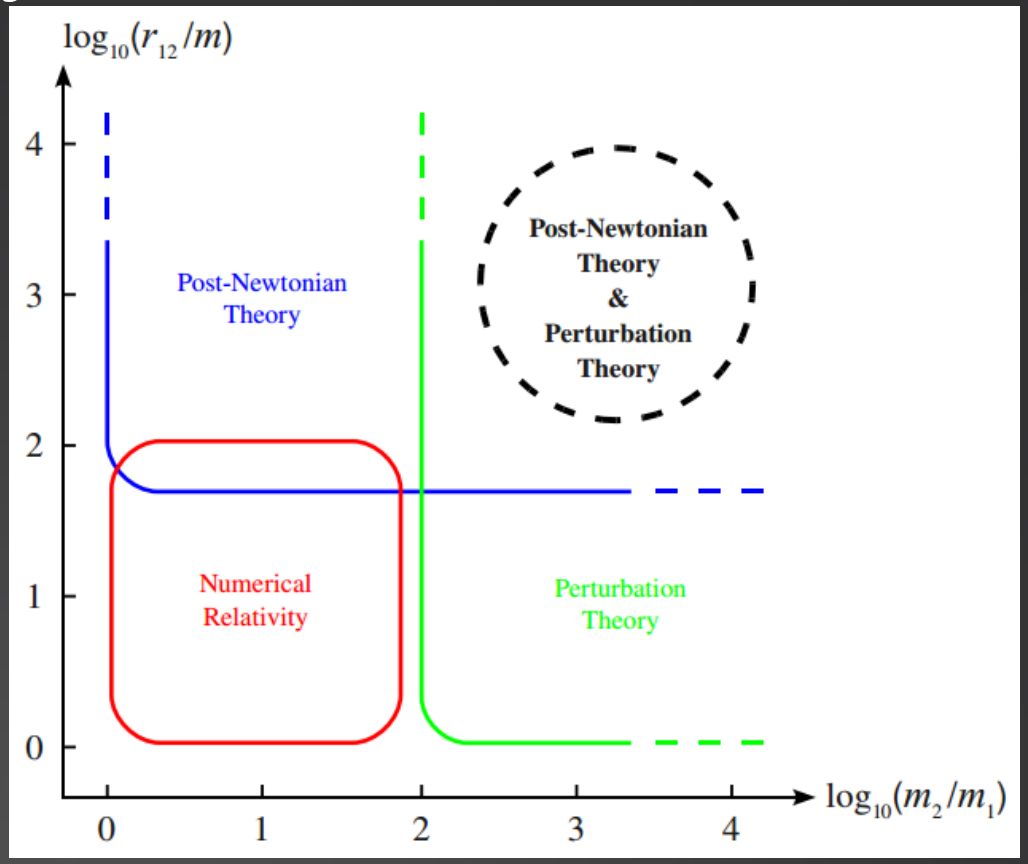
$$E = \frac{1}{2}\mu v_\infty^2$$

- Critical impact parameter & Cross section

$$b = \frac{L}{\mu v_\infty} = \frac{L}{\sqrt{2\mu E}}, \qquad \sigma = \pi b^2$$

Approaches

- Post Newtonian
& Numerical Relativity



Blanchet et al.,
Phys. Rev. D 81, 064004 (2010)

03

Post-Newtonian (PN)

- Post-Newtonian
  - Approximate solution of Einstein field equations
  - Expansion in $1/c^2$
  - Point mass
  - GW radiation from Mass quadrupole moment

$$\frac{dE}{dt} = -\frac{1}{5}\left(\frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3} - \frac{1}{3}\frac{d^3Q_{ii}}{dt^3}\frac{d^3Q_{jj}}{dt^3}\right)$$

where $Q_{ij} \equiv \sum_a m^a x_a^i x_a^j$

Approach 1. Exact Parabolic orbit (EPO)

Newtonian orbit

Approach 2. PN corrected orbit (PNCO)

PN equation of motion

03

Post-Newtonian (PN)

- Using Exact Parabolic Orbit (EPO)
  - Based on Peters (1964) - GW Radiation from elliptic orbit
  - Push the limit to parabolic orbit : e→1

$$\Delta E = \frac{170\pi}{3}\frac{G^7}{c^5}\frac{m_1^9 m_2^9}{(m_1+m_2)^3}\frac{1}{L^7}$$

  - Neglecting the orbit changes
  - Weak encounter
  - Most favored method (e.g., Quinlan & Shapiro 1989, Mouri & Taniguchi 2002, O'Leary et al. 2009, Hong & Lee 2015, Bird et al. 2016, Clesse et al. 2016)

03

Post-Newtonian (PN)

- Using PN Corrected Orbit (PNCO)
  - Gradually changing orbit
  - PN equation of motion

$$\boldsymbol{a} = \boldsymbol{a}_N + \frac{1}{c^2}\boldsymbol{a}_{1PN} + \frac{1}{c^3}\boldsymbol{a}_{1.5PN,SO} + \frac{1}{c^4}(\boldsymbol{a}_{2PN} + \boldsymbol{a}_{2PN,SS}) + \frac{1}{c^5}(\boldsymbol{a}_{2.5PN} + \boldsymbol{a}_{2.5PN,SO}) + \frac{1}{c^6}\boldsymbol{a}_{3PN} + \frac{1}{c^7}\boldsymbol{a}_{3.5PN}$$

  - Two body code
  - Mass quadrupole formula

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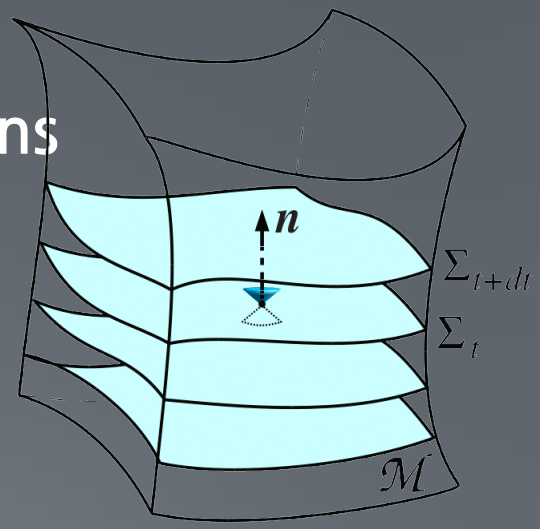
Numerical Relativity (NR)



- Numerical solution of Einstein field Equations
- Most accurate method
- Cover all the phases of BH merging
- Expensive

- 3+1 decomposition (ADM formalism, Arnowitt et al. 1962)
  - 4 dimensional spacetime equation → 3 dimensional space + 1 dimensional time
  - Constraint equations
    - Hamiltonian (1) & Momentum (3)
  - Evolution equations (6)
  - Gauge choice (4)

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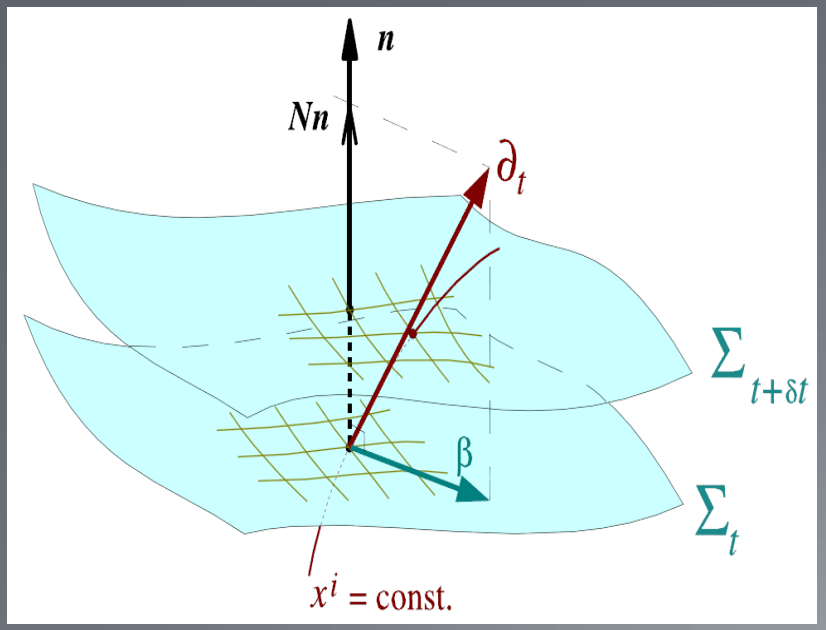
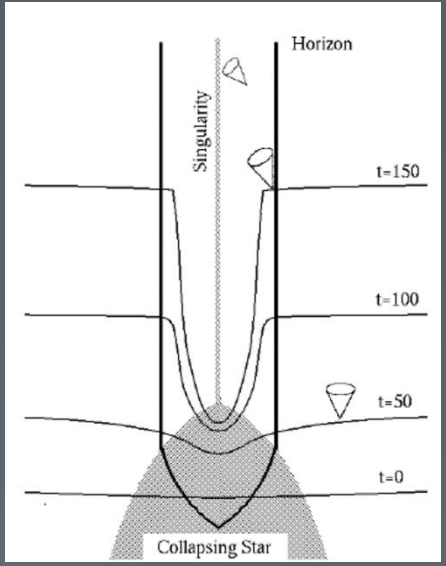
Gauge choice



- Gauge (Coordinate) choice
  - Lapse (α) & Shift (β^i)
- Geodesic slicing ($\alpha = 1,\ \beta^i = 0$)
  - Singularity
- Maximal slicing ($K = 0$)
  - Singularity avoidance
- 1+log slicing ($\alpha = 1 + \ln \gamma$)
  - Easy to implement and fast to solve
  - Singularity avoidance

03

Avoiding Singularity



- Singularity avoiding coordinates
  - Postpone the singularity until $t = \infty$
  - Grid stretching problem along black hole throat
  - Hard to identify singularity avoiding coordinates that lead long & stable evolution for orbiting binaries
- Black hole excision (Pretorius 2005)
  - Cut out the inside of the horizon
- Moving Puncture method (Baker et al. 2006, Campanelli et al. 2006)
  - Avoid the singularity through the intelligent choice of gauge
  - 1+log slicing, Gamma-driver shift

03

Weyl Scalar

- Weyl Scalar
  - Newman-Penrose formalism (Newman & Penrose 1962, 1963)
  - Obtained by contracting Weyl tensor with complex null tetrad
  - The quantities related with the gravitational waves at infinity
  - $\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$
  - Radiated energy

$$\frac{dE}{dt} = -\lim_{r\to\infty}\frac{r^2}{16\pi}\int d\Omega\left|\int_{-\infty}^{t} dt'\Psi_4\right|^2$$

03

ADM mass & Geometrized unit

- ADM mass (energy)
  - Total mass (energy) measured on a spatial infinite surface at a certain time
  - In Newtonian limit

$$M_{ADM} = M_0 + T + W$$

  - Parabolic orbit : $M_{ADM} = 1$
- Geometrized units
  - $c = G = 1$
  - Time, mass, energy : M
  - Velocity : no unit
  - Orbit, spin angular momentum : M^2
  - 10-10 $M_\odot$ BH : unit length = 30km, unit time = 0.1ms

03

Software & Hardware

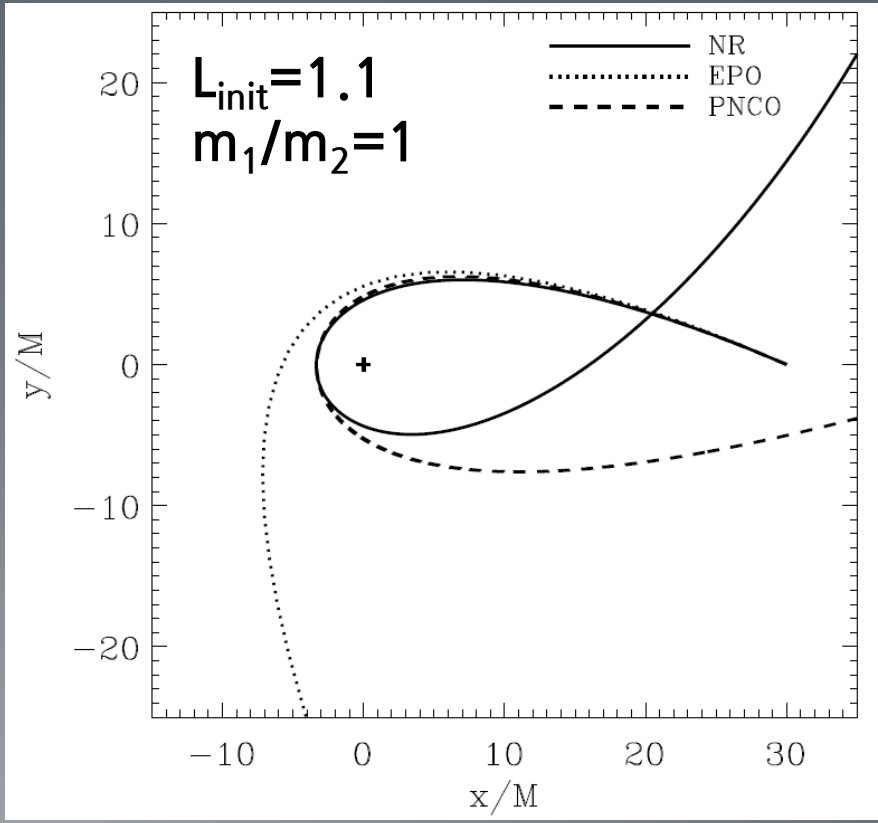
- Software - Einstein Toolkit
  - Collection of software components and tools for simulation and analysis for general relativistic phenomena
  - Free, open source (http://einsteintoolkit.org)
  - Based on Cactus code
    - Flesh (central core) + Thorns (application module)
    - Over 100 thorns for simulation management
  - HDF5 parallel file I/O, AMR (Adaptive Mesh Refinement), ...

- Hardware - Tachyon2 (KISTI) & Gmunu (SNU)

03

PN vs NR

	Orbit	Size of mass	Spin
EPO	Exact parabolic	Point	No
PNCO	Initially parabolic	Point	Yes
NR	Initially parabolic	Extended	Yes



Results 1

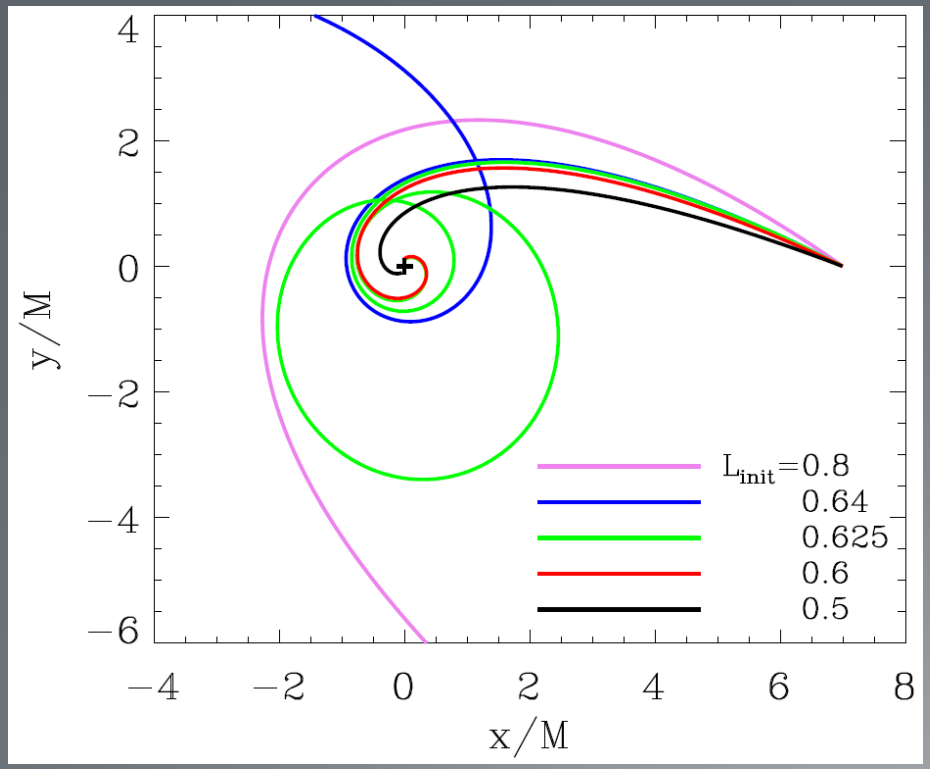
Unequal mass BHs

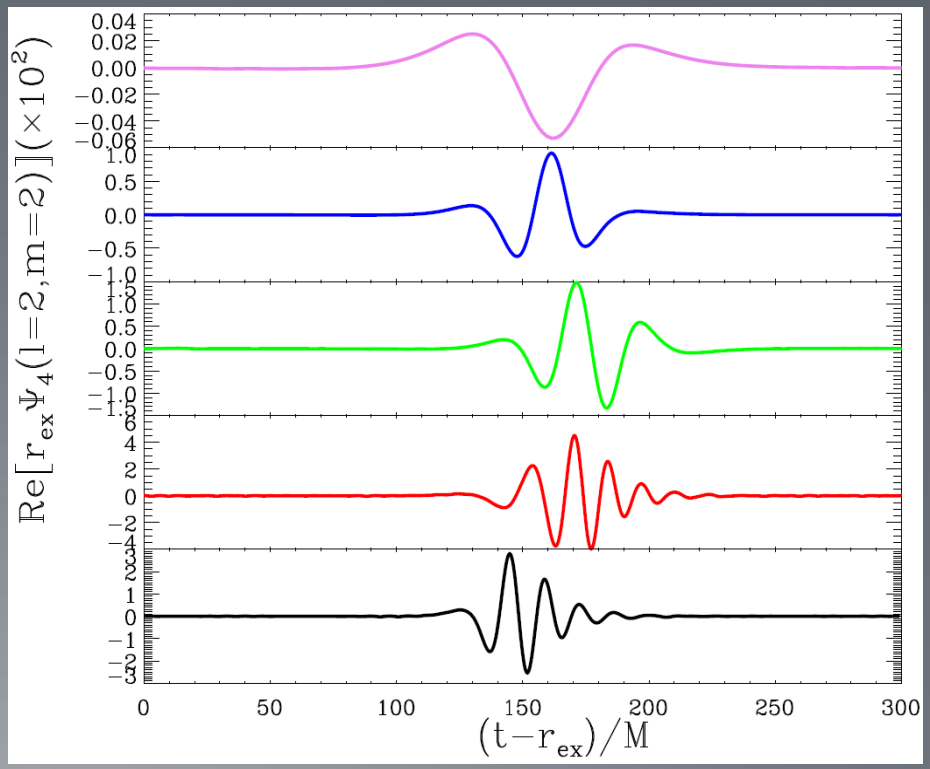
- Effect of BH mass spectrum

04

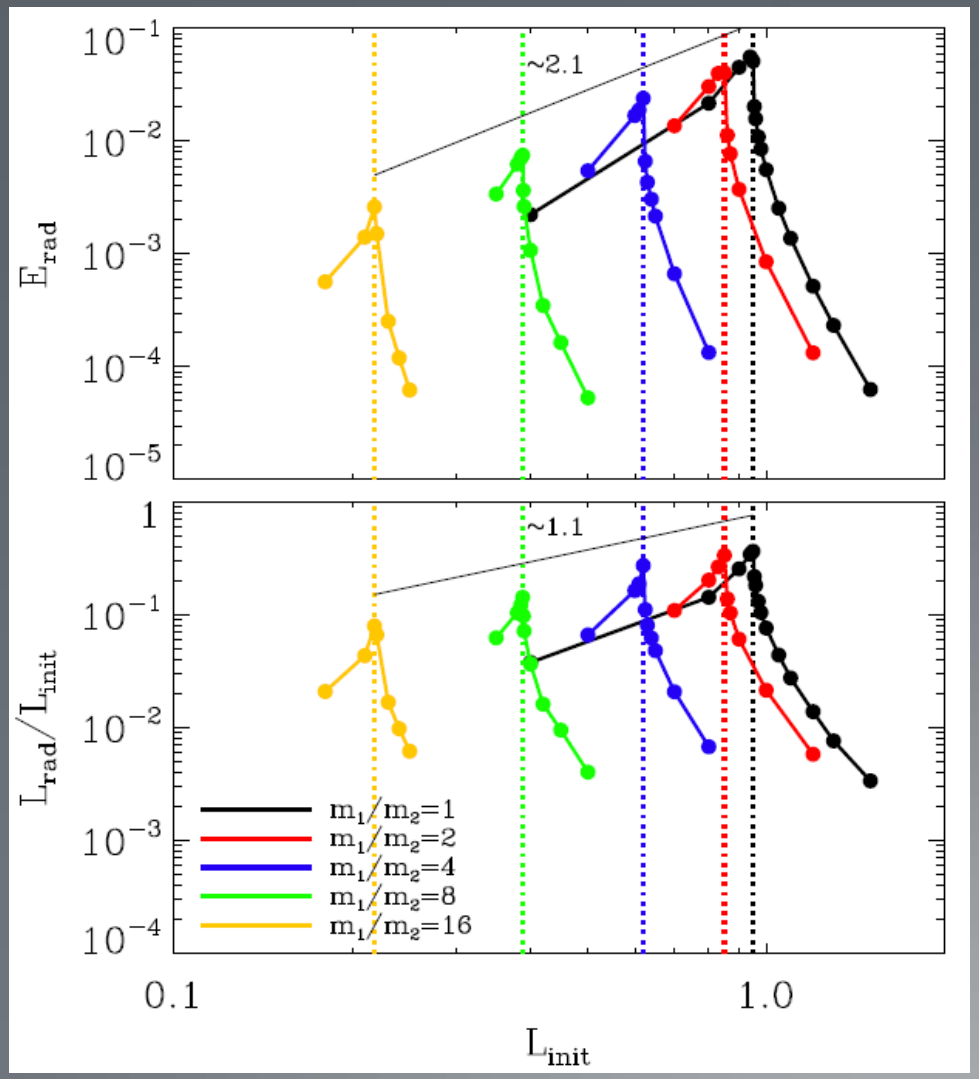
Orbits & Waves - $m_1/m_2=4$

- Orbit of heavier BH





Radiated E & L

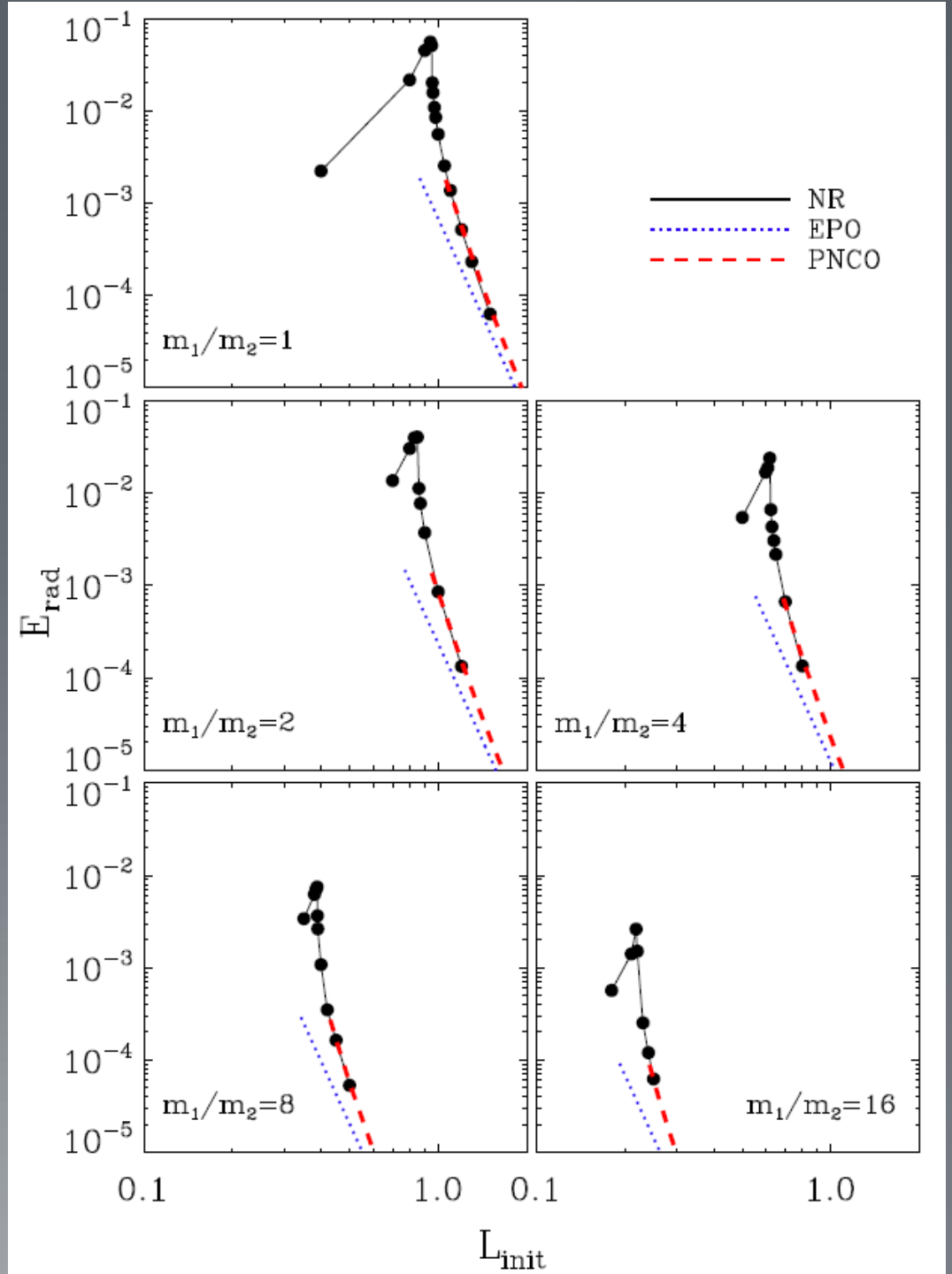


- Left : Direct merging
- Right: fly-by

- Most GW radiations on the boundary
- Shift of peak point
- fly-by orbit
  - Can be used for parabolic approximation
  - Less energy for higher mass ratio at fixed L_{init}

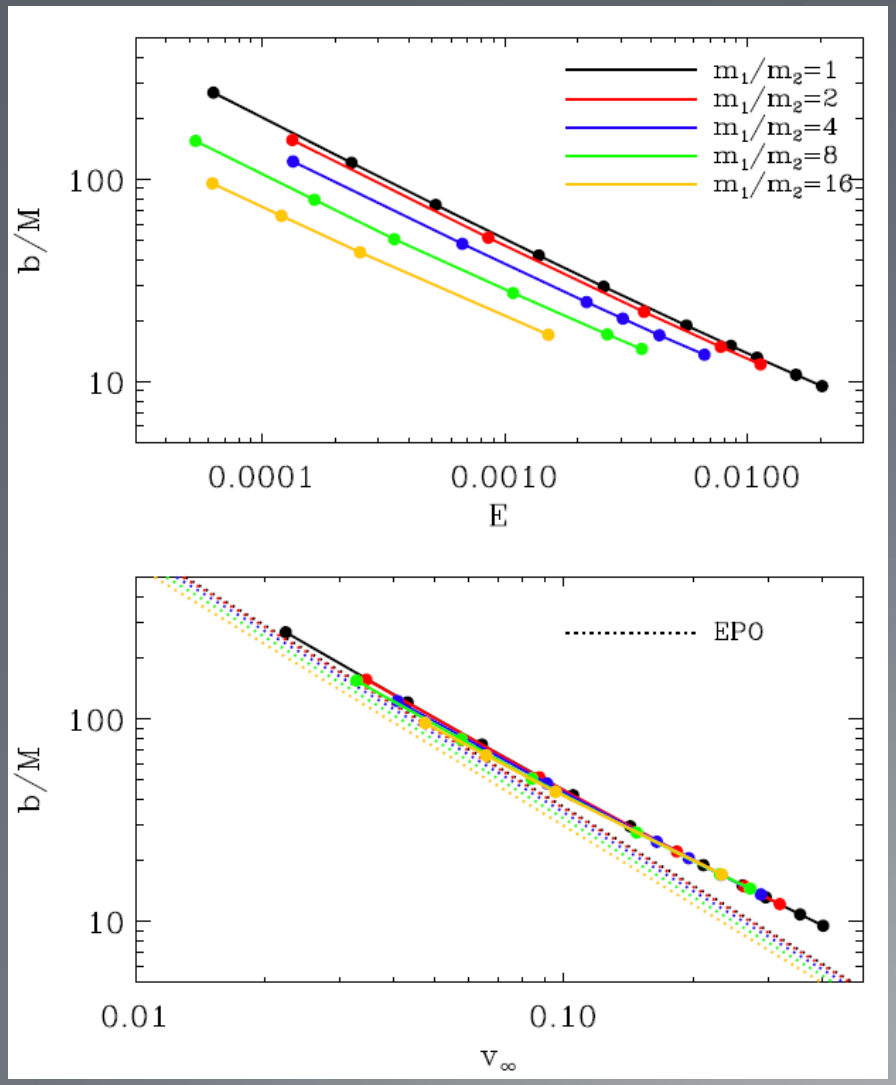
04

Comparison with PN



04

Impact parameter



- Radiated E in parabolic orbit = Energy of hyperbolic orbit
- EPO (Exact Parabolic Orbit)

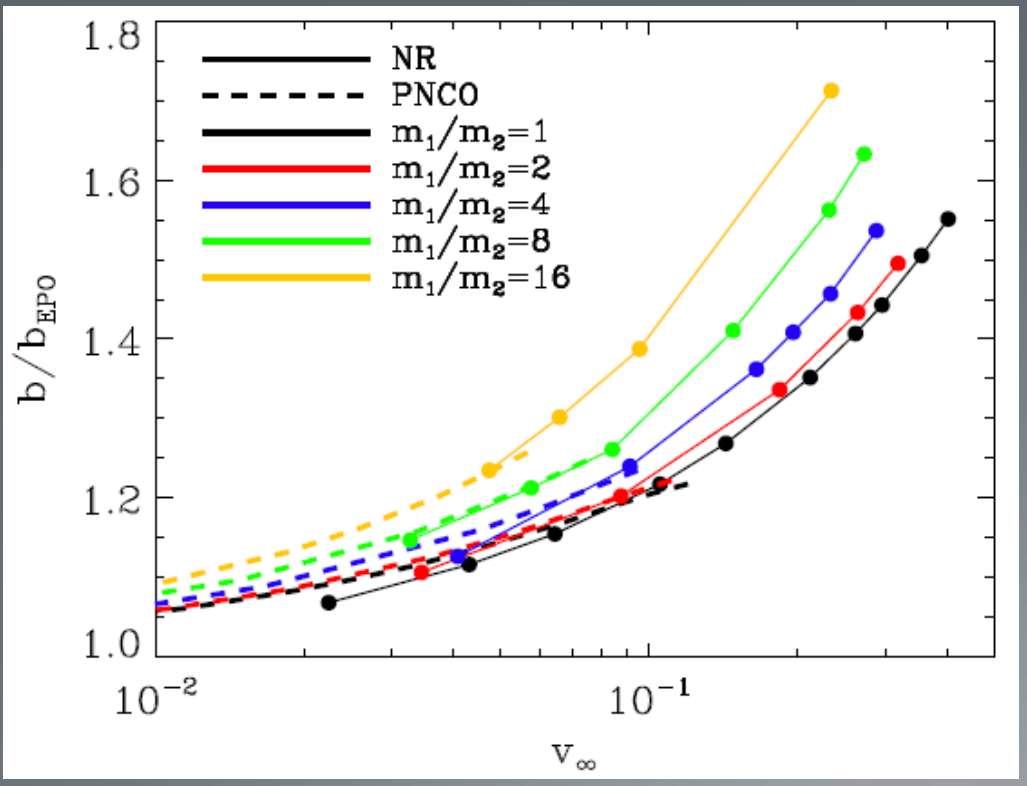
$$E = \frac{1}{2}\mu v_\infty^2$$

$$b = \left(\frac{340\pi}{3}\frac{m_1 m_2 (m_1+m_2)^5}{v_\infty^9}\right)^{1/7}$$

- Difficult to capture the high energy BHs → small impact parameter
- Upper limit of v_∞ for fixed impact parameter
- Gentle slope in high energy

04

Impact parameter



- Impact parameters divided by EPO's
- Deviations in high velocity for both PNCO and NR
- EPO : $v_\infty \lesssim 0.01$
- PNCO : $0.01 \lesssim v_\infty \lesssim 0.1$
- NR : $0.1 \lesssim v_\infty$
- Conspicuous for higher mass ratio

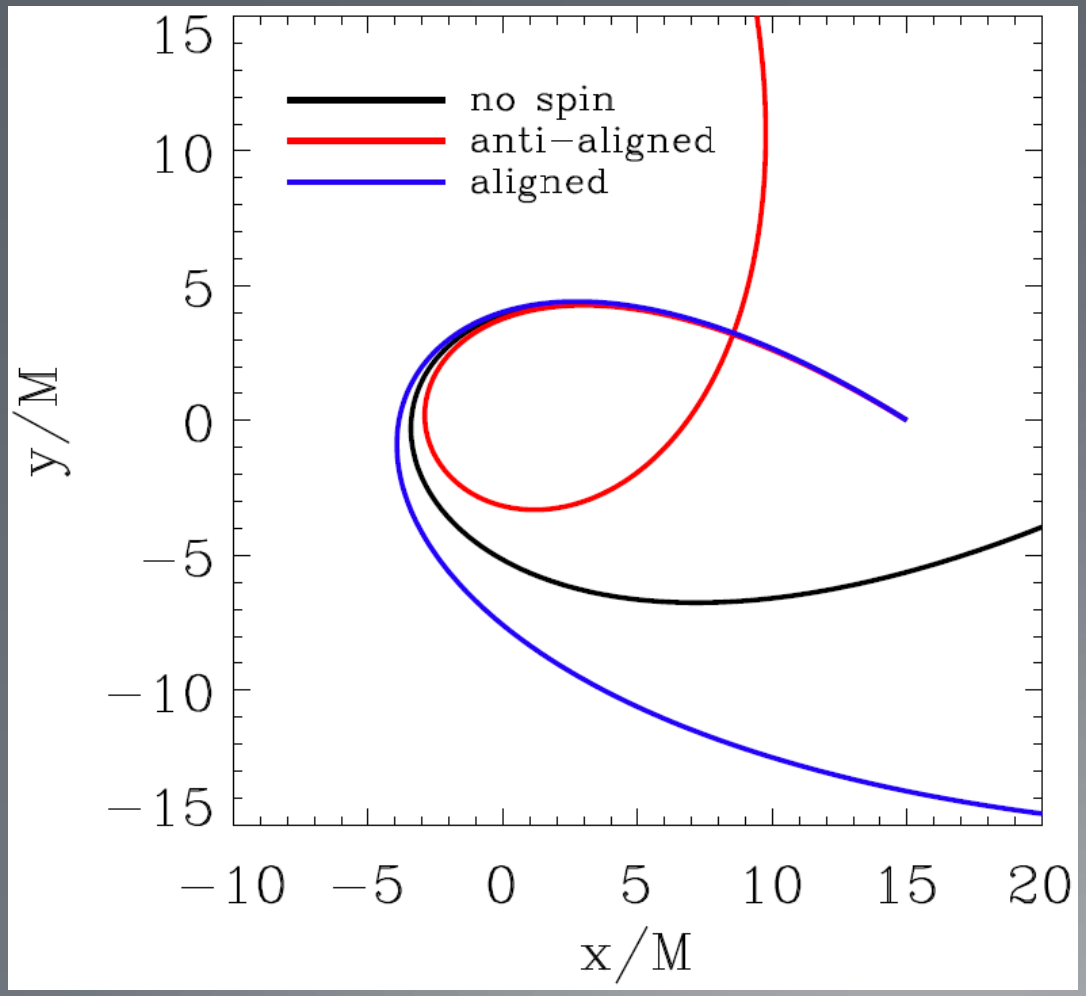
Results 2

Spinning BHs

- Frame dragging effect

05

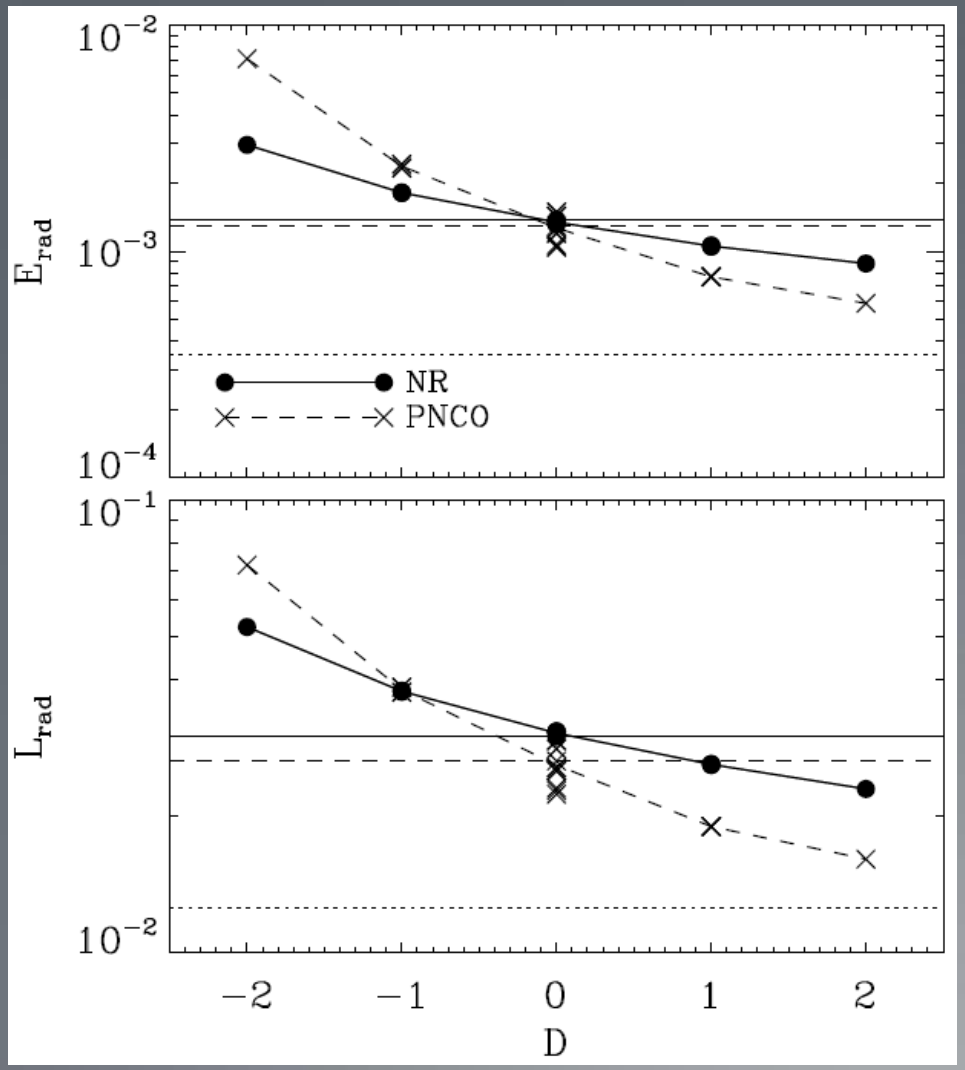
Spin Direction



- More winding orbit for anti-aligned spins → shorter pericenter distance

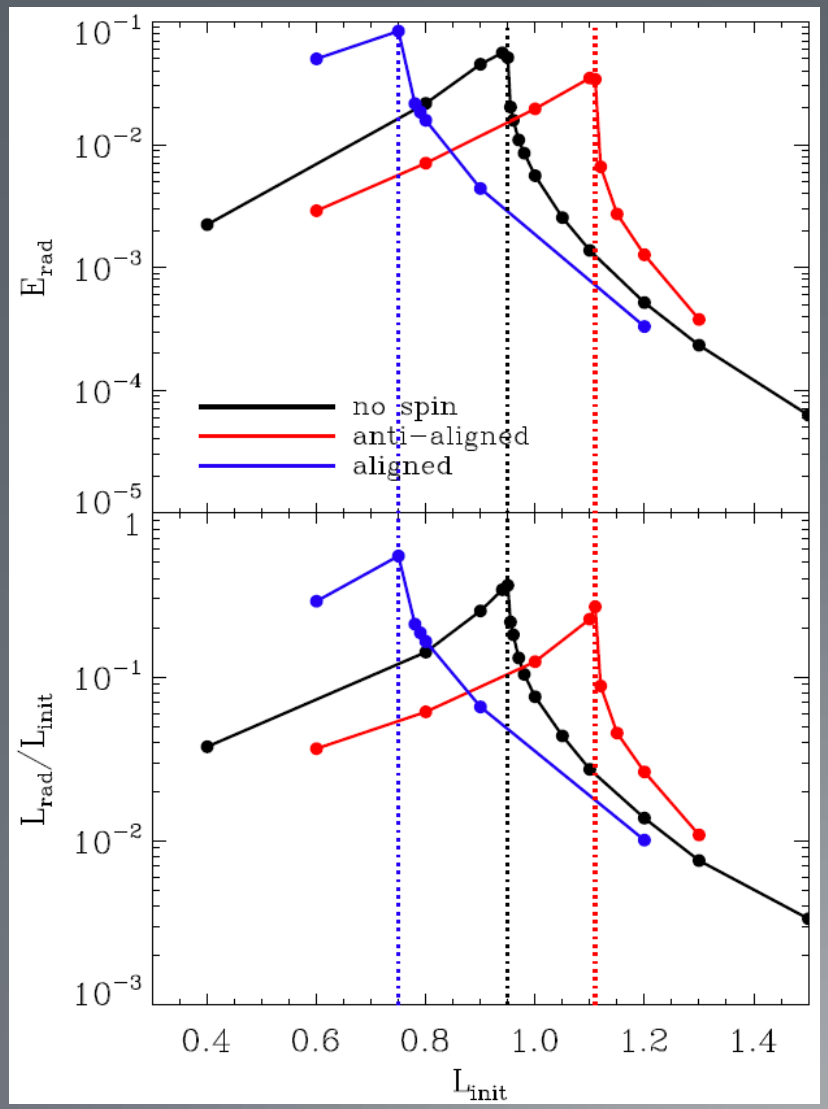
- Loosely bound orbit for aligned spins → longer pericenter distance

Spin Direction



- Dependence on spin direction
  - $m_1/m_2=1$, $L_{init}=1.1$
  - Fixed spin magnitude : $S/m^2=0.5$
  - Various spin combinations with x, y and z direction
  - Radiated E & L depend on $D = \hat{L} \cdot (\hat{S}_1 + \hat{S}_2)$
  - PNCO is not good for this case.

Radiated E & L

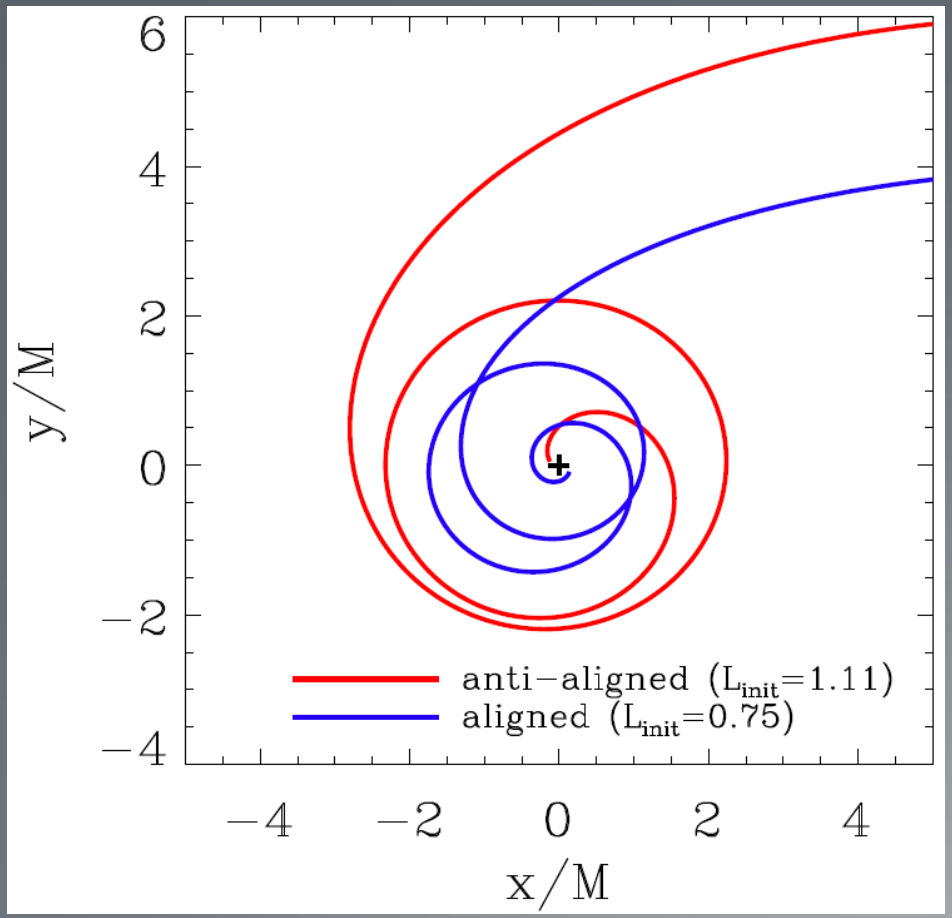


- For aligned and anti-aligned spin directions
- Spin magnitudes: $S/m^2=0.9$

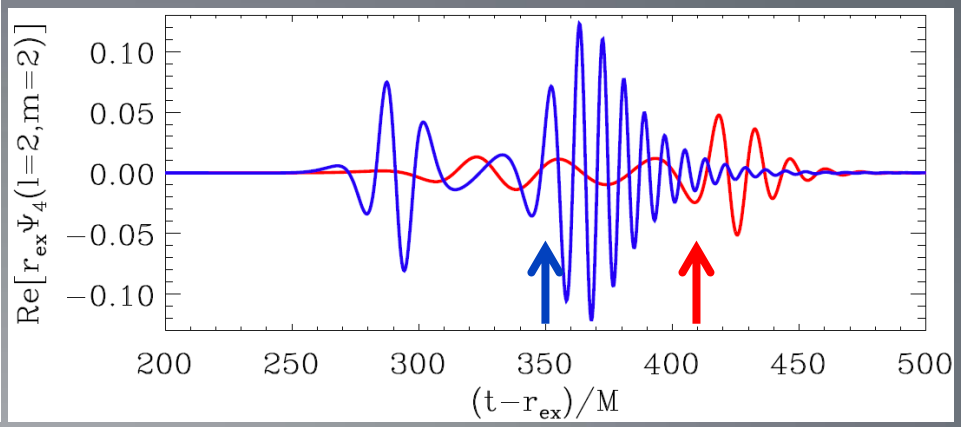
- More GW radiation for anti-aligned BHs at fixed L_{init}
- More GW radiation for aligned BHs at the peak
- ~10% of energy radiation

05

Orbits & Waves at the peak

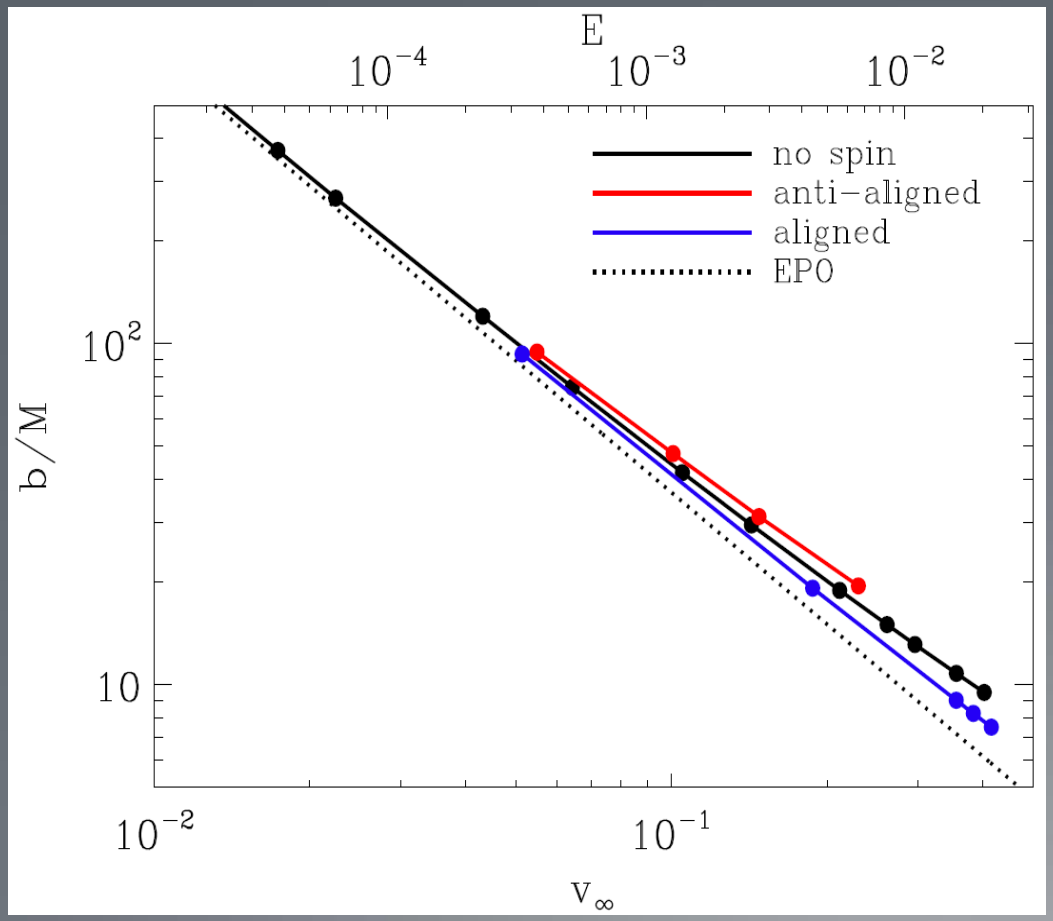


- Anti-aligned BHs have
  - More whirly orbit before merging
  - Longer duration of ringdown phase



05

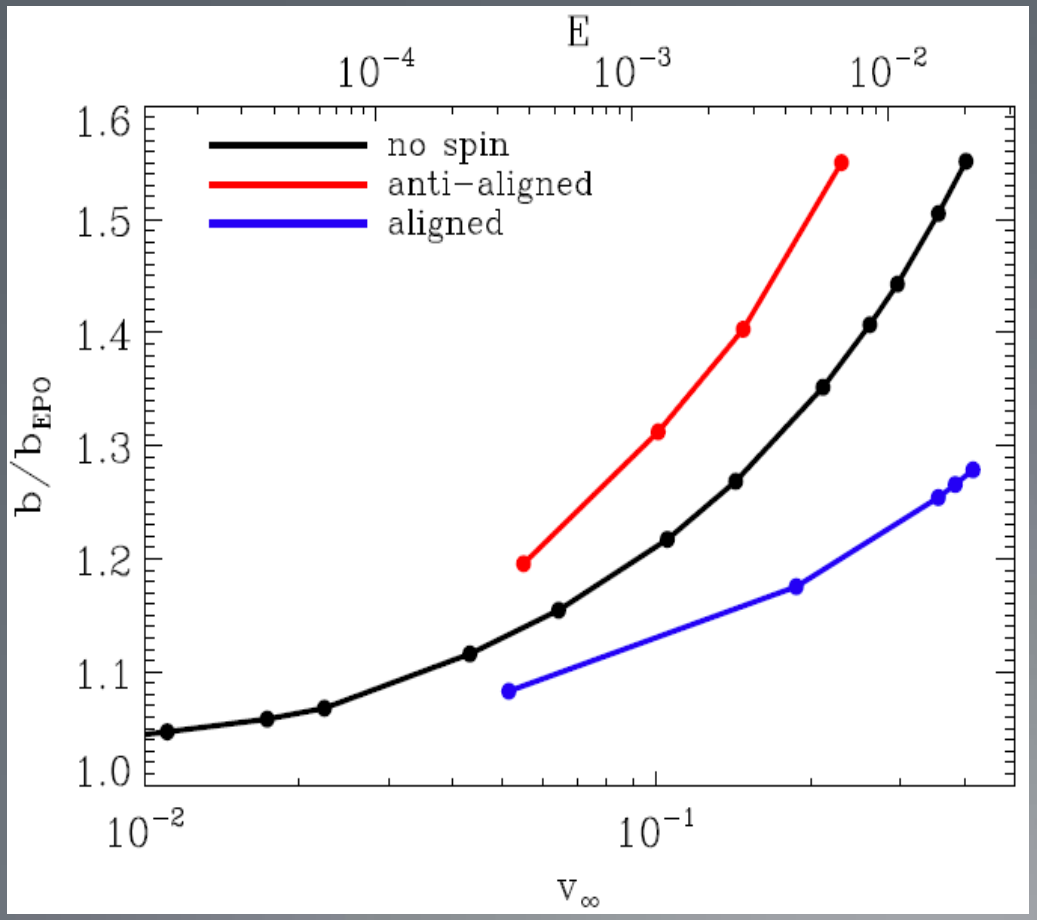
Impact parameter



- Different slopes for different spin directions in high energy
- More gentle slope for anti-aligned spins
- Higher speed BH capture for anti-aligned spins
- Larger impact parameters than EPO

05

Impact parameter



- Larger deviation from EPO for anti-aligned spin
- Larger than 10% deviations from non-spinning at $v_\infty = 0.2$

Discussions

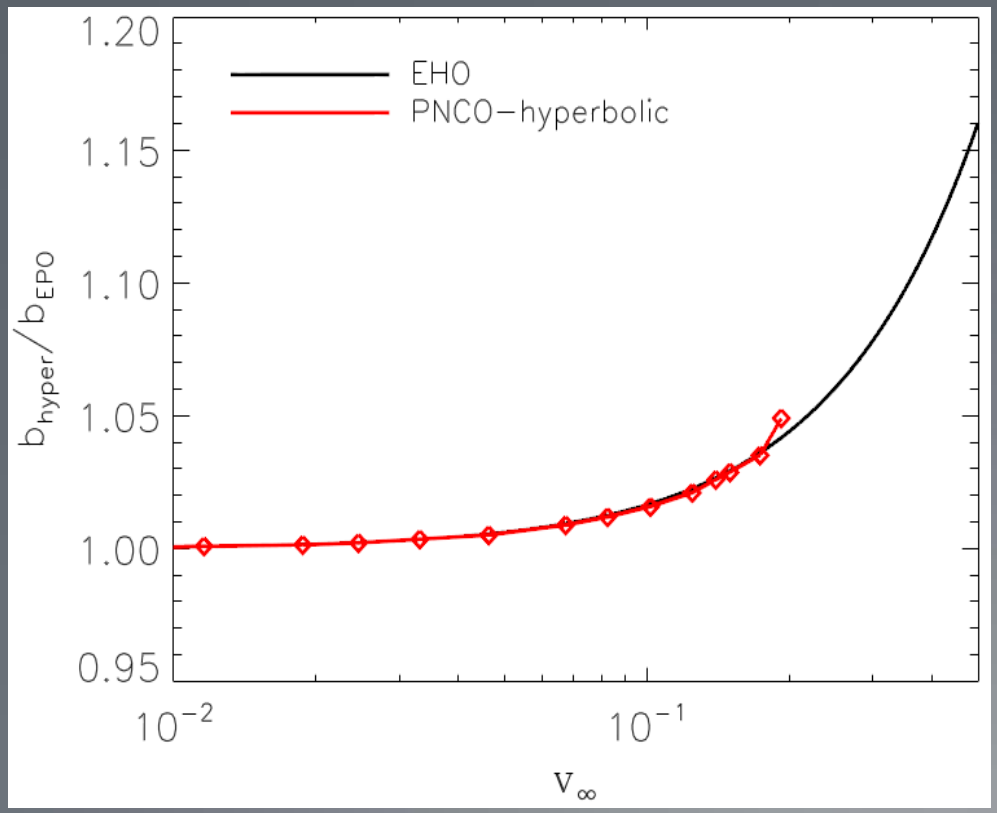
06

Validity of Parabolic approximation

- True hyperbolic orbit is different from the parabolic orbit.
- True hyperbolic orbit with PN
  - Exact Hyperbolic Orbit (EHO)
    - GW radiations from hyperbolic orbit (Hansen 1972)
    - Orbital energy = Radiated E
  - PNCO - hyperbolic
    - PN equation of motion
    - Repetition of the initial conditions

06

Validity of Parabolic approximation



- Parabolic approximation
  - Valid within 5% up to $v_\infty = 0.2$.
  - Can make the impact parameter to be underestimated in high energy

07

Conclusion & Summary

- The most energetic encounter
  - Around the boundary between direct merging and fly-by orbit
  - A few percent of total ADM energy
- More GW radiations
  - Equal mass and anti-aligned spins at the same L_{init}
  - Equal mass and aligned spins at the peak point
- Larger impact parameters
  - Equal mass and anti-aligned spins
  - Deviate from PN in high energy
- NR is required for $\Delta E \gtrsim 10^{-3}$
  - Strong encounter with short pericenter
  - Eccentric merger

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