Gravitational Radiation Capture of Black Holes

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Introduction

- GW & GR capture
Gravitational Waves (GWs)

- Ripples of spacetime curvature that propagate as waves
- Indirect detection
  - PSR 1913+16 (Hulse & Taylor 1974, Weisberg & Taylor 2005)
- Direct detections
  - aLIGO (advanced Laser Interferometer Gravitational-wave Observatory) on 14 Sep. and 26 Dec. 2015
- New window - GW astronomy has started!
GW sources

- GRB/Supernova, Spinning Neutron star, Cosmological Sources, ...

- Compact Binary Coalescence (CBC)
  - Strongest source (Abadie et al. 2010)
  - Predictable wave forms
  - Detectable frequency & strength for ground-based detectors
Formation of Compact Binaries

- From stellar binary (Postnov & Yungelson 2006)
  - Survived binary even after the stellar evolution

- Three-body process (Aarseth & Heggie 1976, Bae et al. 2014)
  - Encounter of three bodies at the same time

  - Energy loss through the emission of GWs
GR capture

- Energy radiation > initial orbital energy
- Unbound (E>0) → bound (E<0)
- Hyperbolic orbit (e>1) → Elliptic orbit (e<1)
Astrophysical Applications

- Eccentric BH binary merging from GR capture
- BHs in dense region
  - Center of globular clusters
  - Galactic center near the supermassive BH
    - Over $10^4$ BHs within 1pc in the center of Milky Way
- Primordial BHs (Bird et al. 2016, Clesse et al. 2016)
Assumption

- Parabolic approximation
**Parabolic approximation**

- **Difficulties in finding marginally capturing hyperbolic orbit**
  - Full orbit simulation is necessary.

- **Parabolic approximation**
  - Hyperbolic orbit is similar with parabolic orbit at pericenter.
  - Assume that GW radiation from hyperbolic is same with parabolic.
  - Use parabolic orbit instead of hyperbolic orbit.
  - Parabolic orbit simulations $\rightarrow E_{\text{rad}} \rightarrow$ Hyperbolic with $E \approx E_{\text{rad}}$ can be captured.
Critical Impact parameter

- Orbital angular momentum
  \[ L = b \mu v_\infty \]

- Energy
  \[ E = \frac{1}{2} \mu v_\infty^2 \]

- Critical impact parameter & Cross section
  \[ b = \frac{L}{\mu v_\infty} = \frac{L}{\sqrt{2\mu E}}, \quad \sigma = \pi b^2 \]
Approaches

- Post Newtonian & Numerical Relativity

Post-Newtonian (PN)

- Post-Newtonian
  - Approximate solution of Einstein field equations
  - Expansion in $1/c^2$
  - Point mass
  - GW radiation from Mass quadrupole moment

\[
\frac{dE}{dt} = - \frac{1}{5} \left( \frac{d^3 Q_{ij}}{dt^3} \frac{d^3 Q_{ij}}{dt^3} - \frac{1}{3} \frac{d^3 Q_{ii}}{dt^3} \frac{d^3 Q_{jj}}{dt^3} \right)
\]

where $Q_{ij} \equiv \sum_a m^a x^i_a x^j_a$

Approach 1. Exact Parabolic orbit (EPO)
  Newtonian orbit
Approach 2. PN corrected orbit (PNCO)
  PN equation of motion
Post-Newtonian (PN)

- Using **Exact Parabolic Orbit (EPO)**
  - Based on Peters (1964) - GW Radiation from elliptic orbit
  - Push the limit to parabolic orbit: \( e \to 1 \)
    
    \[
    \Delta E = \frac{170\pi G^7}{3} \frac{m_1^9 m_2^9}{c^5 (m_1 + m_2)^3} \frac{1}{L^7}
    \]
  - Neglecting the orbit changes
  - Weak encounter
Post-Newtonian (PN)

- Using PN Corrected Orbit (PNCO)
  - Gradually changing orbit
  - PN equation of motion
    \[ a = a_N + \frac{1}{c^2} a_{1PN} + \frac{1}{c^3} a_{1.5PN, SO} + \frac{1}{c^4} (a_{2PN} + a_{2PN, SS}) + \frac{1}{c^5} (a_{2.5PN} + a_{2.5PN, SO}) + \frac{1}{c^6} a_{3PN} + \frac{1}{c^7} a_{3.5PN} \]

- Two body code
- Mass quadrupole formula
Numerical Relativity (NR)

- Numerical solution of Einstein field Equations
- Most accurate method
- Cover all the phases of BH merging
- Expensive

- 3+1 decomposition (ADM formalism, Arnowitt et al. 1962)
  - 4 dimensional spacetime equation → 3 dimensional space + 1 dimensional time
  - Constraint equations
    - Hamiltonian (1) & Momentum (3)
  - Evolution equations (6)
  - Gauge choice (4)
Gauge choice

- Gauge (Coordinate) choice
  - Lapse ($\alpha$) & Shift ($\beta^i$)

- Geodesic slicing ($\alpha = 1, \beta^i = 0$)
  - Singularity

- Maximal slicing ($K = 0$)
  - Singularity avoidance

- $1 + \log$ slicing ($\alpha = 1 + \ln \gamma$)
  - Easy to implement and fast to solve
  - Singularity avoidance
Avoiding Singularity

- Singularity avoiding coordinates
  - Postpone the singularity until $t = \infty$
  - Grid stretching problem along black hole throat
  - Hard to identify singularity avoiding coordinates that lead long & stable evolution for orbiting binaries

- Black hole excision (Pretorius 2005)
  - Cut out the inside of the horizon

- Moving Puncture method (Baker et al. 2006, Campanelli et al. 2006)
  - Avoid the singularity through the intelligent choice of gauge
  - $1+\log$ slicing, Gamma-driver shift
Weyl Scalar

- Weyl Scalar
  - Obtained by contracting Weyl tensor with complex null tetrad
  - The quantities related with the gravitational waves at infinity
  - $\Psi_4 = \ddot{h}_+ - i\dot{h}_\times$
  - Radiated energy

$$\frac{dE}{dt} = - \lim_{r \to \infty} \frac{r^2}{16\pi} \int d\Omega \left| \int_{-\infty}^{t} dt' \Psi_4 \right|^2$$
ADM mass & Geometrized unit

- ADM mass (energy)
  - Total mass (energy) measured on a spatial infinite surface at a certain time
  - In Newtonian limit
    \[ M_{ADM} = M_0 + T + W \]
  - Parabolic orbit: \( M_{ADM} = 1 \)

- Geometrized units
  - \( c = G = 1 \)
  - Time, mass, energy: \( M \)
  - Velocity: no unit
  - Orbit, spin angular momentum: \( M^2 \)
  - 10-10 M⊙ BH: unit length = 30km, unit time = 0.1ms
Software & Hardware

- **Software - Einstein Toolkit**
  - Collection of software components and tools for simulation and analysis for general relativistic phenomena
  - Free, open source ([http://einstein Toolkit.org](http://einstein Toolkit.org))
  - Based on Cactus code
    - Flesh (central core) + Thorns (application module)
    - Over 100 thorns for simulation management
  - HDF5 parallel file I/O, AMR (Adaptive Mesh Refinement), ...

- **Hardware - Tachyon2 (KISTI) & Gmunu (SNU)**
## PN vs NR

<table>
<thead>
<tr>
<th></th>
<th>Orbit</th>
<th>Size of mass</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPO</td>
<td>Exact parabolic</td>
<td>Point</td>
<td>No</td>
</tr>
<tr>
<td>PNCO</td>
<td>Initially parabolic</td>
<td>Point</td>
<td>Yes</td>
</tr>
<tr>
<td>NR</td>
<td>Initially parabolic</td>
<td>Extended</td>
<td>Yes</td>
</tr>
</tbody>
</table>

\[ L_{\text{init}} = 1.1 \]
\[ m_1/m_2 = 1 \]
Results 1

Unequal mass BHs

- Effect of BH mass spectrum
Orbits & Waves
- $m_1/m_2=4$

- Orbit of heavier BH
Radiated E & L

- Left: Direct merging
- Right: fly-by

- Most GW radiations on the boundary
- Shift of peak point
- fly-by orbit
  - Can be used for parabolic approximation
  - Less energy for higher mass ratio at fixed $L_{\text{init}}$
Comparison with PN
Impact parameter

- Radiated E in parabolic orbit
  - Energy of hyperbolic orbit
- EPO (Exact Parabolic Orbit)
  \[ E = \frac{1}{2} \mu v_\infty^2 \]
  \[ b = \left( \frac{340\pi m_1 m_2 (m_1 + m_2)^5}{3 v_\infty^9} \right)^{1/7} \]
- Difficult to capture the high energy BHs → small impact parameter
- Upper limit of \( v_\infty \) for fixed impact parameter
- Gentle slope in high energy
Impact parameter

- Impact parameters divided by EPO's
- Deviations in high velocity for both PNCO and NR
- EPO: $v_\infty \lesssim 0.01$
- PNCO: $0.01 \lesssim v_\infty \lesssim 0.1$
- NR: $0.1 \lesssim v_\infty$
- Conspicuous for higher mass ratio
Results 2

Spinning BHs

- Frame dragging effect
Spin Direction

- More winding orbit for anti-aligned spins → shorter pericenter distance
- Loosely bound orbit for aligned spins → longer pericenter distance
Spin Direction

- Dependence on spin direction
  - $m_1/m_2=1$, $L_{\text{init}}=1.1$
  - Fixed spin magnitude: $S/m^2=0.5$
  - Various spin combinations with x, y and z direction
  - Radiated E & L depend on $D = \mathbf{L} \cdot (\hat{S}_1 + \hat{S}_2)$
  - PNCO is not good for this case.
Radiated E & L

- For aligned and anti-aligned spin directions
- Spin magnitudes: $S/m^2=0.9$
- More GW radiation for anti-aligned BHs at fixed $L_{\text{init}}$
- More GW radiation for aligned BHs at the peak
- $\sim10\%$ of energy radiation
Orbits & Waves at the peak

- Anti-aligned BHs have
  - More whirly orbit before merging
  - Longer duration of ringdown phase
Impact parameter

- Different slopes for different spin directions in high energy
- More gentle slope for anti-aligned spins
- Higher speed BH capture for anti-aligned spins
- Larger impact parameters than EPO
Impact parameter

- Larger deviation from EPO for anti-aligned spin
- Larger than 10% deviations from non-spinning at \( \nu_\infty = 0.2 \)
Discussions
Validity of Parabolic approximation

- True hyperbolic orbit is different from the parabolic orbit.
- True hyperbolic orbit with PN
  - Exact Hyperbolic Orbit (EHO)
    - GW radiations from hyperbolic orbit (Hansen 1972)
    - Orbital energy = Radiated E
  - PNCO - hyperbolic
    - PN equation of motion
    - Repetition of the initial conditions
Validity of Parabolic approximation

- Parabolic approximation
  - Valid within 5% up to $v_\infty = 0.2$.
  - Can make the impact parameter to be underestimated in high energy
Conclusion & Summary

- The most energetic encounter
  - Around the boundary between direct merging and fly-by orbit
  - A few percent of total ADM energy
- More GW radiations
  - Equal mass and anti-aligned spins at the same $L_{init}$
  - Equal mass and aligned spins at the peak point
- Larger impact parameters
  - Equal mass and anti-aligned spins
  - Deviate from PN in high energy
- NR is required for $\Delta E \gtrsim 10^{-3}$
  - Strong encounter with short pericenter
  - Eccentric merger
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