2016.6.17 KNAG Meeting

Density-Magnetic Field Correlation in MHD Turbulence Driven by Different Driving Schemes with Different Correlation Times

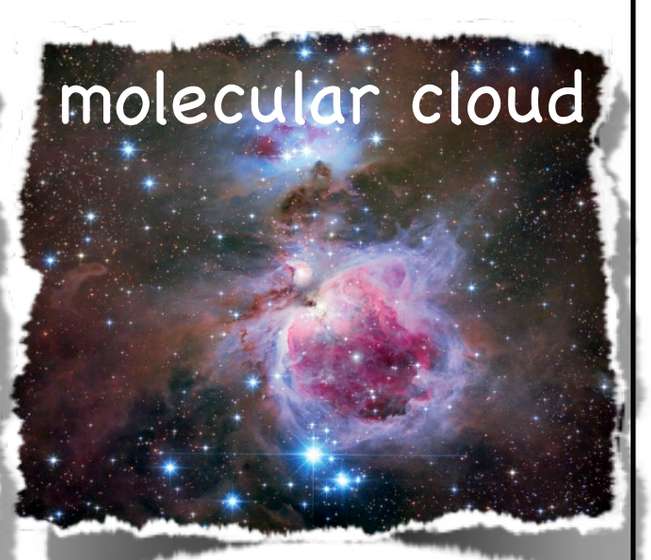
Heesun Yoon[1], Jungyeon Cho[1], and Jongsoo Kim[2,3]

1 Chungnam National University (CNU)
2 Korea Astronomy and Space Science Institute (KASI)
3 University of Science and Technology (UST)

Outlines

1	Introduction	
2	Motivation	
3	Simulation Conditions	
4	Results	Result 1) Driving Schemes Result 2) sonic Mach Numbers (Ms) Result 3) Magnetic Field Strength Result 4) Decaying Turbulence Result 5) Magnetic Prandtl Number (Prm)
5	Discussion	Discussion 1) standard deviation of RM Discussion 2) Comparison of RM/DM & True $B_{\parallel}$
6	Summary	

Introduction



Turbulent & Magnetized

- **Information of Magnetic Field**
  - Zeeman Effect
  - Starlight Polarization
  - Thermal FIR/sub-mm Emission
  - Synchrotron Emission
  - **Faraday Rotation Measure (RM)**

Importance of ρ&B Correlation

- **Estimate average magnetic field along the line of sight**

$$\langle B_{\parallel} \rangle \sim \frac{\int n_e B_{\parallel} ds}{\int n_e ds} = \frac{RM}{DM}$$

where n_e : electron density
$B_{\parallel}$: parallel magnetic field
RM : the rotation measure
DM : the dispersion measure
ds : line of sight

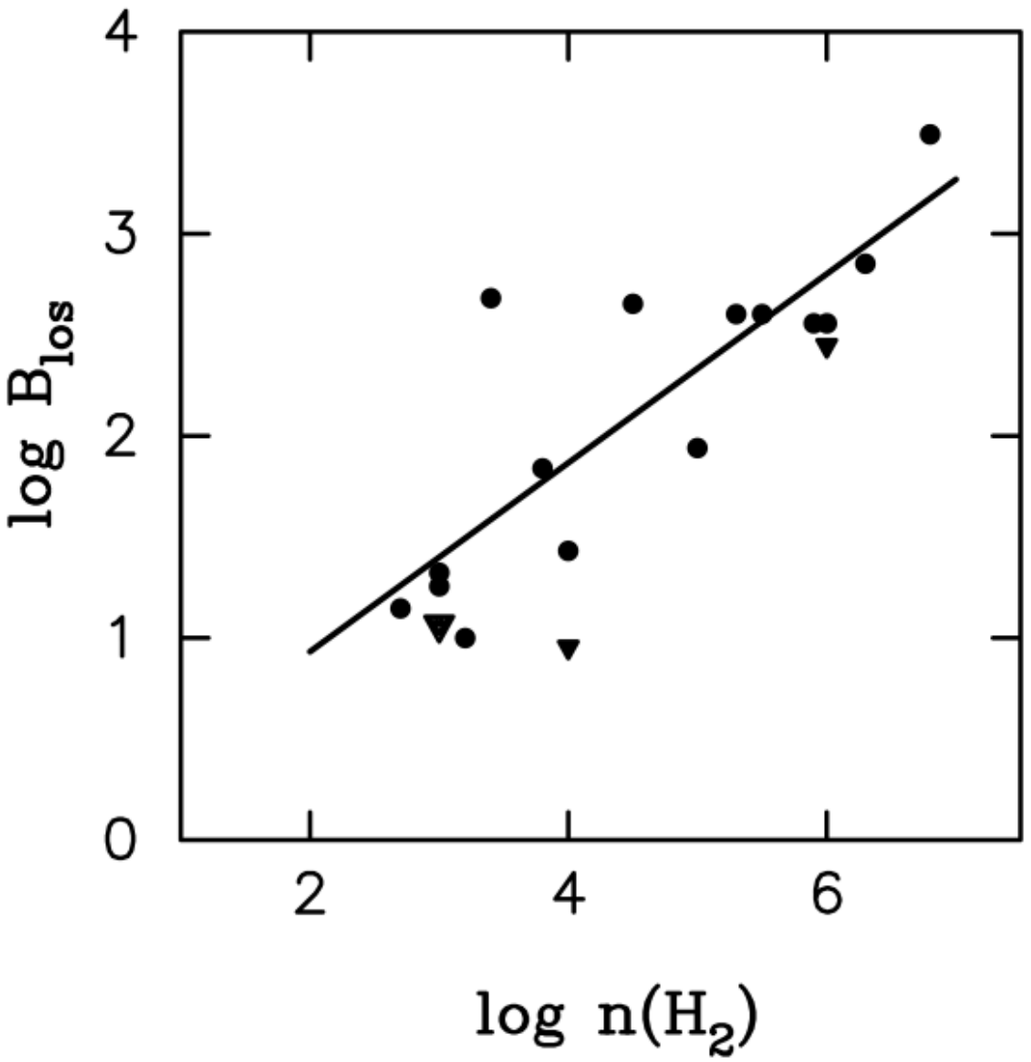
- **ρ(~n_e) & B Correlation**

- Un correlation : True <$B_{\parallel}$>
- Positive Correlation : Overestimate the <$B_{\parallel}$>
- Negative correlation : Underestimate the <$B_{\parallel}$>

Beck et al. 2003

ρ & B Relations in Previous Study

Observation Study (Crutcher 1999)



$|B| \propto n^{\kappa}$

➤ $\kappa = 0.47 \pm 0.08$

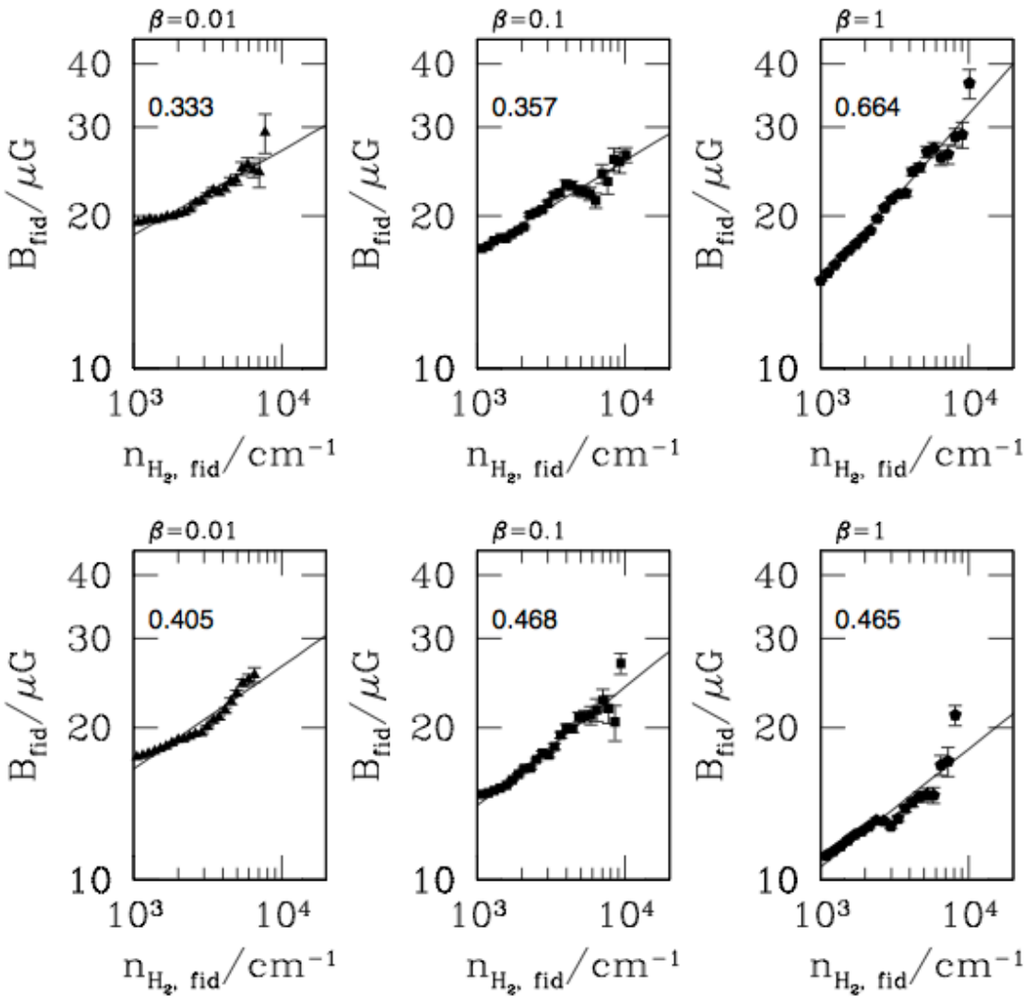
MOLECULAR CLOUD DATA

Cloud	B_{los} (μG)	log N (H_2 cm^{-2})	log n (H_2 cm^{-3})	ΔV (km s^{-1})	T_K	R (parsec)
W3 OH.....	3100	23.7	6.8	1.5	100	0.02
DR 21 OH1.....	710	23.6	6.3	2.3	50	0.05
Sgr B2	480	23.4	3.4	15	70	22
M17 SW.....	450	23.1	4.5	4.0	50	1.0
W3 (main).....	400	23.2	5.5	3	60	0.12
S106	400	22.8	5.3	1.6	30	0.07
DR 21 OH2.....	360	23.3	6.0	2.3	50	0.05
OMC-1	360	23.2	5.9	1.4	100	0.05
NGC 2024	87	22.9	5.0	1.5	25	0.2
S88 B	69	22.3	3.8	3.0	40	0.7
B1	27	21.9	4.0	1.0	12	0.2
W49 B	21	21.6	3	1.5	10	1
W22.....	18	22.2	3	0.9	10	4
W40.....	14	22.0	2.7	2.6	10	5
ρ Oph 1	10	21.7	3.2	1.4	25	0.8
OMCN-4.....	<280	23.1	6	1.4	35	0.03
Tau G	<16	21.6	3	1.3	10	1
L183	<16	21.2	3.1	0.6	10	0.3
L1647.....	<15	22.1	3	2.3	10	3
ρ Oph 2	<14	21.6	3	1.3	25	0.9
TMC-1	<13	21.9	3	1.5	10	1.9
L1495 W	<13	21.6	3	1.0	10	0.9
L134	<11	21.3	3.2	0.7	10	0.3
TMC-1C	<9	21.9	4.0	0.5	10	0.2
L1521.....	<9	21.7	3	1.0	10	1.2
L889	<7	22.0	3	2.1	13	2.4
Tau 16.....	<7	21.7	3	1.0	10	1.2

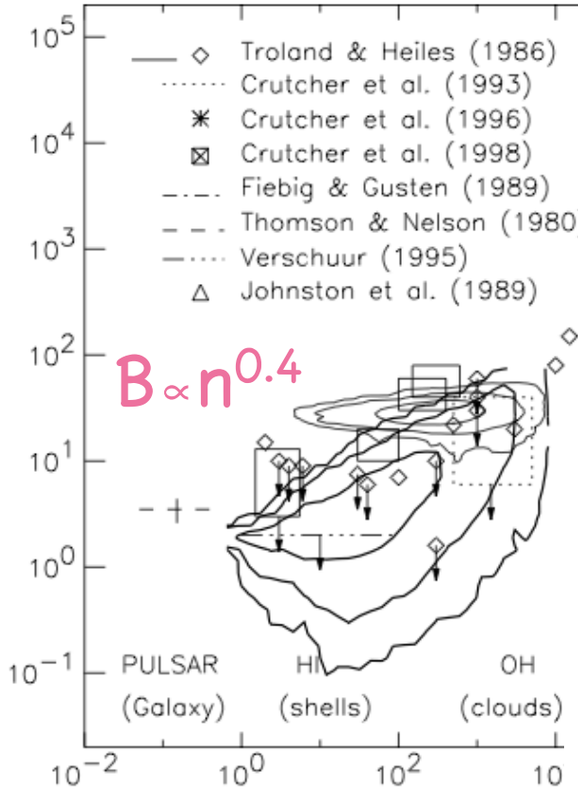
- Zeeman measurements of magnetic field strength in MC
- Evidence that magnetic field is important

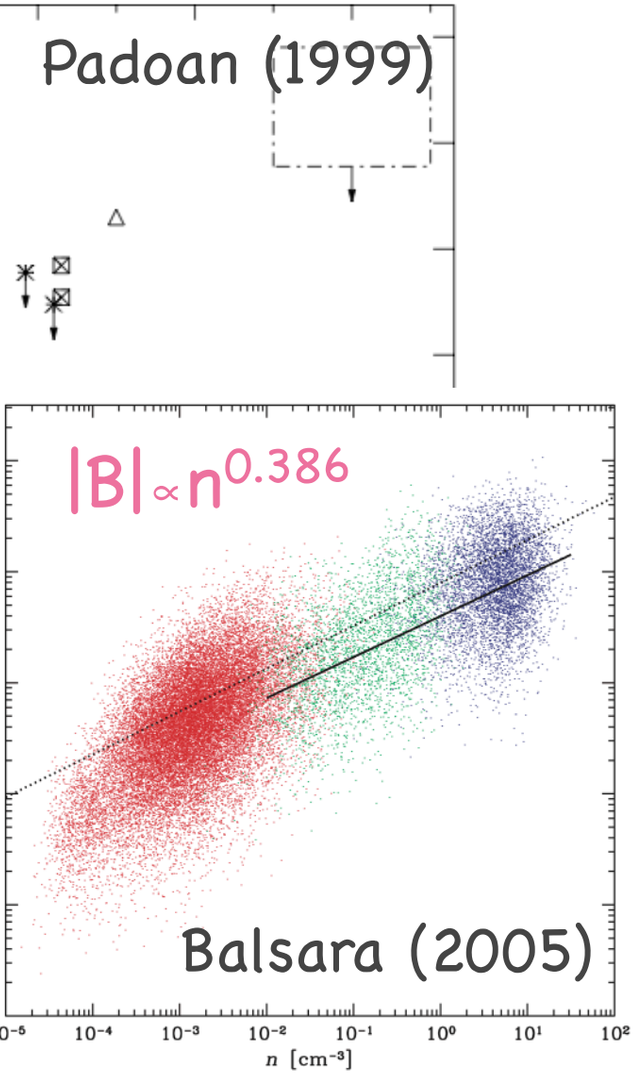
ρ & B Relations in Previous Study

Numerical Study



Ostriker (2001)

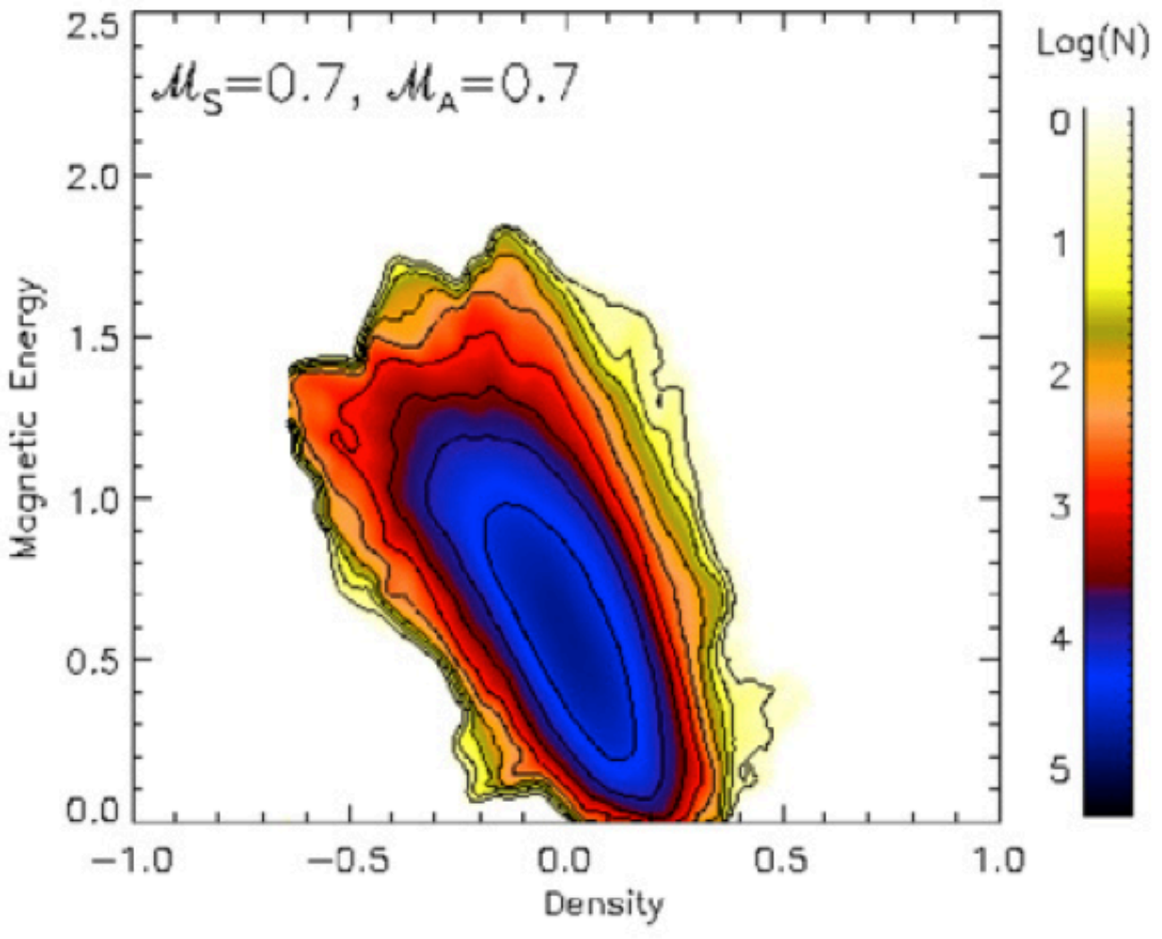




- solve the MHD equation in 3D
- Molecular Clouds
- B-n relations similar to that of Crutcher (1999)

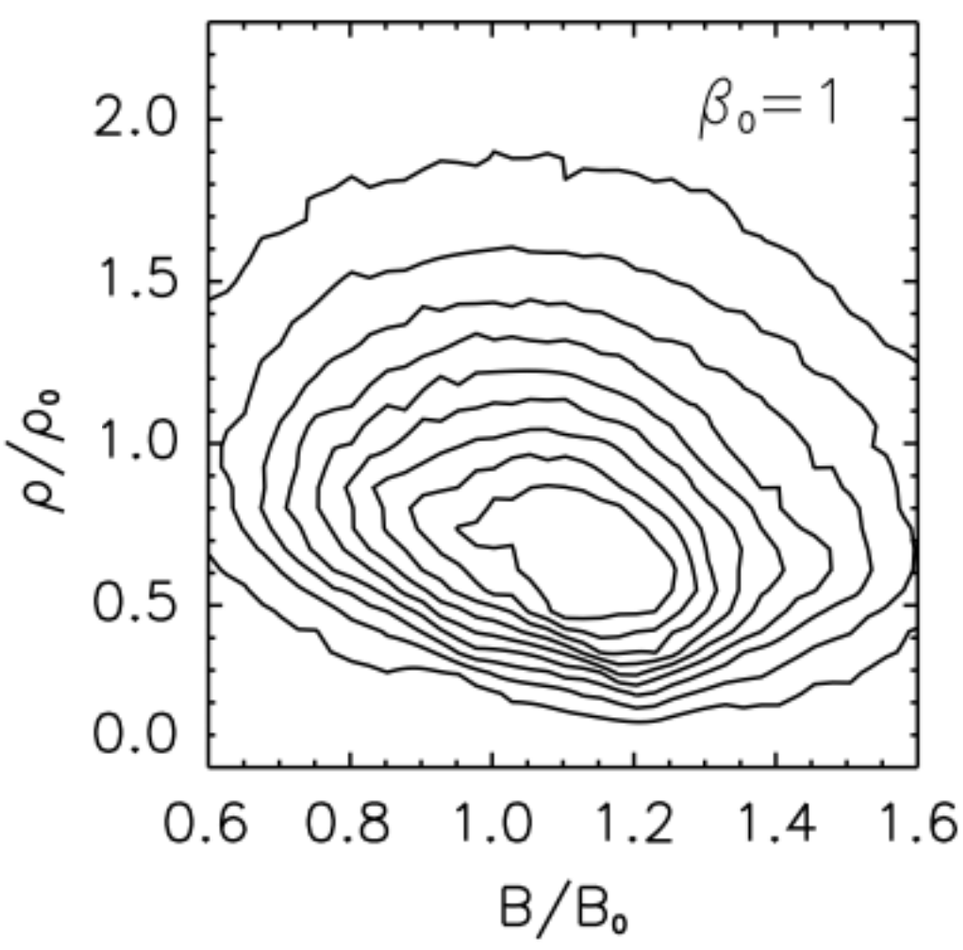
Motivation

Burkhart et al. 2009



- **strong anti-correlation** between ρ & B^2

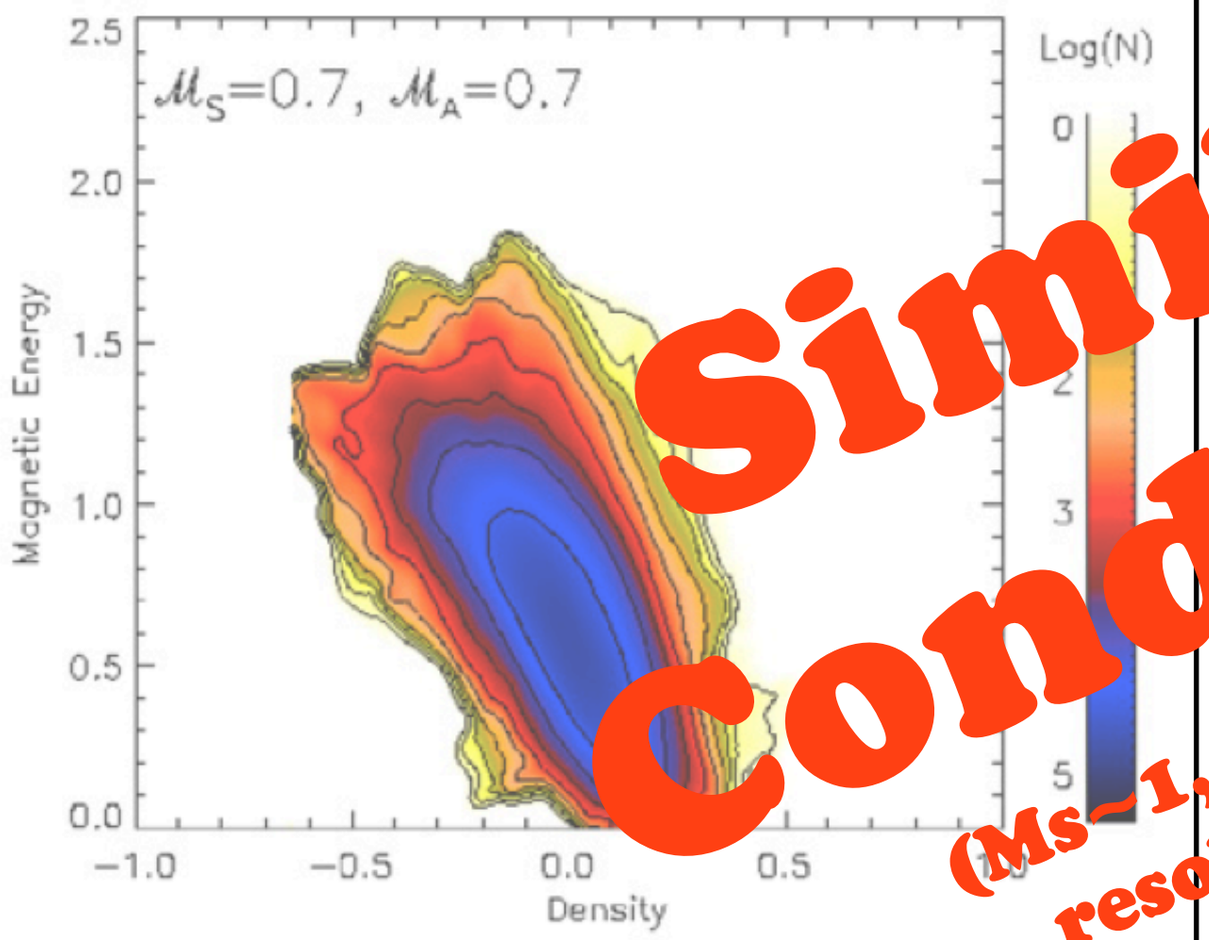
Wu et al. 2009



- **no strong correlation** between ρ & B

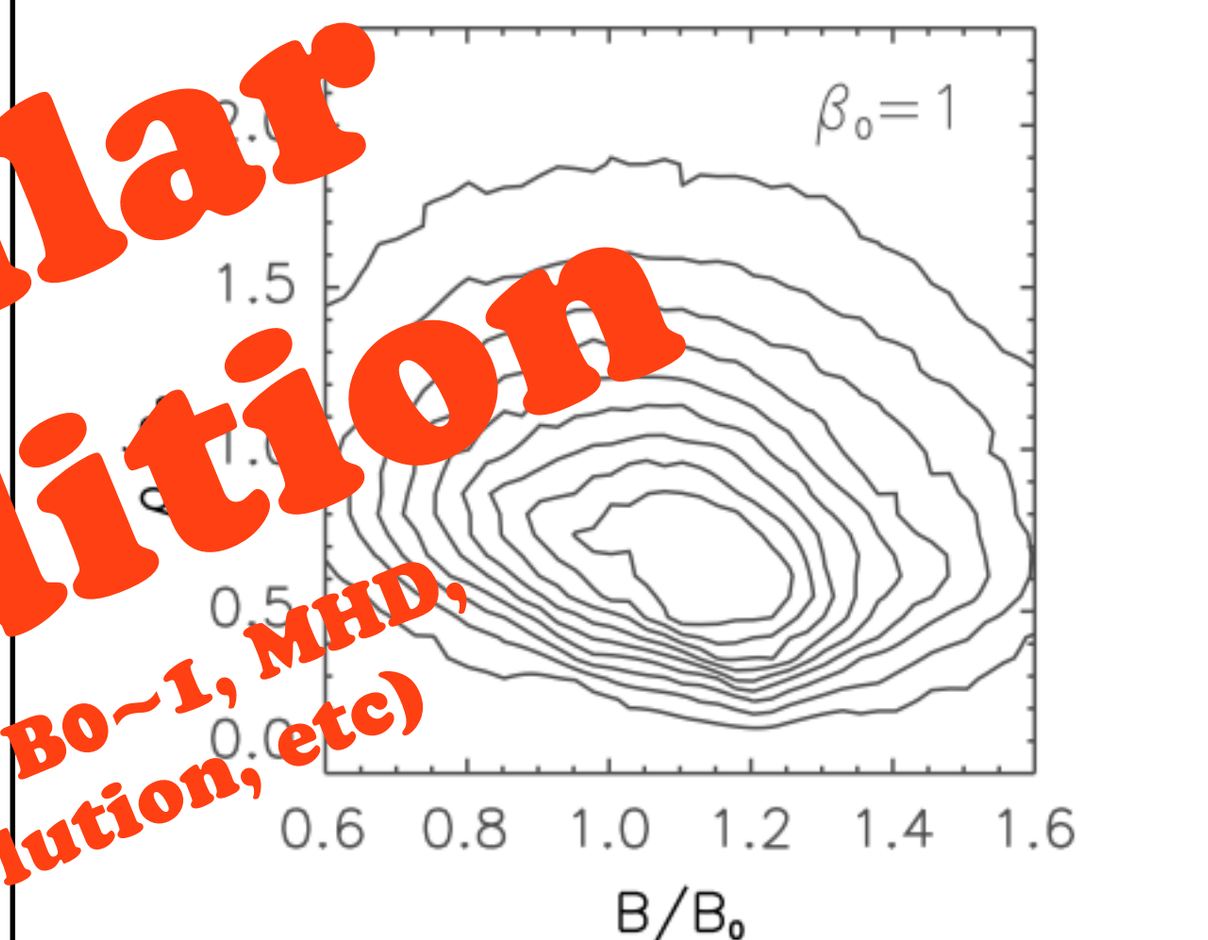
Motivation

Burkhart et al. 2009



- strong anti-correlation
  between ρ & B^2

Wu et al. 2009

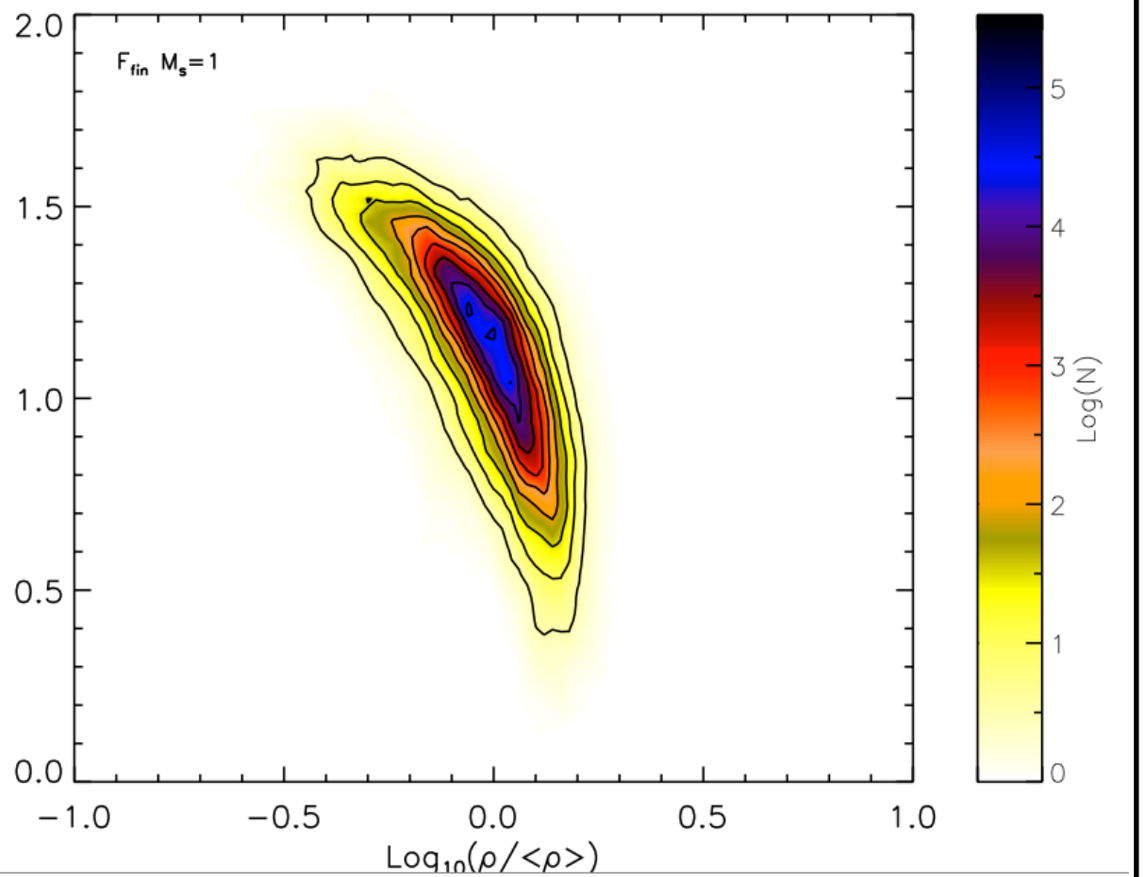


- no strong correlation
  between ρ & B

Similar condition
(Ms~1, B0~1, MHD, resolution, etc)

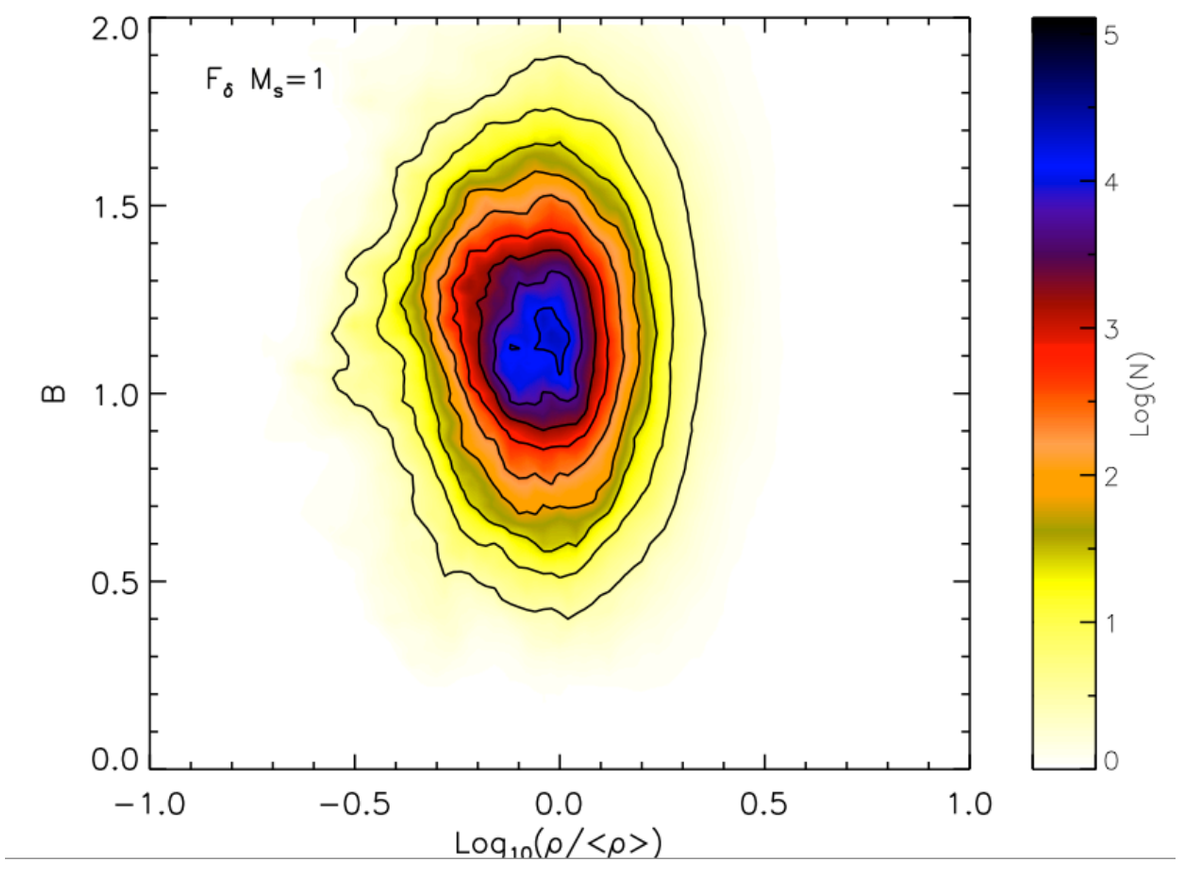
Motivation

Burkhart et al. 2009



- **strong anti-correlation** between ρ & B

Wu et al. 2009



- **no strong correlation** between ρ & B

Simulation Conditions

- solve the MHD equations in a periodic box of size 1

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \mathbf{0}, \quad (1)$$

$$\rho(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}) + \mathbf{c_s^2} \nabla \rho - (\nabla \times \mathbf{B}) \times \mathbf{B} = \rho \mathbf{f}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \mathbf{0}, \quad (3)$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}, \quad (4)$$

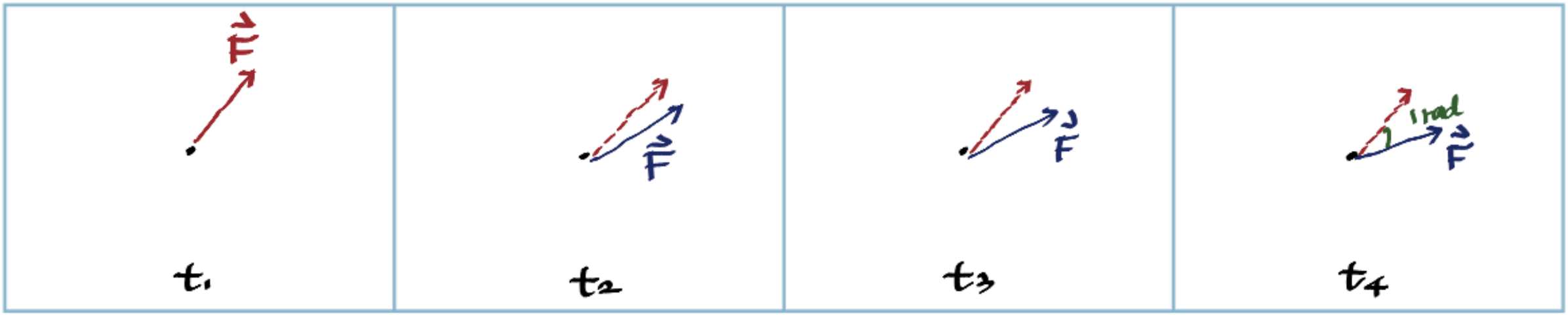
TABLE 1
SIMULATIONS

Run	Code[a]	Driving[b]	M_A [c]	M_s [d]	Resolution	r[e]
F1	ENO	$F_{\rm fin}$	0.7	0.7	512^3	-0.798
F2	ENO	$F_{\rm fin}$	7.0	0.7	256^3	-0.494
F3	TVD	$F_{\rm fin}$	1.0	1.0	512^3	-0.716
F4	TVD	$F_{\rm fin}$	1.0	4.0	512^3	-0.039
F5	TVD	$F_{\rm fin}$	1.0	8.0	512^3	0.111
F6	TVD	$F_{\rm fin}$	10.0	1.0	512^3	-0.444
F7	TVD	$F_{\rm fin}$	10.0	1.0	256^3	-0.315
F8	TVD	$F_{\rm fin}$	1000.0	1.0	256^3	-0.021
F9[f]	ENO	$F_{\rm fin}$	7.0	1.0	256^3	-0.730
D1	ENO	F_δ	1.0	1.0	512^3	-0.073
D2	ENO	F_δ	10.0	1.0	256^3	0.209
D3	TVD	F_δ	1.0	1.0	512^3	-0.015
D4	TVD	F_δ	1.0	4.0	512^3	0.166
D5	TVD	F_δ	1.0	8.0	512^3	0.235
D6	TVD	F_δ	10.0	1.0	512^3	0.217
D7	TVD	F_δ	10.0	1.0	256^3	0.254
D8	TVD	F_δ	1000.0	1.0	256^3	0.353
D9[f]	ENO	F_δ	10.0	1.0	256^3	0.145

- $B0_x$ = 1.0, 0.1, and 0.001
- Ms ~ 1, 4, and 8
- 256^3 & 512^3 grid zones
- use two different codes : ENO Code (Cho & Lazarian 2002)
  TVD code (Kim et al. 1999) and
- use two different driving scheme : finite-correlated driving Scheme(F_{fin})
  and delta-correlated driving scheme(F_δ)
  (main difference : time scale of the forcing)

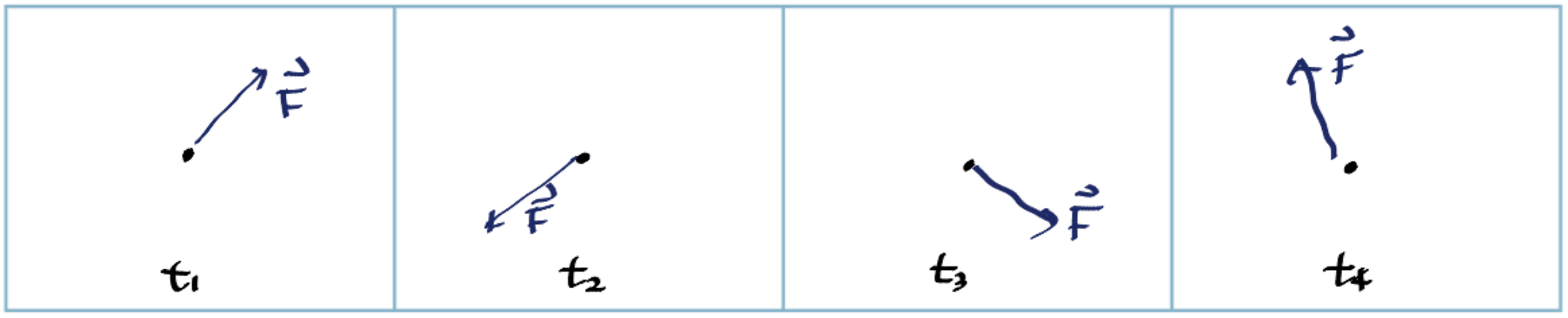
Finite- vs. Delta- correlated driving scheme

Finite-correlated driving scheme



1 correlation time ~ 1 large eddy turnover time

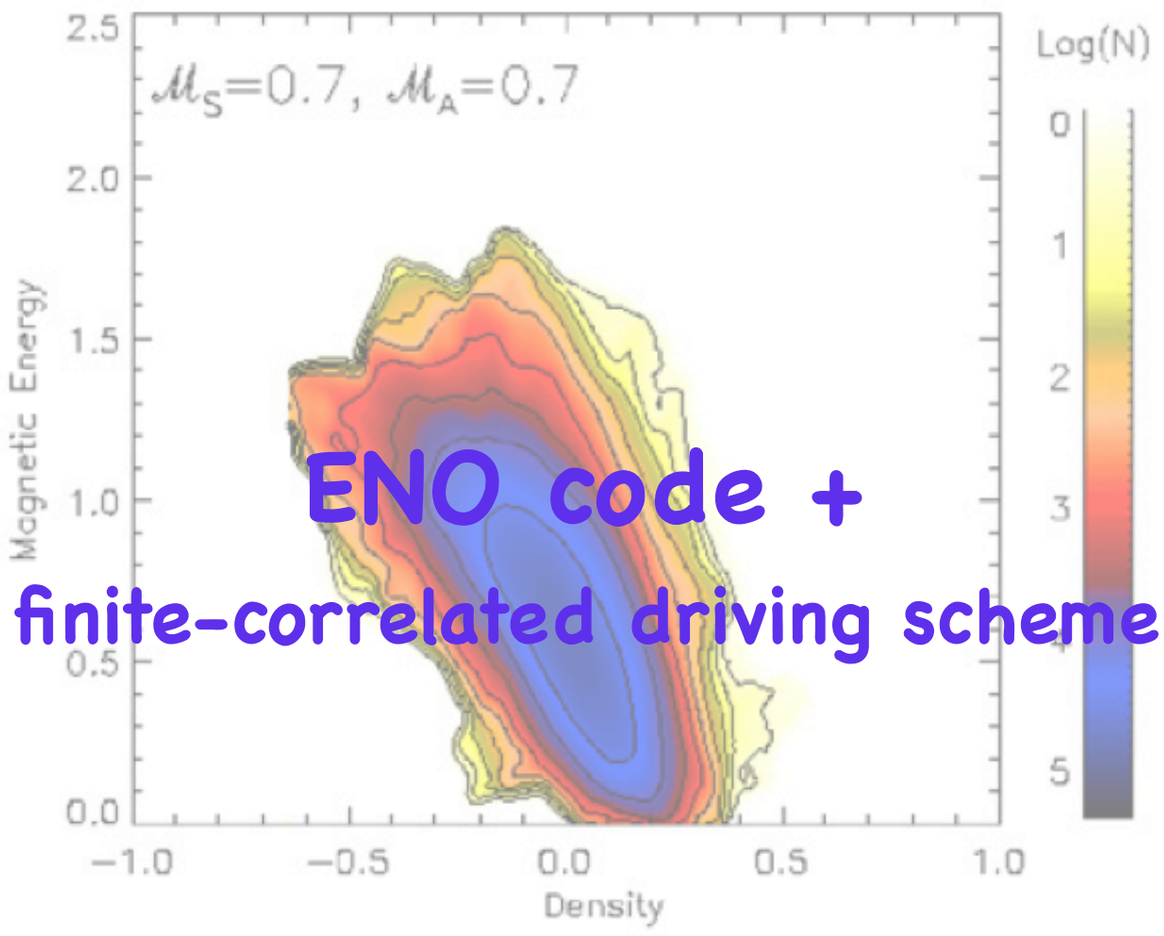
Delta-correlated driving scheme



1 correlation time ~ 0

Motivation

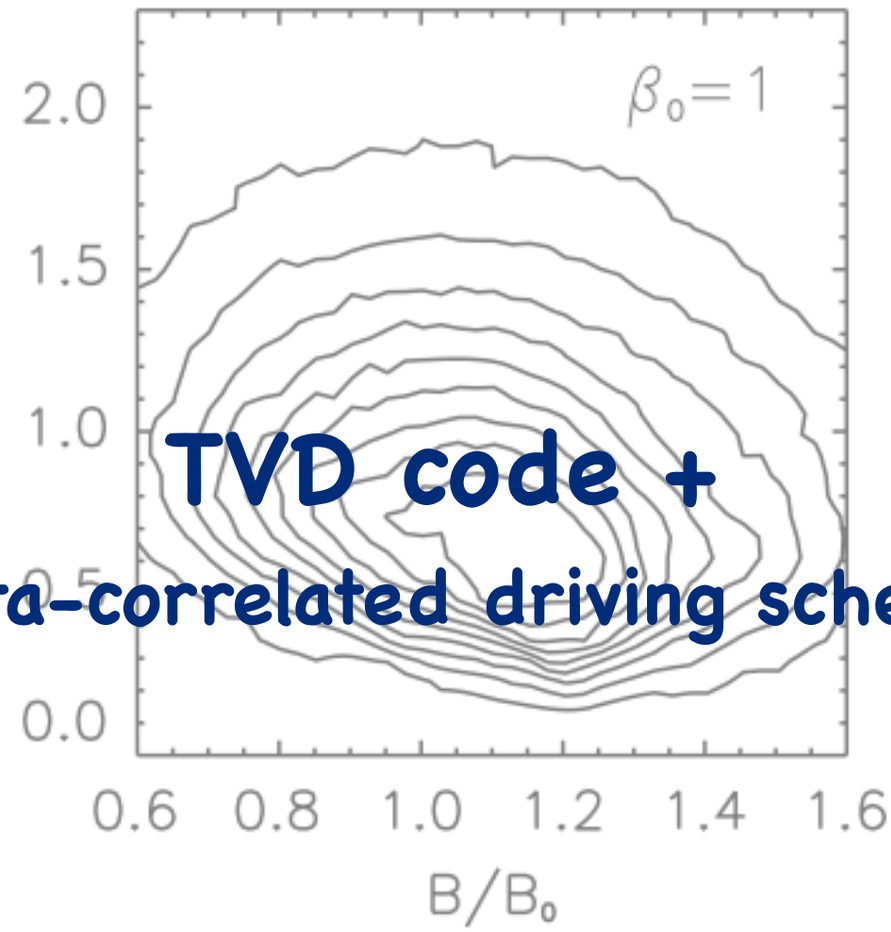
Burkhart et al. 2009



ENO code +
finite-correlated driving scheme

- **strong anti-correlation**
  between ρ & B^2

Wu et al. 2009



TVD code +
delta-correlated driving scheme

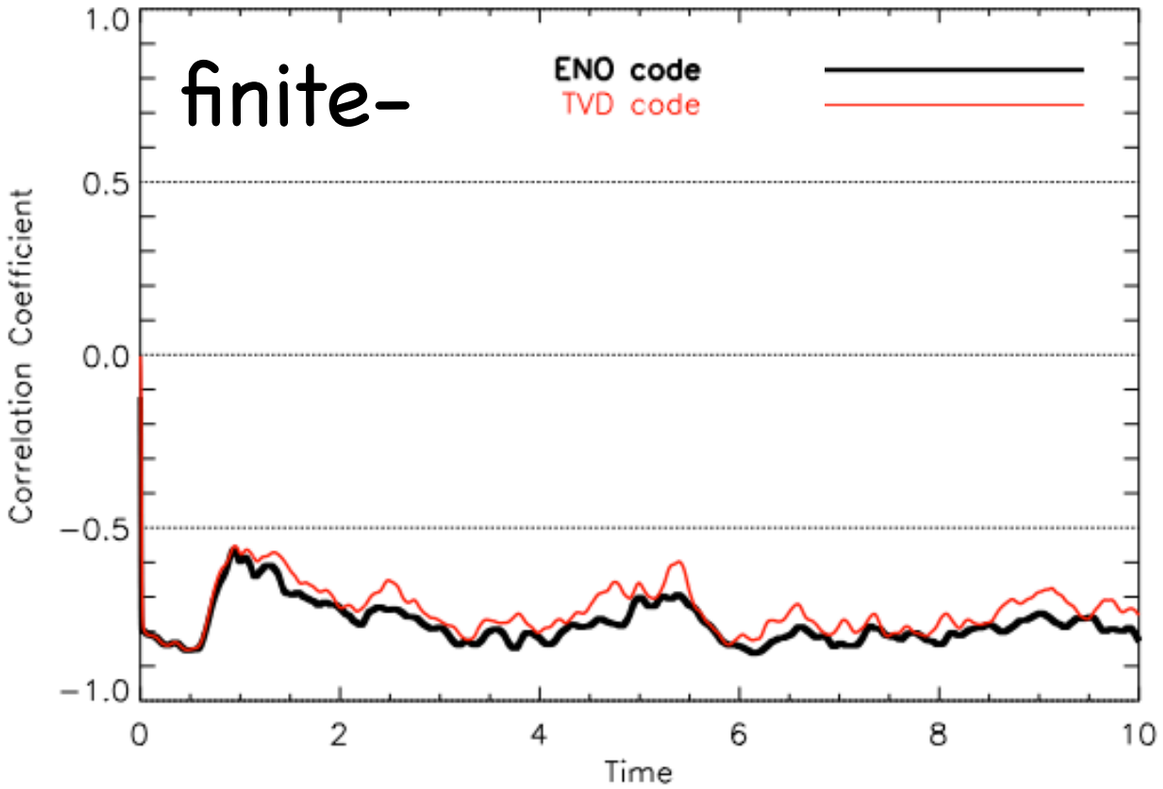
- **no strong correlation**
  between ρ & B

Result 1. Dependence on the Driving Scheme

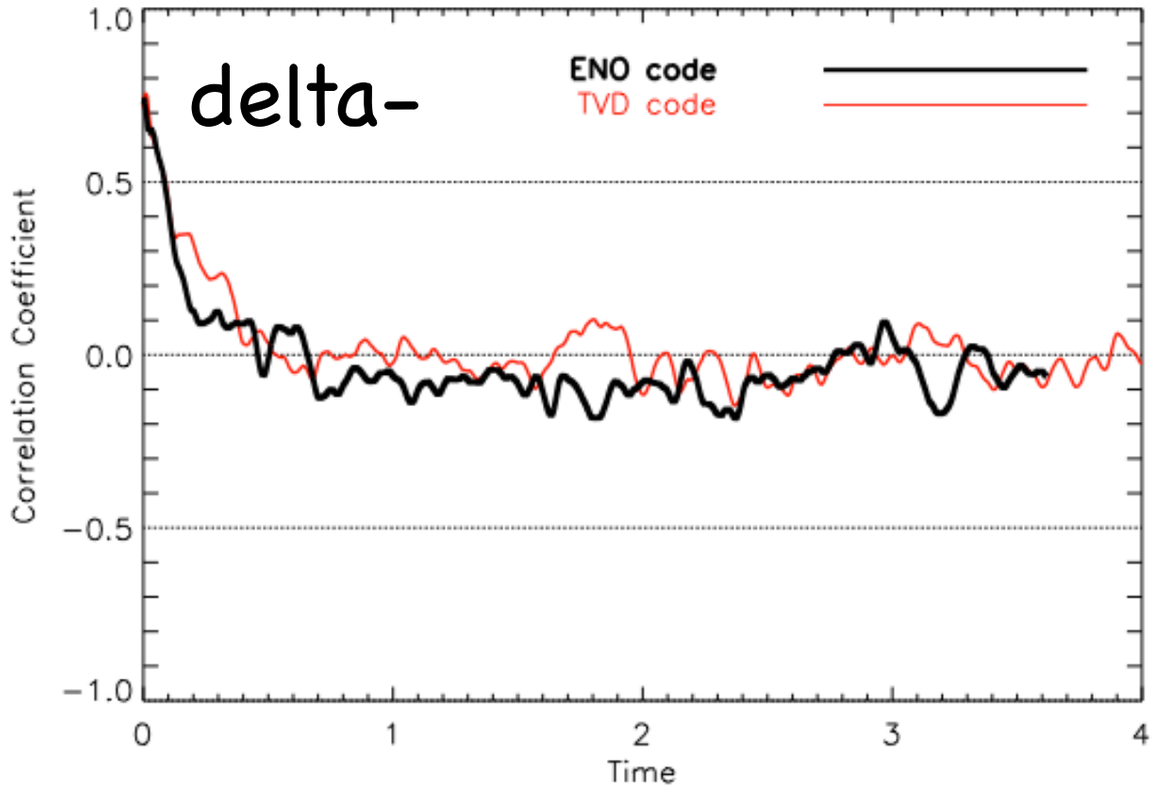
B_0 = 1 & Ms~1

use : ENO & TVD codes

submitted to ApJ (Yoon et al. 2016)



finite-correlated driving scheme



delta-correlated driving scheme

- finite-correlated driving : strong anti-correlation between ρ & B
- delta-correlated driving : no/weak correlation between ρ & B
- pressure equilibrium (Pgas + Pmag = const.)

Correlation Coefficient

$$r(\rho, B) = \frac{\sum_{i,j,k} (\rho_{i,j,k} - \langle\rho\rangle)(B_{i,j,k} - \langle B\rangle)}{\sqrt{\sum_{i,j,k} (\rho_{i,j,k} - \langle\rho\rangle)^2} \sqrt{\sum_{i,j,k} (B_{i,j,k} - \langle B\rangle)^2}},$$

ρ : Density

<ρ> : Mean Density

B: Magnetic Field Strength

<B> : Mean Magnetic Field Strength

\+ : Positive Correlation

\- : Negative (or Anti-) correlation

|r|~0 : No Correlation

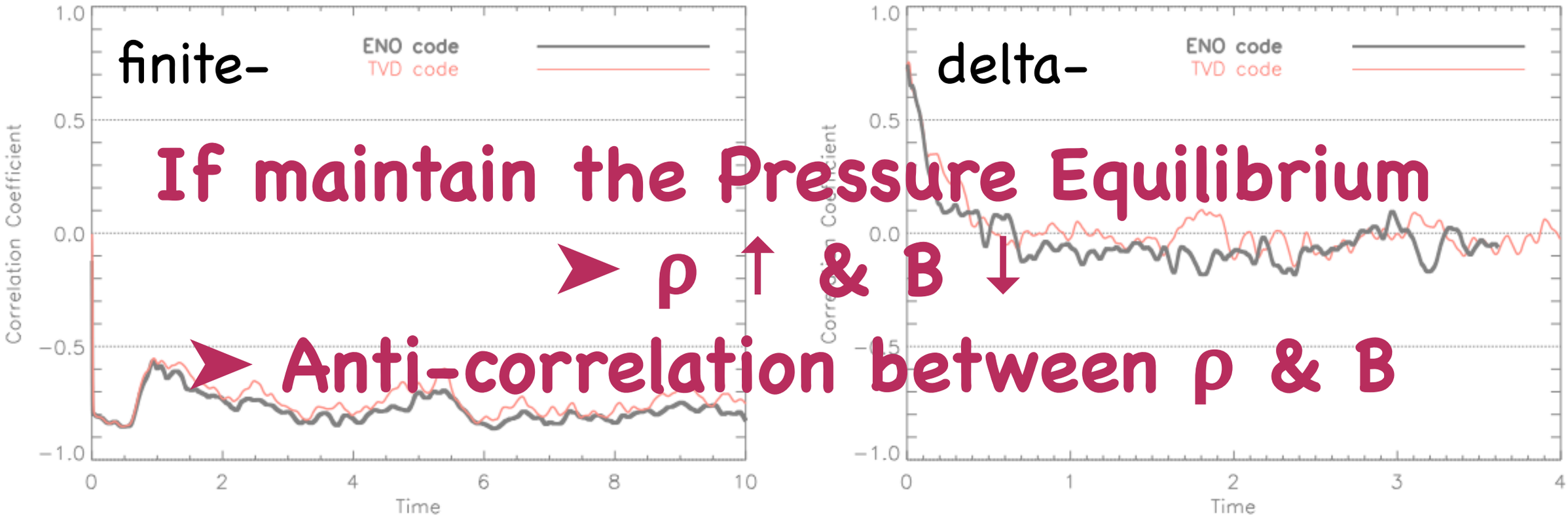
|r|~1 : Strong Correlation

Result 1. Dependence on the Driving Scheme

$B_0 = 1$ & Ms~1

use : ENO & TVD codes

submitted to ApJ (Yoon et al. 2016)



If maintain the Pressure Equilibrium

➤ ρ ↑ & B ↓

➤ Anti-correlation between ρ & B

finite-correlated driving scheme

delta-correlated driving scheme

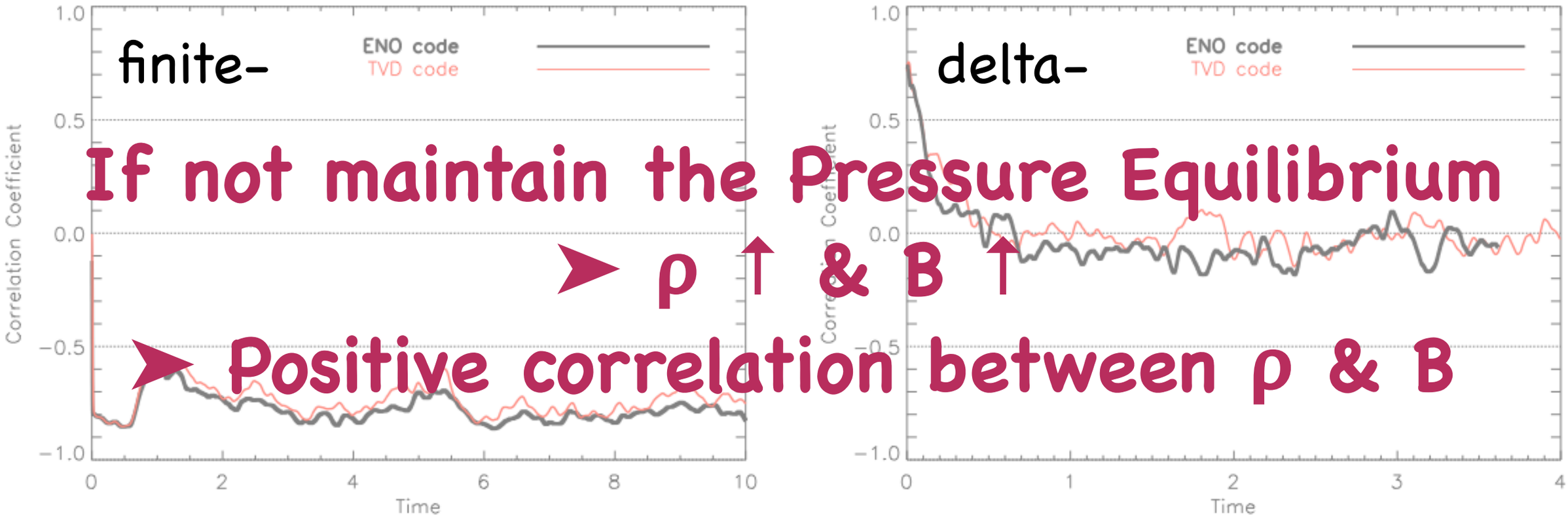
- finite-correlated driving : strong anti-correlation between ρ & B
- delta-correlated driving : no/weak correlation between ρ & B
- pressure equilibrium (Pgas + Pmag = const.)

Result 1. Dependence on the Driving Scheme

B_0 = 1 & Ms~1

use : ENO & TVD codes

submitted to ApJ (Yoon et al. 2016)



If not maintain the Pressure Equilibrium

➤ ρ ↑ & B ↑

➤ Positive correlation between ρ & B

finite-correlated driving scheme

delta-correlated driving scheme

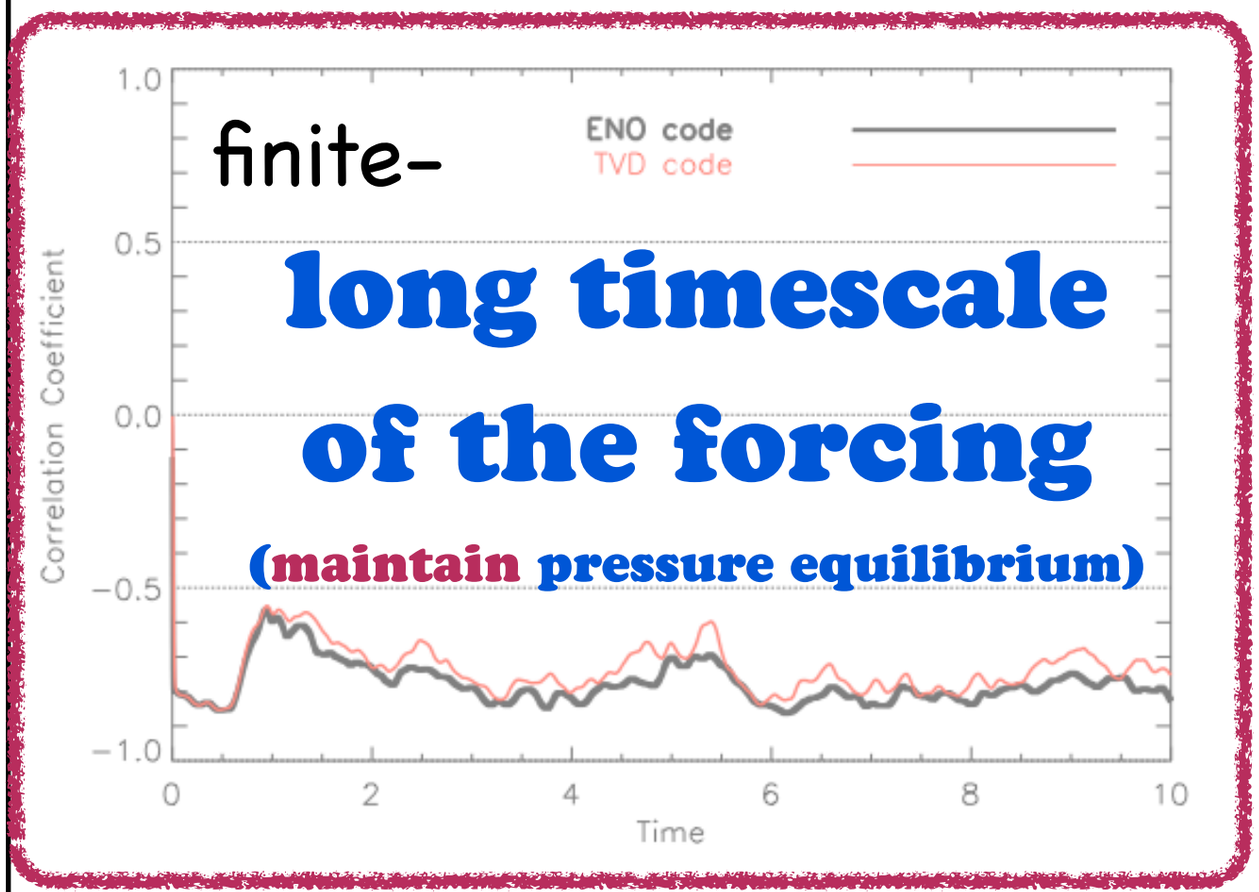
- finite-correlated driving : strong anti-correlation between ρ & B
- delta-correlated driving : no/weak correlation between ρ & B
- pressure equilibrium (Pgas + Pmag = const.)

Result 1. Dependence on the Driving Scheme

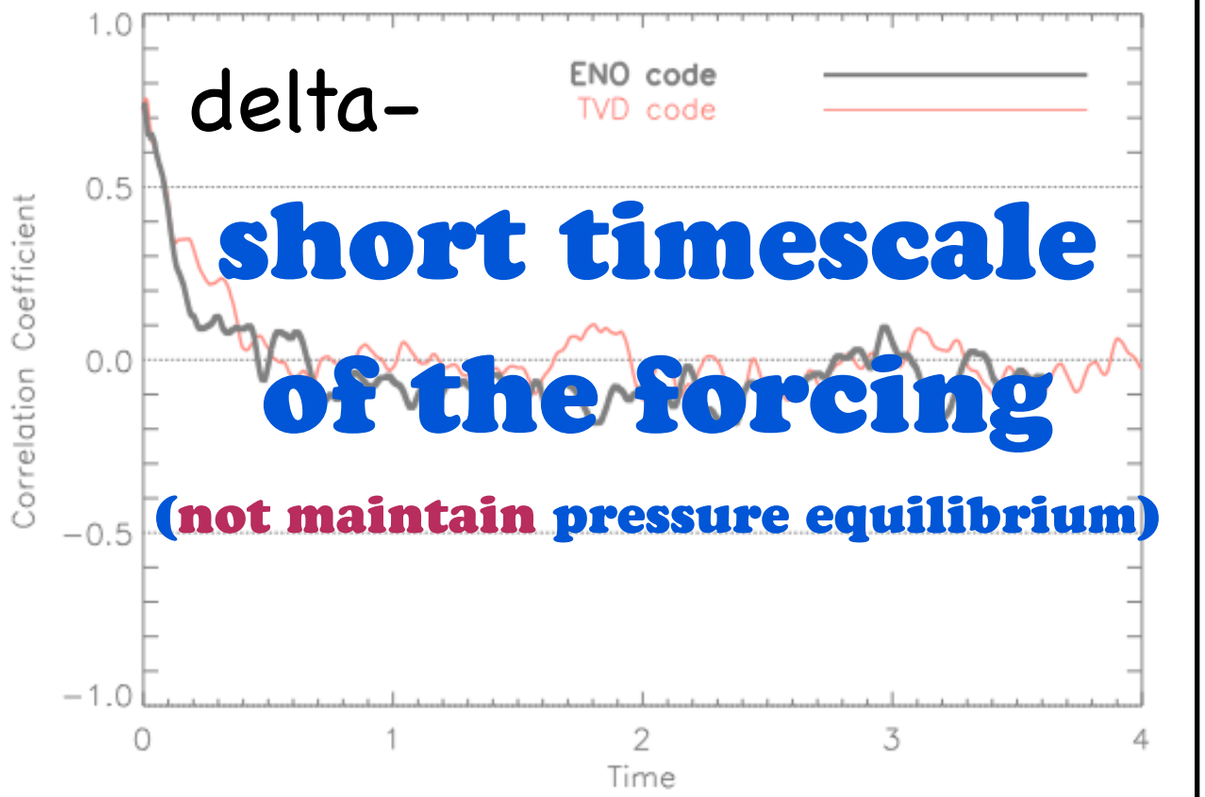
B_0 = 1 & Ms~1

use : ENO & TVD codes

submitted to ApJ (Yoon et al. 2016)



finite-correlated driving scheme



delta-correlated driving scheme

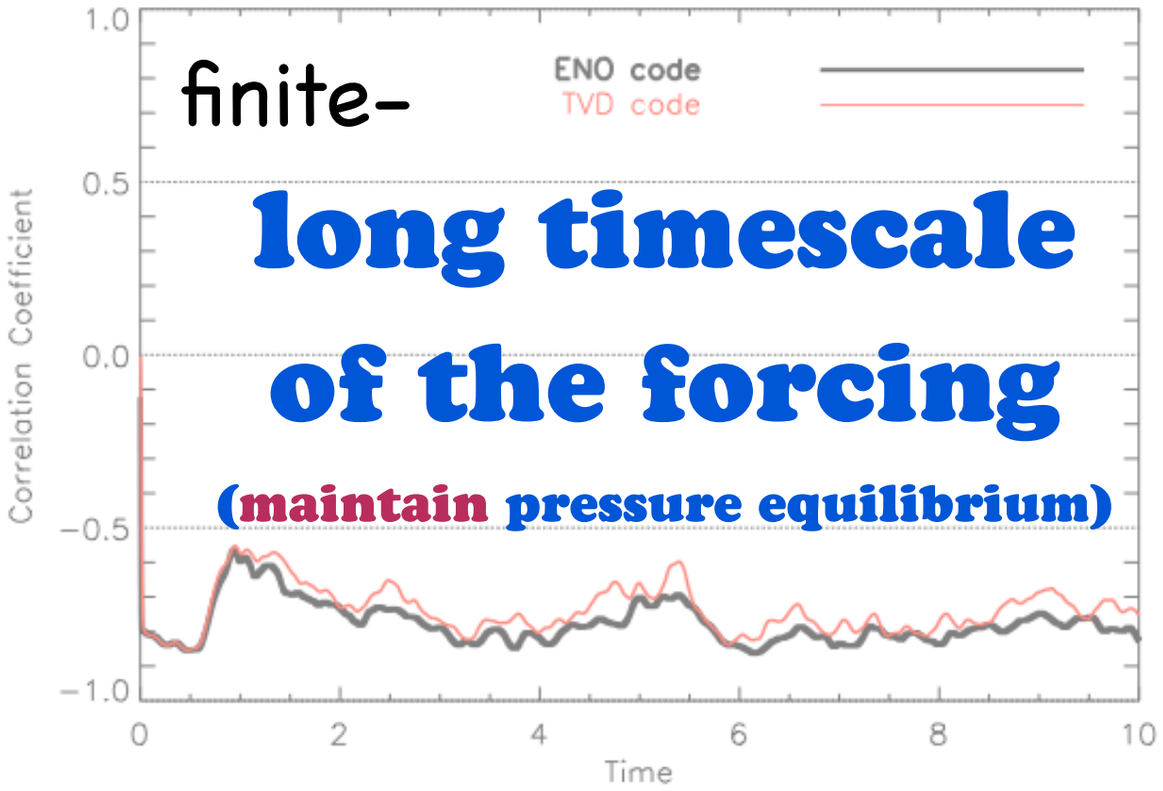
- finite-correlated driving : strong anti-correlation between ρ & B
- delta-correlated driving : no/weak correlation between ρ & B
- pressure equilibrium (Pgas + Pmag = const.)

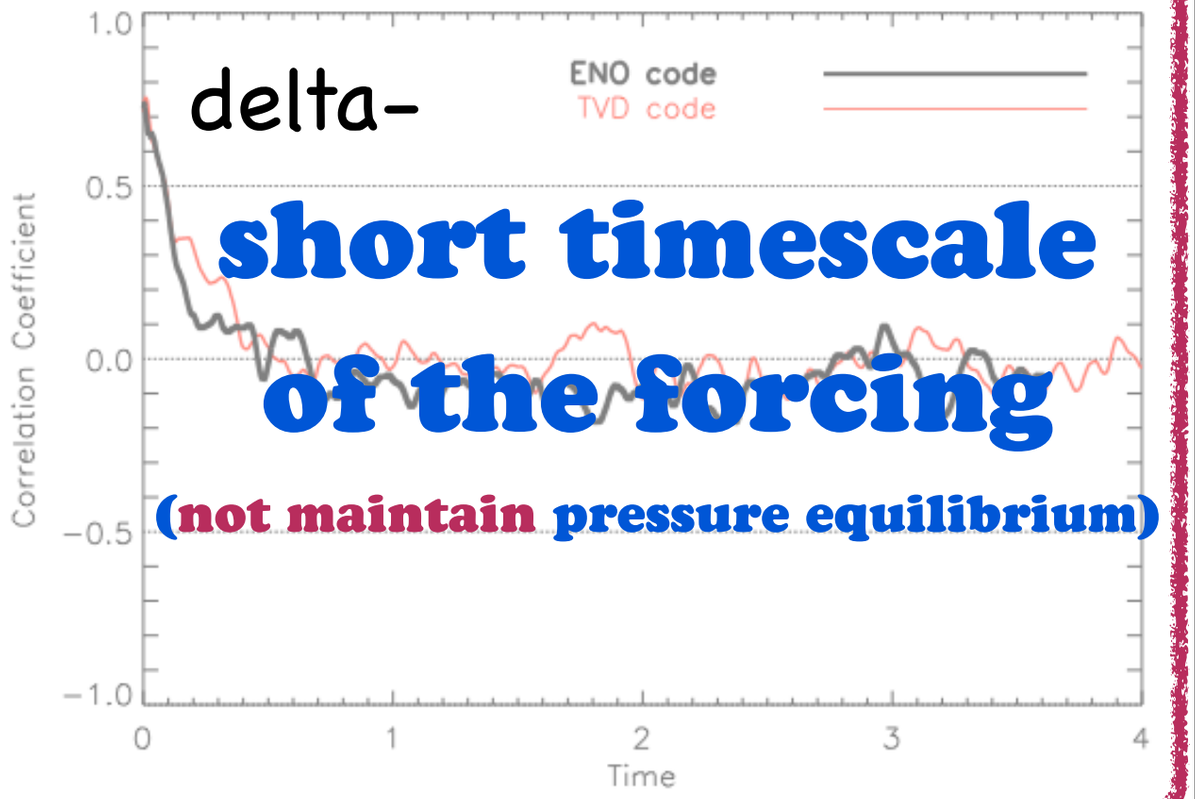
Result 1. Dependence on the Driving Scheme

B_0 = 1 & Ms~1

use : ENO & TVD codes

submitted to ApJ (Yoon et al. 2016)





finite-correlated driving scheme

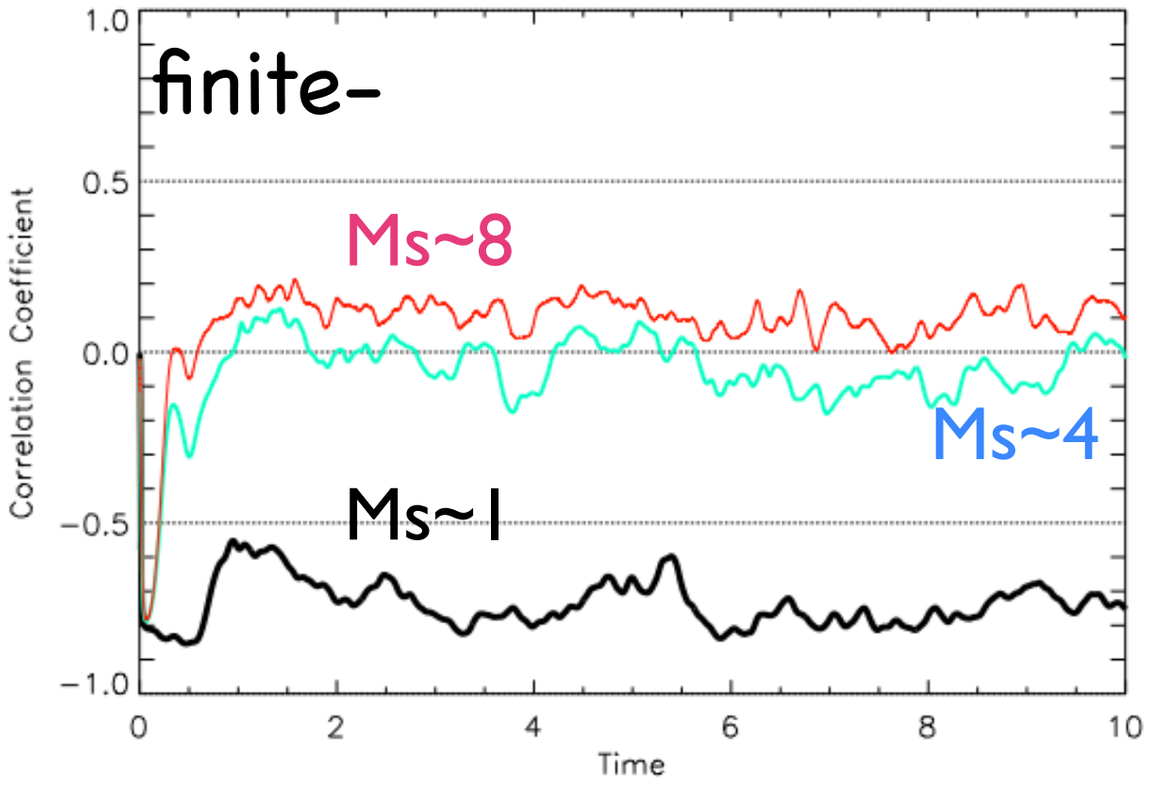
delta-correlated driving scheme

- finite-correlated driving : strong anti-correlation between ρ & B
- delta-correlated driving : no/weak correlation between ρ & B
- pressure equilibrium (Pgas + Pmag = const.)

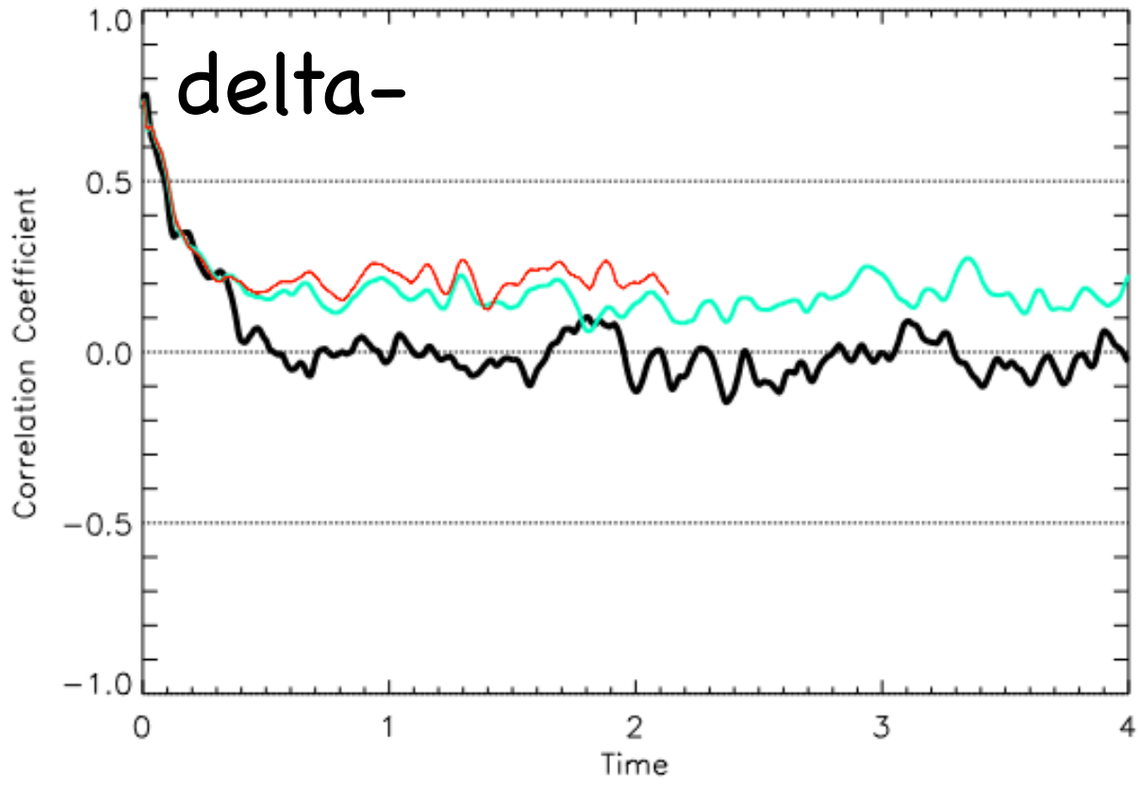
Result 2. Dependence on the Ms

B_0 = 1 & different Ms

submitted to ApJ (Yoon et al. 2016)



finite-correlated driving scheme



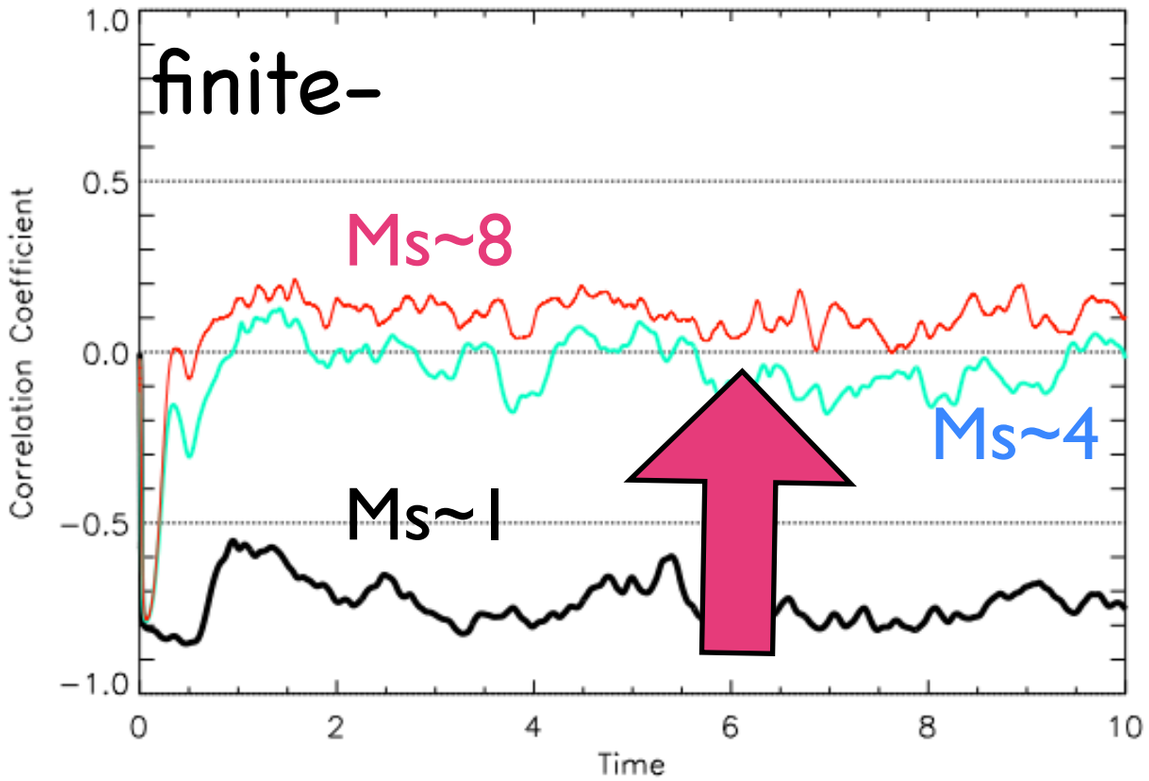
delta-correlated driving scheme

- finite-correlated driving : correlation coefficient change -1 to 0
- delta-correlated driving : very little change, slightly larger than 0
- pressure balance is not well maintained

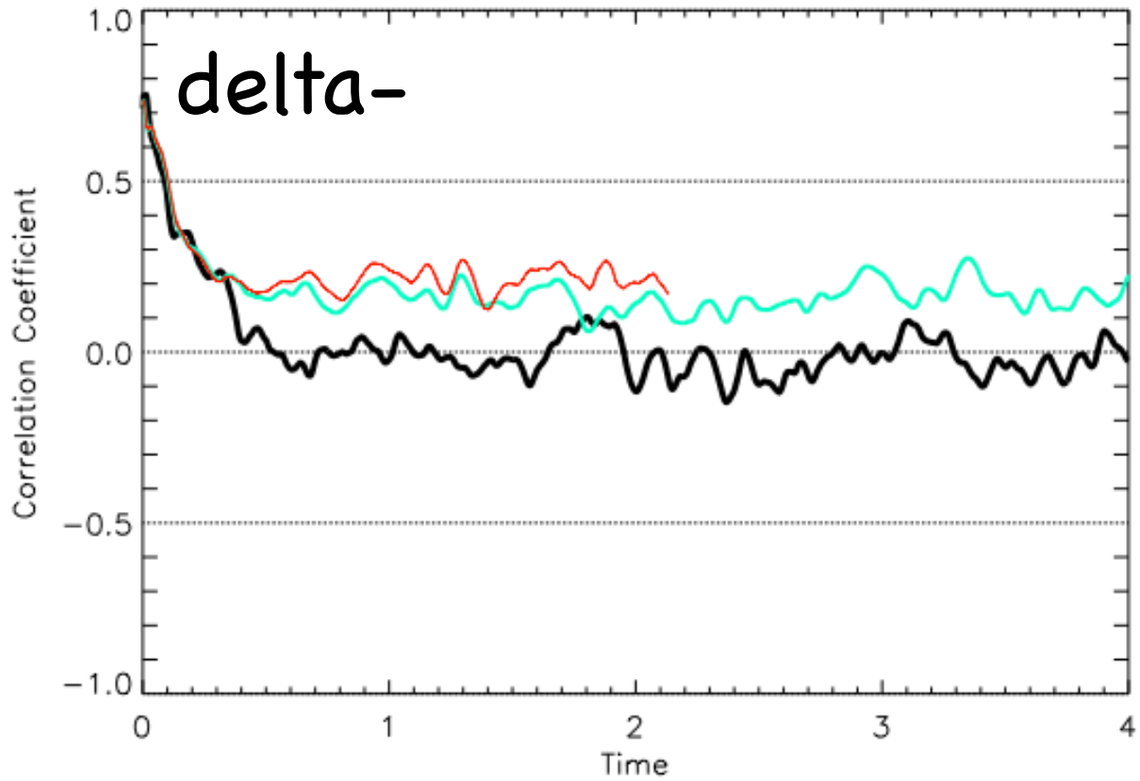
Result 2. Dependence on the Ms

B_0 = 1 & different Ms

submitted to ApJ (Yoon et al. 2016)



finite-correlated driving scheme

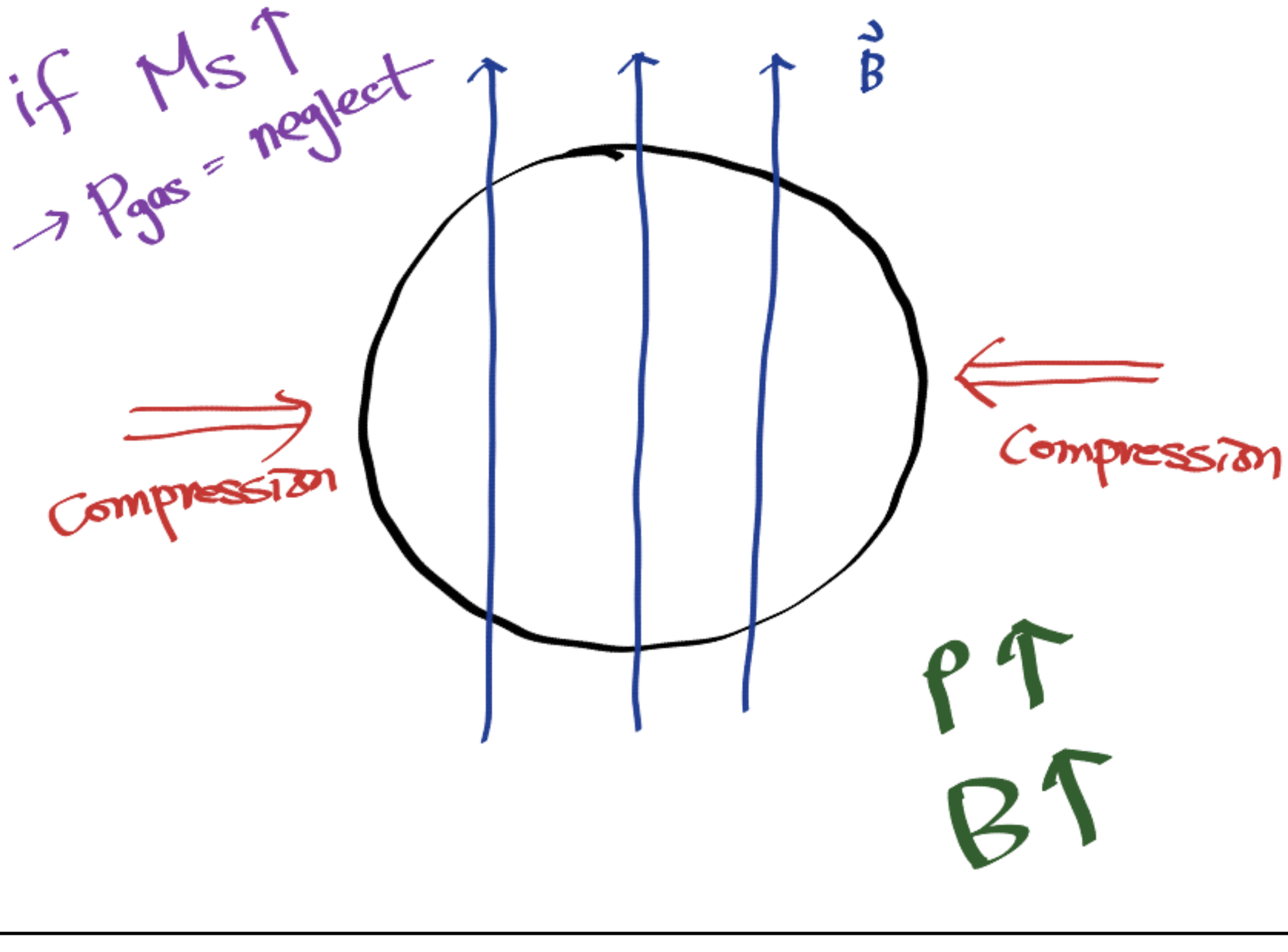


delta-correlated driving scheme

- finite-correlated driving : correlation coefficient change -1 to 0
- delta-correlated driving : very little change, slightly larger than 0
- pressure balance is not well maintained

Result 2. Dependence on the Ms

B_0 = 1 & different Ms



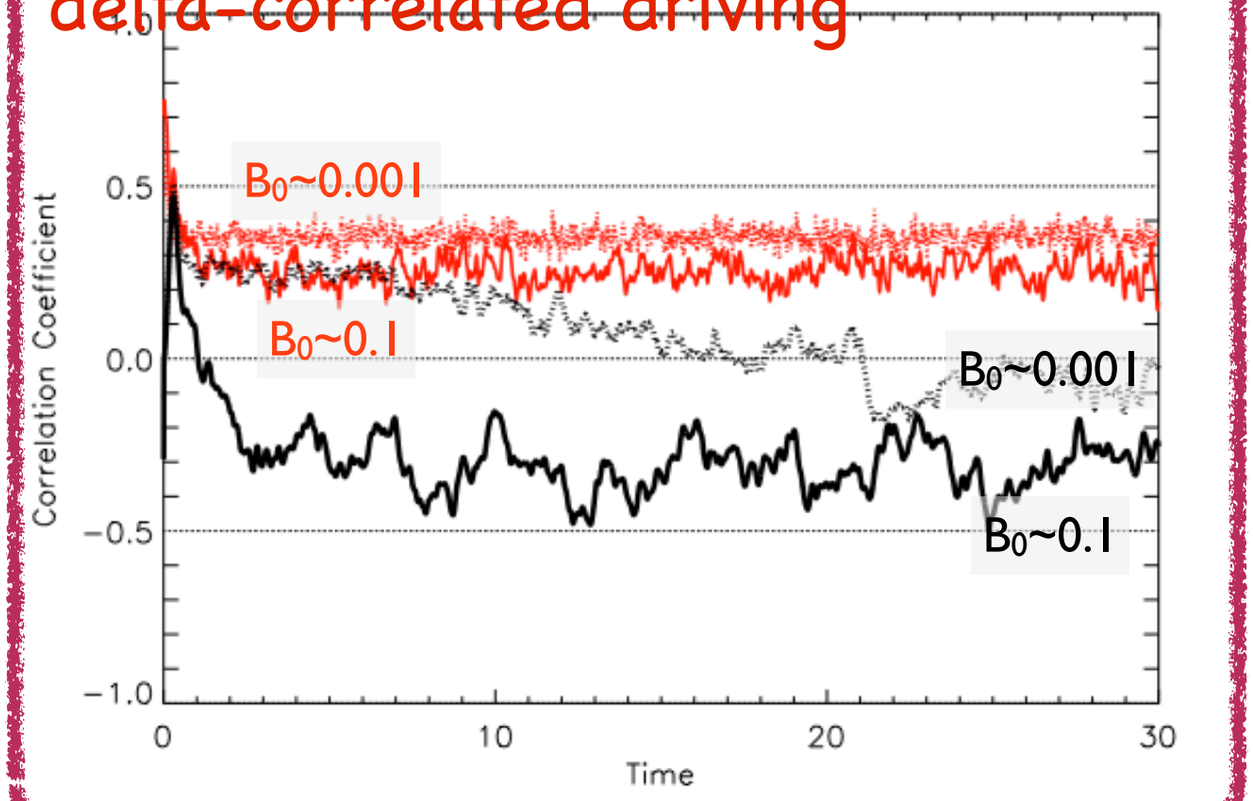
Result 3. Dependence on the B_0

B_0 = 0.1, 0.001 & Ms~1

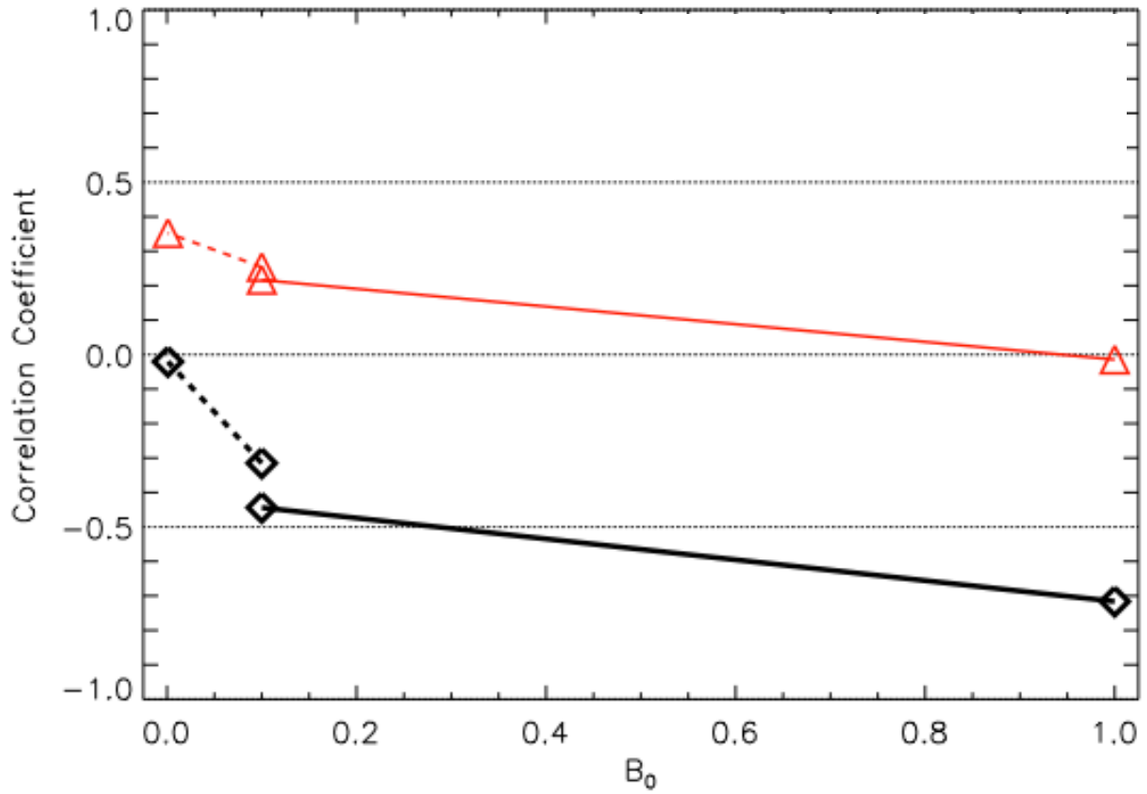
finite-correlation driving

submitted to ApJ (Yoon et al. 2016)

delta-correlated driving



Time Evolution of Correlation Coeff.



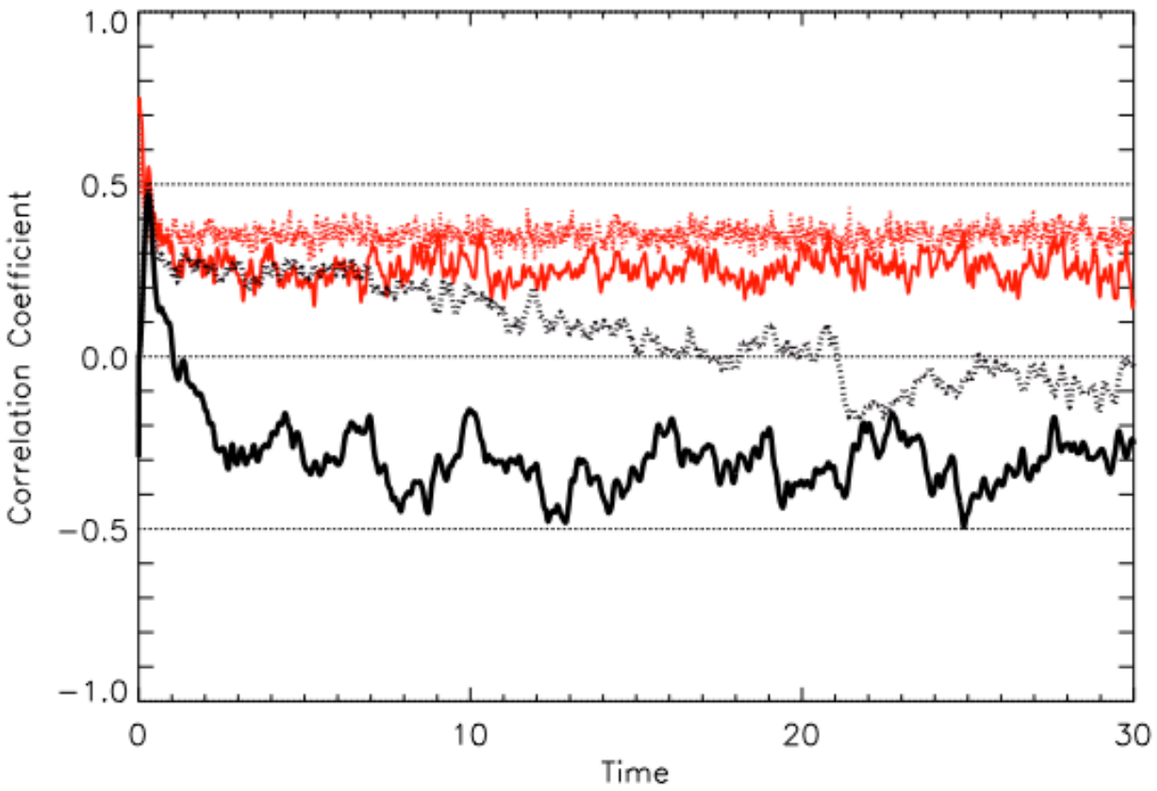
Average Values of Correlation Coeff.

- finite-correlated driving : still anti-correlation between ρ & B
- delta-correlated driving : positive correlation between ρ & B
- B decrease ➤ moves toward positive correlation between ρ & B

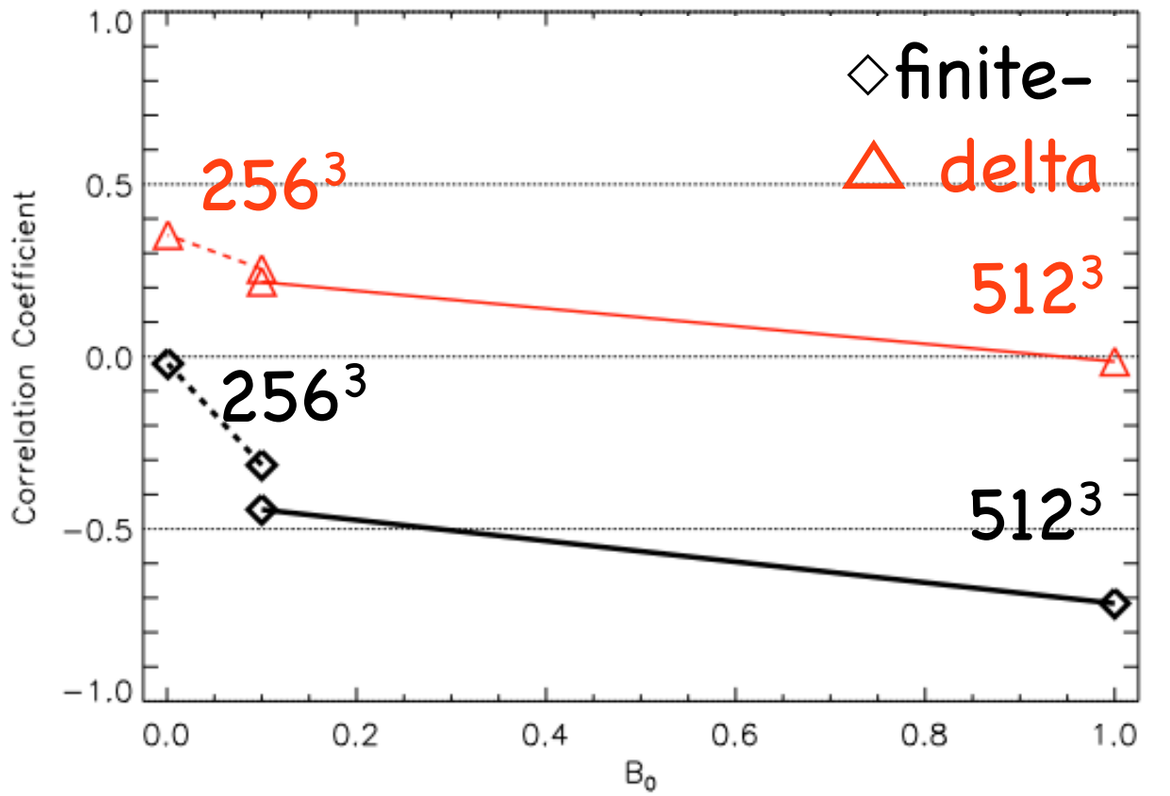
Result 3. Dependence on the B_0

B_0 = 0.1, 0.001 & Ms~1

submitted to ApJ (Yoon et al. 2016)



Time Evolution of Correlation Coeff.

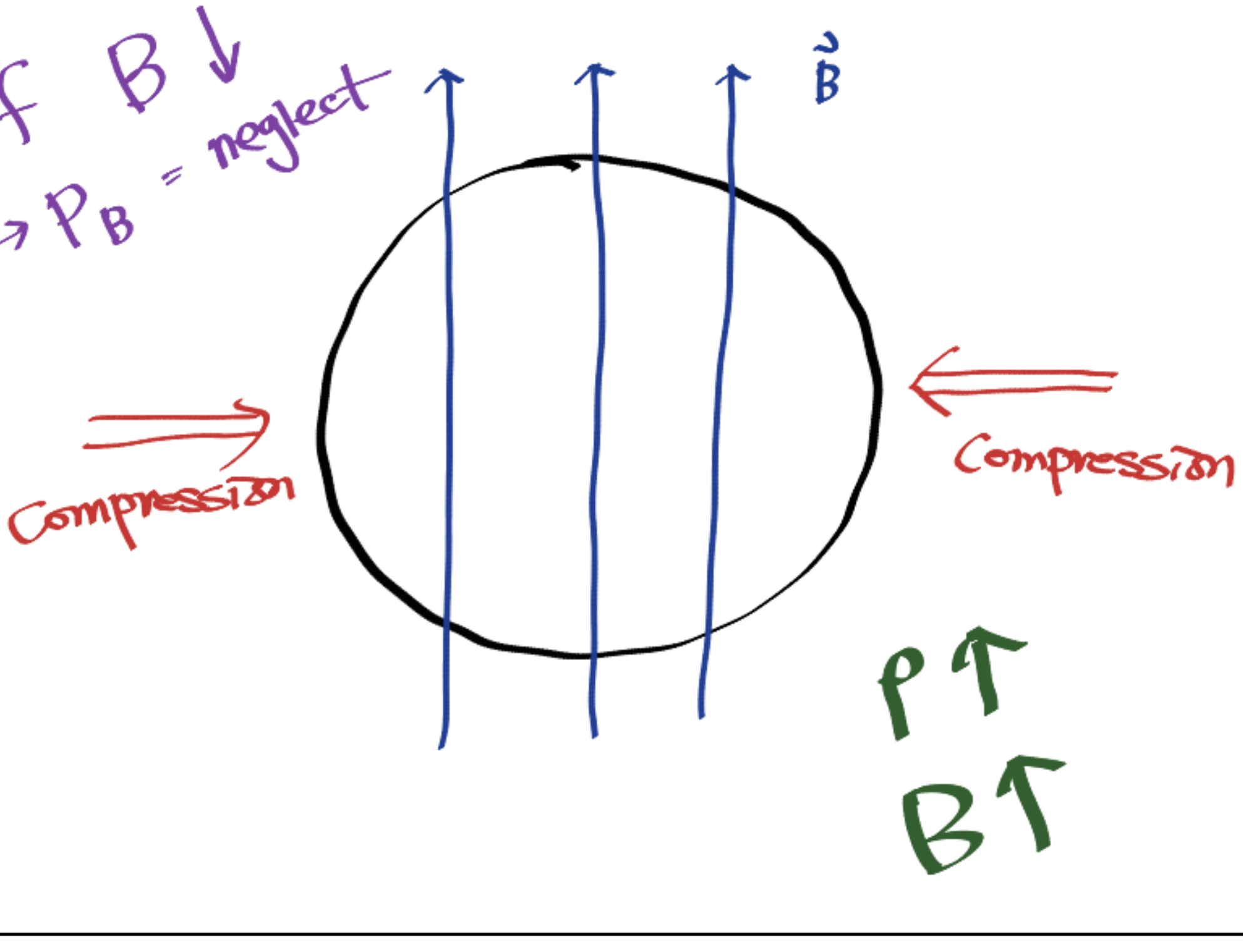


Average Values of Correlation Coeff.

- finite-correlated driving : still anti-correlation between ρ & B
- delta-correlated driving : positive correlation between ρ & B
- B decrease ➤ moves toward positive correlation between ρ & B

Result 3. Dependence on the B_0

B_0 = 0.1, 0.001 & Ms~1

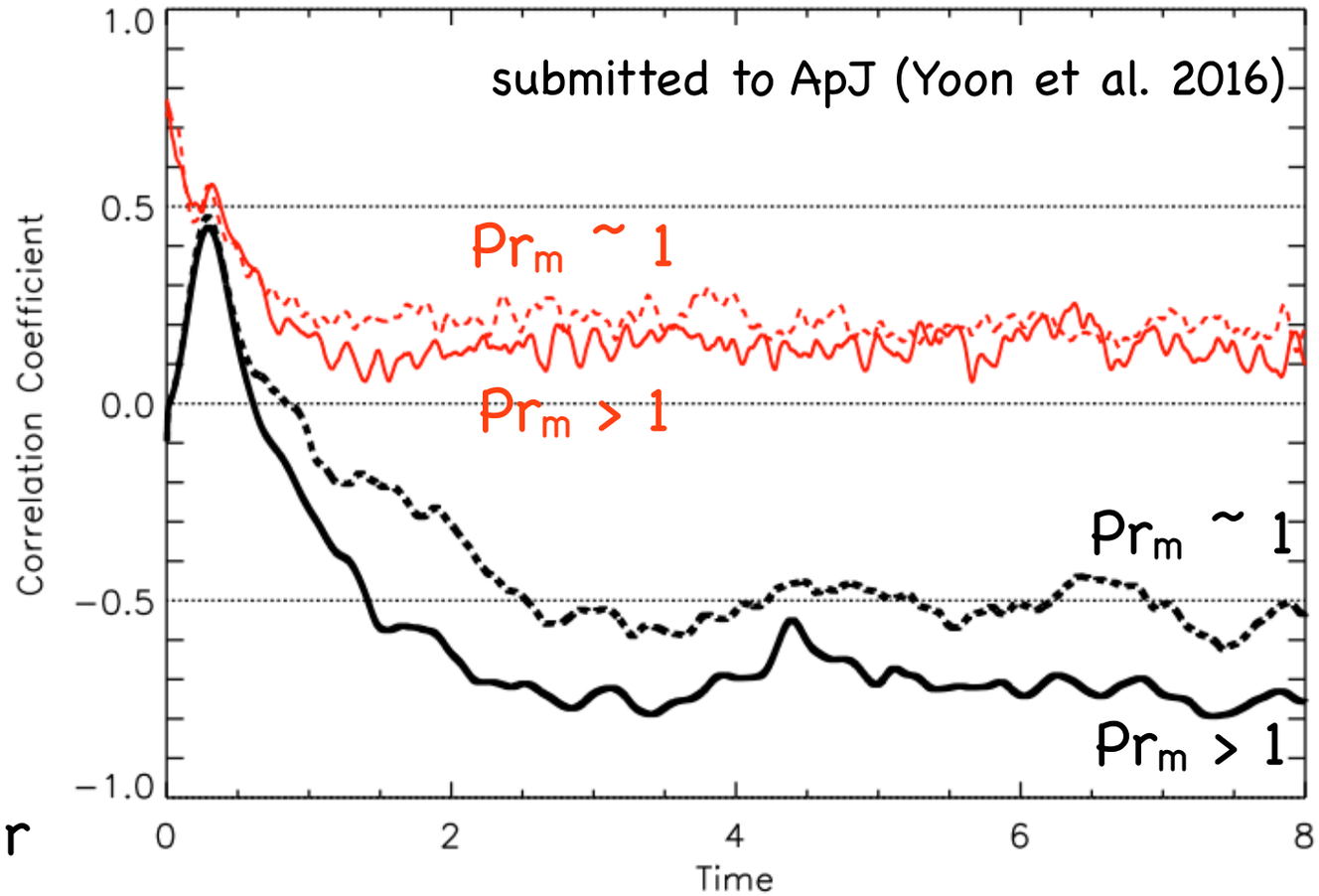


Result 4. Dependence on the Pr_m

B_0 = 0.1 & Ms~1 High Magnetic Prandtl Number

finite-correlation driving

delta-correlated driving



$Pr_m = \nu/\eta$: magnetic Prandtl Number

where ν : viscosity

η : magnetic diffusivity

Time Evolution of Correlation Coeff.

- finite-correlated driving scheme : anti-correlation stronger
- delta-correlated driving scheme : do not make much difference
- forcing timescale vs. timescale for structure change

Result 4. Dependence on the Pr_m

B_0 = 0.1 & Ms~1 High Magnetic Prandtl Number

$T_{forcing} > T_{structure_change}$

➤ maintain pressure equilibrium

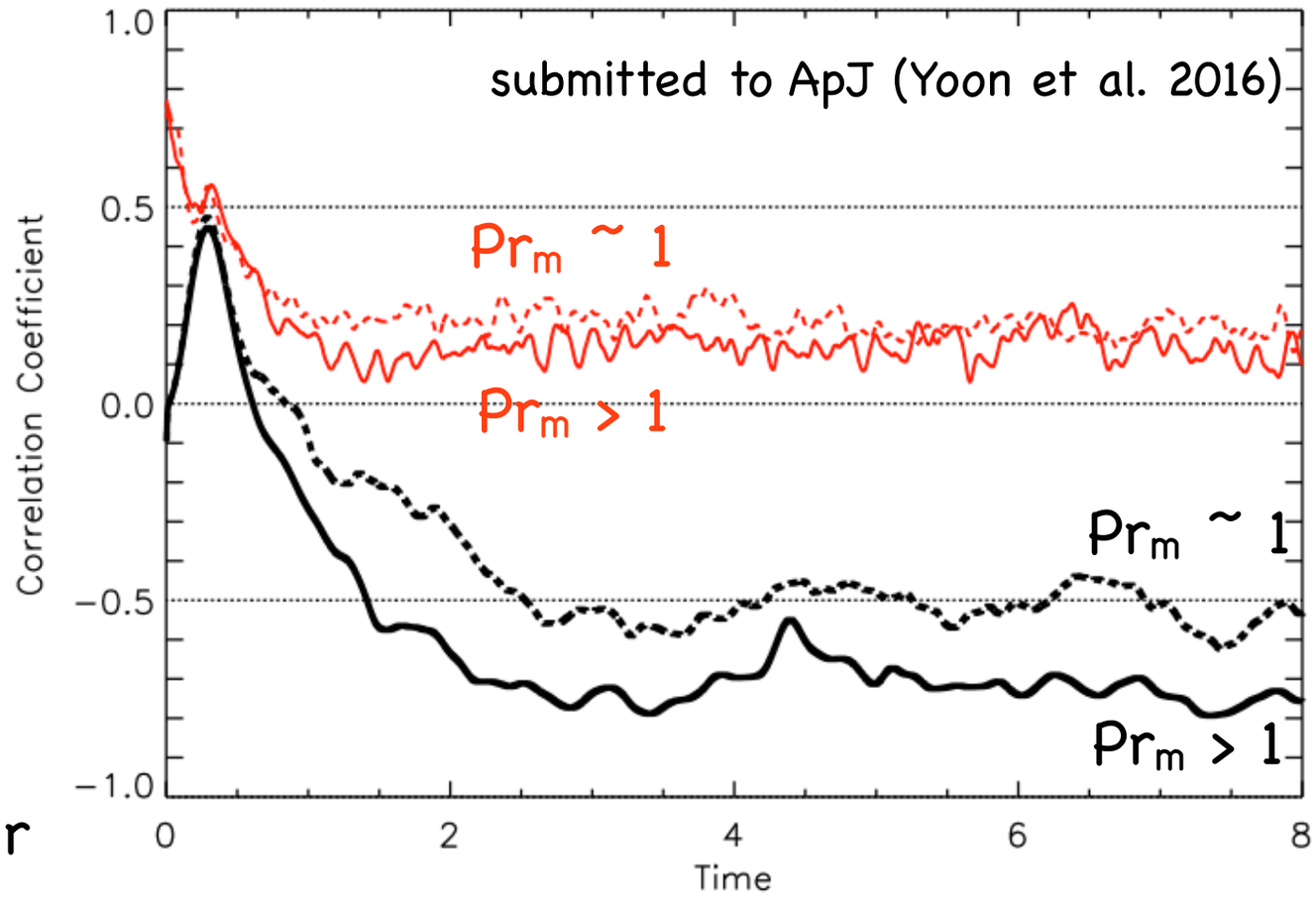
$T_{forcing} < T_{structure_change}$

➤ not maintain pressure equilibrium

$Pr_m = \nu/\eta$: magnetic Prandtl Number

where ν : viscosity

η : magnetic diffusivity



Time Evolution of Correlation Coeff.

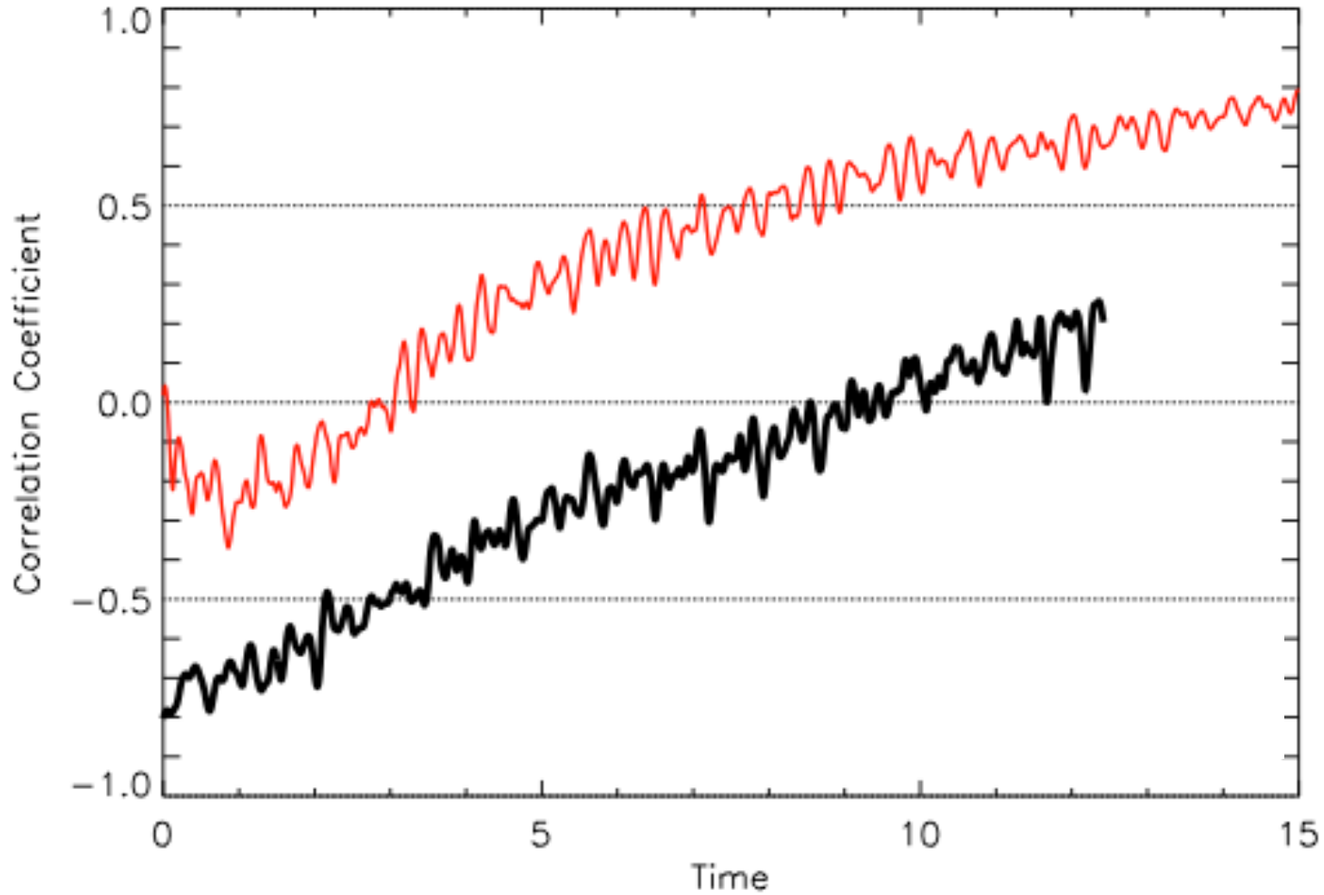
- finite-correlated driving scheme : anti-correlation stronger
- delta-correlated driving scheme : do not make much difference
- forcing timescale vs. timescale for structure change

Result 5. Decaying Turbulence

B_0 = 1 & Ms~1 Decaying Turbulence

finite-correlation driving

delta-correlated driving



Time Evolution of Correlation Coeff.

submitted to ApJ (Yoon et al. 2016)

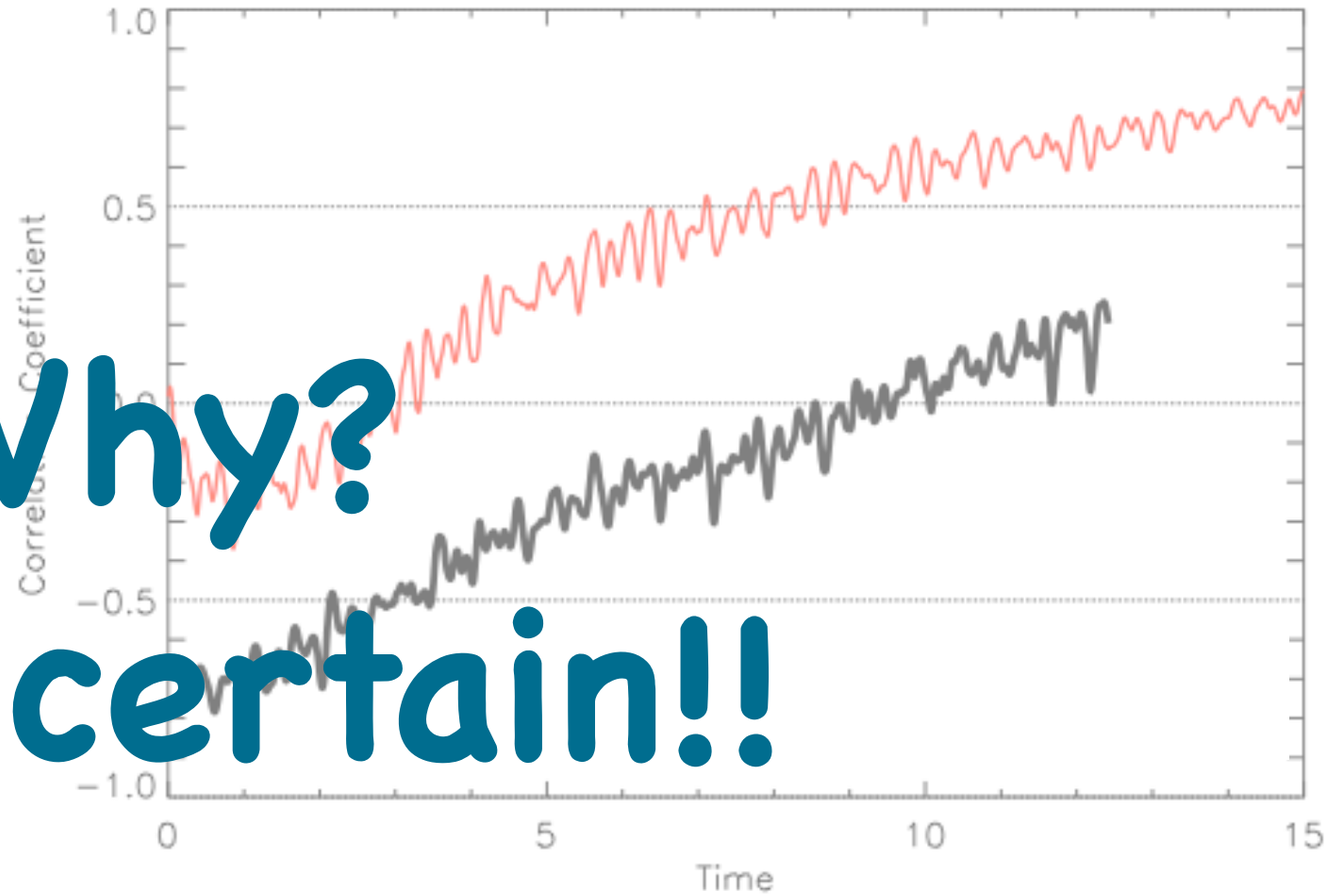
- As turbulence decays, the correlation coefficients for both driving scheme increase
- The reason for this is uncertain

Result 5. Decaying Turbulence

B_0 = 1 & Ms~1 Decaying Turbulence

finite-correlation driving

delta-correlated driving



Why?

➤ Uncertain!!

Time Evolution of Correlation Coeff.

submitted to ApJ (Yoon et al. 2016)

- As turbulence decays, the correlation coefficients for both driving scheme increase
- The reason for this is uncertain (fast mode??) ➤ more study

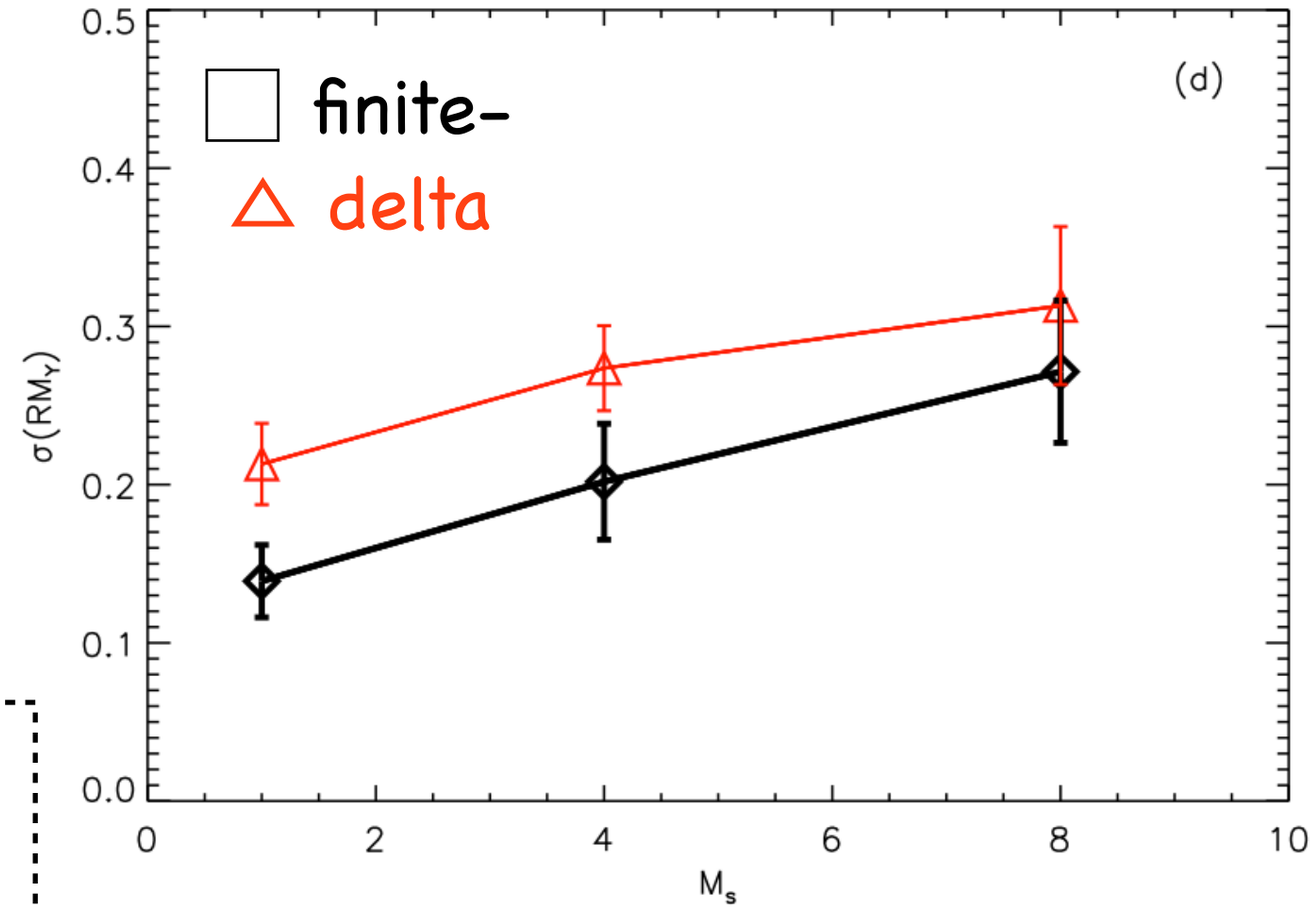
Discussion I : Standard Deviation of RM

Standard Deviation of RM

$$RM = \sum_j B_{\parallel,j}\rho_j,$$

- ρ & B Correlation (Beck et al. 2003)
  - Anti-correlation : Underestimate the $\langle B_\parallel \rangle$
    ➤ σ_{RM} Reduced

submitted to ApJ (Yoon et al. 2016)

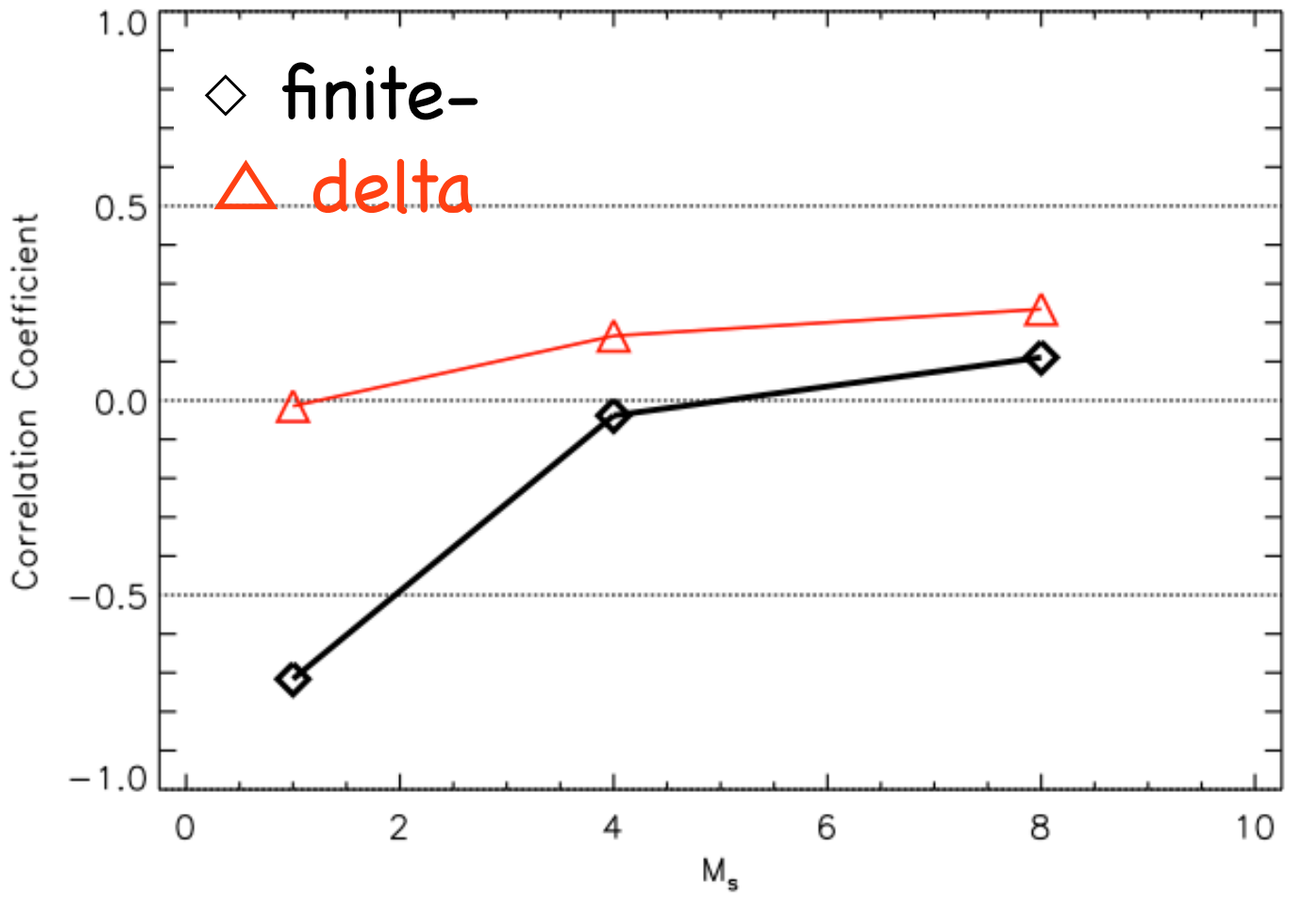


Standard Deviation of RM

- RM is integrated along the y-axis (perpendicular to the mean magnetic field)
- consistent with Beck et al. (2003)
  ➤ anti-correlation between ρ & B tends to reduce the σ_{RM}

Discussion I : Standard Deviation of RM

Standard Deviation of RM



ρ & B Correlation Coefficient vs. Ms

submitted to ApJ (Yoon et al. 2016)

- RM is integrated along the y-axis (perpendicular to the mean magnetic field)
- consistent with Beck et al. (2003)
  - ➤ anti-correlation between ρ & B tends to reduce the σ_{RM}

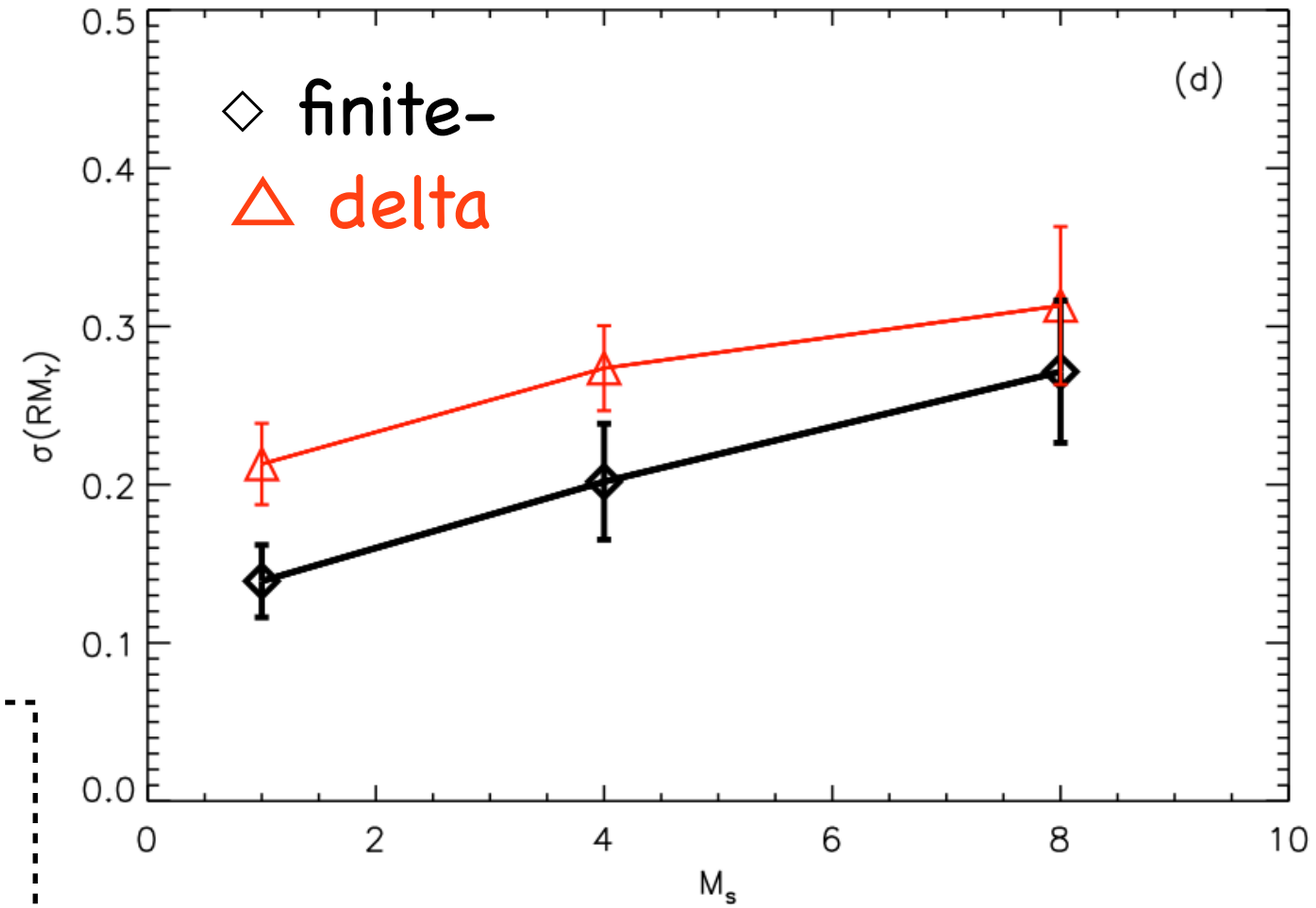
Discussion I : Standard Deviation of RM

Standard Deviation of RM

$$RM = \sum_j B_{\parallel,j}\rho_j,$$

- ρ & B Correlation (Beck et al. 2003)
  - Anti-correlation : Underestimate the $\langle B_\parallel \rangle$
    ➤ σ_{RM} Reduced

submitted to ApJ (Yoon et al. 2016)



Standard Deviation of RM

- RM is integrated along the y-axis (perpendicular to the mean magnetic field)
- consistent with Beck et al. (2003)
  ➤ anti-correlation between ρ & B tends to reduce the σ_{RM}

Discussion 2 : Comparison of RM/DM & True <$B_{\parallel}$>

- **Estimate average magnetic field along the line of sight**

$$\langle B_{\parallel} \rangle \sim \frac{\int n_e B_{\parallel} ds}{\int n_e ds} = \frac{RM}{DM}$$

where n_e : electron density
$B_{\parallel}$: parallel magnetic field
RM : the rotation measure
DM : the dispersion measure
ds : line of sight

- **ρ(~n_e) & B Correlation**

  - Un correlation : True <$B_{\parallel}$>
  - Positive Correlation : Overestimate the <$B_{\parallel}$>
  - Negative correlation : Underestimate the <$B_{\parallel}$>

Beck et al. 2003

Discussion 2 : Comparison of RM/DM & True <$B_{||}$>

<RM/DM> vs. <$B_{||}$>

submitted to ApJ (Yoon et al. 2016)

$$\langle B_{||} \rangle \sim \frac{\int n_e B_{||} ds}{\int n_e ds} = \frac{RM}{DM}$$

COMPARISON OF <RM/DM> & TRUE <$B_{||}$>

Run	Code[a]	Driving[b]	$\langle RM_x/DM_x \rangle$ [c]	True $\langle B_{||} \rangle$ [d]
F3	TVD	$F_{\rm fin}$	0.943	1.000
F4	TVD	$F_{\rm fin}$	0.950	1.000
F5	TVD	$F_{\rm fin}$	0.963	1.000
D3	TVD	F_{δ}	0.975	1.000
D4	TVD	F_{δ}	0.992	1.000
D5	TVD	F_{δ}	0.987	1.000

[a-b] Same as in Table 1
[c] Average Value of RM/DM integrated along the x-direction
[d] Average value of true $B_{||}$

F3 & D3 : Ms~1
F4 & D4 : Ms~4
F5 & D5 : Ms~8

- close to the true value of <$B_{||}$> in all driving scheme
- Ms~1 : finite-correlated driving scheme slightly underestimate
- Ms~4 & ~8 : become similar

Discussion 2 : Comparison of RM/DM & True <$B_{||}$>

<RM/DM> vs. <$B_{||}$>

submitted to ApJ (Yoon et al. 2016)

$$\langle B_{||} \rangle \sim \frac{\int n_e B_{||} ds}{\int n_e ds} = \frac{RM}{DM}$$

COMPARISON OF <RM/DM> & TRUE <$B_{||}$>

Run	Code[a]	Driving[b]	$\langle RM_x/DM_x \rangle$ [c]	True $\langle B_{||} \rangle$ [d]
F3	TVD	$F_{\rm fin}$	0.943	1.000
F4	TVD	$F_{\rm fin}$	0.950	1.000
F5	TVD	$F_{\rm fin}$	0.963	1.000
D3	TVD	F_δ	0.975	1.000
D4	TVD	F_δ	0.992	1.000
D5	TVD	F_δ	0.987	1.000

[a-b] Same as in Table 1
[c] Average Value of RM/DM integrated along the x-direction
[d] Average value of true $B_{||}$

F3 & D3 : Ms~1
F4 & D4 : Ms~4
F5 & D5 : Ms~8

- close to the true value of <$B_{||}$> in all driving scheme
- Ms~1 : finite-correlated driving scheme slightly underestimate
- Ms~4 & ~8 : become similar

Discussion 2 : Comparison of RM/DM & True $\langle B_{\parallel} \rangle$

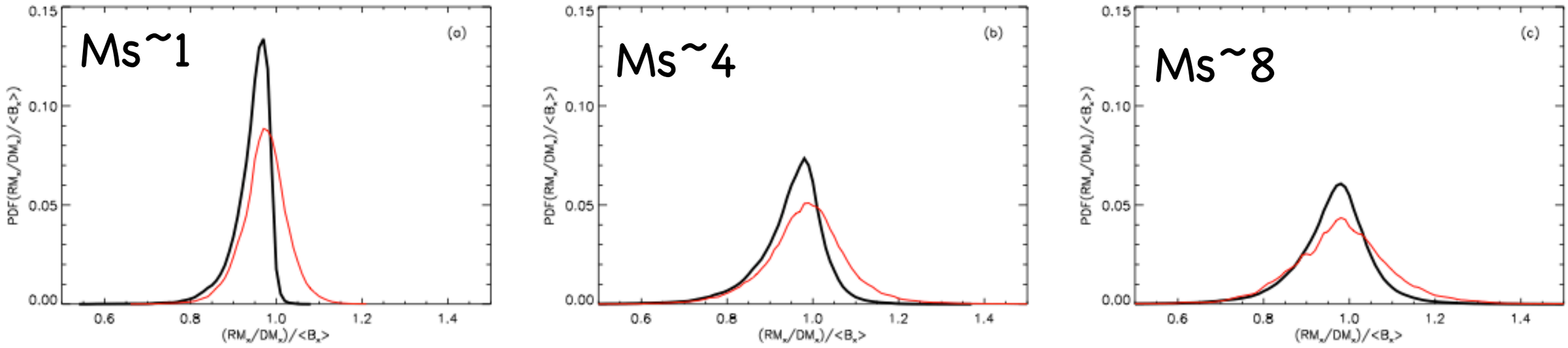
$\langle RM/DM \rangle$ vs. $\langle B_{\parallel} \rangle$

finite-correlation driving

delta-correlated driving

submitted to ApJ (Yoon et al. 2016)

$(RM_x/DM_x)/\langle B_x \rangle$



PDFs of $(RM/DM)/\langle B_{\parallel} \rangle$

- RM & DM are integrated the x-axis (parallel to the mean magnetic field)
- both driving scheme are sharp peaks near 1 ($RM/DM \sim \langle B_{\parallel} \rangle$)
- finite-correlated driving scheme : a bit skewed to the left
- delta-correlated driving scheme : more or less symmetric

Discussion 2 : Comparison of RM/DM & True $\langle B_{\parallel} \rangle$

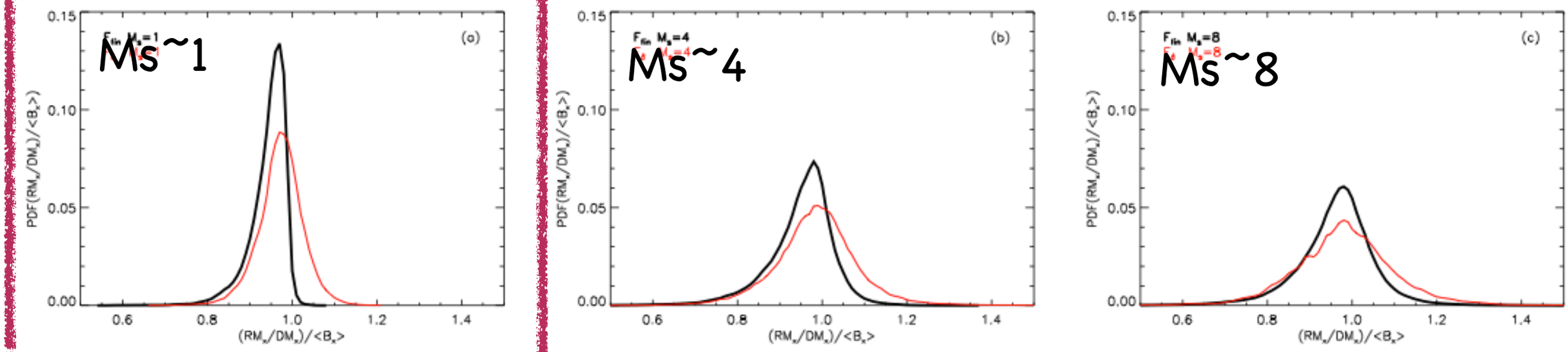
$\langle RM/DM \rangle$ vs. $\langle B_{\parallel} \rangle$

finite-correlation driving

delta-correlated driving

submitted to ApJ (Yoon et al. 2016)

$(RM_x/DM_x)/\langle B_x \rangle$



PDFs of $(RM/DM)/\langle B_{\parallel} \rangle$

- RM & DM are integrated the x-axis (parallel to the mean magnetic field)
- both driving scheme are sharp peaks near 1 ($RM/DM \sim \langle B_{\parallel} \rangle$)
- finite-correlated driving scheme : a bit skewed to the left
- delta-correlated driving scheme : more or less symmetric

Summary

✔ We study about correlation between ρ & B in MHD turbulence

✔ Several factors affect that correlation between ρ & B
1) Driving scheme (correlation timescale of forcing)
2) sonic Mach Number
3) Mean Magnetic Feld Strength
4) Magnetic Prandtl number
5) Decaying Turbulence

✔ Pressure equilibrium is key factor that determines the ρ & B correlation

✔ Consistent with Beck et al. (2003)
1) Reduced the Standard Deviation of RM
2) Anti-correlation Between ρ & B ➤ Underestimate the <B>

Summary

✔ We study about correlation between ρ & B in MHD turbulence

✔ Several factors affect that correlation between ρ & B
1) Driving scheme (correlation timescale of forcing)
2) sonic Mach Number
3) Mean Magnetic Feld Strength
4) Magnetic Prandtl number
5) Decaying Turbulence

✔ Pressure equilibrium is key factor that determines the ρ & B correlation

✔ Consistent with Beck et al. (2003)
1) Reduced the Standard Deviation of RM
2) Anti-correlation Between ρ & B ➤ Underestimate the <B>

Thank you for your attention

THE END

@ Micro Beach, Saipan