Transport in Tangled Magnetic Fields

Zhibin Guo

Peking University

Collaborators:
Dongsu Ryu(CNU), P. H. Diamond(NFRI, UCSD)

1st KNAG meeting, CNU, 13/09/2012

OUTLINE

(1) Background: cooling flow problem, Prandtl number problem...

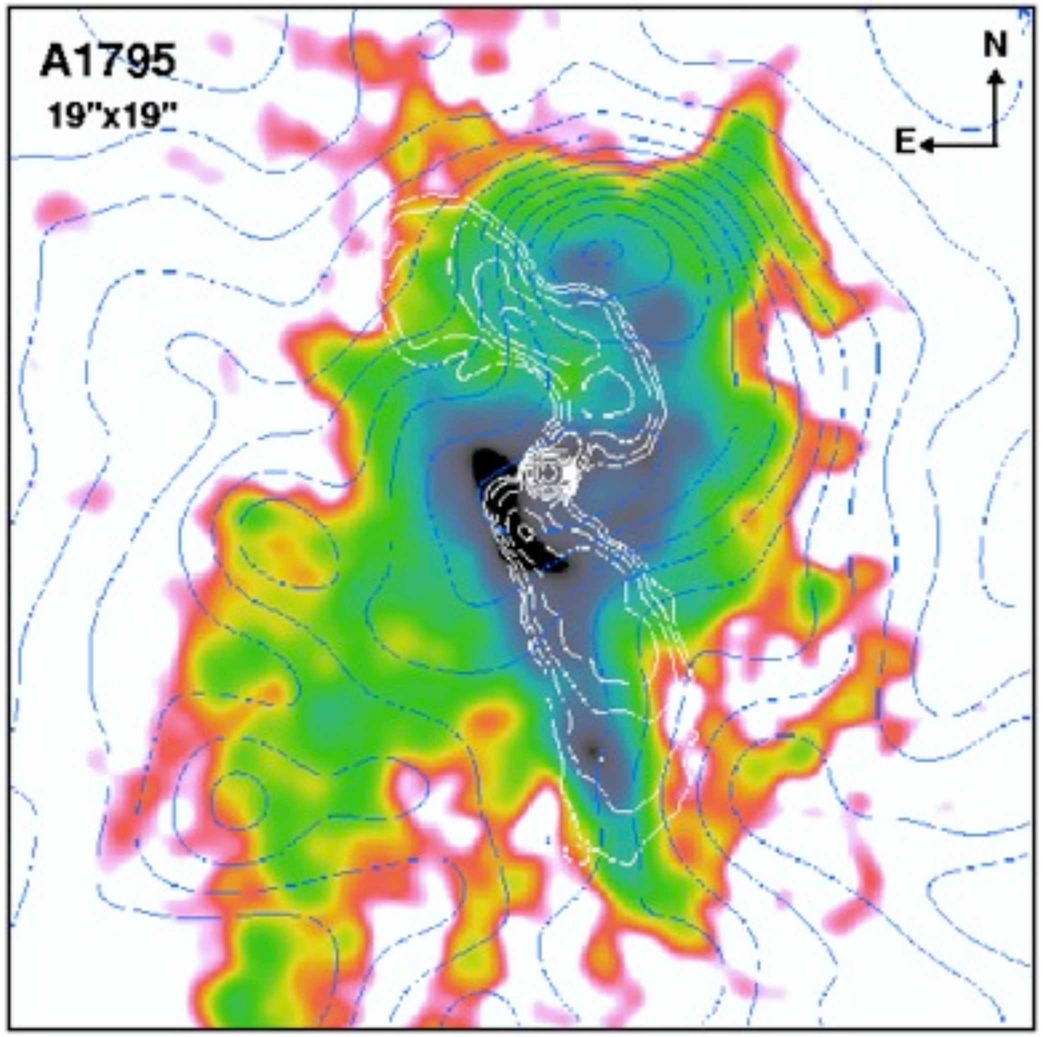
(2) theoretical considerations:

- Kubo number
- Magnetic field lines' relative dispersion in turbulence
- Electrons' conductivity
- Ions' viscosity

(3) factors not included in the current model

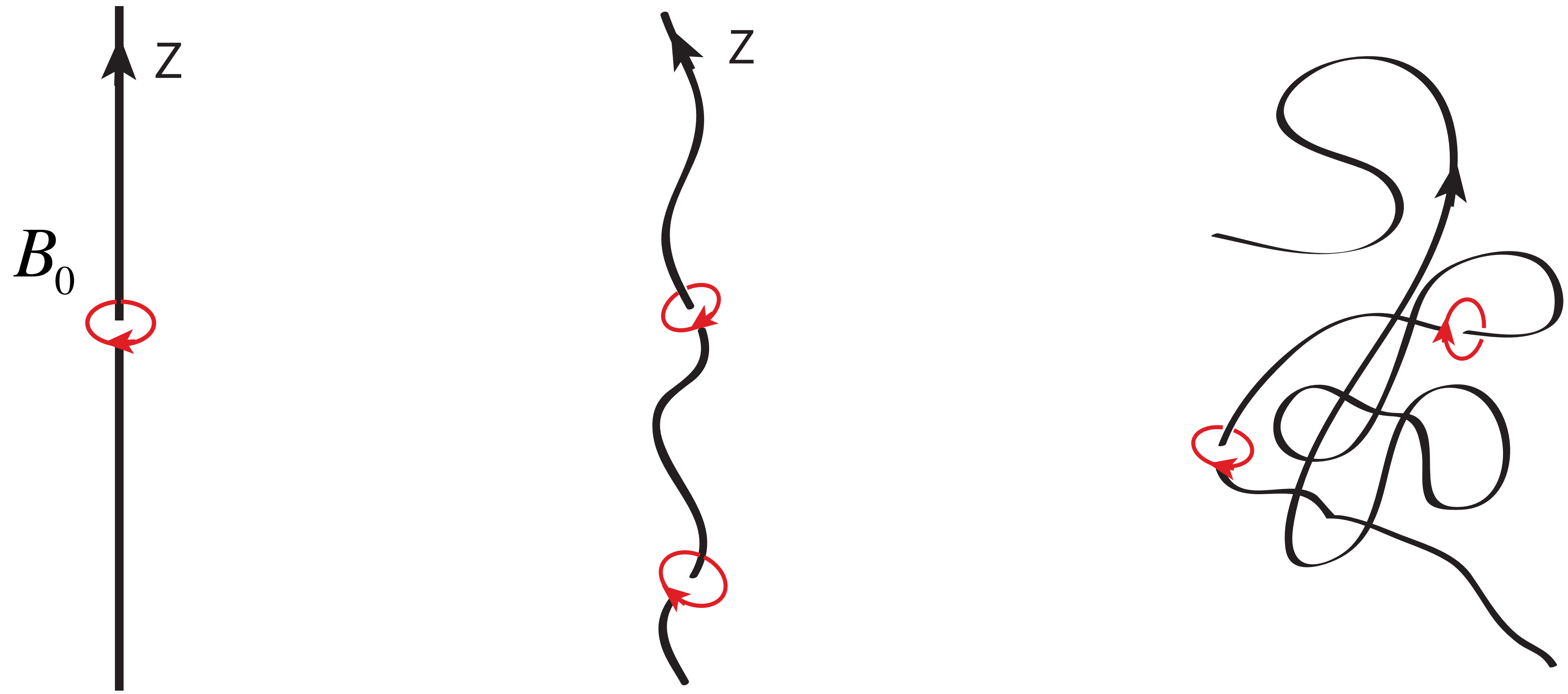
- Dynamics, percolation...

Cooling flow



www.rit.edu/cos/astrophysics/astro.html

Theoretical considerations



$\delta B = 0$ $\delta B \ll B_0$ $\delta B \gg B_0$

$$\chi_\perp \simeq \frac{\chi_{Sp}}{(\omega_{ce}\tau_{ee})^2} \sim \frac{1}{B_0^2}$$

$$\chi_{\perp,eff} \simeq (\chi_\parallel \chi_\perp)^{1/2} = \rho_e v_{the} \sim \frac{1}{B_0}$$

$$\chi_{\perp,eff} = ??$$

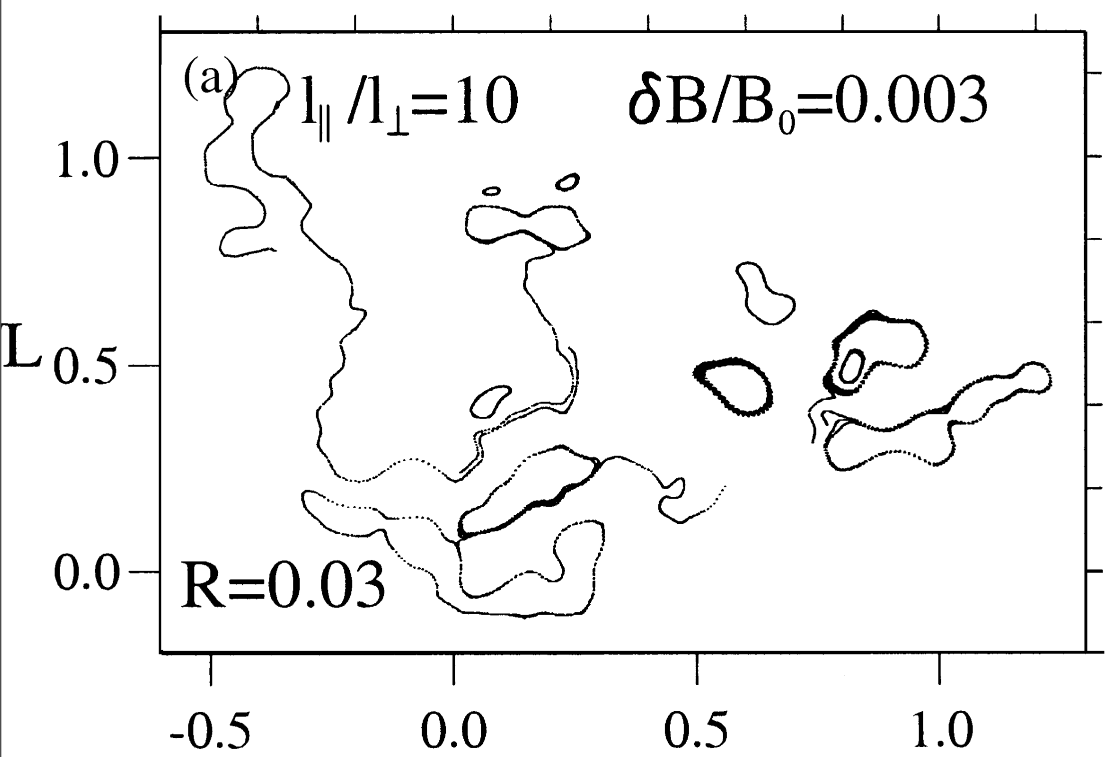
length of "random walk":

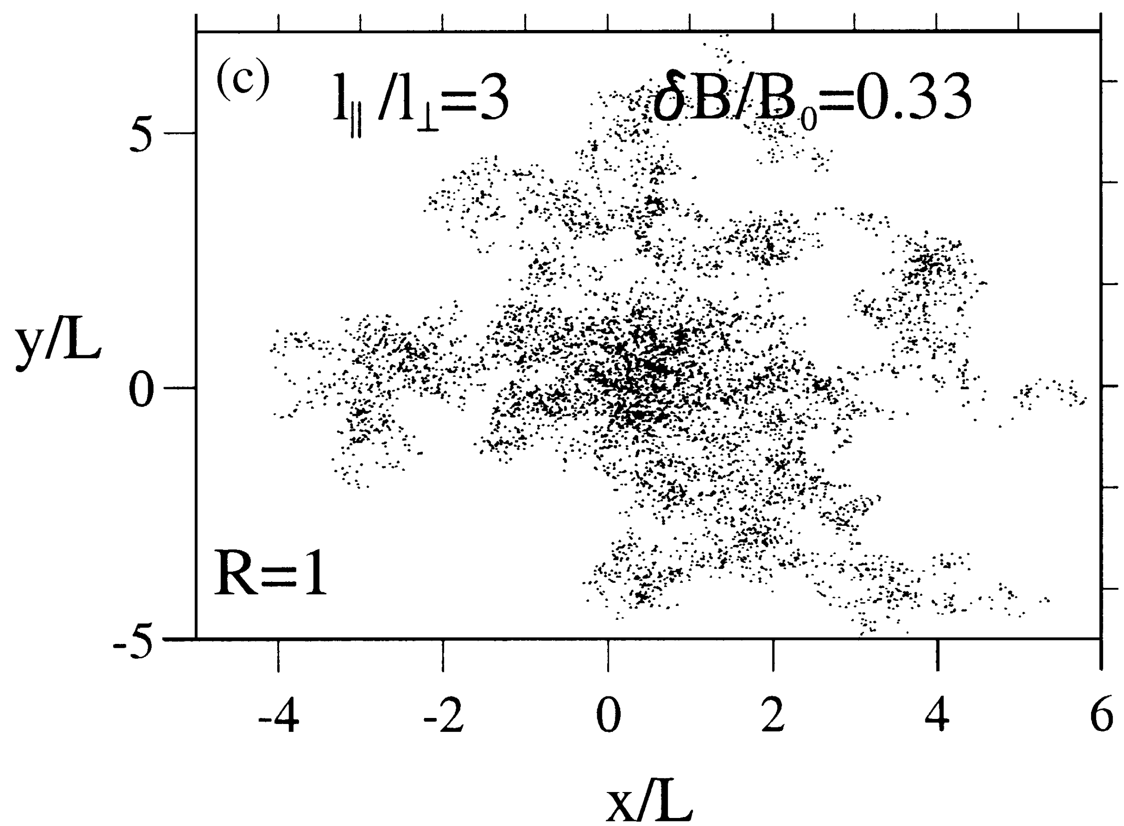
$$\rho_e(\omega_{ce}\tau_{ee})^{-1} \Rightarrow \rho_e(\omega_{ce}\tau_{ee})^{0} \Rightarrow \rho_e(\omega_{ce}\tau_{ee})^{1} = l_{ee}\,??$$

Kubo number

conventional definition $$\kappa \equiv \frac{\delta B}{B_0} \frac{l_\parallel}{l_\perp}$$

$\kappa = 0.03$ $\kappa = 1$

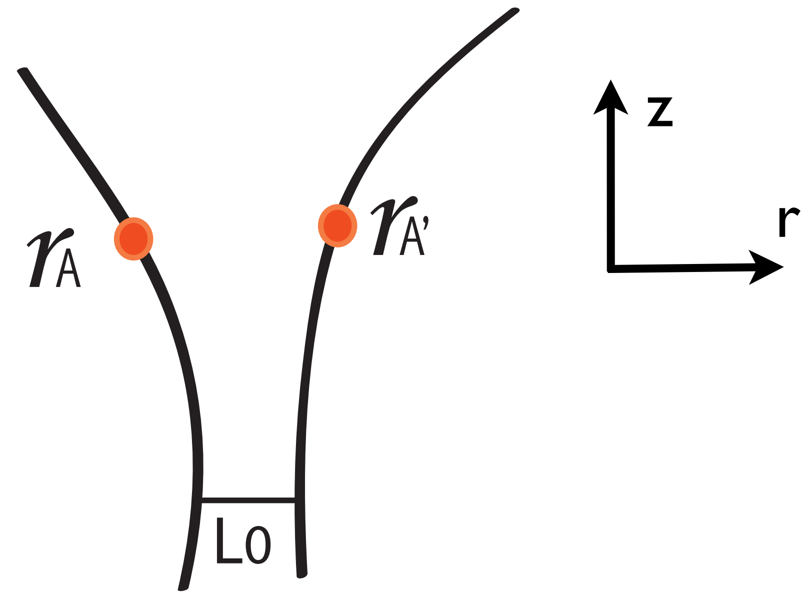




In ICM, $\delta B \gg B_0$, $l_\parallel \simeq l_\perp$, thus $\kappa \gg 1$

magnetic field lines are highly tangled when observing from the system scale

field line dispersion



field line equation

$$\frac{dz}{B_L + B_\parallel} = \frac{dr}{B_\perp}$$

$$\frac{d}{dz}|r_A - r_{A'}| = \frac{|B_\perp(r_A) - B_\perp(r_{A'})|}{B_L + B_\parallel} = \frac{\delta B(l)}{B_{l_B}}, \qquad l = |r_A - r_{A'}|$$

In turbulence, spectrum of is power-law form

$$\delta B_\perp^2 = c_K \epsilon^{2/3} l^{2/3}$$

Then,

$$l(z) = c_R l_B^{-1/2} z^{3/2}$$

Scale of the smallest random structure is l_B (correlation length of magnetic fields)

$$\langle \Delta r^2 \rangle \simeq (\frac{z}{l_B}) l_B^2 = z l_B$$

collisional case, $l_B \gg l_{ee}$

$$z = (\chi_\| t)^{1/2},$$

$$\chi_{eff} \simeq \frac{z_B l_B}{t_B^2} = c_R^{2/3} \chi_\| = \frac{1}{3} c_R^{2/3} \chi_{Sp}$$

Collisionless case, $l_B \ll l_{ee}$

$$z_{cls} = v_{the} t_{cls}$$

$$\chi_{eff,cls} = c_R^{2/3} v_{the} l_B$$

In summary

Strongly tangled magnetic field medium($\kappa \gg 1$) is highly conductive.

The effective heat conductivity depends on the power form of tangled fields, BUT is independent of the specific power exponent

Ions' viscosity

In ICM, the ions are also strongly magnetized

$$l_B \gg ion's \quad gyroradius$$

Thus, we expect ions' viscosity behaves similarly with electron conductivity under the assumption of static magnetic configuration.

Collisional case: $\nu_{i,eff} = c_R^{2/3} \nu_{\|} = \frac{1}{3} c_R^{2/3} \nu_{Spitzer}$

Collisionless case: $\nu_{i,eff,cls} = c_R^{2/3} v_{thi} l_B$

Influence on Prandtl number $P_r \equiv \frac{\nu_i}{\eta}$

$\eta \sim d_e^2 \nu_{ei}$ is not influenced by B.

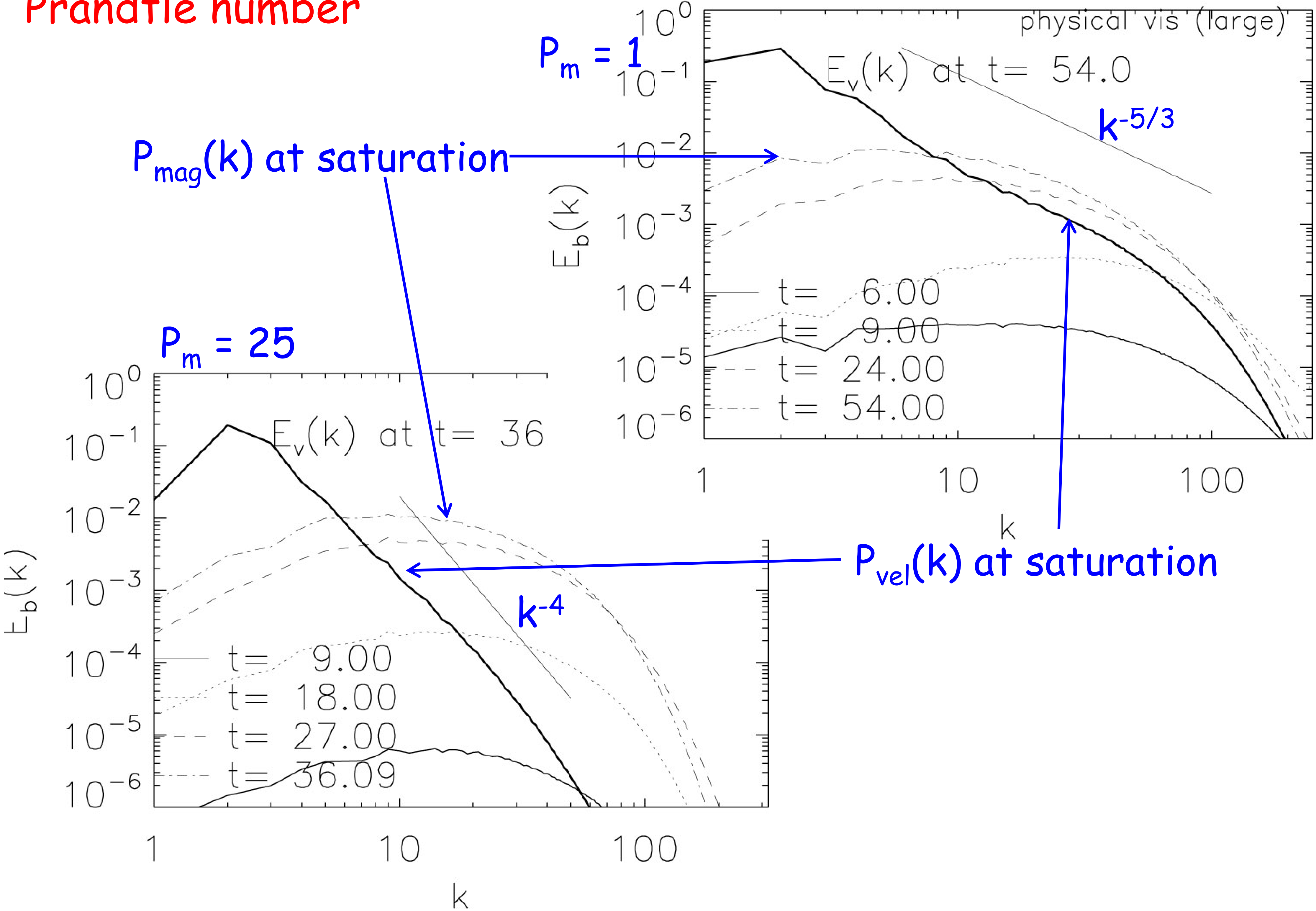
When $\kappa \ll 1$, $\nu_{i,eff} \simeq (\nu_{\|}\nu_{\perp})^{1/2} \simeq \rho_i v_{thi} \sim \frac{1}{B_0}$

Ion's viscosity is strongly suppressed by the regular field, Pr is constrained to a small value.

When $\kappa \gg 1$, $\nu_{i,eff} \simeq \nu_{Sp} \simeq l_{ii} v_{thi} \gg \rho_i v_{thi}$

Thus, P_r is increased largely.

Incompressible MHD turbulence with different magnetic Prandtle number



Ryu, ATCTP, Korea, 2011

Percolation

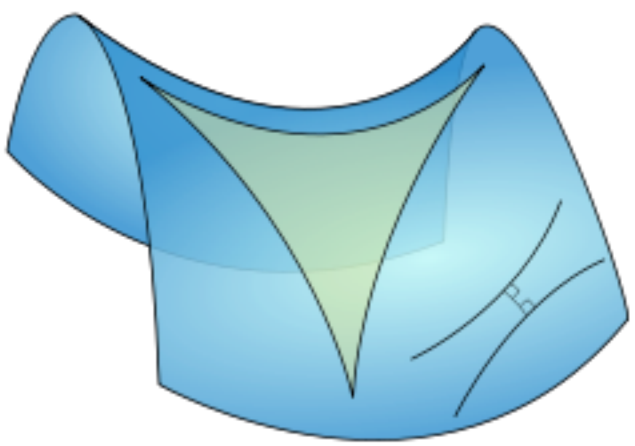
Okubo-Weiss criterion(2D):

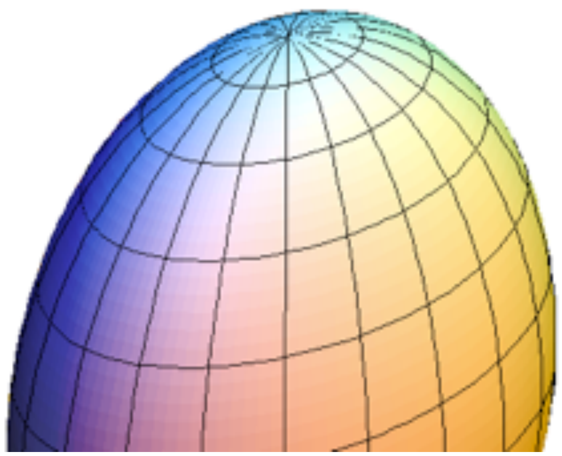
distinction between regular and irregular

$$Q = \frac{c^2}{4}\left[\left(\frac{\partial^2 A}{\partial x \partial y}\right)^2 - \frac{\partial^2 A}{\partial x^2}\frac{\partial^2 A}{\partial y^2}\right]$$

A --- magnetic potential function

Q = -(Gaussian curvature)





Q>0 → "X" point

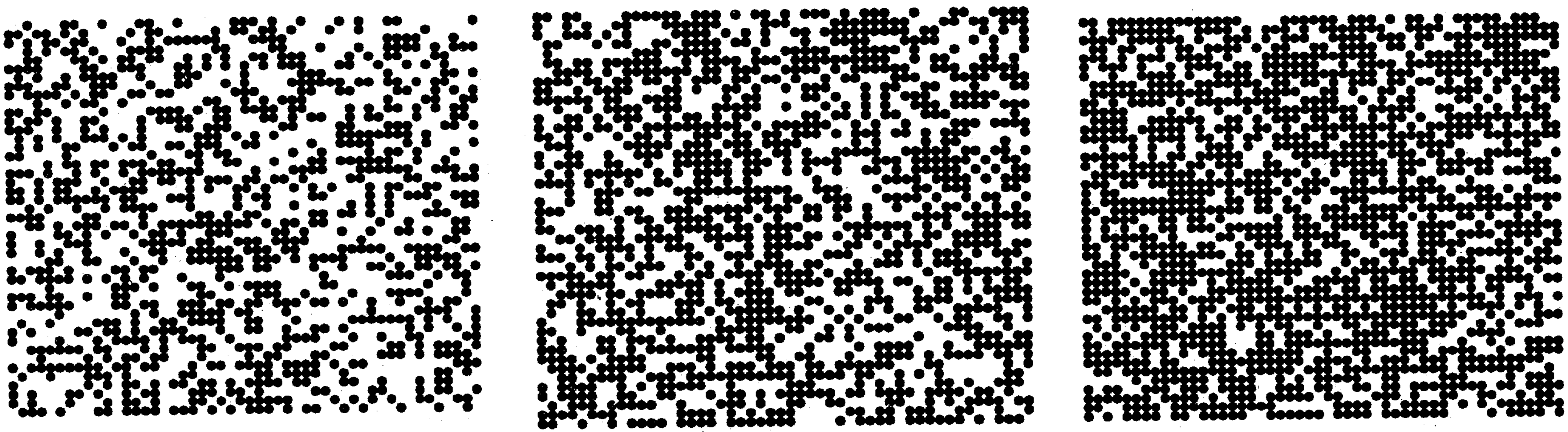
Q<0 → "O" point

Percolation

(1) critical behavior: infinite correlation length

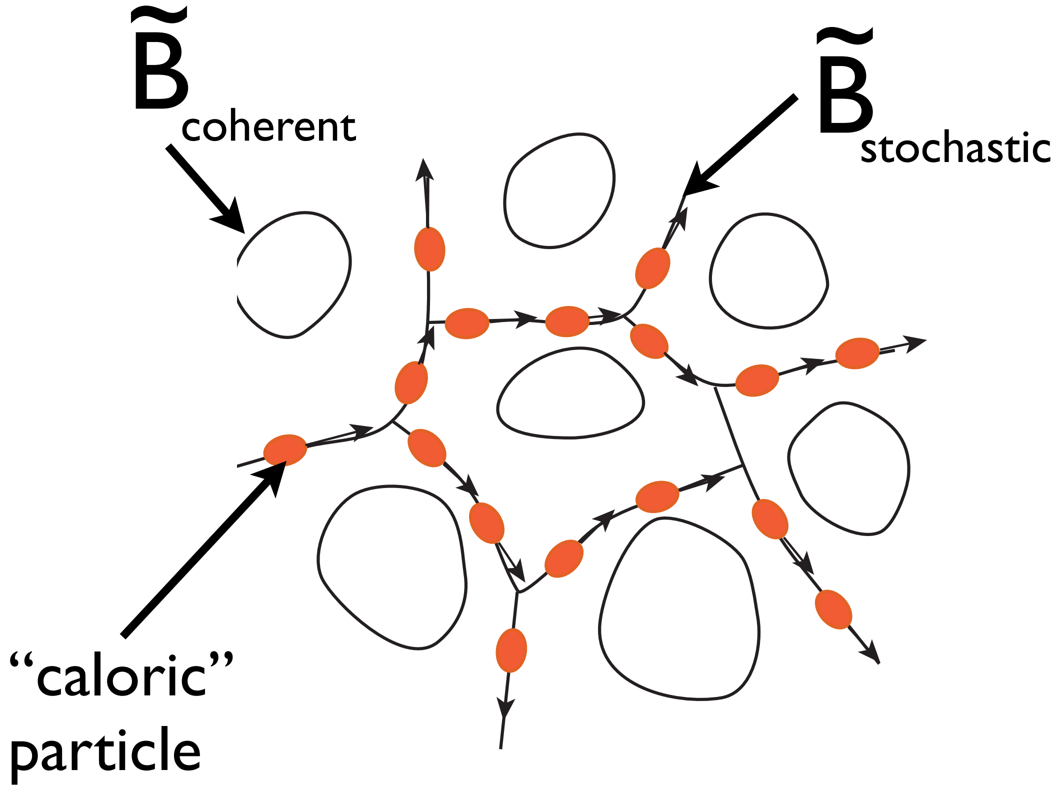
(2) fractal structure

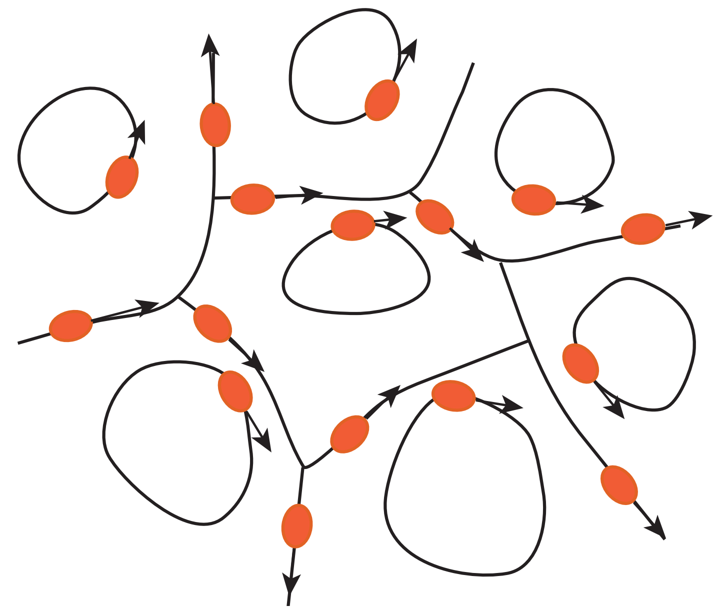
(Isichenko1992)



$p<p_c$ $p=p_c$ $p>p_c$

p_c---critical exponent





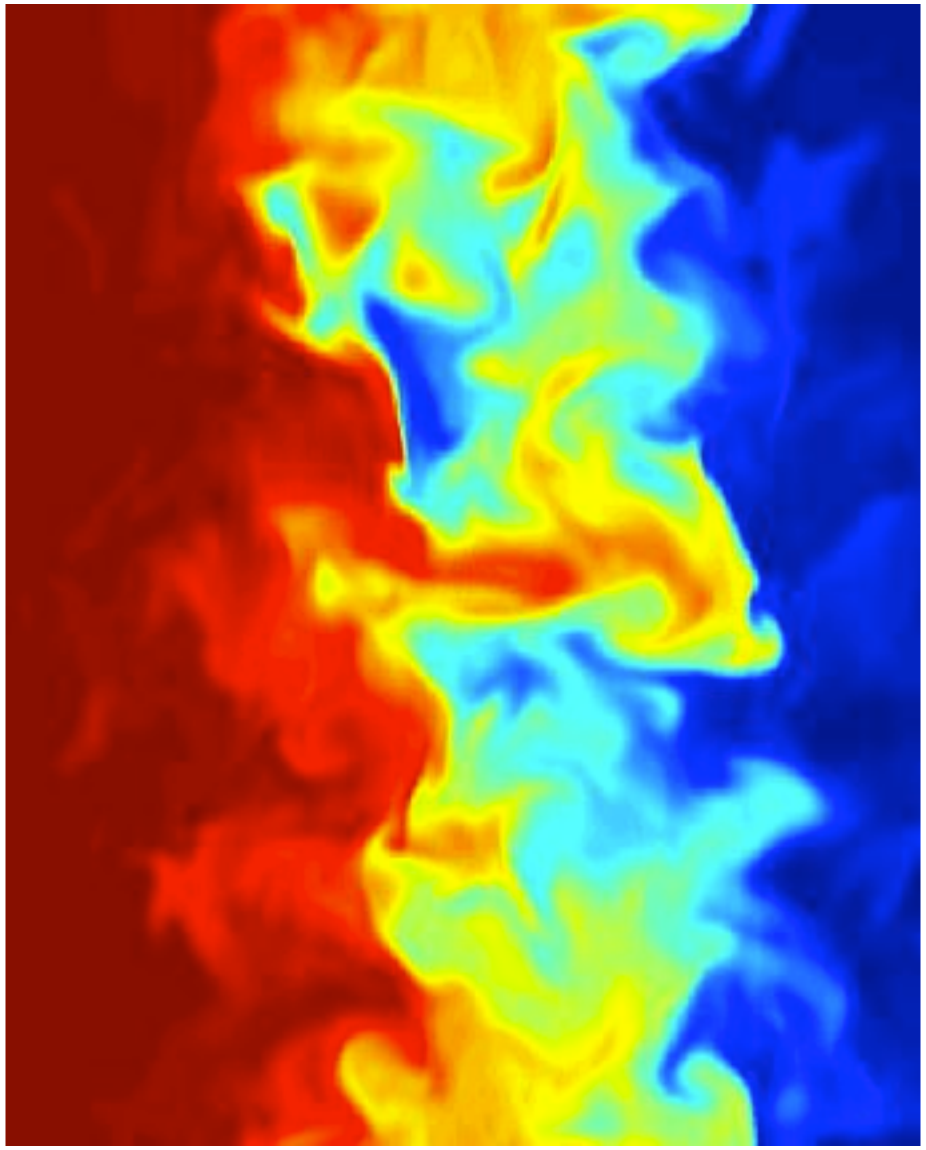
$$\langle \tilde{B}_{coh} \tilde{B}_{sto} \rangle = 0$$

$$\langle \tilde{B}_{coh} \tilde{B}_{sto} \rangle \neq 0$$

heat flux
“trapping”

Dynamic effect: turbulent mixing

$\sim v_{turb} l_c$



Summery

1, Electron's heat / ion's momentum is enhanced by tangled magnetic fields;

2, $\chi_{e,eff} / \nu_{i,eff}$ only depends on power law of B spectrum, and is insensitive to the specific power exponent.

THANK YOU !