A Scaling of Velocity and Magnetic field in Decaying Turbulence in Expanding/Collapsing Media

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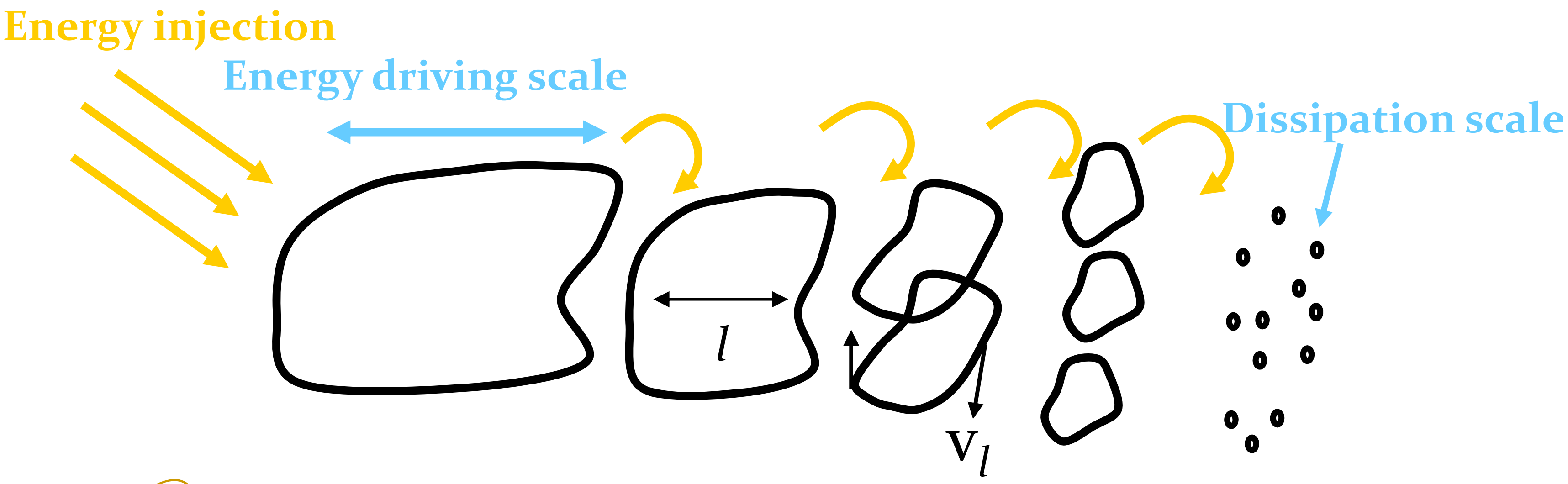
Purpose of Study

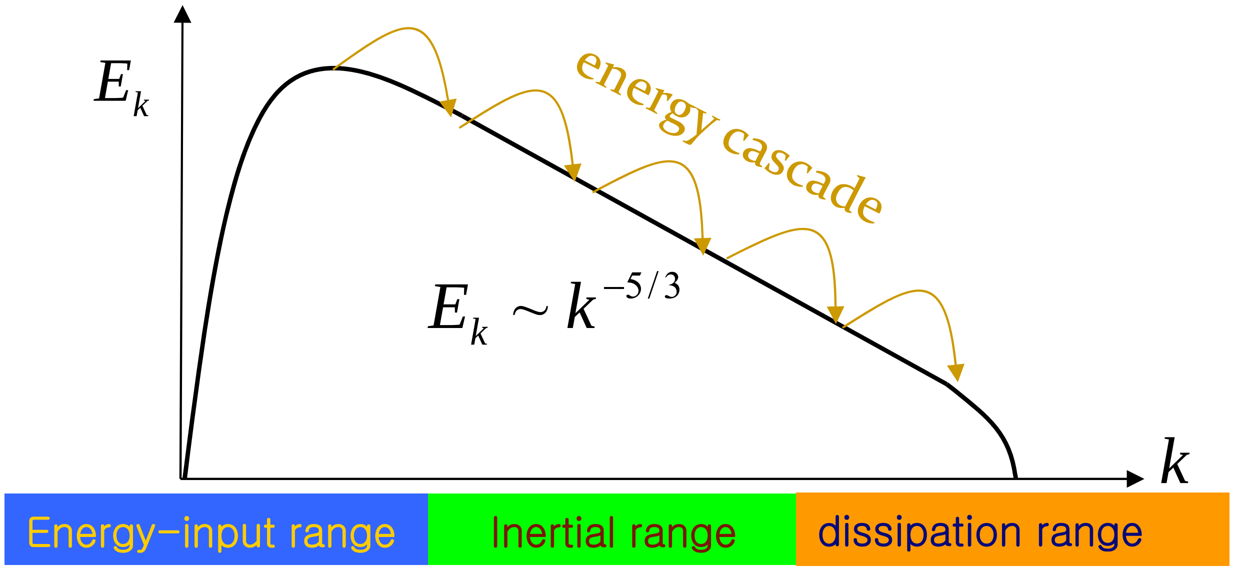
1. We investigated magnetohydrodynamic (MHD) decaying turbulence by including the effects of expansion and collapse of background medium.

2. We examine the properties of turbulence in the regimes of $t_{eddy} < t_{exp\text{-}coll}$ and $t_{eddy} > t_{exp\text{-}coll}$. Based on it, we derive a scaling for the time evolution of flow velocity and magnetic field.

Contents

Energy cascade and Kolmogorov spectrum



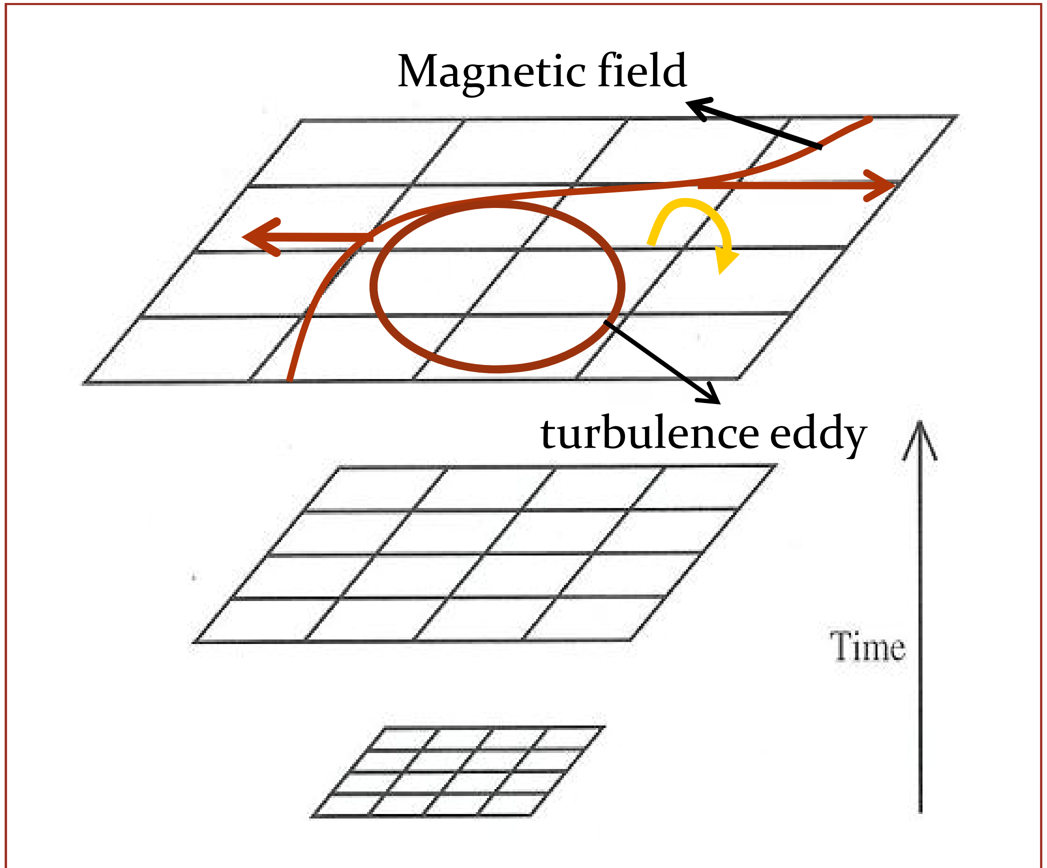


$$\frac{V_l^2}{t_{cas}} = \text{const}, \quad t_{cas} = l/V_l \quad \Bigg\} \quad \frac{V_l^3}{l} = \text{const}$$

- The "Kolmogorov spectrum" is generally observed in turbulence.
- The velocity dispersion ($\sim v_l$) decreases with scale as $v_l \propto l^{1/3}$

Fluid in expanding/collapsing coordinate

Expansion in the comoving coordinate system



- comoving coordinate system

$$\mathbf{r} = a(t)\mathbf{x}$$

r is physical distance
x is comoving distance
a(t) is scale factor

$$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$$

$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$

- $\mathbf{x}\dot{a}$ → Proper velocity
- $a\dot{\mathbf{x}}$ → peculiar velocity ($\mathbf{v} = a\dot{\mathbf{x}}$)

- When the matter expand, the magnetic field in the matter is expand with comoving coordinate system.

The MHD equations in expanding/collapsing media

p' is the pressure , $J = \nabla \times \mathbf{B}$ is the current , ν is the viscosity, η is the magnetic diffusion, $v' = \sqrt{\rho}v$ is velocity and **B** is magnetic field. Where f is a random driving force.

$$\frac{\partial v'}{\partial t} = \sqrt{a}\, v' \times (\nabla \times v') - \frac{5}{2}\frac{\dot{a}}{a} v' + \frac{1}{a^2\sqrt{a}} J \times B + \nu \nabla^2 v' + \nabla p' + f$$

$$\frac{\partial B}{\partial t} = \sqrt{a}\, \nabla \times (v' \times B) - 2\frac{\dot{a}}{a} B + \eta \nabla^2 B$$

Where, $v' = \sqrt{\rho}v \propto \frac{1}{a\sqrt{a}} v$

$$a(t) = \left(\frac{t}{t_{exp-coll}} + 1\right)^{a_p} \text{ with } a_P = 1 \text{ or } -1$$

when, $a_P = 1$

$\Rightarrow \dot{a} = 1 / t_{exp-coll}$

and $a_P = -1$

$\Rightarrow \dot{a} = -t_{exp-coll} / (t + t_{exp-coll})^2$

We have simulated using $t_{exp-coll}$ with 1, 10

Scaling for flow velocity in expanding/collapsing media

- Two time scales

  t_{eddy} , $t_{exp\text{-}coll}$

- Expanding/collapsing dominated

  - expanding media

$$\delta v \propto \left(\frac{t_{exp} + t}{t_{exp}} \right)^{\alpha}$$

  - collapsing media

$$\delta v \propto \left(\frac{t_{coll}}{t_{coll} + t} \right)^{\alpha}$$

- Turbulence dominated

  - expanding media

$$\delta v \propto \left(1 + \frac{t}{t_{eddy}} \right)^{\beta} = \left[1 + \frac{t_{exp}}{t_{eddy}} (a - 1) \right]^{\beta}$$

  - collapsing media

$$\delta v \propto \left(1 + \frac{t}{t_{eddy}} \right)^{\beta} = \left[1 + \frac{t_{coll}}{t_{eddy}} (a^{-1} - 1) \right]^{\beta}$$

Where,

$$a(t) = \left(\frac{t_{exp} + t}{t_{exp}} \right)$$ in expanding media

$$a(t) = \left(\frac{t_{coll}}{t_{coll} + t} \right)$$ in collapsing media

Scaling for flow velocity in expanding/collapsing media

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Where, $\alpha = -1$

$t_{exp} = 10$ in expanding media, $t_{coll} = 1$ in collapsing media

$\beta \sim -0.7$ in HD regime (Dilevsen et al. 2004)

$\beta \sim -0.5$ in HMD regime (Biskamp & Muller 1999)

Scaling for magnetic field in expanding/collapsing media

- Two time scales

  t_{eddy} , $t_{exp\text{-}coll}$

- Expanding/collapsing dominated

  - expanding media

$$\delta b \propto \left(\frac{t_{exp} + t}{t_{exp}} \right)^{\alpha}$$

  - collapsing media

$$\delta b \propto \left(\frac{t_{coll}}{t_{coll} + t} \right)^{\alpha}$$

- Turbulence dominated

  - expanding media

$$\delta b \propto \left(1 + \frac{t}{t_{eddy}} \right)^{\beta} = \left[1 + \frac{t_{exp}}{t_{eddy}} (a - 1) \right]^{\beta}$$

  - collapsing media

$$\delta b \propto \left(1 + \frac{t}{t_{eddy}} \right)^{\beta} = \left[1 + \frac{t_{coll}}{t_{eddy}} (a^{-1} - 1) \right]^{\beta}$$

Where, $\alpha = -2$

$t_{exp} = 10$ in expanding media, $t_{coll} = 1$ in collapsing media

$\beta = 0$ in HD regime

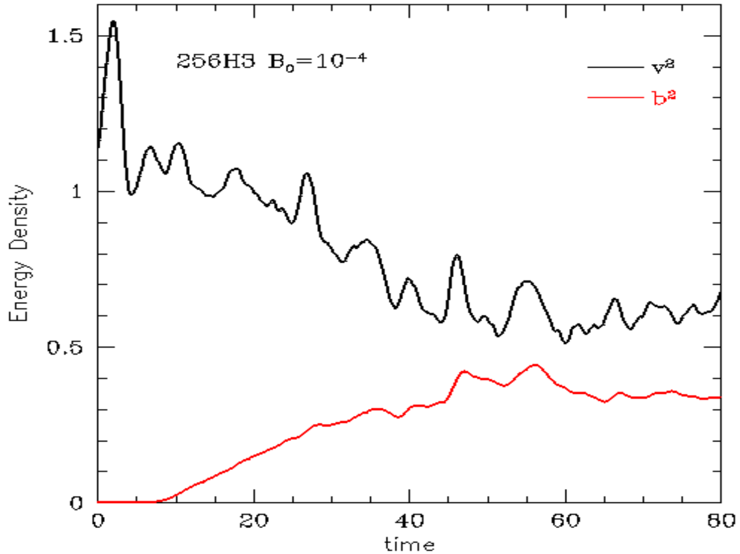
$\beta \sim -0.5$ in HMD regime (Biskamp & Muller 1999)

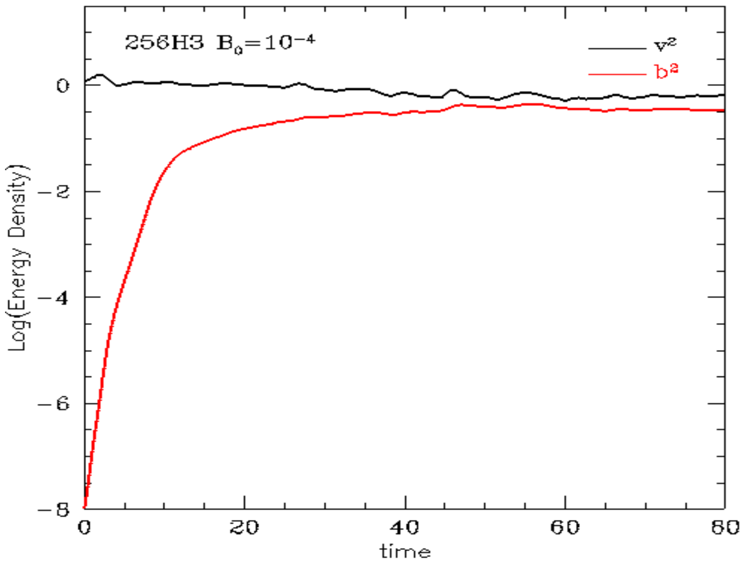
Simulation –initial condition

- Resolution : 256^3 grid (periodic box size = 2) π
- Incompressibility is assumed, and ∇ B= ∇ v = 0. .
- We use a pseudo-spectral method to solve the incompressible MHD equations
- Have considered only case of
- Have considered hyperviscosity, hyperdi $\nu = \eta$ n
- At t=0, Mean magnetic field strength is
- Considered density in the kinetic energy . $B_0 = 0.0001$

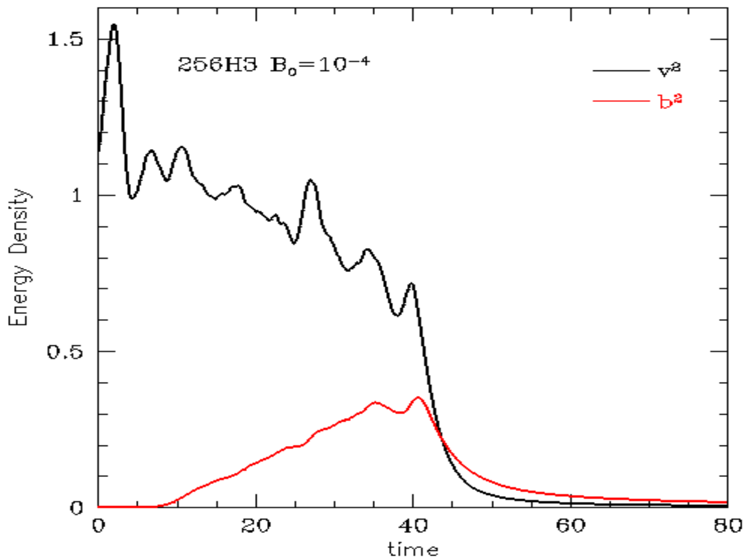
$$E_k = \frac{1}{2}\rho v^2 = \frac{1}{2}\frac{v^2}{a^3} \qquad \left(\rho \propto \frac{1}{a^3}\right)$$

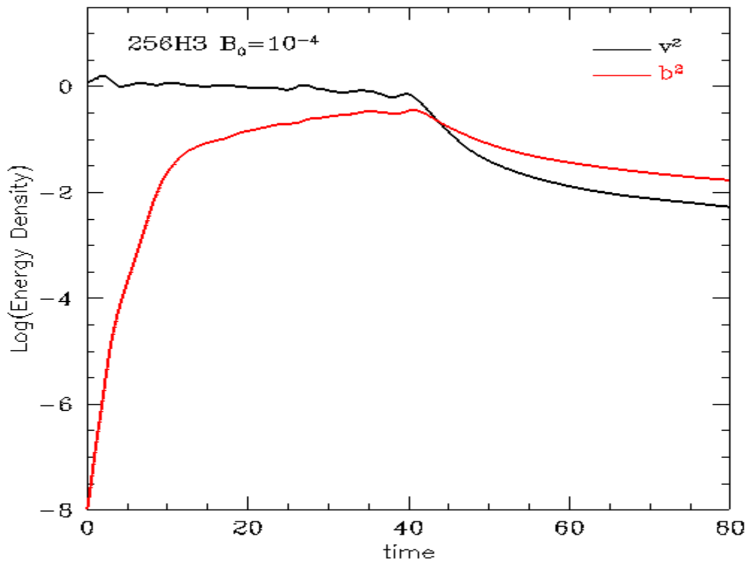
Time evolution of kinetic and magnetic energy density without expanding/collapsing effect





After the turbulence has reached a stationary state, we turn off the random driving forces.

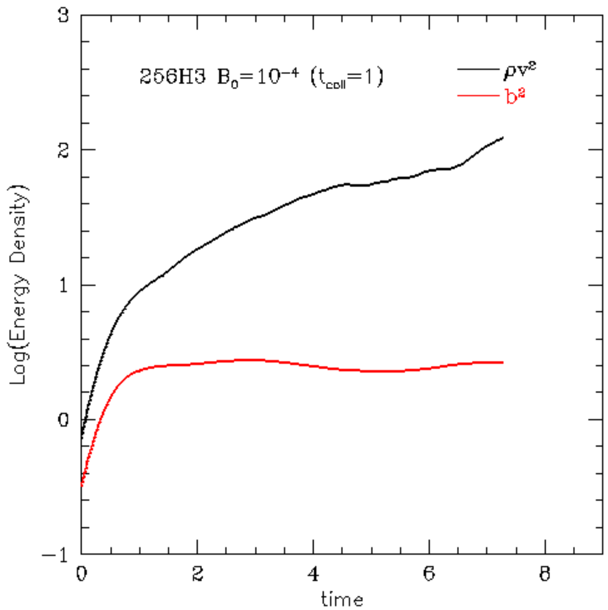


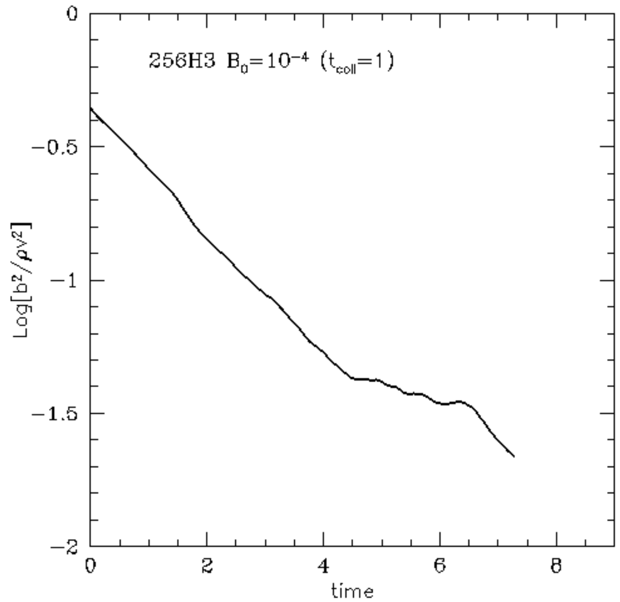


- From this point , we has injected the effect of the expansion/collapse.

Energy density and Energy ratio

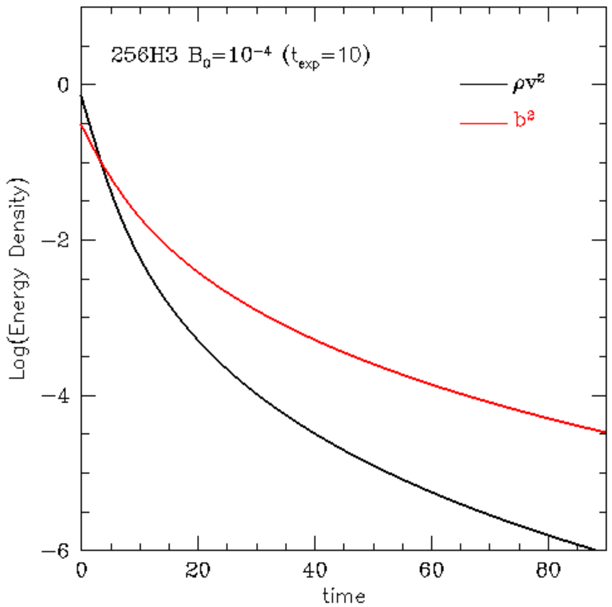
$$f = 0 \Rightarrow \text{viscous term} \propto \sqrt{a}$$

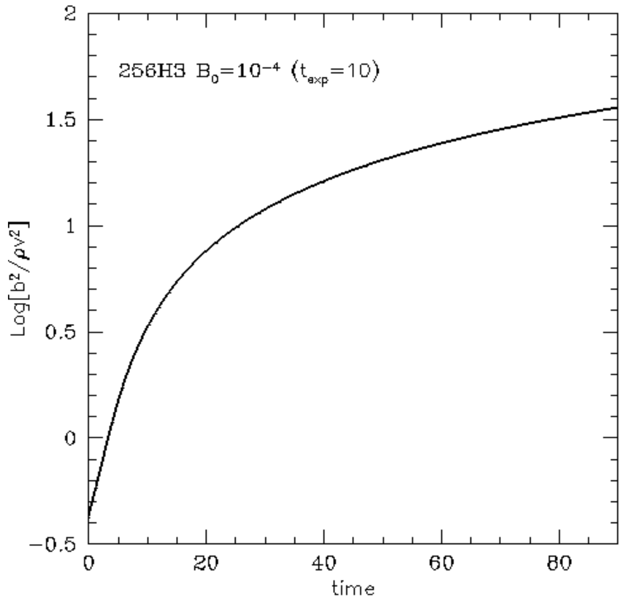




▣ collapsing media

- The initial time in the graph represents the beginning of the decaying turbulence.
- Kinetic energy densities are more dominant than magnetic energy densities.





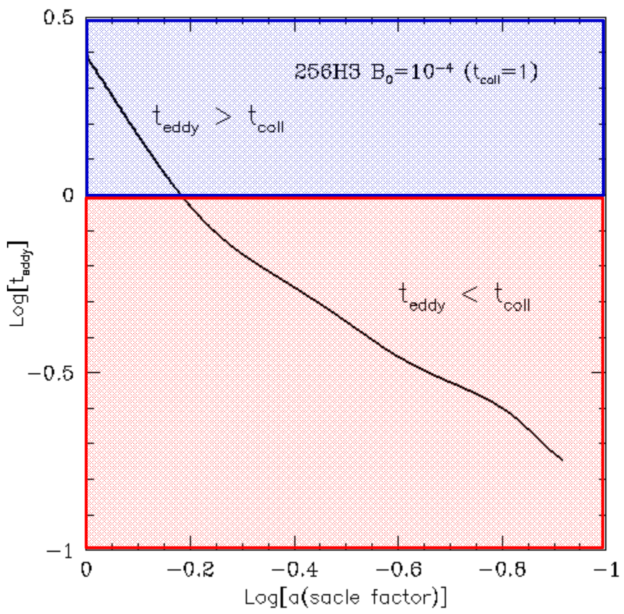
▣ expanding media

- Magnetic energy densities are more dominant than kinetic energy densities.
- Ratio of magnetic energy and kinetic density does not saturate;

  either it keeps decreasing or increasing with time.

Eddy turn over time in expanding/collapsing media

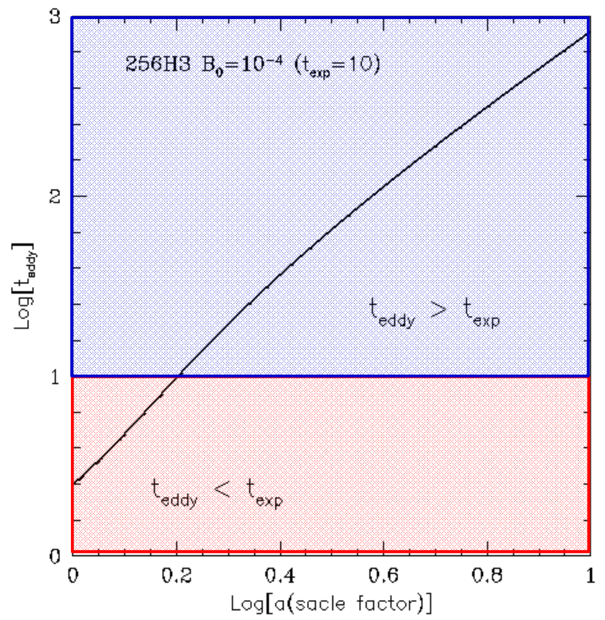
$$t_{eddy} = aL_0 / (a^3 v'^2 + b^2)^{1/2}$$



Red zone
=> turbulence dominated
Blue zone
=> expanding
/collapsing dominated

collapsing media

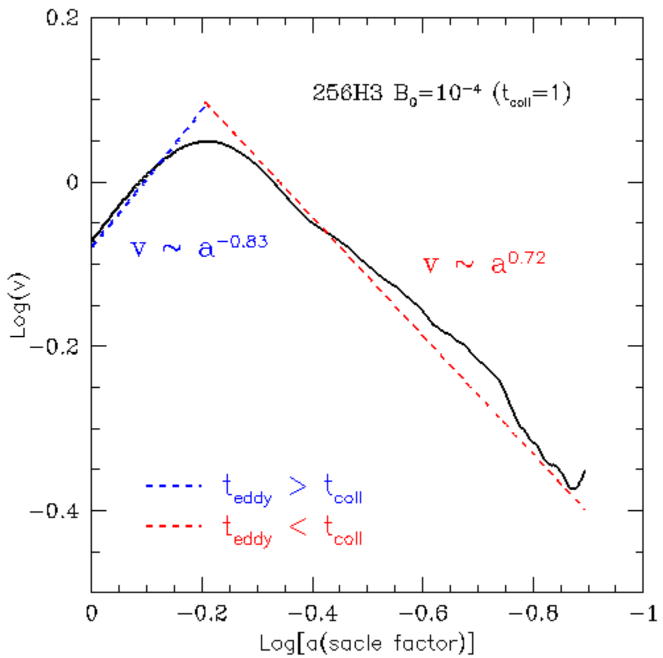
- t_{eddy} is decrease with the time evolution.
- $t_{eddy} > t_{exp\text{-}coll}$ change to $t_{eddy} < t_{exp\text{-}coll}$ at a ≈ -0.2



expanding media

- t_{eddy} is increase with the time evolution.
- $t_{eddy} < t_{exp\text{-}coll}$ change to $t_{eddy} > t_{exp\text{-}coll}$ at a ≈ 0.2

A scaling of velocity in collapsing media



■ Simulation results

- collapsing dominated ($t_{eddy} > t_{coll}$)

  $v \sim a^{-0.83}$

- turbulence dominated ($t_{eddy} < t_{coll}$)

  $v \sim a^{0.72}$

- collapsing dominated

$$\delta v \propto \left(\frac{t_{coll}}{t_{coll} + t} \right)^{\alpha}$$

- turbulence dominated

$$\delta v \propto \left[1 + \frac{t_{coll}}{t_{eddy}} (a^{-1} - 1) \right]^{\beta}$$

$t_{eddy} >> t_{coll}$ ⟹ $\delta v \propto a^{\alpha}$

$t_{eddy} << t_{coll}$ & $a \sim 1$ ⟹ $\delta v \propto (a^{-1} - 1)^{\beta}$ unless $a - 1 << \frac{t_{coll}}{t_{eddy}}$

$a << 1$ ⟹ $\delta v \propto a^{-\beta}$

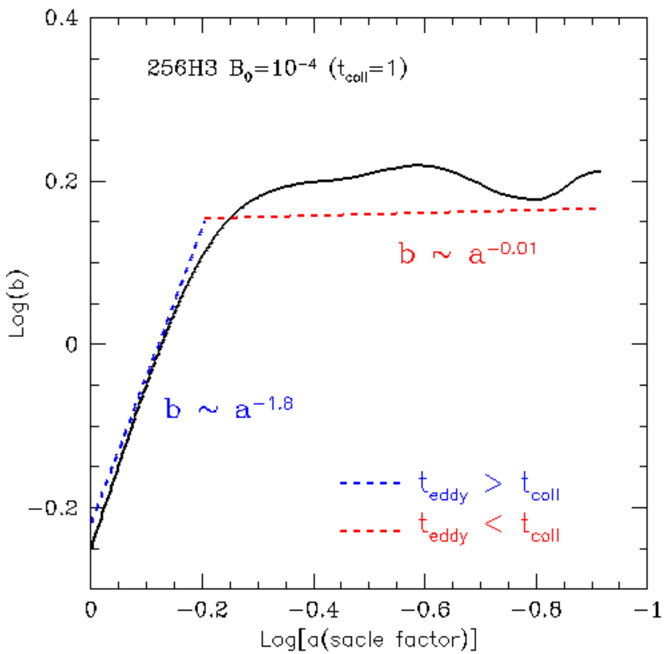
$t_{eddy} \sim t_{coll}$ & $a \sim 1$ ⟹ constant

$a << 1$ ⟹ $\delta v \propto a^{-\beta}$

$\alpha \sim -1$

$\beta \sim -0.7$ in HD regime

A scaling of magnetic field in collapsing media



■ Simulation results

- collapsing dominated
  ($t_{eddy} > t_{coll}$)

  $b \sim a^{-1.8}$

- turbulence dominated
  ($t_{eddy} < t_{coll}$)

  $b \sim a^{-0.01}$

- collapsing dominated

$$\delta b \propto \left(\frac{t_{coll}}{t_{coll} + t} \right)^{\alpha}$$

- turbulence dominated

$$\delta b \propto \left[1 + \frac{t_{coll}}{t_{eddy}} (a^{-1} - 1) \right]^{\beta}$$

$t_{eddy} >> t_{coll}$ ⟹ $\delta b \propto a^{-\alpha}$

$t_{eddy} << t_{coll}$ & $a \sim 1$ ⟹ $\delta b \propto (a^{-1} - 1)^{\beta}$ unless $a - 1 << \frac{t_{coll}}{t_{eddy}}$

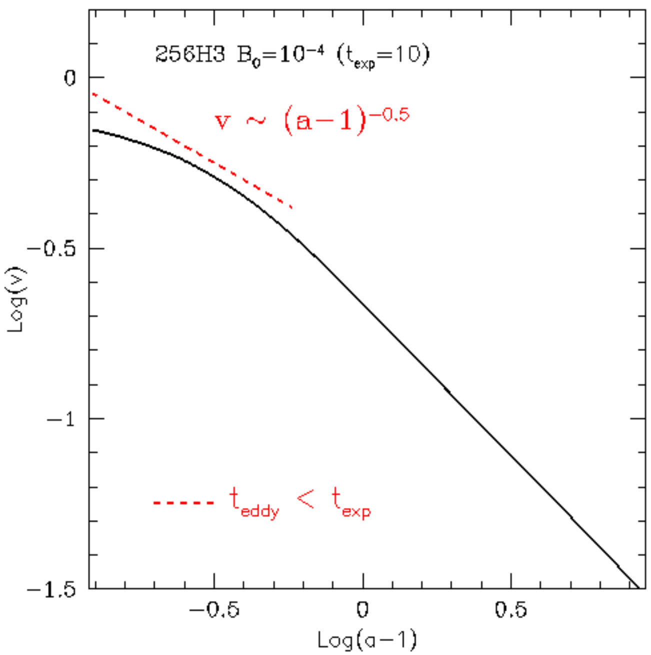
$a << 1$ ⟹ $\delta b \propto a^{-\beta}$

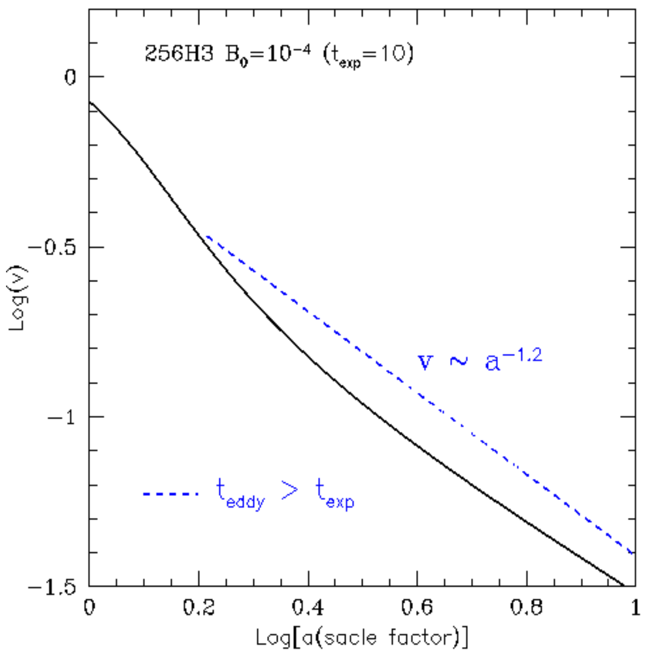
$t_{eddy} \sim t_{coll}$ & $a \sim 1$ ⟹ constant

$a << 1$ ⟹ $\delta b \propto a^{-\beta}$

$\alpha \sim -2$
$\beta = 0$ in HD regime

A scaling of velocity in expanding media





■ Simulation results

- expanding dominated ($t_{eddy} > t_{exp}$)

  $v \sim a^{-1.2}$

- turbulence dominated ($t_{eddy} < t_{exp}$)

  $v \sim (a-1)^{-0.5}$

- expanding dominated

$$\delta v \propto \left(\frac{t_{exp}}{t_{exp} + t} \right)^{\alpha}$$

- turbulence dominated

$$\delta v \propto \left[1 + \frac{t_{exp}}{t_{eddy}} (a-1) \right]^{\beta}$$

$t_{eddy} >> t_{exp}$ ⟹ $\delta v \propto a^{\alpha}$

$t_{eddy} << t_{exp}$ & $a \sim 1$ ⟹ $\delta v \propto (a-1)^{\beta}$ unless $a - 1 << \frac{t_{exp}}{t_{eddy}}$

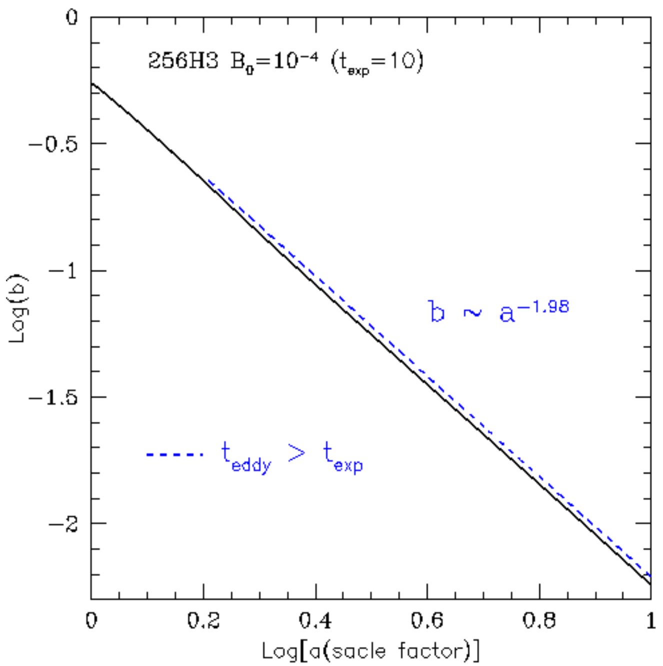
$a << 1$ ⟹ $\delta v \propto a^{\beta}$

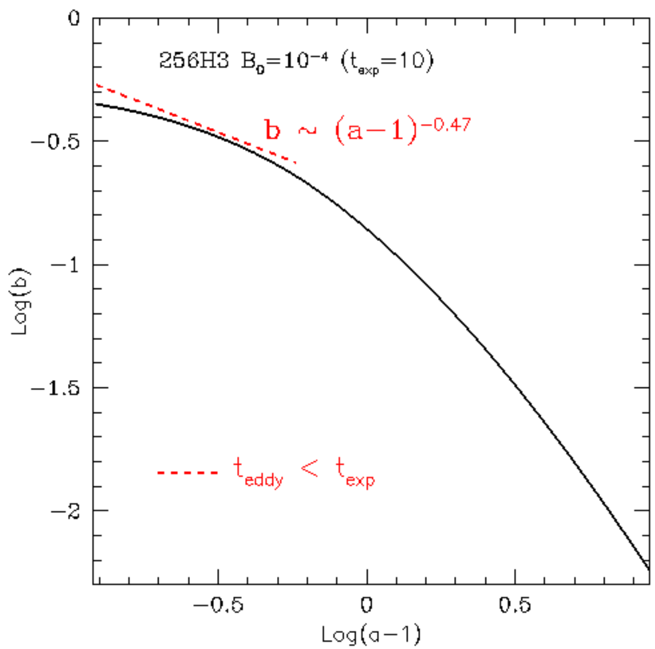
$t_{eddy} \sim t_{exp}$ & $a \sim 1$ ⟹ constant

$a << 1$ ⟹ $\delta v \propto a^{\beta}$

$\alpha \sim -1$
$\beta \sim -0.5$ in MHD regime

A scaling of magnetic field in expanding media





■ Simulation results

- expanding dominated ($t_{eddy} > t_{exp}$)

  $b \sim a^{-1.98}$

- turbulence dominated ($t_{eddy} < t_{exp}$)

  $b \sim (a-1)^{-0.47}$

- expanding dominated

$$\delta b \propto \left(\frac{t_{exp}}{t_{exp} + t}\right)^{\alpha}$$

- turbulence dominated

$$\delta b \propto \left[1 + \frac{t_{exp}}{t_{eddy}}(a-1)\right]^{\beta}$$

$t_{eddy} >> t_{exp}$		⇒	$\delta b \propto a^{\alpha}$
$t_{eddy} << t_{exp}$ &	$a \sim 1$	⇒	$\delta b \propto (a-1)^{\beta}$ unless $a-1 << \frac{t_{exp}}{t_{eddy}}$
	$a << 1$	⇒	$\delta b \propto a^{\beta}$
$t_{eddy} \sim t_{exp}$ &	$a \sim 1$	⇒	constant
	$a << 1$	⇒	$\delta b \propto a^{\beta}$

$\alpha \sim -2$

$\beta \sim -0.5$ in MHD regime

Conclusion

- We performed a preliminary study of incompressible MHD decaying turbulence by including the effect of expansion and collapse.
- Ratio of energy density does not saturate; either it keeps decreasing or increasing with time.
- Scaling for velocity and magnetic field in expanding/collapsing media

$t_{eddy} > t_{exp\text{-}coll}$ ⇨ turbulence expanding/collapsing dominated

$v \sim a^{-1.2}, \quad b \sim a^{-1.98}$ in expanding media

$v \sim a^{-0.82}, \quad b \sim a^{-1.8}$ in collapsing media

$t_{eddy} < t_{exp\text{-}coll}$ ⇨ turbulence dominated

$v \sim (a-1)^{-0.5}, \quad b \sim (a-1)^{-0.47}$ in expanding media

$v \sim a^{0.72}, \quad b \sim \text{constant}$ in collapsing media

- The specific results would depend on t_{eddy} and $t_{exp\text{-}coll}$.
  We will explore those in future.