MHD turbulence in expanding/collapsing media

Junseong Park ,Chungnam National University
Dongsu Ryu, Chungnam National University
Jungyeon Cho, Chungnam National University

KNAG meeting, October 22, 2010

Purpose of Study

1. We investigate driven magnetohydrodynamic (MHD) turbulence by including the effects of expansion and collapse of background medium.

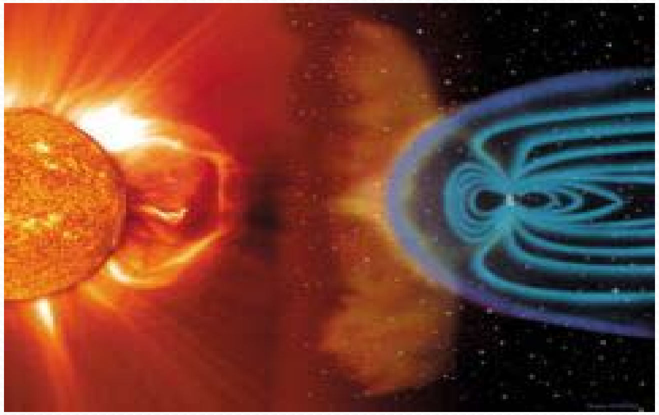
2. The main goal is to quantify the evolution and saturation of strength and characteristic lengths of magnetic fields in expanding and collapsing media.

Contents

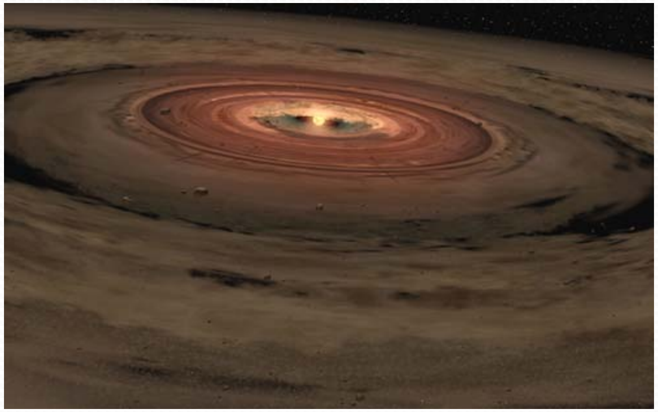
- Introduction
  - Energy cascade and Kolmogorov scaling
  - Fluid in expanding/collapsing coordinate
- The MHD equations in expanding/collapsing media
- Simulation –initial condition
- Simulation results
- conclusion and Further Work

What is turbulence?

Turbulence is a flow regime characterized by high momentum convection, low momentum diffusion, and pressure and velocity variation with time.



MHD turbulence in the solar wind



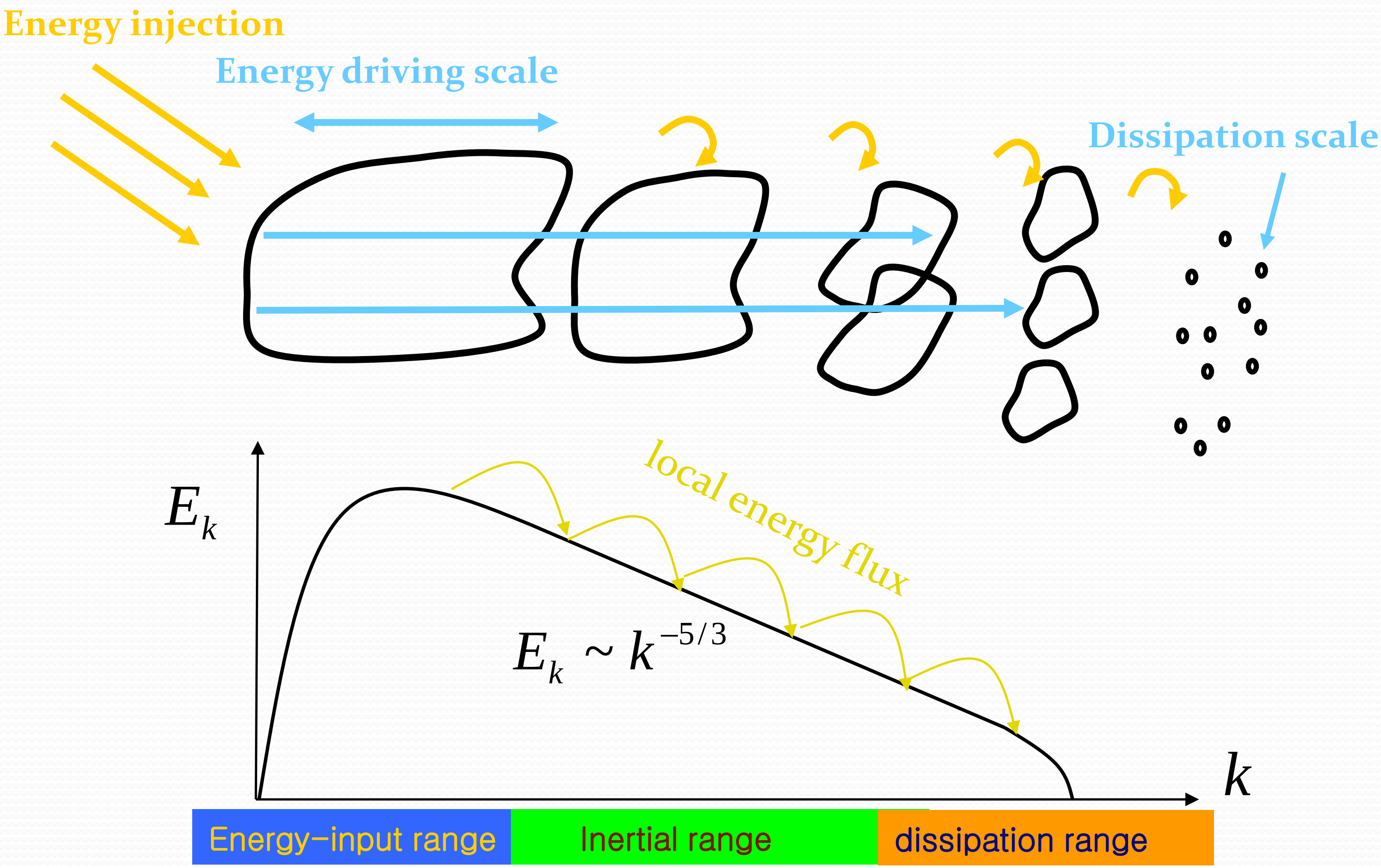
accretion flow

The Reynolds number characterizes whether flow conditions lead to turbulence or not.

$$\mathrm{Re} \sim \frac{\mathrm{v}L}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}} > \sim 100 - 1{,}000 \longrightarrow \text{turbulent}$$

Astrophysical fluids are turbulent and magnetized ($\mathrm{Re} > 10^{10}$)

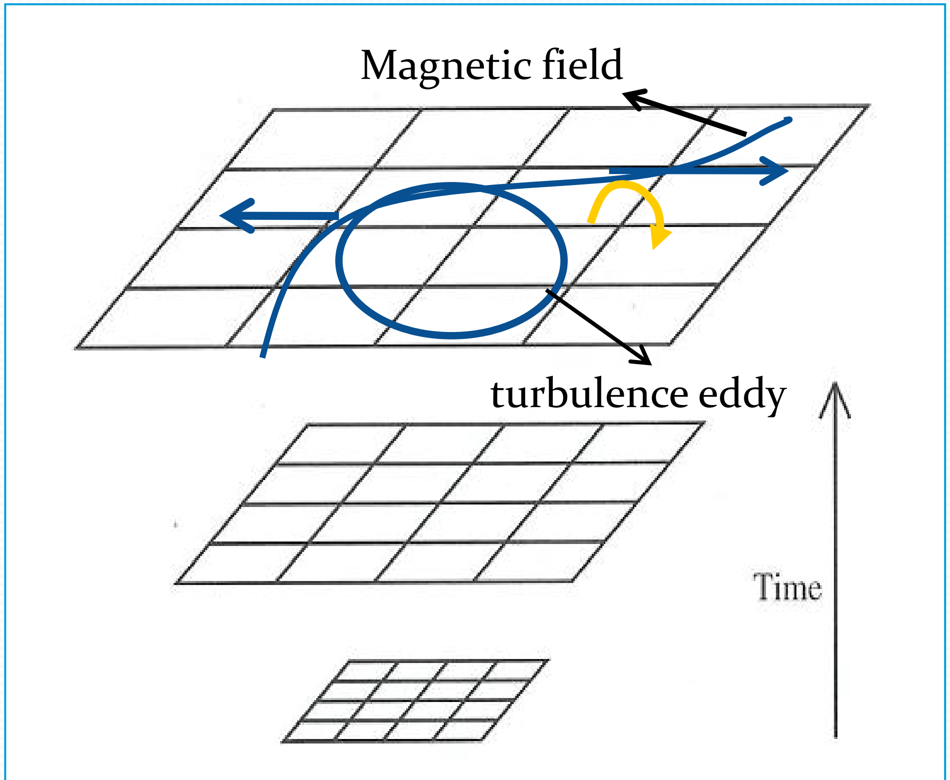
energy cascade and Kolmogorov spectrum



- The "Kolmogorov -5/3 spectrum" is generally observed in turbulence.
- Kolomogorov theory is simple and good for describing energy cascade in incompressible turbulence.

Fluid in expanding/collapsing coordinate

Expansion in the comoving coordinate system



- comoving coordinate system

$$\mathbf{r} = a(t)\mathbf{x}$$

r is physical distance
x is comoving distance
a(t) is scale factor

$$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$$

$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$

Proper velocity

peculiar velocity ($\mathbf{v} = a\dot{\mathbf{x}}$)

- When the matter expand, the magnetic field in the matter is expand with comoving coordinate system

The MHD equations in expanding/collapsing media

- p' is the pressure , $J = \nabla \times \mathbf{B}$ is the current , ν is the viscosity, η is the magnetic diffusion, **v** is the peculiar velocity ($\mathbf{v} = a\dot{\mathbf{x}}$) and **B** is magnetic field. where f is a random driving force. incompressibility is assumed. ($\nabla \cdot \mathrm{B} = \nabla \cdot \mathrm{v} = 0$)

$$\frac{\partial \mathrm{v}}{\partial t} = \frac{1}{a}\mathrm{v}\times(\nabla\times\mathrm{v}) - \frac{\dot{a}}{a}\mathrm{v} + \frac{1}{a}\mathrm{J}\times\mathrm{B} + \frac{1}{a^2}\nu\nabla^2\mathrm{v} + \nabla p' + f$$

$$\frac{\partial \mathrm{B}}{\partial t} = \frac{1}{a}\nabla\times(\mathrm{v}\times\mathrm{B}) - 2\frac{\dot{a}}{a}\mathrm{B} + \frac{1}{a^2}\eta\nabla^2\mathrm{B}$$

$$a(t) = \left[\frac{t}{\tau_H} + 1\right]^{a_P}$$

where, $a_P = 1$

$\Rightarrow \dot{a} = 1/\tau_H$

and $a_P = -1$

$\Rightarrow \dot{a} = -\tau_H/(t+\tau_H)^2$

we simulated with τ_H is 100, 50, 30,20,10.

Simulation –initial condition

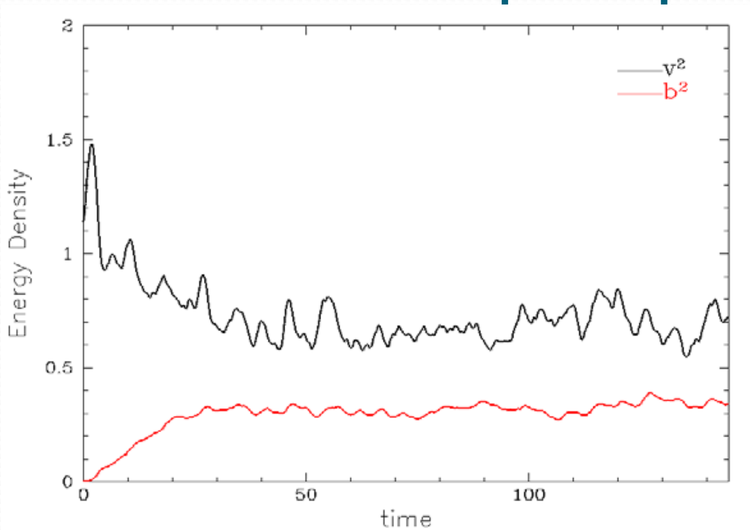
- Resolution : 128^3 grid (periodic box size = 2π)
- Incompressibility is assumed, and $\nabla \cdot B = \nabla \cdot v = 0$.
- We use a pseudo-spectral method to solve the incompressible MHD equations.
- Have considered only case of $\nu = \eta$
- Have considered hyperviscosity, hyperdiffusion
- At t=0, Mean magnetic field strength is $B_0 = 0.0001$
- Considered density in the kinetic energy .

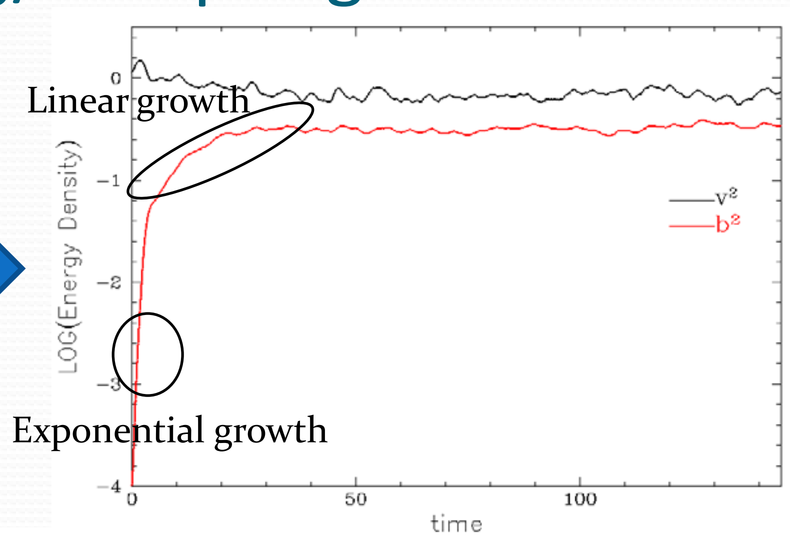
$$E_k = \frac{1}{2}\rho V^2 = \frac{1}{2}\frac{V^2}{a^3} \qquad \left(\rho \propto \frac{1}{a^3}\right)$$

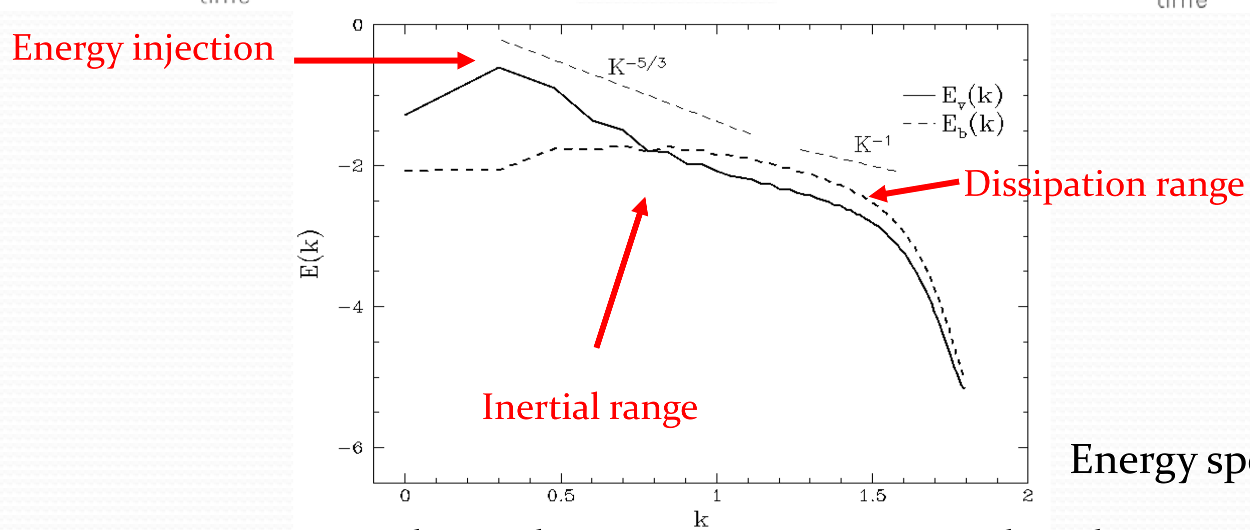
$$\left\langle V_{forcing} \right\rangle = \text{constant}$$

$$\left\langle E_{forcing} \right\rangle \propto \rho \propto \frac{1}{a^3}$$

Time evolution of kinetic and magnetic energy densities except expanding/collapsing effect.





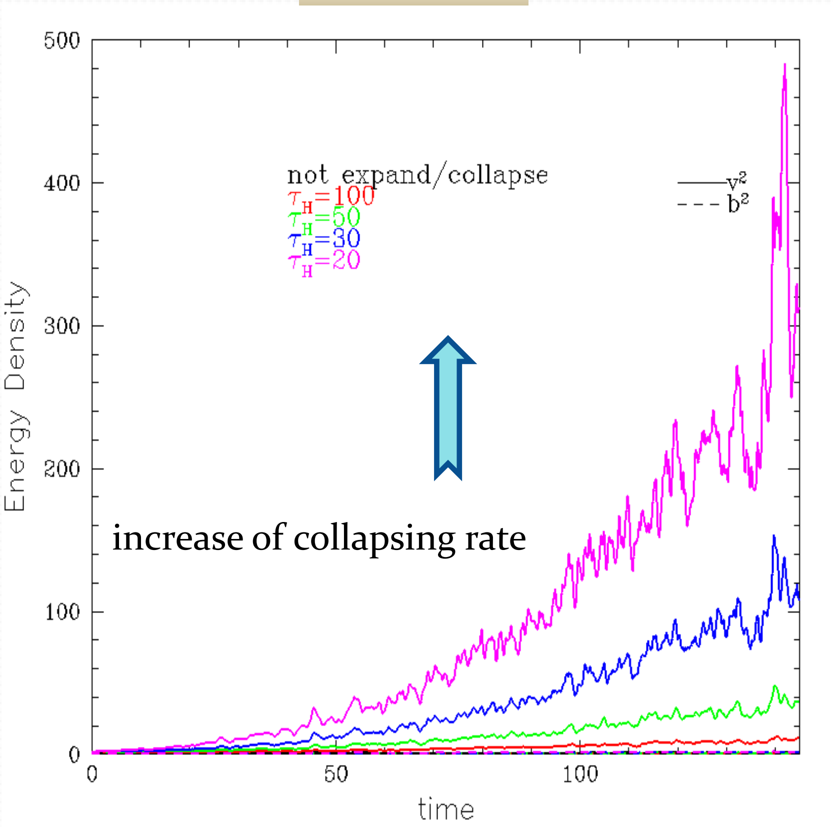


Energy spectra

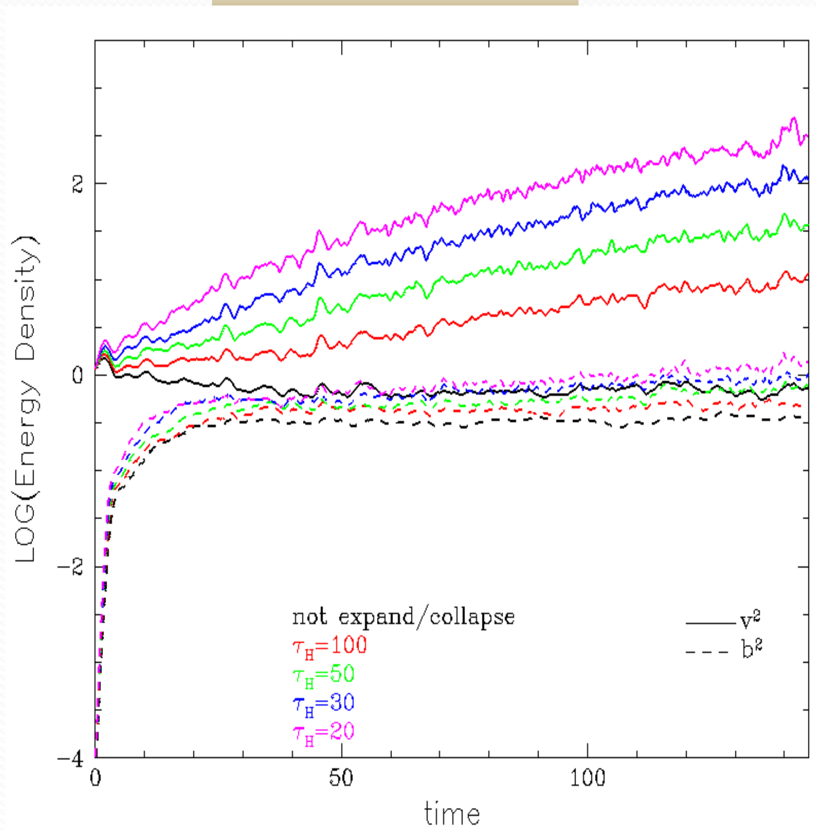
Kinetic energy spectrum peaks at the energy injection scale. The magnetic energy spectrum peaks at wave number larger than the energy injection scale.

Time evolution of kinetic and magnetic energy densities in collapsing media.

Real scale



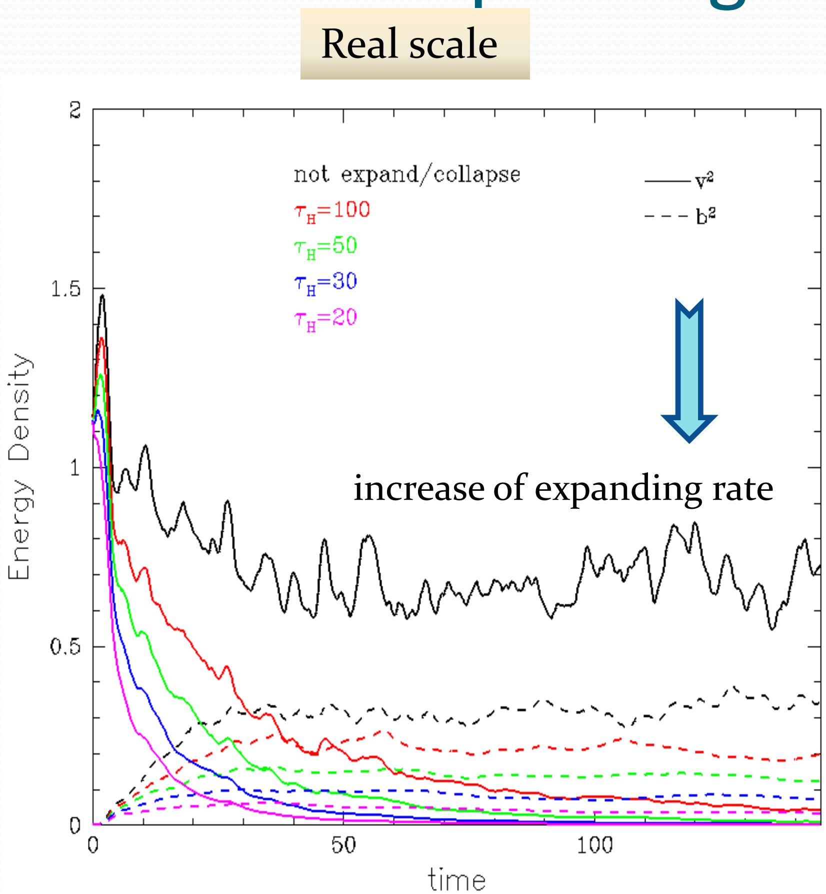
Logarithmic scale

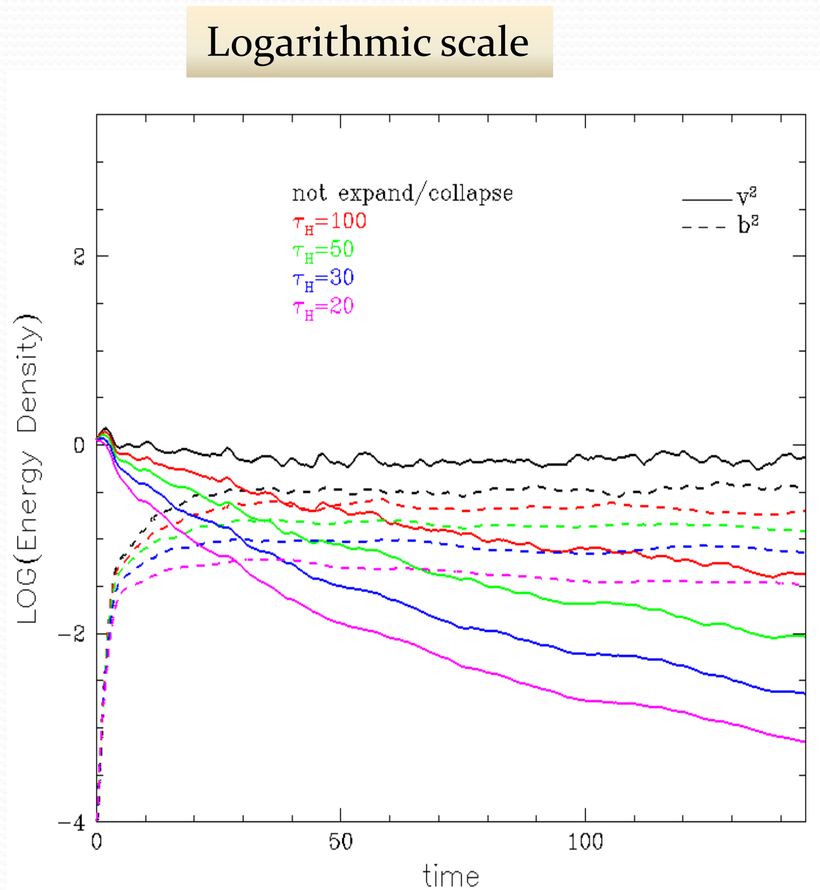


- When the collapsing rate gets increase, the magnetic energy and kinetic energy densities increase.
- The kinetic energy densities strongly depends on the value of collapsing rate.

However, magnetic energy shows that collapsing rate does not strongly affect.

Time evolution of kinetic and magnetic energy densities in expanding media.



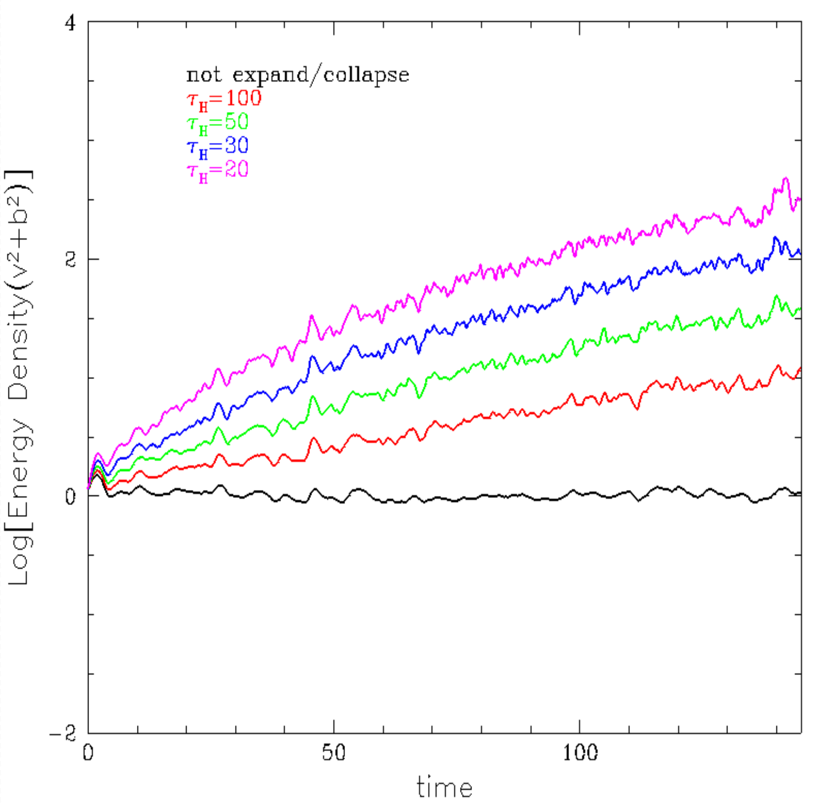


- When the expanding rate gets increase, the magnetic energy and kinetic energy densities decrease.
- The kinetic energy densities strongly depends on the value of collapsing rate.

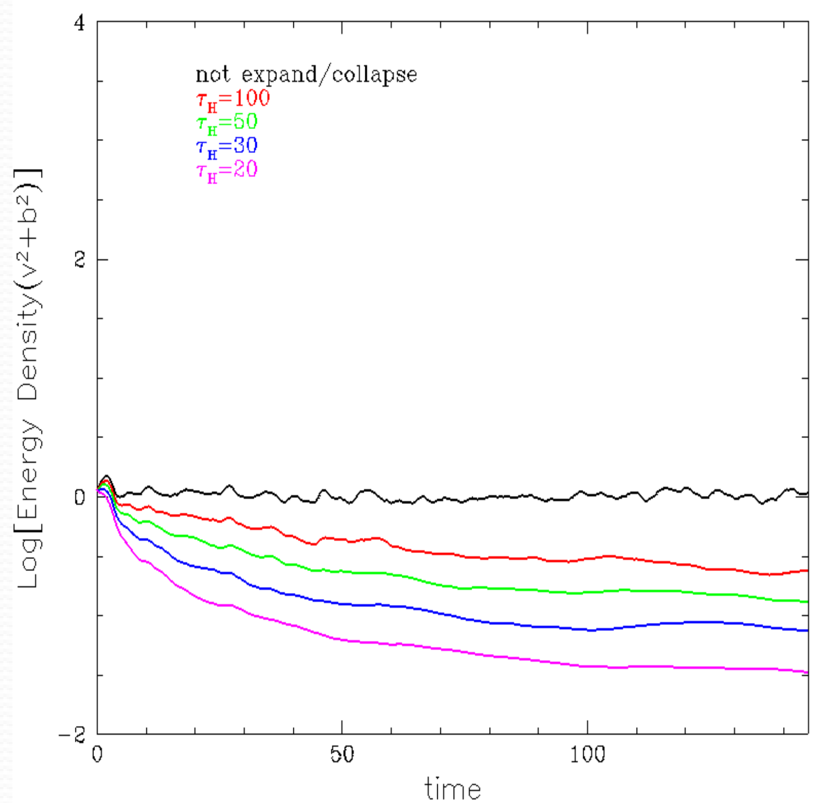
However, magnetic energy shows that collapsing rate does not strongly affect.

Time evolution of total energy densities in expanding/collapsing media.

collapsing media



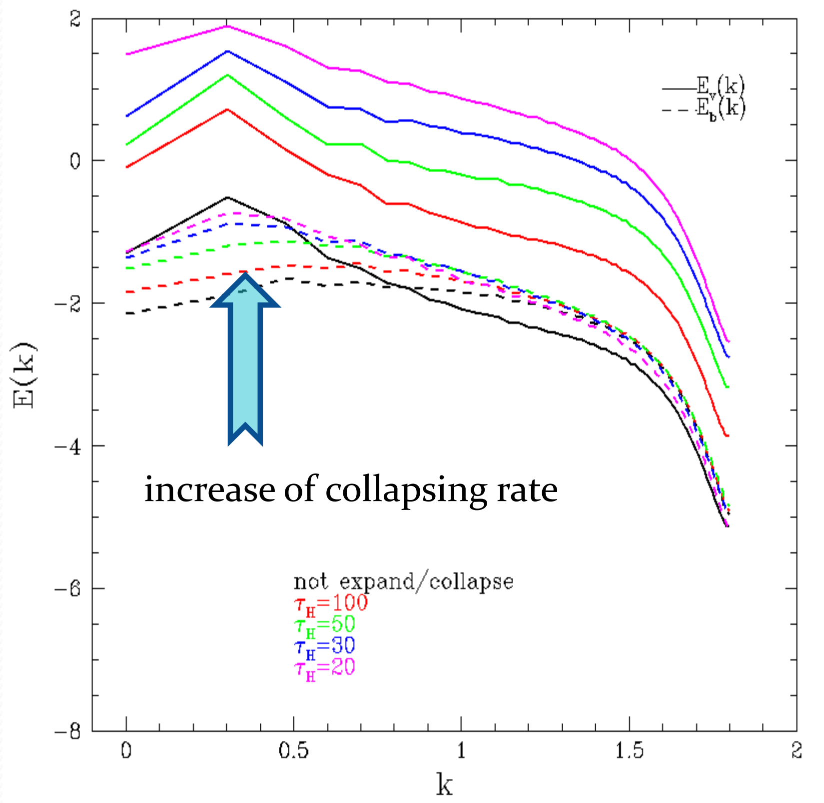
Expanding media



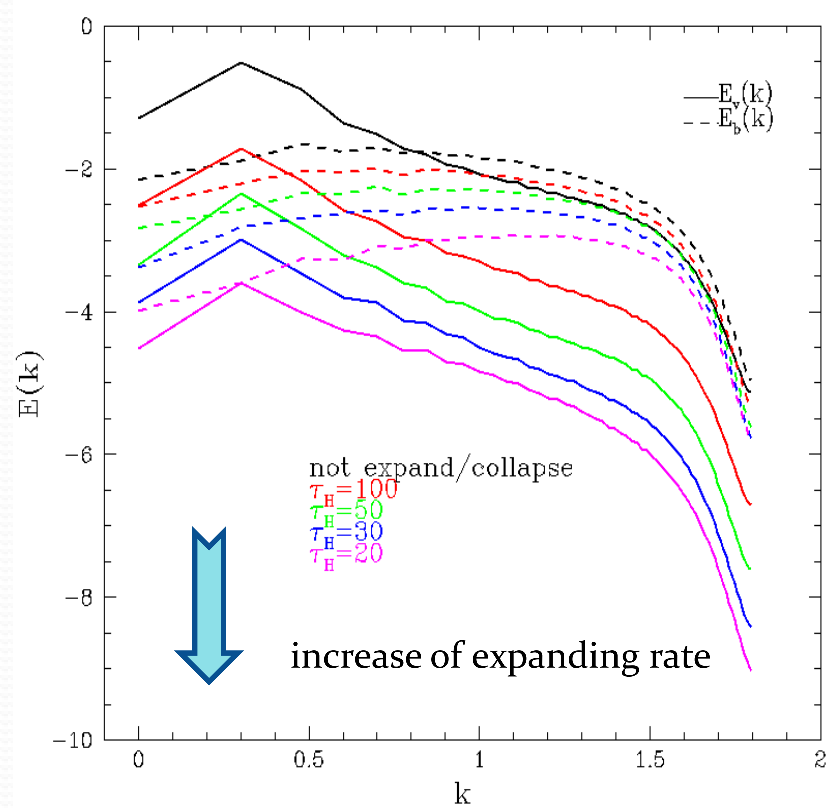
- When the collapsing rate gets increase, the total energy densities increase. Otherwise the total energy densities decrease.
- The level of total energy density strongly depends on the value of expanding /collapsing rate.

Energy spectrum in expanding/collapsing media

collapsing media



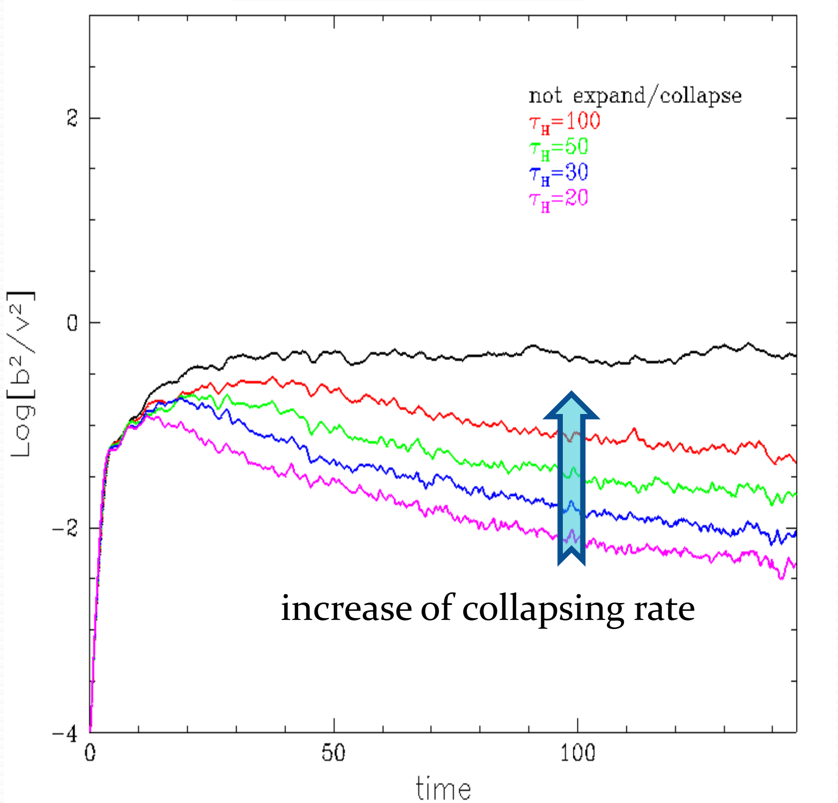
Expanding media



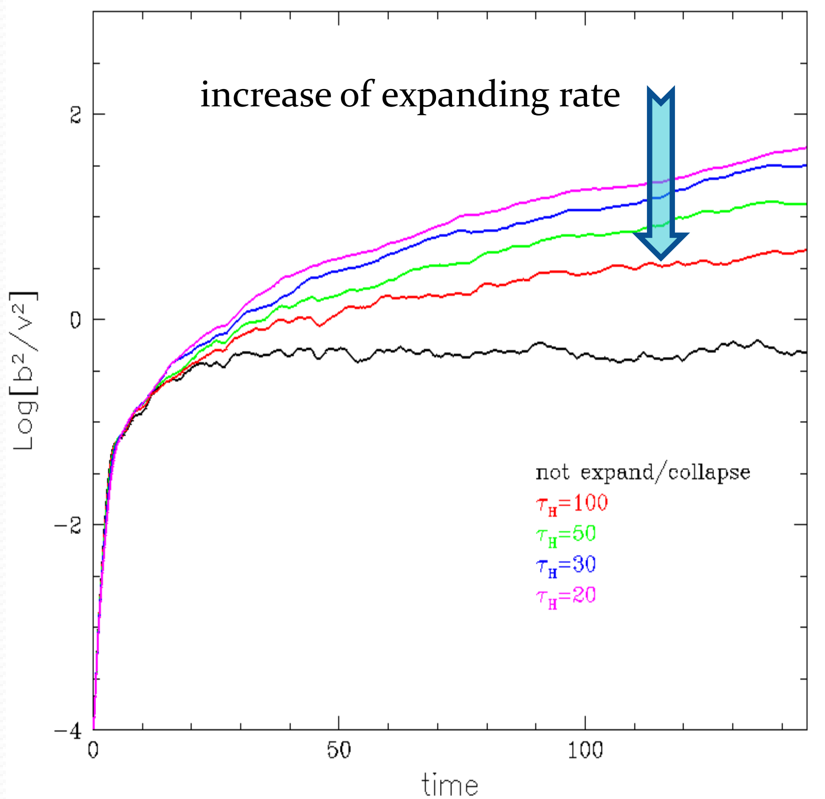
- The magnetic energy spectrum, as expanding /collapsing rate is increase. There is no a great change at small scale, but the energy spectrum great change at large scale.
- The magnetic energy spectrum are not very sensitive to the value of τ_H while the kinetic energy spectrum do show a strong dependence on τ_H

Ratio of magnetic energy and kinetic energy densities with time evolution.

collapsing media



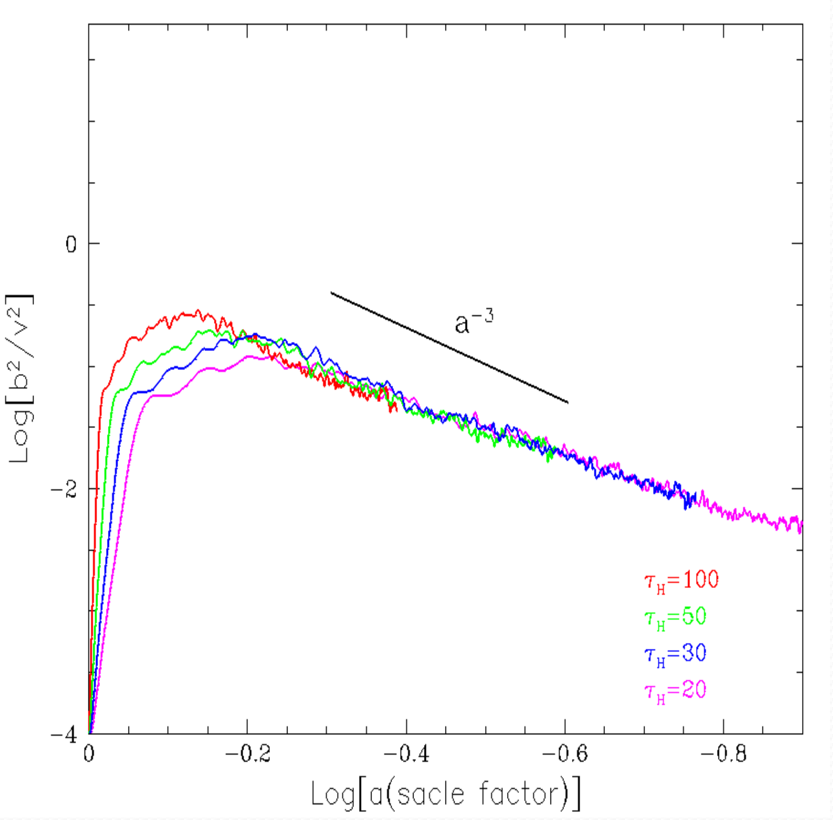
Expanding media



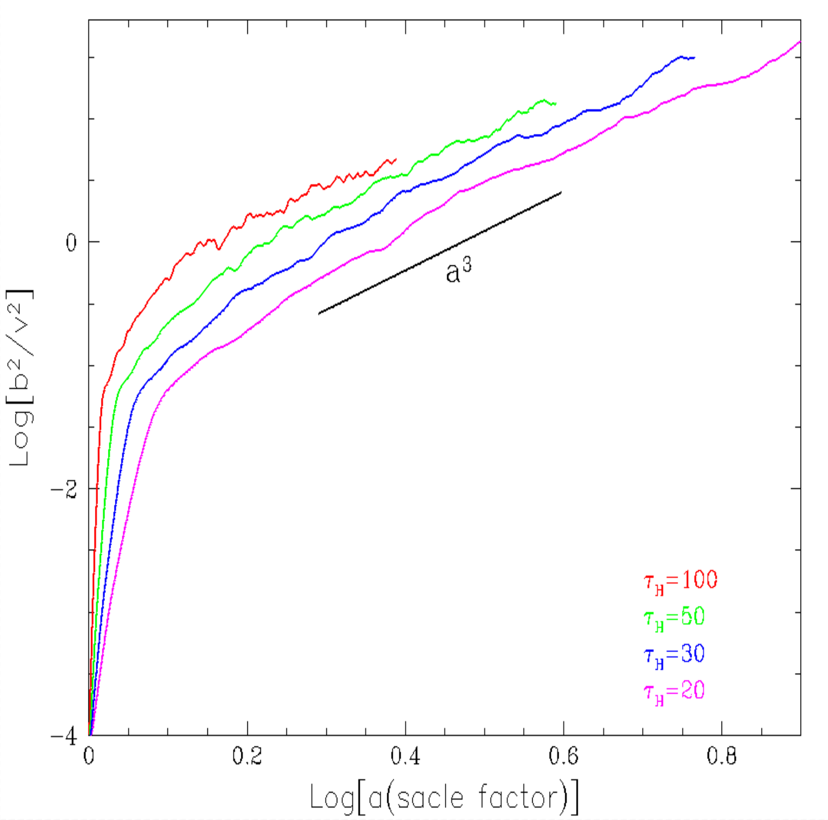
- When the collapsing rate gets increase, the ratio b^2 and v^2 decrease. Otherwise the ratio increase.
- Expanding / collapsing rate does strongly affect the ratio of magnetic energy and kinetic energy densities.

Ratio of magnetic energy and kinetic energy densities with a(t).

collapsing media



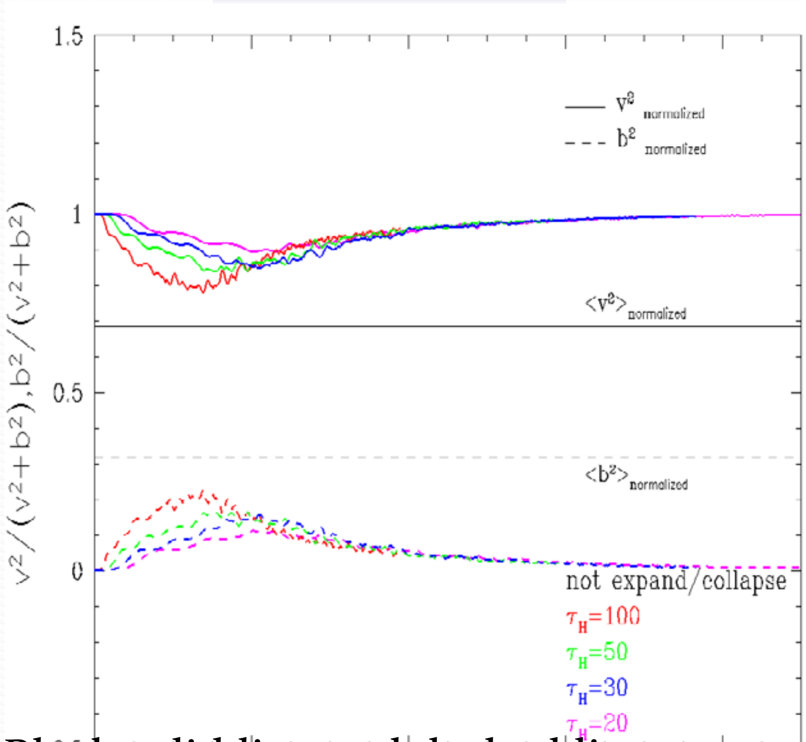
Expanding media



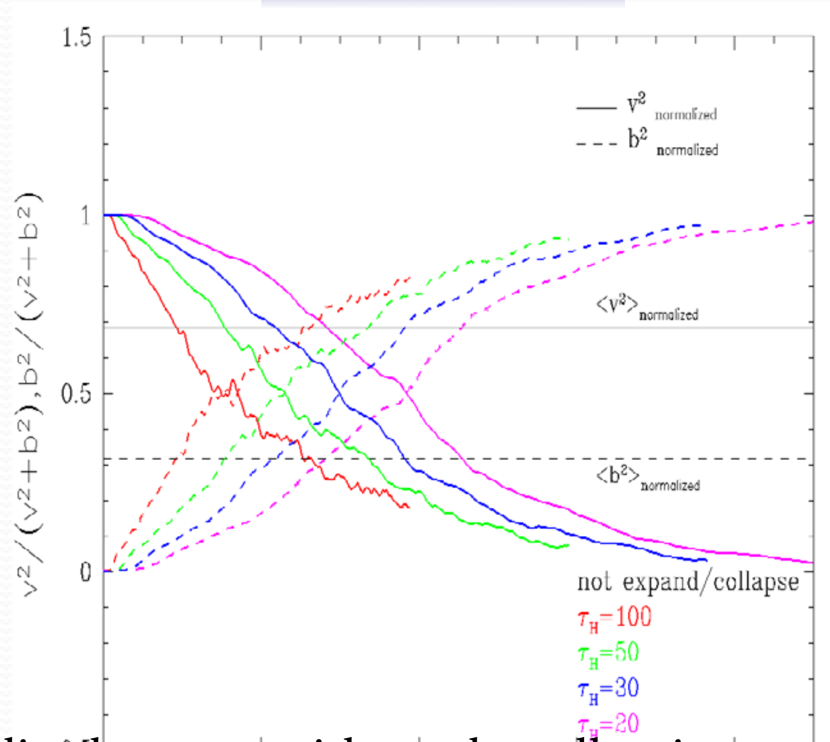
- When the collapsing rate gets increase, the ratio b^2 and v^2 decrease. Otherwise the ratio increase.
- The growth pattern between the b^2/v^2 and scale factor : $\frac{b^2}{v^2} \sim a^{\pm 3}(t)$

Scale factor evolution of Normalized magnetic energy and kinetic energy densities.

collapsing media

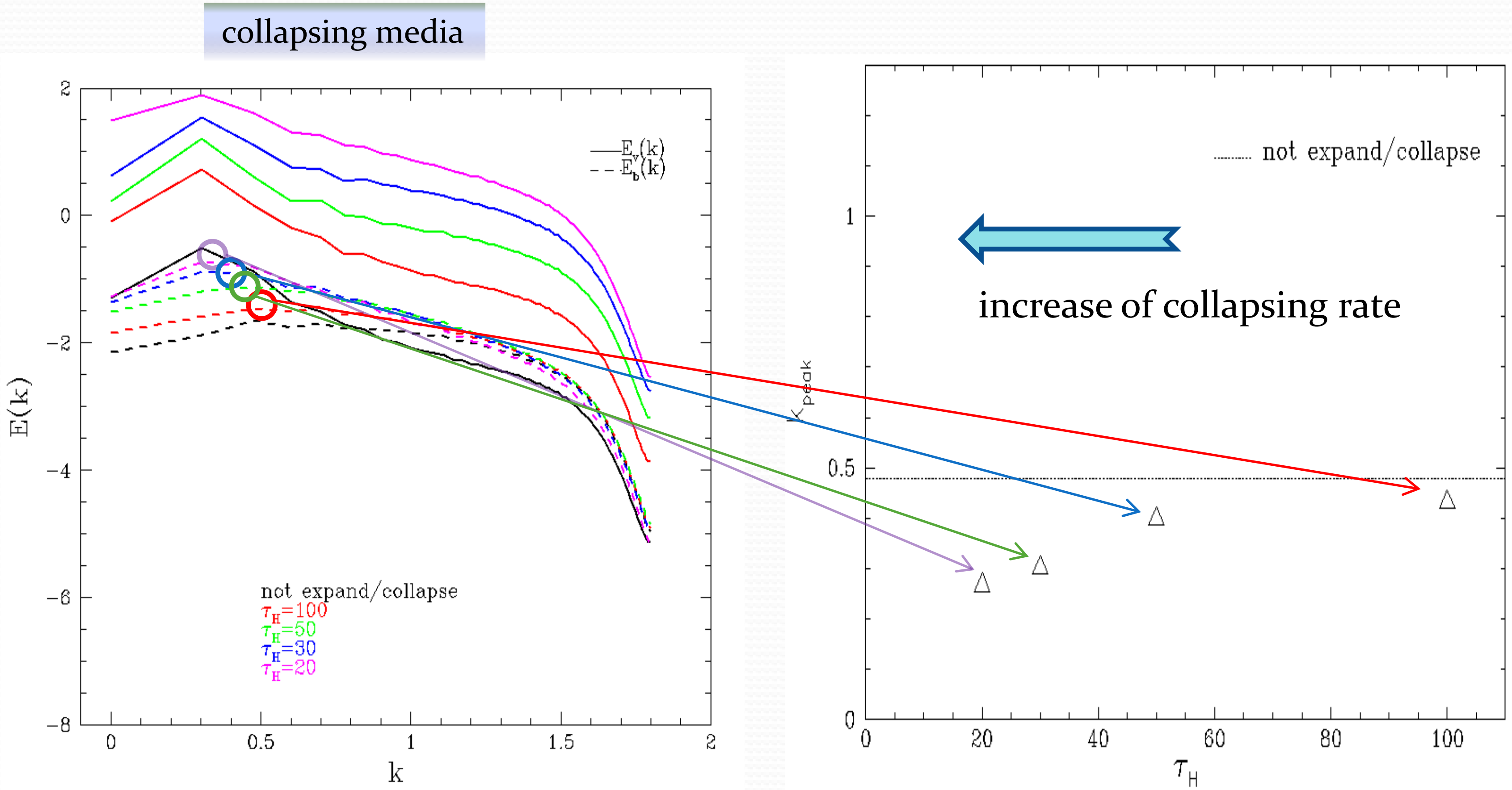


Expanding media



- Black solid line and dashed line are normalized average without the collapsing and Expanding effect. In this case average is taken after turbulence has reached a saturation state.
- In the collapsing media and Expanding media, Normalized Kinetic energy and magnetic energy density are different each other.
- However, We can know that Normalized kinetic energy density is closer plus 1, and magnetic energy dentisy is closer 0 with incresing scale factor.

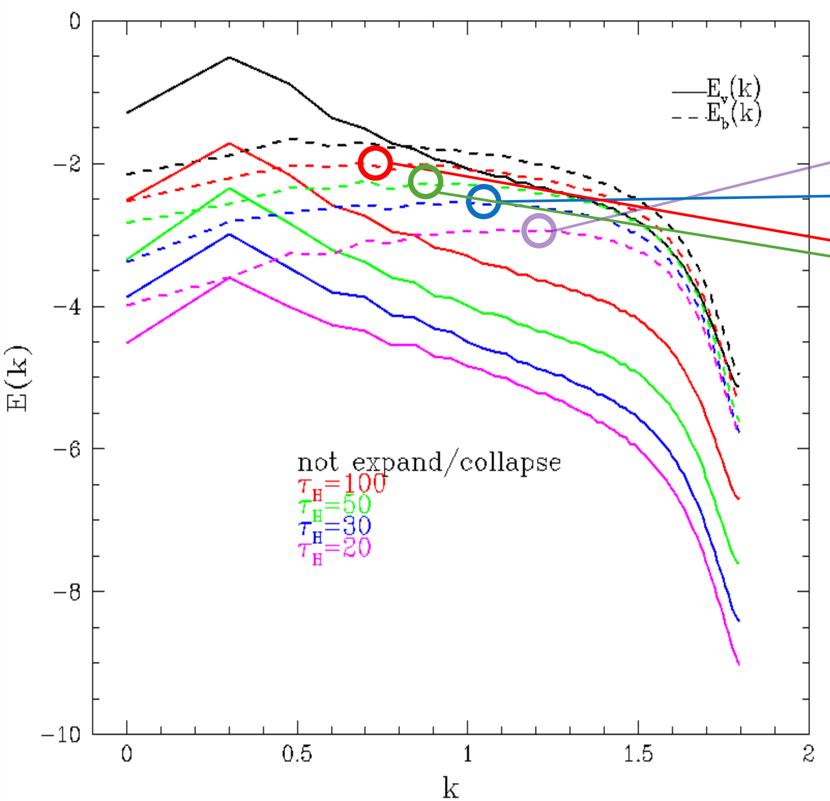
The location of peak magnetic energy in Energy spectra

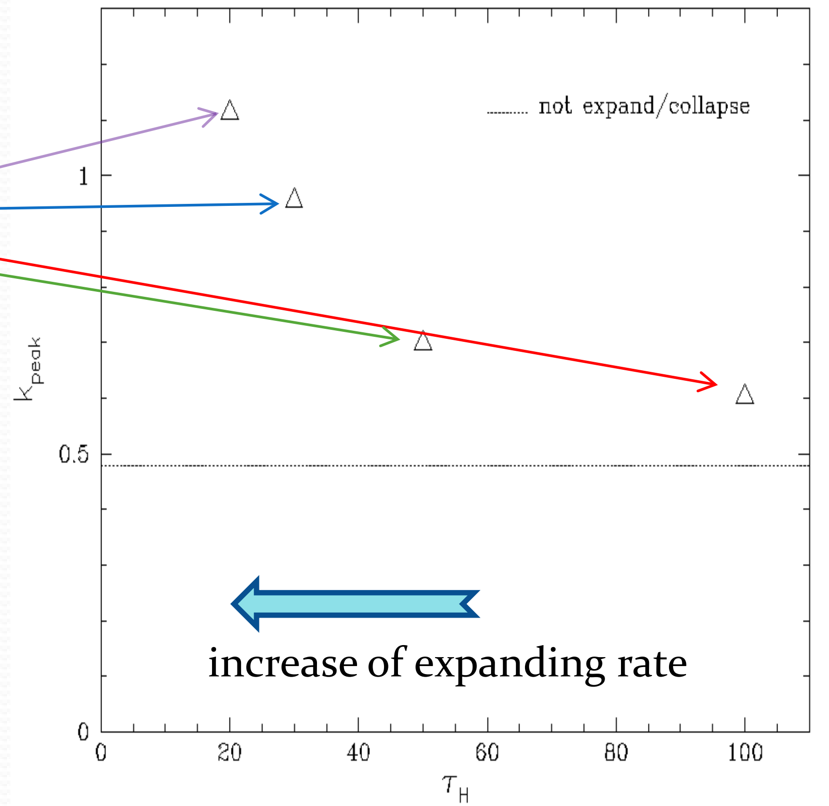


- When the collapsing rate gets increase, the location of the point of maximum magnetic energy moves toward smaller wave number.
- Right figure shows that wave number of peak of magnetic energy is smaller with increasing of collapsing rate.

The location of peak magnetic energy in Energy spectra

Expanding media

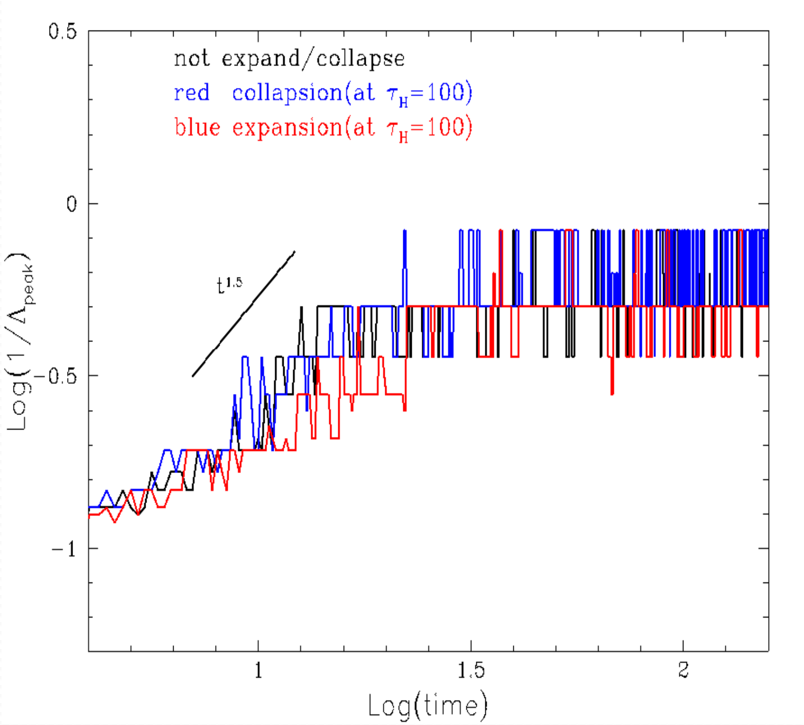


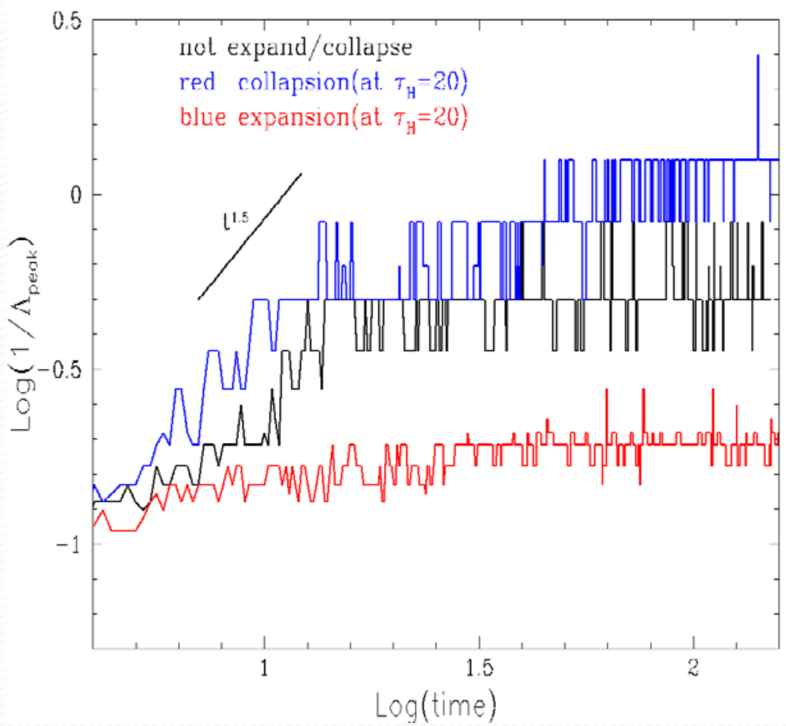


- When the expanding rate gets increase, the location of the point of maximum magnetic energy moves toward larger wave number.
- Right figure shows that wave number of peak of magnetic energy is lager with increasing of expanding rate.

Characteristic lengths of Magnetic Field in MHD turbulence

Peak scale of spectrum of magnetic field





- Where, $\wedge_{peak} = k_{peak}/L_o$ ($L_o = 2.5$; energy injection scale)
- The growth pattern in the linear growth stage is seems to be $\sim t^{1.5}$
- When the MHD turbulence get collapsing effect, the slope is more steep in the linear growth stage. Otherwise the slope is normally gentle.

conclusion and Further Work

- The energy spectrum, as expanding /collapsing rate is increase there is no a great change at small scale, but the energy spectrum great change at large scale.
- magnetic energy density does not saturate; either it keeps decreasing or increasing with time.
- The kinetic energy densities strongly depends on the value of collapsing and expanding rate. However, magnetic energy shows that collapsing and expanding rate does not strongly affect.
- We studied characteristic lengths of peak scale of spectrum of magnetic field. When the collapsing rate gets increase , the slope is more steep in the linear growth stage. Otherwise the slope is normally gentle.
- Resolution : 256^3 with different τ_{H}.