Dynamical Friction in a Gaseous Medium

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• Gravitational wake
  – An astronomical object in orbital motion interacts with its surrounding medium, producing overdensity at the downstream direction

• Dynamical friction (DF).
  – Gravitational interaction between a perturber and its wake pulls it backward, providing a negative torque
  – The perturber loses orbital angular momentum, resulting in orbital decay.
DF in a Collisionless Background

- Chandrasekhar (1943) formula for a DF force on a object with mass \( m \) and velocity \( v_m \) in a homogeneous medium with density \( \rho_0 \) and velocity dispersion \( \sigma \):

\[
F = -\frac{4\pi \rho_0 G^2 m^2 \ln \Lambda}{v_m^2} \left[ \text{erf}(\mathcal{M}) - \frac{2\mathcal{M}}{\sqrt{\pi}} e^{-\mathcal{M}^2} \right]
\]

where

\[
\mathcal{M} = \frac{v_m}{(\sqrt{2}\sigma)} \quad \Lambda = \frac{r_{\text{max}}}{r_{\text{min}}}
\]

- Orbital decay of satellite galaxies orbiting their host galaxies (Tremaine 1976; Lin & Tremaine 1983; Weinberg 1989; Hashimoto et al. 2003; Fujii et al. 2006)

- Dynamical fates of globular clusters near the Galactic center (e.g., Kim&Morris 2003; McMillan & Portegies Zwart 2003; Kim et al. 2004)

- Galaxy formation within the framework of a hierarchical clustering scenario (e.g., Zentner & Bullock 2003; Bullock& Johnston 2005)

- Formation of Kuiper Belt binaries (Goldreich et al. 2002), and planet migrations (Del Popolo et al. 2003) via interactions with planetesimals
DF in a Gaseous Medium

• …May be important in
  – Massive BH mergers in galactic nuclei (Escala et al. 2004, 2005; Dotti et al. 2006, 2007)
Steady-State DF

- Steady-state, linear theory (Dokuchaev 1964; Ruderman & Spiegel 1971; Rephaeli & Salpeter 1980).
- For supersonic motions,

\[
F_{ss} = \frac{4\pi (GM_p)^2 \rho_0}{V^2} \ln \left( \frac{r_{\text{max}}}{r_{\text{min}}} \right)
\]

- For subsonic motions, \( F_{ss} = 0 \) because of the front-back symmetry of the density wake about the perturber.
- What is happening at \( M=1 \)?
Ostriker (1999) considered a situation where a perturber is introduced at $t=0$, and obtained analytic expressions for the density wake and the drag force.

Subsonic wake:
- Density distribution is symmetric w.r.t. the perturber.
- Constant density counters are ellipses in the $R$-$z$ plane.
- Causal region is a sphere with radius $(R^2+z^2)^{1/2}$ outside of which sonic perturbations are not reached yet.
- All the region inside the sonic sphere receives a sonic perturbation just once.
Supersonic Wake

• A shape of a loaded ice cone
  – Sonic sphere + Mach waves

• All the region inside the sonic sphere receives a perturbation once.

• The region inside the mach cone but outside the sonic sphere receives sonic perturbations twice.
  – Larger density than in the sonic sphere

• Asymmetric density wake makes the drag force large.
Drag Force

- DF force on a perturber moving on a straight-line trajectory through a uniform medium.

\[ F_{DF} = -\frac{4\pi \rho_0 (GM_p)^2}{V_p^2} I \]

where

\[ I = \begin{cases} 
\frac{1}{2} \ln \left( \frac{1+\mathcal{M}}{1-\mathcal{M}} \right) - \mathcal{M}, & \text{if } \mathcal{M} < 1 \\
\frac{1}{2} \ln \left( 1 - \mathcal{M}^{-2} \right) + \ln \left( \frac{V_p t}{r_{min}} \right), & \text{if } \mathcal{M} > 1 
\end{cases} \]

\[ \ln(c_s t/r_{min}) = 4, 6, \ldots, 16 \]
Circular-Orbit Perturber

- Perturbers in real astronomical systems are likely to follow curvilinear orbits.
  - motions of galaxies in galaxy clusters, binary black holes near the central parts of galaxies, compact stars in accretion disks
- Even for objects experiencing orbital decay, a near-circular orbit is a good approximation since the associated friction time is longer than the orbital time.

- Consideration of a circular-orbit perturber may be important, since it will allow the perturber, if supersonic, to overtake the backside of its wake, which was created about an orbital period earlier.

Escala et al. (2004)
Density Wake of a Circular-Orbit Perturber

- A linear perturbation theory + semi-analytic computation
  (Kim & Kim 2007)

\[ M=2 \]
Number of sonic perturbations received by the wake

\[ M < 4.60 : \text{up to three sonic perturbations} \]
\[ M < 7.79 : \text{up to five perturbations} \]
Density Wake vs. M

(a) $M = 0.5$

(b) $M = 2.0$

(c) $M = 4.0$

(d) $M = 5.0$
Drag Force on Circular-orbit Perturbers

- $F_\phi$ is well fitted to Osrik'er's formula with $V_p t = 2R_p$.

- $F_R$ plays an insignificant role in the orbital decay – it changes the orbital eccentricity more than the semimajor axis.
DF of a Binary

- A binary consisting of two point-mass objects at the opposite sides of the system center in a uniform gaseous medium (Kim et al. 2008).

- DF experienced by an object is a combination of its own wake as well as the wakes of its companion.
Drag Force

- $I_{\text{comp}}$ has an opposite sign to $I_{\text{self}}$, reducing the net DF force

- $|I_{\text{comp}} / I_{\text{self}}| \sim 0.4$ on average
  - 0.12 at $M=1.2$
  - $0 \rightarrow 0$ as $M \rightarrow 0$
The results of Escala et al. (2004) that showed that it takes about 1.5 Myr for a binary to decay from 7 to 0.7 pc is reproduced when the effect of the companion wake is considered.
Nonlinear DF

• DF discussed so far is based on the assumption that density wakes remain in the linear regime.

• The strength of gravitational perturbations due to a body with mass $M_p$ can be measured by the dimensionless parameter

$$\mathcal{A} = \frac{G M_p}{a_\infty^2 r_s}$$

  – corresponds to the perturbed density at a distance $r_s$ from the perturber relative to the background density
  – equal to the Bondi radius $r_B = \frac{G M_p}{a_\infty^2}$ relative to $r_s$.

• $\mathcal{A} \sim 0.1–1$ for galaxies embedded in typical intracluster media
  $\sim 10–100$ for protoplanets in protostellar disks
  $\sim 10^6–10^8$ for supermassive black holes near galaxy centers
Numerical Method

• FLASH 3.0 code
• Two-dimensional simulations (R,z) assuming axial symmetry
• Resolution $> 5$ grids / $r_s$

• Perturber is modeled by a Plummer sphere with softening radius $r_s$
  – Does not possess a defined boundary though which matter is either accreted or reflected (cf. Bondi-Hoyle-Littleton accretion problem)

• Consider cases with straight-line trajectories only.
• Adiabatic with index $\gamma = 5/3$
• Units of length, velocity, and time: $r_s$, $a_\infty$, and $t_{\text{cross}} = r_s / a_\infty$
  – Models are completely characterized by $M$ and $A$

• 58 model simulations
  – Different $M = 0.5 - 4.0$ and $A = 0.01 - 100$
  – Run until $t < 600$ $t_{\text{cross}}$
  – Largest grid models have $3072 \times 12,288$ zones in $(R, z)$
Linear Cases

- Analytic theories adopt a point-mass perturber corresponding to $r_s = 0$. Need to introduce the cutoff radius $r_{\text{min}}$ in the force formula.

- In numerical simulations, one needs to assign a non-zero value to $r_s$, making it unnecessary to use $r_{\text{min}}$.
- Want to find a proper relationship between $r_s$ and $r_{\text{min}}$ that makes the numerical and analytical results consistent with each other when $A \ll 1$.

\[ r_{\text{min}} = 0.35 \mathcal{M}^{0.6} r_s \]
Supersonic Model with $M=1.5$ & $A=20$

- Mach waves turn into a detached bow shock.
- The gravitational potential is so deep that the material arriving at the rear side of the perturber can be pulled back toward the perturber, creating a vortex (ring) around the perturber.
- The vortex with low density rises buoyantly toward high-$R$ region and is eventually swept away by the background flow, leaving a nearly-hydrostatic gas near the perturber.
• When $A \sim 1$, the density wake is shifted toward the perturber compared with the linear case and still located preferentially at the rear side.
  — the resulting drag force will be larger than the linear counterpart.
• When $A >> 1$, the shock is detached and the density wake around the perturber is nearly spherically symmetric.
  — Nonlinear DF force is smaller than the linear counterpart.
- Distribution of density near the perturber is spherically symmetric and in near-hydrostatic equilibrium

HSE under the Plummer potential:

\[
\rho = \rho_0 \left\{ 1 + \frac{(\gamma - 1)\mathcal{A}a_\infty^2}{a_0^2} \left[ \frac{r_s}{(r^2 + r_s^2)^{1/2}} - 1 \right] \right\}^{1/(\gamma - 1)}
\]
Detached Shock Distance

- Good correlations between $\delta$ and $\eta$, where $\eta$ is the nonlinearity parameter defined by

$$\eta \equiv \frac{\mathcal{A}}{\mathcal{M}^2 - 1}$$

$$\frac{\delta}{r_s} = \eta$$

$$\frac{\delta}{r_s} = 2(\eta/2)^{2.8}$$
• For $A \gg 1$, KE of the incident flow is almost entirely converted to TE of the postshock flow that supports a hydrostatic envelope against the gravitational potential of the perturber.

• For strong shocks ($M \gg 1$), the postshock $\text{TE} \propto a^2_{\infty}M^2$, and the gravitational potential energy $\propto -GM_p/\delta$ at the shock location,
  \[ \delta/r_s \propto A/M^2. \]

• In the limit of $M \to 1$, the flow does not in principle produce a shock, corresponding to $\delta \to \infty$.

\[
\begin{align*}
TE &= \frac{3}{2} \rho_2 a_2^2 \propto \rho_2 M^2 a_{\infty}^2 \\
GE &\propto \rho_2 \frac{GM_p}{\delta}
\end{align*}
\]

\[
KE = \frac{1}{2} \rho_{\infty} M^2 a_{\infty}^2
\]
Gravitational Drag Force

- The drag forces for nonlinear cases with high $A$ also have a logarithmic time dependence similarly to linear cases.

- Some early fluctuations in response to the oscillations of detached bow shocks before a quasi-steady state is attained.

- The (normalized) drag force decreases with increasing $A$, indicating that the nonlinear effect makes the DF force smaller than the linear estimate.
- \( F/F_{\text{lin}} \approx 1 \) for \( \eta < 0.7 \): a bow shock that barely forms is attached to a perturber.

- For \( \eta \sim 0.7-2 \), DF force slightly larger (by less than 20%) than the linear counterpart: most of the material in the wake is still located behind of, but closer to, the perturber.

- For \( 100 > \eta > 2 \), the presence of a large hydrostatic envelope makes the drag force reduced considerably.

\[
F = F_{\text{lin}} \times \left( \frac{\eta}{2} \right)^{-0.45}
\]

- This result appears to be independent of the BCs at the surface of a perturber.
Discussion

- Escala et al. (2005) carried out numerical simulations of a DF-induced merger of supermassive binary black holes, finding that $\tau_{\text{decay}} \propto M_p^{-0.3}$ in their simulations.

- The linear theory gives $\tau_{\text{decay}} = (M_p V_p)/F_{\text{DF}} \propto M_p^{-1}$.

- Our nonlinear result predict $\tau_{\text{decay}} \propto M_p^{-0.55}$ if the perturber is sufficiently massive.
  - the delayed orbital decay of supermassive black holes in Escala et al. (2005) is partly due to the nonlinear effect.

- Many other factors that may change $\tau_{\text{decay}}$
  - Background: rotating, radially stratified, self-gravitating disk
  - Orbits of BHs: curvilinear, possibly eccentric, orbits
Summary

- **Circular orbit** causes the induced wake to bend around the perturber in a trailing spiral fashion
  - The wake outside $2R_p$ has a negligible contribution to the DF force.

- In a decay of an equal-mass **binary**, the wake of one object can provide a countervailing force to the drag of the other, amounting to 40% on average.

- **Nonlinear effect** of a massive perturber reduce the gravitational drag force $F/F_{\text{lin}} = \eta^{-0.45}$ due to the more-or-less symmetric distribution of the density wake near the perturber.